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## Abstract

International monetary economists have difficulty explaining the behaviour of exchange rates and inflation using dynamic general equilibrium models with nominal rigidities. We develop an open economy model with nominal rigidity in goods markets and matching frictions in labour markets - a framework which has considerable success generating persistence in a closed economy context. We find that i) the exchange rate channel introduced in an open economy context does not mitigate this account of inflation and output persistence; ii) this combination of rigidities generates a plausible explanation of exchange rate behaviour; iii) the model is better able to account for labour market variables.

KEYWORDS: Exchange rates; nominal rigidity; search and matching, unemployment. JEL Classification: E24, E32, E52, F41, J64.

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# 1 Introduction.

In recent years, researchers in international monetary economics have developed a number of smallscale quantitative dynamic general equilibrium (DGE) models of open economies. In these models attention focuses on nominal frictions in the form of sluggish price adjustment. Papers typically analyse the dynamic effects of monetary policy disturbances and other shocks, see, inter alia, Chari, Kehoe and McGrattan (2002), Gali and Monacelli (2002), McCallum and Nelson (1999) hereafter CKM, GM and MN respectively. This research has the potential to shed light on the nature of the shocks hitting the economy, the propagation mechanisms at work, and, ultimately, the design and conduct of macroeconomic policy.

Successful analysis of policy relies on the adequacy of the underlying structural model, but current models typically fail to account for the persistence properties of inflation, exchange rates and output data. In this paper I develop and analyse a model which incorporates both nominal rigidity and search and matching frictions in the labour market. This framework not only helps to account for the persistence properties observed in the data but allows us to examine how labour market variables respond to shocks - an issue which appears to concern policymakers and which cannot meaningfully be addressed using the frictionless Walrasian labour market set up of existing models.

The inability of current models to account for the behaviour of exchange rates and inflation is well documented. For instance, a baseline two-country model with nominal rigidities is unable simultaneously to account for both the volatility and persistence of real exchange rates, CKM (2002). In an analogous small open economy model nominal rigidities explain only some 40-50% of historical exchange rate variation of exchange rate data, Kollmann (2001). MN (2000) document the inability of the small open economy model of GM (2002) to account for the inflation persistence.

These problems reflect the well-known absence of strong internal propagation mechanisms within DGE models in the real business cycle tradition, Cogley and Nason (1995). In current models (international) monetary economists typically remove capital from, and introduce money

and nominal rigidity (in the form of Calvo price adjustment) into, an otherwise frictionless DGE model.<sup>1</sup> In this class of model, price adjustment is sluggish but inflation is a jump variable and responds directly to current and future marginal costs, see for example Gali (2002). Due to the frictionless nature of labour markets, marginal costs are proportional to current and future output. As a result, neither inflation nor output exhibits persistence or humped shaped behaviour in response to monetary shocks - Calvo price rigidity alone (the principal source of rigidity in current monetary DGE models) does not provide a strong internal propagation mechanism.

Exchange rates, inflation and price setting decisions are related - this manifests itself in several ways in the literature. In the Dornbusch (1976) model of exchange rate dynamics, sluggish price adjustment is sufficient to generate overshooting in the exchange rate - a phenomenon that finds empirical support in the work of Eichenbaum and Evans (1995). The modern DGE literature relates the overshooting result to the extent of exchange rate pass through, which depends on the pricing decisions of firms. Betts and Devereux (2000) show that there is no exchange rate overshooting under the producer currency pricing - which leads to complete exchange rate pass-through, whereas they obtain exchange rate overshooting when prices are set in local currency (incomplete exchange rate pass-through). Goldberg and Knetter (1997) summarise evidence supporting the incomplete exchange rate pass-through. An alternative perspective is that the presence of an exchange rate channel in the monetary transmission mechanism can alter the speed with which monetary shocks are transmitted to real variables. The exchange rate can affect the domestic price level directly by altering the domestic currency price of imports (assuming some exchange rate pass-through), and can also alter relative prices (when prices display nominal rigidity), thereby influencing aggregate demand and supply - loosely speaking openness makes the Phillips curve steeper, see Lane (1997). Thus international linkages may mitigate against finding exchange rate and inflation persistence. Simulation work supports this insight: for a small open economy model under Calvo price rigidity, calibrated to US data, MN (2000) show that inflation (and by

implication output) display greater persistence under incomplete exchange rate pass-through than  $\frac{1}{1}$  One justification for the omission of capital is that variation in capital stock is unimportant at business cycle frequencies, McCallum and Nelson (1999). Another is that with or without adjustment costs, capital adjustment does not greatly augment the propagation mechanism of the canonical RBC model, Cogley and Nason (1995).

is present in the complete exchange rate pass-through set up of GM (2002). Even so MN (1999) are unable adequately to capture exchange rate and inflation behaviour fully without the introduction of backward-looking price-setting behaviour.

A variety of 'solutions' have been proposed to generate greater persistence.<sup>2</sup> One approach introduces backward-looking elements into the standard model in a more or less ad hoc fashion, for example Gali and Gertler (1999) generate inflation persistence by assumption, rather than allowing it to emerge as a consequence of forward-looking behaviour on the part of firms in a way that is consistent with the evidence on infrequent price adjustment at the microeconomic level. An alternative is to incorporate other (real) sources of friction in the hope that the interaction of real and nominal rigidity might help to explain observed persistence. Examples of this approach include Christiano, Eichenbaum and Evans (2001), and Dotsey and King (2001).

A further problem with existing international monetary models lies in their frictionless structure for the labour market. Research on labour market behaviour has made substantial progress using the search-and-matching-mediated equilibrium unemployment framework, Mortensen and Pissarides (1999).<sup>3</sup> While this approach was initially geared towards understanding the labour market behaviour as an end in itself, it turns out to have attractive properties fore students of the business cycle since it provides a strong internal propagation mechanism, Merz (1995), Andolfatto (1996) and Den Haan, Ramey and Watson (1999) - hereafter DHRW.<sup>4</sup>. Matching frictions in the labour market also offer insights in an open economy context. Using a small open economy (nonmonetary) DGE model Feve and Langot (1996) find that matching frictions can account for the cyclical pattern of (French) labour market variables rather better than can the standard Walrasian approach. The inclusion of labour market search and matching frictions improves the ability of a two-country business cycle model to match the real fluctuations, Hairault (2002).

Returning to monetary economies, the strong propagation mechanism with matching frictions

in labour markets has recently led researchers to investigate whether, combined with nominal  $\frac{1}{2}$  An alternative explanation introduces persistence through the response of policy makers to shocks, Woodford

 $<sup>\</sup>binom{(1999)}{3}$  Indeed Hall (1999), discussing labour market frictions (of all varieties) and business cycles, in the chapter preceding that of Mortensen and Pissarides (1999) virtually declares search-and-matching to be the approach of choice to modelling labour market frictions. <sup>4</sup> Cogley and Nason (1995) had previously argued that frictions associated with adjusting labour input increase the

<sup>4</sup> Cogley and Nason (1995) had previously argued that frictions associated with adjusting labour input increase the strength with which (technology) shocks are propagated. Their employment adjustment costs story can be seen as a reduced form precursor to the search and matching based models.

rigidities in goods markets, these frictions help explain the relationship between inflation and unemployment as well as other standard features of business cycle fluctuations. Maintaining the assumption that prices are perfectly flexible while imposing rigidities on (nominal) portfolio adjustment, Cooley and Quadrini (1999) find a negative correlation between inflation and unemployment. Combining equilibrium unemployment and quadratic costs of price adjustment in a DGE framework, Cheron and Langot (2000) find a negative correlation between unemployment and inflation (the Phillips curve) and a negative correlation between unemployment and vacancies (the Beveridge curve). Walsh (2003) demonstrates that labour market matching frictions with Calvo-style nominal rigidities can account the hump-shaped response of output to monetary shocks, whilst reducing the required degree of nominal rigidity to match estimates based on micro data.<sup>5</sup>

Drawing on these diverse insights we develop an open economy model with nominal rigidities in goods markets and labour market matching frictions. Using this framework we address several issues pertaining to the structure of the economy as a prelude to policy analysis. We consider whether incorporating an exchange rate channel to the monetary transmission mechanism mitigates against the attractive persistence properties of a model with labour market matching frictions, and, relatedly, whether persistence can be achieved using parameterisations of nominal rigidity consistent with microeconomic data. Secondly we ask whether the incorporation of matching frictions improves the persistence properties of exchange rates. Thirdly, we examine how monetary disturbances affect labour market variables. Finally we examine the plausibility of the mechanisms at work in this model. The layout of the paper is as follows. In the next section we outline a small open economy with Calvo-style nominal rigidities in the goods market and matching frictions in the labour market and characterise the equilibrium. In Section 3 we calibrate the model to U.S. data investigate the dynamic responses to monetary shocks and compare these to those obtained from an equivalent model with a frictionless Walrasian labour market set up.

Section 4 contains a summary, a conclusion and suggestions for further work.

 $<sup>\</sup>frac{1}{5}$  Bils and Klenow (2002) using US data find that prices last on average for roughly six months. Standard estimates based on aggregate data suggest that on average a price is set for 1 year, Gali and Gertler (1999).

# 2 Model

In our simulation work we compare the behaviour of our model with search and matching frictions (hereafter SM) with a standard framework with frictionless Walrasian labour markets (hereafter FLEX). Here we briefly summarise the main features of the FLEX approach, and the relationship to the SM model. The remainder of this section contains a detailed discussion of the SM model. There are a variety of issues underlying the modelling assumptions of current DGE international monetary models. Our FLEX model, see Figure 1, is based on the approach of MN (1999), (2000) so we point out the key assumptions regarding openness.

There are 5 players in the *FLEX* economy: households, retailers, wholesalers, government and the rest of the world (RoW). Government issues money, collects seigniorage revenues and rebates this to consumers. It undertakes no other function. RoW variables are exogenous - this is one sense in which the economy is small. Households supply labour to wholesale goods firms, borrow from (lend to) RoW, and purchase differentiated final goods from monopolistically competitive retailers (using domestic currency). Retailers supply not only domestic households but also export to RoW. Retailers set prices in domestic currency according to a Calvo price adjustment rule. They produce final goods by costlessly differentiating the homogeneous wholesale good. The wholesale good is produced by combining labour input and imports according to a Cobb-Douglas production function. Wholesalers are price takers in product and factor markets. All variation in labour input, due to changes in demand or in factor prices, occurs along the intensive margin.

The nature of firms pricing decisions affect the extent of exchange rate pass through. Our approach, following MN (1999), in which imports enter only as inputs to production provides a representation of incomplete pass-through at the final goods level. This is consistent with empirical evidence Goldberg and Knetter (1997) - although exchange rate pass-through to imports is complete whereas empirical evidence suggests a figure of 0.5. The MN approach has the advantage of parsimony as the domestic price index depends only on the prices of domestic final goods. The assumption that the economy is a price taker in import markets but can set prices for its exports (which form a negligible component of RoW consumption) is another sense in which the economy

is small. Another important set of assumptions concerns the specification of asset markets and the role of the current account. MN (1999) assume that asset markets are incomplete: households' opportunities to pool risk with RoW are limited - they can hold domestic and foreign currency denominated bonds.<sup>6</sup> Asset market incompleteness implies that current account dynamics matter as they lead to wealth effects associated with changes in net foreign asset position. MN proceed under the (implicit) assumption that these wealth effects are small and can be neglected.<sup>7</sup> For ease of comparison, given the other attractive features of their model, we follow MN's approach.

The key differences between the *SM* and *FLEX* models arise in wholesale production and labour markets, see Figure 2. Wholesale production now occurs in matches: firm-worker pairs the line joining households and wholesalers now corresponds to labour supply in existing matches. However, labour supply is assumed inelastic within a match, so variation in labour input requires changes in the number of matches. Creation of new matches is governed by a constant returns to scale aggregate matching function - where the probability of a firm filling a vacancy, and the probability of an unemployed worker finding a job depend on the relative numbers of these two types. Reduction in the number of existing matches is achieved by job-destruction. Existing matches are subject to idiosyncratic productivity disturbances. Those with low productivity may choose to split up. On doing so the firm and worker enter the matching pool. This is the mechanism underlying changes in employment. The incentive to change employment by changing the rate of job creation or destruction arises because of the quasi-rents which accrue to existing matches in the face of shocks to demand because matching frictions prevent instantaneous adjustment in the labour market.

Given this basic outline of the model let us now fill in some detail by discussing in turn the decision problems of households, wholesale firms, retail firms, our assumptions about the actions

of the government and finally characterising the equilibrium for the economy.

 $<sup>\</sup>overline{_6}$  Here smallness is captured by assuming that RoW holds only RoW issued bonds - thus acting as a large open economy. 7 As is well known, a technical problem, the non-stationarity of consumption, arises under market incompleteness.

Schmitt-Grohe and Uribe (2003) discuss a number of "solutions" to this problem but show that at business cycle frequencies, model dynamics are affected little whether or not the non-stationarity problem is ignored.

## 2.1 Households

We assume that the domestic economy contains a continuum of households of unit mass indexed by  $j \in [0, 1]$ . When employed, these households supply labour inelastically. They own all firms and carry cash balances to the goods market to purchase consumption goods, for which they are subject to a cash in advance constraint. They can also hold domestic and/or foreign bonds. To avoid the distributional issues that arise because some firms and workers are unmatched, it is assumed that workers pool their income at the end of the period and choose aggregate consumption to maximise the expected utility function of a representative worker <sup>8</sup>

$$U_{\tau} = E_{\tau} \left[ \sum_{t=\tau} \beta^{s-\tau} \left[ u(C_t) + (1-\chi_t) h - \chi_t a \right] \right].$$
 (1)

where  $\beta$  gives the discount factor, h is the utility value of (non-tradable) home production, a is the disutility of work. For any individual household  $j \in [0, 1]$ ,  $\chi_t^j$  is an indicator function taking the value 1 when the agent is employed and zero otherwise.  $C_t$  is the composite consumption index consumed by the representative domestic household in period s, this index consists of all differentiated goods sold by the monopolistically competitive retailers. We assume that there is a continuum of such firms of unit mass, and define the composite consumption index by the constant elasticity of substitution (CES) function

$$C_{t} \equiv \left[ \int_{0}^{1} c_{t} \left( z \right)^{\frac{e-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \varepsilon > 0$$

Where  $\varepsilon$  represents the elasticity of demand for product z. The price deflator P for nominal money balances corresponding to this index is the consumption-based money price index:  $P = \left[\int_0^1 p(z')^{1-\varepsilon} dz'\right]^{\frac{1}{1-\varepsilon}}$ , and demand for good z is  $c(z) = \left(\frac{p(z)}{P}\right)^{-\varepsilon} C$ .

Domestic households maximise this objective function (1) subject to a cash in advance (CIA) constraint

$$\int_{0}^{1} p_t(z) c_t(z) dz = P_t C_t \le M_{t-1} + P_t T_t$$
(2)

where  $M_t$  is the representative household's holding of nominal money at the end of period t, and

 $T_t$  denotes a lump-sum transfer expressed in units of the consumption index. This implies that a  $\overline{s}$  This assumption is a common simplification in the literature on business cycle fluctuations under labour market search designed to facilitate tractability, see e.g. Andolfatto (1996). There is of course an issue surrounding the incentive compatibility of participation which we assume away here.

household's current income is unavailable for purchasing domestic goods in the current period, and also that only the domestic currency required for purchases of domestic retail goods by domestic households need be held in advance.

The representative domestic household's budget constraint can be written in units of domestic currency as

$$M_t + P_t C_t + P_t B_t + S_t P_t^* B_t^* = P_t Y_t^l + P_t D_t + M_{t-1} + R_t^n P_t B_{t-1} + R_t^{n*} S_t P_t^* B_t^* + P_t T_t$$
(3)

 $P_tB_t$  represents expenditure by the representative household on domestic 1-period bonds  $B_t$ , acquired during period t. Domestic bonds held between dates t-1 and t offer gross nominal return  $R_t^n$ .  $S_tP_t^*B_t^*$  represents nominal expenditure in units of domestic currency by the representative domestic household on foreign bonds,  $B_t^*$ ,  $S_t$  is the nominal exchange rate. Foreign bonds held between dates t-1 and t offer gross nominal return  $R_t^{n*}$ .  $Y_t^l$  is the household's real labour income and  $D_t$  is its share of real aggregate profits from wholesale and retail firms.

The representative household chooses a sequence of consumption, money holdings and holdings of foreign and domestic bonds. The first order conditions for the household's problem can be reduced to a standard Euler equation, which for the case of constant relative risk aversion (CRRA) instantaneous utility,  $u(C_t) = \frac{C_t^{1-\phi}}{1-\phi}$ , is:

$$1 = \beta R_t^n E_t \left[ \frac{P_t}{P_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\phi} \right]$$
(4)

and also an uncovered interest parity condition:

$$E_t \left[ R_{t+1}^n \right] = E_t \left[ \frac{R_{t+1}^{n*} S_{t+1}}{S_t} \right].$$
(5)

## 2.2 Goods and Labour Markets

Business activity occurs in retail (final) and wholesale (intermediate) sectors. Production occurs in the wholesale sector, in firm-worker pairs. These employment relationships are formed through an aggregate matching process. Output produced in the wholesale sector is sold in a competitive market to retail firms. Retailers costlessly transform the wholesale output into retail goods. Both the domestic market and the export market for domestic retail goods are monopolistically competitive, so retail prices display a markup over wholesale prices. Retail prices are sticky.

## 2.2.1 The Wholesale sector

**Production** Production of intermediate goods takes place in the wholesale sector through in matched firm-worker pairs - or, for notational ease, *matches*. Each match consists of 1 worker and 1 firm, who together engage in production until the employment relationship is severed. Both firms and workers are restricted to a single employment relationship at any given time. At date tmatch i can use imported goods,  $IM_{it}$  to produce

$$Y_{it}^w = X_{it} I M_{it}^\alpha$$

units of wholesale goods, where  $X_{it}$  represents a non-negative idiosyncratic productivity disturbances, with mean of unity.<sup>9</sup> We assume that idiosyncratic productivity disturbances are serially uncorrelated. Matches act as price takers and sell their wholesale output at (nominal) price  $P_t^w$ . Match *i* chooses the flexible factor, imports, to maximise the value of current profits.<sup>10</sup>

$$\max_{IM_{it}} \left\{ \frac{P_t^w X_{it} I M_{it}^\alpha - S_t P_t^* I M_{it}}{R_t^n P_t} \right\}$$

The optimal choice of inputs for match i at date t is

$$IM_{it} = \left(\frac{\alpha X_{it}}{\mu_t Q_t}\right)^{\frac{1}{1-\alpha}}$$

where  $\mu_t = \frac{P_t}{P_t^w}$  is the markup (of retail prices over wholes ale prices) and

$$Q_t = \frac{S_t P_t^*}{P_t} \tag{6}$$

is the real exchange rate. Thus the value of date t production by match i is

$$(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}\left(R_t^n\right)^{-1}\left[\frac{X_{it}}{\mu_t Q_t^\alpha}\right]^{\frac{1}{1-\alpha}}$$

Despite the competitive nature of the wholesale goods market, the presence of frictions associated with the formation of matches allows existing production units to earn rents. The expected value of an existing match that produces in date t is the value of current profits, less the utility cost of working, a, plus the continuation value,  $\Gamma^{J}$ . This continuation value represents the present  $\frac{9}{3}$  Allowing for aggregate productivity shocks is straightforward, but is omitted here as our focus is on the impact of monetary disturbances.

of monetary disturbances.  $_{10}$ Here the nominal interest rate term in the denominator arises because the CIA constraint dictates that current profits are only available for consumption next period.

value of expected future rents associated with being part of an ongoing productive relationship. So the value of an existing match that does produce in period t is

$$(1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}}\,(R_t^n)^{-1}\left[\frac{X_{it}}{\mu_t Q_t^\alpha}\right]^{\frac{1}{1-\alpha}} - a + \Gamma_{it}^J$$

which is increasing in X and  $\Gamma^J$  and decreasing in  $\mu$ , a, Q and  $R^n$ . Thus the exchange rate and interest rate affect the value of a match, output and employment decisions.

Separation, Matching and Labour Market Variables A match will break up (separate) endogenously if its value is less than the value of the outside options available to the constituent firm and worker. Any firm can post a vacancy, so free entry ensures that the value of this option, a matched firm's outside option, is zero. By contrast, the value of the worker's opportunities outside the match is the sum of the value of home production, h, and the present value of future worker opportunities (probability weighted value of future employment relationships and future spells of unemployment), denoted as  $\Gamma_{it}^{U}$ . Define the surplus for match i at date t,  $SU_{it}$ , as the difference between the value of a match and the value of the outside options available to the firm and worker:

$$SU_{it} = (1 - \alpha) \alpha^{\frac{\alpha}{1 - \alpha}} (R_t^n)^{-1} \left[ \frac{X_{it}}{\mu_t Q_t^\alpha} \right]^{\frac{1}{1 - \alpha}} - a + \Gamma_{it}^J - \left( h + \Gamma_{it}^U \right)$$
(7)

Endogenous separation occurs when  $SU_{it} \leq 0$ . Therefore define a threshold value of idiosyncractic productivity,  $\bar{X}_{it}$ , such that separation occurs if

$$X_{it}^{\frac{1}{1-\alpha}} \leq \bar{X}_{it}^{\frac{1}{1-\alpha}} = \frac{\left(\mu_t Q_t^{\alpha}\right) R_t^n}{\left(1-\alpha\right) \alpha^{\frac{\alpha}{1-\alpha}}} \left(h + \Gamma_{it}^U + a - \Gamma_{it}^J\right)$$

Finally note that the temporal independence of the idiosyncratic shock allows the *i* subscript to be dropped from the terms  $\bar{X}_t = \bar{X}_{it}$ ,  $\Gamma^U_t = \Gamma^U_{it}$ ,  $\Gamma^J_t = \Gamma^J_{it}$ , so the threshold value for idiosyncratic productivity can be rewritten

$$X_{it}^{\frac{1}{1-\alpha}} \leq \bar{X}_t^{\frac{1}{1-\alpha}} = \frac{(\mu_t Q_t^{\alpha}) R_t^n}{(1-\alpha) \alpha^{\frac{1}{1-\alpha}}} \left(h + \Gamma_t^U + a - \Gamma_t^J\right)$$
(8)

Having described efficient endogenous separation we are in position to describe the timing of employment and separation decisions. Let us define the number of matches at the beginning of period t as  $N_t \in [0, 1]$ . We assume that quits are exogenous and capture this by allowing a

fraction,  $\rho^x$ , of matches to separate exogenously prior to the realisation of period t (productivity) shocks. Subsequently, idiosyncratic productivity disturbances are realised, and a match may choose to break up if the value of the match surplus is negative. Endogenous separation occurs with probability  $\rho_{it}^n$  where

$$\rho_{it}^n = \rho_t^n = \int_{-\infty}^{\bar{X}_t} f(X) \, dX \tag{9}$$

and  $f(\cdot)$  is the probability density function over  $X_{it}$ .<sup>11</sup> The overall separation rate in period t is

$$\rho_{it} = \rho^x + (1 - \rho^x) \,\rho_t^n. \tag{10}$$

If the match does not sever then date t production occurs. Aggregate output of wholesale goods,  $Y_t^w$ , is therefore

$$Y_t^w = (1 - \alpha) \,\alpha^{\frac{\alpha}{1 - \alpha}} \,(1 - \rho^x) \,N_t \mu_t^{\frac{-1}{1 - \alpha}} Q_t^{\frac{-\alpha}{1 - \alpha}} \cdot \int_{\bar{x}_t}^{\infty} X^{\frac{1}{1 - \alpha}} f(X) \,dX \tag{11}$$

while aggregate imports are

$$IM_{t} = \alpha^{\frac{1}{1-\alpha}} \left(1 - \rho^{x}\right) N_{t} \left(\mu_{t} Q_{t}\right)^{\frac{-1}{1-\alpha}} \cdot \int_{\bar{x}_{t}}^{\infty} X^{\frac{1}{1-\alpha}} f\left(X\right) dX.$$
(12)

Next we turn to the matching frictions. We model this rigidity using an aggregate matching function. Matching occurs at the same time as production. We assume a continuum of potential firms, with infinite mass, and a continuum of workers of unit mass. Unmatched firms choose whether or not to post a vacancy given that it costs C per period to post a vacancy. Free entry of firms determines the size of the vacancy pool. Define the mass of firms posting vacancies to be  $V_t$ . Let the mass of searchers, unmatched workers, be  $U_t$ . All unmatched workers may enter the matching market in period t - even if their match dissolved at the start of period t. So

$$U_t = 1 - (1 - \rho_t) N_t \tag{13}$$

New matches in date t begin production in date t + 1, while unmatched workers remain in the worker matching pool. The flow of successful matches created in period t is given by

$$\mathcal{M}_t = m U_t^{\gamma} V_t^{1-\gamma}. \tag{14}$$

<sup>&</sup>lt;u>11Note that this endogenous separation rate</u> represents the probability that a match severs given i) the date t realisations of the productivity shocks and ii) that the match has not separated exogenously during period t. It is an increasing function of  $\bar{X}_t$ .

where  $\gamma \in (0, 1)$  and m > 0. Thus the number of employment relationships at the start of period t + 1 is

$$N_{t+1} = (1 - \rho_t) N_t + \mathcal{M}_t.$$
(15)

Denote the probability that a vacancy is filled in date t as

$$\kappa_t^f = \frac{\mathcal{M}_t}{V_t} \tag{16}$$

and the probability that an unemployed worker enters employment in period t as

$$\kappa_t^w = \frac{\mathcal{M}_t}{U_t}.\tag{17}$$

Gross job destruction is the employment relationships that separate less exogenous separations that rematch within period

$$DES_{t} = \frac{\left[\rho^{x} + (1 - \rho^{x}) F\left(\bar{X}_{t}\right)\right] N_{t} - \kappa_{t}^{f} \rho^{x} N_{t}}{N_{t}} = \rho^{x} + (1 - \rho^{x}) F\left(\bar{X}_{t}\right) - \kappa_{t}^{f} \rho^{x}$$
(18)

Gross job creation is the flow of new matches (as a fraction of existing employment) less matches due to firms that filling vacancies that resulted from exogenous separations

$$CRE_t = \frac{\mathcal{M}_t - \kappa_t^f \rho^x N_t}{N_t} = \frac{\mathcal{M}_t}{N_t} - \kappa_t^f \rho^x \tag{19}$$

State transitions and the value of  $\Gamma_t^J$  and  $\Gamma_t^U$  Suppose that firms and workers obtain fixed shares of any non-negative match surplus,  $SU_t$ , where  $\eta$  is the worker's share. To determine the equilibrium values of  $\Gamma_t^U$  and  $\Gamma_t^J$  we need to consider the possible period t + 1 outcomes for unmatched firms, unmatched workers and ongoing firm-worker pairs. The value of the surplus for match *i* from production in period t + 1, is

$$SU_{it+1} = (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} \left(R_{t+1}^n\right)^{-1} \left[\frac{X_{it+1}}{\mu_{t+1}Q_{t+1}^\alpha}\right]^{\frac{1}{1-\alpha}} - a + \Gamma_{t+1}^J - \left(h + \Gamma_{t+1}^U\right)$$

Now consider a worker in the unemployment pool at date t. Her future payoff is  $h + \Gamma_{t+1}^U$  either if the worker is unsuccessful in the matching market at date t, or if she successfully matches at date t, but severs (exogenously or endogenously) prior to production at date t + 1. However, if she successfully matches in date t and the relationship survives to date t + 1, then she obtains

 $\eta SU_{t+1} + h + \Gamma_{t+1}^U$ . Appropriately discounted, the date t value of the unemployed worker's expected future payoffs is therefore

$$\Gamma_t^U = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\phi} \left[ \kappa_t^w \left( 1 - \rho^x \right) \int_{\bar{X}_{t+1}}^\infty \eta S U_{it+1} f\left( X \right) dX + h + \Gamma_{t+1}^U \right] \right], \tag{20}$$

where  $\kappa_t^w$  is the probability that she successfully matches in period t. The worker obtains  $\eta SU_{t+1}$ with probability  $\kappa_t^w (1 - \rho^x) (1 - \rho_{t+1}^n)$ , reflecting the probability that she matches in period t and that the match survives to t + 1.

Due to free entry, the value of a firm in the period t vacancy pool must be 0, so

$$0 = -\mathcal{C} + \kappa_t^f E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\phi} (1 - \rho^x) \int_{\bar{X}_{t+1}}^{\infty} (1 - \eta) S_{it+1} f(X) \, dX \right]$$
(21)

where  $\kappa_t^f$  represents the probability that the firm matches in in period t.

Finally, the present value,  $\Gamma_t^J$ , of the expected future joint returns to an ongoing employment relationship which produces both at date t and date t + 1 is

$$\Gamma_t^J = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\phi} \left[ (1 - \rho^x) \int_{\bar{X}_{t+1}}^{\infty} SU_{it+1} f\left( X \right) dX + h + \Gamma_{t+1}^U \right] \right]$$
(22)

## 2.2.2 Retail Sector

There is a continuum of retailers, with unit mass. Retail firm z acquires the wholesale good at price  $P_t^w$  and costlessly transform it into the divisible retail good z which is then either sold to domestic households or exported to the rest of the world. The market for retail goods is characterised by monopolistic competition. The aggregate demand for good z in period t is

$$Y_t(z) = c_t(z) + ex_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon} C_t + \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon} EX_t$$
$$= \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon} [C_t + EX_t]$$

where  $C_t$  denotes the composite (domestic) consumption index and  $EX_t$  denotes aggregate exports and we have assumed that the elasticity of substitution between heterogeneous domestic final goods is identical in the domestic economy and in the rest of the world.<sup>12</sup> Aggregation across goods, z, gives an expression for aggregate demand

$$Y_t = C_t + EX_t. (23)$$

 $<sup>\</sup>frac{12}{12}$  Note that the exchange rate does not appear in the export demand term because it cancels from both the numerator and the denominator.

We assume that aggregate exports are an increasing function of the real exchange rate and (exogenous) rest-of-world income  $Y_t^*$ :

$$EX_t = Q_t^{b_q} Y_t^{*b_y} \qquad a, b > 0.$$
(24)

Suppose output of final good z is demand determined and that retail goods prices exhibit nominal rigidities and follow a Calvo style adjustment scheme. Let the probability that a profitmaximising retail firm adjusts its price in a given period be  $1 - \omega$ , and define the price of retail good z at date t be  $p_t(z)$ . All firms setting price at date t face the same expected future demand and cost conditions and so choose the same price independent of z, so we write the price set by firms which adjust (price) in date t as  $p_t^*$ . The retailer's problem may be expressed as

$$\min_{p_t^*} E_t \sum_{j=0}^\infty \left(\beta\omega\right)^j \left(\frac{C_{t+j+1}}{C_t}\right)^{-\phi} \left[ \left(\frac{p_t^*}{P_{t+j}}\right)^{1-\varepsilon} - \mu_{t+j}^{-1} \left(\frac{p_\tau^*}{P_{t+j}}\right)^{-\varepsilon} \right] Y_{t+j}.$$

The first order condition for this problem is:

$$\left(\frac{p_t^*}{P_t}\right) = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \left(\beta\omega\right)^j \left(\frac{C_{t+j+1}}{C_t}\right)^{-\phi} \left[\mu_{t+j}^{-1} \left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon}\right] Y_{t+j}}{E_t \sum_{t=\tau}^{\infty} \left(\beta\omega\right)^{t-\tau} \left(\frac{C_{t+1}}{C_t}\right)^{-\phi} \left[\left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon-1}\right] Y_{t+j}}$$
(25)

The aggregate retail price index evolves according to

$$P_t^{1-\varepsilon} = (1-\omega) \left(p_t^*\right)^{1-\varepsilon} + \omega P_{t-1}^{1-\varepsilon}.$$
(26)

## 2.3 Monetary and Fiscal Policy

We assume that the government spending is zero and the government maintains a balanced budget by rebating seigniorage revenues to households in the form of lump-sum transfers. The government budget constraint is thus

$$P_t T_t = M_t^s - M_{t-1}^s$$

where  $M_t^s$  is the aggregate money stock. Money supply growth rate is assumed to evolve according to the AR(1) process

$$\theta_t = \rho_\theta \theta_{t-1} + \varepsilon_{\theta,t}. \tag{27}$$

## 2.4 Equilibrium

In equilibrium  $M_t = M_t^s$  and the government budget constraint holds so  $P_t T_t = M_t - M_{t-1}$  and the cash in advance constraint becomes

$$C_t = \frac{M_t}{P_t}.$$
(28)

Holdings of domestic bonds are normalised to zero for simplicity:  $B_t = 0$ . Following MN (2000), we assume that the wealth effects associated with net foreign asset accumulation are small. Then in equilibrium aggregate demand,  $Y_t = EX_t + C_t$ , equals aggregate wholesale output less expenditures on vacancies

$$Y_t = Y_t^w - \mathcal{C}V_t$$

Combining this information

$$C_t + EX_t = Y_t^w - \mathcal{C}V_t \tag{29}$$

Thus the system of equations governing equilibrium in the economy consists of the numbered equations (4) through to equation (29).

# 3 Simulations

We log-linearise the model about its (zero-inflation, zero growth) steady state and use impulse response analysis and dynamic simulations to tease out the dynamic structure of the economy. Parameter values are chosen to match U.S. data and are taken from previous studies.

## 3.1 Steady State & Calibration

Model calibration involves choice of several sets of parameters governing steady state values of labour market variables; nominal rigidity; household preferences and international linkages. We also specify the processes governing the evolution of idiosyncratic productivity and money supply growth. Where feasible, in order to facilitate comparison, we follow the parameterisations used by DHRW (2000), Walsh (2003) and for open economy parameters MN (2000). These authors choose values to match the behaviour of the US economy. The parameter values are summarised in Table 1, the following sections contain some discussion of the rationale for and origins of these choices.

## 3.1.1 Labour Markets, Matching and Separation

We specify the following labour market parameters  $\rho$ , N, CRE,  $\gamma$ ,  $\kappa^{f}$ , and  $\eta$ .

In steady state the probability of separation,  $\rho$ , can be written as  $\rho = \rho^x + (1 - \rho^x) F(\bar{X})$ . Substituting into equation (13) the steady state value of the number of workers searching for work in any given period is

$$U = 1 - (1 - \rho) N$$

This can be rearranged to obtain an expression for the ratio of searchers to employment U/N as

$$\frac{U}{N} = \frac{1}{N} - (1 - \rho).$$
(30)

DHRW (2000), note that, for the US, around 10% of employment relationships separate each quarter:  $\rho = 0.1$ . They assume that the fraction of workers in employment (in steady state) is set as N = 0.94, (so steady state unemployment is 6%). Therefore the ratio of workers searching for jobs to employed workers, U/N, is 0.154.

Another aspect of steady-state is that the rate of job creation (19) equals the rate of job destruction (18)

$$CRE = DES = \rho - \rho^x \kappa^f.$$

DHRW (2000) set steady state job creation, CRE equal 5.2%, and the average probability of filling a vacancy,  $\kappa^f$ , at 0.7, so that the probability of exogenous separation,  $\rho^x = 0.068$ . In steady state, the average probability of filling a vacancy depends on the ratio of workers engaged in search to posted vacancies: equation (16) becomes

$$\kappa^{f} = \frac{mU^{\gamma}V^{1-\gamma}}{V} = m\left(\frac{U}{N}\right)^{\gamma}\left(\frac{V}{N}\right)^{-\gamma}.$$
(31)

The steady state condition for the employment evolution equation (15) is

$$\rho = m \left(\frac{U}{N}\right)^{\gamma} \left(\frac{V}{N}\right)^{1-\gamma}.$$

Using equation (31) and re-arranging we determine the ratio of vacancies to employment as

$$\frac{V}{N} = \frac{\rho}{\kappa^f}.$$

Then  $\frac{V}{N} = \frac{1}{7}$ . We assume, following Petrongolo and Pissarides (2001), that the parameter  $\gamma$  in the matching function takes the value 0.6. Therefore m = 0.68.

The steady state value of the separation threshold can be determined from equations (9) and (10). This determines the steady state endogenous probability of separation  $\rho^n = 0.036$ . By specifying the distribution of idiosyncratic productivity shocks (see below) the value of  $\bar{X}$  is determined.

The share of any match surplus obtained by workers is set, following the literature, at  $\eta = 0.5$ . Then the steady state of equation (21) can be used to determine the cost of posting a vacancy, C. To do this we also need to know the value of the elasticity of output with respect to imports,  $\alpha$ . We return to this after discussing price setting and preferences.

## 3.1.2 Price Rigidity & Price Setting

Calibration of nominal rigidities and price setting by retailers involves specification of  $\omega$ , P and  $\varepsilon$ .

The extent of nominal rigidity in the goods market is completely determined by  $\omega$ , which captures the fraction of retailers each period that do not adjust their price. Empirical evidence from studies using aggregate data suggests that prices last for 9-12 months on average - consistent with  $\omega \in [2/3, 3/4]$ . We take  $\omega = 3/4$  as a baseline value.

In steady state, from equation (26), price-setting firms set price,  $p^*$ , equal to the aggregate price level, P,

$$p^* = P.$$

We normalise the aggregate price level to unity. Combining this with steady state optimal price setting behaviour (from equation (25)) determines the steady state mark up as

$$\mu = \frac{\varepsilon}{(\varepsilon - 1)}.$$

We assume that,  $\varepsilon$ , the elasticity of demand equals 11, so  $\mu = 1.1$ .

## 3.1.3 Preferences

Following Walsh (2003), the preferences of the representative household are characterised by the parameters  $\phi, \beta$  and h.

In steady state, the Euler equation (4) becomes

$$1 = \beta R^n. \tag{32}$$

The discount factor reflecting the subjective rate of time preference is standard in DGE models. Under the assumption that 1 period represents 1 quarter we set  $\beta = 0.989$ . This determines  $\mathbb{R}^n$ . We assume a CRRA form for the instantaneous utility function:  $u(C_t) = \frac{C_t^{1-\phi}}{1-\phi}$ , and set  $\phi = 2$ .

The value of home production, h, is normalised to zero. Given assumptions about the probability of separation, the import elasticity of output  $\alpha$ , and the parameters of the distribution of idiosyncratic shocks, this will determine the disutility of work, a, from the steady state of equation (8), the condition which defines the endogenous separation threshold.

## 3.1.4 International Linkages

International linkages involve choice of steady state values for Q, S, and  $P^*$ , and  $\frac{IM}{Y}$  and parameters  $b_{Y^*}$  and  $b_q$ .

The steady state version of the uncovered interest parity condition (5) requires the domestic and rest of world nominal interest rates be tied together.

$$R^n = R^{n*}. (33)$$

For simplicity the steady state values of the real exchange rate, Q, the nominal exchange rate S, the foreign price level,  $P^*$  are set equal to 1.

The assumption that imports are used only as factors of production follows directly from MN (1999). They assume that output is a general CES function of imports and labour inputs, but assume that both inputs can be adjusted costlessly. To retain a reasonably simple characterisation given our more complex labour market frictions we specialise further to a Cobb-Douglas production function. Drawing on US data MN (1999) assume the steady state share of imports as a fraction of domestic GDP,  $\frac{IM}{Y}$  is 0.11.

The steady state version of the economy wide resource constraint (29) is  $Y \equiv C + EX = Y^w - CV$ . Under the assumption that  $CV \ll Y^w$ ,  $Y \simeq Y^w$ , the import:output ratio  $IM/Y \simeq IM/Y^w$ .

Parameter / Variable	Value
$\rho$	0.1
N	0.94
CRE	0.052
$\kappa^f$	0.7
$\gamma$	0.6
$\eta$	0.5
ω	0.75
$P, Q, S, P^*$	1
ε	11
$\beta$	0.989
$\phi$	2
h	0
$\frac{IM}{V}$	0.11
$b_q^I, b_{y^*}$	1
$ ho_{ heta}$	0.5
$\sigma_X$	0.15

Exchange Rates, Nominal Rigidities and Equilibrium Unemployment

Table 1: Calibration

Dividing (12) by (11), and proceeding to the steady state gives

$$\frac{IM}{Y} \simeq \frac{\alpha}{1-\alpha}.$$
(34)

This implies a value of  $\alpha$  of 0.1. So we can compute a and C as discussed in previous paragraphs. We also require that

$$IM = EX.$$

The aggregate export equation (24) takes the same form as in MN (1999). Parameters,  $b_{Y^*}$  and  $b_Q$  governing the elasticity of domestic exports with respect to the real exchange rate and foreign output are set equal to 1.

## 3.1.5 Monetary Policy & Productivity Shocks

The money supply growth process is assumed to follow an AR(1) process with the autoregressive parameter  $\rho_{\theta} = 0.5$ , following Walsh (2003). Idiosyncratic productivity shocks are log-normally distributed with mean unity. Idiosyncratic shocks are independently identically distributed across time. The standard deviation of idiosyncratic productivity shocks is set at 0.15, following Walsh (2003).

## 3.2 Results

This section describes **i**) the impulse responses of the model to a monetary shock and **ii**) evidence on key business cycle statistics obtained from stochastic simulations. The aim of these experiments is to contrast the behaviour of the *FLEX* and *SM* models and shed light on the mechanisms at work in these two economies. The parameterisation of the *FLEX* economy follows that described above except that the production function function for wholesale goods is Cobb-Douglas with constant returns to scale,  $Y_t^w = N_t^{1-\alpha} I M_t^{\alpha}$  and abstracts from idiosyncratic productivity disturbances. Household period utility takes the form  $\frac{C^{1-\phi}}{1-\phi} + \zeta \frac{(1-N)^{1-\xi}}{1-\xi}$ , where  $\xi = 1$  represents elasticity of labour supply,  $\zeta$  is an arbitrary constant and steady state hours n = 0.3 are 30% of the total time-endowment. Note that despite these modifications, the Walrasian labour market of the MN model is not, as formulated, nested as a special case of the model with labour market frictions. So our aim is simply to contrast the behaviour of the sticky price - endogenous job destruction model with a standard DGE open economy model.<sup>13</sup>

## 3.2.1 Impulse Responses

Here we examine impulse responses to a shock to money supply growth. Figure (3) shows the response in the FLEX model; Figures (4) and (5) document the behaviour of the SM model. Each time period corresponds to one quarter. We begin by discussing key characteristics of the impulse response function, and proceed to try to understand the mechanisms underlying these results.

A 1% shock to money supply growth should ultimately raise the nominal exchange rate by 2%. This occurs in both models. In the short-run positive demand shocks leads to an expansion of output and employment. Adjustment towards steady state is quicker in the *FLEX* economy. The half life of employment, inflation and the exchange rates in Figure (3) is 2-3 quarters, whereas in the *SM* economy it is 3-8 quarters. The qualitative nature of the responses exhibit similarities and

differences. In both models, inflation is front loaded, In *FLEX* the impact effect is a 0.6% rise.  $\overline{_{13}}$ There are a number of differences from MN (1999) which facilitate comparison betweem *FLEX* and *SM*. They do not differentiate between wholesale and retailers (they adopt a Yeoman-Farmer set up). They assume a CES production function. They introduce money directly into the utility function, and allow for habit persistence in consumption. They use McCallum's p-bar price adjustment rule. Finally, they adopt a Taylor style interest rate rule, where we assume an exogenous money supply growth process.

In SM it is lower at 0.37% and inflation returns to zero more slowly - this is consistent with the common overall rise in the price level of 2%. In both *FLEX* and *SM*, the nominal exchange rate exhibits delayed overshooting, consistent with Eichenbaum and Evans (1995) findings. Consistent with the differences in the inflation responses, the degree of nominal overshooting is larger when price adjustment is slower. The magnitude of the employment response is similar in both models. In *FLEX* employment is front-loaded, but in *SM* the response is hump-shaped reflecting labour market frictions.<sup>14</sup> The output response is similar to that of employment in both models and so is consistent with the stylised response of output to monetary innovations obtained from VAR studies. In one sense it is clear that augmenting the standard framework with an extra source of rigidity slows down adjustment , but we can gain insight by considering the mechanisms at work in more detail.

By inducing exchange rate depreciation, the monetary expansion raises export demand. This tends to raise demand for inputs, including imports. There is a trade-off here because the exchange rate depreciation raises the cost of imports. The exchange rate depreciates less in the *FLEX* model so imports fall less (or rise more) than in the *SM* model. The front-loaded nature of the inflation response reflects the forward looking nature of price adjustment. Under Calvo price setting, inflation is the discounted present value of marginal costs. So in both models this discounted sum declines as one scrolls forward through time. However, the impact effect is smaller under *SM* and inflation is more drawn out. This suggests that marginal costs do not rise as much in the immediate aftermath of a monetary expansion under search matching frictions as in the *FLEX* economy. Initially this seems puzzling: matching frictions delay labour market adjustment, which should force wholesalers to rely on imports which have risen in price due to more extensive exchange rate depreciation, yet as discussed imports are likely to rise more in the *FLEX* model (as the real exchange rate depreciation is smaller). To understand why costs don't rise we need to consider in more detail the labour market response in the *SM* model, Figure 5.

As discussed earlier changes in labour input can be brought about by changes in job-creation

and/or job destruction. Examining the impulse response it is clear that there is an initial spike  $_{14}$ Note that employment is a pre-determined variable under EJD and so reacts to the monetary expansion with a 1-period lag.

of job creation and a sharper, more prolonged decline in job destruction. In recognition of the increased rents (temporarily) available to matches firms open up new vacancies, some of which result in matches. At the same time job destruction declines as existing low productivity matches remain in activity - since idiosyncratic productivity shocks are transitory there is a large incentive to maintain an existing match. As a result, by the following period there are more matches to produce wholesale output, so there is little need to create more new matches after the initial burst of matching activity. Furthermore the pool of unemployed workers searching for jobs is relatively small, which reduces the probability of a firm filling a vacancy - consequently the number of vacancies declines. So the persistent employment response in the SM model lies in the large and persistent reduction in job destruction.

The job destruction decision depends on future profitability - due to the costs of (the delays in) forming new matches. This is captured in the continuation value of an existing match. This is high partly because of the disconnected nature of the price adjustment and employment decisions. Price setters, taking labour market variables as given, do not adjust prices as quickly because labour market adjustment is incomplete, and matches, taking prices as given, do not complete labour market adjustment quickly because price adjustment is incomplete. In turn this suggests that adjustment might be more rapid if the matches made both employment and price-setting decisions. The rise in labour input on the extensive margin does not lead workers to require higher compensation - which is partly responsible for the rise in marginal costs in the *FLEX* model.

## 3.2.2 Dynamic Simulations

Here we analyse the business cycle statistics obtained from stochastic simulations. Following the approach of Chari, Kehoe and McGrattan (2002), we focus on the ability of monetary shocks alone to account for observed volatility of key variables at business cycle frequencies. In each case the standard deviation of monetary shocks  $\sigma_{\varepsilon_{\theta}}$  is chosen to match the volatility of (HP-detrended) output in US data. Business cycle statistics are obtained by averaging across 100 simulations.

Table 2 reports exchange rate and inflation data for US data,<sup>15</sup> results for *FLEX* and *SM* for the  $_{15}$ Labour market data is taken from DHRW (2000), for the period 1959:1 - 1996:4. Exchange rate data comes from CKM (2002)) and covers the period 1973:1 - 1994:4.

baseline parameterisation where prices last on average for 1 year, and an alternative parameterisation of SM where prices last on average for 6 months, consistent with Bils and Klenow's (2002) evidence. Table 3 reports labour market data for the same parameterisations.

In terms of ability to match the observed volatility of real and nominal exchange rate, the EJD model performs substantially better than the MN model, while producing substantial persistence in both real and nominal exchange rate fluctuations in response to monetary shocks. Indeed, on the baseline parameter values (with a level of price stickiness not supported by the micro data), the EJD model produces too much persistence in the exchange rate. While none of the models captures the contemporaneous cross-correlation patterns of real and nominal exchange rates- although here too the EJD model seems to outperform the others.

The EJD model comes closest to matching historical patterns of volatilities of employment and jobs flows. Compared with the MN model, EJD recreates more accurately the volatility patterns for employment: In particular, employment simply responds too much to monetary shocks. Equally without the endogenous job destruction feature the SM model produces lower employment variability than in either the EJD approach or the data. Compared with the SM model which lacks the endogenous job destruction feature, the benchmark EJD model provides an improved match to the volatility of job destruction, but predicts job creation volatility to be almost twice the value observed in the data whereas the SM model matches the job creation volatility in US data but produces a job destruction volatility only one half as large as that observed in the data. This tendency of models with endogenous job destruction to produce overly volatile job-creation is noted by Den Haan, Ramey and Watson in the context of productivity shocks; clearly this anomaly carries over to an environment (with price-stickiness) in which monetary shocks are the prime driving force for economic fluctuations. While the origins of the job creation volatility anomaly is interesting in its own right it is not the principal focus here. Suffice to say that combined with the improved persistence properties outlined in the impulse response analysis of the previous section it appears that endogeneity of job destruction appears an attractive starting point for understanding labour market behaviour at business cycle frequencies in monetary economies.

It appears that while the EJD model comes closest to matching US data, all the models fail

Statistic	Data	FLEX	$SM \\ \omega = 3/4$	$SM$ $\omega = 1/2$
Standard Deviation				
w.r.t. output				
Inflation	4.362	2.502	4.039	5.891
Real Exchange Rate	4.676	2.004	23.522	6.643
Nominal Exchange Rate	0.671	0.405	0.332	0.455
1 <sup>st</sup> Order Serial Correlation				
Real Exchange Rate	0.831	0.985	0.788	0.901
Nominal Exchange Rate	0.866	0.983	0.999	0.957
Cross Correlation				
Real-Nominal Exchange Rate	0.991	0.499	0.344	0.627

Exchange Rates, Nominal Rigidities and Equilibrium Unemployment

Table 2: Business Cycle Statistics

to a reasonable job of matching the (relative) volatility of consumption, at least in comparison to the work of Chari, Kehoe and McGrattan. However, their work incorporates capital stock and allows for adjustment costs in investment activity. In fact they choose the capital adjustment cost parameter in order to match the relative volatility of consumption and investment to GDP.

Next we examine the robustness of the EJD results to variations in the model and the baseline parameterisation. With such a rich framework there are numerous avenues for explanation. We focus here on just two - the impact of shocks from other sources and of the extent of nominal rigidity. The results are summarised in Table 3.

Clearly monetary shocks are not the only source of variation in an economy. Here we allow for aggregate productivity shocks (using a baseline RBC parameterisation with  $\rho_z = 0.95$  and  $\sigma_{\varepsilon_z} = 0.007$ .) The standard deviation of monetary shocks is then chosen to match US output variability, other parameters (in particular the degree of price rigidity) remain at the baseline values. Productivity shocks and monetary shocks are assumed to be independent. The introduction of (persistent) aggregate productivity shocks tends to reduce the variability of all the variables presumably to match US levels of real and nominal exchange rate variability one should reduce the extent of price stickiness. Finally, and unsurprisingly, the greater persistence of the exogenous forcing variables raises the persistence of the simulated exchange rate series.

The last three columns of Table 3 report the results of allowing for variation in the extent of price stickiness (with only monetary shocks). The final column corresponds to the baseline

Standard Devaition w.r.t. output	Data	FLEX	$SM\\ \omega = 3/4$	$SM \\ \omega = 1/2$
Employment	0.671	2.161	0.157	0.455
Job Creation	2.895	-	2.704	4.918
Job Destruction	4.868	-	2.163	4.863

Exchange Rates, Nominal Rigidities and Equilibrium Unemployment

Table 3: Labour Market Statistics: Cross Model Comparison

parameterisation of the EJD model in Table 2. Only monetary shocks are present and it appears that reducing the extent of price stickiness to value consistent with the microeconomic data tends to impair the EJD model's ability to explain the real and nominal exchange rate volatility, the persistence properties of the nominal exchange rate, the contemporaneous correlation of real and nominal exchange rates and net export volatility.

# 4 Conclusions

In this paper we examined the dynamic behaviour of a small open economy model with search and matching friction in the labour market and nominal rigidities in the goods market. The model we developed is rich enough to address a wide range of issues, but, in the spirit of Dornbusch (1976), we focussed on the behaviour of exchange rates in response to (monetary) shocks. The combination of matching frictions in the labour market and nominal price rigidity in the goods market provides a much better account of the persistence of real and nominal variables than available in the current generation of DGE monetary models based on nominal price stickiness alone, e.g. Gali and Monacelli (2003), McCallum and Nelson (1999). Reassuringly, the existence of an exchange rate channel for monetary transmission does not overturn the persistent response to monetary shocks obtained in an analogous closed economy model, Walsh (2003). Instead the model outperforms existing international monetary DGE models with nominal rigidities in terms of its ability to replicate both the persistence and the volatility of real (and nominal) exchange rates. So it appears that a combination of real and nominal rigidities may help to understand a variety of exchange rate puzzles posed by the existing literature. However work remains to be done as the model poses difficulties for understanding the response of some variables including net exports.

Future work should focus on deepening our understanding of the mechanisms at work within

the model and analysing the robustness of the results. For instance, there are several channels for monetary transmission within the model: the traditional interest rate channel, the exchange rate channel and the cost channel. It would seem to be important to analyse the extent to which each contributes to the persistence and volatility results obtained above, as well as the consider the robustness of these in the face of parameter variations. Another issue to be addressed is the behaviour of the exchange rate when monetary policy follows an interest rate rule, for example a Taylor rule. Besides the impact of domestic monetary disturbances , one might also study response of the economy to disturbances such as deviations from UIP or foreign price / output shocks. Finally, the present paper makes specific assumptions about the form of exchange rate pass through, it would be of interest to examine how robust the results are to variations in the extent of exchange rate pass-through.

# References

- Andolfatto, D., (1996), 'Business Cycles and Labour Market Search', American Economic Review, 86 (1), pp112-132.
- [2] Betts, C. and M. B. Devereux, (1999), "Exchange Rate Dynamics in a Model of Pricing to Market", Journal of International Economics, 50, 215-244.
- [3] Bils, M. and P. Klenow, "Some Evidence on the Importance of Sticky Prices", NBER Working Paper # 9069.
- [4] Christiano, L., M. Eichenbaum and C. Evans (2001), "Nominal Rigidities and the Dynamic Effects of Shock to Monetary Policy.", NBER Working Paper 8403.
- [5] Chari, V. V., P. J. Kehoe and E. R. McGrattan, (2002), "Can Sticky Price Models generate Volatile and Persistent Real Exchange Rates?" *Review of Economic Studies*, 69, pp533-563.
- [6] Cheron, A. and P. Langot, (1999), "The Phillips Curve and Beveridge Curves Revisited", *Economics Letters*, 69, pp371-376.
- [7] Cogley, T. and J. Nason, (1995), "Output Dynamics in Real Business Cycle Models", American Economic Review, 85, pp492-511.
- [8] Cooley, T., and V. Quadrini (1999), "A Neoclassical model of the Phillips Curve Relation", Journal of Monetary Economics, 44(2), pp165-193.
- [9] Den Haan, W. J., G. Ramey and J. Watson, (2000), "Job Destruction and the Propagation of Shocks", American Economic Review, 90(3) pp482-498.
- [10] Dornbusch, R., (1976, "Expectations and Exchange Rate Dynamics", Journal of Political Economy, 84, 1161-76.
- [11] Dotsey, M., and R. G. King (2001), "Pricing, Production and Persistence", NBER Working Paper 8407.
- [12] Eichenbaum, M. and C. Evans, (1995), "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates", *Quarterly Journal of Economics*, **110**, 975-1004.
- [13] Feve, P. and F. Langot, (1996), "Unemployment and the Business Cycle in a Small Open Economy", Journal of Economic Dynamics and Control, 20, pp1609-1639.
- [14] Gali, J., (2003), "New Perspectives on Monetary Policy, Inflation and the Business Cycle", in M. Dewatripont, L. Hansen and S. Turnovsky (Eds.), Advances in Economic Theory, Cambridge University Press, Cambridge, UK.
- [15] Gali, J., and M. Gertler, (1999), "Inflation Dynamics: A Structural Econometric Analysis", Journal of Monetary Economics, 44, 195-222.
- [16] Gali, J. and T. Monacelli, (2002), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", NBER Working Paper #8905.
- [17] Goldberg, P. K., and M. Knetter, (1997), "Goods Prices and Exchange Rates: What Have We Learned?", Journal of Economic Literature, 35, pp1243-1272.
- [18] Hall, R. E., (1999), "Labour Market Frictions and Employment Fluctuations", in J. Taylor and M. Woodford, *Handbook of Macroeconomics*, Vol. 1C, North Holland, Amsterdam
- [19] Hairault, J-O. (2002), "Labour Market Search and International Business Cycles", Review of Economic Dynamics, 5, pp535-558.
- [20] Kollmann, R., (2001) "The Exchange Rate in a Dynamic Optimising Model with Nominal Rigidities: A Quantitative Investigation", *Journal of International Economics*, 55, 243-262.
- [21] Lane, P. (1997), "Inflation in Open Economies", Journal of International Economics, 42, pp327-47.
- [22] McCallum, B., and E. Nelson, (1999), "Nominal Income Targeting in an Open-economy Optimising Model", Journal of Monetary Economics, 43, 553-578.

- [23] McCallum, B., and E. Nelson, (2000), "Monetary Policy for an Open Economy: An Alternative Framework with Optimising Agents and Sticky Prices", Oxford Review of Economic Policy, 16, (4), pp 74-91.
- [24] Merz, M., (1995), Search in the Labour Market and the Real Business Cycle", Journal of Monetary Economics, 36, (2), 269-300.
- [25] Mortensen, D., and C. Pissarides, (1999), "Job Reallocation, Employment Fluctuations and Unemployment", in J. Taylor and M. Woodford, *Handbook of Macroeconomics*, Vol. 1C, North Holland, Amsterdam.
- [26] Petrongolo, B., and C. Pissarides, (2001), "Looking into the Black Box: A Survey of the Matching Function", *Journal of Economic Literature*, **39**, 390-431.
- [27] Walsh, C. E. (2003), "Labour Market Search and Monetary Shocks", in S. Altug, J. Chadha and C. Nolan, (Eds.) *Elements of Dynamic Macroeconomic Analysis*, Cambridge University Press, Cambridge, UK.



Figure 1:



Figure 2:



FIGURE 3: FLEX RESPONSES TO UNIT SHOCK TO MONEY GROWTH

Figure 3:



FIGURE 4: SM RESPONSES TO UNIT SHOCK TO MONEY GROWTH

Figure 4:



Figure 5: