Exercise 6A

Question 1:

Solution:

Distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(i) A (9, 3) and B (15, 11)

AB = $\sqrt{(15-9)^2 + (11-3)^2}$
=$\sqrt{(6)^2 + 8^2}$
= $\sqrt{100}$
=10 units

(ii)A (7, -4) and B (-5, 1)

AB = $\sqrt{(-5-7)^2 + (1-(-4))^2}$
= $\sqrt{(-12)^2 + 5^2}$
= $\sqrt{169}$
=13 units

(iii)A (-6, -4) and B (9, -12)

AB = $\sqrt{(9\ -(-6))^2 + ((-12)-(-4))^2}$
= $\sqrt{(15)^2 + (-8)^2}$
= $\sqrt{289}$
=17 units

(iv) A (1, -3) and B (4, -6)

AB = $\sqrt{(4-1)^2 + ((-6)-(-3))^2}$

= $\sqrt{(3)^2 + (-3)^2}$
= $\sqrt{18}$
=3 $\sqrt{2}$units

(v) P (a + b, a – b) and Q (a – b, a + b)

PQ = $\sqrt{((a-b)-(a+b))^2 + ((a+b)-(a-b))^2}$

= $\sqrt{(2b)^2 + (2b)^2}$
= $\sqrt{8b^2}$
= 2b $\sqrt{2}$ units

(vi) P (a sin α, a cos α) and Q (a cos α, -a sin α)

PQ = $\sqrt{(\text{a} \cos \alpha - \text{a} \sin \alpha)^2 + (-\text{a} \sin \alpha - \text{a} \cos \alpha)^2}$
=$\sqrt{a^2(\cos \alpha - \sin \alpha)^2 + a^2 (\sin \alpha - \cos \alpha)^2}$

= $a\sqrt{2 \sin^2 \alpha + 2\, cos^2 \alpha}$
= a $\sqrt{2}$ units

Question 2:

Solution:

Distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance from origin O (0, 0) and the given points (x, y) is

Distance formula: $\sqrt{x^2 + y^2}$

(i)A (5, -12)
OA = $\sqrt{(5)^2 + (-12)^2}$
= $\sqrt{25 + 144}$
= √169
= 13 units

(ii) B (-5, 5)
OB = $\sqrt{(-5)^2 + (5)^2}$
= $\sqrt{25 + 25}$
= √50
= 5√2 units

(iii) C (-4, -6

OC = $\sqrt{(-4)^2 + (-6)^2}$
= $\sqrt{16 + 36}$
= √52
= 2√13 units

Question 3:

Solution:

Given: Points A (x, -1), B (5, 3) and AB = 5 units

Distance formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(5-x)^2 + (3+1)^2}$

Squaring both sides:

$25 - 10x + x^2 + 16 = 25$

$x^2 - 10x + 16 = 0$

$x^2 - 2x - 8x + 16 = 0$

$x(x - 2) - 8(x - 2) = 0$

$(x - 2)(x - 8) = 0$

Either (x-2) = 0 or (x-8) – 0

x = 2 or x = 8

Question 4:

Solution:

Given: Points A (2, -3), B (10, y) and AB = 10

Distance formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB^2 = $(x_2 - x_1)^2 + (y_2 - y_1)^2$

$(10)^2 = (10 - 2)^2 + (y + 3)^2$

$100 = (8)^2 + y^2 + 6y + 9$

$y^2 + 6y + 9 + 64 = 100$

$y^2 + 6y + 73 - 100 = 0$

$y^2 + 6y - 27 = 0$

$y^2 + 9y - 3y - 27 = 0$

$y(y + 9) - 3(y + 9) = 0$

$(y + 9)(y - 3) = 0$

Either, $y + 9 = 0$, then $y = -9$

or $y - 3 = 0$, then $y = 3$

$y = 3, -9$

Question 5:

Solution:

Given: Points P (x, 4), Q (9, 10) and PQ = 10

Distance formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

PQ^2 = $(x_2 - x_1)^2 + (y_2 - y_1)^2$

100 = (9 – x)^2 + (10 – 4)^2

= 81 + x^2 – 18x + 36

= 117 + x^2 – 18x

100 = 117 + x^2 – 18x

x^2 – 18x + 17x = 0 (Solve this equation)

(x – 1)(x – 17)

x = 1 or x = 17

Question 6:

Solution:

Given: Point A (x, 2) is equidistant from B (8, -2) and C (2, -2)

Which implies:

AB = AC

Squaring both sides

AB^2 = AC^2

Using distance formula:

Distance formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

We have,

$(8 - x)^2 + (-2 - 2)^2 = (2 - x)^2 + (-2 - 2)^2$

$(8 - x)^2 + (-4)^2 = (2 - x)^2 + (-4)^2$

$64 - 16x + x^2 = 4 - 4x + x^2$

$64 - 4 = -4x + 16x$

$12x = 60 \Rightarrow x = \frac{60}{12} = 5$

$AB = \sqrt{(8-5)^2 + (-4)^2}$

$= \sqrt{3^2 + (-4)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ units

$x = 5$, AB = 5 units

Question 7:

Solution:

Given: A (0, 2) is equidistant from B (3, p) and C(p, 5)

which implies: AB = AC

or AB^2 = AC^2

Using distance formula:

Distance formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

We have,

$(3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$(3)^2 + (p-2)^2 = p^2 + (3)^2$

$p^2 = p^2 - 4p + 4$

$4p = 4 \Rightarrow p = \frac{4}{4} = 1$

and AB $= \sqrt{(3-0)^2 + (1-2)^2}$

$= \sqrt{9+1} = \sqrt{10}$ units

Question 8:

Solution:

Let point P(x, 0) is on x-axis and equidistant from A(2, -5) and B (-2, 9)

PA = PB

or PA^2 = PB^2

(2 – x)^2 + (– 5 – 0)^2 = (– 2 – x)^2 + (9 – 0)^2

(2 – x)^2 + (– 5)^2 = (– 2 – x)^2 + (9)^2

29 + x^2 – 4x = 85 + x^2 + 4x

56 = – 8x

or x = – 7

The point on ×-axis is (– 7, 0)

Question 9:

Solution:

Let the points on x-axis be P(x, 0) and Q(y, 0) which are at distance of 10 units from point A(11, -8).

Which implies:

PA = QA

or PA^2 = QA^2

$$(11 - x_1)^2 + (-8)^2 = (11 - x_2)^2 + (-8)^2 = 10^2$$
$$(11 - x)^2 + (-8)^2 = 10^2$$
$$121 - 22x + x^2 + 64 = 100$$
$$x^2 - 22x + 85 = 0$$
$$x^2 - 12x - 5x + 85 = 0$$
$$x(x - 17) - 5(x - 17) = 0$$
$$(x - 17)(x - 5) = 0$$

Either (x – 17) = 0 or (x – 5) = 0

x = 17 or x = 5

So, the points are : (17, 0) and (5, 0)

Question 10:

Solution:

Let point P(0, y) is on the y-axis, then

PA = PB

or PA^2 = PB^2

$$(6 - 0)^2 + (5 - y)^2 = (-4 - 0)^2 + (3 - y)^2$$
$$36 + 25 - 10y + y^2 = 16 + 9 - 6y + y^2$$
$$61 - 10y = 25 - 6y$$
$$61 - 25 = -6y + 10y$$

36 = 4y

or y = 9

The required point is (0,9).

Question 11.

Solution:

Since P (x, y) is equidistant from A (5, 1) and B (-1, 5), then

PA = PB

or PA^2 = PB^2

(5 – x)^2 + (1 – y)^2 = (– 1 – x)^2 + (5 – y)^2

(25 + x^2 – 10x) + (1 + y^2 – 2y) = (1 + x^2 + 2x + 25 + y^2 – 10y)

26 + x^2 – 10x + y^2 – 2y = (26 + x^2 + 2x + y^2 – 10y)

12x = 8y

3x = 2y

Hence proved.

Question 12:

Solution:

Since P (x, y) is equidistant from A(6, -1) and B(2, 3), then

PA = PB

or PA^2 = PB^2

(6 – x)^2 + (-1 – y)^2 = (2 – x)^2 + (3 – y)^2

(36 + x^2 – 12x) + (1 + y^2 + 2y) = (4 + x^2 – 4x + 9 + y^2 – 6y)

37 – 12x + 2y = 13 – 4x – 6y

8x = 8y + 24

x – y = 3

Hence proved.

Question 13:

Solution:

Let the coordinates of the point be O(x, y), then

OA = OB = OC

or OA^2 = OB^2 = OC^2

$$OA^2 = (5 - x)^2 + (3 - y)^2$$
$$OB^2 = (5 - x)^2 + (-5 - y)^2$$
$$OC^2 = (1 - x)^2 + (-5 - y)^2$$
$$(5 - x)^2 + (3 - y)^2 = (5 - x)^2 + (-5 - y)^2$$
$$9 - 6y + y^2 = 25 + 10y + y^2$$
$$9 - 25 = 10y + 6y \Rightarrow 16y = -16$$
$$y = \frac{-16}{16} = -1$$
$$\text{and } (5 - x)^2 + (-5 - y)^2 = (1 - x)^2 + (-5 - y)^2$$
$$25 - 10x + x^2 = 1 - 2x + x^2$$
$$-10x + 2x = 1 - 25$$

-8x = -24

or x = 3

So, coordinates of the point is (3,–1).

Question 14:

Solution:

Given: Points A (4, 3) and B (x, 5) lie on a circle with centre O (2, 3)

To find: value of x

OA = OB

or OA^2 = OB^2

$$OA = OB \Rightarrow OA^2 = OB^2$$
$$OA^2 = (2-4)^2 + (3-3)^2 \text{ and}$$
$$OB^2 = (2-x)^2 + (3-5)^2$$
$$(2-4)^2 + (3-3)^2 = (2-x)^2 + (3-5)^2$$
$$(-2)^2 + 0^2 = 4 - 4x + x^2 + (-2)^2$$
$$4 = 4 - 4x + x^2 + 4$$
$$x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0$$
$$x = 2$$

The value of x is 2.

Question 15:

Solution:

Given: Point C(-2, 3) is equidistant from points A(3, -1) and B(x,8).

Then

CA = CB

or CA^2 = CB^2

$CB^2 = (x + 2)^2 + (8 - 3)^2$

$CA^2 = (3 + 2)^2 + (-1 - 3)^2$

$(x + 2)^2 + (8 - 3)^2 = (3 + 2)^2 + (-1 - 3)^2$

$(x + 2)^2 + 5^2 = 5^2 + (-4)^2$

$x^2 + 4x + 4 + 25 = 25 + 16$

$x^2 + 4x + 29 - 41 = 0$

$x^2 + 4x - 12 = 0$

$x^2 + 6x - 2x - 12 = 0$

$x(x + 6) - 2(x + 6) = 0$

$(x + 6)(x - 2) = 0$

This implies: x = 2 or x =-6

NOW: AC = $\sqrt{5^2 + (-4)^2} = \sqrt{41}$

Therefore: AC = $\sqrt{41}\ units$

Question 16:

Solution:

Given: Point P(2, 2) is equidistant from the two points A(-2, k) and B(-2k, -3)

PA =PB or PA^2 = PB^2

$(2 + 2)^2 + (2 - k)^2 = (2 + 2k)^2 (2 + 3)^2$

$4^2 + 4 - 4k + k^2 = 4 + 8k + 4k^2 + 5^2$

$16 + 4 - 4k + k^2 = 4 + 8k + 4k^2 + 25$

$4k^2 + 8k + 29 - 20 + 4k - k^2 = 0$

$3k^2 + 12k + 9 = 0$

$k^2 + 4k + 3 = 0$

$k^2 + k + 3k + 3 = 0$

k(k+1) + 3(k+1) = 0

(k + 1)(k + 3) = 0

thus, k = -1 or k = -3

If k = -1

AP^2 = 20 - 4k + k^2

= 20 + 4 + 1

=25

AP = 5 units

If k = -3

AP^2 = 20 - 4k + k^2

= 20 + 12 + 9

= 41AP = √41 units

Question 17:

Solution:

(i)

Let point P(x, y) is equidistant from A(a + b, b – a) and B(a – b, a + b), then

$AP = BP$ or $AP^2 = BP^2$

$((a + b) - x)^2 + ((a - b) - y)^2 = ((a - b) - x)^2 + ((a + b) - y)^2$

$(a + b)^2 + x^2 - 2(a + b)x + (a - b)^2 + y^2 - 2(a - b)y = (a - b)^2 + x^2 - 2(a - b)x + (a + b)^2 + y^2 - 2(a + b)y$

$(a^2 + b^2 + 2ab + x^2 - 2(a + b)x + b^2 + a^2 - 2ab + y^2 - 2(a - b)y = (a^2 + b^2 - 2ab + x^2 - 2(a - b)x + b^2 + a^2 + 2ab + y^2 - 2(a + b)y$

$\Rightarrow -2(a + b)x - 2(a - b)y = -2(a - b)x - 2(a + b)y$

$\Rightarrow ax + bx + ay - by = ax - bx + ay + by$

$\Rightarrow bx = ay$

(ii)

Point P(x, y) is equidistant from the points A(5, 1) and B(– 1, 5), means $PA = PB$ or $PA^2 = PB^2$

$(5 - x)^2 + (1 - y)^2 = (-1 - x)^2 + (5 - y)^2$

$(25 + x^2 - 10x) + (1 + y^2 - 2y) = (1 + x^2 + 2x + 25 + y^2 - 10y)$

$26 + x^2 - 10x + y^2 - 2y = (26 + x^2 + 2x + y^2 - 10y)$

$12x = 8y$

$3x = 2y$

Hence proved.

Question 18:

Solution:

Points are collinear if sum of any two of distances is equal to the distance of the third.

(i)Let A (1, -1), B (5, 2), C (9, 5)

A, B and C are collinear if AB + BC = AC

$AB = \sqrt{(5-1)^2+(2+1)^2} = \sqrt{4^2+3^2}$

$= 16 + 9 = \sqrt{25} = 5$ units

$BC = \sqrt{(9-5)^2+(5-2)^2} = \sqrt{4^2+3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ units

$AC\ \sqrt{(9-1)^2+(5+1)^2}$

$= \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

$= 10$ units

From above, we can see that

AB + BC = 10 = AC

Therefore, A, B and C are collinear

(ii) Let A(6, 9), B(0, 1) and C(-6, -7)
A, B and C are collinear if AB + BC = AC

$$AB = \sqrt{(0-6)^2+(1-9)^2} = \sqrt{(-6)^2+(-8)^2}$$
$$= \sqrt{36+64} = \sqrt{100} = 10 \text{ units}$$
$$BC = \sqrt{(-6-0)^2+(-7-1)^2}$$
$$= \sqrt{(-6)^2+(-8)^2} = \sqrt{36+64}$$
$$= \sqrt{100} = 10 \text{ units}$$
$$CA = \sqrt{(6+6)^2+(9+7)^2} = \sqrt{12^2+16^2}$$
$$= \sqrt{144+256} = \sqrt{400} = 20 \text{ units}$$

From above, we can see that
AB + BC = 10 + 10 = 20 = CA
Therefore, A, B and C are collinear.

(iii) let A(-1, -1), B(2, 3) and C(8, 11)
A, B and C are collinear if AB + BC = AC

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(2+1)^2+(3+1)^2} = \sqrt{3^2+4^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$
$$BC = \sqrt{(8-2)^2+(11-3)^2} = \sqrt{6^2+8^2}$$
$$= \sqrt{36+64} = \sqrt{100} = 10 \text{ units}$$
$$CA = \sqrt{(8+1)^2+(11+1)^2} = \sqrt{9^2+12^2}$$
$$= \sqrt{81+144} = \sqrt{225} = 15 \text{ units}$$

From above, we can see that
AB + BC = 5 + 10 = 15 = AC
Therefore, A, B and C are collinear.

(iv) Let A(-2, 5), B(0, 1) and C(2, -3).

A, B and C are collinear if AB + BC = AC

$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(0+2)^2+(1-5)^2} = \sqrt{2^2+(-4)^2}$

$= \sqrt{4+16} = \sqrt{20}$

$BC = \sqrt{(2-0)^2+(-3-1)^2} = \sqrt{2^2+(-4)^2}$

$= \sqrt{4+16} = \sqrt{20}$

$= \sqrt{4\times5} = 2\sqrt{5}$ untis

$CA = \sqrt{(2+2)^2+(-3-5)^2} = \sqrt{4^2+(-8)^2}$

$= \sqrt{16+64} = \sqrt{80} = \sqrt{16\times5} = 4\sqrt{5}$

From above, we can see that

$AB + BC = \sqrt{20} + 2\sqrt{5} = 2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5} = AC$

Therefore, A, B and C are collinear.

Question 19:

Solution:

Given points are A(7, 10), B(-2, 5) and C(3, -4)

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (-2 - 7)^2 + (5 - 10)^2 = (-9)^2 + (-5)^2$

$= 81 + 25 = 106$

$BC^2 = (3 + 2)^2 + (-4 - 5)^2 = (5)^2 + (-9)^2$

$= 25 + 81 = 106$

$CA^2 = (7 - 3)^2 + (10 + 4)^2 = (4)^2 + (14)^2$

$= 16 + 196 = 212$

$AB^2 + BC^2 = 106 \Rightarrow AB = BC$

From above, two of the sides are of equal length, so triangle ABC is an isosceles triangle.

Check for: Isosceles right triangle

Sum of square of two sides = Square of third side

$AB^2 + BC^2 = 106 + 106 = 212 = CA^2$

Hence given points are vertices of an isosceles right triangle.

Question 20:

Solution:

Given points are A (3, 0), B (6, 4) and C (-1, 3)

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (6 - 3)^2 + (4 - 0)^2 = 3^2 + 4^2$$
$$= 9 + 16 = 25$$
$$BC^2 = (-1 - 6)^2 + (3 - 4)^2 = (-7)^2 + (-1)^2$$
$$= 49 + 1 = 50$$
$$CA^2 = (3 + 1)^2 + (0 - 3)^2 = 4^2 + 3^2$$
$$= 16 + 9 = 25$$
$$AB^2 = CA^2 = 25$$
$$AB = CA$$

From above, two of the sides are of equal length, so triangle ABC is an isosceles triangle.

Check for : Isosceles right triangle

Sum of square of two sides = Square of third side

AB^2 + AC^2 = 25 + 25 = 50 = CB^2

Hence given points are vertices of an isosceles right triangle.

Exercise 6B

Question 1:

Solution:

If a point P(x,y) divides a line segment having end points coordinates (x_1, y_1) and (x_2, y_2), then coordinates of the point P can be find using below formula:

$x=m_1x_2+m_2x_1m_1+m_2$ $y=m_1y_2+m_2y_1m_1+m_2$

(i) Let P(x, y) be the point which divides the line joining the points A (-1, 7) and B (4, -3) in the ratio 2 : 3. then

x = (2 × 4 + 3 × (– 1))/(2 + 3)

= (8 – 3) /5

= 5/5

= 1

x = 1.

y = (2 × – 3 + 3 × 7)/ 5

= (– 6 + 21)/5

= 15 / 5

= 3

y = 3

Therefore, required point is (1, 3).

x = (7 × 4 + 2 × (– 5))/(7 + 2)

= (28 – 10) /9

= 18/9

= 2

y = (7 × (– 7) + 2 × 11)/ 9

= (– 49 + 22)/9

= – 27 / 9

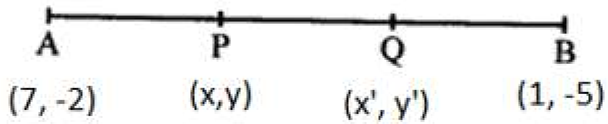
= – 3

Therefore, required point is (2, – 3)

Question 2:

Solution:

Let A(7, -2) and B(1, -5) be the given points and P(x, y) and Q (x', y') are the points of trisection.



Step 1: Find the coordinate of P

Point P divides AB internally in the ratio 1:2

$$(x, y) = \left[\frac{1(1)+2(7)}{1+2}, \frac{1(-5)+1(-2)}{1+2}\right]$$

$$= \left(\frac{1+14}{3}, \frac{-5-4}{3}\right) = \left(\frac{15}{3}, \frac{-9}{3}\right) = (5, -3)$$

Step 2: Find the coordinate of Q

Point Q is the mid-point PB.

(x', y') = ((5+1)/2, (-3-5)/2) = (3, -4)

Therefore, the coordinates of the points of trisection are (5, -3) and (3, -4)

Question 3

Solution:

Coordinate of point P(x, y) can be calculated by using below formula:

Now,

$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$ $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

x = ((3 × 2) + 4(– 2))/ (3 + 4)

= (6 – 8)/7

= – 2/7

y = (3(– 4) + 4(– 2))/ 7

= (– 12 – 8)/ 7

= – 20 / 7

Point P is (-2/7, -20/7)

Question 4:

Solution:

Let the point A(x, y) which lies on line joining P(6, -6) and Q(-4, -1) such that PA/PQ = 2/5

Line segment PQ is divided by the point A in the ratio 2:3.

$x=m_1x_2+m_2x_1m_1+m_2$ $y=m_1y_2+m_2y_1m_1+m_2$

Step 1: Find coordinates of A(x, y)

x = (2(– 4) + 3(6))/(2 + 3)

= (– 8 + 18) /5

= 10/5 = 2

y = (2(– 1) + 3(– 6))/ 5

= (–2 – 18)/5

= – 20 / 5

= – 4

Step 2: Point A also lies on the line 3x + k(y + 1) = 0

3(2) + k(– 4 + 1) = 0

6 – 3k = 0

or k = 2

Question 5:

Solution:

Given: Points P, Q, R and S divides a line segment joining the points A (1, 2) and B (6, 7) in 5 equal parts.

We know that:

$x=m_1x_2+m_2x_1m_1+m_2$ $y=m_1y_2+m_2y_1m_1+m_2$

Now,

Step 1: Find coordinates of P.

P(x, y) divides AB in the ratio 1:4

x = (1 × 6 + 4 × 1)/1 + 4

= (6 + 4) /5

= 10/5

= 2

y = (1x 7 + 4 × 2)/5

= (7 + 8)/5

= 15 / 5

= 3

So, P(x, y) = P(2, 3)

Step 2: Find coordinates of Q.

Q divides the segment AB in ratio 2:3

x = (2x 6 + 3x 1)/5

= (12 + 3) /5

= 15/5 = 3

y = (2 × 7 + 3 × 2)/ 5

= (14 + 6)/5

= 20 / 5 = 4

So, Q(x, y) = Q(3,4)

Step 3: Find coordinates of R.

R divides the segment AB in ratio 3:2

x = (3 × 6 + 2 × 1)/5

= (18 + 2) /5

= 20/5

= 4

y = (3 × 7 + 2 × 2)/ 5

= (21 + 4)/5

= 25 / 5

= 5

So, R(x, y) = R(4,5)

Question 6:

Solution:

Given: Points P, Q and R in order divide a line segment joining the points A (1, 6) and B (5, -2) in four equal parts.

Using formulas:

$x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$ $y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$

Step 1: Find coordinates of P.

P(x, y) divides AB in the ratio of 1:3

x = (5+3)/4 = 8/4 = 2

y = (-2+18) = 16/4 = 4

So P(x, y) = P(2, 4)

Step 2: Find coordinates of Q.

Q divides the segment AB in ratio 2:2 or 1:1. So Q ia midpoint of AB

So Q((1+5)/2, (6-2)/2) = (3, 2)

So, Q(x, y) = Q(3,2)

Step 3: Find coordinates of R.

R divides the segment AB in ratio 3:1

x = (3 × 5 + 1 × 1)/4

= 4

y = (3x(-2) + 1×6)/4

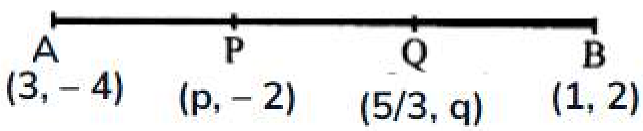
= 0

So, R(x, y) = R(4,0)

Question 7:

Solution:

The line segment joining the point A(3, -4) and B(1, 2) is trisected by the points P(p, -2) and Q(1/2, q). (given)



Step 1: Find x coordinate of P which is p

P(p, -2) divides AB in the ratio of 1:2

p = (1+6)/3 = 7/3

Step 2: Find coordinates of Q

Q divides the segment AB in ratio 2:1

x = (2×1 + 1×3)/3

= (2 + 3) /3

= 5/3

y = (2×2 + 1(– 4))/3

= (4 – 4)/3

= 0/ 3

= 0

= q

Therefore, p = 7/3 and q = 0

Question 8:

Solution:

(i) Midpoint of the line segment joining A (3, 0) and B (-5, 4):

Midpoint = ((x_1 + x_2)/2 , (y_1+y_2)/2)

= ((3-5)/2, (0+4)/2)

= (-1, 2)

(ii) Midpoint of the line segment joining P (-11, -8) and Q (8, -2)

PQ midpoint = ((-11+8)/2, (-8-2)/2)

= (-3/2, -5)

Question 9:

Solution:

Given: (2, p) is the mid point of the line segment joining the points A (6, -5), B (-2, 11)

To find: the value of p

p = (-5+11)/2 = 6/2 = 3

Question 10:

Solution:

Mid point of the line segment joining the points A(2a, 4) and B (-2, 3b) is C(1, 2a + 1)

Mid point of AB = ((2a-2)/2, (4+3b)/2) ….(1)

Mid point of AB = (1, 2a + 1) ….(2) (given)

Now, from (1) and (2)

1 = (2a-2)/2

=> a = 2

and 2a + 1 = (4+3b)/2

10-4 = 3b

or b = 2

Answer: a = 2 and b = 2

Question 11:

Solution:

The line segment joining the points A(-2, 9) and B(6, 3) is a diameter of a circle with centre C.

Which means C is the midpoint of AB.

let (x, y) be the coordinates of C, then

x = (-2 + 6)/2 = 2 and

y = (9+3)/2 = 6

So, coordinates of C are (2, 6).

Question 12:

Solution:

Given:

AB is diameter of a circle with centre C.

Coordinates of C(2, -3) and other point is B (1, 4)

Point C is the midpoint of AB.

Let (x, y) be the coordinates of A, then

2 = (x+1)/2

4 = x +1

x = 3

and

-3 = (y+4)/2

-6 =y +4

or y = -10

So, coordinates of A are (3, -10).

Question 13:

Given: P (2, 5) divides the line segment joining the points A(8,2) and B(-6, 9).

Let P divides the AB in the ratio m : n

$$2 = \frac{mx_2 + nx_1}{m+n} = \frac{m(-6) + n \times 8}{m+n}$$

$$2m + 2n = -6m + 8n$$

$$2m + 6m = 8n - 2n \Rightarrow 8m = 6n$$

$$\frac{m}{n} = \frac{6}{8} = \frac{3}{4}$$

$$\text{Ratio} = 3 : 4$$

Question 14:

Solution:

Let P divides the line segment joining the points A and B in the ratio m:n.

$$\frac{3}{4} = \frac{mx_2 + nx_1}{m+n} = \frac{m \times 2 + n \times \frac{1}{2}}{m+n}$$

$$\frac{3}{4} = \frac{2m + \frac{n}{2}}{m+n}$$

$$3m + 3n = 8m + 2n$$

3n – 2n = 8m – 3n

or m/n = 1/5

Therefore, required ratio is 1:5.

Question 15:

Solution:

Let P divides the join of A and B in the ratio k : 1, then

Step 1: Find coordinates of P:

6 = (k × 8 + 1×3)/ (k+1)

=> 6k + 6 = 8k + 3

or k = 3/2

P divides the join of A and B in the ratio 3:2

Step 2: Find the value of m

m = (2k-4)/(k+1) = (2×3/2 – 4)/(3/2+1)

= -1/(5/2)

= -2/5

Question 16:

Solution:

Let point P divides the join of A and B in the ratio m : n, then

$$-3 = \frac{m(-2)+n(-5)}{m+n}$$

$$-3m - 3n = -2m - 5n$$

$$-5n + 3n = -3m + 2m$$

$$-2n = -m \Rightarrow \frac{m}{n} = \frac{-2}{-1} = \frac{2}{1}$$

ratio = m:n = 2:1

Now,

$$k = \frac{m\times 3+n\times(-4)}{m+n}$$

$$= \frac{2\times 3+1\times(-4)}{2+1}$$

$$= \frac{6-4}{3} = \frac{2}{3}$$

The value of k is 2/3.

Question 17:

Solution:

Let point P on the x-axis divides the line segment joining the points A and B the ratio m : n

Consider P lies on x-axis having coordinates (x, 0).

x = (m × 5 + n x2)/ (m + n)

x = (5m + 2n) / (m + n)

5m + 2n = x(m + n)

(5 – x)m + (2 – x)n = 0 ….(1)

And,

y = 0 = (m × 6 + n(– 3))/ (m+n)

0 = (6m – 3n) /(m + n)

6m – 3n = 0

6m = 3n

or m/n = 3/6 = 1/2

P divides AB in the ratio 1:2.

=> m = 1 and n = 2

From (2)

(5 – x) + (2 – x)(2) = 0

5 – x + 4 – 2x = 0

3x = 9

x = 3

Hence coordinates are (3,0)

Question 18:

Solution:

Given: A (-2, -3) and B (3, 7) divided by the y-axis

Let P lies on y-axis and dividing the line segment AB in the ratio m : n

So, the coordinates of P be (0, y)

$$0 = \frac{m \times 3 + n \times (-2)}{m+n} \Rightarrow 0 = 3m - 2n$$

$$3m = 2n \Rightarrow \frac{m}{n} = \frac{2}{3}$$

Ratio = 2 : 3

$$\text{and } y = \frac{2 \times 7 + 3 \times (-3)}{2+3} = \frac{14-9}{5} = \frac{5}{5} = 1$$

Therefore, the coordinates of P be (0, 1).

Question 19:

Solution:

Given: A line segment joining the points A (3, -1) and B (8, 9) and another line x – y – 2 = 0.

Let a point P (x, y) on the given line x – y – 2 = 0 divides the line segment AB in the ratio m : n

To find: ratio m:n

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{m \times 8 + n \times 3}{m+n} = \frac{8m+3n}{m+n}$$

$$\text{and } y = \frac{my_2 + ny_1}{m+n} = \frac{m \times 9 + n \times (-1)}{m+n}$$

$$= \frac{9m-n}{m+n}$$

Since point P lies on x – y – 2 = 0, so

$$\frac{8m+3n}{m+n} - \frac{9m-n}{m+n} - 2 = 0$$

$$\frac{8m+3n}{m+n} - \frac{9m-n}{m+n} = 2$$

$$8m + 3n - 9m + n = 2m + 2n$$

$$-m + 4n = 2m + 2n$$

$$-m - 2m = +2n - 4n$$

$$-3m = -2n$$

$$\frac{m}{n} = \frac{-2}{-3} = \frac{2}{3}$$

The required ratio is 2:3.

Question 20:

Solution:

Given: Vertices of ∆ABC are A(0, -1), B(2, 1) and C(0, 3)

Let AD, BE and CF are the medians of sides BC, CA and AB respectively, then

Step 1: Find Coordinates of D, E and F
Coordinates of D:

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{4}{2}\right) = (1, 2)$$

Coordinates of E:

$$\left(0, \frac{2}{2}\right) = (0, 1)$$

Coordinates of F:

$$\left(\frac{2}{2}, 0\right) = (1, 0)$$

Step 2: Find the length of AD, BE and CF
Using

Distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

length of AD

$$= \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{1^2 + 3^2}$$

$$= \sqrt{1+9} = \sqrt{10} \text{ units}$$

Length of BE

$$= \sqrt{(2-0)^2 + (1-1)^2}$$

$$= \sqrt{2^2 + (0)^2}$$

$$= 2 \text{ units}$$

Length of CF

$$= \sqrt{(1-0)^2 + (0-3)^2}$$

$$= \sqrt{1+9} = \sqrt{10} \text{ units}$$

Exercise 6C

Question 1:

Solution:

Area of ∆ABC whose vertices are is (x_1,y_1),$(x_2,y_2)$and (x_3,y_3) are

$$\text{Area of } \Delta ABC=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

(i) In ∆ABC, vertices are A (1, 2), B (-2, 3) and C (-3, -4)

Area of triangle = 1/2(1(3 + 4)–2(–4–2)–3(2–3))

= 1/2(7 + 12 + 3) = 22/2

= 11 sq units

(ii) A (-5, 7), B (-4, -5) and C (4, 5)

Area of triangle = 1/2(–5(–5–5)–4(5–7) + 4(7 + 5))

= 1/2(–50 + 8 + 48)

= 5 sq units

(iii) A (3, 8), B (-4, 2) and C (5, -1)

Area of triangle = 1/2(3(2 + 1)–4(–1–8) + 5(8–2))

= 1/2(9 + 36 + 30)

= 1/2(75)

= 37.5 sq units

(iv) A (10, -6), B (2, 5) and C (-1, 3)

Area of triangle = 1/2(10(5–3) + 2(3 + 6)–1(–6–5))

= 1/2(20 + 18 + 11)

= 1/2(49)

= 24.5 sq units

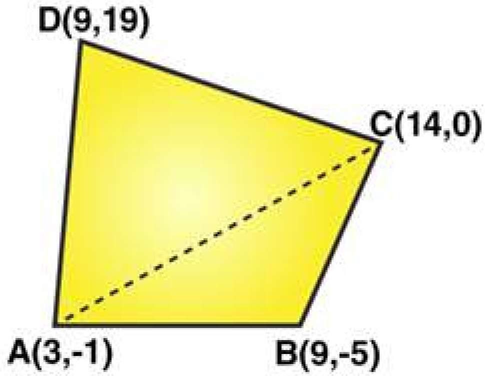
Question 2:

Solution: Vertices of quadrilateral ABCD are A(3,-1), B(9, -5), C (14, 0) and D(9, 19)

Construction: Join diagonal AC.

We know that:

$$\text{Area of } \Delta ABC=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$



Area of triangle ABC:

$= \frac{1}{2}[3(-5-0) + 9(0+1) + 14(-1+5)]$

$= \frac{1}{2}[-15 + 9 + 14(4)]$

$= \frac{1}{2}[-15 + 9 + 56]$

= 1/2 x 50
= 25 sq. units

Area of triangle ADC:

$= \frac{1}{2}[3(0-19) + 14(19+1) + 9(-1+0)]$

$= \frac{1}{2}[3(-19) + 14 \times 20 + 9 \times (-1)]$

$= \frac{1}{2}[-57 + 280 - 9]$

= 1/2 x 214
=107 sq. units

Now, Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ADC

= 25 + 107
= 132 sq. units

Question 3:

Given: PQRS is a quadrilateral whose vertices are P(-5, -3), Q(-4, -6), R(2, -3) and S(1, 2)

Construction: Join PR

We know that:

$$\text{Area of } \triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of triangle PQR:

$$= \frac{1}{2}[-5(-6 + 3) + (-4)(-3 + 3) + 2(-3 + 6)]$$

$$= \frac{1}{2}[-5(-3) + (-4)(0) + 2(3)]$$

$$= \frac{1}{2}[15 + 0 + 6] = \frac{21}{2} \text{ sq. units}$$

Area of triangle PSR.

$$= \frac{1}{2}[-5(-3 - 2) + 2(2 + 3) + 1(-3 + 3)]$$

$$= \frac{1}{2}[-5(-5) + 2 \times 5 + 1 \times 0]$$

$$= \frac{1}{2}[25 + 10 + 0] = \frac{35}{2} \text{ sq. units}$$

Now, Area of quadrilateral PQRS = Area of triangle PQR + Area of triangle PSR

= 21/2 + 35/2

= 28 sq. units

Question 4:

Solution:

Given: ABCD is a quadrilateral whose vertices are A (-3, -1), B (-2, -4), C (4, -1) and D (3, 4).

By Joining AC, we get two triangles ABC and ADC

We know that:

Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of triangle ABC:

$= \frac{1}{2}[-3(-4 + 1) + (-2)(-1 + 1) + 4(-1 + 4)]$

$= \frac{1}{2}[-3(-5) + (-2) \times 0 + 4 \times 3]$

$= \frac{1}{2}[15 - 0 + 12] = \frac{1}{2} \times 27 = \frac{27}{2}$ sq. units

Area of triangle ADC.

$= \frac{1}{2}[-3(-4) + 4(4 + 1) + (3)(-1 + 1)]$

$= \frac{1}{2}[-3(-3) + 4 \times 5 + 3 \times (0)]$

$= \frac{1}{2}[9 + 20 - 0] = \frac{1}{2} \times 29 = \frac{29}{2}$ sq. units

Now, Area of quadrilateral PQRS = Area of triangle ABC + Area of triangle ADC
= 27/2 + 29/2
= 28 sq. units

Question 5:

Solution:

Given: ABCD is a quadrilateral whose vertices are A (-7, 5), B (-6, -7), C (-3, -8) and D (2, 3).
By Joining AC, we get two triangles ABC and ADC

We know that:

Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of triangle ABC:

$= \frac{1}{2}[-7(-7 - (-8)] + (-6)\,[(-8) - 5] + (-3)\,[5 - (-7)]$

$= \frac{1}{2}[-7 \times (1) + (-6) \times (-13) + (-3) \times 12]$

$= \frac{1}{2}[-7 + 78 - 36]$

$= \frac{1}{2} \times 35 = \frac{35}{2}$ sq. units

Area of triangle ADC.

$= \frac{1}{2}[-7(-8 - 3) + (-3)\,(3 - 5) + (2)\,(-5\,(-8)]$

$= \frac{1}{2}[(-7) \times (-11) + (-3) \times (-2) + 2 \times 13]$

$= \frac{1}{2}[77 + 6 + 26] = \frac{109}{2}$ sq. units

Now, Area of quadrilateral PQRS = Area of triangle ABC + Area of triangle ADC

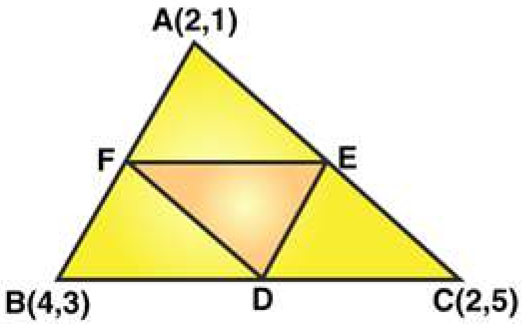
= 35/2 + 109/2
= 72 sq. units

Question 6:

Solution:

Given: A triangle whose vertices are A(2, 1), B(4, 3) and C(2, 5)

Let D, E and F are the midpoints of the sides CB, CA and AB respectively of ΔABC, as shown in the below figure.



Find vertices of D, E and F:

Midpoint formula: (x, y) = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Vertices of D:

$= \left(\frac{4+2}{2}, \frac{3+5}{2}\right)$

$= \left(\frac{6}{2}, \frac{8}{2}\right) = (3, 4)$

Vertices of E:

$= \left(\frac{2+2}{2}, \frac{5+1}{2}\right)$

$= \left(\frac{4}{2}, \frac{6}{2}\right) = (2, 3)$

Vertices of F:

$= \left(\frac{2+4}{2}, \frac{1+3}{2}\right)$

$= \left(\frac{6}{2}, \frac{4}{2}\right) = (3, 2)$

Area of triangle DEF:

We know that:

Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of triangle DEF =

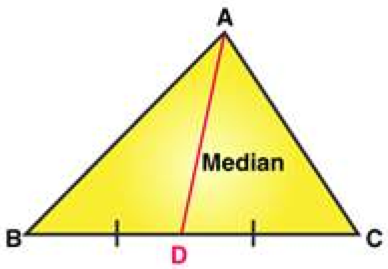
$= \frac{1}{2}[3(3-2) + 2(2-4) + 3(4-3)]$

$= \frac{1}{2}[3 \times 1 + 2 \times (-2) + 3 \times 0]$

$= \frac{1}{2} \times 2 = 1$ sq. units

Question 7:

Solution:



D is midpoint of BC, So find its coordinates using below:

Midpoint formula: (x, y) = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

D = ((3 + 5)/2,(3–1)/2) = (4, 1)

Find area of triangle ABD:

We know that:

Area of a triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

So,

Area of triangle ABD:

= 1/2(7(3–1) + 5(1+3) + 4(–3–3))

= 1/2(14 + 20–24)

= 1/2(10)

= 5 sq. units …(1)

Area of triangle ACD:

= 1/2(7(–1–1) + 3(1 + 3) + 4(–3 + 1))

= 1/2(–14 + 12–8)

= 1/2(10)

= 5 sq. units ….(2)

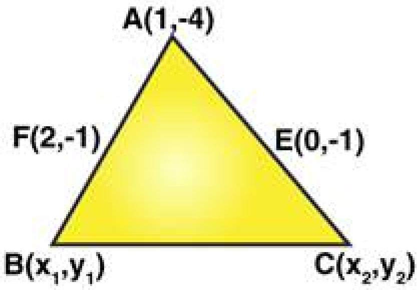
From (1) and (2), we conclude that Area of triangle ABD and ACD is equal.

Hence proved.

Question 8:

Solution:

Given: A ∆ABC with A(1, -4)



Let F and E are the midpoints of AB and AC respectively
Let Coordinates of F are (2, -1) and Coordinates of E are (0, -1)
Let coordinates of B be (x_1 , y_1) and Coordinates of C be (x_2 , y_2)

Find coordinate of B:

using section formula:

$$2 = \frac{1+x_1}{2} \Rightarrow x_1 = 4 - 1 = 3$$

$$-1 = \frac{4+y_1}{2} \Rightarrow y_1 = -2 + 4 = 2$$

Coordinate of B are (3,2)

Find coordinate of C:
using section formula:

$$0 = \frac{1+x_2}{2} \Rightarrow 1 + x_2 = 0 \Rightarrow x_2 = -1$$

$$-1 = \frac{-4+y_2}{2} \Rightarrow -4 + y_2 = -2$$

$$\Rightarrow y_2 = -2 + 4 = 2$$

Coordinate of C are (-1,2)
Now,

Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of triangle ABC:

$$= \frac{1}{2}[1(2 - 2) + 3(2 + 4) + (-1)(-4 - 2)]$$

$$= \frac{1}{2}[1 \times 0 + 3 \times 6 + (-1) \times (-6)]$$

$$= \frac{1}{2}[0 + 18 + 6] = \frac{24}{2} = 12 \text{ sq. units}$$

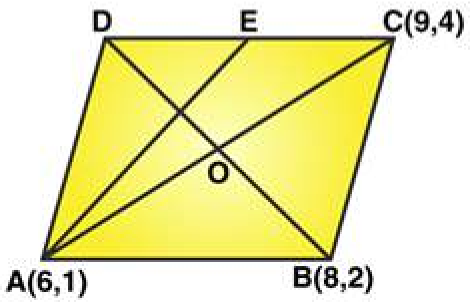
Question 9:

Solution:

A(6, 1), B(8, 2) and C(9, 4) are the three vertices of a parallelogram ABCD.

E is the midpoint of DC.

Join AE, AC and BD which intersects at O, where O is midpoint of AC.



Midpoint formula: $(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Find coordinates of midpoints D and E:

$$\frac{x+8}{2} = \frac{15}{2} \Rightarrow x + 8 = 15$$

$$x = 15 - 8 = 7$$

and

$$\frac{y+2}{2} = \frac{5}{2} \Rightarrow y + 2 = 5$$

$$y = 5 - 2 = 3$$

Coordinate of D are (7, 3)

And

$$= \left(\frac{7+9}{2}, \frac{3+4}{2}\right)$$

$$= \left(8, \frac{7}{2}\right)$$

Coordinate of E are (7, 3)

Now, We know that:

Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of ΔADE

$= \frac{1}{2}[6\left(3-\frac{7}{2}\right) + 7\left(\frac{7}{2}-1\right) + 8(1-3)]$

$= \frac{1}{2}[6 \times \left(\frac{-1}{2}\right) + 7 \times \frac{5}{2} + 8(-2)]$

$= \frac{5}{2}[-3 + \frac{35}{2} - 16]$

$= \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$ sq. units

Question 10:

(ii) The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, -2). If the third vertex is (7/2, y), find the value of y.

Solution:

We know that:

Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

(i) Vertices of a ΔABC are A (1, -3), B (4, p) and C (-9, 7) and
Area = 15 sq. units

$$15 = \frac{1}{2}[1(p - 7) + 4(7 + 3) + (-9)(-3 - p)]$$
$$30 = (p - 7 + 40 + 27 + 9p)$$
$$30 = 10p + 60$$
$$10p = 30 - 60 = -30 \Rightarrow p = \frac{-30}{10} = -3$$

(ii)

$$5 = \frac{1}{2}[2(-2 - y) + 3(y - 1) + \frac{7}{2}(1 + 2)]$$
$$10 = \left[-4 - 2y + 3y - 3y + \frac{7}{2} + 7\right]$$
$$10 = \left[y + \frac{7}{2}\right] \quad 10 - \frac{7}{2} = y \Rightarrow y = \frac{13}{2}$$

Question 11:

Solution:

Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$6 = \frac{1}{2}[(k + 1)(-3 + k) + 4(-k - 1) + 7(1 + 3)]$$
$$6 \times 2 = [-3k + k^2 - 3 + k - 4k - 4 + 28]$$
$$12 = [k^2 - 6k + 21]$$
$$k^2 - 6k + 21 - 12 = 0 \Rightarrow k^2 - 6k + 9 = 0$$
$$(k - 3)^2 = 0 \Rightarrow k - 3 = 0$$
$$k = 3$$

The value of k is 3.

Question 12:

Solution:

Area of triangle = 53 square units

Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$53 = \frac{1}{2}[-2(-4 - 10) + k(10 - 5) + (2k + 1)(5 + 4)]$

$53 = \frac{1}{2}[-2 \times (-14) + k \times 5 + (2k + 1) \times 9]$

$106 = 23k + 37$

$k = \frac{69}{23} = 3$

The value of k is 3.

Question 13:

Solution:

Points are collinear if the area of a triangle is equal to zero.

Areaof△ABC=12[x1(y2–y3)+x2(y3–y1)+x3(y1–y2)]

(i) A (2, -2), B (-3, 8) and C (-1, 4)

Δ = 1/2{2 (8– 4) + (–3) (4 + 2) –1 (2– 8)}

Δ = 1/2 {8–18 + 10}

Δ = 0

Hence points are collinear.

(ii) A (-5, 1), B (5, 5) and C (10, 7)

Δ = 1/2{–5(5– 7) + 5 (7–1) + 10 (1–5)}

Δ = 1/2{10 + 30–40}

Δ = 0

Hence points are collinear.

(iii) A (5, 1), B (1, -1) and C (11, 4)

Δ = 1/2{5(–1– 4) + 1 (4– 1) + 11 (1 + 1)}

= 1/2{–25 + 3 + 22}

= 0

Hence points are collinear.

(iv) A (8, 1), B (3, -4) and C (2, -5)

Δ = 1/2{8(–4 + 5) + 3 (–5–1) + 2 (1 + 4)}

= 1/2{8–18 + 10}

= 0

Hence points are collinear.

Question 14:

Solution:

Points are A (x, 2), B (-3, -4) and C (7, -5) are collinear.

Which means area of triangle ABC = 0

$$\text{Area of } \triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[x(-4+5) + (-3)(-5-2) + 7(2+4)]$$

$$= \frac{1}{2}[x \times 1 + (-3) \times (-7) + 7 \times 6]$$

$$= \frac{1}{2}[x + 21 + 42] = \frac{1}{2}(x + 63)$$

Since points are collinear:

½(x + 63) = 0

Or x = -63

Question 15:

Solution:

Points are A (-3, 12), B (7, 6) and C (x, 9) are collinear.

Which means area of triangle ABC = 0

$$\text{Area of } \triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-3(6-9) + 7(9-12) + x(12-6)]$$

$$= \frac{1}{2}[-3 \times (-3) + 7 \times (-3) + x \times 6]$$

$$= \frac{1}{2}[9 - 21 + 6x]$$

$$= \frac{1}{2}[6x - 12]$$

Since points are collinear:

½(6x -12) = 0

Or x = 2

Question 16:

Solution:

Points are P (1, 4), Q (3, y) and R (-3, 16) are collinear.

Which means area of triangle PQR = 0

Area of $\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[1(y-16) + 3(16-4) + (-3)(4-y)]$$

$$= \frac{1}{2}[y - 16 + 3 \times 12 - 12 + 3y]$$

$$= \frac{1}{2}[4y - 16 + 36 - 12] = \frac{1}{2}[4y + 8]$$

Since points are collinear:

½(4y + 8) = 0

Or y = -2

Question 17:

Solution:

Points are A (-3, 9), B (2, y) and C (4, -5) are collinear.

Which means area of triangle ABC = 0

Areaof$\triangle$ABC=12[x1(y2–y3)+x2(y3–y1)+x3(y1–y2)]

$$= \frac{1}{2}[-3(y + 5) + 2(-5 - 9) + 4(9 - y)]$$

$$= \frac{1}{2}[-3y - 15 + 2 \times (-14) + 36 - 4y]$$

$$= \frac{1}{2}[-7y - 15 - 28 + 36]$$

$$= \frac{1}{2}[-7y - 7]$$

Since points are collinear:

½(-7y – 7) = 0

Or y = -1

Question 18:

Solution:

Points are A (8, 1), B (3, -2k) and C (k, -5) are collinear.

Which means area of triangle ABC = 0

Areaof$\triangle$ABC=12[x1(y2–y3)+x2(y3–y1)+x3(y1–y2)]

$$= \frac{1}{2}[8(-2k + 5) + 3(-5 - 1) + k(1 + 2k)]$$

$$= \frac{1}{2}[-16k + 40 + 3(-6) + k + 2k^2]$$

$$= \frac{1}{2}[-16k + 40 - 18 + k + 2k^2]$$

$$= \frac{1}{2}[22 - 15k + 2k^2]$$

Since points are collinear:

½(2k^2 – 15k + 22) = 0

Or $2k^2 - 15k + 22 = 0$

$2k^2 - 11k - 4k + 22 = 0$

$k(2k - 11) - 2(2k - 11) = 0$

$(k-2)(2k - 11) = 0$

$k = 2$ or $k = 11/2$. Answer.

Question 19:

Solution:

Points are A(2, 1), B(x, y) and C(7, 5) are collinear.

Which means area of triangle ABC = 0

$$\text{Area of } \triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[2(y-5) + x(5-1) + 7(1-y)]$$

$$= \frac{1}{2}[2y - 10 + 4x + 7 - 7y]$$

$$= \frac{1}{2}[4x - 5y - 3]$$

Since points are collinear:

$\frac{1}{2}(4x - 5y - 3) = 0$

$4x - 5y - 3 = 0$

Relationship between x and y.

Question 20:

Solution:

Points are A(x, y), B(-5, 7) and C(-4, 5) are collinear.

Which means area of triangle ABC = 0

$$\text{Area of } \triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[x(7-5) + (-5)(5-y) + (-4)(y-7)]$$

$$= \frac{1}{2}[x \times 2 - 25 + 5y - 4y + 28]$$

$$= \frac{1}{2}[2x + y + 3]$$

Since points are collinear:

½(2x +y + 3) = 0

2x +y + 3 = 0

Relationship between x and y.

Question 21:

Solution:

Points are A(a, 0), B(0, b) and C(1, 1) are collinear.

Which means area of triangle ABC = 0

Area of $\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= ½[(a(b-1) + 0(1-0) + 1(0-b)]

= ½[ab – a + 0 – b]

Since points are collinear:

½(ab -a-b) = 0

ab – a – b = 0

Divide each term by "ab", we get

1 – 1/b -1/a = 0

or 1/a + 1/b = 1. hence proved.

Question 22:

Solution:

Points are P(-3, 9), Q(a, b) and R(4, -5) are collinear.

Which means area of triangle = 0

Area of $\triangle ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[-3(b+5) + a(-5-9) + 4(9-b)]$$

$$= \frac{1}{2}[-3b - 15 - 5a - 9a + 36 - 4b]$$

$$= \frac{1}{2}[-14a - 7b + 21]$$

Since points are collinear, we have

½(-14a – 7b + 21) = 0

-14a – 7b + 21 = 0

2a + b =3 …..(1)

a + b = 1 …..(2) (given)

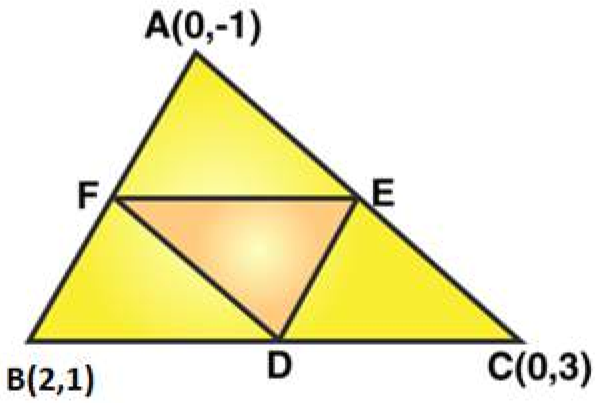
From (1) and (2)

a = 2 and b = -1

Question 23:

Solution:

Vertices of ∆ABC are A (0, -1), B (2, 1) and C (0, 3)



Area of ΔABC = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of triangle ABC:

$= \frac{1}{2}[0(1 - 3) + 2(3 + 1) + 0(-1 - 1)]$

$= \frac{1}{2}[0 + 2 \times 4 + 0] = \frac{1}{2} \times 8 = 4$ sq. units

From figure: Points D, E and F are midpoints of sides BC, CA and AB respectively.

Find the coordinates of D, E and F

coordinates of D

$= \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$

=(1,2)

coordinates of E

$= \left(\frac{0+0}{2}, \frac{3-1}{2}\right)$

= (0, 1)

coordinates of F

$= \left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$

=(1,0)

Act
Go t

Area of triangle DEF:

= 1/2[1+0+1]

= 1 sq. units

Therefore,

Ratio in the area of triangles ABC and DEF = 4/1 = 4:1.

Question 24:

Solution:

Let A (a, a^2), B (b, b^2) and C (0, 0) are the vertices of a triangle.

Let us assume that that points are collinear, then area of ∆ABC must be zero.

Now, area of ∆ABC

$$= \frac{1}{2}[a(b^2 - 0) + b(0 - a^2) + 0(a^2 - b^2)]$$

$$= \frac{1}{2}(ab^2 - bc^2)$$

$$= \frac{ab}{2}(b - a)$$

$$\neq 0$$

Which is contraction to our assumption.

This implies points are not be collinear. Hence proved.

Exercise 6D

Question 1:

Solution:

Points A(-1, y) and B(5, 7) lie on a circle with centre O(2, -3y).

Which means: OA = OB or $OA^2 = OB^2$

using distance formula, we get

$(-1-2)^2 + (y-(-3y))^2 = (5-2)^2 + (7-(-3y))^2$

$9 + 16y^2 = 9 + (7 + 3y)^2$

$16y^2 = 49 + 42y + 9y^2$

$7y^2 – 42y – 49 = 0$

$7(y^2-6y-7) = 0$

$y^2-7y + y-7 = 0$

$y(y-7) + 1(y-7) = 0$

$(y + 1)(y-7) = 0$

Therefore, y = 7 or y = -1

Possible values of y are 7 or -1.

Question 2:

Solution:

A (0, 2) is equidistant from the points B (3, p) and C (p, 5)

Which means: AB = AC or $AB^2 = AC^2$

using distance formula, we get

$(0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$

$9 + 4 + p^2 – 4p = p^2 + 9$

$4p-4 = 0$

$p = 1$

Therefore, the value of p is 1.

Question 3:

Solution:

ABCD is a rectangle whose three vertices are B (4, 0), C (4, 3) and D (0, 3).

Find length of one of its diagonal, say BD: using distance formula, we get

$$BD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-0)^2 + (0-3)^2}$$

$$= \sqrt{(4)^2 + (-3)^2} = \sqrt{16+9}$$

$$= \sqrt{25} = 5 \text{ units}$$

Therefore, length of one of its diagonal is 1.

Question 4:

Solution:

Point P (k – 1, 2) is equidistant from the points A (3, k) and B (k, 5).

PA = PB or PA^2 = PB^2

$$(3 - k + 1)^2 + (k - 2)^2 = (k - k + 1)^2 + (5 - 2)^2$$
$$(4 - k)^2 + (k - 2)^2 = 1^2 + 3^2$$
$$16 - 8k + k^2 + k^2 - 4k + 4 = 1 + 9$$
$$2k^2 - 12k + 20 = 10$$
$$2k^2 - 12k + 20 - 10 = 0$$
$$2k^2 - 12k + 10 = 0$$
$$k^2 - 6k + 5 = 0$$
$$k^2 - k - 5k + 5 = 0$$

k(k-1)-5(k-1) = 0

(k-5)(k-1) = 0

k = 1 or k = 5

Question 5:

Solution:

If point P (x, 2) divides the join of A (12, 5) and B (4, -3), then

using section formula, we get

2 = (m x (-3) + n x (5)) / (m + n)

2m + 2n = -3m + 5n

5m = 3n

m/n = m:n = 3:5

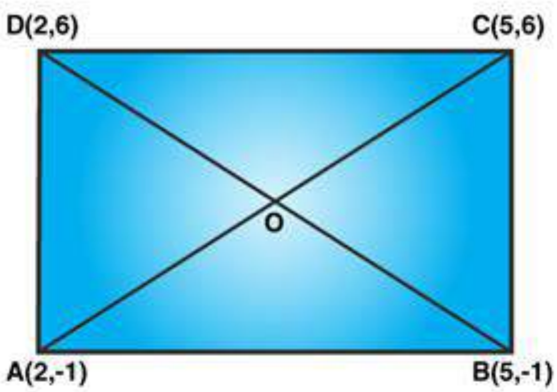
The required ratio is 3:5.

Question 6:

Solution:

Vertices f a rectangle ABCD are A(2, -1), B(5, -1), C(5, 6) and D(2, 6)

To prove: Diagonals of the rectangle are equal and bisect each other.



Diagonal AC

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-2)^2 + (6+1)^2}$$

$$= \sqrt{9+49} = \sqrt{58}$$

Diagonal BD

$$\sqrt{(5-2)^2 + (-1-6)^2}$$

$$= \sqrt{3^2 + (-7)^2}$$

$$= \sqrt{58}$$

AC and BD are equal in length. Thus, Diagonals are equal.
Now,
Consider that O is the midpoint of AC then its coordinate are
Midpoint formula:

$$(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

= ((2+5)/2, (-1+6)/2)

= (7/2, 5/2)

If point O divides AC in the ratio m:n, then

$$\frac{7}{2} = \frac{mx_2+nx_1}{m+n} = \frac{m\times2+n\times5}{m+n}$$

$$= \frac{2m+5n}{m+n}$$

$7m + 7n = 4m + 10n$

$7m - 4m = 10n - 7n \Rightarrow 3m = 3n$

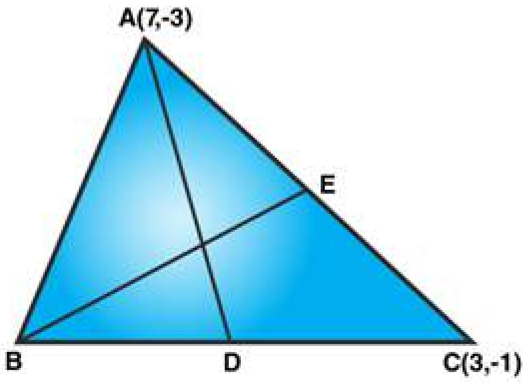
$m = n$

Which shows, O is the midpoint of diagonals.

Question 7:

Solution:

Vertices of ΔABC are A(7, -3), B(5, 3) and C(3, -1)



From figure: BE and AD are the medians of triangle.

Find Coordinates of E and D:
Coordinates of E =

$$\left(\frac{3+7}{2}, \frac{-1-3}{2}\right)$$

= (5, -2)

Coordinates of D=

$$\left(\frac{3+5}{2}, \frac{-1+3}{2}\right)$$

= (4, 1)

Find AD and BE using distance formula:

$$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(7-4)^2+(3+1)^2} = \sqrt{3^2+4^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

$$\text{and } BE = \sqrt{(5-5)^2+(3+2)^2} = \sqrt{0^2+5^2}$$

$$= \sqrt{25} = 5 \text{ units}$$

Therefore, BE = 5 units and AD = 5 units (both are equal)

Question 8:

Solution:

C (k, 4) divides the join of A (2, 6) and B (5, 1) in the ratio 2 : 3.

Using Section Formula:

$$k = \frac{\frac{2}{3} \times (5) + 1 \times (2)}{\frac{2}{3} + 1}$$

After simplifying, we get k = 16/5

The value of k is 16/5.

Question 9:

Solution:

Since point lies on x-axis, y-coordinate of the point will be zero.

Let P (x, 0) be on x-axis which is equidistant from A (-1, 0) and B (5, 0)

Using section formula:

$$x = \frac{1 \times (5) + 1 \times (-1)}{1 + 1}$$

Or x = 2

Thus, the required point is (2, 0).

Question 10:

Solution:

Using distance formula, we have

$$= \sqrt{\left(\frac{2}{5} + \frac{8}{5}\right)^2 + (2-2)^2}$$

$$= \sqrt{\left(\frac{10}{5}\right)^2 + 0^2}$$

$$= \sqrt{2^2 + 0^2}$$

$$= \sqrt{4} = 2 \text{ units}$$

Question 11:

Solution:

The points (3, a) lies on the line 2x – 3y = 5.

Put value of x = 3 and y = a in given equation,

2 x 3 – 3 x a = 5

6 – 3a = 5

3a = 6 – 5

a = 1/3

Question 12:

Solution:

Points A (4, 3) and B (x, 5) lie on the circle with centre O(2, 3)

Which means: OA = OB

=> OA^2 = OB^2

$$(2-4)^2 + (3-3)^2 = (2-x)^2 + (3-5)^2$$
$$(-2)^2 + 0^2 = (2-x)^2 + (-2)^2$$
$$(2-x)^2 = 0$$

2-x = 0

x = 2

The value of x is 2.

Question 13:

Solution:

P(x, y) is equidistant from the point A(7, 1) and B(3, 5)

PA = PB

=> PA^2 = PB^2

$$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$$
$$x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$$

-8x + 8y = -16

x - y = 2

Relation between x and y is x - y = 2

Question 14:

Solution:

Centroid of ∆ABC having vertices A (a, b), B (b, c) and C (c, a) is the origin.

Let O (0, 0) is the centroid of ∆ABC.

a + b + c = 0

And

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$$

Question 15:

Centroid of ∆ABC whose vertices are A(2, 2), B(-4, -4) and C(5, -8).

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$$

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{2-4+5}{3}, \frac{2-4-8}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{-10}{3}\right)$$

$$= \left(1, \frac{-10}{3}\right)$$

Question 16:

Solution:

Point C(4, 5) divide the join of A(2, 3) and B(7, 8)

Let point C(4, 5) divides the AB in the ratio m : n

Using section formula:

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$4 = \frac{m(7) + n(2)}{m + n}$$

4m + 4n = 7m + 2n

3m = 2n

m:n = 2:3

The required ratio is 2:3.

Question 17:

Solution:

Points A(2, 3), B(4, k) and C(6, -3) are collinear.

Area of triangle having vertices A, B and C = 0

Area of a triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of given Δ ABC = 0

$= \frac{1}{2}[(2(k - (-3)) + 4(-3 - 3) + 6(3 - k))] = 0$

2k + 6 - 24 + 18 - 6k = 0

-4k = 0

or k = 0

The value of k is zero.