COUNCIL FOR QUALITY ASSURANCE IN GENERAL AND FURTHER EDUCATION AND TRAINING

Exemplar Book on Effective Questioning

Mathematics

Compiled by the Statistical Information and Research (SIR) Unit

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PREFACE

The National Senior Certificate (NSC) examinations are set and moderated in part using tools that specify the types of cognitive demand and the content deemed appropriate for Mathematics at Grade 12 level. Until recently, the level of cognitive demand made by a question was considered to be the main determinant of the overall level of cognitive challenge of an examination question.

However, during various examination evaluation projects conducted by Umalusi from 2008-2012, evaluators found the need to develop more complex tools to distinguish between questions which were categorised at the same cognitive demand level, but which were not of comparable degrees of difficulty. For many subjects, for each type of cognitive demand a three-level degree of difficulty designation, *easy, moderate and difficult* was developed. Evaluators first decided on the type of cognitive process required to answer a particular examination question, and then decided on the degree of difficulty, *as an attribute of the type of cognitive demand*, of that examination question.

Whilst this practice offered wider options in terms of *easy, moderate* and *difficult* levels of difficulty for each type of cognitive demand overcame some limitations of a one-dimensional cognitive demand taxonomy, other constraints emerged. Bloom's Taxonomy of Educational Objectives (BTEO)(Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956) and the Revised Bloom's Taxonomy are based on the assumption that a cumulative hierarchy exists between the different categories of cognitive demand (Bloom *et al.,* 1956; Bloom, Hastings & Madaus, 1971). The practice of 'levels of difficulty' did not necessarily correspond to a hierarchical model of increasing complexity of cognitive demand. A key problem with using the level of difficulty as an attribute of the type of cognitive demand of examination questions is that, questions recognised at a higher level of cognitive demand are not necessarily categorised as more difficult than other questions categorised at lower levels of cognitive demand. For example, during analyses a basic recognition or recall question could be considered more difficult than an easy evaluation question.

Research further revealed that evaluators often struggled to agree on the classification of questions at so many different levels. The finer categorization for each level of cognitive demand and the process of trying to match questions to pre-set definitions of levels of difficulty made the process of making judgments about cognitive challenge overly procedural. The complex two-dimensional multi-level model also made findings about the cognitive challenge of an examination very difficult for Umalusi's Assessment Standards Committee (ASC) to interpret.

In an Umalusi Report, *Developing a Framework for Assessing and Comparing the Cognitive Challenge of Home Language Examinations* (Umalusi, 2012), it was recommended that the type and level of cognitive demand of a question and the level of a question's difficulty should be analysed separately. Further, it was argued that the ability to assess cognitive challenge lay in experts' abilities to recognise subtle interactions and make complicated connections that involved the use of multiple criteria simultaneously. However, the tacit nature of such judgments can make it difficult to generate a common understanding of what constitutes criteria for evaluating the cognitive challenge of examination questions, despite descriptions given in the policy documents of each subject.

The report also suggested that the Umalusi external moderators and evaluators be provided with a framework for thinking about question difficulty, which would help them identify where the main sources of difficulty or ease in questions might reside. Such a framework should provide a common language for evaluators and moderators to discuss and justify decisions about question difficulty. It should also be used for building the capacity of novice or less experienced moderators and evaluators to exercise the necessary expert judgments by making them more aware of key aspects to consider in making such judgments.

The revised Umalusi examination moderation and evaluation instruments for each subject draw on research and literature reviews, together with the knowledge gained through the subject workshops. At these workshops the proposed revisions were discussed with different subject specialists to attain a common understanding of the concepts, tools and framework used; and to test whether the framework developed for thinking about question difficulty 'works' for different content subjects. Using the same framework to think about question difficulty across subjects will allow for greater comparability of standards across subjects and projects.

An important change that has been made to the revised examination evaluation instrument is that the analysis of *the type of cognitive demand* of a question and analysis of *the level of difficulty* of each question are now treated as two separate judgments involving two different processes. Accordingly, the revised examination evaluation instrument now includes assessment of difficulty *as well as* cognitive demand.

LIST OF ABBREVIATIONS

Abbreviation	Full name
ASC	Assessment Standards Committee
BTEO	Bloom’s Taxonomy of Educational Objectives
CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
FET	Further Education and Training
IEB	Independent Examinations Board
NSC	National Senior Certificate
NQF	National Qualifications Framework
QAA	Quality Assurance of Assessment
QCC	Qualifications, Curriculum and Certification
SIR	Statistical Information and Research

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ACKNOWLEDGEMENTS

This Mathematics exemplar book is informed by Umalusi Research Reports of previous years, especially the report by Reeves (Umalusi, 2012) titled '*Developing a framework for assessing and comparing the cognitive challenge of Home Language examinations*'.

In addition, Mathematics subject experts and practitioners are acknowledged for their contribution to the content of this exemplar book. Included in this group are: Umalusi External Moderators and Maintaining Standards Subject Teams and Team Leaders; together with the South African Comprehensive Assessment Institute and the Independent Examinations Board (IEB) Examiners and Internal Moderators.

We also acknowledge the contributions of the members of the Umalusi Quality Assurance of Assessment (QAA); Qualifications, Curriculum and Certification (QCC) and Statistical Information and Research (SIR) Units. We specifically acknowledge the contribution made by the individuals listed below:

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This exemplar book was prepared by Prof. Rajendran Govender.

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1. INTRODUCTION

The rules of assessment are essentially the same for all types of learning because, to learn is to acquire knowledge or skills, while to assess is to identify the level of knowledge or skill that has been acquired (Fiddler, Marienau & Whitaker, 2006). Nevertheless, the field of assessment in South Africa and elsewhere in the world is fraught with contestation. A review of the research literature on assessment indicates difficulties, misunderstanding and confusion in how terms describing educational measurement concepts, and the relationships between them, are used (Frisbie, 2005).

Umalusi believes that if all role players involved in examination processes can achieve a common understanding of key terms, concepts and processes involved in setting, moderating and evaluating examination papers, much unhappiness can be avoided. This exemplar book presents a particular set of guidelines for both novice and experienced Mathematics national examiners, internal and external moderators, and evaluators to use in the setting, moderation and evaluation of examinations at the National Senior Certificate (NSC) level.

The remainder of the exemplar book is organised as follows. First, the context in which the exemplar book was developed is described (Part 2), followed by a statement of its purpose (Part 3). Brief summaries of the roles of moderation and evaluation (Part 4) and cognitive demand (Part 5) in assessment follow. Examination questions selected from the NSC Mathematics examinations of assessment bodies, the Department of Basic Education (DBE), and/or the Independent Examinations Board (IEB) are used to illustrate how to identify different levels of cognitive demand as required by the Curriculum and Assessment Policy Statement (CAPS) Mathematics document (Part 6). Part 7 explains the protocols for identifying different levels of difficulty within a question paper. Application of the Umalusi framework for determining difficulty described in Part 7 is illustrated, with reasons, by another set of questions from a range of Mathematics examinations (Part 8). Concluding remarks complete the exemplar book (Part 9).

2. CONTEXT

Umalusi has the responsibility to quality assure qualifications, curricula and assessments of National Qualification Framework (NQF) Levels 1 – 5. This is a legal mandate assigned by the *General and Further Education and Training Act (58 of 2001)* and the *National Qualification Framework Act (67 of 2008)*. To operationalize its mandate, Umalusi, amongst other things, conducts research and uses the findings of this research to enhance the quality and standards of curricula and assessments.

Since 2003, Umalusi has conducted several research studies that have investigated examination standards. For example, Umalusi conducted research on the NSC examinations, commonly known as 'Matriculation' or Grade 12, in order to gain an understanding of the standards of the new examinations (first introduced in 2008) relative to those of the previous NATED 550 Senior Certificate examinations (Umalusi, 2009a, 2009b). Research undertaken by Umalusi has assisted the organisation to arrive at a more informed understanding of what is meant by assessing the cognitive challenge of the examinations and of the processes necessary for determining whether the degree of cognitive challenge of examinations is comparable within a subject, across subjects and between years.

Research undertaken by Umalusi has revealed that different groups of examiners, moderators and evaluators do not always interpret cognitive demand in the same way, posing difficulties when comparisons of cognitive challenge were required. The research across all subjects also showed that using the type and level of cognitive demand of a question *only* as measure for judging the cognitive challenge of a question is problematic because cognitive demand levels on their own do not necessarily distinguish between degrees of difficulty of questions.

The new Umalusi framework for thinking about question difficulty described in this exemplar book is intended to support all key role players in making complex decisions about what makes a particular question challenging for Grade 12 examination candidates.

3. THE PURPOSE OF THE EXEMPLAR BOOK

The overall goal of this exemplar book is to ensure consistency of standards of examinations across the years in the Further Education and Training (FET) sub-sector and Grade 12 in particular. The specific purpose is to build a shared understanding among teachers, examiners, moderators, evaluators, and other stakeholders, of methods used for determining the type and level of cognitive demand as well as the level of difficulty of examination questions.

Ultimately, the common understanding that this exemplar book seeks to foster is based on the premise that the process of determining the type and level of cognitive demand of questions and that of determining the level of difficulty of examination questions, are two separate judgments involving two different processes, both necessary for evaluating the cognitive challenge of examinations. This distinction between cognitive demand and difficulty posed by questions needs to be made in the setting, moderation, evaluation and comparison of Mathematics examination papers.

The exemplar book includes an explanation of the new Umalusi framework which is intended to provide all role-players in the setting of Mathematics examinations with a common language for thinking and talking about question difficulty. The reader of the exemplar book is taken through the process of evaluating examination questions, first in relation to determining the type and level of cognitive demand made by a question; and then in terms of assessing the level of difficulty of a question. This is done by providing examples of a range of questions, which make different types of cognitive demands on candidates, and examples of questions at different levels of difficulty.

Each question is accompanied by an explanation of the reasoning behind why it was judged as being of a particular level of cognitive demand or difficulty, and the reasoning behind the judgements made is explained. These examples of examination questions provided were sourced by Mathematics moderators from previous DBE and the IEB Mathematics question papers, pre- and post- the implementation of CAPS during various Umalusi workshops.

This exemplar book is an official document. The process of revising the Umalusi examination evaluation instrument and of developing a framework for thinking about question difficulty for both moderation and evaluation purposes has been a consultative one, with the DBE and the IEB assessment bodies. The new framework for thinking about question difficulty is to be used by Umalusi in the moderation and evaluation of Grade 12 Mathematics examinations, and by all the assessment bodies in the setting of the question papers, in conjunction with the CAPS documents.

4. MODERATION AND EVALUATION OF ASSESSMENT

A fundamental requirement, ethically and legally, is that assessments are fair, reliable and valid (American Educational Research Association [AERA], American Psychological Association [APA] and National Council on Measurement in Education [NCME], 1999). Moderation is one of several quality assurance assessment processes aimed at ensuring that an assessment is fair, reliable and valid (Downing & Haladyna, 2006). Ideally, moderation should be done at all levels of an education system, including the school, district, provincial and national level in all subjects.

The task of Umalusi examination **moderators** is to ensure that the quality and standards of a particular examination are maintained each year. Part of this task is for moderators to alert examiners to details of questions, material and/or any technical aspects in examination question papers that are deemed to be inadequate or problematic and that therefore, challenge the validity of that examination. In order to do this, moderators need to pay attention to a number of issues as they moderate a question paper – these are briefly described below.

Moderation of the technical aspects of examination papers includes checking correct question and/or section numbering, and ensuring that visual texts and/or resource material included in the papers are clear and legible. The clarity of instructions given to candidates, the wording of questions, the appropriateness of the level of language used, and the correct use of terminology need to be interrogated. Moderators are also expected to detect question predictability, for example, when the same questions regularly appear in different examinations, and bias in examination papers. The adequacy and accuracy of the marking memorandum (marking guidelines) needs to be checked to ensure that it reflects and corresponds with the requirements of each question asked in the examination paper being moderated.

In addition, the task of moderators is to check that papers adhere to the overall examination requirements as set out by the relevant assessment body with regard to the format and structure (including the length, type of texts or reading selections prescribed) of the examination. This includes assessing compliance with assessment requirements with regard to ensuring that the content is examined at an appropriate level and in the relative proportions (weightings) of content and/or skills areas required by the assessment body.

The role of Umalusi examination **evaluators** is to perform analysis of examination papers after they have been set and moderated and approved by the Umalusi moderators. This type of analysis entails applying additional expert judgments to evaluate the quality and standard of finalised examination papers before they are written by candidates in a specific year. However, the overall aim of this evaluation is to judge the comparability of an examination against the previous years' examination papers to ensure that consistent standards are being maintained over the years.

The results of the evaluators' analyses, and moderators' experiences provide the Umalusi's Assessment Standards Committee (ASC) with valuable information which, is used in the process of statistical moderation of each year's examination results. Therefore this, information forms an important component of essential qualitative data informing the ASC's final decisions in the standardisation of the examinations.

In order for the standardisation process to work effectively, efficiently and fairly, it is important that examiners, moderators and evaluators have a shared understanding of how the standard of an examination paper is assessed, and of the frameworks and main instruments that are used in this process.

5. COGNITIVE DEMANDS IN ASSESSMENT

The *Standards for educational and psychological testing* (AERA, APA, & NCME, 1999) require evidence to support interpretations of test scores with respect to cognitive processes. Therefore, valid, fair and reliable examinations require that the levels of cognitive demand required by examination questions are appropriate and varied (Downing & Haladyna, 2006). Examination papers should not be dominated by questions that require reproduction of basic information, or replication of basic procedures, and under-represent questions invoking higher level cognitive demands.

Accordingly, the Grade 12 CAPS NSC subject examination specifications state that examination papers should be set in such a way that they reflect proportions of marks for questions at various level of cognitive demand. NSC examination papers are expected to comply with the specified cognitive demand levels and weightings. NSC examiners have to set and NSC internal moderators have to moderate examination papers as reflecting the proportions of marks for questions at different levels of cognitive demand as specified in the documents. Umalusi's external moderators and evaluators are similarly tasked with confirming compliance of the examinations with the CAPS cognitive demand levels and weightings, and Umalusi's revised examination evaluation instruments continue to reflect this requirement.

Despite subject experts, examiners, moderators and evaluators being familiar with the levels and explanations of the types of cognitive demand shown in the CAPS documents, Umalusi researchers have noted that individuals do not always interpret and classify the categories of cognitive demand provided in the CAPS the same way. In order to facilitate a common interpretation and classification of the cognitive demands made by questions, the next section of this exemplar book provides a clarification of each cognitive demand level for Mathematics followed by illustrative examples of examination questions that have been classified at that level of cognitive demand.

6. EXPLANATIONS AND EXAMPLES OF QUESTIONS ASSESSED AT THE DIFFERENT COGNITIVE DEMAND LEVELS IN THE MATHEMATICS TAXONOMY ACCORDING TO CAPS

The taxonomies of cognitive demand for each school subject in the CAPS documents are mostly based on the Revised Bloom's Taxonomy (Anderson and Krathwohl, 2001) but resemble the original Bloom's taxonomy in that categories of cognitive demand are arranged along a single continuum. Bloom's Taxonomy of Educational Objectives (BTEO) (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956) and the Revised Bloom's Taxonomy imply that each more advanced or successive category of cognitive demand subsumes all categories below it. The CAPS Taxonomies of Cognitive Demand make a similar assumption (Crowe, 2012).

Note:
In classifying the type and level of cognitive demand, each question is classified at the highest level of cognitive process involved. Thus, although a particular question involves recall of knowledge, as well as comprehension and application, the question is classified as an 'analysis' question if that is the highest level of cognitive process involved. If 'evaluating' is the highest level of cognitive process involved, the question as a whole should be classified as an 'evaluation' question. On the other hand, if one of more sub-sections of the question and the marks allocated for each sub-section can stand independently, then the level of cognitive demand for each sub-section of the question should be analysed separately.

The CAPS documents for many subjects also give examples of descriptive verbs that can be associated with each of the four levels of cognitive demand. However, it is important to note that such 'action verbs' can be associated with more than one cognitive level depending on the context of a question.

In CAPS Mathematics, the four cognitive levels used to guide all assessment tasks are based on those suggested in the TIMSS study of 1999 (Table 1).

TABLE 1: THE TAXONOMY OF COGNITIVE DEMAND LEVELS FOR MATHEMATICS NSC EXAMINATIONS ACCORDING TO CAPS

Level of cognitive demand	**Type of cognitive demand**	**Explanation of categorization** **Questions which require students:**
1	Knowledge	• Straight recall (or reproduction) of previously learnt facts, formulae, rules or definition. • Straight immediate use of mathematical facts. • Identification of correct formula on the information sheet (no changing of the subject). • Reading values directly from a graph. • Appropriate use of mathematical vocabulary. • Knowledge of formulae and substitution of values into formula.
2	Routine Procedures	• Estimation and appropriate rounding of numbers. • Proofs of prescribed theorems and derivation of formulae. • Identification and direct use of correct formula on the information sheet (no changing of the subject). • Perform well-known procedures. • Simple applications and calculations which might involve a few steps. • Derivation from given information may be involved. • Identification and use (after changing of the subject) of correct formula. • Problems tend to be familiar and are generally similar to those encountered in class. • Little ambiguity about what needs to be done. • Identifying and manipulating of formulae.
3	Complex Procedures	• Problems are mainly unfamiliar. • Procedures involve higher order reasoning and/or complex calculations. • Problems mostly do not have a direct route to the solution. • These problems either require fairly complex procedures in finding the solutions or require the learner to understand and/or interpret the mathematical concept.
4	Problem Solving	• Solving non-routine, unseen problems by demonstrating higher level understanding and cognitive processes.

		• Solve problems where there is no obvious starting point or route to the solution. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Might require the ability to break the problem down into its constituent parts.

Source: CAPS (DBE, 2011a, p53.)

To facilitate reading of this section, each of the above cognitive demand levels in the Mathematics Taxonomy are explained, and the explanation is followed by at least **three** examples of questions from previous Mathematics NSC examinations classified at each of the levels of cognitive demand shown in Table 1 above. These examples were selected to represent the **best and clearest** examples of each level of cognitive demand that the Mathematics experts could find. The discussion below each example question explains the reasoning processes behind the classification of the question at that particular type of cognitive demand (Table 2 to Table 5).

Note:
Be mindful that analyses of *the level of cognitive process* of a question and *the level of difficulty* of each question are to be treated as two separate judgments involving two different processes. Therefore, whether the question is easy or difficult should not influence the categorisation of the question in terms of the type and level of cognitive demand. Questions should NOT be categorised as higher order evaluation/synthesis questions because they are difficult questions. Some questions involving the cognitive process of recall or recognition may be more difficult than other recall or recognition questions. Not all comprehension questions are easier than questions involving analysis or synthesis. Some comprehension questions may be very difficult, for example explanation of complex scientific processes. For these reasons, you need to categorise the level of difficulty of questions separately from identifying the type of cognitive process involved.

TABLE 2: EXPLANATION AND EXAMPLES KNOWDGE LEVEL 1 OF COGNITIVE DEMAND QUESTIONS USING THE MATHEMATICS TAXONOMY

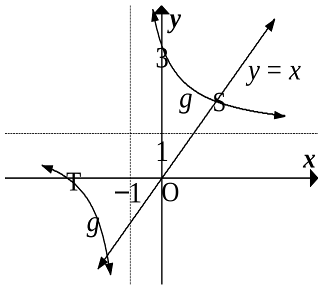
Example 1:	
Question 11.1, DBE Mathematics 2015, Paper 2 Complete the following statement: If the sides of two triangles are in the same proportion, then the triangles are. (1)	
Discussion: The question requires candidates to remember a theorem associated with similar triangles, which they would have done in class. They are expected to recognise the theorem and recall the enunciation of the theorem from memory, and use it to complete the given geometrical statement. This question is therefore categorised as knowledge.	
Memorandum/Marking guidelines Equiangular or similar	(✓answer)
Example 2:	
Question 2.1, DBE Mathematics 2015, Paper 1 The following geometric sequence is given: 10; 5; 2,5 ; 1,25 ; … Calculate the value of the 5th term, T_5, of this sequence.	

Discussion:
In this question candidates are explicitly informed that the given sequence is a geometric sequence and all they need to remember from the extensive examples done in class is that they would need to use the nth term formula for a geometric sequence to assist them to calculate the value of the 5th term, T_5, of this given geometric sequence. Moreover, the candidates need to identify/select the nth term formula, $T_n = ar^{n-1}$, for a geometric sequence which is given the given formula sheet and use it to calculate the value of the 5th term. To proceed with the substitution into the formula candidates need to identify that $n = 5$, $a = 10$, and recall that r, which is the common ratio of a geometric sequence can be easily obtained by dividing T_2 by T_1, i.e. $r = \frac{5}{10} = \frac{1}{2}$. As this question primarily requires the candidate to have knowledge of the nth term formula of a geometric sequence, and substitute the values of n, a and r appropriately into the formula to calculate T_5, this question is therefore categorised as 'Knowledge'.

Memorandum/Marking guideline	
$r = \frac{T_2}{T_1}$ $= \frac{5}{10}$ $= \frac{1}{2}$	✓ $r = \frac{1}{2}$
$T_5 = 1{,}25\left(\frac{1}{2}\right)$ $= \frac{5}{8}$ or $0{,}625$ **OR/*OF*** $T_5 = 10\left(\frac{1}{2}\right)^4$ $= \frac{5}{8}$ or $0{,}625$	✓answer (2)

Example 3:
Question 4.1, DBE Mathematics 2014, Paper 1   The diagram below shows the hyperbola g defined by $g(x) = \frac{2}{x+p} + q$

with the asymptotes *y* = 1 and x = – 1. The graph of *g* intersects the *x*-axis at T and the *y*-axis at (0; 3). The line *y* = *x* intersects the hyperbola in the first quadrant at S.



4.1 Write down the values of *p* and *q*. (2)

<u>Discussion</u>:

Question 4.1 is a sub-question amongst a broader range of questions that varies in cognitive demand. In particular, Question 4.1 requires candidates to know that for any hyperbola of the form $g(x) = \frac{a}{x+p} + q$ has two asymptotes:

- A vertical asymptote given by $x = -p$
- A horizontal asymptote given by $y = q$.

In this particular case, candidates are informed that the given diagram shows the hyperbola *g* defined by $g(x) = \frac{2}{x+p} + q$ with the asymptotes y = 1 and x = –1.

Hence, in this case candidates should work backwards to recognize that: $p = 1$ since the vertical asymptote $x = -1$, and $q = 1$ since the horizontal asymptote is y=1.

Furthermore, Grade 12 learners would have been exposed to this type of questions extensively in class, and they simply need to recall how to establish the values of p and q when given the asymptotes of the hyperbola. Therefore, this question is categorized as 'knowledge'.

<u>Memorandum/Marking guideline</u>	
$p = 1$ $q = 1$	✓*p* value ✓*q* value (2)

TABLE 3: EXPLANATION AND EXAMPLES OF THE ROUTINE PROCEDURES LEVEL 2 OF COGNITIVE DEMAND QUESTIONS USING THE MATHEMATICS TAXONOMY

Example 1:

Question 8.1, DBE Mathematics 2015, Paper 1

If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

Discussion:

Determining $f'(x)$ from first principles can be accomplished through performing well known procedures as outlined in the following steps:

Step 1: Candidates must write down the formula for finding the gradient from fist principles, namely: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Note in this instance, this formula is given in the formula sheet, and candidates should recognise it and copy it correctly into the answer book.

Step2: Write down the given $f(x)$ and then determine $f(x+h)$:

$f(x) = x^2 - 3x$

$\therefore f(x+h) = x^2 + 2xh + h^2 - 3x - 3h$

Step 3: Determine $f(x+h) - f(x)$:

$$f(x+h) - f(x) = x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)$$
$$= 2xh + h^2 - 3h$$

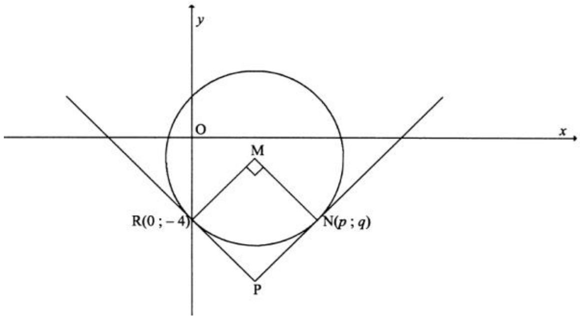
Step 4: Substitute the expression for $f(x+h) - f(x)$ into the formula and then simplify the expression to evaluate the limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$
$$= \lim_{h \to 0} \frac{h(2x + h - 3)}{h}$$
$$= \lim_{h \to 0} (2x + h - 3)$$
$$= 2x - 3$$

This kind of question is presented across most prescribed textbooks and discussed by most teachers in their classrooms generally using above kind of well- known steps, which embraces substitution into a formula, simplification of expressions and evaluation of a limit, this Question 8.1 can be categorized as 'Routine Procedure'.

Memorandum/Marking guideline	
	✓finding $f(x+h)$

$f(x+h)=(x+h)^2-3(x+h)$ $=x^2+2xh+h^2-3x-3h$ $f(x+h)-f(x)=x^2+2xh+h^2-3x-3h-(x^2-3x)$ $=2xh+h^2-3h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$ $=\lim_{h\to 0}(2x+h-3)$ $=2x-3$ **OR** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$ $=\lim_{h\to 0}(2x+h-3)$ $=2x-3$	✓ $2xh+h^2-3h$ ✓formula ✓factorisation ✓answer (5) ✓formula ✓finding *f*(x+h) ✓ $2xh+h^2-3h$ ✓factorisation ✓answer (5)
Example 2:	
<u>Question 6.1, DBE Mathematics 2013, Paper 2</u> In the diagram below, M is the centre of the circle having the equation. $x^2+y^2-6x+2y-8=0$. The circle passes through R(0;-4) and N$(p;q)$. $R\widehat{M}N=90°$. The tangents drawn to the circle at R and N meet at P.	



Show that M is the point (3;-1).

Discussion:

The given information and diagram is given to answer 7 sub-questions (Questions 6.1-6.7) in Question 6. However, at first read, one would immediately realize that all of the given information including the diagram is not really essential to answer Question 6.1.

The given information, namely M is the centre of the circle having the equation $x^2 + y^2 - 6x + 2y - 8 = 0$, is necessary and sufficient to answer Q6.1. To find the coordinates of M, which is the centre of the circle, candidates are expected to manipulate the equation $x^2 + y^2 - 6x + 2y - 8 = 0$ to the form $(x - a)^2 + (y - b)^2 = r^2$, by using the process of completing the square.

To do this, candidates must first move the constant term over to the RHS of the equation and group the terms having the variable x together and those having the variable y together respectively on the LHS as follows:

$\therefore x^2 - 6x + y^2 + 2y = 8$

Secondly, candidates are expected to complete the square for the expressions in x and y separately (i.e. make $x^2 - 6x$ into a square and $y^2 + 2y$ into a square).

To do this, candidates must first calculate the square of half of the coefficient of x and square of half of the coefficient of y as follows:

$[\frac{1}{2} coefficient\ of\ x]^2$ = $[\frac{1}{2}(-6)]^2$ =$[(-3)]^2 = 9$ and
$[\frac{1}{2} coefficient\ of\ y]^2$=$[\frac{1}{2}(2)]^2$= 1

Now candidates are expected to add the above constants to both sides of the equation as follows:

$\therefore x^2 - 6x + 9 + y^2 + 2y + 1\ = 8 + 9 + 1$

Then candidates are expected to form both perfect squares on LHS and simplify the RHS and then compare it to the form $(x - a)^2 + (y - b)^2 = r^2$ to see that the coordinates of M are $(3; 1)$, as illustrated hereunder:

$(x - 3)^2 + (y + 1)^2 = 18$

$\therefore (x-3)^2 + (y-(-1))^2 = 18$, which is in the form$(x-a)^2 + (y-b)^2 = r^2$,
$\therefore M(3; 1)$

Now determining the coordinates of M, which is centre of the circle, using the given information, is generally similar to those encountered in class and most textbooks, and embraces the use of a set of well-known procedures to manipulate the equation $x^2 + y^2 - 6x + 2y - 8 = 0$ to the form$(x-a)^2 + (y-b)^2 = r^2$ by using the process of completing the square. Hence, this question is categorized as 'Routine Procedure'.

Memorandum/Marking guideline

$$x^2 + y^2 - 6x + 2y - 8 = 0$$
$$x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$$
$$(x-3)^2 + (y+1)^2 = 18$$
$$\therefore \text{M}(3\ ; -1)$$

If only $(x-3)^2 + (y+1)^2 = r^2\ (r^2 \neq 18)$, then 2 marks

✓ $x^2 - 6x + 9$
✓ $y^2 + 2y + 1$
✓ $(x-3)^2$
✓ $(y+1)^2$
(4)

OR

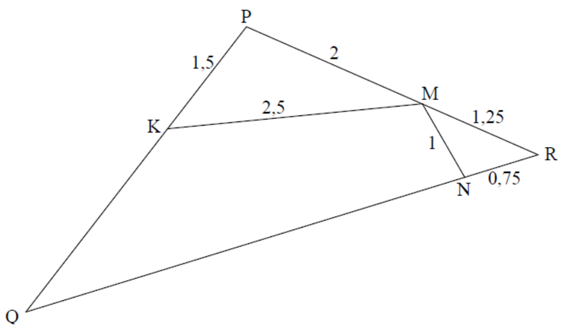
$$x_M = -\frac{1}{2}(\text{coefficient of } x)$$
$$x_M = -\frac{1}{2}(-6)$$
$$x_M = 3$$
$$y_M = -\frac{1}{2}(\text{coefficient of } y)$$
$$y_M = -\frac{1}{2}(2)$$
$$y_M = -1$$
$$\therefore \text{M}(3\ ; -1)$$

✓ $x_M = -\frac{1}{2}(-6)$
✓ $x_M = 3$
✓ $y_M = -\frac{1}{2}(2)$
✓ $y_M = -1$
(4)

Example 3:

Question 11.2.1, DBE Mathematics 2015, Paper 2

11.2	In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ∆PQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1,25 and NR = 0,75.



Prove that ∆KPM | | | ∆RNM (3)

<u>Discussion</u>:
In this question candidates are given the lengths of the sides of triangles KPM and RNM respectively. They need to reflect on typical examples done in class, and realize that they can prove that $\Delta KPM///\Delta RNM$ by showing that corresponding sides of these two triangles are in the same proportion. This demonstration involves a well-known procedure of calculating the ratio of the corresponding sides by taking the longest side length of ΔKPM and dividing by the longest side length of ΔRNM, then dividing shortest side length of ΔKPM by the shortest side length of ΔRNM, followed the division of the remaining side length ΔKPM by the remaining side length of ΔRNM, and finally concluding that the all three ratios of corresponding sides are equal (if they indeed are so). As this constitutes a well-known procedure embracing a simple application of a similar triangle theorem and calculations which involve a few steps, which are familiar and generally similar to those encountered in class, this question is categorised as routine procedure.

<u>Memorandum/Marking guideline</u>	
$\frac{KP}{RN}=\frac{1,5}{0,75}=2$; $\frac{PM}{NM}=\frac{2}{1}=2$; $\frac{KM}{RM}=\frac{2,5}{1.25}=2$	✓✓✓
$\therefore \frac{KP}{RN}=\frac{PM}{NM}=\frac{KM}{RM}$	all 3 statements
∴ ∆KPM | | | ∆RNM [Sides of ∆ in prop]	(3)
OR/*OF*	

$\frac{RN}{KP}=\frac{0,75}{1,5}=\frac{1}{2}$; $\frac{NM}{PM}=\frac{1}{2}$; $\frac{RM}{KM}=\frac{1,25}{2,5}=\frac{1}{2}$ $\therefore \frac{RN}{KP}=\frac{NM}{PM}=\frac{RM}{KM}$ ∴ ΔKPM | | | ΔRNM [Sides of Δ in prop] **OR** In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90°$ [converse Pyth theorem] cos $P\hat{K}M=\frac{1,5}{2,5}=\frac{3}{5}$ and cos $\hat{R}=\frac{0,75}{1,25}=\frac{3}{5}$ $\therefore P\hat{K}M=\hat{R}$ In ΔKPM and ΔRNM $P\hat{K}M=\hat{R}$ [proved] $\hat{P}=M\hat{N}R$ [proved] ∴ΔKPM | | |ΔRNM [∠;∠;∠ OR 3rd ∠]	✓✓✓ all 3 statements (3) ✓ $\hat{P}=M\hat{N}R$ ✓ $P\hat{K}M=\hat{R}$ ✓[∠;∠;∠ OR 3rd ∠] (3)

TABLE 4: EXPLANATION AND EXAMPLES OF THE COMPLEX PROCEDURES LEVEL 3 OF COGNITIVE DEMAND QUESTIONS USING THE MATHEMATICS TAXONOMY

<table>
<tr><td colspan="2">Example 1:</td></tr>
<tr><td colspan="2"><u>Question 11.2.3, DBE Mathematics 2015, Paper 1</u>

The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

The arrangements must start with a vowel and end in a consonant and repetition of letters is allowed.</td></tr>
<tr><td colspan="2"><u>Discussion</u>:

This problem involves higher order reasoning as there is no obvious route to the solution. Candidates need to analyse the given in formation and conceptually understand what they must do in order to distil a strategy to solve this problem. In this problem candidates need to distinguish the vowels from the consonants in the given word and then establish that there are three vowels and 4 consonants. Candidates need to reason that as there are three vowels, there are then 3 options for the first position. Similarly, as there are 4 consonants there are then 4 options of the last position. More importantly and complex, is that candidates must come to see that there are five remaining positions and that these positions can be filled in by the five remaining letters in 5 ways (i.e. 5x 4 x4x2x1 ways) and this requires conceptual understanding of the arrangement of the new 'word', which consists of seven letters abiding to the given conditions. Finally, within the context of the counting principle candidates need to put all the sub-aspects and elements together, to finally establish that the number of different arrangements can be expressed as follows: $3\times(5\times4\times3\times2\times1)\times4=1440$.
Hence, this question is categorised as 'Complex Procedure'.</td></tr>
<tr><td><u>Memorandum/Marking guideline</u>

There are 3 vowels $\Rightarrow$3 options for first position

There are 4 consonants $\Rightarrow$4 options for last position

The remaining 5 letters can be arranged in $5\times4\times3\times2\times1$ ways

$3\times(5\times4\times3\times2\times1)\times4=1440$</td><td>✓×3

✓×4

✓$5\times4\times3\times2\times1$

✓answer

(4)</td></tr>
</table>

Example 2:	
Question 8.4, DBE Mathematics 2014, Paper 1 Given: $f(x)=2x^3-2x^2+4x-1$. Determine the interval on which *f* is concave up. (4)	
Discussion: This question requires in-depth understanding of the concept of concavity. Candidates need to have both a visual and conceptual understanding of what it means to say that a function is concave up in a given interval or concave down in a given interval. To answer this question, candidates need to be conversant with the logical procedure of determining the interval on which a function is concave up. This higher order reasoning process entails the following procedural steps: Candidates must first determine $f'(x)$, and then calculate $f''(x)$, and finally solve $f''(x)>0$. Using the steps described, the candidate is expected to do produce the response like the one presented in the marking guideline. As this question, which revolves a new concept in the CAPS curriculum, involves higher order reasoning and invokes the use of the second derivative test to determine the interval on which f is concave up, it is categorised as 'Complex Procedure'.	
Memorandum/Marking guideline $f(x)=2x^3-2x^2+4x-1$ $f'(x)=6x^2-4x+4$ $f''(x)=12x-4$ f is concave up when $f''(x)>0$ $\therefore 12x-4>0$ $12x>4$ $x>\frac{1}{3}$	✓first derivative ✓ second derivative ✓ $f''(x)>0$ ✓ $x>\frac{1}{3}$ (4)
Example 3:	
Question 5(b), IEB Mathematics 2014, Paper 2 Determine the general solution to: $3sin\theta.sin22°=3cos\theta.cos22°+1$ (6)	
Discussion: In determining the general solution to $3sin\theta.sin22°=3cos\theta.cos22°+1$, there is not an obvious route to the solution. Candidates need to first establish how they can simplify the given trigonometric equation to a form where the sine	

of angle = a number or the cosine of an angle = a number or tan of an angle = a number.

Now this invokes higher order reasoning and intuitive manipulations. Firstly, candidates need to transpose the trigonometric term $3cos\theta.cos22°$ to the LHS to give the following resultant equation: $3\sin\theta.\sin 22° - 3\cos\theta.\cos 22° = 1$. The candidate needs to make a significant connection between the compound angle formula, $\cos(\alpha+\beta) = cos\alpha cos\beta - sin\alpha sin\beta$, and $3sin\theta.sin22° - 3cos\theta.cos22$. In order to do this the candidate needs to first recognize that $3sin\theta.sin22° - 3cos\theta.cos22$ can be expressed in the following form: -3$(-sin\theta.sin22° + cos\theta.cos22)$ = -3$(cos\theta.cos22 - sin\theta.sin22°)$. On succeeding with this manipulation, the candidate is most likely to see that he could apply the compound angle formulae $\cos(\alpha+\beta) = cos\alpha cos\beta - sin\alpha sin\beta$ to reduce $cos\theta.cos22° - sin\theta.sin\ 22°$ to the form $(\cos(\theta + 22°)$ and consequently arrive at $-3(\cos(\theta+22°)) = 1$, which is significant step in path to finding the required general solution. Thereafter, the candidate writes $-3(\cos(\theta+22°)) = 1$ in the form $\cos(\theta+22°) = -\frac{1}{3}$, which is in the form where

the cosine of an angle = a number. Now from here, the candidate needs to know that since cosine of the angle, $(\theta + 22°)$, is negative they must determine the reference angle (which in this case is 70,5°), and hence use it solve for θ in both quadrants 2 and 3, which will then enable them to derive the general solution as shown below:

$\theta + 22 = 109,5 + k.360$ or $\theta + 22 = 109,5 + k.360$

$\therefore\ \theta = 87,5° + k.360°, k \in Z$ or $\therefore\ \theta = 228,5° + k.360°, k \in Z$

Alternatively:

$\theta + 22° = \pm 109,5° + k.360°;\ k \in Z$

$\theta = 87,5° + k.360°;\ k \in Z$ or $\theta = -131,5° + k.360°;\ k \in Z$

As discussed above, we see that this question requires higher order reasoning connected with the use of compound angles to help to reduce the given trigonometric equation to the form $\cos(\theta+22°) = -\frac{1}{3}$, and the subsequent solving of the trigonometric equation, underpinned by good degree of conceptual understanding and higher order reasoning involving multiple steps. Hence, this question is classified as 'Complex Procedure'.

Memorandum/Marking guideline

$3\sin\theta.\sin 22° = 3\cos\theta.\cos 22° + 1$ **OR** $\theta + 22 = 109,5 + k.360$

$3\sin\theta.\sin 22° - 3\cos\theta.\cos 22° = 1$ ✓A	$\theta + 22 = 250,5 + k.360$
$-3(\cos(\theta + 22°)) = 1$ ✓A	$\therefore \theta = 87,5 + k.360$ ✓A
$\cos(\theta + 22°) = -\frac{1}{3}$ ✓A	or $\theta = 228,5 + k.360; k \in Z$ ✓A
$\theta + 22° = \pm 109,5° + k.360°;\ k \in Z$ ✓A	
$\theta = 87,5° + k.360°;\ k \in Z$ or $\theta = -131,5° + k.360°;\ k \in Z$ ✓A✓A	(6)

TABLE 5: EXPLANATION AND EXAMPLES OF THE PROBLEM-SOLVING LEVEL 4 OF COGNITIVE DEMAND QUESTIONS USING THE MATHEMATICS TAXONOMY

Example 1:

Question 1.1.4, DBE Mathematics 2015, Paper 1

Given: $(3x - y)^2 + (x - 5)^2 = 0$

Solve for x and y.

Discussion:

This question covers the sub-topic simultaneous equations. Normally candidates are exposed to simultaneous equation in two unknowns, wherein the two equations (either both linear equations or one linear and one quadratic) are explicitly given, for example like the following Question 1.2 that appeared in DBE Mathematics 2009, Paper 1: Solve simultaneously for x and y in the following set of equations:

$x - y = 3$
$x^2 - xy - 2y^2 - 7 = 0.$

In retrospect, nearly all textbooks, previous examination papers and teachers themselves present/discuss the solving of simultaneous equations, where there are always two distinct equations which must be solved simultaneously in two unknowns. Consequently, learners only experience the solving of simultaneous equation in two unknowns, wherein both equations are explicitly stated. Hence, majority of learners come to believe that there must be two distinct equations like Q1.2 that appeared in DBE Mathematics 2009, Paper 1. So, within the context of the limited learner experiences just described, Q.1.4 which requires candidates solve for x and y given $(3x - y)^2 + (x - 5)^2 = 0$, can be classified as non-routine in nature as the two equations, which candidates must solve simultaneously are not explicitly given nor easily discernible.

In order to distil the two equations $(3x-y)^2+(x-5)^2=0$, which candidates must solve for x and y simultaneously, candidates needed to know and conceptually understand the following mathematical statement and deduction:

'The square of any number is always positive or zero. So, for the sum of two squares to be zero, both squares must be zero'. The latter deduction demands higher order reasoning and is pivotal in enabling the candidate to know why $(3x^2-y)^2=0$ and $(x-5)^2=0$, and also why it follows that $3x-y=0$ and $x-5=0$. As these are critical steps that are required to solve for x and y not commonly discussed in most classrooms and prescribed school textbooks, makes this problem non-routine in nature.

Now having $3x-y=0$ and $x-5=0$, candidates can easily from $x-5=0$ get $x=5$ They would then substitute $x=5$ in $3x-y=0$ and solve to get $y=15$.

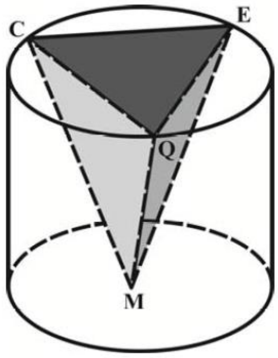
On the basis of the above discussion Q1.2 characterizes a non-routine problem, which demands a higher order reasoning and processes, and hence it can be categorized as 'Problem Solving'.

<u>Memorandum/Marking guideline</u>	
The square of any number is always positive or zero So, for the sum of two squares to be zero, both squares must be zero, i.e. $(3x-y)^2=0 \quad \text{and/}en \quad (x-5)^2=0$ $3x-y=0 \quad \text{and/}en \quad x-5=0$ $x=5$ $3(5)-y=0$ $y=15$	✓ $3x-y=0$ ✓ $x-5=0$ ✓ $x=5$ ✓ $y=15$ (4)

Example 2:

<u>Question 5(D), IEB Mathematics 2014, Paper 2</u>

In the diagram below, ΔECQ is equilateral with sides 20 cm. Vertices E, C and Q lie on the circumference of the circle at the top of a cylinder.
M is the centre of the base of the cylinder.
CEQM forms a triangular pyramid and is cut out of the wooden cylinder.
The volume of the triangular pyramid is 3 000 cm^3.



Calculate the volume of the remaining wood correct to the nearest cubic centimetre.
Use formulae: $V = \pi r^2 h$ and $V = \frac{1}{3} \times \text{Area of base} \times \perp \text{height}$. (7)

Discussion:

In the main candidates need to calculate the volume of the wooden cylinder, using the formula $V_{cylinder} = \pi r^2 h$, and then subtract the volume of the triangular pyramid (which is cut out of the wooden cylinder) to get the volume of the remaining wood.

Now in order calculate the $V_{cylinder} = \pi r^2 h$, the candidate needs to first determine the values of h and *r*.

To calculate h,candidates need to use the formula

$$V_{pyramid} = \frac{1}{3} \times \text{Area of base} \times \perp \text{height}$$

In doing so candidates need to calculate the area of the base of the pyramid, which is in the shape of an equilateral triangle (namely equilateral ΔECQ with sides 20cm). To calculate the area of equilateral triangle ΔECQ, candidates should know that they are given each side is 20cm and that each angle of an equilateral triangle is $60°$. With this information at hand, candidates need to realize that they could make connections with the sine rule and use it as follows to calculate the area of ΔECQ;

Area of $\Delta ECQ = \frac{1}{2} \times 20 \times 20 \times \sin 60° = 10\sqrt{3}$

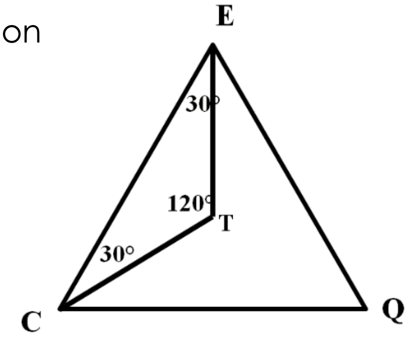
Now piecing all the available information up to this stage, candidates can proceed as follows to finally determine h:

$$V_{pyramid} = \frac{1}{3} \times \text{Area of base} \times \perp \text{height}$$

$$\therefore 3\ 000 = \frac{1}{3} \times \left(\frac{1}{2} \times 20 \times 20 \times \sin 60° \right) \times \perp h$$

$$\therefore 3000 = \frac{1}{3}\left(10\sqrt{3}\right) \times (\perp \text{h})$$

$$\therefore \perp \text{h} = 51,961524 \ldots$$



Now with candidates having a visual representation as the one alongside, they will be able to see that ET= r, which is
the radius of the wooden cylinder, and also that EC = 20cm as it is a side of equilateral ΔECQ.
With this information at hand, candidates may see that they can use the sine rule to help calculate the value of r as follows:

$$\frac{\text{r}}{\sin 30°} = \frac{20}{\sin 120°}$$

$$\therefore \text{r} = 11,54700 \ldots$$

Now substituting the values of r and h in to the formula, $V_{cylinder} = \pi \text{ r}^2\text{h}$, candidates will get $V_{cylinder} = \pi \text{ r}^2\text{h} = \pi \times 11,547 \ldots^2 \times 51,9615 \ldots = 21765,592 \ldots$
Finally, the Volume of wood remaining = 21 765,592 -3000 = 18 765,592

$$V_{remaining} = 18766 \text{ cm}^3 .$$

From the above discussion, it is evident that for candidates to successfully answer this question they must understand the given information, and what is asked for in the question. Working with the given diagram and given formulae, candidates must via higher order reasoning and processes analyse for themselves the sequence of mathematical procedures/processes that will be followed in order to arrive at the desired solution to the given problem. In doing so candidates will have to break the problem into constituent parts and then piece (synthesize) together the relevant parts in order to establish a feasible solution. This includes the ability to reason creatively as illustrated in the development of the solution. Hence Q5(d) can be categorized as 'Problem Solving'.

Memorandum/Marking guidelines

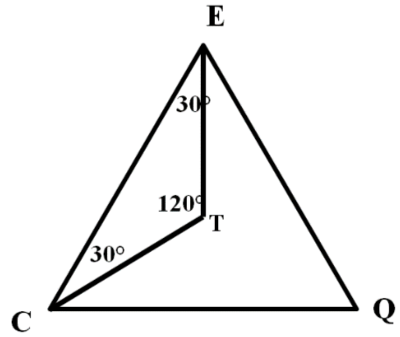
$$V_{cylinder} = \pi \text{ r}^2\text{h}$$

$$V_{pyramid} = \frac{1}{3} \times \text{Area of base} \times \perp \text{height}$$

$$\therefore 3\ 000 = \frac{1}{3} \times \left(\frac{1}{2} \times 20 \times 20 \times \sin 60°\right) \times \perp \text{h} \quad ✓\text{A}$$

$$\therefore \perp \text{h} = 51,961524 \ldots \quad ✓\text{A}$$

✓M



$$\frac{r}{\sin 30°} = \frac{20}{\sin 120°}$$ ✓A

$\therefore r = 11,54700\ ...$ ✓A

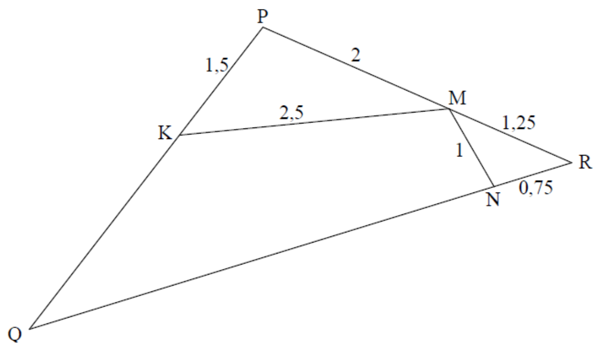
$V_{cylinder} = \pi\ r^2 h = \pi \times 11,547\ ...^2 \times 51,9615\ ... = 21765,592\ ...$ ✓A

$V_{remaining} = 18766\ cm^3$ ✓A (7)

Example 3:

Question 11.2.2, DBE Mathematics 2015, Paper 2

In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ΔPQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1,25 and NR = 0,75



Discussion:

An analysis of problem involves breaking it down into its component parts, examining each in turn. More importantly, it involves developing a strategy of how to solve the given problem, and in particular developing a sequence of mathematical steps (procedures, calculations, proofs, reasoning) that must be performed in order to solve the given problem in valid ways. Closely associated with analysis is the synthesis behaviour, which involves 'piecing together' of the relevant parts in order to establish a structure not clearly visible before, and also includes the ability to think creatively. In general, analysis and synthesis require much more than straightforward application of previously learnt principles and concepts.

The problem of determining the length of NQ in Q11.2.2 can be pursued in various ways, depending on how a candidate reads and understands the problem and strategy he chose to develop and enact. So, in essence, the are other routes/ways to solve the given problem. For the purpose of this discussion I will focus on a strategy that is underpinned by the use of similar triangles.

In calculating the length of NQ in Q11.2.2, the candidate must think of how he can establish the length of RQ, as it will enable him to determine the length of NQ since NQ= RQ- NR and NR is given to be 0,75 units. The inherent challenge for the candidate is to build up a strategy of how to determine the length of QR, which is not immediately straightforward. In doing so, the candidate is expected to refer to his proof in Q11.2.1, wherein he/she proved that $\Delta KPM///\Delta RNM$, and marked the corresponding angles that are consequently equal. Thereafter, the candidate is expected to see and select on his own two other related triangles which could be proved to be similar such that equal ratios of their corresponding sides, wherein RQ is one of the sides, can be logically deduced and be used to calculate the length of RQ after relevant substitution of the lengths of the sides making up the proportion. All this entails is for the candidate to identify that he could prove $\Delta RPQ///\Delta KPM$ on the basis of their corresponding angles being equal. Now to do this, the candidate must first deduce from Q11.2.1, wherein he proved that $\Delta KPM///\Delta RNM$, that $P\widehat{K}M = N\widehat{R}M$, and mark this on his diagram. The looking and focussing on $\Delta RPQ \; and \; \Delta KPM$, the candidate is expected to see that since $P\widehat{K}M = N\widehat{R}M$, it naturally follows that $P\widehat{K}M = \widehat{R}$, and also establish that $\widehat{P}$ is common. As the minimal conditions for the similarity of $\Delta RPQ \; and \; \Delta KPM$ to exist have been met, the candidate is expected to deduce that $\Delta RPQ \; and \; \Delta KPM$ are similar on the grounds that they are equiangular. As soon as the candidate has successfully proved ΔRPQ /// ΔKPM, he is expected to realize that the corresponding sides are in proportion using the applicable similar triangle theorem, and more importantly he needs to at least use a pair of ratios to form a proportion, wherein RQ is one of the sides and the dimension of remaining corresponding sides are at least known or can be calculated or derived in some other ways. Thereafter candidate can proceed to calculate RQ, and then proceed to calculate NQ using the fact that NQ = RQ- NR.

Piecing all of the constituent parts together through creative justified thinking and reasoning embedded in the use of previously accepted axioms, definitions and theorems, the candidate is expected to synthesise the coherent solution to a non-routine problem, which is properly argued and explained as illustrated in the memorandum/marking guidelines hereunder. Hence, this question is categorized as problem solving.

Memorandum/Marking guidelines	
$P\hat{K}M = \hat{R}$ [ΔKPM | | | ΔRNM]	✓ S

∴ $\hat{P}$ is common ∴ ΔRPQ | | | ΔKPM [∠∠∠] $\frac{RP}{KP} = \frac{RQ}{KM}$ [ΔRPQ | | | ΔKPM] ∴ $\frac{3,25}{1,5} = \frac{RQ}{2,5}$ ∴ RQ = $\frac{2,5 \times 3,25}{1,5}$ = 5,42 or $5\frac{5}{12}$ ∴ NQ = 5,42 – 0,75 = 4,67 or $4\frac{2}{3}$ [NB: There are other alternative solutions to this problem, which the reader can consider as well]	✓ ΔRPQ|||ΔKPM ✓ S ✓ subst. correctly ✓ RQ = $5\frac{5}{12}$ ✓ NQ = answer (6)

To accomplish the goal of discriminating between high achievers, those performing very poorly, and all candidates in between, examiners need to vary the challenge of examination questions. Until recently, the assumption has been that 'alignment' with the allocated percentage of marks for questions at the required cognitive demand levels meant that sufficient examination questions were relatively easy; moderately difficult; and difficult for candidates to answer.

However, research and candidate performance both indicate that a range of factors other than type of cognitive demand contribute to the cognitive challenge of a question. Such factors include the level of content knowledge required, the language used in the question, and the complexity or number of concepts tested. In other words, cognitive demand levels on their own do not necessarily distinguish between degrees of difficulty of questions.

This research helps, to some extent, explain why, despite that some NSC examination papers have complied with the specified cognitive demand weightings stipulated in the policy, they have not adequately distinguished between candidates with a range of academic abilities in particular between higher ability candidates. As a result, examiners, moderators and evaluators are now required to assess the difficulty level of each examination question in addition to judging its cognitive demand.

Section 7 below explains the new protocol introduced by Umalusi for analysing examination question difficulty.

7. ANALYSING THE LEVEL OF DIFFICULTY OF EXAMINATION QUESTIONS

When analysing the level of difficulty of each examination question, there are six important protocols to note. These are:

1. Question difficulty is **assessed independently** of the type and level **of cognitive demand**.
2. Question difficulty is assessed against **four levels of difficulty**.
3. Question difficulty is determined against the assumed capabilities of the **ideal** '**envisaged'** Grade 12 Mathematics NSC examination **candidate**.
4. Question difficulty is determined using **a common framework** for thinking about question difficulty.
5. Question difficulty entails **distinguishing unintended sources of difficulty** or ease **from intended sources of difficulty** or ease.
6. Question difficulty entails identifying **differences** in levels of difficulty **within a single question**.

Each of the above protocols is individually explained and discussed below.

7.1 Question difficulty is assessed independently of the type and level of cognitive demand

As emphasised earlier in this exemplar book, the revised Umalusi NSC examination evaluation instruments separate the analysis of the type of cognitive demand of a question from the analysis of the level of difficulty of each examination question. Cognitive demand describes the *type of cognitive process* that is required to answer a question, and this does not necessarily equate or align with the *level of difficulty* of other aspects of a question, such as the difficulty of the content knowledge that is being assessed. For example, a recall question can ask a candidate to recall very complex and abstract scientific content. The question would be categorised as Level 1 in terms of the cognitive demand taxonomy but may be rated as 'difficult' (Level 3 Table 6 below).

> ***Note:***
> Cognitive demand is just one of the features of a question that can influence your comparative judgments of question difficulty. The type and level of cognitive process involved in answering a question does not necessarily determine how difficult the question would be for candidates. Not all evaluation/synthesis/analysis questions are more difficult than questions involving lower-order processes such as comprehension or application.

7.2 Question difficulty is assessed at four levels of difficulty

The revised Umalusi NSC examination evaluation instruments require evaluators to exercise expert judgments about whether each examination question is 'Easy', 'Moderately difficult', 'Difficult' or 'Very difficult' for the envisaged Grade 12 learner to answer. Descriptions of these categories of difficulty are shown in Table 6.

TABLE 6: LEVELS OF DIFFICULTY OF EXAMINATION QUESTIONS

1	2	3	4
Easy for the envisaged Grade 12 student to answer.	**Moderately difficult** for the envisaged Grade 12 student to answer.	**Difficult** for the envisaged Grade 12 student to answer.	**Very difficult** for the envisaged Grade 12 student to answer. The skills and knowledge required to answer the question allow for the top students (*extremely* high-achieving/ability students) to

			be discriminated from other high achieving/ability students).

Note :
The forth level, 'very difficult' has been included in the levels of difficulty of examination questions to ensure that there are sufficient questions that discriminate well amongst higher ability candidates.

7.3 Question difficulty is determined against the assumed capabilities of the ideal 'envisaged' Grade 12 Mathematics NSC examination candidate

The revised Umalusi NSC examination evaluation instruments require evaluators to exercise expert judgments about whether each examination question is 'Easy', 'Moderately difficult', 'Difficult' or 'Very difficult' for the '**envisaged**' Grade 12 learner to answer (Table 6). In other words, assessment of question difficulty is linked to a particular target student within the population of NSC candidates, that is, the Grade 12 candidate of average intelligence or ability.

The Grade 12 learners that you may have taught over the course of your career cannot be used as a benchmark of the 'envisaged' candidate as we cannot know whether their abilities fall too high, or too low on the entire spectrum of all Grade 12 Mathematics candidates in South Africa. The revised Umalusi NSC examination evaluation instruments thus emphasise that, when rating the level of difficulty of a particular question, your conception of the 'envisaged' candidate needs to be representative of the entire population of candidates for all schools in the country, in other words, of the overall Grade 12 population.

Most importantly, the conception of this 'envisaged' candidate is a learner who has been taught the whole curriculum adequately by a teacher who is qualified to teach the subject, in a functioning school. There are many disparities in the South African education system that can lead to very large differences in the implementation of the curriculum. Thus this 'envisaged' learner is not a typical South African Grade 12 learner – it is an intellectual construct (an imagined person) whom you need to imagine when judging the level of difficulty of a question. This ideal 'envisaged' Grade 12 learner is an aspirational ideal of where we would like all Mathematics) learners in South Africa to be.

> ***Note:***
> The concept of the ***ideal envisaged Grade 12 candidate is*** that of an imaginary learner who has the following features:
> a. Is of average intelligence or ability
> b. Has been taught by a competent teacher
> c. Has been exposed to the entire examinable curriculum
>
> This envisaged learner represents an imaginary person who occupies the middle ground of ability and approaches questions *having had all the necessary schooling.*

7.4 Question difficulty is determined using a common framework for thinking about question difficulty

Examiners, moderators and evaluators **in all subjects** are now provided with a common framework for thinking about question difficulty to use when identifying sources of difficulty or ease in each question, and to provide their reasons for the level of difficulty they select for each examination question.

The framework described in detail below provides the main sources of difficulty or 'ease' inherent in questions. The four sources of difficulty which must be considered when thinking about the level of difficulty of examination questions in this framework are as follows.

1. '**Content difficulty'** refers to the difficulty inherent in the subject matter and/or concept/s assessed.
2. '**Stimulus difficulty'** refers to the difficulty that candidates confront when they attempt to read and understand the question and its source material. The demands of the reading required to answer a question thus form an important element of 'stimulus difficulty'.
3. '**Task difficulty'** refers to the difficulty that candidates confront when they try to formulate or produce an answer. The level of cognitive

demand of a question forms an element of 'Task difficulty', as does the demand of the written text or representations that learners are required to produce for their response.

4. **'Expected response difficulty'** refers to difficulty imposed by examiners in a marking guideline, scoring rubric or memorandum. For example, mark allocations affect the amount and level of answers students are expected to write.

This framework derived from Leong (2006) was chosen because it allows the person making judgments about question difficulty to grapple with nuances and with making connections. The underlying assumption is that judgment of question difficulty is influenced by the interaction and overlap of different aspects of the four main sources of difficulty. Whilst one of the above four sources of difficulty may be more pronounced in a specific question, the other three sources may also be evident. Furthermore, not all four sources of difficulty need to be present for a question to be rated as difficult.

The four-category conceptual framework is part of the required Umalusi examination evaluation instruments. Each category or source of difficulty in this framework is described and explained in detail below (Table 7). Please read the entire table very carefully.

TABLE 7: FRAMEWORK FOR THINKING ABOUT QUESTION DIFFICULTY

CONTENT/CONCEPT DIFFICULTY
Content/concept difficulty indexes the difficulty in the **subject matter, topic or conceptual knowledge** assessed or required. In this judgment of the item/question, difficulty exists in the **academic and conceptual demands** that questions make and/or the **grade level** boundaries of the various 'elements' of domain/subject knowledge (topics, facts, concepts, principles and procedures associated with the subject).
For example:
Questions that assess '**advanced content**', that is, subject knowledge that is considered to be in advance of the grade level curriculum, are *likely* to be difficult or very difficult for most candidates. Questions that assess subject knowledge which forms part of the core curriculum for the grade are *likely* to be moderately difficult for most candidates. Questions that assess **'basic content'** or subject knowledge

candidates would have learnt at lower grade levels, and which would be familiar to them are *unlikely* to pose too much of a challenge to most candidates. Questions that require general everyday knowledge or knowledge of 'real life' experiences are *often* easier than those that test more **specialized** school **knowledge**. Questions involving only concrete objects, phenomena, or processes are *usually* easier than those that involve more **abstract constructs, ideas, processes or modes.** Questions which test learners' understanding of theoretical or **de-contextualised issues or topics**, rather than their knowledge of specific examples or contextualised topics or issues *tend* to be more difficult. Questions involving familiar, contemporary/current contexts or events are *usually* easier than those that are more **abstract or** involve **'imagined' events** (e.g. past/future events) or **contexts** that are **distant from learners' experiences.** Content difficulty may also be varied by changing **the number of knowledge elements or operations assessed**. *Generally,* the difficulty of a question increases with the number of knowledge elements or operations assessed. Questions that assess learners on two or more knowledge elements or operations are *usually* (but not always) more difficult than those that assess a single knowledge element or operation. Assessing learners on **a combination of knowledge elements or operations that are seldom combined** *usually* increases the level of difficulty.
EXAMPLES OF INVALID OR UNINTENDED SOURCE OF CONTENT DIFFICULTY
• Testing obscure or unimportant concepts or facts that are not mentioned in the curriculum, or which are unimportant to the curriculum learning objectives. • Testing very advanced concepts or operations that candidates are extremely unlikely to have had opportunities to learn.

STIMULUS DIFFICULTY
Stimulus difficulty refers to the difficulty of the linguistic **features of the question** (**linguistic** complexity) and the challenge that candidates face when they attempt to read, interpret and understand the words and phrases in the question AND when they attempt to read and understand the **information or 'text' or source material (diagrams, tables and graphs, pictures, cartoons, passages, etc.) that accompanies the question.**
For example:
Questions that contain words and phrases that require only simple and straightforward comprehension are *usually* easier than those that require the candidate to understand **subject specific phraseology and terminology** (e.g. idiomatic or grammatical language not usually encountered in everyday language), or that require more technical comprehension and specialised

command of words and language (e.g. everyday words involving different meanings within the context of the subject).

Questions that contain information that is 'tailored' to an expected response, that is, questions that contain no irrelevant or distracting information, are *generally* easier than those that require candidates to select relevant and appropriate information or **unpack a large amount of information** for their response. A question **set in a very rich context** *can* increase question difficulty. For example, learners *may* find it difficult to select the correct operation when, for example, a mathematics or accountancy question is set in a context-rich context.

Although the level of difficulty in examinations is *usually* revealed most clearly through the questions, text complexity or the degree of **challenge or complexity in written or graphic texts** (such as a graph, table, picture, cartoon, etc.) that learners are required to read and interpret in order to respond *can* increase the level of difficulty. Questions that depend on reading and selecting content from a text *can* be more challenging than questions that do not **depend on actually reading the accompanying text** because they test reading comprehension skills as well as subject knowledge. Questions that require candidates to **read a lot** *can* be more challenging than those that require limited reading. Questions that tell learners where in the text to look for relevant information are *usually* easier than those where **learners are not told where to look**.

The level of difficulty *may* increase if texts set, and reading passages or other **source material** used are challenging for the grade level, and make **high reading demands** on learners at the grade level. Predictors of textual difficulty include

- **semantic content** – for example, if vocabulary and words used are typically outside the reading vocabulary of Grade 12 learners, 'texts' (passage, cartoon, diagram, table, etc.) are *usually* more difficult. 'Texts' are *generally* easier if words or images are made accessible by using semantic/context, syntactic/structural or graphophonic/visual cues.
- **syntactic or organisational structure** – for example, sentence structure and length. For example, if learners are likely to be *familiar with the structure* of the 'text' or resource, for example, from reading newspapers or magazines, etc. 'texts' are *usually* easier than when the structure is unfamiliar.
- **literary techniques** – for example, abstractness of ideas and imagery – and **background knowledge required**, for example, to make sense of allusions.
- if the **context** is **unfamiliar** or remote, or if candidates do not have or are **not provided with access to the context** which informs a text (source material, passage, diagram, table, etc.) they are expected to read, and which informs the question they are supposed to answer and the answer they are expected to write, then constructing a response is *likely* to be more difficult than when the context is provided or familiar.

Questions which require learners to **cross-reference different sources** are *usually* more difficult than those which deal with one source at a time.

Another factor in stimulus difficulty is presentation and visual appearance. For example, type face and size, use of headings, and other types of textual organisers

etc. can aid **'readability'** and make it easier for learners to interpret the meaning of a question.
EXAMPLES OF INVALID OR UNINTENDED SOURCES OF STIMULUS DIFFICULTY
• Meaning of words unclear or unknown. • Difficult or impossible to work out what the question is asking. • Questions which are ambiguous. • Grammatical errors in the question that could cause misunderstanding. • Inaccuracy or inconsistency of information or data given. • Insufficient information provided. • Unclear resource (badly drawn or printed diagram, inappropriate graph, unconventional table). • Dense presentation (too many important points packed in a certain part of the stimulus).

TASK DIFFICULTY
Task difficulty refers to the **difficulty that candidates confront when they try to formulate or produce an answer**.
For example:
In most questions, to generate a response, candidates have to work through the steps of a solution. *Generally*, questions that **require more steps in a solution** are more difficult than those that require fewer steps. Questions involving only one or two steps in the solution are *generally* easier than those where several operations required for a solution**.** Task difficulty may also be mediated by the **amount of guidance present in the question**. Although question format is not necessarily a factor and difficult questions can have a short or simple format, questions that provide guided steps or cues (e.g. a clear and detailed framework for answering) are *generally* easier than those that are more open ended and require candidates to form or tailor their **own response strategy** or argument, work out the steps **and maintain the strategy for answering** the question by themselves. A high degree of prompting (a high degree of prompted recall, for example) *tends* to reduce difficulty level. Questions that test specific knowledge are *usually* less difficult that **multi-step, multiple-concept or operation questions**. A question that requires the candidate to **use** a **high level of** appropriate **subject specific, scientific or specialised terminology in** their **response** *tends* to be more difficult than one which does not. A question requiring candidates to **create a complex abstract (symbolic or graphic) representation** is *usually* more challenging than a question requiring candidates to create a concrete representation.

A question requiring writing a one-word answer, a phrase, or a simple sentence is *often* easier to write than **responses that require more complex sentences, a paragraph or a full essay or composition**. Narrative or descriptive writing, for example where the focus is on recounting or ordering a sequence of events chronologically, is *usually* easier than **writing discursively** (**argumentatively or analytically**) where ideas need to be developed and ordered logically. Some questions reflect task difficulty simply by **'creating the space' for A-Grade candidates** to demonstrate genuine insight, original thought or good argumentation, and to write succinctly and coherently about their knowledge. Another element is the **complexity in structure of the required response**. When simple connections between ideas or operations are expected in a response, the question is *generally* easier to answer than a question in which the significance of the relations between the parts and the whole is expected to be discussed in a response. In other words, a question in which an unstructured response is expected is *generally* easier than a question in which **a relational response** is required. A response which involves **combining or linking a number of complex ideas or operations** is *usually* more difficult than a response where there is no need to combine or link ideas or operations. On the other hand, questions which require continuous prose or extended writing *may* also be easier to answer correctly or to get marks for than questions that require no writing at all or single letter answer (such as multiple choice), or a brief response of one or two words or short phrase/s because they **test very specific knowledge**. The **cognitive demand** or **thinking processes** required form an aspect of task difficulty. Some questions test thinking ability, and learners' capacity to deal with ideas, etc. Questions that assess inferential comprehension or application of knowledge, or that require learners to take ideas from one context and use it in another, for example, *tend* to be more difficult than questions that assess recognition or retrieval of basic information. On the other hand, questions requiring recall of knowledge are *usually* more difficult than questions that require simple recognition processes. When the **resources for answering** the question are included in the examination paper, then the task is *usually* easier than when candidates have to **use and select their own internal resources** (for example, their own knowledge of the subject) or transform information to answer the question. Questions that require learners to take or **transfer** ideas, **skills or knowledge from one context/subject area and use them in another** *tend* to be more difficult.
EXAMPLES OF INVALID OR UNINTENDED SOURCES OF TASK DIFFICULTY
• Level of detail required in an answer is unclear. • Context is unrelated to or uncharacteristic of the task than candidates have to do.

- Details of a context distract candidates from recalling or using the right bits of their knowledge.
- Question is unanswerable.
- Illogical order or sequence of parts of the questions.
- Interference from a previous question.
- Insufficient space (or time) allocated for responding.
- Question predictability or task familiarity. If the same question regularly appears in examination papers or has been provided to schools as exemplars, learners are likely to have had prior exposure, and practised and rehearsed answers in class (for example, when the same language set works are prescribed each year).
- Questions which involve potential follow-on errors from answers to previous questions.

EXPECTED RESPONSE DIFFICULTY
Expected response difficulty refers to difficulty imposed by examiners in a **mark scheme and memorandum**. This location of difficulty is more applicable to 'constructed' response questions, as opposed to 'selected' response questions (such as multiple choice, matching/true-false).
For example:
When examiners expect few or no details in a response, the question is *generally* easier than one where the mark scheme implies that **a lot of details are expected**. A further aspect of expected response difficulty is the clarity of the **allocation of marks**. Questions are *generally* easier when the allocation of marks is explicit, straight-forward or logical (i.e. 3 marks for listing 3 points) than when the **mark allocation is indeterminate or implicit** (e.g. when candidates need all 3 points for one full mark or 20 marks for a discussion of a concept, without any indication of how much and what to write in a response). This aspect affects difficulty because candidates who are unclear about the mark expectations in a response may not produce sufficient amount of answers in their response that will earn the marks that befit their ability. Some questions are more difficult/easy to mark accurately than others. Questions that are **harder to mark and score objectively** are *generally* more difficult for candidates than questions that require simple marking or scoring strategies on the part of markers. For example, recognition and recall questions are *usually* easier to test and mark objectively because they usually require the use of matching and/or simple scanning strategies on the part of markers. More complex questions requiring analysis (breaking down a passage or material into its component parts), evaluation (making judgments, for example, about the worth of material or text, or about solutions to a problem), synthesis (bringing together parts or elements to form a whole), and creativity (presenting own ideas or original thoughts) are *generally* harder to mark/score objectively. The best way to test for analysis, evaluation, synthesis and creativity is usually through extended writing. Such extended writing *generally* requires the use of more cognitively demanding *marking* strategies such as interpreting and evaluating the logic of what the candidate has written.

Questions where **a wide range of alternative answers or response/s** is possible or where the correct answer may be arrived at through different strategies *tend* to be more difficult. On the other hand, questions may be so open-ended that learners will get marks even if they engage with the task very superficially.
EXAMPLES OF INVALID OR UNINTENDED SOURCES OF EXPECTED RESPONSE DIFFICULTY
• Mark allocation is unclear or illogical. The weighting of marks is important in questions that comprise more than one component when components vary in levels of difficulty. Learners may be able to get the same marks for answering easy component/s of the item as other learners are awarded for answering the more difficult components. • Mark scheme and questions are incongruent. For example, there is no clear correlation between the mark indicated on the question paper and the mark allocation of the memorandum. • Question asked is not the one that examiners want candidates to answer. Memorandum spells out expectation to a slightly different question, not the actual question. • Impossible for candidate to work out from the question what the answer to the question is (answer is indeterminable). • Wrong answer provided in memorandum. • Alternative correct answers from those provided or spelt out in the memorandum are also plausible. • The question is 'open' but the memo has a closed response. Memo allows no leeway for markers to interpret answers and give credit where due.

The framework described above does not provide you with explicit links between the different sources of difficulty, or show relationships and overlaps between the different categories and concepts in the framework. This is because it is impossible to set prescribed rules or pre-determined combinations of categories and concepts used for making judgments about the source of difficulty in a particular examination question.

The intention behind the framework is to allow you to exercise your sense of judgment as an expert. The complexity of your judgment lies in your ability as an expert to recognise subtle interactions and identify links between different categories of a question's difficulty or ease. For example, a question that tests specific knowledge of your subject can actually be more difficult that a multi-step question because it requires candidates to explain a highly abstract concept, or very complex content. In other words, although questions that test specific knowledge are *usually* less difficult than multiple-concept or operation questions, the level of difficulty of the content knowledge required to answer a question can make the question more difficult than a multi-step or multi-operation question.

Not all one-word response questions can automatically be assumed to be easy. For example, multiple-choice questions are not automatically easy because a choice of responses is provided – some can be difficult. As an expert in your subject, you need to make these types of judgments about each question.

> ***Note:***
> It is very important that you become extremely familiar with the framework explained in Table 7, and with each category or source of difficulty provided (i.e. content difficulty, task difficulty, stimulus difficulty, and expected response difficulty). You need to understand the examples of questions which illustrate each of the four levels (Table 8 to Table 11). This framework is intended to assist you in discussing and justifying your decisions regarding the difficulty level ratings of questions. You are expected to **refer to all four categories or sources of difficulty** in justifying your decisions.
>
> When considering question difficulty ask:
> - How difficult is the **knowledge** (content, concepts or procedures) that is being assessed for the envisaged Grade 12 candidate? (*Content difficulty*)
> - How difficult is it for the envisaged I Grade 12 candidate to formulate the answer to the question? In considering this source of difficulty, you should **take into account the type of cognitive demand** made by the task. (*Task difficulty*)
> - How difficult is it for the envisaged Grade 12 candidate to **understand the question and the source material** that need to be read to answer the particular question? (*Stimulus difficulty*)
> - What does the **marking memorandum and mark scheme** show about the difficulty of the question? (*Expected response difficulty*)

7.5 Question difficulty entails distinguishing unintended sources of difficulty or ease from intended sources of difficulty or ease

Close inspection of the framework for thinking about question difficulty (Section 7.4, Table 7) above, shows that, for each general category or source of difficulty, the framework makes a distinction between 'valid' or intended, and 'invalid' or unintended sources of question difficulty or ease. Therefore, defining question difficulty entails identifying whether sources of difficulty or ease in a question were intended or unintended by examiners. Included in Table 7 are examples of unintended sources of difficulty or ease for each of the four categories.

Valid difficulty or 'easiness' in a question has its source in the requirements of the question, and is ***intended*** by the examiner (Ahmed and Pollit, 1999). Invalid sources of difficulty or 'easiness' refer to those features of question difficulty or 'easiness' that were ***not intended*** by the examiner. Such unintended 'mistakes' or omissions in questions can prevent the question from assessing what the examiner intended, and are likely to prevent candidates from demonstrating their true ability or competence, and can result in a question being easier or more difficult than the examiner intended.

For example, grammatical errors in a question that could cause misunderstanding for candidates are unintended sources of question difficulty because the difficulty in answering the question could lie in the faulty formulation of the question, rather than in the intrinsic difficulty of the question itself (for example, because of stimulus difficulty). Candidates "may misunderstand the question and therefore not be able to demonstrate what they know" (Ahmed and Pollit, 1999, p.2). Another example is question predictability (when the same questions regularly appear in examination papers or textbooks) because familiarity can make a question which was intended to be difficult, less challenging for examination candidates.

Detecting unintended sources of difficulty or ease in examinations is largely the task of moderators. Nevertheless, evaluators also need to be vigilant about detecting sources which could influence or alter the intended level of question difficulty that moderators may have overlooked.

Note:
When judging question difficulty, you should distinguish **unintended sources of question difficulty or ease** from those sources that are intended, thus ensuring that examinations have a range of levels of difficulty. The framework for thinking about question difficulty allows you to systematically identify technical and other problems in each question. Examples of problems might be: unclear instructions, poor phrasing of questions, the provision of inaccurate and insufficient information, unclear or confusing visual sources or illustrations, incorrect use of terminology, inaccurate or inadequate answers in the marking memorandum, and question predictability. You should **not** rate a question as difficult/easy if the source of difficulty/ease lies in the 'faultiness' of the question or memorandum. Instead, as moderators and evaluators, you need to alert examiners to unintended sources of difficulty/ease so that they can improve questions and remedy errors or sources of confusion before candidates write the examination.

7.6 Question difficulty entails identifying differences in levels of difficulty within a single question

An examination question can incorporate more than one level of difficulty if it has subsections. It is important that the components of such questions are 'broken down' into to their individual levels of difficulty.

Note:
Each subsection of a question should be analysed separately so that the percentage of marks allocated at each level of difficulty and the weighting for each level of difficulty can be ascertained as accurately as possible for that question.

8. EXAMPLES OF QUESTIONS AT DIFFERENT LEVELS OF DIFFICULTY

This section provides at least **three** examples of questions from previous Mathematics NSC examinations (Table 8 to Table 11) categorised at each of the four levels of difficulty described in Section 7 (Table 6) above. These examples were selected to represent the **best and clearest** examples of each level of difficulty that the Mathematics experts could find. The discussion below each example question tries to explain the reasoning behind the judgments made about the categorisation of the question at that particular level of difficulty.

TABLE 8: EXAMPLES OF QUESTIONS AT DIFFICULTY LEVEL 1 – EASY

Example 1:

Question 1.1.2 (a), DBE Mathematics 2013, Paper 1

Solve for x: $2x^2 - 5x - 11 = 0$ (leave your answer correct to TWO decimal places). (4)

Discussion:

- The **content** on solving quadratic equations is an essential part of the curriculum topic dealing with the use of the quadratic formula. Candidates should be familiar with the use of the quadratic formula to solve the given type of quadratic equation, which is in standard form, to two decimal places as it is also used repeatedly throughout the Grade 11-12 curriculum in other topics, for example functions and graphs.
- The instruction is very specific and clear, and it is not difficult to work out what the question asking. The terms used in the question such as solve, and leave your answer correct to TWO decimal places should be familiar to Grade 12 candidates **(stimulus)**.
- The **task** is to solve the given quadratic equation correctly to two decimal places. The statement 'leave your answer correct to TWO decimal places' immediately signals to the candidate that quadratic equation does not factorize neatly into factors, and hence the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, must be used, with the aid of the calculator as an option. In this case the quadratic formula is provided in the formula sheet, hence all the candidate needs to do is to identify it, select it and use it. Now in order to use the quadratic formula, the candidate must ensure/write the quadratic in standard form, namely in the form $ax^2 + bx + c = 0$, and then read of the values of a, b and c, which must be correspondingly substituted into the quadratic formula. However, in this case the quadratic equation is already given in standard form, so reading of the values of a, b and c is actually straightforward. So, once candidates have substituted the values of a, b and c into the quadratic formula, they could then do the necessary calculations and use a calculator if necessary to solve correctly to two decimal places.
- The first step of the answer (see marking guideline) requires the correct substitution of b into the correct formula for one mark, and the correct substitution of a and c into correct formula, and finally each correct decimal answer is allocated one mark each (**expected response**).

This question is an 'easy' question in regard to all four sources of difficulty.

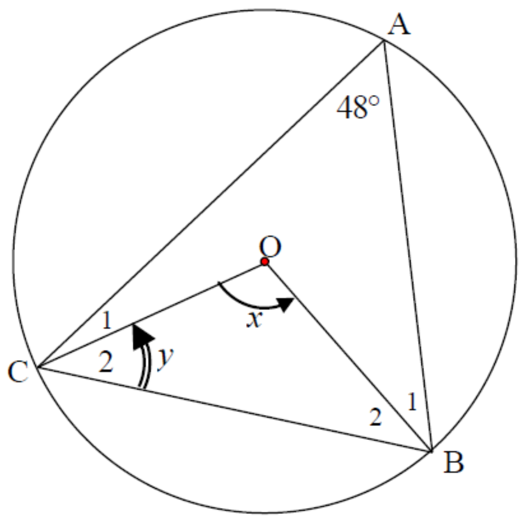
Memorandum/Marking guidelines		
$2x^2-5x-11=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(-11)}}{2(2)}$ $=\frac{5\pm\sqrt{113}}{4}$ $=3{,}91$ or $-1{,}41$	**NOTE:** Wrong formula: 0/4 marks	✓ correct substitution of b into correct formula ✓ correct substitution of a and c into correct formula ✓✓ decimal answers (4)

Example 2:

Question 8.1, DBE Mathematics 2014, Paper 2

In the diagram, O is the centre of the circle passing through A, B and C.

$C\hat{A}B = 48°$, $C\hat{O}B = x$ and $\hat{C}_2 = y$.



Determine with reasons, the size of:

8.1.1 x

8.1.2 y

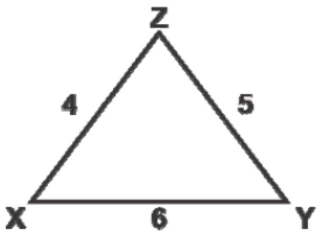
Discussion:

- The **content** on geometry forms an essential part of the curriculum topic dealing with Euclidean Geometry. The diagram provided in this question is quite a standard diagram that appears in most school textbooks and classwork activities, and should therefore be familiar to Grade 12 candidates. In particular this question has two sub-questions (8.1 & 8.2), the first question focuses on the direct use of the angle at centre theorem (which is Grade 11 work) and the latter question 8.2 focuses on the theorem that states that sum of the angles of any triangle is 180° coupled with knowledge that the radii of a given circle is always constant (or equal) and that in a triangle the angles opposite equal sides are equal, which is work done in Grade 9. The candidates should be familiar with the use of the aforementioned theorems and geometrical facts, since they are sufficiently exposed to the knowledge and use of the angle at centre theorem throughout their Grade 11 and 12 curricula, and the others since being in Grade 9.

- The question is written in simple mathematical language, the diagram is clear with appropriate labels, and it is not difficult to work out what the question is asking. The diagram is simple to read as it does not contain too many lines and angles, thus making it easy for the candidate to clearly see that chord BC subtends $C\hat{O}B$ at the centre of the circle, and moreover to easily see that BC also subtends $\hat{A}$ at the circumference, and hence easily see there is a relationship between both angles, namely $C\hat{O}B = 2\hat{A}$. Moreover, as per given simple diagram containing minimal information, candidates can easily see that OC= OB as they radii of circle O, and thereafter easily deduce using their diagram that in $\Delta OCB, \hat{C}_2 = \hat{B}_2 = y$. Now having, $x = C\hat{O}B = 2\hat{A} = 2(48°) = 96°$, and $\hat{C}_2 = \hat{B}_2 = y$, the candidate can then easily apply the sum of the angles of triangle theorem to calculate the value of y (**stimulus**).
- In 8.1, the **task** is to calculate x, which is the value of an $C\hat{O}B$. It should be easy for Grade 12 learners to formulate a response as the question is a routine direct application of the angle at centre theorem. In fact
- as chord BC subtends $C\hat{O}B$ and correspondingly only subtends one angle, namely $\hat{A}$ at the circumference, the calculation is rather easy. In Question 8.2, candidates are merely required to remember and use the following basic Grade 9 theorems and geometrical facts to determine the value of y, which they have been working with mostly from Grade 9 onwards: that radii of a given circle are always equal; the angles opposite equal sides of a triangle are always equal; and the sum of the angles of a triangle is $180°$.
- Simple mathematical statements with appropriate reasons are required to be provided with calculations to earn marks. In Q8.11 one mark is allocated for the statement and one mark for the reason. In Q8.1.2 a mark is allocated for each mathematical statement and no marks are allocated for reasons as this is entails Grade 9 work. In the main the envisaged Grade 12 learner should find it easy to prepare the required answers and have no problem achieving the marks allocated (4), as indicated below (**expected Response**).

This question is an 'easy' question in regard to all four sources of difficulty.

<u>**Memorandum/Marking guidelines**</u>	
8.1.1 $x = 96°$ $\angle$ at centre = 2$\angle$ at circumference)	✓ S ✓ R (2)
8.1.2 $\hat{C}_2 + \hat{B}_2 = 180° - 96° = 84°$ (sum of $\angle$s in Δ) $y = \hat{B}_2 = 42°$ ($\angle$s opp = sides)	✓ S ✓ S (2)

Example 3:
Question 5(c), IEB Mathematics 2014, Paper 2 ΔXYZ has lengths 4, 5 and 6 as shown in the diagram.    Use the cosine rule, show that $cos\,\hat{Y} + cos\,\hat{Z} = \frac{7}{8}$.
<u>Discussion</u>: • Although the question is posed as a problem for learners to solve, the information required to show that $cos\,\hat{Y} + cos\,\hat{Z} = \frac{7}{8}$ is familiar to candidates as the use of the cosine rule to calculate the size of an angle of a triangle when the lengths of its three sides are given. It is used repeatedly throughout the Grade 11-12 curriculum in doing calculations within two dimensional contexts. In addition, the cosine formula is provided in the diagram sheet, so all learners need to do is identify it, adapt it to the given diagram and then in familiar ways, make the appropriate substitutions and manipulations to obtain the desired value for $cos\,\hat{Y}$ and $cos\,\hat{Z}$, which could then be finally added to produce the desired outcome (**content**). • The question is clear, short and concisely written, and it is easy to discern what the question requires. The diagram provided is a clear, singular standard diagram of a triangle having the dimensions of all its sides clearly inserted and vertices clearly labelled. The latter helps make the necessary required substitutions into the adapted cosine formula easier (**stimulus**). • The **task** is to use the cosine rule to show that $cos\,\hat{Y} + cos\,\hat{Z} = \frac{7}{8}$. Thus, any candidate who has learnt and used the cosine rule in particular to calculate the size of an angle of a triangle when the lengths of its 3 sides are given, should have no problem in doing this question as they merely have to determine $cos\,\hat{Y}$ and $cos\,\hat{Z}$ and then find its sum. • The envisaged Grade 12 candidate should find it easy to prepare the required solution (as shown in the marking guideline) and have no problems in achieving that allocated 5 marks as indicated hereunder (**expected response**). This question is an 'easy' question in regard to all four sources of difficulty.
<u>Memorandum/Marking guidelines</u> $4^2 = 5^2 + 6^2 - 2(5)(6)\cos Y$ ✓A $\therefore 16 = 25 + 36 - 60\cos Y$

$\therefore \cos Y = \frac{45}{60}$ $\therefore \cos Y = \frac{3}{4}$ ✓A $6^2 = 4^2 + 5^2 - 2(4)(5)\cos Z$ ✓A $\therefore \cos Z = \frac{5}{40}$ $\cos Z = \frac{1}{8}$ ✓A $\therefore \cos\hat{Y} + \cos\hat{Z} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ ✓A (5)

TABLE 9: EXAMPLES OF QUESTIONS AT DIFFICULTY LEVEL 2 – MODERATELY DIFFICULT

Example 1:
Question 3(b), IEB Mathematics 2013, Paper 1 Determine $f'(x)$ given $f(x) = \frac{3x^4 + 7x^2 - 5x}{2x^2}$ Leave your answer with positive exponents. (4)
<u>Discussion</u>: • The question is generally similar to those routinely encountered in class and previous examination papers, and it assesses the subject knowledge which forms part of the core curriculum for Grade 12. However, as the number of knowledge elements includes the simplification of algebraic expressions, the subsequent application of the rules of differentiation, and the final writing of the derivative free of negative exponents, this question can be regarded as moderately difficult for the envisaged Grade 12 learner (**content**). • The instruction is short and easy to read and understand. The notation and terms used in this question should be familiar to the envisaged Grade 12 candidates (**stimulus**). • In this question candidates are first expected to divide each term in the polynomial $3x^4 + 7x^2 - 5x$ by the monomial $2x^2$ to obtain the expression in the form $\frac{3}{2}x^2 + \frac{7}{2} - \frac{5}{2}x^{-1}$. Thereafter, candidates are expected to use the formula $\frac{d}{dx}(ax^n) = anx^{n-1}$ (for any real number n) together with the rules below to differentiate the resultant function, $f(x) = \frac{3}{2}x^2 + \frac{7}{2} - \frac{5}{2}x^{-1}$, in terms of x: ▪ $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ ▪ $\frac{d}{dx}[k] = 0,\ where\ k\ is\ a\ constant$

- $\frac{d}{dx}[k(f(x)] = k\frac{d}{dx}[(f(x)], where\ k\ is\ a\ constant.$

As the above question, require candidates to perform routine calculations which involve a few steps such as division of a polynomial by a monomial, the recall and application of relevant rules of differentiation and the writing of the final answer free of negative exponents, this question is moderately difficult for the envisaged Grade 12 learner (**task**).

- The marking guidelines suggest how candidates, should almost proceed in finding the derivative of $f(x)$

In the marking guideline one mark (MA= Method Accuracy) is allocated for mathematical accurate division of $3x^4 + 7x^2 - 5x$ by the monomial $2x^2$ to obtain the expression in the form $\frac{3}{2}x^2 + \frac{7}{2} - \frac{5}{2}x^{-1}$ (see step 2). Two method (M) marks is allocated for using the relevant rules of differentiation to get $3x + \frac{5}{2}x^{-2}$, and this means that if learners simplify f(x) in correctly, markers will have to follow through to check if learners applied rules of differentiation to obtain their derivative. This does make the marking of this question slightly more **tedious**. Finally, an accuracy (A)mark is allocated of for the answer free of negative exponents. This means that candidates will automatically loose the very last mark if they do not have the final answer $3x + \frac{5}{2x^2}$, which is written with positive exponents. So, although the mark allocation is relatively straight forward, it does require the learner to simplify $f(x)$ correctly and to must definitely have the final answer (**expected response**).

Despite the easy stimulus, this question is considered to be moderately difficult because of the levels of content, task and expected response difficulty.

Memorandum/Marking guidelines	
$f(x) = \frac{3x^4 + 7x^2 - 5x}{2x^2}$	
$= \frac{3}{2}x^2 + \frac{7}{2} - \frac{5}{2}x^{-1}$	$\frac{3}{2}x^2 + \frac{7}{2} - \frac{5}{2}x^{-1}$ (MA✓)
$\therefore f'(x) = (\frac{3}{2}x^2 + \frac{7}{2} - \frac{5}{2}x^{-1})$	
$= \frac{d}{dx}(\frac{3}{2}x^2) + \frac{d}{dx}(\frac{7}{2}) - \frac{d}{dx}(\frac{5}{2}x^{-1})$ [step4]	candidates could omit writing this 4th step
$=(2)(\frac{3}{2})x^{2-1} + 0 - (-1)(\frac{5}{2})x^{-1-1}$ [step5]	candidates could omit writing this 5th step
$= 3x + \frac{5}{2}x^{-2}$ [step6]	$3x + \frac{5}{2}x^{-2}$ (M✓✓)
$= 3x + \frac{5}{2x^2}$ (A✓)	$3x + \frac{5}{2x^2}$ (A✓)

Example 2:

Question 7.1.3, DBE Mathematics 2012, Paper 1

The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.

Calculate the value of the monthly payment into the sinking fund. (5)

Discussion:

- The **content** assessed forms part of the core curriculum for Grade 12. However, the knowledge/concepts and formula involved in answering this question are moderately difficult for the envisaged Grade 12 candidate. As for determining the value of the monthly payment into the sinking find, the candidate needs to know that he could use the future value annuity formula, $F = \frac{x\left[(1+i)^n - 1\right]}{i}$ to solve this problem, and must be able to establish that $F =$ R90 000, $i = \frac{0{,}085}{12}$, and $n = 61$ in order solve for x correctly. In addition, establishing *n*, which is the number of monthly instalments, needs more thinking and reasoning, and consequently makes the problem more content difficult.
- The question contains subject specific mathematical words, phrases and terms that need more technical comprehension and interpretation in order for the candidate to select the appropriate formula to use within a sinking fund context, and to ascertain/determine the values of F, i, and n that could be substituted into the future value annuity formula. The phrasing of the question is subject to the candidates' interpretation and makes room for candidate to alternatively work out the present value of the sinking fund needed, and then use the present value formula to determine x, which will give the monthly payment into the sinking fund (**stimulus**).
- The **task** requires candidates to read the problem based in a sinking fund context and determine the value of the monthly payment into the sinking fund such that R90 000 can be realized at the end of five years under given instalment and interest rate conditions. A candidate's choice of strategy to solve the problem can vary depending on whether he chooses to solve the problem using the future value annuity formula or whether he chooses to first find the present value of the sinking fund and then use the present value formula. All of this plus the fact that the candidate needs to establish correctly number of monthly payments from the given information (i.e. the first payment will be made immediately and the last payment will be made at the end of the five-year period) makes this task **moderately difficult**, even though both formulae (future and present) can be identified from those given in the formula sheet and the respective calculations can be done with the aid of a calculator.
- In the expected response using future value formula the mark allocation is clear in that a mark is allocated for each of the correct values F_v, i and n,and their respective substitution into the correct formula, and one mark is allocated for the final answer of $x = R\,1\,184{,}68$ (see Method 1 in marking guideline).

  According to the above marking guidelines, for method 1 a candidate can only earn a maximum of two out of 5 marks if he uses the wrong formula.

  Across Method 2, which is shown in the marking guideline, consistency in marking as per method 1 does prevail. However, as obtaining the number of monthly instalments is slightly tricky, it is possible that the envisaged Grade 12 candidates may not be able to earn all the marks in this question, even though the expected responses are clear.

This question is considered to be moderately difficult because of the levels of content, stimulus, task and expected response difficulty.

Memorandum/Marking guidelines

Method 1

Sinking fund needed: $F_v = \text{R } 90\ 000$	✓ $F_v = \text{R } 90\ 000$
$F_v = \dfrac{x[(1+i)^n - 1]}{i}$	✓ $i = \dfrac{0{,}085}{12} = \dfrac{17}{2400}$ in annuity formula
$90\ 000 = \dfrac{x\left[\left(1+\dfrac{0{,}085}{12}\right)^{61} - 1\right]}{\dfrac{0{,}085}{12}}$	✓ $n = 61$ ✓ subs into correct formula
$x = \text{R}1\ 184{,}68$ **NOTE:** Incorrect formula award max 2/5 marks	✓ answer (5)

Method 2

Present value of sinking fund needed: $90000 = P_v\left(1+\dfrac{0{,}085}{12}\right)^{61}$ $P_v = \text{R}58513{,}03$	✓ $i = \dfrac{0{,}085}{12} = \dfrac{17}{2400}$ in annuity formula ✓ $n = 61$ ✓ $P_v = \text{R}58513{,}03$
Using the present value formula: $P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $58513{,}03 = \dfrac{x\left[1-\left(1+\dfrac{0{,}085}{12}\right)^{-61}\right]}{\dfrac{0{,}085}{12}}$	✓ subs into correct formula
$x = \text{R}1\ 184{,}68$	✓ answer (5)

Example 3:

Question 8.3, DBE Mathematics March 2013, Paper 2

Prove that $\dfrac{cos^2x\, sin^2x + cos^4x}{1 - sin\, x} = 1 + \sin x$

Discussion:

- Candidates would have been exposed to proving identities and setting out of proofs of identities in class as the **content** assessed forms part of the core curriculum for Grade 12, but identifying which strategy to use and which trigonometric identities and ratios to invoke to reduce the LHS to the RHS is moderately difficult for the envisaged Grade 12 learner with regard to this question.
- Even though the phrasing of the question as a proof type of question appear to be complex and difficult, this type of question has appeared in past NSC examinations and is adequately discussed across most school textbooks. Therefore, the proof type of instruction in this question should be relatively familiar and not as difficult as it might appear (**stimulus**).
- Answering this question does not simply depend on recalling, recognizing or retrieving information but requires analytical thinking. To formulate their answer candidates need to study both trigonometric expressions on the either side of the equation and decide on the strategy they should use to prove that the RHS=LHS. In this instance they need to work with LHS and simplify it via the use of relevant trigonometric identities and ratios until they arrive at $1 + \ sinx$, which is on the LHS, but this requires a relatively high degree of skill and insight. For example, learners need to see that they need to factorize the denominator on the LHS to get $\frac{cos^2x(sin^2x+cos^2x)}{1-sinx}$. Immediately therafter, candidates need to realize from work on trigonometric identities that $sinx^2 + cosx^2 = 1$, and hence simplify $\frac{cos^2x(sin^2x+cos^2x)}{1-sinx}$ to get $\frac{cosx^2(1)}{1-sinx}$. The latter needs further analysis and realization that they should use the identity $cos^2x = 1 - sin^2x$ to get the expression $\frac{1-sin^2x}{1-sinx}$. Candidates are then expected to factorize $1 - sin^2x$ to get $(1 + sinx)\ (1 - sinx)$, and hence express $\frac{1-sin^2x}{1-sinx}$ as $\frac{(1+sinx)\ (1-sinx)}{1-sinx}$, and subsequently obtain $1 + \ sin\,x$ after cancelling the like factors. As evident in the sequence of steps, this question requires higher order thinking processes and test learners' ability to think and to deal with ideas, particularly on the selection and use of appropriate trigonometric identities coupled with a relevant proof strategy (**task**).
- A coherent and justified simplification of the LHS to $1 + sinx$ is expected from candidates, and the marks as illustrated in the marking guideline, are awarded for critical steps and is relatively straightforward. However, it may be moderately difficult for the envisaged Grade 12 candidate to produce the expected response as it requires some degree of insight regarding choice of proof strategy and the appropriate use of elementary trigonometric identities (**expected response**).

This question is considered to be moderately difficult because of the levels of content, task and expected response difficulty.

Memorandum/Marking guidelines	
$LHS = \dfrac{\cos^2 x(\sin^2 x + \cos^2 x)}{1-\sin x}$	✓ factorisation
$= \dfrac{\cos^2 x.(1)}{1-\sin x}$	✓ 1
$= \dfrac{(1-\sin^2 x)}{1-\sin x}$	✓ $1-\sin^2 x$
$= \dfrac{(1+\sin x)(1-\sin x)}{1-\sin x}$	✓ factors
$=1+\sin x$	
$= RHS$	(4)

TABLE 10: EXAMPLES OF QUESTIONS AT DIFFICULTY LEVEL 3 – DIFFICULT

Example 1:

Question 9.4, DBE Mathematics 2015, Paper 1

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = 12x. P\ and$ Q(2;10) are the turning points of h. The graph of h passes through the origin.

9.1 …

9.2 …

9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

9.4 Explain the significance of the change in Q9.3 with respect to *h*. (1)

[NB: The focus of discussion is on Q9.4]

Discussion:

The question that is being presented for discussion is Q9.4.

- Question 9.4 is located within the sub-topic 'the use of the second derivative test to determine the points of concavity and points of inflection' and this constitutes 'advanced content', that is, subject knowledge that is considered to be in advance of the Grade 12 level curriculum. In this regard, the question tests whether candidates can use the established result in Q9.3, wherein they showed that the concavity of h changes at $x = \frac{1}{2}$, to substantiate that h has a point of inflection at $x = \frac{1}{2}$. As the response to this question requires deep conceptual understanding and higher order application of knowledge, it can be classified as difficult **(content)**.
- The phrasing of the question with the use of the word 'significance' may confuse the envisaged Grade 12 learner **(stimulus)**.
- There is a change in concavity at $x = \frac{1}{2}$. Candidates are expected to use that point as the ground for explaining that the graph of h has a point of inflection at $x = \frac{1}{2}$. As this question tests the application of knowledge within an inferential comprehension context, it is classified as **task** difficult.

- The output is to write that h has a point of inflection at $x = \frac{1}{2}$, and one mark is allocated for this response. It is possible that an envisaged Grade 12 learner may be able to explain the significance of the change in concavity at $x = \frac{1}{2}$ with respect to h **(expected response)**.

This knowledge level question considered to be difficult because of the levels of content, stimulus, task and difficulty.

<u>Memorandum/Marking guidelines</u>	
The graph of *h* has a point of inflection at $x = \frac{1}{2}$	✓answer (1)
OR	
The graph of *h* changes from concave up to concave down at $x = \frac{1}{2}$	✓answer (1)
OR	
The graph of *h* changes concavity at $x = \frac{1}{2}$	✓answer (1)

Example 2:

<u>Question 11.3, DBE Mathematics 2015, Paper 1</u>

There are *t* orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

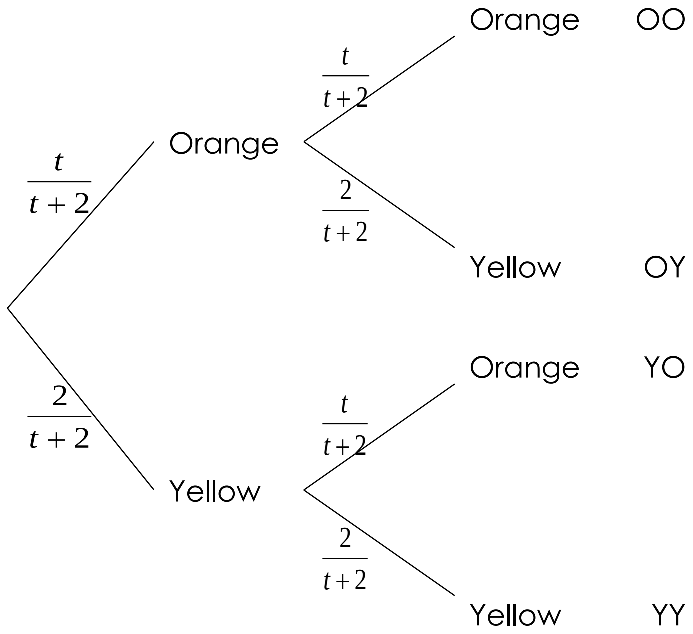
Calculate how many orange balls are in the bag. (10)

Discussion:

- This question assesses the use of a tree diagram to solve a probability problem, which falls within the spectrum of 'advanced **content**' of Grade 12 curriculum. Tree diagrams in a probability context are challenging. However, since the use of tree diagrams forms part of the Grade 11-12 curriculum, and is done across most classrooms and discussed in most school textbooks, we may say that this question is pitched at a difficult level and therefore cannot be considered very difficult **(content).**
- Although the instructions are specific and clear, candidates are provided with large amount of information, which they have to interpret to select relevant and appropriate ways of working. The **stimulus** material is abstract and complex as candidates are expected to work with t orange balls and 2 yellow balls in a bag. All the information is presented in numerical and verbal terms, from which candidates must construct a tree diagram showing the respective probabilities on each branch in terms of the variable t. This combination contributes to the degree of challenge in completing the tree diagram with appropriate probability values and extracting the desired probability equation.
- The **task** of working with tree diagrams in a probability context is cognitively demanding in itself and the added challenge of the candidate working with an unknown (t) number of orange balls in a bag makes the problem quite difficult. In this question, the candidates are expected to read, understand and interpret the given information and map it onto tree diagram in terms of the variable t, and then obtain the probability equation and solve it algebraically to obtain $t = 3$, which will then enable candidates to conclude that there are 3 orange balls in the bag. All this suggests that this question requires a fairly high degree of original and creative thought in a context, which is rather unfamiliar to the envisaged Grade 12 candidates. Hence, this question makes high conceptual and cognitive demands on Grade 12 candidates.
- Allocation of marks is clearly suggested in the marking guideline below. In the main, a mark is allocated for $\frac{t}{t+2}$ and another mark is allocated for $\frac{2}{t+2}$, which permeates the branches. A further two marks are allocated for constructing the probability equation,$\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right) + \left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right) = \frac{52}{200}$. A mark is allocated for simplifying the equation to read $3t^2 - 13t - 12 = 0$, and a mark is allocated for $t = 3$. The envisaged Grade 12 candidate should find it difficult to achieve all the marks due to the steps that are involved in working out the answer **(expected response)**.

This question is considered to be difficult because of the levels of content, stimulus, task and expected response.

Memorandum/Marking guidelines



✓P(O) = $\left(\frac{t}{t+2}\right)$

✓P(Y) = $\left(\frac{2}{t+2}\right)$

$$\text{P(Orange, Orange)} + \text{P(Yellow, Yellow)} = \frac{52}{100}$$

$$\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right)+\left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right)=\frac{52}{100}$$

✓P(O,O) = $\left(\frac{t}{t+2}\right)^2$

✓ P(Y,Y) = $\left(\frac{2}{t+2}\right)^2$

✓ $\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right)+\left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right)=\frac{52}{100}$

$$\frac{t^2}{t^2+4t+4}+\frac{4}{t^2+4t+4}=\frac{13}{25}$$

$$25\left(t^2+4\right)=13\left(t^2+4t+4\right)$$
$$3t^2-13t+12=0$$
$$(3t-4)(t-3)=0$$
$$t=3$$

✓ $t=3$ (no CA)

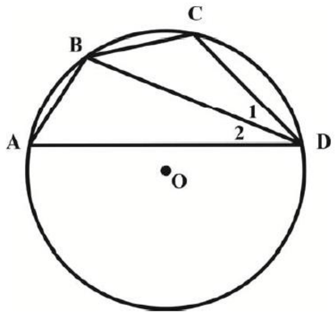
(6)

There are 3 orange balls in the bag.

Example 3:

Question 8(c), IEB Mathematics 2014, Paper 2

In the diagram below, AD is a chord of the circle with radius 3 unit.
AB and BC are equal chords of length 2 units each.



Determine the size of $\widehat{D}_2$

Discussion:

- To answer this question the candidate needs to use their geometry and trigonometry skills and knowledge which entail legitimate geometrical constructions, knowledge and application of the angle at centre theorem as well as the cosine rule. Even though the candidates may have sufficient knowledge of how to use and apply the angle at centre theorem as well as the cosine rule when given information in a given diagram **(content)**, the challenge in this question is to first read, understand and analyse the problem and then develop a strategy of how to solve this particular problem, and this is what elevates the content to a higher difficulty level **(task)**. This, coupled with the fact that geometry has to be used in combination with trigonometry, which is a seldom the practice in our classrooms increases the level of difficulty. Hence, this content is difficult.
- The wording of the given information and the question in relation to the labelled diagram is explicit and clear. However, the candidate need to insert all the given information onto the given diagram, and thereafter fathom out on his own that the radii OB and OA must be constructed. If the candidates are not able to recognize the importance of this construction and hence make the construction, then they would not be able to develop a strategy to solve this problem or move any further, and this step in itself raise the **stimulus** difficulty of this question.
- The **task** skills and knowledge required to answer this question are often demonstrated by high achieving ability students, who are able to self-identify the mandatory construction, namely join OA and OB, and work with knowledge elements from two or more topics (like geometry and trigonometry). In the main the question requires candidates to join OA and OB, and realize that if they know $B\hat{O}A$ then they could use the angle at centre theorem to assist to find $\widehat{D}_2$ - but this requires a candidate to use a high level of appropriate subject specific knowledge and reasoning skills. Moreover, to invoke the cosine rule from trigonometry, which is another area of school mathematics in order to calculate the size of $B\hat{O}A$ makes this question much more challenging for the envisaged Grade 12 candidate. In retrospect, candidates need to realize that OB=OA = 3units since they are radii of the same circle, and consequently see on their diagram that they have the length of all three sides of ΔBOA, and hence realize the cosine rule can be used to determine $B\hat{O}A$. As this question tests candidates thinking ability and candidates' capacity to deal with complex ideas and conjecture up a creative strategy to calculate $\widehat{D}_2$, this task is question is deemed to be difficult for the envisaged Grade 12 learner.

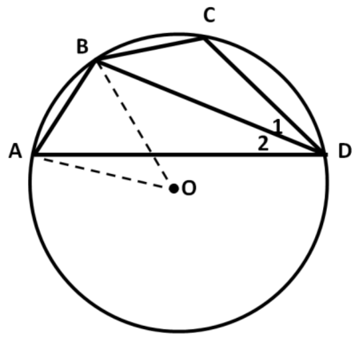
- The **expected response** and mark allocation is clearly articulated in the marking guideline hereunder. Candidates are allocated a mark for their self-identified mandatory construction. A mark is allocated for the appropriate substitution of the values of the lengths of sides of ΔBOA into the cosine formula to determine $\cos B\hat{O}A$, and a mark is given for simplifying and obtaining $\cos B\hat{O}A = \frac{7}{9}$. The final mark is given for using the angle at centre theorem to obtain $\hat{D}_2 = 19,5°$

The envisaged Grade 12 candidate should find it difficult to achieve all the marks due to the steps that are involved in working out the answer (expected response).

Hence, in totality this question is considered to be difficult because of the levels of content, stimulus, and task and difficulty.

Memorandum/Marking guidelines

Join O to B and O to A. ✓A



$D_2 = \frac{1}{2}\hat{O}$; angle at centre

In ΔBOA, $2^2 = 3^2 + 3^2 - 2.3.3.\cos\hat{O}$ ✓A

$\cos\hat{O} = \frac{3^2+3^2-2^2}{2\times3\times3} = \frac{7}{9}$ ✓A

$\therefore \hat{O} = 38,9°$

$\hat{D}_2 = 19,5°$; ✓A angle at centre. (4)

TABLE 11: EXAMPLES OF QUESTIONS AT DIFFICULTY LEVEL 4 – VERY DIFFICULT

Example 1:

Question 8(c), IEB Mathematics 2013, Paper 2

Prove that the radius of the circle having equation

$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$ can never exceed $\sqrt{13}$ for any value of θ.

(5)

Discussion:

- The **content** of this question s relates to topics in analytical geometry, trigonometry, and algebra. These include at least writing the given equation in centre radius form, $(x-a)^2 + (x-a)^2 = r^2$; using the completing square method; the use of the Pythagorean identity, $sin^2\theta + cos^2\theta = 1$, and simplification of trigonometric expressions; and having the intuitive understanding that $sin^2\theta \leq 1$ for all values of θ. As the number of knowledge elements this question invokes is more than two, the content difficulty is inherently increased. Moreover, this kind of question is generally unfamiliar to candidates and assessing candidates on a combination of knowledge elements in a complex proof context makes this question highly content difficult.

- The question is relatively complex with concise conditions and instructions and it is very difficult for students to work out what they must do to answer this question (stimulus). In particular it is not easy for candidates to see what strategy, skills and knowledge they must use to prove the required result, but instead it is only possible after careful interrogation of the question, insightful deep thinking, higher order analysis and synthesis that candidates can distil a strategy to prove the given result **(stimulus)**.
- The **task** set out in this question requires a high level cognitive demand, and it is highly challenging as student need to read and understand the question, identify the content and relevant strategies that can be used and connected to show the required result, and then enact the plan. Sifting out the analytical and trigonometric skills and knowledge needed to answer this question is quite complex, Hence, this task allows for extremely high achieving students to be discriminated from other high achieving students. This includes the ability to reason creatively as illustrated in the development of the solution.
- The **expected response** (which is shown in the marking guidelines) requires candidates to complete the square, write the LHS as perfect squares, and establish that $r^2 = 4cos^2\theta + 16sin^2\theta$-3. However, the decision to express $4cos^2\theta + 16sin^2\theta$-3 in terms of $sin\theta$ can be quite a novel idea for candidates to think about and pursue. The most challenging step is for candidates to argue on logical grounds that $r^2 = 1 + 12sin^2\theta \leq \theta$ for all value of θ. Failure to get this step, will make it absolutely very difficult for candidates to show that the radius of the given circle will never exceed √13 for any value of θ. Producing a coherent logical answer of this kind is very difficult for most Grade 12 candidates.

This question is judged to be very difficult in relation to all four sources of difficulty.

<u>Memorandum/Marking guidelines</u>

$x^2 + y^2 + 4x.cos\theta + 8y.sin\theta + 3 = 0$

$x^2 + 4x.cos\theta + 4cos^2\theta + y^2 + 8y.sin\theta + 16sin^2\theta = -3 + 16sin^2\theta + 4cos^2\theta$ (MA✓)

$(x + 2cos\theta)^2 - 4cos^2\theta + (y + 4sin\theta)^2 - 16sin^2\theta + 3 = 0$ (A✓)

$(x + 2cos\theta)^2 + (y + 4sin\theta)^2 = 4cos^2\theta + 16sin^2\theta - 3$ (MA✓)

$r^2 = 4(1 - sin^2\theta) + 16sin^2\theta - 3$ (A✓)

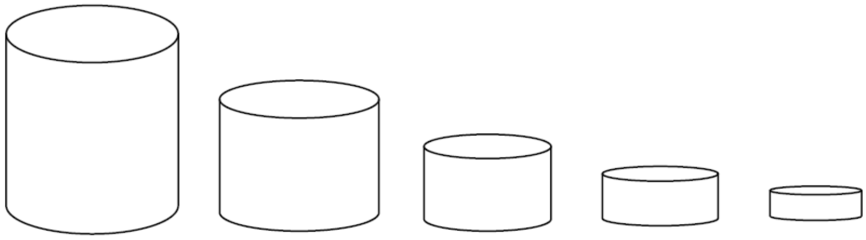
$r^2 = 1 + 12sin^2\theta \leq 13\ for\ all\ values\ of\ \theta$ (MA✓)

$\therefore r \leq \sqrt{13}$

Example 2:

<u>Question 3.2, DBE, Mathematics 2012, Paper 1</u>

Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would it possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

Discussion:

- The **content** namely sum of a geometric sequence and sum to infinity of a geometric sequence, forms an essential part of the Grade 12 curriculum. Although the content would have been learnt in class, the problem is very difficult for the envisaged Grade 12 candidate in that it involves a context that is different from candidates' experiences wherein they are required to diligently perform salient calculations using higher order reasoning and hence use it creatively to justify their response.
- Although the given information, diagram and instructions are clear, the structure of the question is unfamiliar and is set in a rich context. Candidates have to unpack the given information and in the process, may find it difficult to select the correct methods and strategies to assist them to develop a position on the posited challenge as the question is quite open ended and un-scaffold **(stimulus)**.
- The **task** requires candidates to logically argue through the use of appropriate and relevant mathematical knowledge as to whether the first water tank, which is empty, can hold the water from the remaining 19 tanks given that the volume of each tank is half the volume of the previous tank. This question test thinking ability, and learners' ability to deal with complex ideas in a problem-solving context. This question involves extensive analysis of complex information together with interpretation of the facts to distil a strategy to solve this open worded problem with a justified explanation. Grade 12 candidates are very unlikely to have prior exposure to this type of question in class, thus this is a type of question that 'sifts' out the candidates who have insight from simply those who have learnt in class.
- As per marking guidelines (**expected response**) below, there are a range of alternative answers that are possible depending on how candidates read, understand, interpret and analyse the problem. To get full marks should be very difficult for the envisaged Grade 12 candidate as they have to present a coherent logical argument justified by relevant calculations and valid reasons.

All of the above sources of difficulty make the question very difficult for the envisaged Grade 12 learner.

Memorandum/Marking guidelines	
Let V be the volume of the first tank.	
$\frac{V}{2};\frac{V}{4};\frac{V}{8}\ldots$	✓ $\frac{V}{2}$
$S_{19}=\frac{\frac{V}{2}\left[1-(\frac{1}{2})^{19}\right]}{1-\frac{1}{2}}$	✓ substitute into correct formula
$=\frac{524287}{524288}V$	
$=0{,}9999980927V$	✓ answer
$< V$	
Yes, the water will fill the tank without spilling	✓conclusion
OR	(4)
Let V be the volume of the first tank.	
$\frac{V}{2};\frac{V}{4};\frac{V}{8}\ldots$	✓ $\frac{V}{2}$
$S_{19}=\frac{\frac{V}{2}\left[1-(\frac{1}{2})^{19}\right]}{1-\frac{1}{2}}$	✓ substitute into correct formula
$=V\left[1-(\frac{1}{2})^{19}\right]$	
$< V.1$	✓ observes that
$< V$	$[1-\left(\frac{1}{2}\right)^{19}]<1$
Yes, the water will fill the tank without spilling	✓conclusion
OR	
Let V be the volume of the first tank.	(4)
$\frac{V}{2};\frac{V}{4};\frac{V}{8}\ldots$	✓ $\frac{V}{2}$
$S_{\infty}=\frac{\frac{V}{2}}{1-\frac{1}{2}}$	
$=V$	✓ substitute into correct formula
Since the first tank will hold the water from infinitely many tanks without spilling over, certainly:	

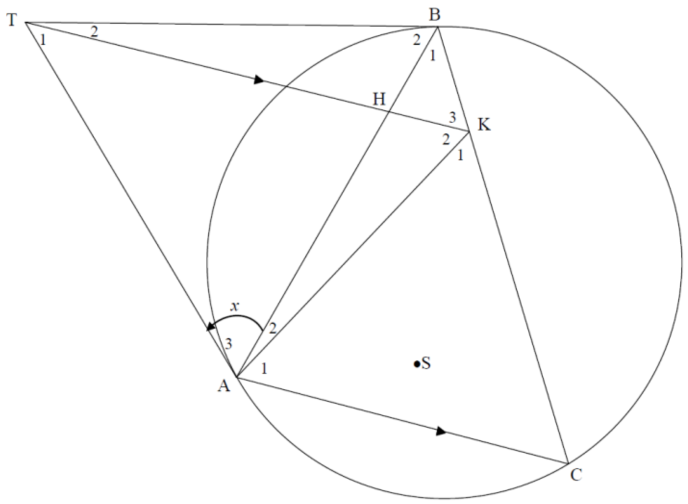
Yes, the first tank will hold the water from the other 19 tanks without spilling water. **OR** If the tanks are emptied one by one, starting from the second, each tank will fill only half the remaining space, so the first tank can hold all the water from the other 19 tanks.	✓✓ correct argument (4) ✓(explicit or understood from the argument) ✓✓✓ argument) (4)

Example 3:

Question 9.5, DBE Mathematics 2015, Paper 2

[As the solving for question 9.55, does depend on the use of items proved earlier in Questions 9.1-9,4, the entire question is presented for use of easy reference]

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC. AK is drawn. Let $\hat{A}_3 = x$.



9.1 Prove that $\hat{K}_3 = x$.

9.2 Prove that AKBT is a cyclic quadrilateral.

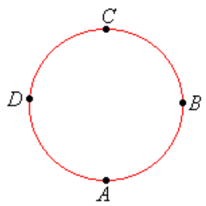
9.3 Prove that TK bisects $A\hat{K}B$.

9.4 Prove that TA is a tangent to the circle passing through the points A, K and H.

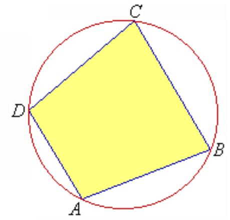
9.5 S is a point in the circle such that the points A, S, K and B are concyclic. Explain why A, S, B and T are also concyclic.

Discussion:

- Candidates require specialized knowledge about concyclic points, that is they need to know that points that lie on the same circle are said to be **concyclic**. For example, *A*, *B*, *C* and *D* are **concyclic points**.



Furthermore, candidates need to know that If the vertices of a quadrilateral lie on a circle, then the quadrilateral is said to be **cyclic**. For example, *ABCD* is a **cyclic quadrilateral** since the vertices *A*, *B*, *C* and *D* lie on the circle (that is the same as saying that points A, B, C and D are concyclic).



This knowledge is extremely important to answer this question as candidates need to realize that in order to explain why A, S, B and T are also concyclic they effectively need to logically argue that they lie on the same circle. Even though the envisaged Grade 12 candidates may at least know of the ways to prove that a quadrilateral is cyclic, and may have extensively used them to prove that a quadrilateral is cyclic within given geometrical figures in their classrooms, they are very unlikely to have had exposure to this type of question including the given information in some instances. Moreover, it is highly unlikely that the envisaged candidate will know that a unique circle can be drawn through minimum of three points in a plane, which can also be used as grounds to argue and explain why A, S, B and T are concyclic. All this coupled with the fact that the candidates have to work within a complex diagram makes the question **content**-wise highly difficult for the envisaged Grade 12 candidate.
- The diagram is rather complex as it includes a combination of geometrical objects (namely circle, parallel lines, tangents, chords, segments, etc.), which candidates need to systematically analyse in order to see relationships that could be used to explain why A, S, B and T are also concyclic. This coupled with the fact that the candidates have to make connection with prior established proofs (Questions 9.1-9.4) within the context of highly dense diagram makes it very difficult for envisaged Grade 12 candidates to appropriate the required information and select the correct methods/strategies to show that A, S, B and T are also concyclic **(stimulus)**.
- The **task** set out in this question requires high cognitive demand- student need to understand this content and select relevant information from the given information in this question and also invoke the prior results proved in

earlier questions (9.1) and appropriately connect it to what they know about concyclic points and subsequently use it to logically explain why A, S, B and T are also concyclic. This question demands that candidates either know that a unique circle can pass through a minimum of three points in a plane, or the ways of proving that a quadrilateral is cyclic. In the case of the unique circle passing through a minimum of 3 points in a plane, candidates are expected to use it as grounds to argue as follows to show that A, S, B and T are concyclic:

> The circle passing through points A, K and B contains the point S on the circumference (since A, S, K and B are concyclic)
>
> The circle passing through points A, K and B contains the point T on the circumference
> $\therefore$ points A, S, B and T are also concyclic.

Even though one can prove that a quadrilateral is cyclic in 3 possible ways, the candidate needs to study the available information and realize that they can show that A, S, B and T are concyclic by showing that the opposite angles of quadrilateral A, S, B and T are supplementary in order to conclude that points A, S, B and T are also concyclic. See below:

> $B\hat{S}A = B\hat{K}A = 2x$ [A, S, K & B concyclic]
> $A\hat{T}B = 180° - 2x$ [A, T, B & K concyclic]
> $\therefore$ points A, S, B and T are also concyclic
> [since opp $\angle$s of quad = 180°]

Now in the development of either of the two explanations, which is cognitively demanding, candidates need not only have a reasonably in-depth knowledge about concyclic points and ways to prove that a quadrilateral cyclic information, but also be able to 'sift' out necessary and sufficient information and use them efficiently and diligently to build a logically justified explanation. This is extremely difficult for the envisaged Grade 12 leaner **(task).**

- In order to earn full marks candidates need to produce, a short, concise and succinct coherent argument backed by valid mathematical statements and reasons generated through using the complex diagram in association with both the given information in Q9.5 and established information from Question 9.1-9.4, as well as established geometrical definitions and theorems. Hence, the expected response, which is discussed under in the prior paragraph is very difficult for the envisaged Grade 12 candidates **(expected response)**.

This question is judged to be difficult for all four sources of difficulty.

<u>**Memorandum/Marking guidelines**</u>	
$B\hat{S}A = B\hat{K}A = 2x$ [A, S, K & B concyclic]	✓ S (both statements)

$A\hat{T}B = 180° - 2x$ [A, T, B & K concyclic] ∴ points A, S, B and T are also concyclic [opp ∠s of quad = 180°] **OR** The circle passing through points A, K and B contains the point S on the circumference (A, S, K and B concyclic). The circle passing through A, K and B contains the point T on the circumference (proven in 9.2). ∴ points A, S, B and T are also concyclic	✓ R (2) ✓ S ✓ S (2)

9. CONCLUDING REMARKS

This exemplar book is intended to be used as a training tool to ensure that all role players in the Mathematics Examinations are working from a common set of principles, concepts, tools and frameworks for assessing cognitive challenge when examinations are set, moderated and evaluated. We hope that the discussion provided and the examples of questions shown by level and type of cognitive demand and later by level of difficulty assist users of the exemplar book to achieve this goal.

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