# Review Exercise Set 6

Exercise 1: Graph two periods of the given tangent function.

$$y = -\tan\left(\frac{1}{3}x\right)$$

Exercise 2: Graph two periods of the given cotangent function.

$$y = 3\cot\left(x - \pi\right)$$

Exercise 3: Graph two periods of the given secant function.

$$y = \frac{2}{3}\sec\left(x + \frac{\pi}{4}\right)$$

Exercise 4: Graph two periods of the given cosecant function.

$$y = -2\csc\left(3x - \pi\right)$$

Exercise 5: Graph the given trigonometric function over two complete periods.

$$y = \frac{5}{4}\sec\left(x + \frac{\pi}{2}\right) - 3$$

# Review Exercise Set 6 Answer Key

Exercise 1: Find the period of the tangent function and then graph it over two periods.

$$y = -\tan\left(\frac{1}{3}x\right)$$

Find the period

$$period = \frac{\pi}{B} = \frac{\pi}{\frac{1}{3}} = \pi \times 3 = 3\pi$$

Find the asymptotes at the beginning and end of the first period

$$-\frac{\pi}{2} < \frac{1}{3}x < \frac{\pi}{2}$$
$$-\frac{3\pi}{2} < x < \frac{3\pi}{2}$$

Find the asymptote at the end of the second period

= last asymptote + period

$$\frac{3\pi}{2} + 3\pi = \frac{3\pi}{2} + \frac{6\pi}{2} = \frac{9\pi}{2}$$

Find the x-intercepts in both periods

$$x = \frac{-\frac{3\pi}{2} + \frac{3\pi}{2}}{2} = \frac{0}{2} = 0$$

1st x-intercept = (0, 0)

x = 1st x-intercept + period

$$x = 0 + 3\pi = 3\pi$$

2nd x-intercept = (3π, 0)

Find points that are $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the asymptotes

Since the coefficient of the tangent function is negative, the $\frac{1}{4}$ point will have a positive y-value and the $\frac{3}{4}$ point will have a negative y-value.

Exercise 1 (Continued):

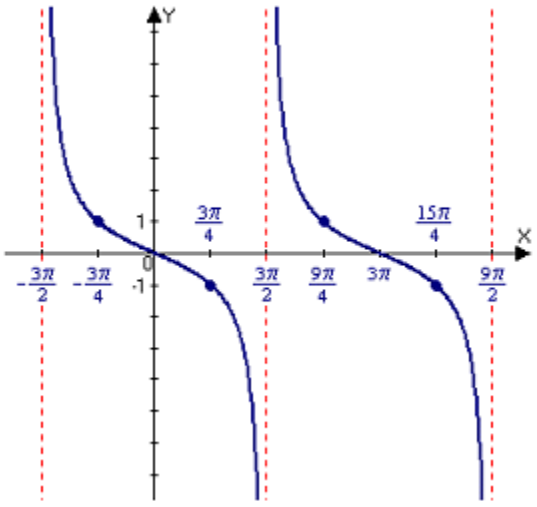
1st period

| One-quarter point | Three-quarter point |
|---|---|
| $x = -\frac{3\pi}{2} + \frac{1}{4}(3\pi)$ $= -\frac{6\pi}{4} + \frac{3\pi}{4} = -\frac{3\pi}{4}$ $(x, A) = \left(-\frac{3\pi}{4}, 1\right)$ | $x = -\frac{3\pi}{2} + \frac{3}{4}(3\pi)$ $= -\frac{6\pi}{4} + \frac{9\pi}{4} = \frac{3\pi}{4}$ $(x, A) = \left(\frac{3\pi}{4}, -1\right)$ |

2nd period

| One-quarter point | Three-quarter point |
|---|---|
| $x = \frac{3\pi}{2} + \frac{1}{4}(3\pi)$ $= \frac{6\pi}{4} + \frac{3\pi}{4} = \frac{9\pi}{4}$ $(x, A) = \left(\frac{9\pi}{4}, 1\right)$ | $x = \frac{3\pi}{2} + \frac{3}{4}(3\pi)$ $= \frac{6\pi}{4} + \frac{9\pi}{4} = \frac{15\pi}{4}$ $(x, A) = \left(\frac{15\pi}{4}, -1\right)$ |

Graph the function



Exercise 2: Graph two periods of the given cotangent function.

$$y = 3\cot\left(x - \pi\right)$$

Find the period

$$period = \frac{\pi}{B} = \frac{\pi}{1} = \pi$$

Find the asymptotes at the beginning and end of the first period

$$0 < x - \pi < \pi$$
$$\pi < x < 2\pi$$

Find the asymptote at the end of the second period

= last asymptote + period
= 2π + π
= 3π

Find the x-intercepts in both periods

$$x = \frac{\pi + 2\pi}{2} = \frac{3\pi}{2}$$

1st x-intercept = $\left(\frac{3\pi}{2}, 0\right)$

x = 1st x-intercept + period

$$x = \frac{3\pi}{2} + \pi = \frac{3\pi}{2} + \frac{2\pi}{2} = \frac{5\pi}{2}$$

2nd x-intercept = $\left(\frac{5\pi}{2}, 0\right)$

Find points that are $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the asymptotes

Since the coefficient of the cotangent function is positive, the $\frac{1}{4}$ point will have a positive y-value and the $\frac{3}{4}$ point will have a negative y-value.

Exercise 2 (Continued):

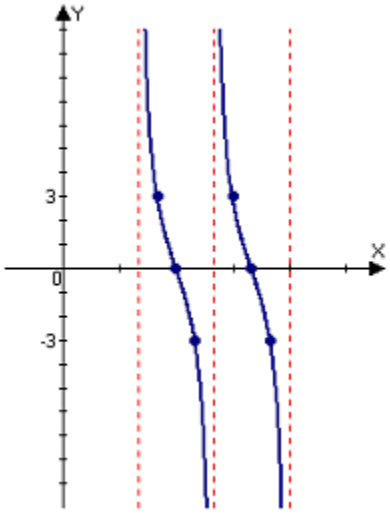
1st period

| One-quarter point | Three-quarter point |
|---|---|
| $x = \pi + \frac{1}{4}(\pi)$ $= \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$ $(x, A) = \left(\frac{5\pi}{4}, 3\right)$ | $x = \pi + \frac{3}{4}(\pi)$ $= \frac{4\pi}{4} + \frac{3\pi}{4} = \frac{7\pi}{4}$ $(x, A) = \left(\frac{7\pi}{4}, -3\right)$ |

2nd period

| One-quarter point | Three-quarter point |
|---|---|
| $x = 2\pi + \frac{1}{4}(\pi)$ $= \frac{8\pi}{4} + \frac{\pi}{4} = \frac{9\pi}{4}$ $(x, A) = \left(\frac{9\pi}{4}, 3\right)$ | $x = 2\pi + \frac{3}{4}(\pi)$ $= \frac{8\pi}{4} + \frac{3\pi}{4} = \frac{11\pi}{4}$ $(x, A) = \left(\frac{11\pi}{4}, -3\right)$ |

Graph the function



Exercise 3: Graph two periods of the given secant function.

$$y = \frac{2}{3}\sec\left(x + \frac{\pi}{4}\right)$$

Replace the secant function with its reciprocal function

$$y = \frac{2}{3}\cos\left(x + \frac{\pi}{4}\right) = \frac{2}{3}\cos\left(x - \left(-\frac{\pi}{4}\right)\right)$$

Find the amplitude, period, and phase shift

| Amplitude | Period | Phase shift |
|---|---|---|
| $amplitude = \lvert A \rvert$ $= \left\lvert \frac{2}{3} \right\rvert$ $= \frac{2}{3}$ | $period = \frac{2\pi}{B}$ $= \frac{2\pi}{1}$ $= 2\pi$ | $phase\ shift = \frac{C}{B}$ $= \frac{-\frac{\pi}{4}}{1}$ $= -\frac{\pi}{4}$ |

Find the x-values of the five key points

$$\text{interval width} = \frac{period}{4}$$
$$= \frac{2\pi}{4} = \frac{\pi}{2}$$

The first key point will start at $x = -\frac{\pi}{4}$ and increase by $\frac{\pi}{2}$

$$x_1 = -\frac{\pi}{4} \qquad x_2 = -\frac{\pi}{4} + \frac{\pi}{2} \quad x_3 = \frac{\pi}{4} + \frac{\pi}{2} \qquad x_4 = \frac{3\pi}{4} + \frac{\pi}{2} \quad x_5 = \frac{5\pi}{4} + \frac{\pi}{2}$$

$$x_1 = -\frac{\pi}{4} \qquad x_2 = \frac{\pi}{4} \qquad x_3 = \frac{3\pi}{4} \qquad x_4 = \frac{5\pi}{4}$$
$$x_5 = \frac{7\pi}{4}$$

Exercise 3 (Continued):

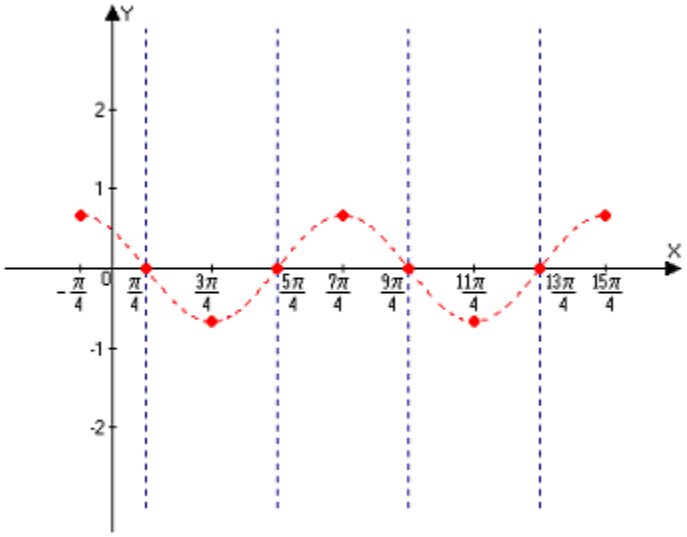
Find the y-values of the five key points

| x-value | y-value | key point |
|---|---|---|
| $-\frac{\pi}{4}$ | $y=\frac{2}{3}\cos\left(x+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos\left(-\frac{\pi}{4}+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos(0)=\frac{2}{3}$ | $\left(-\frac{\pi}{4},\frac{2}{3}\right)$ |
| $\frac{\pi}{4}$ | $y=\frac{2}{3}\cos\left(x+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos\left(\frac{\pi}{2}\right)=0$ | $\left(\frac{\pi}{4},0\right)$ |
| $\frac{3\pi}{4}$ | $y=\frac{2}{3}\cos\left(x+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos\left(\frac{3\pi}{4}+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos(\pi)=-\frac{2}{3}$ | $\left(\frac{3\pi}{4},-\frac{2}{3}\right)$ |
| $\frac{5\pi}{4}$ | $y=\frac{2}{3}\cos\left(x+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos\left(\frac{5\pi}{4}+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos\left(\frac{3\pi}{2}\right)=0$ | $\left(\frac{5\pi}{4},0\right)$ |
| $\frac{7\pi}{4}$ | $y=\frac{2}{3}\cos\left(x+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos\left(\frac{7\pi}{4}+\frac{\pi}{4}\right)$<br>$=\frac{2}{3}\cos(2\pi)=\frac{2}{3}$ | $\left(\frac{7\pi}{4},\frac{2}{3}\right)$ |

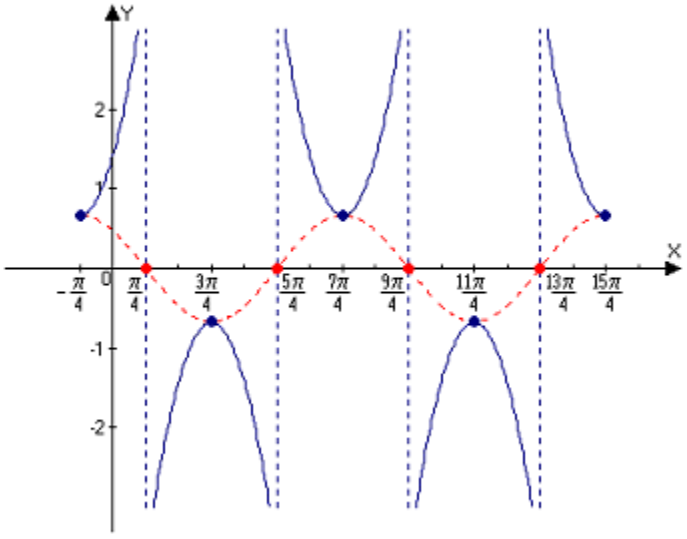
Graph the reciprocal function of $y = \frac{2}{3}\cos\left(x + \frac{\pi}{4}\right)$

The vertical asymptotes for the secant function will occur where the cosine function is equal to zero (crosses the x-axis)

Once the first period is graph repeat the pattern over the second period.




Graph the secant function using the graph of the cosine function as a guide



Exercise 4: Graph two periods of the given cosecant function.

$$y = -2\csc\left(3x - \pi\right)$$

Replace the cosecant function with its reciprocal function

$$y = -2\sin\left(3x - \pi\right)$$

Find the amplitude, period, and phase shift

| Amplitude | Period | Phase shift |
|---|---|---|
| $amplitude = \lvert A \rvert$ $= \lvert -2 \rvert$ $= 2$ | $period = \frac{2\pi}{B}$ $= \frac{2\pi}{3}$ | $phase\ shift = \frac{C}{B}$ $= \frac{\pi}{3}$ |

Find the x-values of the five key points

$$\text{interval width} = \frac{period}{4}$$
$$= \frac{\frac{2\pi}{3}}{4} = \frac{2\pi}{3} \times \frac{1}{4} = \frac{\pi}{6}$$

The first key point will start at x = $\frac{\pi}{3}$ and increase by $\frac{\pi}{6}$

$$x_1 = \frac{\pi}{3} \qquad x_2 = \frac{\pi}{3} + \frac{\pi}{6} \qquad x_3 = \frac{\pi}{2} + \frac{\pi}{6} \qquad x_4 = \frac{2\pi}{3} + \frac{\pi}{6} \qquad x_5 = \frac{5\pi}{6} + \frac{\pi}{6}$$

$$x_1 = \frac{\pi}{3} \qquad x_2 = \frac{\pi}{2} \qquad x_3 = \frac{2\pi}{3} \qquad x_4 = \frac{5\pi}{6} \qquad x_5 = \pi$$

Find the y-values of the five key points

| x-value | y-value | key point |
|---|---|---|
| $\frac{\pi}{3}$ | $y = -2\sin\left(3\left(\frac{\pi}{3}\right) - \pi\right)$ $= -2\sin\left(0\right) = 0$ | $\left(\frac{\pi}{3}, 0\right)$ |
| $\frac{\pi}{2}$ | $y = -2\sin\left(3\left(\frac{\pi}{2}\right) - \pi\right)$ $= -2\sin\left(\frac{\pi}{2}\right) = -2$ | $\left(\frac{\pi}{2}, -2\right)$ |

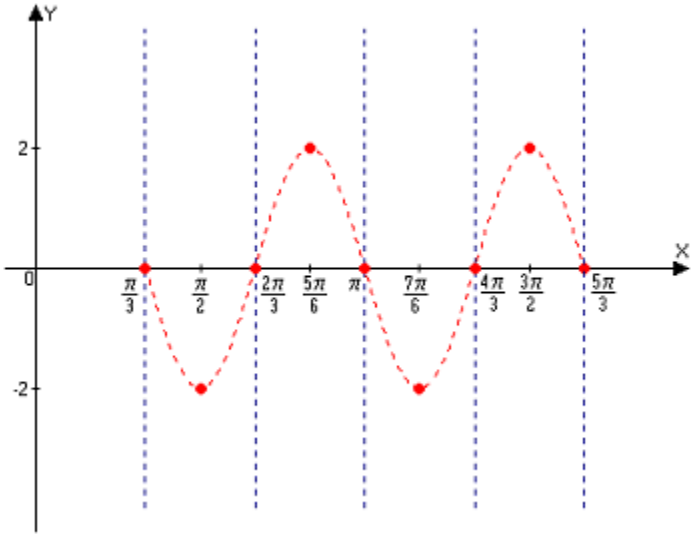
Exercise 4 (Continued):

| x-value | y-value | key point |
|---|---|---|
| $\frac{2\pi}{3}$ | $y=-2\sin\left(3\left(\frac{2\pi}{3}\right)-\pi\right)$ $=-2\sin(\pi)=0$ | $\left(\frac{2\pi}{3},0\right)$ |
| $\frac{5\pi}{6}$ | $y=-2\sin\left(3\left(\frac{5\pi}{6}\right)-\pi\right)$ $=-2\sin\left(\frac{3\pi}{2}\right)=2$ | $\left(\frac{5\pi}{6},2\right)$ |
| π | $y=-2\sin(3(\pi)-\pi)$ $=-2\sin(2\pi)=0$ | $(\pi,0)$ |

Graph the reciprocal function of y = -2 sin (3x - π)

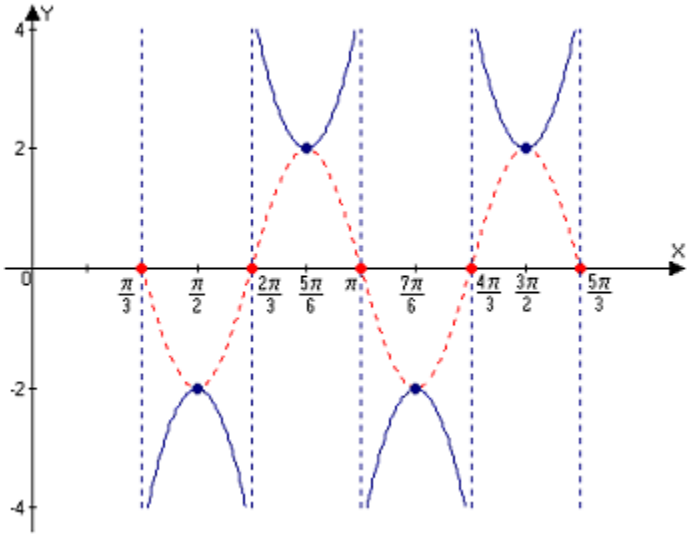
The vertical asymptotes for the cosecant function will occur where the sine function is equal to zero (crosses the x-axis)

Once the first period is graph repeat the pattern over the second period.



Exercise 4 (Continued):

Graph the secant function using the graph of the cosine function as a guide




Exercise 5: Graph the given trigonometric function over two complete periods.

$$y=\frac{5}{4}\sec\left(x+\frac{\pi}{2}\right)-3$$

Replace the secant function with its reciprocal function

$$y=\frac{5}{4}\cos\left(x+\frac{\pi}{2}\right)-3$$
$$=\frac{5}{4}\cos\left(x-\left(-\frac{\pi}{2}\right)\right)-3$$

Find the amplitude, period, and phase shift

| Amplitude | Period | Phase shift |
|---|---|---|
| $amplitude=\lvert A\rvert$ $=\left\lvert\frac{5}{4}\right\rvert$ $=\frac{5}{4}$ | $period=\frac{2\pi}{B}$ $=\frac{2\pi}{1}$ $=2\pi$ | $phase\ shift=\frac{C}{B}$ $=\frac{-\frac{\pi}{2}}{1}$ $=-\frac{\pi}{2}$ |

Exercise 5 (Continued):

Find the x-values of the five key points

$$\text{interval width} = \frac{period}{4}$$
$$= \frac{2\pi}{4} = \frac{\pi}{2}$$

The first key point will start at x = $-\frac{\pi}{2}$ and increase by $\frac{\pi}{2}$

$$x_1 = -\frac{\pi}{2} \quad x_2 = -\frac{\pi}{2} + \frac{\pi}{2} = 0 \quad x_3 = 0 + \frac{\pi}{2} = \frac{\pi}{2} \quad x_4 = \frac{\pi}{2} + \frac{\pi}{2} = \pi \quad x_5 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$

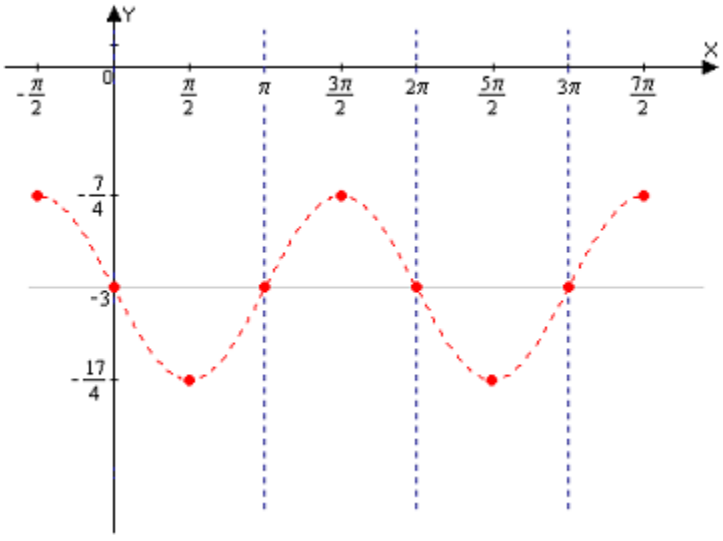
Find the y-values of the five key points

| x-value | y-value | key point |
|---|---|---|
| $-\frac{\pi}{2}$ | $y = \frac{5}{4}\cos\left(x + \frac{\pi}{2}\right) - 3$<br>$= \frac{5}{4}\cos\left(-\frac{\pi}{2} + \frac{\pi}{2}\right) - 3$<br>$= \frac{5}{4}\cos(0) - 3$<br>$= \frac{5}{4} - 3 = -\frac{7}{4}$ | $\left(-\frac{\pi}{2}, -\frac{7}{4}\right)$ |
| 0 | $y = \frac{5}{4}\cos\left(x + \frac{\pi}{2}\right) - 3$<br>$= \frac{5}{4}\cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right) - 3$<br>$= \frac{5}{4}\cos(\pi) - 3$<br>$= -3$ | (0, -3) |
| $\frac{\pi}{2}$ | $y = \frac{5}{4}\cos\left(x + \frac{\pi}{2}\right) - 3$<br>$= \frac{5}{4}\cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right) - 3$<br>$= \frac{5}{4}\cos(\pi) - 3$<br>$= -\frac{5}{4} - 3 = -\frac{17}{4}$ | $\left(\frac{\pi}{2}, -\frac{17}{4}\right)$ |

Exercise 5 (Continued):

| x-value | y-value | key point |
|---|---|---|
| π | $y=\frac{5}{4}\cos\left(x+\frac{\pi}{2}\right)-3$<br>$=\frac{5}{4}\cos\left(\pi+\frac{\pi}{2}\right)-3$<br>$=\frac{5}{4}\cos\left(\frac{3\pi}{2}\right)-3$<br>$=-3$ | (π, -3) |
| $\frac{3\pi}{2}$ | $y=\frac{5}{4}\cos\left(x+\frac{\pi}{2}\right)-3$<br>$=\frac{5}{4}\cos\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)-3$<br>$=\frac{5}{4}\cos(2\pi)-3$<br>$=\frac{5}{4}-3=-\frac{7}{4}$ | $\left(\frac{3\pi}{2},-\frac{7}{4}\right)$ |

Graph the reciprocal function of $y=\frac{5}{4}\cos\left(x+\frac{\pi}{2}\right)-3$



Exercise 5 (Continued):

Graph the secant function using the graph of the cosine function as a guide