- Determine the size of a given matrix.
- Identify the row vectors and column vectors of a given matrix.
- Perform the arithmetic operations of matrix addition, subtraction, scalar multiplication, and multiplication.
- Determine whether the product of two given matrices is defined.
- Compute matrix products using the row-column method, the column method, and the row method.
- Express the product of a matrix and a column vector as a linear combination of the columns of the matrix.
- Express a linear system as a matrix equation, and identify the coefficient matrix.
- Compute the transpose of a matrix.
- Compute the trace of a square matrix.

## Exercise Set 1.3

**1.** Suppose that $A$, $B$, $C$, $D$, and $E$ are matrices with the following sizes:

$$\begin{array}{ccccc} A & B & C & D & E \\ (4 \times 5) & (4 \times 5) & (5 \times 2) & (4 \times 2) & (5 \times 4) \end{array}$$

In each part, determine whether the given matrix expression is defined. For those that are defined, give the size of the resulting matrix.

(a) $BA$

(b) $AC + D$

(c) $AE + B$

(d) $AB + B$

(e) $E(A + B)$

(f) $E(AC)$

(g) $E^T A$

(h) $(A^T + E)D$

**Answer:**

(a) Undefined

(b) $4 \times 2$

(c) Undefined

(d) Undefined

(e) $5 \times 5$

(f) $5 \times 2$

(g) Undefined

(h) $5 \times 2$

**2.** Suppose that $A$, $B$, $C$, $D$, and $E$ are matrices with the following sizes:

$$\begin{array}{ccccc} A & B & C & D & E \\ (3\times 1) & (3\times 6) & (6\times 2) & (2\times 6) & (1\times 3) \end{array}$$

In each part, determine whether the given matrix expression is defined. For those that are defined, give the size of the resulting matrix.

(a) $EA$

(b) $AB^T$

(c) $B^T\left(A+E^T\right)$

(d) $2A+C$

(e) $\left(C^T+D\right)B^T$

(f) $CD+B^TE^T$

(g) $\left(BD^T\right)C^T$

(h) $DC+EA$

**3.** Consider the matrices

$$A=\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix},\quad B=\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix},\quad C=\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},\quad D=\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix},\quad E=\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

In each part, compute the given expression (where possible).

(a) $D+E$

(b) $D-E$

(c) $5A$

(d) $-7C$

(e) $2B-C$

(f) $4E-2D$

(g) $-3(D+2E)$

(h) $A-A$

(i) $\mathrm{tr}(D)$

(j) $\mathrm{tr}(D-3E)$

(k) $4\ \mathrm{tr}(7B)$

(l) $\mathrm{tr}(A)$

**Answer:**

(a) $\begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 15 & 0 \\ -5 & 10 \\ 5 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} -7 & -28 & -14 \\ -21 & -7 & -35 \end{bmatrix}$

(e) Undefined

(f) $\begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix}$

(g) $\begin{bmatrix} -39 & -21 & -24 \\ 9 & -6 & -15 \\ -33 & -12 & -30 \end{bmatrix}$

(h) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

(i) 5

(j) $-25$

(k) 168

(l) Undefined

**4.** Using the matrices in Exercise 3, in each part compute the given expression (where possible).

(a) $2A^T + C$

(b) $D^T - E^T$

(c) $(D - E)^T$

(d) $B^T + 5C^T$

(e) $\frac{1}{2}C^T - \frac{1}{4}A$

(f) $B - B^T$

(g) $2E^T - 3D^T$

(h) $\left(2E^T - 3D^T\right)^T$

(i) $(CD)E$

(j) $C(BA)$

(k) $\mathrm{tr}(DE^T)$

(l) $\mathrm{tr}(BC)$

**5.** Using the matrices in Exercise 3, in each part compute the given expression (where possible).

(a) $AB$

(b) $BA$

(c) $(3E)D$

(d) $(AB)C$

(e) $A(BC)$

(f) $CC^T$

(g) $(DA)^T$

(h) $\left(C^T B\right)A^T$

(i) $\mathrm{tr}(DD^T)$

(j) $\mathrm{tr}\left(4E^T - D\right)$

(k) $\mathrm{tr}\left(C^T A^T + 2E^T\right)$

(l) $\mathrm{tr}\left(\left(EC^T\right)^T A\right)$

**Answer:**

(a) $\begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$

(b) Undefined

(c) $\begin{bmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$

(f) $\begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}$

(g) $\begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}$

(h) $\begin{bmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{bmatrix}$

(i) 61

(j) 35

(k) 28

(l) 99

**6.** Using the matrices in Exercise 3, in each part compute the given expression (where possible).

(a) $\left(2D^T - E\right)A$

(b) $(4B)C + 2B$

(c) $(-AC)^T + 5D^T$

(d) $\left(BA^T - 2C\right)^T$

(e) $B^T(CC^T - A^TA)$

(f) $D^TE^T - (ED)^T$

**7.** Let

$$A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

Use the row method or column method (as appropriate) to find

(a) the first row of *AB*.

(b) the third row of *AB*.

(c) the second column of *AB*.

(d) the first column of *BA*.

(e) the third row of *AA*.

(f) the third column of *AA*.

**Answer:**

(a) $[67\ 41\ 41]$

(b) $[63\ 67\ 57]$

(c) $\begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix}$

(d) $\begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix}$

(e) $[24\ 56\ 97]$

(f) $\begin{bmatrix} 76 \\ 98 \\ 97 \end{bmatrix}$

**8.** Referring to the matrices in Exercise 7, use the row method or column method (as appropriate) to find

(a) the first column of *AB*.

(b) the third column of *BB*.

(c) the second row of *BB*.

(d) the first column of *AA*.

(e) the third column of *AB*.

(f) the first row of *BA*.

**9.** Referring to the matrices A and B in Exercise 7, and Example 9,

(a) express each column vectorof *AA* as a linear combination of the column vectors of *A*.

(b) express each column vector of *BB* as a linear combination of the column vectors of *B*.

**Answer:**

(a) $\begin{bmatrix}-3\\48\\24\end{bmatrix}=3\begin{bmatrix}3\\6\\0\end{bmatrix}+6\begin{bmatrix}-2\\5\\4\end{bmatrix}$; $\begin{bmatrix}12\\29\\56\end{bmatrix}=-2\begin{bmatrix}3\\6\\0\end{bmatrix}+5\begin{bmatrix}-2\\5\\4\end{bmatrix}+4\begin{bmatrix}7\\4\\9\end{bmatrix}$; $\begin{bmatrix}76\\98\\97\end{bmatrix}=7\begin{bmatrix}3\\6\\0\end{bmatrix}+4\begin{bmatrix}-2\\5\\4\end{bmatrix}+9\begin{bmatrix}7\\4\\9\end{bmatrix}$

(b) $\begin{bmatrix}64\\21\\77\end{bmatrix}=6\begin{bmatrix}6\\0\\7\end{bmatrix}+7\begin{bmatrix}4\\3\\5\end{bmatrix}$; $\begin{bmatrix}14\\22\\28\end{bmatrix}=-2\begin{bmatrix}6\\0\\7\end{bmatrix}+\begin{bmatrix}-2\\1\\7\end{bmatrix}+7\begin{bmatrix}4\\3\\5\end{bmatrix}$; $\begin{bmatrix}38\\18\\74\end{bmatrix}=4\begin{bmatrix}6\\0\\7\end{bmatrix}+3\begin{bmatrix}-2\\1\\7\end{bmatrix}+5\begin{bmatrix}4\\3\\5\end{bmatrix}$

**10.** Referring to the matrices A and B in Exercise 7, and Example 9,

(a) express each column vector of *AB* as a linear combination of the column vectors of *A*.

(b) express each column vector of *BA* as a linear combination of the column vectors of *B*.

**11.** In each part, find matrices *A*, **x**, and **b** that express the given system of linear equations as a single matrix equation $A\mathbf{x}=\mathbf{b}$, and write out this matrix equation.

(a) $$\begin{aligned}2x_1-3x_2+5x_3&=7\\9x_1-x_2+x_3&=-1\\x_1+5x_2+4x_3&=0\end{aligned}$$

(b) $$\begin{aligned}4x_1\qquad\;\,-3x_3+x_4&=1\\5x_1+x_2\qquad\;\,-8x_4&=3\\2x_1-5x_2+9x_3-x_4&=0\\3x_2-x_3+7x_4&=2\end{aligned}$$

**Answer:**

(a) $\begin{bmatrix}2&-3&5\\9&-1&1\\1&5&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}7\\-1\\0\end{bmatrix}$

(b) $\begin{bmatrix}4&0&-3&1\\5&1&0&-8\\2&-5&9&-1\\0&3&-1&7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}1\\3\\0\\2\end{bmatrix}$

**12.** In each part, find matrices *A*, **x**, and **b** that express the given system of linear equations as a single matrix equation $A\mathbf{x}=\mathbf{b}$, and write out this matrix equation.

(a) $$\begin{aligned}x_1-2x_2+3x_3&=-3\\2x_1+x_2\qquad&=0\\-3x_2+4x_3&=1\\x_1\qquad+x_3&=5\end{aligned}$$

(b) $$\begin{aligned}3x_1+3x_2+3x_3&=-3\\-x_1-5x_2-2x_3&=3\\-4x_2+x_3&=0\end{aligned}$$

**13.** In each part, express the matrix equation as a system of linear equations.

(a) $\begin{bmatrix}5&6&-7\\-1&-2&3\\0&4&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}2\\0\\3\end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 5 & -3 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -9 \end{bmatrix}$

**Answer:**

(a)
$$\begin{aligned} 5x_1 + 6x_2 - 7x_3 &= 2 \\ -x_1 - 2x_2 + 3x_3 &= 0 \\ 4x_2 - x_3 &= 3 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 &= 2 \\ 5x_1 - 3x_2 - 6x_3 &= -9 \end{aligned}$$

**14.** In each part, express the matrix equation as a system of linear equations.

(a) $\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 7 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -2 & 0 & 1 \\ 5 & 0 & 2 & -2 \\ 3 & 1 & 4 & 7 \\ -2 & 5 & 1 & 6 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

In Exercises 15–16, find all values of *k*, if any, that satisfy the equation.

**15.** $[k \quad 1 \quad 1]\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix}\begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$

**Answer:**

$-1$

**16.** $[2 \quad 2 \quad k]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$

In Exercises 17–18, solve the matrix equation for *a*, *b*, *c*, and *d*.

**17.** $\begin{bmatrix} a & 3 \\ -1 & a+b \end{bmatrix} = \begin{bmatrix} 4 & d-2c \\ d+2c & -2 \end{bmatrix}$

**Answer:**

$a = 4,\ b = -6,\ c = -1,\ d = 1$

**18.** $\begin{bmatrix} a-b & b+a \\ 3d+c & 2d-c \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$

**19.** Let *A* be any $m \times n$ matrix and let *0* be the $m \times n$ matrix each of whose entries is zero. Show that if $kA = 0$, then $k = 0$ or $A = 0$.

**20.** (a) Show that if *AB* and *BA* are both defined, then *AB* and *BA* are square matrices.

(b) Show that if *A* is an $m \times n$ matrix and *A*(*BA*) is defined, then *B* is an $n \times m$ matrix.

**21.** Prove: If $A$ and $B$ are $n \times n$ matrices, then $\mathrm{tr}(A+B) = \mathrm{tr}(A) + \mathrm{tr}(B)$.

**22.** (a) Show that if $A$ has a row of zeros and $B$ is any matrix for which $AB$ is defined, then $AB$ also has a row of zeros.

(b) Find a similar result involving a column of zeros.

**23.** In each part, find a $6 \times 6$ matrix $[a_{ij}]$ that satisfies the stated condition. Make your answers as general as possible by using letters rather than specific numbers for the nonzero entries.

(a) $a_{ij} = 0 \text{ if } i \neq j$

(b) $a_{ij} = 0 \text{ if } i > j$

(c) $a_{ij} = 0 \text{ if } i < j$

(d) $a_{ij} = 0 \text{ if } |i - j| > 1$

**Answer:**

(a) $\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$

(b) $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$

(c) $\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}$

(d) $\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$

**24.** Find the $4 \times 4$ matrix $A = [a_{ij}]$ whose entries satisfy the stated condition.

(a) $a_{ij} = i + j$

(b) $a_{ij} = i^{j-1}$

(c) $a_{ij} = \begin{cases} 1 & \text{if } |i-j| > 1 \\ -1 & \text{if } |i-j| \le 1 \end{cases}$

**25.** Consider the function $y = f(x)$ defined for $2 \times 1$ matrices *x* by $y = Ax$, where

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Plot *f*(*x*) together with *x* in each case below. How would you describe the action of *f*?

(a) $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) $x = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

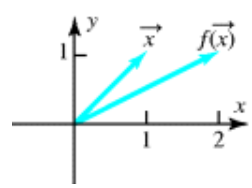
(c) $x = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(d) $x = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

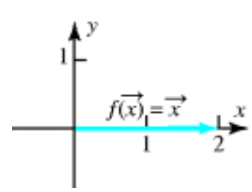
**Answer:**

$f\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 \end{pmatrix}$

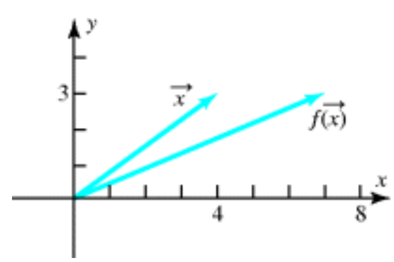
(a) $f\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$



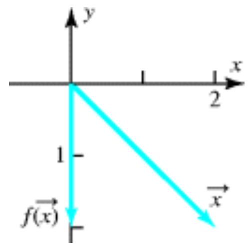

(b) $f\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$




(c) $f\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$




(d) $f\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$




**26.** Let $I$ be the $n \times n$ matrix whose entry in row $i$ and column $j$ is

$$\begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Show that $AI = IA = A$ for every $n \times n$ matrix $A$.

**27.** How many $3 \times 3$ matrices $A$ can you find such that

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ x-y \\ 0 \end{bmatrix}$$

for all choices of $x$, $y$, and $z$?

**Answer:**

One; namely, $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

**28.** How many $3 \times 3$ matrices $A$ can you find such that

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xy \\ 0 \\ 0 \end{bmatrix}$$

for all choices of $x$, $y$, and $z$?

**29.** A matrix $B$ is said to be a ***square root*** of a matrix $A$ if $BB = A$.

(a) Find two square roots of $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$.

(b) How many different square roots can you find of $A = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$?

(c) Do you think that every $2 \times 2$ matrix has at least one square root? Explain your reasoning.

**Answer:**

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

(b) Four; $\begin{bmatrix} \sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} -\sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} \sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$, $\begin{bmatrix} -\sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$

**30.** Let $0$ denote a $2 \times 2$ matrix, each of whose entries is zero.

(a) Is there a $2 \times 2$ matrix $A$ such that $A \neq 0$ and $AA = 0$? Justify your answer.

(b) Is there a $2 \times 2$ matrix $A$ such that $A \neq 0$ and $AA = A$? Justify your answer.

## True-False Exercises

In parts (a)–(o) determine whether the statement is true or false, and justify your answer.

**(a)** The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ has no main diagonal.

**Answer:**

True

**(b)** An $m \times n$ matrix has *m* column vectors and *n* row vectors.

**Answer:**

False

**(c)** If *A* and *B* are $2 \times 2$ matrices, then $AB = BA$.

**Answer:**

False

**(d)** The *i* th row vector of a matrix product *AB* can be computed by multiplying *A* by the *i*th row vector of *B*.

**Answer:**

False

**(e)** For every matrix *A*, it is true that $\left(A^T\right)^T = A$.

**Answer:**

True

**(f)** If *A* and *B* are square matrices of the same order, then $\operatorname{tr}(AB) = \operatorname{tr}(A)\operatorname{tr}(B)$.

**Answer:**

False

**(g)** If *A* and *B* are square matrices of the same order, then $(AB)^T = A^T B^T$.

**Answer:**

False

**(h)** For every square matrix *A*, it is true that $\operatorname{tr}\left(A^T\right) = \operatorname{tr}(A)$.

**Answer:**

True

**(i)** If *A* is a $6 \times 4$ matrix and *B* is an $m \times n$ matrix such that $B^T A^T$ is a $2 \times 6$ matrix, then $m = 4$ and $n = 2$.

**Answer:**

True

**(j)** If $A$ is an $n \times n$ matrix and $c$ is a scalar, then $\mathrm{tr}(cA) = c\ \mathrm{tr}(A)$.

**Answer:**

True

**(k)** If $A$, $B$, and $C$ are matrices of the same size such that $A - C = B - C$, then $A = B$.

**Answer:**

True

**(l)** If $A$, $B$, and $C$ are square matrices of the same order such that $AC = BC$, then $A = B$.

**Answer:**

False

**(m)** If $AB + BA$ is defined, then $A$ and $B$ are square matrices of the same size.

**Answer:**

True

**(n)** If $B$ has a column of zeros, then so does $AB$ if this product is defined.

**Answer:**

True

**(o)** If $B$ has a column of zeros, then so does $BA$ if this product is defined.

**Answer:**

False