

Review 5 Exponents and Logarithms**Multiple Choice**

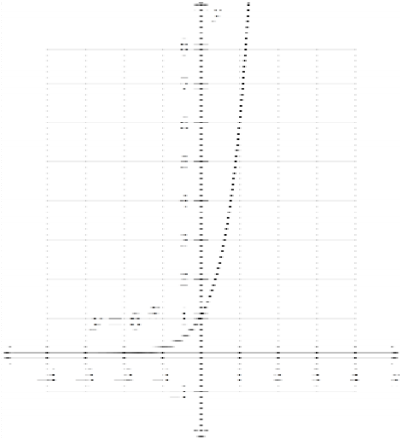
Identify the choice that best completes the statement or answers the question.

- _____ 1. Determine the missing value in this table of values for the function $y = 2^x$.

x	$y = 2^x$
-1	0.5
0	
1	2

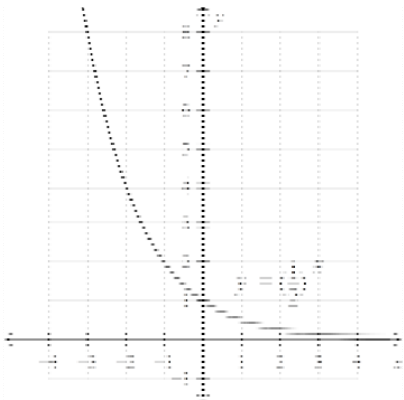
- A. 1
B. -1
C. 0
D. 2

- _____ 2. Determine the range of $y = 6^x$.



- A. $x > 0$
B. $y \in \mathbb{R}$
C. $y > 0$
D. $y < 0$

- _____ 3. Determine the y-intercept of the graph of $y = \left(\frac{1}{2}\right)^x$.



- A. $\frac{1}{2}$
 B. 0
 C. 1
 D. 2
- _____ 4. Which exponential function is decreasing?

- A. $y = \left(\frac{1}{3}\right)^x$
 B. $y = 1.383^x$
 C. $y = 7.7^x$
 D. $y = \left(\frac{5}{2}\right)^x$

- _____ 5. This table of values represents an exponential function. Determine the missing value.

x	y
0	1
1	0.01
2	

- A. 0.000001
 B. 0.0002
 C. -0.98
 D. 0.0001
- _____ 6. What is the range of the function $y = 3(2^{x-7})$?
- A. $y > 7$
 B. $y \in \mathbb{R}$
 C. $y > 0$
 D. $y > 3$
- _____ 7. What is the y-intercept of the graph of $y = 4^{3x} - 2$?
- A. 62
 B. 1
 C. -1
 D. -2

_____ 8. Determine the equation of the horizontal asymptote of the graph of $y - k = c(a)^{d(x-h)}$, $a > 0$, $c \neq 0$, $d \neq 0$.

A. $y = 0$

C. $y = -k$

B. $y = 1$

D. $y = k$

_____ 9. Write $\frac{1}{343}$ as a power of 7.

A. $7^{\frac{1}{3}}$

C. 7^{-49}

B. 7^{-3}

D. 7^3

_____ 10. Write $\sqrt{343}$ as a power of 7.

A. 7^6

C. $7^{\frac{2}{3}}$

B. $7^{\frac{7}{2}}$

D. $7^{\frac{3}{2}}$

_____ 11. Which number below cannot be written as a power of 2?

A. $\sqrt{32}$

C. 1

B. $\frac{1}{256}$

D. 6

_____ 12. Write $49^4\sqrt{7}$ as a power of 7.

A. $7^{\frac{7}{4}}$

C. $7^{\frac{1}{2}}$

B. $7^{\frac{9}{4}}$

D. 7^8

_____ 13. Solve: $343 = 49^{x+5}$

A. $x = \frac{7}{2}$

C. $x = -\frac{7}{2}$

B. $x = \frac{338}{7}$

D. $x = 338$

_____ 14. Solve: $2^x = 32^{x-16}$

A. $x = -11$

C. $x = 4$

B. $x = 20$

D. $x = -16$

____ 15. Solve: $343^{x-5} = 49^{x-3}$

A. $x = -\frac{1}{9}$

C. $x = 9$

B. $x = -9$

D. $x = \frac{1}{9}$

____ 16. Solve: $6^x = 36^4 \sqrt{6}$

A. $x = \frac{7}{4}$

C. $x = \frac{1}{2}$

B. $x = 8$

D. $x = \frac{9}{4}$

____ 17. The expression $\log x$ represents the common logarithm of x .
What is the value of the base of $\log x$?

A. 1

C. e

B. 0

D. 10

____ 18. Evaluate $\log_2 256$.

A. 8

C. -8

B. 254

D. 128

____ 19. Write this logarithmic expression as an exponential expression: $\log_5 \left(\frac{1}{125} \right) = -3$

A. $5 = \left(\frac{1}{125} \right)^{-3}$

C. $\frac{1}{125} = (-3)^5$

B. $5^{\frac{1}{125}} = -3$

D. $\frac{1}{125} = 5^{-3}$

____ 20. Write this logarithmic expression as an exponential expression: $\log_9 \left(\sqrt[3]{81} \right) = \frac{2}{3}$

A. $\sqrt[3]{81} = 9^{\frac{2}{3}}$

C. $9^{\sqrt[3]{81}} = \frac{2}{3}$

B. $\sqrt[3]{81} = \left(\frac{2}{3} \right)^9$

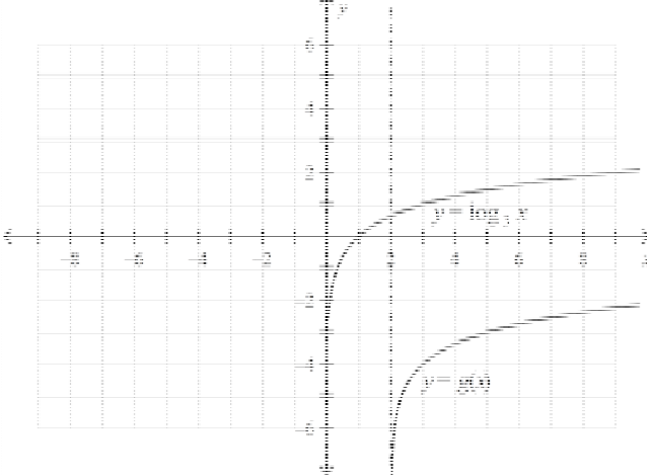
D. $9 = \left(\sqrt[3]{81} \right)^{\frac{2}{3}}$

Name: _____

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- _____ 21. For which value of x is $y = \log_9 x$ not defined?
- A. $x = -9$ C. $x = \frac{1}{9}$
B. $x = 1$ D. $x = 81$
- _____ 22. Given $\log_b a = c$, which statement is true?
- A. $a = b^c$ C. $c = b^a$
B. $c = a^b$ D. $a = c^b$
- _____ 23. Simplify: $\log 2 + \log 3$
- A. $\log 1$ C. $\log 5$
B. $\log \frac{2}{3}$ D. $\log 6$
- _____ 24. Simplify: $2 \log 9$
- A. $\log 18$ C. $\log 81$
B. $\log 11$ D. $\log \frac{2}{9}$
- _____ 25. Which of these expressions is NOT equal to $\log 4096$?
- A. $2 \log 64$ C. $3 \log 16$
B. $4 \log 1024$ D. $6 \log 4$
- _____ 26. Write as a single logarithm: $\frac{1}{2} \log 16 + 3 \log 1$
- A. $\log 5$ C. $\log 24$
B. $\log 4$ D. $\log 18$
- _____ 27. Write as a single logarithm: $3 \log_4 3 - \log_4 12 + 2$
- A. $\log_4 \frac{3}{2}$ C. $\log_4 225$
B. $\log_4 (-1)$ D. $\log_4 36$
- _____ 28. Evaluate: $\log_6 25 - 6 \log_6 5 + 4 \log_6 30$
- A. 4 C. $\frac{115}{6}$
B. 1296 D. $\frac{50}{3}$

- ___ 29. The graph of $y = \log_3(7(x+4))$ is the image of the graph of $y = \log_3 x$ after it has been
- compressed horizontally by a factor of $\frac{1}{7}$, and then translated 4 units left.
 - stretched horizontally by a factor of 7, and then translated 4 units right.
 - compressed horizontally by a factor of $\frac{1}{7}$, and then translated 4 units right.
 - stretched horizontally by a factor of 7, and then translated 4 units left.
- ___ 30. The graph of $y = \log_2 x$ is compressed horizontally by a factor of $\frac{1}{7}$, and then translated 8 units down. Identify the equation of the image graph.
- $y - 8 = -\log_2(-7x)$
 - $y = -8 \log_2(x - 7)$
 - $y = 7 \log_2(x + 8)$
 - $y + 8 = \log_2(7x)$
- ___ 31. What is the domain of the function $y = \log_2(-3x) - 5$?
- $x < 0$
 - $x < -5$
 - $x \in \mathbb{R}$
 - $x > 0$
- ___ 32. The graphs of $y = \log_3 x$ and its transformation image $y = g(x)$ are shown. Identify the transformations.



- A translation of 2 units right and 4 units down
- A horizontal compression by a factor of $\frac{1}{2}$
- A reflection in the x -axis
- A vertical stretch by a factor of 2

- _____ 33. Which logarithm is equal to $\log_6(x-3) + \log_6 x$?
- A. $\log_6(x^2 - 3x)$ C. $\log_{12}(x^2 - 3x)$
B. $\log_6(2x - 3)$ D. $\log_6(-x)$
- _____ 34. Which logarithm is equal to $\log_8(3x-1) - 5\log_8(x)$?
- A. $\log_8\left(\frac{3x-1}{x^5}\right)$ C. $\log_8\left(\frac{3x-1}{5x}\right)$
B. $\log_{16}\left(\frac{3x-1}{x^5}\right)$ D. $\log_8(x^5 + 3x - 1)$
- _____ 35. Solve: $\log 30 - \log 5 = \log x$
- A. $x = 25$ C. $x = 35$
B. $x = 150$ D. $x = 6$
- _____ 36. Solve: $3\log 10 = \log x$
- A. $x = 13$ C. $x = \frac{3}{10}$
B. $x = 1000$ D. $x = 30$
- _____ 37. Solve: $32^{x+3} = 16^{x-8}$
- A. $x = -14$ C. $x = -\frac{11}{16}$
B. $x = -11$ D. $x = -47$
- _____ 38. Solve: $\log x + \log(x+21) = 2$
- A. $x = 25$ C. $x = 25, -4$
B. $x = 4$ D. $x = -25, 4$
- _____ 39. Solve: $2922 = 5^x$
Give the solution to the nearest hundredth.
- A. $x \doteq 3.47$ C. $x \doteq 584.4$
B. $x \doteq 0.2$ D. $x \doteq 4.96$
- _____ 40. To repay a loan, Chloe makes payments weekly for 5 years.
How many payments does she make?
- A. 260 C. 5
B. 52 D. 57

- _____ 41. Use the equation $600 = 300(1.0075)^{4t}$ to determine the time in years (to the nearest year) it will take an investment of \$300 to double when it is invested in an account that pays 3% annual interest, compounded quarterly.
- A. 23 years
B. 12 years
C. 24 years
D. 1 year
- _____ 42. The Richter scale measures the intensity of an earthquake. The magnitude, M , of an earthquake can be determined using the function $M = \log\left(\frac{I}{S}\right)$, where I microns is the intensity of the earthquake, and S microns is the intensity of a standard earthquake.

In June 2010, California experienced an earthquake with magnitude 5.7.

In October 2010, Indonesia experienced an earthquake with magnitude 7.7.

How many times as intense as the California earthquake was the Indonesia earthquake?

- A. 10^2 times as intense
B. 3 times as intense
C. 10^3 times as intense
D. Approximately 1.4 times as intense
- _____ 43. The pH scale measures the acidity or alkalinity of a solution. A solution that has a pH of 7 is neutral. For each increase of 1 pH, a solution is 10 times as alkaline. For each decrease of 1 pH, a solution is 10 times as acidic.

A sample of soap has a pH of 9.5. A sample of household ammonia has a pH of 11.4.

To the nearest whole number, how many times as alkaline as the soap is the ammonia?

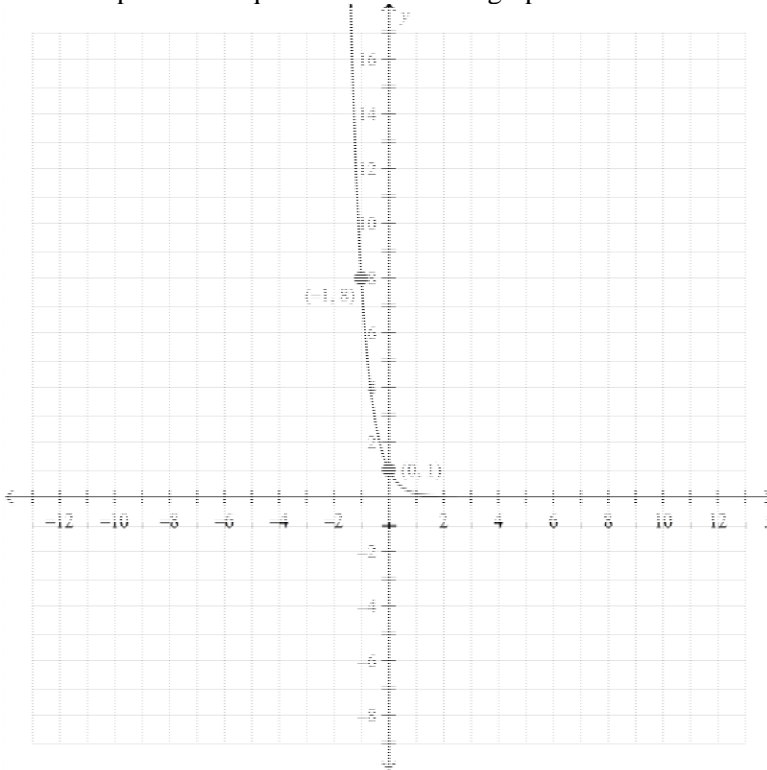
- A. 79 times as alkaline
B. 12 times as alkaline
C. 40 times as alkaline
D. 2 times as alkaline
- _____ 44. The present value formula is used when an amount, PV dollars, is borrowed and then repaid through a series of equal payments at equal time intervals, and the compounding period of the interest is equal to the time interval for the payments. The first payment is made after a time equal to the compounding period. The formula is: $PV = \frac{R[1 - (1 + i)^{-n}]}{i}$, where R dollars is the regular payment, i is the interest rate per compounding period, and n is the number of payments.

A person has a balance of \$274.51 on a credit card. The credit card charges 20% annual interest, compounded monthly. The minimum payment is \$10 per month. If the person does not make any more purchases using the card, and pays only the minimum payment each month, how long will it take before the balance is paid off, to the nearest month?

- A. 23 months
B. 40 months
C. 17 months
D. 37 months
- _____ 45. Which set of properties does the function $y = 4^x$ have?
- A. no x -intercept, no y -intercept
B. x -intercept is 1, no y -intercept
C. no x -intercept, y -intercept is 1
D. x -intercept is 0, y -intercept is 0

- _____ 46. Which set of properties is correct for the function $y = 4^x$?
- | | |
|----------------------------------------------------------------------------|-------------------------------------------------------------------------------|
| A. domain $\{x x \in \mathbb{R}\}$, range $\{y y > 0, y \in \mathbb{R}\}$ | C. domain $\{x x \in \mathbb{R}\}$, range $\{y y \geq 0, y \in \mathbb{R}\}$ |
| B. domain $\{x x \in \mathbb{R}\}$, range $\{y y < 0, y \in \mathbb{R}\}$ | D. domain $\{x x \in \mathbb{R}\}$, range $\{y y \leq 0, y \in \mathbb{R}\}$ |

- _____ 47. Which exponential equation matches the graph shown?

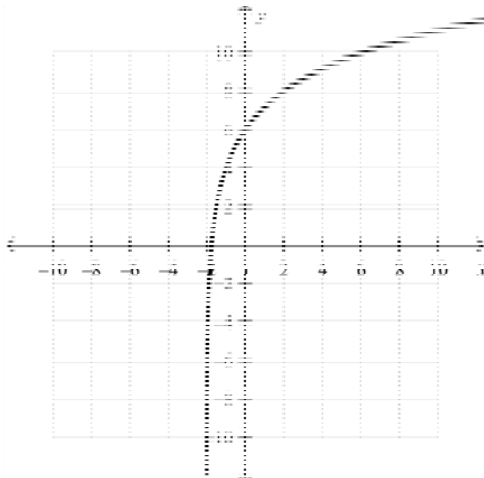


- | | |
|-------------------------------------|--------------------------------------|
| A. $y = \left(\frac{1}{8}\right)^x$ | C. $y = -\left(\frac{1}{8}\right)^x$ |
| B. $y = 8^x$ | D. $y = -8^x$ |
- _____ 48. An investment of \$500 is placed into an account that earns interest, compounded annually, at a rate of 6% for 10 years. The amount, A , in the account can be modelled by the function $A = 500(1.06)^t$, where t is the time, in years. What is the domain of this function?
- | | |
|-----------------------------------------------|-----------------------------------------|
| A. $\{t t \geq 10, t \in \mathbb{R}\}$ | C. $\{t 0 < t < 10, t \in \mathbb{R}\}$ |
| B. $\{t 0 \leq t \leq 10, t \in \mathbb{R}\}$ | D. $\{t t > 10, t \in \mathbb{R}\}$ |
- _____ 49. The equation $A = 30\left(\frac{1}{7}\right)^n$ can also be written as
- | | |
|---------------------|---------------------|
| A. $A = 30(7)^{-n}$ | C. $A = 30(7)^n$ |
| B. $A = 15(7)^n$ | D. $A = 15(7)^{-n}$ |

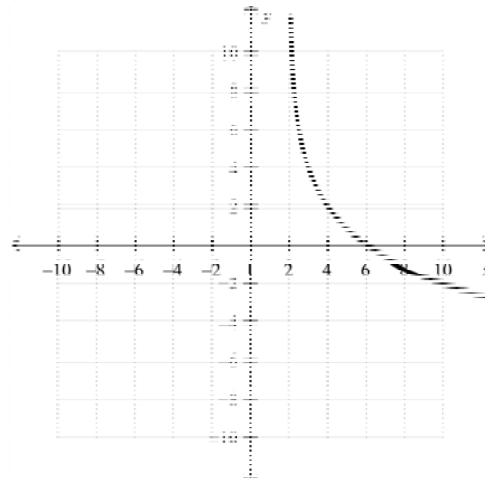
- _____ 50. Which function results when the graph of the function $y = 9^x$ is reflected in the y -axis, compressed vertically by a factor of $\frac{1}{9}$, and shifted 6 units down?
- A. $y = \frac{1}{9}(9)^{-x} - 6$ C. $y = \frac{1}{9}(9)^x + 6$
B. $y = \frac{1}{9}(9)^{-x} + 6$ D. $y = \frac{1}{9}(9)^x - 6$
- _____ 51. Solve for x , to one decimal place.
 $5 = 2^x$
- A. 2.3 C. 4.6
B. 2.5 D. 10.0
- _____ 52. Another way of writing $2^4 = 16$ is
- A. $\log_4 2 = 16$ C. $\log_2 16 = 4$
B. $\log_{16} 2 = 4$ D. $\log_2 4 = 16$
- _____ 53. Which of the following represents $b = \log_2 8$?
- A. $b^2 = 8$ C. $b^8 = 2$
B. $2^8 = b$ D. $2^b = 8$
- _____ 54. What is the equation for the asymptote of the function $f(x) = -2 \log_9 [4(x - 3)] - 4$?
- A. $x = -3$ C. $x = 4$
B. $x = 3$ D. $x = -4$

_____ 55. Which graph represents the function $y = -4 \log_4 [(x - 2)] + 4$?

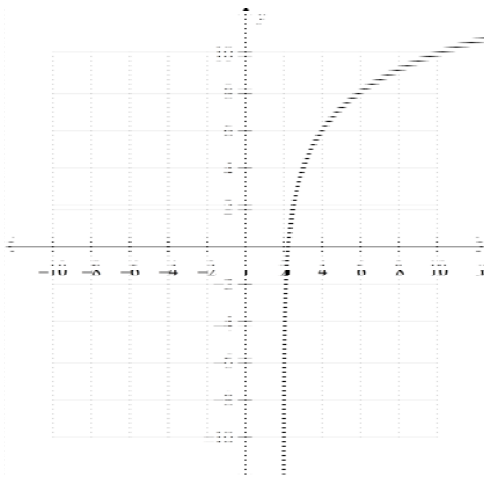
A.



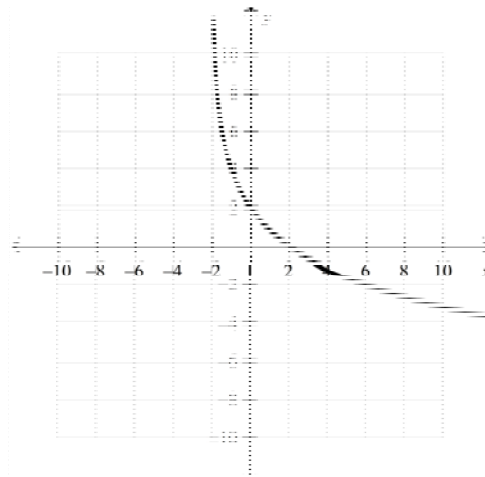
C.



B.



D.



_____ 56. If $\log 3 = s$, $\log 5 = u$, and $\log 7 = y$, an algebraic expression in terms of s , u , and y for $\log \frac{5}{441}$ is

A. $u - 2s + 2y$

C. $u - 2(s + y)$

B. $u - 2(s - y)$

D. $u - 2s + y$

_____ 57. Solve $\log_2 x = \log_2 5 + \log_2 2$.

A. $x = \frac{2}{5}$

C. $x = 10$

B. $x = \frac{5}{2}$

D. $x = 1024$

_____ 58. Solve $4^{7x-9} = 7^{x+4}$. Round your answer to two decimal places.

A. 1.74

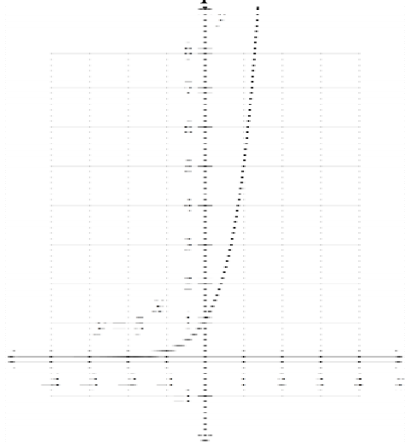
C. -0.60

B. 1.54

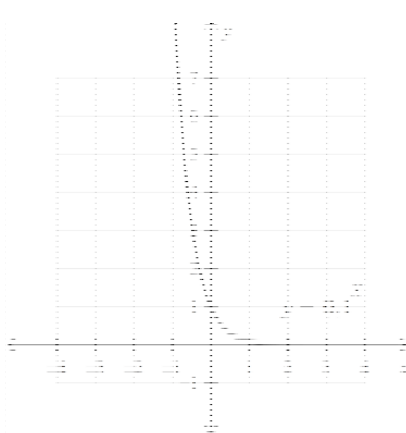
D. 2.61

Short Answer

1. Determine the equation of the horizontal asymptote of the graph of $y = 5^x$.

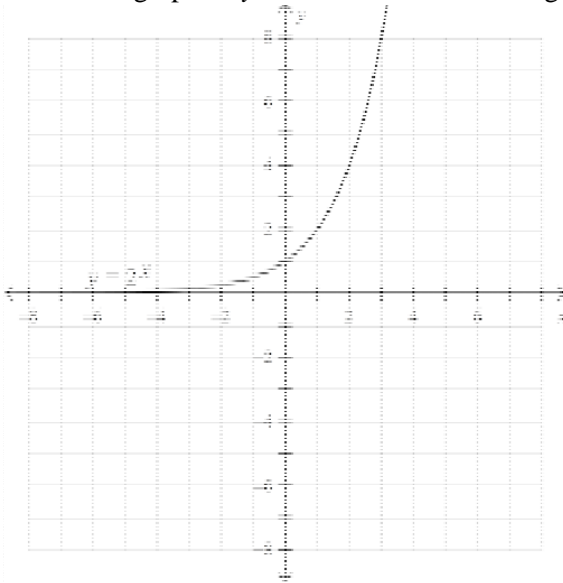


2. The graph of $y = 0.1^x$ is shown below. Identify the y-intercept.



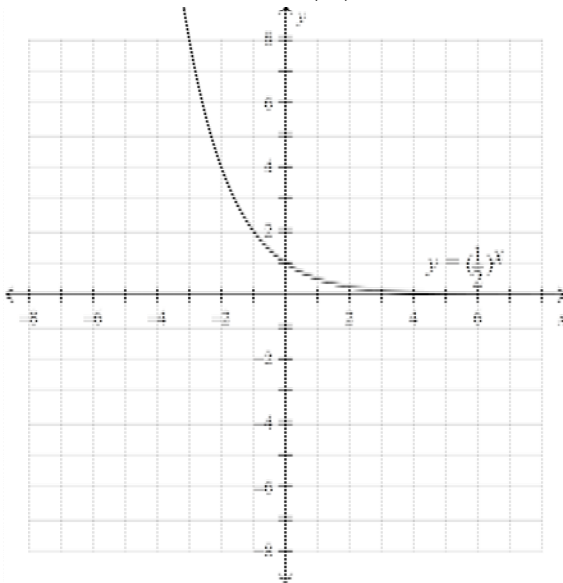
3. The graph of $y = 2^x$ is shown below.

Sketch the graph of $y = 2^{-0.5(x-2)}$ on the same grid.



4. The graph of $y = \left(\frac{1}{2}\right)^x$ is shown below.

Sketch the graph of $y = -\left(\frac{1}{2}\right)^x + 3$ on the same grid.



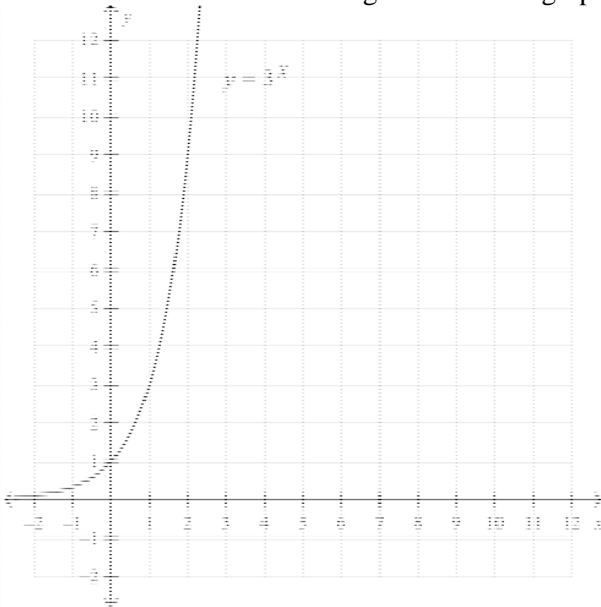
5. Solve: $(\sqrt[4]{6})^{x+4} = \sqrt{216}$

6. Solve: $\frac{\sqrt[3]{36}}{216} = 6^{x+2}$

7. Solve: $\left(\frac{1}{49}\right)^{x+4} = \left(\sqrt{343}\right)^x$

8. Solve: $64^x = 2^{(x^2+8)}$

9. The graph of $y = 3^x$ is shown below.
Sketch its inverse on the same grid. Label the graph with its equation.



10. Evaluate: $\log_4 256$
11. Evaluate: $\log_4 \left(\frac{1}{128}\right)$
12. Evaluate: $\log_2 (64\sqrt{2})$
13. Write 4 as a logarithm with base 3.
14. Write as a single logarithm: $7 \log x - \log y$
15. Write as a single logarithm: $\frac{7}{3} \log x + 3 \log y$

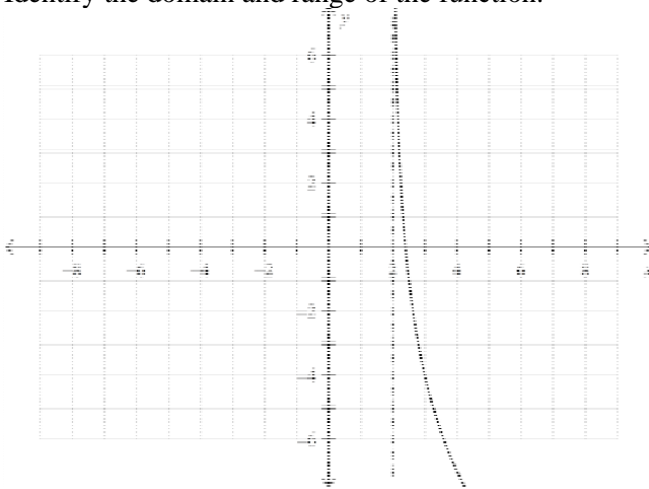
16. Write as a single logarithm: $\frac{9}{2} \log_6 x - 2 \log_6 y - 7 \log_6 z$

17. Write as a single logarithm: $5 + \log_2 x$

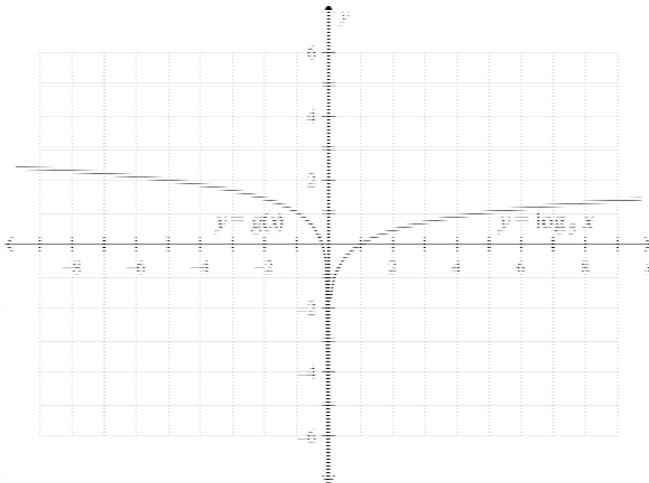
18. Write this expression in terms of $\log a$ and $\log b$.

$$\log \left(\frac{a^6}{b^4} \right)$$

19. The graph of $y + 4 = -3 \log_2 (x - 2)$ is shown below. Identify the domain and range of the function.



20. The graphs of $y = \log_5 x$ and its transformation image $y = g(x)$ are shown. Write an equation for the function $y = g(x)$ in the form $y = \log_5(dx)$.



21. Determine whether $x = 512$ is a root of this equation.
 $\log_8 x + \log_8 (8x) = 7$

22. Determine whether $x = 2$ is a root of this equation.
 $\log(x - 3) + \log(x - 12) = 1$
23. Solve: $2^{x-3} = 7^{x-5}$
Give the solution to the nearest hundredth.
24. Solve: $\log_8(4x + 28) - \log_8(x + 7) = \log_8(x + 5)$
25. Solve: $\log_2 90 = \log_2(x - 3) + \log_2(x - 2)$
26. The future value formula is used when an amount, FV dollars, is saved through a series of equal investments at equal time intervals, and the compounding period of the interest is equal to the time interval for the investments. The formula is: $FV = \frac{R[(1 + i)^n - 1]}{i}$, where R dollars is the regular investment, i is the interest rate per compounding period, and n is the number of investments.

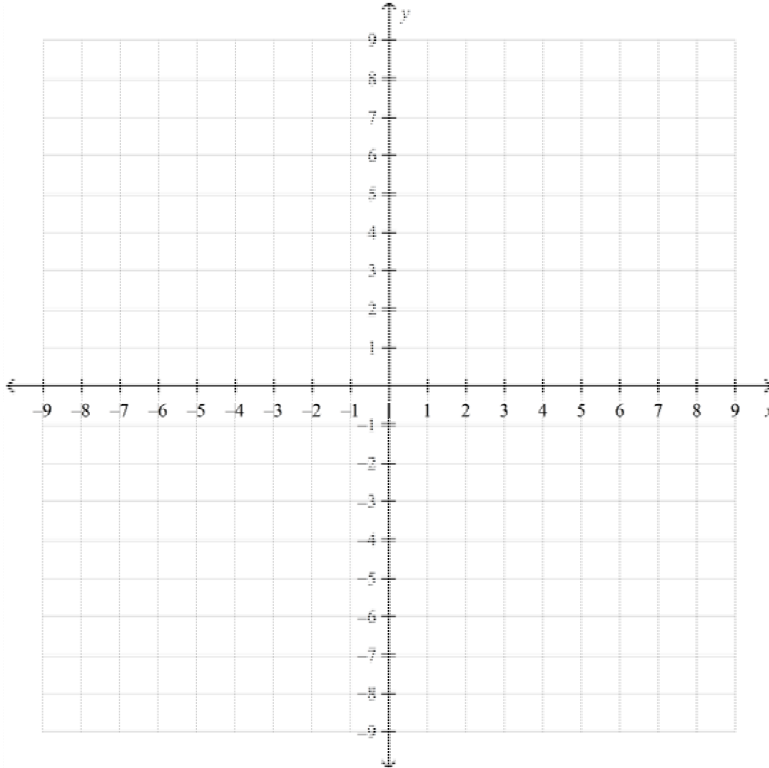
Each month, Raj deposits \$70 into a savings account with an annual interest rate of 3.5%, compounded monthly. How much will Raj have in the account after 4 years?

27. The compound interest formula is used when an amount, A dollars, is saved after making a single investment of A_0 dollars in an account that earns i percent annual interest, with n compounding periods per year, for t years. The formula is: $A = A_0 \left(1 + \frac{i}{n}\right)^{nt}$

To the nearest year, how long will it take an investment of \$700 to double at an annual interest rate of 3%, compounded monthly?

28. For the function $y = \frac{1}{2}(3)^{x-2}$,

- a) describe the transformations of the function when compared to the function $y = 3^x$
 b) sketch the graph of the given function and $y = 3^x$ on the same set of axes



- c) state the domain, the range, and the equation of the asymptote

29. Write the equation for the function that results from each transformation or set of transformations applied to the base function $y = 5^x$.

- a) reflect in the y -axis
 b) shift 3 units to the right
 c) shift 1 unit down and 4 units to the left
 d) reflect in the x -axis and shift 2 units down

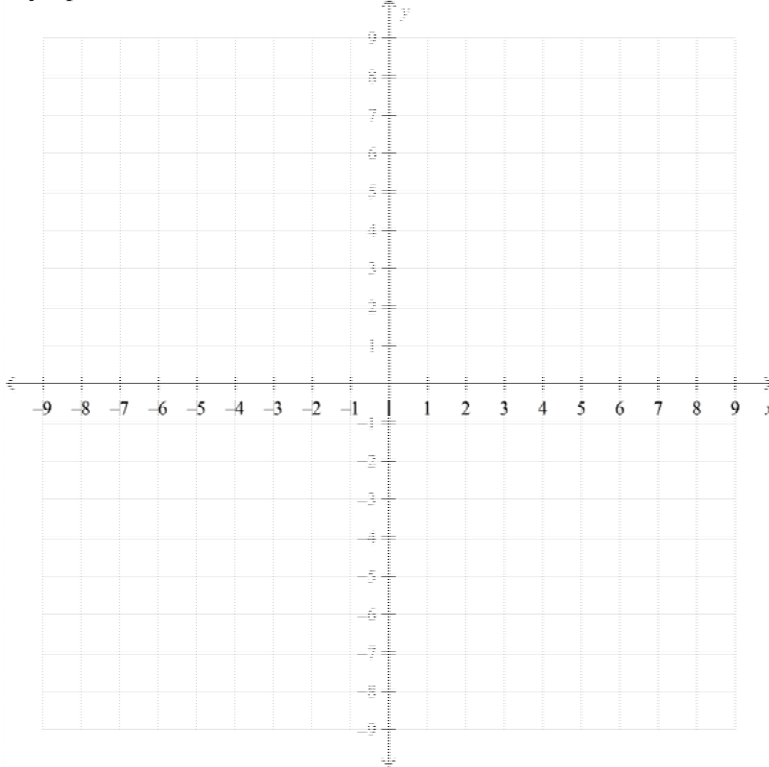
30. Solve for n : $9^{n-1} = \left(\frac{1}{3}\right)^{4n-1}$

31. Evaluate $\log_2 \sqrt[4]{32}$.

32. Evaluate $\log_2 64 + \log_3 27 \times \log_4 \frac{1}{256}$.

33. If $\log_4(m-n) = 0$ and $\log_4(m+n) = 2$, determine the values of m and n .

34. Graph the function $f(x) = -\log(x + 2) - 1$. Identify the domain, the range, and the equation of the vertical asymptote.

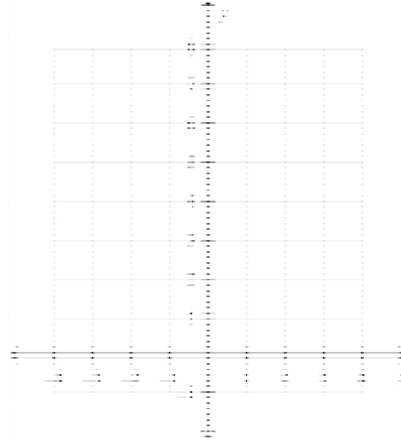


35. Given $\log_2 7 \approx 2.8074$, find the value of $\log_2 14$.
36. Solve the equation $6^{3x+1} = 2^{2x-3}$. Leave your answer in exact form.
37. Solve for x .
 $2\log_4(x + 4) - \log_4(x + 12) = 1$

Problem

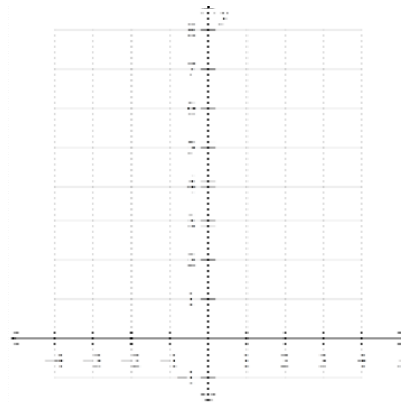
1. Graph the function $y = 3^x$.

Describe these characteristics of the graph:
 x -intercept, y -intercept, equation of the
horizontal asymptote, domain, and range.

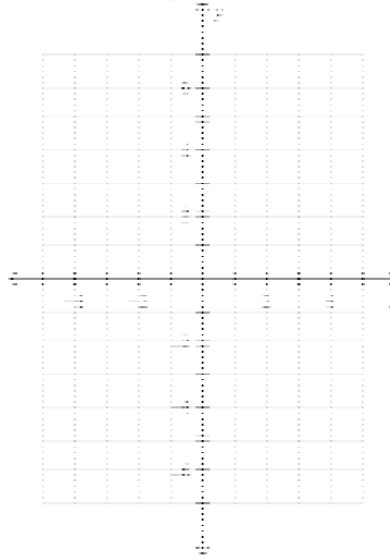


2. Graph the function $y = \left(\frac{1}{6}\right)^x$.

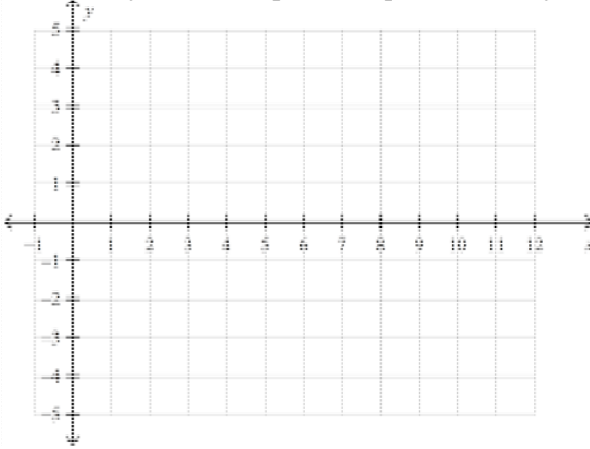
Describe these characteristics of the graph:
 x -intercept, y -intercept, equation of the
horizontal asymptote, domain, and range.



3. a) Use transformations to sketch the graph of the exponential function $y = 2^{-3(x+1)} + 1$.
- b) Determine:
- whether the function is increasing or decreasing
 - the intercepts
 - the equation of the asymptote
 - the domain of the function
 - the range of the function



4. For what values of k does the equation $16^x = 64^{(x^2+k)}$ have no real solution?
5. a) Graph $y = \log_7 x$.
- b) Identify the intercepts, the equations of any asymptotes, and the domain and range of the function.



6. Use two different strategies to write $3(\log x - \log y)$ as a single logarithm. Which strategy do you prefer? Explain.
7. Write as a single logarithm: $2 \log(x+2) + 2 \log(x-7) - \log(x^2 - 5x - 14)$
8. Write as a single logarithm: $\log(2x^2 + 3x + 1) - \log(x^2 - 1)$
9. Determine the exact value of x .
 $\log_2(x-8) + \log_2(x+1) = 3$

10. A colony of ants starts with an initial population of 50 and doubles every week for 8 weeks.
- Create a table of values for weeks 0 to 8 for the population of the colony.
 - Graph the data from your table of values.
 - Is the relationship between the ant population and the number of weeks exponential? Explain.
 - Model the information using an equation.
11. A radioactive sample with an initial mass of 72 mg has a half-life of 10 days.
- Write a function to relate the amount remaining, A , in milligrams, to the time, t , in days.
 - What amount of the radioactive sample will remain after 20 days?
 - What amount of the radioactive sample was there 30 days ago?
 - How long, to the nearest day, will it take for there to be 0.07 mg of the initial sample remaining?
12. a) Write the equation for a transformed exponential function with a base of 4 that passes through the point $\left(0, \frac{3}{2}\right)$.
- Write two equations, different from the one in part a), that satisfy these criteria.
 - Use algebraic and/or graphical reasoning to explain why each equation is a solution.
13. Solve the equation $2^{3x} = 4$.
14. List the steps and explain the effect of each transformation to graph the function $y = -3 \log[-2(x-1)] + 4$.
15. Given $\log 7 \approx 0.8451$ and $\log 2 \approx 0.3010$, find the value of $\log 28$.
16. Prove that $\log a + \log a^2 + \log a^3 - \log a^6 = \log 1$.
17. Solve the equation $\log \sqrt[3]{x^2 + 48x} = \frac{2}{3}$.

Review 5 Exponents and Logarithms

Answer Section

MULTIPLE CHOICE

1. ANS: A PTS: 1 DIF: Easy
REF: 5.1 Math Lab: Graphing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Procedural Knowledge
2. ANS: C PTS: 1 DIF: Easy
REF: 5.1 Math Lab: Graphing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Conceptual Understanding
3. ANS: C PTS: 1 DIF: Easy
REF: 5.1 Math Lab: Graphing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Procedural Knowledge
4. ANS: A PTS: 1 DIF: Easy
REF: 5.2 Analyzing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Conceptual Understanding
5. ANS: D PTS: 1 DIF: Easy
REF: 5.2 Analyzing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Procedural Knowledge
6. ANS: C PTS: 1 DIF: Moderate
REF: 5.2 Analyzing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Conceptual Understanding
7. ANS: C PTS: 1 DIF: Moderate
REF: 5.2 Analyzing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
8. ANS: D PTS: 1 DIF: Moderate
REF: 5.2 Analyzing Exponential Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Conceptual Understanding
9. ANS: B PTS: 1 DIF: Easy REF: 5.3 Solving Exponential Equations
LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge
10. ANS: D PTS: 1 DIF: Easy REF: 5.3 Solving Exponential Equations
LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge
11. ANS: D PTS: 1 DIF: Easy REF: 5.3 Solving Exponential Equations
LOC: 12.RF9 TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding
12. ANS: B PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge
13. ANS: C PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge
14. ANS: B PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge
15. ANS: C PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge
16. ANS: D PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge

17. ANS: D PTS: 1 DIF: Easy
REF: 5.4 Logarithms and the Logarithmic Function LOC: 12.RF7
TOP: Relations and Functions KEY: Procedural Knowledge
18. ANS: A PTS: 1 DIF: Easy
REF: 5.4 Logarithms and the Logarithmic Function LOC: 12.RF7
TOP: Relations and Functions KEY: Procedural Knowledge
19. ANS: D PTS: 1 DIF: Easy
REF: 5.4 Logarithms and the Logarithmic Function LOC: 12.RF7
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
20. ANS: A PTS: 1 DIF: Easy
REF: 5.4 Logarithms and the Logarithmic Function LOC: 12.RF7
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
21. ANS: A PTS: 1 DIF: Easy
REF: 5.4 Logarithms and the Logarithmic Function LOC: 12.RF7
TOP: Relations and Functions KEY: Conceptual Understanding
22. ANS: A PTS: 1 DIF: Easy
REF: 5.4 Logarithms and the Logarithmic Function LOC: 12.RF7
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
23. ANS: D PTS: 1 DIF: Easy REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
24. ANS: C PTS: 1 DIF: Easy REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
25. ANS: B PTS: 1 DIF: Easy REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
26. ANS: B PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
27. ANS: D PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
28. ANS: A PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
29. ANS: A PTS: 1 DIF: Moderate
REF: 5.6 Analyzing Logarithmic Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Conceptual Understanding
30. ANS: D PTS: 1 DIF: Moderate
REF: 5.6 Analyzing Logarithmic Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Conceptual Understanding
31. ANS: A PTS: 1 DIF: Moderate
REF: 5.6 Analyzing Logarithmic Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
32. ANS: A PTS: 1 DIF: Moderate
REF: 5.6 Analyzing Logarithmic Functions LOC: 12.RF9
TOP: Relations and Functions KEY: Conceptual Understanding
33. ANS: A PTS: 1 DIF: Easy
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF8
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
34. ANS: A PTS: 1 DIF: Easy
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF8
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding

35. ANS: D PTS: 1 DIF: Easy
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF10
TOP: Relations and Functions KEY: Procedural Knowledge
36. ANS: B PTS: 1 DIF: Easy
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF10
TOP: Relations and Functions KEY: Procedural Knowledge
37. ANS: D PTS: 1 DIF: Moderate
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF10
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
38. ANS: B PTS: 1 DIF: Moderate
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF10
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
39. ANS: D PTS: 1 DIF: Easy
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF10
TOP: Relations and Functions KEY: Procedural Knowledge
40. ANS: A PTS: 1 DIF: Easy
REF: 5.8 Solving Problems with Exponents and Logarithms LOC: 12.RF10
TOP: Relations and Functions KEY: Conceptual Understanding
41. ANS: A PTS: 1 DIF: Easy
REF: 5.8 Solving Problems with Exponents and Logarithms LOC: 12.RF10
TOP: Relations and Functions KEY: Procedural Knowledge
42. ANS: A PTS: 1 DIF: Moderate
REF: 5.8 Solving Problems with Exponents and Logarithms LOC: 12.RF10
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
43. ANS: A PTS: 1 DIF: Moderate
REF: 5.8 Solving Problems with Exponents and Logarithms LOC: 12.RF10
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
44. ANS: D PTS: 1 DIF: Moderate
REF: 5.8 Solving Problems with Exponents and Logarithms LOC: 12.RF10
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
45. ANS: C PTS: 1 DIF: Easy OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: intercepts | exponential function
46. ANS: A PTS: 1 DIF: Average OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: domain | range
47. ANS: A PTS: 1 DIF: Average OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: equation | graph | exponential function
48. ANS: B PTS: 1 DIF: Easy OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: range | domain | exponential function
49. ANS: A PTS: 1 DIF: Easy OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: exponential function | negative exponents
50. ANS: A PTS: 1 DIF: Average OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: transformations of exponential functions

51. ANS: A PTS: 1 DIF: Average OBJ: Section 7.3
 NAT: RF10 TOP: Solving Exponential Equations
 KEY: exponential equation | systematic trial
52. ANS: C PTS: 1 DIF: Easy OBJ: Section 8.1
 NAT: RF7 TOP: Understanding Logarithms KEY: logarithm | exponential function
 NOT: Draft
53. ANS: D PTS: 1 DIF: Easy OBJ: Section 8.1
 NAT: RF7 TOP: Understanding Logarithms KEY: logarithm | exponential function
 NOT: Draft
54. ANS: B PTS: 1 DIF: Average OBJ: Section 8.2
 NAT: RF8 TOP: Transformations of Logarithmic Functions
 KEY: horizontal translation | asymptote
55. ANS: C PTS: 1 DIF: Average OBJ: Section 8.2
 NAT: RF8 TOP: Transformations of Logarithmic Functions
 KEY: horizontal translation | vertical translation | vertical stretch | horizontal stretch
56. ANS: C PTS: 1 DIF: Average OBJ: Section 8.3
 NAT: RF9 TOP: Laws of Logarithms
 KEY: product law | laws of logarithms | quotient law
57. ANS: C PTS: 1 DIF: Average OBJ: Section 8.4
 NAT: RF10 TOP: Logarithmic and Exponential Equations
 KEY: logarithmic equation
58. ANS: D PTS: 1 DIF: Average OBJ: Section 8.4
 NAT: RF10 TOP: Logarithmic and Exponential Equations
 KEY: logarithmic equation | exponential equation

SHORT ANSWER

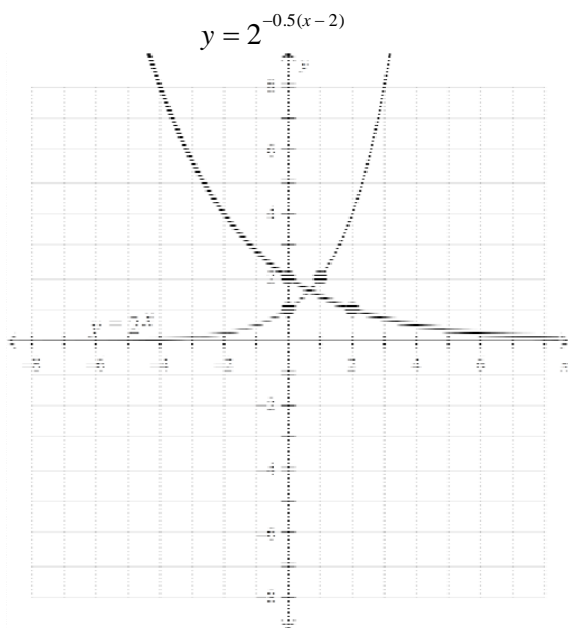
1. ANS:
 $y = 0$

PTS: 1 DIF: Easy REF: 5.1 Math Lab: Graphing Exponential Functions
 LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge

2. ANS:
 The y -intercept is 1.

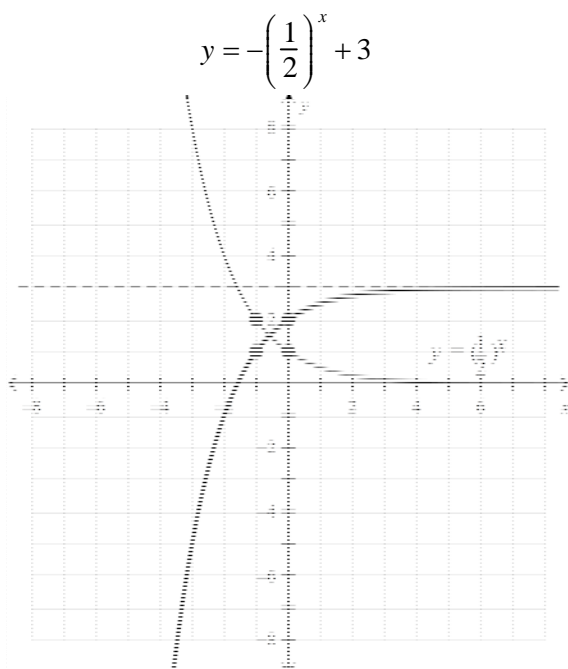
PTS: 1 DIF: Easy REF: 5.2 Analyzing Exponential Functions
 LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

3. ANS:



PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
 LOC: 12.RF9 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge | Communication

4. ANS:



PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
 LOC: 12.RF9 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge | Communication

5. ANS:

$$x = 2$$

PTS: 1

LOC: 12.RF10

DIF: Moderate REF: 5.3 Solving Exponential Equations

TOP: Relations and Functions

KEY: Procedural Knowledge

6. ANS:

$$x = -\frac{13}{3}$$

PTS: 1

LOC: 12.RF10

DIF: Moderate REF: 5.3 Solving Exponential Equations

TOP: Relations and Functions

KEY: Procedural Knowledge

7. ANS:

$$x = -\frac{16}{7}$$

PTS: 1

LOC: 12.RF10

DIF: Moderate REF: 5.3 Solving Exponential Equations

TOP: Relations and Functions

KEY: Procedural Knowledge

8. ANS:

$$x = 4 \text{ or } x = 2$$

PTS: 1

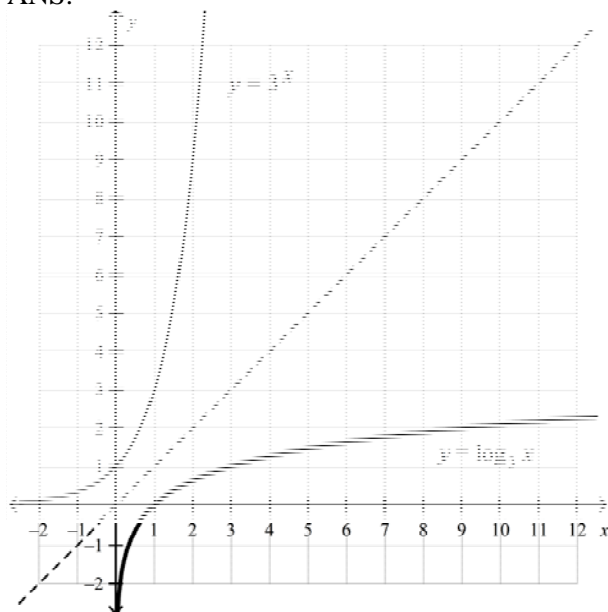
LOC: 12.RF10

DIF: Moderate REF: 5.3 Solving Exponential Equations

TOP: Relations and Functions

KEY: Procedural Knowledge

9. ANS:



PTS: 1

LOC: 12.RF9

DIF: Easy

TOP: Relations and Functions

REF: 5.4 Logarithms and the Logarithmic Function

KEY: Communication | Conceptual Understanding | Procedural Knowledge

10. ANS:

$$\log_4 256 = 4$$

PTS: 1

LOC: 12.RF7

DIF: Easy

REF: 5.4 Logarithms and the Logarithmic Function

TOP: Relations and Functions

KEY: Procedural Knowledge

11. ANS:

$$\log_4 \left(\frac{1}{128} \right) = -\frac{7}{2}$$

PTS: 1

LOC: 12.RF7

DIF: Moderate

REF: 5.4 Logarithms and the Logarithmic Function

TOP: Relations and Functions

KEY: Procedural Knowledge

12. ANS:

$$\log_2 \left(64\sqrt{2} \right) = \frac{13}{2}$$

PTS: 1

LOC: 12.RF7

DIF: Moderate

REF: 5.4 Logarithms and the Logarithmic Function

TOP: Relations and Functions

KEY: Procedural Knowledge

13. ANS:

$$4 = \log_3 81$$

PTS: 1

LOC: 12.RF7

DIF: Moderate

REF: 5.4 Logarithms and the Logarithmic Function

TOP: Relations and Functions

KEY: Conceptual Understanding

14. ANS:

$$\log \left(\frac{x^7}{y} \right)$$

PTS: 1

LOC: 12.RF8

DIF: Easy

REF: 5.5 The Laws of Logarithms

TOP: Relations and Functions

KEY: Procedural Knowledge

15. ANS:

$$\log \left(x^{\frac{7}{3}} y^3 \right)$$

PTS: 1

LOC: 12.RF8

DIF: Easy

REF: 5.5 The Laws of Logarithms

TOP: Relations and Functions

KEY: Procedural Knowledge

16. ANS:

$$\log_6 \left(\frac{x^{\frac{9}{2}}}{y^2 z^7} \right)$$

PTS: 1

LOC: 12.RF8

DIF: Moderate

REF: 5.5 The Laws of Logarithms

TOP: Relations and Functions

KEY: Procedural Knowledge

17. ANS:
 $\log_2 32x$
- PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms
 LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
18. ANS:
 $6\log a - 4\log b$
- PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms
 LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
19. ANS:
 domain: $x > 2$
 range: $y \in \mathbb{R}$
- PTS: 1 DIF: Easy REF: 5.6 Analyzing Logarithmic Functions
 LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding
20. ANS:
 $y = \log_5(-5x)$
- PTS: 1 DIF: Easy REF: 5.6 Analyzing Logarithmic Functions
 LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding
21. ANS:
 $x = 512$ is a root of the equation.
- PTS: 1 DIF: Easy REF: 5.7 Solving Logarithmic and Exponential Equations
 LOC: 12.RF10 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
22. ANS:
 $x = 2$ is not a root of the equation.
- PTS: 1 DIF: Easy REF: 5.7 Solving Logarithmic and Exponential Equations
 LOC: 12.RF10 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
23. ANS:
 $x \doteq 6.11$
- PTS: 1 DIF: Moderate REF: 5.7 Solving Logarithmic and Exponential Equations
 LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge
24. ANS:
 $x = -1$
- PTS: 1 DIF: Moderate REF: 5.7 Solving Logarithmic and Exponential Equations
 LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge
25. ANS:
 $x = 12$
- PTS: 1 DIF: Moderate REF: 5.7 Solving Logarithmic and Exponential Equations
 LOC: 12.RF10 TOP: Relations and Functions
 KEY: Procedural Knowledge | Conceptual Understanding

26. ANS:

Raj will have \$3600.95.

PTS: 1 DIF: Moderate REF: 5.8 Solving Problems with Exponents and Logarithms

LOC: 12.RF10 TOP: Relations and Functions

KEY: Procedural Knowledge | Conceptual Understanding

27. ANS:

Approximately 23 years

PTS: 1 DIF: Moderate REF: 5.8 Solving Problems with Exponents and Logarithms

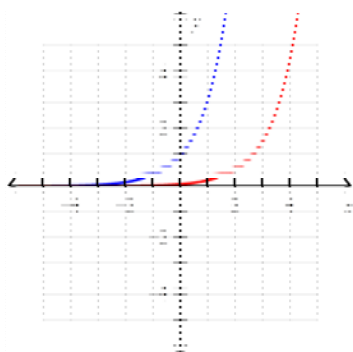
LOC: 12.RF10 TOP: Relations and Functions

KEY: Procedural Knowledge | Conceptual Understanding

28. ANS:

a) a vertical compression by a factor of $\frac{1}{2}$ and a translation of 2 units to the right

b) The graph of $y = 3^x$ is shown in blue and the graph of $y = \frac{1}{2}(3)^{x-2}$ is shown in red.



c) domain $\{x|x \in \mathbf{R}\}$, range $\{y|y > 0, y \in \mathbf{R}\}, y = 0$

PTS: 1 DIF: Average OBJ: Section 7.2 NAT: RF9

TOP: Transformations of Exponential Functions

KEY: graph | transformations of exponential functions

29. ANS:

a) $y = 5^{-x}$

b) $y = 5^{x-3}$

c) $y = 5^{x+4} - 1$

d) $y = -5^x - 2$

PTS: 1 DIF: Average OBJ: Section 7.2 NAT: RF9

TOP: Transformations of Exponential Functions

KEY: equation | transformations of exponential functions

30. ANS:

$$9^{n-1} = \left(\frac{1}{3}\right)^{4n-1}$$

$$\left(3^2\right)^{n-1} = \left(3^{-1}\right)^{4n-1}$$

$$3^{2n-2} = 3^{1-4n}$$

Equate the exponents:

$$2n - 2 = 1 - 4n$$

$$6n = 3$$

$$n = \frac{1}{2}$$

PTS: 1 DIF: Average
TOP: Solving Exponential Equations

OBJ: Section 7.3 NAT: RF10
KEY: change of base

31. ANS:

$$\frac{5}{4}$$

PTS: 1 DIF: Average
TOP: Understanding Logarithms

OBJ: Section 8.1 NAT: RF7
KEY: logarithm NOT: Draft

32. ANS:

$$\begin{aligned} \log_2 64 + \log_3 27 \times \log_4 \left(\frac{1}{256}\right) &= 6 + 3(-4) \\ &= 6 - 12 \\ &= -6 \end{aligned}$$

PTS: 1 DIF: Difficult
TOP: Understanding Logarithms
NOT: Draft

OBJ: Section 8.1 NAT: RF7
KEY: logarithm | exponential function

33. ANS:

Solve the system of equations:

$$m - n = 4^0 \text{ and } m + n = 4^2$$

$$m - n = 1$$

$$m + n = 16$$

Add these equations to find m :

$$2m = 17$$

$$m = 8.5$$

Subtract the first equation from the second to find n :

$$2n = 15$$

$$n = 7.5$$

PTS: 1

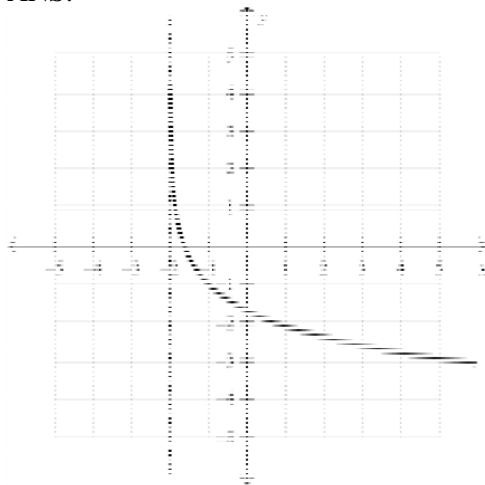
DIF: Difficult

OBJ: Section 8.1 NAT: RF7

TOP: Understanding Logarithms

KEY: logarithm | simultaneous equations

34. ANS:

domain: $\{x|x > -2, x \in \mathbb{R}\}$ range: $\{y|y \in \mathbb{R}\}$ equation of vertical asymptote: $x = -2$

PTS: 1

DIF: Difficult

OBJ: Section 8.2 NAT: RF8

TOP: Transformations of Logarithmic Functions

KEY: transformation | vertical translation | asymptote | graph

35. ANS:

$$\log_2 14 = \log_2 (2 \times 7)$$

$$= \log_2 2 + \log_2 7$$

$$\approx 1 + 2.8074$$

$$= 3.8074$$

PTS: 1

DIF: Difficult

OBJ: Section 8.3 NAT: RF9

TOP: Laws of Logarithms

KEY: power law | laws of logarithms

36. ANS:

$$6^{3x+1} = 2^{2x-3}$$

$$\log(6^{3x+1}) = \log(2^{2x-3})$$

$$(3x + 1) \log 6 = (2x - 3) \log 2$$

$$3x \log 6 + \log 6 = 2x \log 2 - 3 \log 2$$

$$x(3 \log 6 - 2 \log 2) = -3 \log 2 - \log 6$$

$$x = \frac{-(3 \log 2 + \log 6)}{3 \log 6 - 2 \log 2}$$

PTS: 1

DIF: Average

OBJ: Section 8.3 | Section 8.4

NAT: RF9

TOP: Laws of Logarithms | Logarithmic and Exponential Equations

KEY: exponential equation | laws of logarithms

37. ANS:

$$2 \log_4(x+4) - \log_4(x+12) = 1$$

$$\log_4(x+4)^2 - \log_4(x+12) = 1$$

$$\log_4 \frac{(x+4)^2}{(x+12)} = \log_4 4^1$$

$$\frac{(x+4)^2}{(x+12)} = 4$$

$$(x+4)^2 = 4x + 48$$

$$x^2 + 8x + 16 = 4x + 48$$

$$x^2 + 4x - 32 = 0$$

$$(x+8)(x-4) = 0$$

$$x = -8, x = 4$$

Since $x = -8$ is an extraneous root, the solution is $x = 4$.

PTS: 1

DIF: Average

OBJ: Section 8.4

NAT: RF10

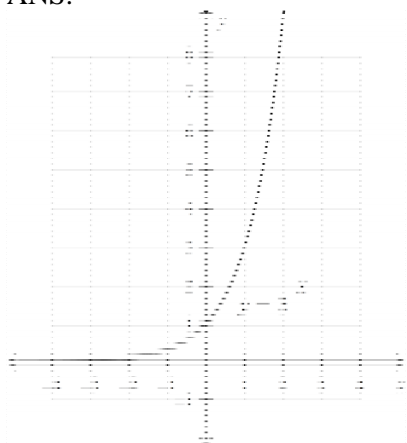
TOP: Logarithmic and Exponential Equations

KEY: logarithmic equation | laws of logarithms

NOT: Draft

PROBLEM

1. ANS:



The graph does not intersect the x -axis, so it does not have an x -intercept.

The y -intercept is 1.

The x -axis is the horizontal asymptote, so the equation of the horizontal asymptote is $y = 0$.

The domain of $y = 3^x$ is $x \in \mathbb{R}$.

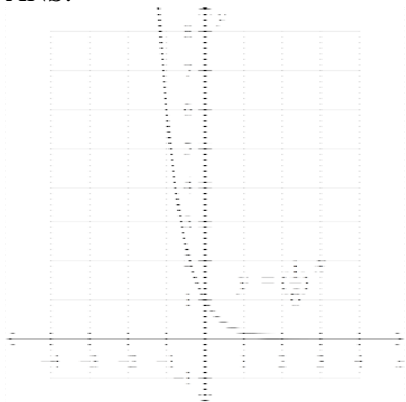
The range of $y = 3^x$ is $y > 0$.

PTS: 1 DIF: Moderate REF: 5.1 Math Lab: Graphing Exponential Functions

LOC: 12.RF9 TOP: Relations and Functions

KEY: Procedural Knowledge | Communication

2. ANS:



The graph does not intersect the x -axis, so it does not have an x -intercept.

The y -intercept is 1.

The x -axis is the horizontal asymptote, so the equation of the horizontal asymptote is $y = 0$.

The domain of $y = \left(\frac{1}{6}\right)^x$ is $x \in \mathbb{R}$.

The range of $y = \left(\frac{1}{6}\right)^x$ is $y > 0$.

PTS: 1 DIF: Moderate REF: 5.1 Math Lab: Graphing Exponential Functions

LOC: 12.RF9 TOP: Relations and Functions

KEY: Procedural Knowledge | Communication

3. ANS:

a) Write the function $y = 2^{-3(x+1)} + 1$ as $y - 1 = 2^{-3(x+1)}$.

Compare $y - 1 = 2^{-3(x+1)}$ with $y - k = c(2)^{d(x-h)}$: $k = 1, c = 1, d = -3, h = -1$

The graph of $y = 2^{-3(x+1)} + 1$ is the image of the graph of $y = 2^x$ after a horizontal compression by a factor of $\frac{1}{3}$, a reflection in the y -axis, then a translation of 1 unit left and 1 unit up.

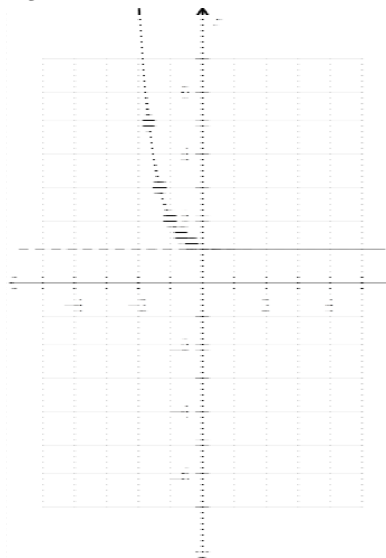
Use the general transformation: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

The point (x, y) on $y = 2^x$ corresponds to the point $(-\frac{1}{3}x - 1, y + 1)$ on $y = 2^{-3(x+1)} + 1$.

Choose points (x, y) on $y = 2^x$.

(x, y)	$(-\frac{1}{3}x - 1, y + 1)$
$(-2, \frac{1}{4})$	$(-\frac{1}{3}, \frac{5}{4})$
$(-1, \frac{1}{2})$	$(-\frac{2}{3}, \frac{3}{2})$
$(0, 1)$	$(-1, 2)$
$(1, 2)$	$(-\frac{4}{3}, 3)$
$(2, 4)$	$(-\frac{5}{3}, 5)$

$$y = 2^{-3(x+1)} + 1$$



b) From the graph:

i) The function is decreasing.

ii) When $x = 0$:

$$y = 2^{-3(0+1)} + 1$$

$$y = 2^{-3} + 1$$

$$y = \frac{9}{8}$$

The y -intercept is: $\frac{9}{8}$

iii) The equation of the horizontal asymptote is $y = 1$.

iv) The domain is $x \in \mathbb{R}$.

v) The range is $y > 1$.

PTS: 1

DIF: Moderate

REF: 5.2 Analyzing Exponential Functions

LOC: 12.RF9

TOP: Relations and Functions

KEY: Procedural Knowledge | Conceptual Understanding | Communication | Problem-Solving Skills

4. ANS:

Write both powers with the same base, then equate the exponents, and simplify the equation.

$$16^x = 64^{(x^2 + k)}$$

$$2^{4x} = 2^{6(x^2 + k)}$$

$$4x = 6x^2 + 6k$$

$$0 = 6x^2 - 4x + 6k$$

This is a quadratic equation.

It has no real roots when the discriminant is less than 0.

Determine when the discriminant is less than 0.

$$(-4)^2 - 4(6)(6k) < 0$$

$$(-4)^2 < 4(6)(6k)$$

$$16 < 144k$$

$$\frac{1}{9} < k$$

So, the equation $16^x = 64^{(x^2 + k)}$ has no real solution when $k > \frac{1}{9}$.

PTS: 1 DIF: Difficult REF: 5.3 Solving Exponential Equations

LOC: 12.RF10 TOP: Relations and Functions

KEY: Conceptual Understanding | Problem-Solving Skills

5. ANS:

a) $y = \log_7 x$ is the inverse of $y = 7^x$, so make a table of values for $y = 7^x$, then interchange the coordinates to make a table of values for $y = \log_7 x$.

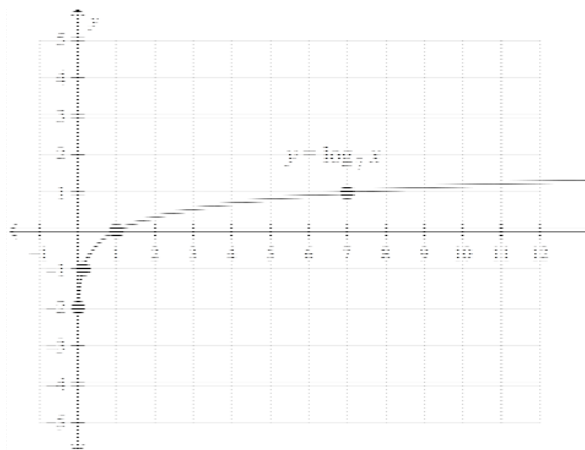
Plot the points for $y = \log_7 x$, then join them with a smooth curve.

For $y = 7^x$:

x	y
-2	$\frac{1}{49}$
-1	$\frac{1}{7}$
0	1
1	7
2	49

For $y = \log_7 x$:

x	y
$\frac{1}{49}$	-2
$\frac{1}{7}$	-1
1	0
7	1
49	2



b) The graph does not intersect the y -axis, so it does not have a y -intercept.

The graph has x -intercept 1.

The y -axis is a vertical asymptote; its equation is $x = 0$.

The domain of the function is $x > 0$.

The range of the function is $y \in \mathbb{R}$.

PTS: 1 DIF: Moderate REF: 5.4 Logarithms and the Logarithmic Function

LOC: 12.RF9 TOP: Relations and Functions

KEY: Procedural Knowledge | Conceptual Understanding | Communication

6. ANS:

Strategy 1: Distribute the common factor first.

$$\begin{aligned} 3(\log x - \log y) &= 3 \log x - 3 \log y \\ &= \log(x^3) - \log(y^3) \\ &= \log\left(\frac{x^3}{y^3}\right) \end{aligned}$$

Strategy 2: Simplify the expression inside the brackets first.

$$\begin{aligned} 3(\log x - \log y) &= 3 \log\left(\frac{x}{y}\right) \\ &= \log\left(\left(\frac{x}{y}\right)^3\right) \end{aligned}$$

Student preferences may vary. For example:

I prefer Strategy 2 because it involves fewer steps.

PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms

LOC: 12.RF8 TOP: Relations and Functions

KEY: Conceptual Understanding | Problem-Solving Skills | Communication

7. ANS:

$$\begin{aligned} &2 \log(x+2) + 2 \log(x-7) - \log(x^2 - 5x - 14) \\ &= \log\left((x+2)^2\right) + \log\left((x-7)^2\right) - \log\left((x+2)(x-7)\right) \\ &= \log\left(\frac{(x+2)^2(x-7)^2}{(x+2)(x-7)}\right) \\ &= \log\left((x+2)(x-7)\right) \\ &= \log\left(x^2 - 5x - 14\right) \end{aligned}$$

PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms

LOC: 12.RF8 TOP: Relations and Functions

KEY: Procedural Knowledge | Conceptual Understanding | Problem-Solving Skills | Communication

8. ANS:

$$\begin{aligned} & \log(2x^2 + 3x + 1) - \log(x^2 - 1) \\ &= \log\left(\frac{2x^2 + 3x + 1}{x^2 - 1}\right) \\ &= \log\left(\frac{(2x + 1)(x + 1)}{(x + 1)(x - 1)}\right) \\ &= \log\left(\frac{2x + 1}{x - 1}\right) \end{aligned}$$

PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms

LOC: 12.RF8 TOP: Relations and Functions

KEY: Procedural Knowledge | Conceptual Understanding | Problem-Solving Skills | Communication

9. ANS:

$$\begin{aligned} \log_2(x - 8) + \log_2(x + 1) &= 3 \\ x > 8 \text{ and } x > -1, \text{ so } x > 8 \end{aligned}$$

$$\log_2(x - 8) + \log_2(x + 1) = 3$$

$$\log_2((x - 8)(x + 1)) = \log_2(2^3)$$

$$(x - 8)(x + 1) = 2^3$$

$$x^2 - 7x - 8 = 8$$

$$x^2 - 7x - 16 = 0$$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 1$, $b = -7$, $c = -16$.

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(-16)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{113}}{2}$$

Since $x > 8$, $x = \frac{7 - \sqrt{113}}{2}$ is extraneous.The solution is $x = \frac{7 + \sqrt{113}}{2}$.

PTS: 1 DIF: Difficult REF: 5.7 Solving Logarithmic and Exponential Equations

LOC: 12.RF10 TOP: Relations and Functions

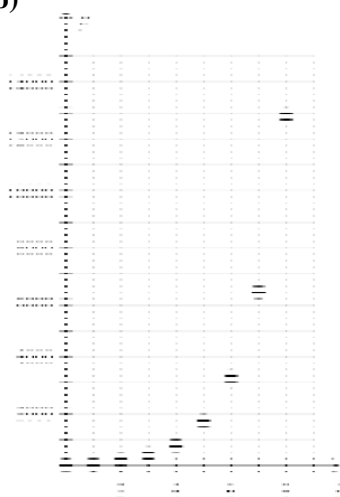
KEY: Procedural Knowledge | Conceptual Understanding

10. ANS:

a)

Time, t (weeks)	Population, P
0	50
1	100
2	200
3	400
4	800
5	1 600
6	3 200
7	6 400
8	12 800

b)



c) The data seem to be exponential, since the graph increases at an increasing rate. The values for population are being multiplied by a factor of 2 between successive terms in the table of values.

d) $P = 50(2)^t$

PTS: 1 DIF: Average OBJ: Section 7.1 NAT: RF9

TOP: Characteristics of Exponential Functions

KEY: graph | modelling | exponential growth

11. ANS:

$$\mathbf{a)} A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$$

$$\mathbf{b)} A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$$

$$= 72\left(\frac{1}{2}\right)^{\frac{20}{10}}$$

$$= 72\left(\frac{1}{2}\right)^2$$

$$= 18$$

There will be 18 mg remaining after 20 days.

$$\mathbf{c)} A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$$

$$= 72\left(\frac{1}{2}\right)^{\frac{-30}{10}}$$

$$= 72\left(\frac{1}{2}\right)^{-3}$$

$$= 576$$

There was 576 mg 30 days ago.

$$\mathbf{d)} A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$$

$$0.07 = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$$

$$\frac{0.07}{72} = \left(\frac{1}{2}\right)^{\frac{t}{10}}$$

Use systematic trial.

$$\frac{0.07}{72} \doteq 0.000972$$

$$\text{For } t = 100, \left(\frac{1}{2}\right)^{10} \doteq 0.000977.$$

It will take approximately 100 days for there to be 0.07 mg remaining.

PTS: 1

DIF: Average

OBJ: Section 7.2 | Section 7.3

NAT: RF9 | RF10

TOP: Transformations of Exponential Functions | Solving Exponential Equations

KEY: modelling | evaluate exponential functions

12. ANS:

Answers may vary. Sample answers:

a) $y = \frac{1}{2}(4)^{2x} + 1$

b) $y = 4^x + 0.5$; $y = -4^x + 2.5$

c) Either graph the equations to show that they all contain the given point or substitute $x = 0$ into each function to show that the result is the value $y = \frac{3}{2}$.

PTS: 1 DIF: Difficult OBJ: Section 7.2 NAT: RF9

TOP: Transformations of Exponential Functions

KEY: transformations of exponential functions | equation

13. ANS:

$$2^{3x} = 2^2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

PTS: 1 DIF: Easy OBJ: Section 7.3 NAT: RF10

TOP: Solving Exponential Equations

KEY: exponential equation | change of base

14. ANS:

1. vertically stretched by a factor of 3

2. horizontally compressed by a factor of $\frac{1}{2}$,

3. reflected in both the x - and y -axes

4. translated 1 unit right and 4 units up

PTS: 1 DIF: Average OBJ: Section 8.2 NAT: RF8

TOP: Transformations of Logarithmic Functions

KEY: transformations | logarithmic functions

15. ANS:

$$\log 28 = \log(7 \times 2^2)$$

$$= \log 7 + 2 \log 2$$

$$\approx 0.8451 + 2(0.3010)$$

$$= 1.4471$$

PTS: 1 DIF: Difficult OBJ: Section 8.3 NAT: RF9

TOP: Laws of Logarithms

KEY: power law of logarithms

16. ANS:

$$\begin{aligned} \text{L. S.} &= \log a + \log a^2 + \log a^3 - \log a^6 \\ &= \log a + 2 \log a + 3 \log a - 6 \log a \\ &= (1 + 2 + 3 - 6) \log a \\ &= 0 \log a \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{R. S.} &= \log 1 \\ &= 0 \end{aligned}$$

L.S. = R.S.Thus, $\log 5 + \log 5^2 + \log 5^3 - \log 5^6 = \log 1$.

PTS: 1

DIF: Average

OBJ: Section 8.3 NAT: RF9

TOP: Laws of Logarithms

KEY: power law | laws of logarithms

17. ANS:

$$\log \sqrt[3]{x^2 + 48x} = \frac{2}{3}$$

$$\log(x^2 + 48x)^{\frac{1}{3}} = \frac{2}{3}$$

$$\frac{1}{3} \log(x^2 + 48x) = \frac{2}{3}$$

$$\log(x^2 + 48x) = 2$$

$$\log(x^2 + 48x) = \log 100$$

$$x^2 + 48x = 100$$

$$x^2 + 48x - 100 = 0$$

$$(x + 50)(x - 2) = 0$$

$$x = -50 \text{ or } x = 2$$

Check the values for extraneous roots.

In this case, both values are possible and solve the equation, so they are both valid.

PTS: 1

DIF: Difficult

OBJ: Section 8.4 NAT: RF10

TOP: Logarithmic and Exponential Equations

KEY: logarithmic equation | extraneous roots