Excel Spreadsheet in Mechanical Engineering Technology Education

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Abstract:

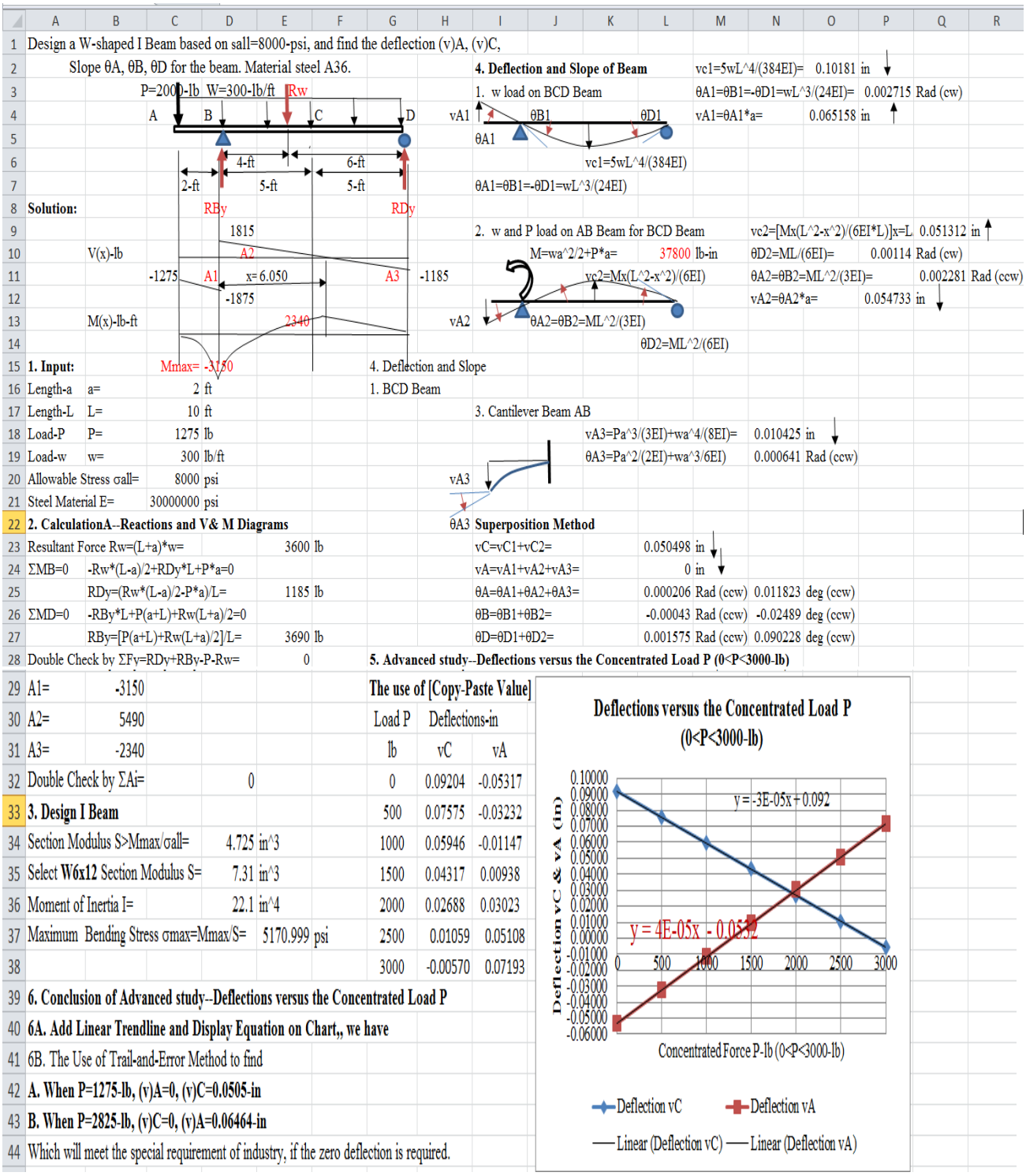
In the last three decades Excel Spreadsheet has become a very popular and effective computational tool for performing engineering calculations. It is a great challenge on educators to apply this tool towards improving our engineering teaching and to provide high quality, learning-center education. Using spreadsheets provide a unique learning experience on the relationship between the component of an equation—an understanding that is essential in engineering analysis. However, the traditional teaching method and manual computation of equations and modelling do not always prove to be effective. Excel Spreadsheet has been successfully been used to promote conceptual change in mechanical system design and analysis. In Excel Spreadsheet Student can perform alternative design and analysis. Student can better understand and interpret the solution using fundamental theoretical and numerical concepts. In this paper, the author is going to introduce his experience how to teach the courses in mechanical engineering technology at RIT using Excel spreadsheets. The case study in engineering mechanics, vibration, machine design, and others will be discussed in this paper.

The case study in this paper is listing below.

Case Study 1. Strength of Materials for Beam (Shaft) Design and Analysis
Case Study 2. Strength of Materials for Combined Stress in unsymmetrical Bending
Case Study 3. Strength of Materials for Combined Stress in column with eccentric load.
Case Study 4. Strength of Materials for Combined Stress in I Beam to find the bending stresses in flange and web of I beam
Case Study 5. Damping Vibration analytical solution
Case Study 6. Gear Box kinematic and shaft design in machine design Case Study
Case Study 7. Numerical Integration for Forced Vibration witn Damping in Spring-Mass System.
Case Study 8. Numerical Differentiation for linkage analysis in Dynamics of Machinery.
Case Study 9. Jet Engine Thermodynamic analysis
Case Study 10. Long-hand-calculation of Stiffness Matrix for two dimensional triangular three-node-element in CAE study.

Case Study 1. Strength of Materials for Beam (Shaft) Design and Analysis

First, create an Input in excel. Which will drive all your calculation and create alternative design and solution by change the input data only. For example, if the value of concentrated force P or uniformly distributed load w change, the alternative solution can be found immediately in excel. Also, you can find the solution of stresses and deflection as functions of load P or w.

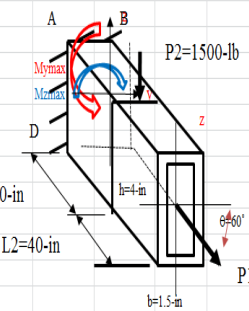


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Design a W-shaped I Beam based on sall=8000-psi, and find the deflection (v)A, (v)C,																	
2	Slope θA, θB, θD for the beam. Material steel A36.								**4. Deflection and Slope of Beam**				vc1=5wL^4/(384EI)=		0.10181	in		
3			P=2000-lb	W=300-lb/ft	Rw				1. w load on BCD Beam				θA1=θB1=-θD1=wL^3/(24EI)=		0.002715	Rad (cw)		
4			A	B		C		D	vA1	θB1		θD1	vA1=θA1*a=		0.065158	in		
5									θA1									
6					4-ft	6-ft					vc1=5wL^4/(384EI)							
7			2-ft		5-ft	5-ft			θA1=θB1=-θD1=wL^3/(24EI)									
8	**Solution:**			RBy			RDy											
9				1815					2. w and P load on AB Beam for BCD Beam					vc2=[Mx(L^2-x^2)/(6EI*L)]x=L		0.051312	in	
10		V(x)-lb		A2						M=wa^2/2+P*a=		37800	lb-in	θD2=ML/(6EI)=	0.00114	Rad (cw)		
11			-1275	A1	x= 6.050		A3	-1185			vc2=Mx(L^2-x^2)/(6EI)			θA2=θB2=ML^2/(3EI)=		0.002281	Rad (ccw)	
12				-1875										vA2=θA2*a=		0.054733	in	
13		M(x)-lb-ft				2340			vA2	θA2=θB2=ML^2/(3EI)								
14												θD2=ML^2/(6EI)						
15	**1. Input:**			Mmax= -3150			4. Deflection and Slope											
16	Length-a	a=	2	ft			1. BCD Beam											
17	Length-L	L=	10	ft					3. Cantilever Beam AB									
18	Load-P	P=	1275	lb							vA3=Pa^3/(3EI)+wa^4/(8EI)=		0.010425	in				
19	Load-w	w=	300	lb/ft							θA3=Pa^2/(2EI)+wa^3/6EI)		0.000641	Rad (ccw)				
20	Allowable Stress σall=		8000	psi					vA3									
21	Steel Material E=		30000000	psi														
22	**2. CalculationA--Reactions and V& M Diagrams**								θA3	**Superposition Method**								
23	Resultant Force Rw=(L+a)*w=				3600	lb				vC=vC1+vC2=		0.050498	in					
24	ΣMB=0	-Rw*(L-a)/2+RDy*L+P*a=0								vA=vA1+vA2+vA3=		0	in					
25		RDy=(Rw*(L-a)/2-P*a)/L=			1185	lb				θA=θA1+θA2+θA3=		0.000206	Rad (ccw)	0.011823	deg (ccw)			
26	ΣMD=0	-RBy*L+P(a+L)+Rw(L+a)/2=0								θB=θB1+θB2=		-0.00043	Rad (ccw)	-0.02489	deg (ccw)			
27		RBy=[P(a+L)+Rw(L+a)/2]/L=			3690	lb				θD=θD1+θD2=		0.001575	Rad (ccw)	0.090228	deg (ccw)			
28	Double Check by ΣFy=RDy+RBy-P-Rw=				0		**5. Advanced study--Deflections versus the Concentrated Load P (0<P<3000-lb)**											
29	A1=		-3150				**The use of [Copy-Paste Value]**											
30	A2=		5490				Load P	Deflections-in										
31	A3=		-2340				lb	vC	vA									
32	Double Check by ΣAi=			0			0	0.09204	-0.05317									
33	**3. Design I Beam**						500	0.07575	-0.03232									
34	Section Modulus S>Mmax/σall=			4.725	in^3		1000	0.05946	-0.01147									
35	Select **W6x12** Section Modulus S=			7.31	in^3		1500	0.04317	0.00938									
36	Moment of Inertia I=			22.1	in^4		2000	0.02688	0.03023									
37	Maximum Bending Stress σmax=Mmax/S=				5170.999	psi	2500	0.01059	0.05108									
38							3000	-0.00570	0.07193									
39	**6. Conclusion of Advanced study--Deflections versus the Concentrated Load P**																	
40	**6A. Add Linear Trendline and Display Equation on Chart,, we have**																	
41	6B. The Use of Trail-and-Error Method to find																	
42	**A. When P=1275-lb, (v)A=0, (v)C=0.0505-in**																	
43	**B. When P=2825-lb, (v)C=0, (v)A=0.06464-in**																	
44	Which will meet the special requirement of industry, if the zero deflection is required.																	

Case Study 2. Strength of Materials for unsymmetrical Bending.

Which is the combination of vertical and horizontal bending. With superposition method, the stresses and deflection could be solved. The stress distribution at the fixed end provide the dangerous stresses for the beam. The combined deflection due to horizontal and vertical bending is solved as well.

Find the stresses at the four corner of the fixed end for a cantilever beam with hollow rectangular cross section h=4-in, b=1.5-in, thickness t=0.25-in.



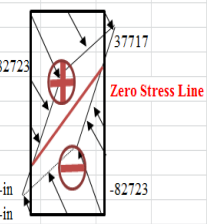
Input:		
Load P1=	500	lb
θ=	60	deg
Load P2=	1500	lb
Length L1=	60	in
Length L2=	40	in
Height h=	4	in
Width b=	1.5	in

5. Combined Stresses

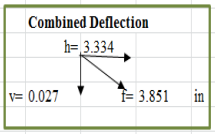
psi	A	B	C	D
σ1	60220	60220	-60220	-60220
σ2	22503	-22503	-22503	22503
Σσ=σ1+σ2	82723	37717	-82723	-37717

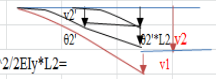
6. Stress Distribution at the Fixed End

Thickness t=	0.25	in		
Material (steel) E=	30000000	psi		
Calculation:				
1. Force Components	P2=P2z=	1500	lb	
	P1z=P1sinθ=	433	lb	
	P1y=P1cosθ=	250	lb	
2. Max. Beading Moment				
Vertical Bending	Mymax=P2z*L1+P1z(L1+L2)=	133299	lb-in	
Horizontal Bending	Mzmax=P1y*(L1+L2)=	25003	lb-in	
3. Moment of Inertia and Section Modulus				
Iy=[bh^3-(b-2t)(h-2t)^3]/12=		4.427	in^4	
Iz=[hb^3-(h-2t)(b-2t)^3^3]/12=		0.833	in^4	
Sy=Iy/(h/2)=		2.214	in^3	
Sz=Iz/((b/2)=		1.111	in^3	
4. Max. Stresses at the Fixed End				
σ1=+/-	Mymax/Sy=+/-	60220	psi	
σ2=+/-	Mzmax/Sz=+/-	22503	psi	
7. Deflection at the Free End				
Vertical Bending	v=v1+v2=(v1)P1z+(v2)P2=v1+[v2'+(θ)2'*L2]=			
Vertical Bending	=P1z*(L1+L2)^3/3EIy+P2*L1^3/3EIy+P2*L1^2/2EIy*L2=			
=	1.08672858 + 0.813176 + 0.027 =			1.927 in
Horizontal Bending	h=h1=P1y*(L1+L2)^3/3EIz=		3.334	in
8. Combined Deflection at the Free End				
f=(v^2+h^2)^0.5=			3.851	in



Combined Deflection

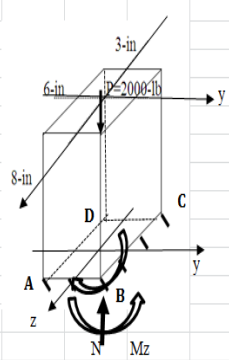


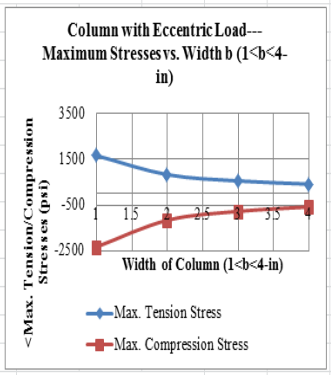


Case Study 3. Strength of column with eccentric load.

Where the stresses of two direction bending and axial compression are calculated separately, and then combined together. The summation of stresses provide the actual stress distribution for column with eccentric load. With the power of Excel, we find the solution of stresses as the function of column width b, which is changed from 1 to 4-in.Both tabular solution and chart form solution are provided.

Find the stress distribution in the column with eccentric load.





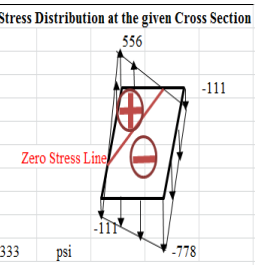
Input Data:		
Load P=	2000	lb
Height h=	6	in
Width b=	3	in
Length L=	8	in
Calculation:		

4. Superposition Method

psi	A	B	C	D
σ1	-111	-111	-111	-111
σ2	-333	-333	333	333
σ3	333	-333	-333	333
Σσ=	-111	-778	-111	556

1. internal Reaction			
Axial Load N= -P=	-2000	lb	
Bending Moment My=P*h/2=	6000	lb-in	
Bending Moment Mz=P*b/2=	3000	lb-in	
2. Cross Section Area and Section Modulus			
A=bh=	18	in^2	
Sy=bh^2/6=	18	in^3	
Sz=hb^2/6=	9	in^3	
3. Combined Stresses			
σ=σ1+σ2+σ3=N/A+/-My/Sy+/-Mz/Sz			
= -111 +/-	333 +/-	333	psi

5. Stress Distribution at the given Cross Section



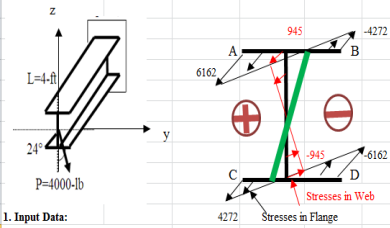
4. Advanced study (The use of [Copy-Paste Value], to find the stresses in Column vs. Width b)

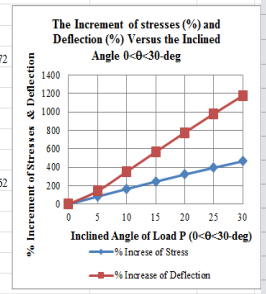
Column Stresses versus width b (1-in<b<4-in)

b=1-in					**b=2-in**				
psi	A	B	C	D	psi	A	B	C	D
σ1	-333	-333	-333	-333	σ1	-167	-167	-167	-167
σ2	-1000	-1000	1000	1000	σ2	-500	-500	500	500
σ3	1000	-1000	-1000	1000	σ3	500	-500	-500	500
Σσ=	-333	-2333	-333	1667	Σσ=	-167	-1167	-167	833
b=3-in					**b=4-in**				
psi	A	B	C	D	psi	A	B	C	D
σ1	-111	-111	-111	-111	σ1	-83	-83	-83	-83
σ2	-333	-333	333	333	σ2	-250	-250	250	250
σ3	333	-333	-333	333	σ3	250	-250	-250	250
Σσ=	-111	-778	-111	556	Σσ=	-83	-583	-83	417

Case Study 4. Strength of Materials for Combined Stress in I Beam to find the bending stresses in flange and web of I beam at dangerous cross section.

Find the maximum stresses and stress distribution in cantilever beam with
Unsymmetrical bending. The I beam is w-shaped (W24x76)





1. Input Data:		
Ibeam W24x76 Section Modulus		
Sy=	176 in^3	
Sz=	18.4 in^3	
Ibeam W24x76 Moment of Inertia		
Iy=	2100 in^4	
Iz=	82.5 in^4	
Load P=	4000 lb	
angle θ=	30 deg	
Length L=	48 in	
Steel Modulus of Elasticity		
E=	30000000 psi	
2. Calculation		

G. Advanced Study

The Increment of stresses (%) and Deflection (%) Versus the Inclined Angle θ

angle θ (deg)	Stresses	Deflection
0	0	0
5	83	143
10	165	353
15	244	566
20	321	776
25	395	979
30	465	1176

* Change Value of θ, and use Copy-Paste value of cell D45 and D46

A. Bending Moment	Vertical	Horizontal		
	Bending	Bending		
	My=P*cosθ*L	My=P*cosθ*L		
=	166281	95994	lb-in	
B. Bending Stress	σ1=+/-My/Sy	σ2=+/-Mz/Sz		
=	945	5217	psi	
C. Combined Stresses				
	A	B	C	D
σ1-psi	945	945	-945	-945
σ2–psi	5217	-5217	5217	-5217
Σσ=(σ1+σ2)-psi	6162	-4272	4272	-6162
D. Deflection				
v=P*cosθ*L^3/(3EIy)=	0.0020	in		
h=P*sinθ*L^3/(3EIz)=	0.0298	in		
Combined Deflection				
f=(v^2+h^2)^0.5=	0.0299	in		
E. Set θ=0 for Vertical Bending only, we have				
f=v=	0.0023	in		
σ=σ1=	1091	psi		
where the maximum bending stress increases			465	%
where the deflection at the free end increases			1176	%

Case Study 5. Damping Vibration analytical solution.

By solving differential equation of damping vibration, the general solution of underdamping vibration of a spring-block system could be found. In Excel, the analytical solution could be solved in both tabular and chart form. By changing the value of damping factor C, the deduction of vibration amplitude as a function of time could be solved and to meet the requirement of industry. In this analysis, the time step 0.02-sec is selected.

Damping Vibration in Microsoft Excel with Analytical Solution							
Input Data:					**g=**	**386.4**	**in/s^2**
Stiffness of Spring	**k=**	**30**	**lb/in**		**e=**	**2.7183**	
Weight of Block	**W=**	**10**	**lb**		**Diff Eq. Damping Vibration**		
Initial Displ.	**x0=**	**0.4**	**in**				
Initial Vel.	**v0=**	**5**	**in/sec**				
PreCalculation for Underdamping:		**Forlulas**	**Results**				
Natural Frequency	**wn=**	**(k/m)^0.5=(k*g/W)^0.5**	**34.047**	**rad/sec**			
Critical Damping Coeff. Ccr=	**Ccr=**	**2m*wn=2W/g*wn=**	**1.764**	**lb/in/sec**			
Damping Factor for Underdamping (Given)		**C=**	**0.120**	**lb/in/sec**		**(1-p^2)^0.5=**	**0.99768**
Ratio Zeta=C/Ccr=	**Zeta=p**	**C/Ccr=**	**0.068**				
Frequency for Damping Vibration	**wd=**	**wn*(1-p^2)^0.5=**	**33.968**	**rad/sec**			
	Zeta*wn=p*wn=		**2.316**	**rad/sec**			
Factor1	**K=((v0+p*wn*x0)/wd)=**		**0.174**				
Period	**T=**	**(2*3.1416)/wd=**	**0.185**	**sec**			
Time Step dt=			**0.020**	**sec**			
Underdamping C=0.12							

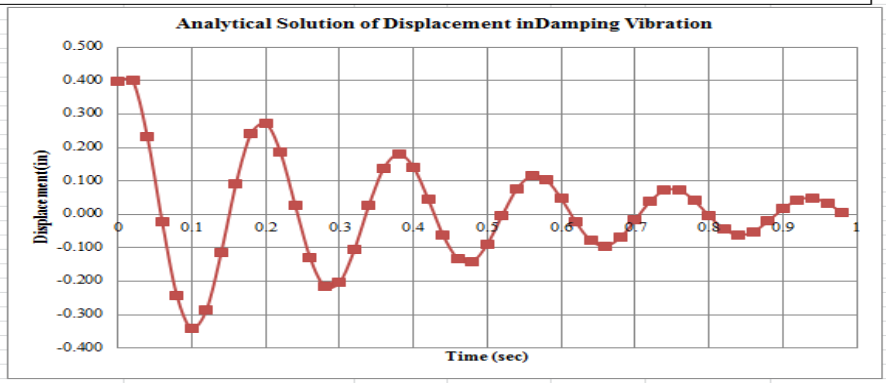
$$x_1(t) = e^{-\varsigma.\omega.t}\left(\frac{v(0)+\varsigma\omega_n x(0)}{\omega_n\sqrt{1-\varsigma^2}}\sin\sqrt{1-\varsigma^2}\,\omega_n t + x(0)\cos\sqrt{1-\varsigma^2}\,\omega_n t\right)$$

$$= e^{-\varsigma * w_n * t}(K * \sin\omega_d t + x(0)\cos\omega_d t)$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

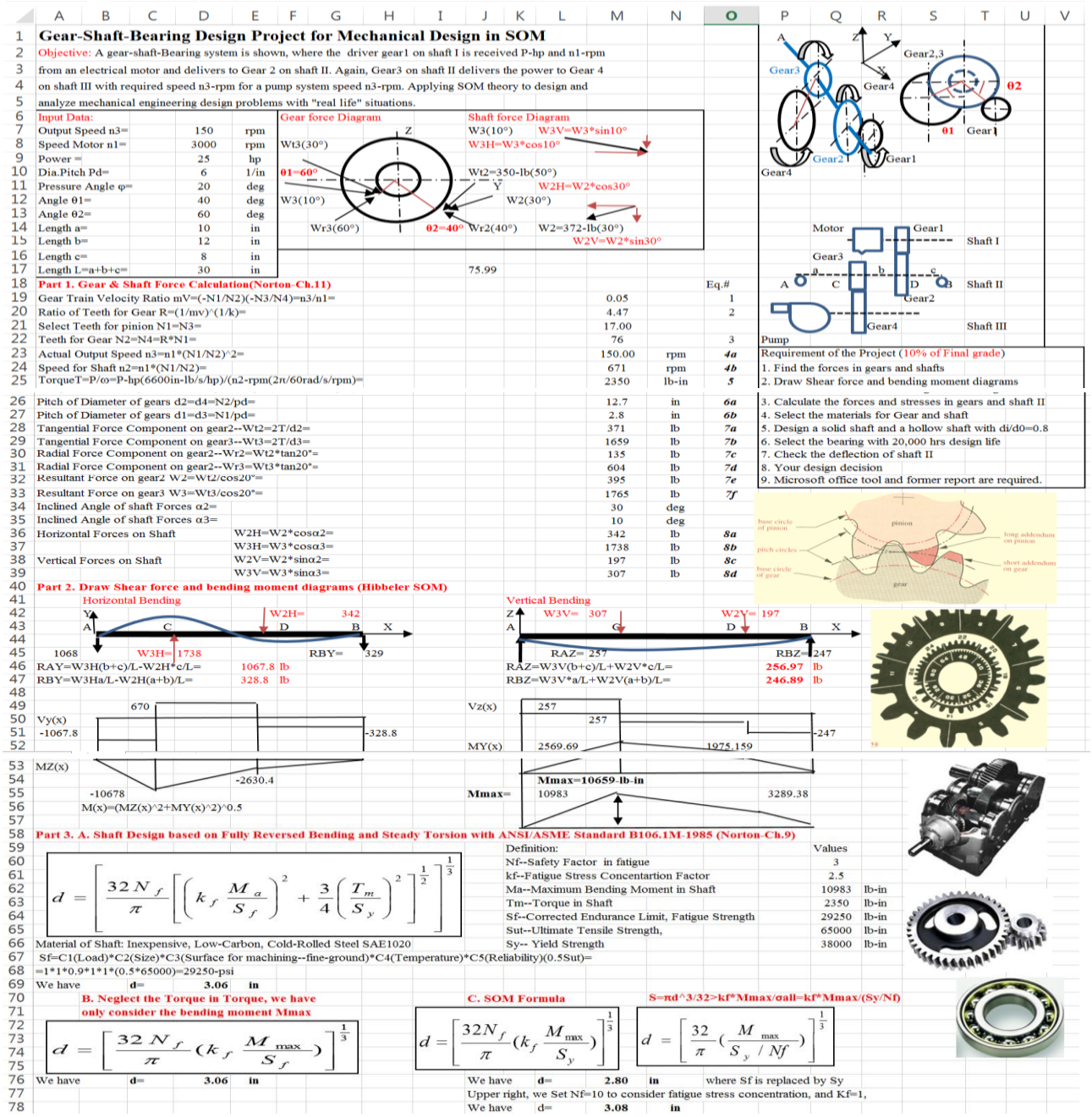
Underdamping

Time t (sec)	Amplitude e^(-pWn*t)	Displ. 1 X1(t)
0	1	0.400
0.020	0.9547361	0.402
0.040	0.91152102	0.232
0.060	0.87026203	-0.021
0.080	0.83087058	-0.243
0.100	0.79326214	-0.342
0.120	0.757356	-0.286
0.140	0.72307512	-0.114
0.160	0.69034592	0.092
0.180	0.65909817	0.241
0.200	0.62926482	0.273
0.220	0.60078184	0.187
0.240	0.57358811	0.028
0.260	0.54762528	-0.128
0.280	0.52283762	-0.216
0.300	0.49917195	-0.204
0.320	0.47657749	-0.106
0.340	0.45500573	0.028
0.360	0.4344104	0.139
0.380	0.41474729	0.181
0.400	0.39597421	0.142
0.420	0.37805087	0.046
0.440	0.36093882	-0.061
0.460	0.34460132	-0.132
0.480	0.32900332	-0.141
0.500	0.31411135	-0.089



Case Study 6. Gear Box kinematic and shaft design in machine design

Which involved 1. The kinematic design of gear box, 2. The three dimensional forces and stresses analysis of the shaft in gear box based on Formulas of American Society of mechanical engineers (ASME) and American National Standard Institute (ANSI). Based on reversed bending and steady torsion for both solid and hollow shaft. 3. The ball bearing selection to find the life of ball bearing Ld. 4. The spur gear design and analysis based on bending stress from AGMA Standard 908-B89 formula.



Gear-Shaft-Bearing Design Project for Mechanical Design in SOM

Objective: A gear-shaft-Bearing system is shown, where the driver gear1 on shaft I is received P-hp and n1-rpm from an electrical motor and delivers to Gear 2 on shaft II. Again, Gear3 on shaft II delivers the power to Gear 4 on shaft III with required speed n3-rpm for a pump system speed n3-rpm. Applying SOM theory to design and analyze mechanical engineering design problems with "real life" situations.

Input Data:		
Output Speed n3=	150	rpm
Speed Motor n1=	3000	rpm
Power =	25	hp
Dia.Pitch Pd=	6	1/in
Pressure Angle φ=	20	deg
Angle θ1=	40	deg
Angle θ2=	60	deg
Length a=	10	in
Length b=	12	in
Length c=	8	in
Length L=a+b+c=	30	in

Gear force Diagram: Wt3(30°), θ1=60°, W3(10°), Wr3(60°), θ2=40°, Wr2(40°), Wt2=350-lb(50°), W2H=W2*cos30°, W2(30°), W2=372-lb(30°), W2V=W2*sin30°

Shaft force Diagram: W3(10°), W3V=W3*sin10°, W3H=W3*cos10°

75.99

Part 1. Gear & Shaft Force Calculation(Norton-Ch.11)			Eq.#
Gear Train Velocity Ratio mV=(-N1/N2)(-N3/N4)=n3/n1=	0.05		1
Ratio of Teeth for Gear R=(1/mv)^(1/k)=	4.47		2
Select Teeth for pinion N1=N3=	17.00		
Teeth for Gear N2=N4=R*N1=	76		3
Actual Output Speed n3=n1*(N1/N2)^2=	150.00	rpm	*4a*
Speed for Shaft n2=n1*(N1/N2)=	671	rpm	*4b*
TorqueT=P/ω=P-hp(6600in-lb/s/hp)/(n2-rpm(2π/60rad/s/rpm)=	2350	lb-in	*5*
Pitch of Diameter of gears d2=d4=N2/pd=	12.7	in	*6a*
Pitch of Diameter of gears d1=d3=N1/pd=	2.8	in	*6b*
Tangential Force Component on gear2--Wt2=2T/d2=	371	lb	*7a*
Tangential Force Component on gear3--Wt3=2T/d3=	1659	lb	*7b*
Radial Force Component on gear2--Wr2=Wt2*tan20°=	135	lb	*7c*
Radial Force Component on gear2--Wr3=Wt3*tan20°=	604	lb	*7d*
Resultant Force on gear2 W2=Wt2/cos20°=	395	lb	*7e*
Resultant Force on gear3 W3=Wt3/cos20°=	1765	lb	*7f*
Inclined Angle of shaft Forces α2=	30	deg	
Inclined Angle of shaft Forces α3=	10	deg	
Horizontal Forces on Shaft W2H=W2*cosα2=	342	lb	*8a*
W3H=W3*cosα3=	1738	lb	*8b*
Vertical Forces on Shaft W2V=W2*sinα2=	197	lb	*8c*
W3V=W3*sinα3=	307	lb	*8d*

Requirement of the Project (10% of Final grade)
1. Find the forces in gears and shafts
2. Draw Shear force and bending moment diagrams
3. Calculate the forces and stresses in gears and shaft II
4. Select the materials for Gear and shaft
5. Design a solid shaft and a hollow shaft with di/d0=0.8
6. Select the bearing with 20,000 hrs design life
7. Check the deflection of shaft II
8. Your design decision
9. Microsoft office tool and former report are required.

Part 2. Draw Shear force and bending moment diagrams (Hibbeler SOM)

Horizontal Bending: W2H= 342, W3H= 1738, 1068, RBY= 329
RAY=W3H(b+c)/L-W2H*c/L= 1067.8 lb
RBY=W3Ha/L-W2H(a+b)/L= 328.8 lb

Vertical Bending: W3V= 307, W2V= 197, RAZ= 257, RBZ= 247
RAZ=W3V(b+c)/L+W2V*c/L= 256.97 lb
RBZ=W3V*a/L+W2V(a+b)/L= 246.89 lb

Vy(x): 670, -1067.8, -328.8
Vz(x): 257, 257, -247
MY(x): 2569.69, 1975.159
MZ(x): -10678, -2630.4
M(x)=(MZ(x)^2+MY(x)^2)^0.5
Mmax=10659-lb-in
Mmax= 10983, 3289.38

Part 3. A. Shaft Design based on Fully Reversed Bending and Steady Torsion with ANSI/ASME Standard B106.1M-1985 (Norton-Ch.9)

$$d = \left[\frac{32N_f}{\pi}\left[\left(k_f\frac{M_a}{S_f}\right)^2 + \frac{3}{4}\left(\frac{T_m}{S_y}\right)^2\right]^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

Definition:	Values	
Nf--Safety Factor in fatigue	3	
kf--Fatigue Stress Concentartion Factor	2.5	
Ma--Maximum Bending Moment in Shaft	10983	lb-in
Tm--Torque in Shaft	2350	lb-in
Sf--Corrected Endurance Limit, Fatigue Strength	29250	lb-in
Sut--Ultimate Tensile Strength,	65000	lb-in
Sy-- Yield Strength	38000	lb-in

Material of Shaft: Inexpensive, Low-Carbon, Cold-Rolled Steel SAE1020
Sf=C1(Load)*C2(Size)*C3(Surface for machining--fine-ground)*C4(Temperature)*C5(Reliability)(0.5Sut)=
=1*1*0.9*1*1*(0.5*65000)=29250-psi
We have d= 3.06 in

B. Neglect the Torque in Torque, we have only consider the bending moment Mmax

$$d = \left[\frac{32N_f}{\pi}\left(k_f\frac{M_{max}}{S_f}\right)\right]^{\frac{1}{3}}$$

We have d= 3.06 in

C. SOM Formula

$$d = \left[\frac{32N_f}{\pi}\left(k_f\frac{M_{max}}{S_y}\right)\right]^{\frac{1}{3}}$$

We have d= 2.80 in

S=πd^3/32>kf*Mmax/σall=kf*Mmax/(Sy/Nf)

$$d = \left[\frac{32}{\pi}\left(\frac{M_{max}}{S_y/Nf}\right)\right]^{\frac{1}{3}}$$

where Sf is replaced by Sy
Upper right, we Set Nf=10 to consider fatigue stress concentration, and Kf=1,
We have d= 3.08 in

The corrected Formula for Hollow Shaft is **Set α=** **0.8**

$$d_o = \left[\frac{32 N_f}{\pi (1-\alpha^4)} \left[\left(k_f \frac{M_a}{S_f} \right)^2 + \frac{3}{4} \left(\frac{T_m}{S_y} \right)^2 \right]^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Outer Diameter d0= **3.647** **in**
Inner Diameter di=α*d0= **2.917** **in**

Select d= **3.2** **in** **for Solid Shaft**
and d0= **3.75** **di=** **3** **in** **for Hollow Shaft**

Part 4. Bearing Selection--Select 6300 Series Ball Bearing for A only, which has the max. reaction force (Norton Ch.10)

RA=(RAY^2+RAZ^2)^0.5= =(1068^2+257^2)^0.5= 1098 lb

Fatigue Life L(in millions of revolutions) is inversely proportional to the third power of the load for ball bearings

Solid Shaft--Bearing Number #6314	Hollow Shaft--Bearing Number #6317
Bore Diameter db=2.7559-in, Dynamic Load Rating C=18000-lb	Bore Diameter db=3.3465-in (70mm), Dynamic Load Rating C=21600-lb
For Bearing A: L=(C/P)^3=(18000/1098)^3=4406e6 Revs	For Bearing A: L=(C/P)^3=(21600/1098)^3=7613e6 Revs
Pumps works 24 hours a day, 24-hrs*60mins*671-rpm=0.966e6 Revs/day,	Ld=7881-days=263-months=22-years
We have the life for Bearing A	
Ld=4561-days=152-months=12.7-years	

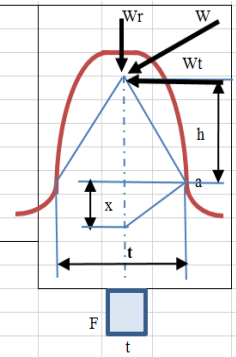
BEARING NUMBER*	BOUNDARY DIMENSIONS BORE mm	BORE inch	O. DIAM. mm	O. DIAM. inch	WIDTH mm	WIDTH inch	SNAP RING DIMENSIONS inches H	S	t	MAX. FILLET RADIUS Shaft & Hsg. inch	APPROX. WEIGHT lb.	S_u LIMITING SPEED r rpm	C DYNAMIC LOAD RATING lb.	C_o STATIC LOAD RATING lb.
6300	10	.3937	35	1.3780	11	.4331	.125	1.562	.044	.025	.13	22000	1400	850
6301	12	.4724	37	1.4567	12	.4724	.125	1.625	.044	.040	.15	20000	1700	1040
6302	15	.5906	42	1.6535	13	.5118	.125	1.821	.044	.040	.20	18000	1930	1200
6303	17	.6693	47	1.8504	14	.5512	.141	2.074	.044	.040	.25	16000	2320	1460
6304	20	.7874	52	2.0472	15	.5906	.141	2.276	.044	.040	.34	14000	3000	1930
6305	25	.9843	62	2.4409	17	.6693	.195	2.665	.067	.040	.58	11000	3800	2550
6306	30	1.1811	72	2.8346	19	.7480	.195	3.091	.067	.040	.83	9500	5000	3400
6307	35	1.3780	80	3.1496	21	.8268	.195	3.406	.067	.060	1.07	8500	5700	4000
6308	40	1.5748	90	3.5433	23	.9055	.226	3.799	.097	.060	1.41	7500	7350	5300
6309	45	1.7717	100	3.9370	25	.9843	.226	4.193	.097	.060	1.95	6700	9150	6700
6310	50	1.9685	110	4.3307	27	1.0630	.226	4.587	.097	.080	2.50	6000	10600	8150
6311	55	2.1654	120	4.7244	29	1.1417	.271	5.104	.111	.080	3.30	5300	12900	10000
6312	60	2.3622	130	5.1181	31	1.2205	.271	5.498	.111	.080	3.81	5000	14000	10800
6313	65	2.5591	140	5.5118	33	1.2992	.304	5.892	.111	.080	4.64	4500	16000	12500
6314	70	2.7559	150	5.9055	35	1.3780	.304	6.286	.111	.080	5.68	4300	18000	14000
6315	75	2.9528	160	6.2992	37	1.4567	.304	6.679	.111	.080	6.60	4000	19300	16300
6316	80	3.1496	170	6.6929	39	1.5354	.346	7.198	.122	.080	9.53	3800	21200	18000
6317	85	3.3465	180	7.0866	41	1.6142	.346	7.593	.122	.100	11.00	3400	21600	18600
6318	90	3.5433	190	7.4803	43	1.6929	.346	7.986	.122	.100	11.60	3400	23200	20000
6319	95	3.7402	200	7.8740	45	1.7717	.346	8.380	.122	.100	13.38	3200	24500	22400
6320	100	3.9370	215	8.4646	47	1.8504	—	—	—	.100	16.34	3000	28500	27000
6321	105	4.1338	225	8.8582	49	1.9291	—	—	—	.100	17.8	2800	30500	30000
6322	110	4.3307	240	9.4488	50	1.9685	—	—	—	.100	21.0	2600	32500	32500
6324	120	4.7244	260	10.2362	55	2.1654	—	—	—	.100	32.3	2400	36000	38000
6326	130	5.1181	280	11.0236	58	2.2835	—	—	—	.12	40.1	2200	39000	43000
6328	140	5.5118	300	11.8110	62	2.4409	—	—	—	.12	48.1	2000	44000	50000
6330	150	5.9055	320	12.5984	65	2.5590	—	—	—	.12	57.8	1900	49000	60000

Part 5. Spur Gear Bending Stresses

AGMA Standard 908-B89 replaced Y by a new geometry factor J, includes effects of stress concentration at root of gear tooth. Many K factors are modifiers to account the various conditions, includes dynamics, load distribution application, size, rim thickness, and idler factors. In first approximation, set the Product of k factors equals 1.

For Gear 3 has the highest value of Wt, we have
The bending geometry Factor for the 20°, 17-tooth pinion in mesh
with the 76-tooth gear found from Chart Y= 0.3 (Juvinall)P.479
Select the middle of the recommended range F=12/pd= 2
We have the bending stress at the low speed gear3
σ= 16588.29 psi

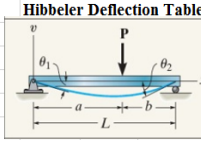
Formulation of Bending Stress (Lewis Eq.) in Spur Gear at the root of the gear. Point a, the bending stress
σ=Mc/I=Wt*h/s=6Wt*h/(Ft^2) (a)
where F--the width of tooth, t--the height of teeth
the range of width of tooth 8/pd<F<16/pd
With similar triangles, (t/2)/x=h/(t/2), or t^2/h=4x (b)
Sub. (b) into (a), we have
σ=6Wt/(4Fx)=Wt*pd/(F*Y)--Lewis Equation '(c)
where Wt is the tengential force on the tooth
pd-the diametral pitch, F-the width of tooth



Part 6. Check the Maximum Deflection of Shaft at section C for Shaft (Hibbeler SOM)

Where Elastic Modulus **E=** **3E+07 psi**
Solid Shaft Moment of Inertia I=πd^4/64= **I=** **5.1446 in^4**
Hollow Shaft Moment of Inertia I=πd0^4(1-α^4)/64= **5.7282 in^4**

Hibbeler Deflection Table for Simply Supported Beam with Concentrated Load



$$\theta_1 = \frac{-Pab(L+b)}{6EIL} \qquad \theta_2 = \frac{Pab(L+a)}{6EIL}$$

$$v\Big|_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$$

$$v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2) \qquad 0 \le x \le a$$

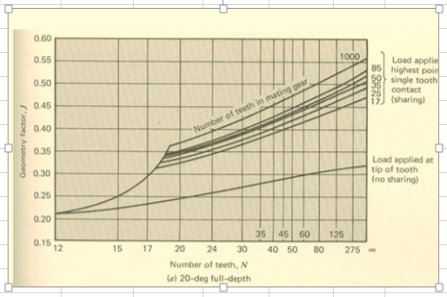
Formulas:

$$vc = \sqrt{(vcy^2 + vcz^2}$$

$$\theta = \tan^{-1}\left(\frac{vcy}{vcz}\right) * 57.3$$

Solid Shaft:
Deflection at C due to Horizontal Bending
vcy=(vcy)W3H+(vcy)W2H
=W3H*(b+c)a/(6EIL)*(L^2-(b+c)^2-a^2)-W2H(b+c)a/(6EIL)(L^2-(b+c)^2-a^2)
=(W3H-W2H)*((b+c)a)(L^2-(b+c)^2-a^2)/(6EIL)= **0.0040 in**
Deflection at C due to Vertical Bending
vcz=(vcz)W3V+(vcz)W2V
=W3V*(b+c)a/(6EIL)*(L^2-(b+c)^2-a^2)+W2V(b+c)a/(6EIL)(L^2-(b+c)^2-a^2)
=(W3V+W2V)*((b+c)a)(L^2-(b+c)^2-a^2)/(6EIL)
=(WV3+WV2)/(WH3-WH2)*C121= **0.0015 in**
Using Cartesian Vector Method to find the total Deflection and its derection at C
Hollow Shaft:
(vc)Hollow=(vc)Solid*(I)Solid/(I)Hollow= **0.0036 in**

vcy= 0.0040
θ
θ= 70.167 deg
vcz= 0.001451



Part 7. Discussion: (Student need answer all the questions given in this Discussion)

1. What is the relation between SOM and Mechanical Design/Analysis?
2. How to reduce the stress and deflection for shaft, gear?
3. How to increase the life of ball bearing?
4. Any suggestion of improvement for this project?
5. Your design decision

Case Study 7. Numerical Integration for Forced Vibration with Damping in Spring-Mass System. Driven by Input Data sheet with different input mass m, spring stiffness k, damping coefficient c, intial displacement x0 and initial velocity v0, external force magnitude F0, frequency w0, and the time step dt for numerical integration, the solution of forced vibration with damping can be solved in a few second for different system. Where the first equation is the differential equation of forced vibration with damping, the second equation is the solution for acceleration at the given time. Then, the velocity and displacement of system could be solved numerically. The use smaller time step will have better numerical solution of the system. The accuracy and convergence of the solution should be considered. In this example, time step dt=0.05-sec. The chart solution shows the displacement, velocity and acceleration solutions in the first 6 min. But the tabular solution shows only in the first 1.2 min due to the space of this paper.

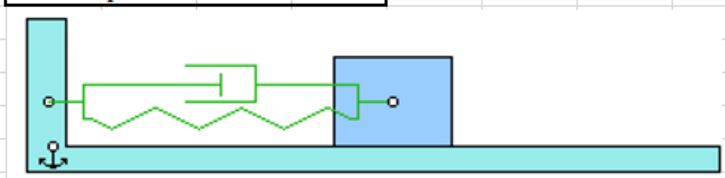
Force Vibration with Numerical Integration

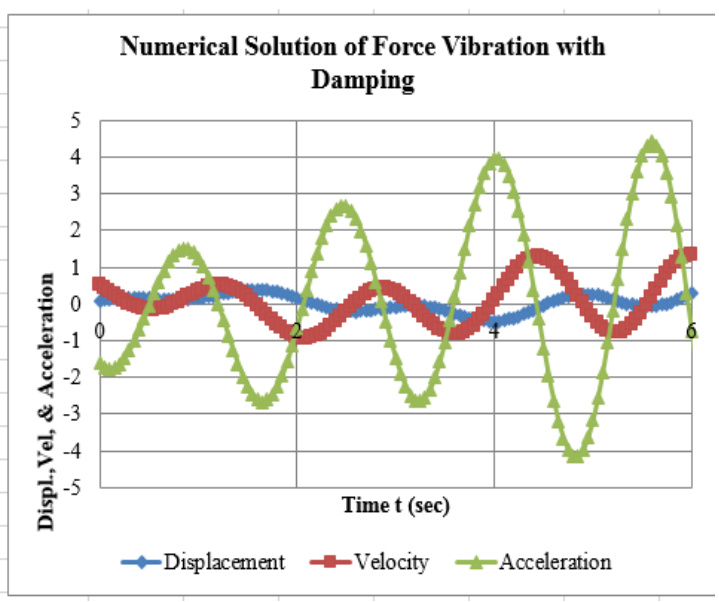
$$m\ddot{x} + c\dot{x} + kx = F_o \sin(\omega_0 t)$$

$$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x + F_0 \sin(\omega_0 t)$$

Input Data:		
Mass m=	10	kg
Spring k=	160	N/m
Damp c=	0.15	N/m/sec
Initial x0=	0.1	m
Initial vo=	0.5	m/sec
Time Step dt=	0.05	sec
Force F0=	4	N
Force Freq w0=	1.3	rad/sec

Time	Displ.	Velocity	Accel
t	x	v	a
sec	m	m/sec	m/sec^2
0	0.1	0.5	-1.6075
0.05	0.1229906	0.419625	-1.714327
0.1	0.141829	0.3339086	-1.755736
0.15	0.1563297	0.2461219	-1.729901
0.2	0.1664734	0.1596268	-1.637647
0.25	0.1724077	0.0777444	-1.482455
0.3	0.1744419	0.0036217	-1.270371
0.35	0.173035	-0.059897	-1.009812
0.4	0.1687779	-0.110387	-0.71127
0.45	0.1623694	-0.145951	-0.386924
0.5	0.1545882	-0.165297	-0.050187
0.55	0.1462606	-0.167806	0.2848166
0.6	0.1382263	-0.153566	0.6037998
0.65	0.1313028	-0.123376	0.8928903
0.7	0.1262501	-0.078731	1.1391938
0.75	0.1237376	-0.021771	1.3313331
0.8	0.1243132	0.0447952	1.4599344
0.85	0.1283778	0.1177919	1.5180445
0.9	0.136165	0.1936941	1.5014572
0.95	0.1477265	0.268767	1.408937
1	0.162926	0.3392139	1.2423279
1.05	0.1814396	0.4013303	1.0065401
1.1	0.2027643	0.4516573	0.7094141
1.15	0.226234	0.487128	0.3614651
1.2	0.2510422	0.5052012	-0.024486





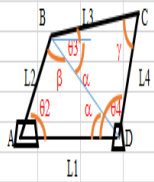
Case Study 8. Numerical Differentiation for linkage analysis in Dynamics of Machinery

In general, the displacement of a given linkage could be found analytically, where a four bar linkage is illustrated below for the mathematical formulas in the first six equations. Link 2 is driven by a constant motor, by selecting time steps, with the input angle of link 2, the angular displacement of Link 3, and 4 can be solved. Then, using center difference formulas equations 7 and 8 for numerical differentiation, the velocity and acceleration of output link 3 and 4 can be solved numerically in tabular form and then plot into chart forms as show.

Position Analytical Analysis of Four-Bar-Linkage

1. Formulas for Position Analytical Analysis of Four-Bar-Linkage with Laws of Cosine and Sine

Triangle ABD,	BD^2=L1^2+L2^2-2L1*L2*cosθ2	
Find BD	BD=(L1^2+L2^2-2L1*L2*cosθ2)^0.5	Eq(1)
Triangle BCD,	BD^2=L3^2+L4^2-2L3*L4*cosγ	
Find γ	γ=cos-1((L3^2+L4^2-BD^2)/2L3*L4)	Eq(2)
Triangle ABD,	sinα/L2=sinβ/BD=sinβ/L1	
Find α	α=sin-1((L2/BD)*sinγ)	Eq(3)
Triangle BCD,	sin(θ4-α)/L3=sinγ/BD=sin(θ3+α)/L4	
Find θ4	θ4=sin-1((L3/BD)*sinγ)+α	Eq(4)
Find θ3	θ3=sin-1((L4/BD)*sinγ)-α	Eq(5)
Double Check by		
Triangle BCD,	θ3+α+γ+θ4-α=0	
	θ3+θ4+γ=180-deg	Eq(6)



Center Difference Formulas for Numerical Differentiation

$$\left(\frac{dy}{dx}\right)_k = \frac{y_{k+1} - y_{k-1}}{2\Delta x_i} \qquad \text{Eq(7)}$$

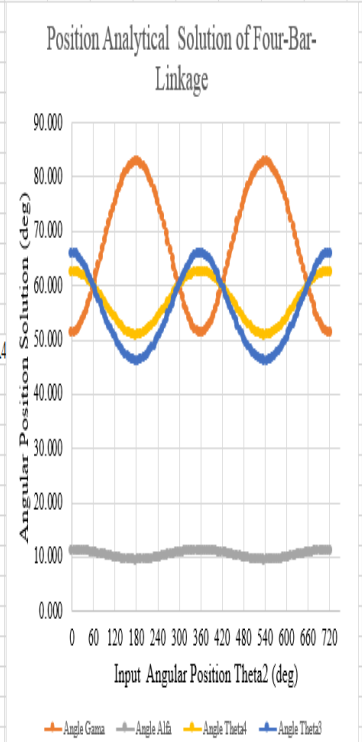
$$\left(\frac{d^2y}{dx^2}\right) = \frac{y_{k+1} - 2y_k + y_{k-1}}{(\Delta x_i)^2} \qquad \text{Eq(8)}$$

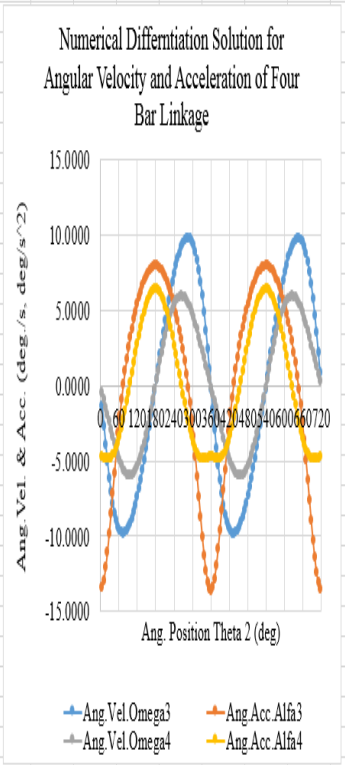
2. Input:

Motor Speed ω2=	1	rad/sec	(ccw)	Four Bar Linkage			
Time step dt=	0.1	sec		L1=	10	in	
dθ2=ω2*dt=	5.73	deg/sec	(ccw)	L2=	2	in	
				L3=	8	in	
				L4=	10	in	

Table of Calculation with Numerical Differentiation for velocity and acceleration

Input:						Double	Numerical Differentiation			
Angle θ2	Length BD	Angle γ	Angle α	Angle θ4	Angle θ3	Check	Ang.Vel ω3	Ang.Acc. α3	Ang.Vel ω4	Ang.Acc. A4
	eq(1)	eq(2)	eq(3)	eq(4)	eq(5)	eq(6)	eq(7)	eq(8)	eq(7)	eq(8)
deg	in	deg	deg	deg	deg	deg	deg/s	deg/s^2	deg/s	deg/s^2
0	8	51.322	11.302	62.6239	66.0677	180.013				
5.73	8.0125	51.413	11.299	62.6003	65.9998	180.013	-1.3509	-13.4223	-0.4734	-4.7404
11.46	8.0497	51.686	11.288	62.5292	65.7976	180.013	-2.6678	-12.9143	-0.9491	-4.7740
17.19	8.1109	52.137	11.270	62.4105	65.4662	180.013	-3.9189	-12.1095	-1.4285	-4.8144
22.92	8.1950	52.756	11.245	62.2435	65.0138	180.013	-5.0776	-11.0640	-1.9114	-4.8419
28.65	8.3004	53.534	11.213	62.0282	64.4507	180.013	-6.1230	-9.8436	-2.3952	-4.8347
34.38	8.4254	54.460	11.174	61.7645	63.7892	180.013	-7.0410	-8.5156	-2.8756	-4.7731
40.11	8.5677	55.518	11.128	61.4531	63.0425	180.013	-7.8238	-7.1416	-3.3463	-4.6420
45.84	8.7253	56.694	11.075	61.0952	62.2244	180.013	-8.4695	-5.7727	-3.8001	-4.4326
51.57	8.8958	57.972	11.015	60.6931	61.3486	180.013	-8.9806	-4.4478	-4.2288	-4.1424
57.3	9.0768	59.336	10.950	60.2495	60.4283	180.013	-9.3626	-3.1929	-4.6246	-3.7741

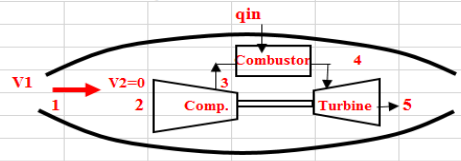


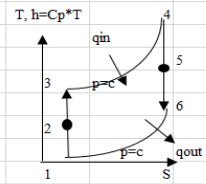


Case Study 9. Jet Engine Thermodynamic analysis.

Jet engine thermodynamic analysis in both SI and English units is processed in Excel--Assume aircraft is stationary and the air is moving towards the aircraft at a velocity of V1=280-m/s. Ideally, the air will leave the diffuser with a negligible velocity (V=0). Air is treated as an ideal gas with constant specific heats Cp=1.005-kJ/kg-K. Student can change any input to find the alternative design in jet engine of aircraft.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Jet Engine--Assume aircraft is stationary and the air is moving towards the aircraft at a velocity of V1=280-m/s													
2	Ideally the air will leave the diffuser with a negligible velocity (V=0)													
3	Air is treated as an ideal gas with constant specific heats Cp=1.005													
4	Input (SI Unit):													
5	Air Inlet Temp. T1=				260	K								
6	Air Inlet Pressure p1=				48	kpa								
7	Air Inlet velocity V1=				280	m/sec								
8	Pressure Ratio PR=p3/p2=				13									
9	Max. Temp T4=				1300	kpa								
10	Specific Heat Cp=				1.005	kJ/kg-K								
11	Ratio Cp/Cv=k=				1.4									
12	Constant Z=(k-1/k)=				0.286									
13														
14	Solution:													
15	**Step 1. Diffuser 1-2**													
16	qdot-Wdot=h2-h1+(V2^2-V1^2)/2				1st Law Thermody.									
17	where qdot=Wdot=V2=0 in diffuser, we have													
18	Cp(T2-T1)-V1^2/2=0													
19	T2=T1+V1^2/(2Cp)=260+(280-m/s)^2/(2Cp)(1-kJ/kg/1000m^2/s^2)=							**Input (English Unit):**						
20					299.0	K								
21	p2=P1*(T2/T1)^(1/Z)=				78.3	kpa		Air Inlet Temp. T1=				420	R	
22	**Step 2. Compressor 2-3**							Air Inlet Pressure p1=				7	psia	
23	p3=p4=p2*PR=				1017.8	kpa		Air Inlet velocity=Aircraft Speed V1=				900	ft/s	
24	T3=T2(p3/p2)^Z=				622.2	K		Pressure Ratio PR=p3/p2=				13		
25	**Step 3. Turbine 4-5**							Max. Temp T4=				2400	R	
26	Wc,in=Wt,out		h3-h2=h4-h5					Specific Heat Cp=				0.24	Btu/lbm-R	
27			T3-T2=T4-T5					Ratio Cp/Cv=k=				1.4		
28	T5=T4-T3+T2=				976.8	K		Constant Z=(k-1/k)=				0.286		
29	p5=p4*(T5/T4)^(1/Z)=				374.2	kpa		1-Btu/lbm=D=				25037	ft^2/s^2	
30	**Step 4. Nozzle 5-6**		p6=p1					Solution:						
31	T6=T5*(p6/p5)^Z=				543.2	K		**Step 1. Diffuser 1-2**						
32	qdot-Wdot=h6-h5+(V6^2-V5^2)/2							qdot-Wdot=h2-h1+(V2^2-V1^2)/2				1st Law Thermody.		
33	qdot=Wdot=V5=0, we have Cp(T6-T5)-V6^2/2=0							where qdot=Wdot=V2=0 in diffuser, we have						
34	V6=((2*Cp(kJ/kg-K)*(T5-T6)K(1000(m^2/s^2)*(1-kJ/kg))^(1/2)=							Cp(T2-T1)-V1^2/2=0						
35					933.5	m/s		T2=T1+V1^2/(2Cp)=420R+(850-ft/s)^2/((2*0.24)(Btu/lbm-R))						
36	**Step 5. Propulsive Work and Propulsive efficiency**							*(1-Btu/lbm/25037-Ft^2/S^2)=				487.4	K	
37	Propulsive Work							p2=P1*(T2/T1)^(1/Z)=				11.8	psia	
38	Wp=(Vexit-Vinlet)*Vaircraft=(V6-V1)(m/s)*V1(m/s)*(1-kJ/kg/(1000-m^2/s^2))							**Step 2. Compressor 2-3**						
39			182.99		kJ/kg			p3=p4=p2*PR=				153.2	psia	
40	Propulsive efficiency		ηp=Wp/qin=		0.269	26.9%		T3=T2(p3/p2)^Z=				1014.3	R	
41	where qin=h4-h3=Cp(T4-T3)=				681.2	kJ/kg		**Step 3. Turbine 4-5**						
42	The fuel comsumption (mdot)fuel=Qdot/qHV=				0.0158	kg		Wc,in=Wt,out		h3-h2=h4-h5				
43	qHV is the heating value of the fuel (kerosene)=				43,000	kJ/kg				T3-T2=T4-T5				
44	where (mdot)fuel/(mdot)=E42/1-kg mass of Ai				0.0158	1.58%		T5=T4-T3+T2=				1873.1	R	
45	**Step 6. Required Propusive Work W* for Aircraft**							p5=p4*(T5/T4)^(1/Z)=				64.3	psia	
46	W*=		4,000		kw			**Step 4. Nozzle 5-6**		p6=p1				
47	The Req. mass flow rate of air mdot=				21.859	kg/s		T6=T5*(p6/p5)^Z=				993.8	R	
48	**which need to design the of Jet Passage Area based on Aerodynamics**							qdot-Wdot=h6-h5+(V6^2-V5^2)/2						
49	The mass flow rate of fuel mfdot=				0.3463	kg/s		qdot=Wdot=V5=0, we have Cp(T5-T6)-V6^2/2=0						
50	Total Mass of fuel in Tank Mf=				20000	kg		V6=((2*0.24(Btu/lbm-R)*(1857.8-965.8)R(25037-Ft^2/S^2/1Btu/lbm))^(1/2)=						
51	Nonstop fly Hours of AirCraft H=				16.0	hours						3250.7	ft/s	
52								**Step 5. Propulsive Work and Propulsive efficiency**						
53								Propulsive Work						
54								Wp=(Vexit-Vinlet)*Vaircraft=(V6-V1)(m/s)*V1(m/s)*(1-Btu/lbm/(25037-ft^2/s^2))						
55										84.50		Btu/lbm		
56								Propulsive efficiency		ηp=Wp/qin=		0.254	25.40%	
57								where qin=h4-h3=Cp(T4-T3)=				332.6	Btu/lbm	
58								The fuel comsumption (mdot)fuel=Qdot/qHV=					0.017	lbm/s
59								qHV is the heating value of the fuel (kerosene)=					19300	Btu/lbm
60								where (mdot)fuel/(mdot)=M58/1-lbm of Air=					0.0172	1.72%





Case Study 10. Long-hand-calculation of Stiffness Matrix for two dimensional triangular three-node-element in CAE study.

There are three topics are show in this spreadsheet. 1. With matrix analysis tool in Excel, the calculation of stiffness matrix for a two dimensional triangular three-node-element is created. 2. For given nodal displacement of the element, to find the stresses in the element. 3. For given nodal forces of the element to find the stresses in the element. Which provides the basic formulation of finite element analysis in two dimensional problems.

Stiffness Matrix for Given Element **Logan Example 6.1.** **(P.322)**

Vector Column Matrices of Stress and Strain

Strain-Displacement Relationship

Constitutive Matrix (6.1.8)

$$[D] = \frac{E}{1-\nu^2}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{pmatrix}$$

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}, \{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \quad (6.1.1,4,5)$$

Nodal Displacement (6.2.1)

$$\{d\} = \begin{Bmatrix} ui \\ vi \\ uj \\ vj \\ um \\ vm \end{Bmatrix}$$

Stress/Strain Relationship

$$\{\sigma\} = [D]\{\varepsilon\} \quad (6.1.7)$$

Constant-Strain Triangular Element

Assumed Displacement Function

u(x,y)=a1+a2*x+a3*y (6.2.2)

v(x,y)=a4+a5*x+a6*y

Strain/Nodal Displacement Relationship (6.2.30)

$$\{\varepsilon\} = 1/2A \begin{pmatrix} \beta i & 0 & \beta j & 0 & \beta m & 0 \\ 0 & \gamma i & 0 & \gamma j & 0 & \gamma m \\ \gamma i & \beta i & \gamma j & \beta j & \gamma m & \beta m \end{pmatrix}\{d\}$$

Where

αi=	xj*ym-yj*xm	βi=	yj-ym	γi	=	xm-xj
αj=	yi*xm-xi*ym	βj=	ym-yi	γj	=	xi-xm
αm=	xi*yj-yi*xj	βm=	yi-yj	γm	=	xj-xi

(6.2.10)

$$\{\sigma\} = [D]\{\varepsilon\}$$

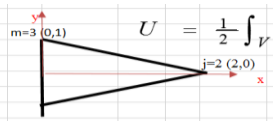
$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

Stress/Nodal Displacement Relationship

$$\{\sigma\} = [D]\{\varepsilon\} = D\ B\ \{d\} \quad (6.2.36)$$

Stiffness Matrix for Given Element **Logan Example 6.1.** p.322

Coordinates		Displacement		Thickness t=	1	in^2
xi=	0	u1=	0	Area A=1/2*Σαi+αj+αm=	2	in^2
yi=	-1	v1=	0.0025	Material (Steel-A36)		
xj=	2	u2=	0.0012	Modulus Elasticity E=	30000000	psi
yj=	0	v2=	0	Poisson's Ratio n=	0.25	
xm=	0	u3=	0			
ym=	1	v3=	0.0025			



$$U = \frac{1}{2}\int_V \{\varepsilon\}^T [D]\{\varepsilon\}\, dV$$

Step 1. Calculate α, β, γ

αi=	2
βi=	-1
γi=	-2
αj=	0
βj=	2
γj	0
αm=	2
βm=	-1
γm=	2

Strain/Nodal Displacement Matrix (6.2.30)

$$B = 1/4 \begin{pmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{pmatrix}$$

Constitutive Matrix (6.1.8)

$$D = 32000000 \begin{pmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{pmatrix}$$

Stiffness Matrix for Given Element (6.2.52)

$$k = t*A*BT*D*B = 1\text{-in}* \ 2\text{-in}^2* \begin{pmatrix} -1 & 0 & -2 \\ 0 & -2 & -1 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \\ -1 & 0 & 2 \\ 0 & 2 & -1 \end{pmatrix} 32000000/16 \begin{pmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{pmatrix} * \begin{pmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{pmatrix}$$

$$= 4E{+}06 \begin{pmatrix} -1 & -0.25 & -0.75 \\ -0.5 & -2 & -0.375 \\ 2 & 0.5 & 0 \\ 0 & 0 & 0.75 \\ -1 & -0.25 & 0.75 \\ 0.5 & 2 & -0.375 \end{pmatrix} * \begin{pmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{pmatrix} = 4000000 \begin{pmatrix} 2.5 & 1.25 & -2 & -1.5 & -0.5 & 0.25 \\ 1.25 & 4.375 & -1 & -0.75 & -0.25 & -3.63 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -1.5 & -0.75 & 0 & 1.5 & 1.5 & -0.75 \\ -0.5 & -0.25 & -2 & 1.5 & 2.5 & -1.25 \\ 0.25 & -3.63 & 1 & -0.75 & -1.25 & 4.375 \end{pmatrix}$$

Case 1. For given Nodal Displacement, to find the Stresses in Element

In Plane Stresses (6.2.36)

$$\{\sigma\} = [D]\{\varepsilon\} = [D]\ B\ \{d\} \quad (6.2.36)$$

$$= 3.2E{+}07 \begin{pmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{pmatrix} * 1/4 \begin{pmatrix} -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & -1 & 0 & 2 & 2 & -1 \end{pmatrix} * \begin{pmatrix} 0.000 \\ 0.003 \\ 0.001 \\ 0.000 \\ 0.000 \\ 0.003 \end{pmatrix}$$

$$= 8000000 \begin{pmatrix} -1 & -0.5 & 2 & 0 & -1 & 0.5 \\ -0.25 & -2 & 0.5 & 0 & -0.25 & 2 \\ -0.75 & -0.375 & 0 & 0.75 & 0.75 & -0.375 \end{pmatrix} * \begin{pmatrix} 0.000 \\ 0.003 \\ 0.001 \\ 0.000 \\ 0.000 \\ 0.003 \end{pmatrix}$$

$$= 8000000 * \begin{pmatrix} 0.0024 \\ 0.0006 \\ -0.001875 \end{pmatrix} = \begin{pmatrix} 19200 \\ 4800 \\ -15000 \end{pmatrix} \text{psi}$$

Principal Stresses and Principal Direction

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{max}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{min} \quad (6.1.2)$$

Also, the **principal angle** θ_p, which defines the normal whose direction is perpendicular to the plane on which the maximum or minimum principal stress acts, is defined by

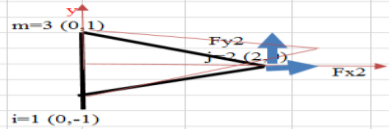
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad (6.1.3)$$

σ1=	28639	psi
σ2=	-4639	psi
θp=	-32.18	deg

Case 2. For given Nodal forces of element to find the stresses

u1=v1=u3=v3=0 Fx2=500-lb, Fy2=200-in

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{16} \\ k_{21} & k_{22} & \cdots & k_{26} \\ \vdots & \vdots & & \vdots \\ k_{61} & k_{62} & \cdots & k_{66} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \qquad (6.2.55)$$



$$\begin{Bmatrix} Fx1 \\ Fy1 \\ 500 \\ 200 \\ Fx3 \\ Fy3 \end{Bmatrix} = 4E{+}06 \begin{bmatrix} 2.5 & 1.25 & -2 & -1.5 & -0.5 & 0.25 \\ 1.25 & 4.375 & -1 & -0.75 & -0.25 & -3.625 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -1.5 & -0.75 & 0 & 1.5 & 1.5 & -0.75 \\ -0.5 & -0.25 & -2 & 1.5 & 2.5 & -1.25 \\ 0.25 & -3.625 & 1 & -0.75 & -1.25 & 4.375 \end{bmatrix} \begin{Bmatrix} u1=0 \\ v1=0 \\ u2=? \\ v2=? \\ u3=0 \\ v3=0 \end{Bmatrix}$$

Solving Sub-Matrix with Sub-matrix Method

$$\begin{Bmatrix} 500 \\ 200 \end{Bmatrix} = 4E{+}06 \begin{bmatrix} 4 & 0 \\ 0 & 1.5 \end{bmatrix} \begin{Bmatrix} u2 \\ v2 \end{Bmatrix}$$

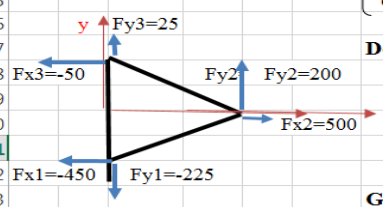
We have

$$\begin{Bmatrix} u2 \\ v2 \end{Bmatrix} = 1/4E{+}06 \begin{bmatrix} 0.25 & 0 \\ 0 & 0.66667 \end{bmatrix} \begin{Bmatrix} 500 \\ 200 \end{Bmatrix} = 1/4E{+}06 \begin{Bmatrix} 125.00 \\ 133.33 \end{Bmatrix} = \begin{matrix} 3.1E{-}05 \text{ in} \\ 3.3E{-}05 \text{ in} \end{matrix}$$

Substituting back to find the unknown nodal forces

$$\begin{Bmatrix} Fx1 \\ Fy1 \\ 500 \\ 200 \\ Fx3 \\ Fy3 \end{Bmatrix} = 4E{+}06 \begin{bmatrix} 2.5 & 1.25 & -2 & -1.5 & -0.5 & 0.25 \\ 1.25 & 4.375 & -1 & -0.75 & -0.25 & -3.625 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -1.5 & -0.75 & 0 & 1.5 & 1.5 & -0.75 \\ -0.5 & -0.25 & -2 & 1.5 & 2.5 & -1.25 \\ 0.25 & -3.625 & 1 & -0.75 & -1.25 & 4.375 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 3.1E{-}05 \\ 3.3E{-}05 \\ 0 \\ 0 \end{Bmatrix}$$

$$= 4E{+}06 \begin{Bmatrix} -0.00011 \\ -5.6E{-}05 \\ 0.00013 \\ 0.00005 \\ -1.3E{-}05 \\ 6.3E{-}06 \end{Bmatrix} = \begin{Bmatrix} -450 \\ -225 \\ 500 \\ 200 \\ -50 \\ 25 \end{Bmatrix} \text{lb}$$



Double check by ΣFx=ΣFy=0

Go back to (6.2.36) to find the stresses

Stress/Nodal Displacement Relationship

$$\{\sigma\} = [D] \{\varepsilon\} = D \; B \; \{d\} \qquad (6.2.36)$$

Conclusion:

Excel spreadsheet in Microsoft Office allows the integration of computer based projects with traditional mechanical engineering topics. Student is enjoying their spreadsheet learning environment and will bring hundreds spreadsheet calculators to deal with their future engineering design and analysis problems. The easy learning Excel spreadsheets allows our student to get the alternative solution of their interested problems by simply changing the inputs in their Excel calculator. Where the most desirable solution for a given problem could be found by considering the given environment and to improve the design quality. With Excel spreadsheet background, MET student get better understanding and great confidence to deal with their future professional challenge.

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Ti-Lin Liu is an associate professor in the Manufacturing and Mechanical Engineering Technology Department at Rochester Institute of Technology. He has been teaching at RIT since 1987. Previously he was the Chair Professor of the Mechanics Department at Shanghai University of Technology. His areas of interest include finite element analysis, computer simulation with working model and Matlab/Simulink, numerical analysis, solid mechanics, heat transfer and system dynamics/vibration. He has been the recipient of teaching awards in China and was the sole recipient of the 2001 Annual Grant Reward from MSC Software Corporation, San Mateo, CA.

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