Exercises in Signals, Systems, and Transforms

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Contents

1 Discrete-Time Signals and Systems

1.1 Signals

1.1.1 Make an accurate sketch of each of the discrete-time signals

(a)

$$x(n) = u(n+3) + 0.5\, u(n-1)$$

(b)

$$x(n) = \delta(n+3) + 0.5\, \delta(n-1)$$

(c)

$$x(n) = 2^n \cdot \delta(n-4)$$

(d)

$$x(n) = 2^n \cdot u(-n-2)$$

(e)

$$x(n) = (-1)^n\, u(-n-4).$$

(f)

$$x(n) = 2\,\delta(n+4) - \delta(n-2) + u(n-3)$$

(g)

$$x(n) = \sum_{k=0}^{\infty} 4\,\delta(n-3\,k-1)$$

(h)

$$x(n) = \sum_{k=-\infty}^{\infty} (-1)^k\, \delta(n-3\,k)$$

1.1.2 Make a sketch of each of the following signals

(a)

$$x(n) = \sum_{k=-\infty}^{\infty} (0.9)^{|k|}\; \delta(n-k)$$

(b)

$$x(n) = \cos(\pi\, n)\, u(n)$$

(c)

$$x(n) = u(n) - 2\, u(n-4) + u(n-8)$$

1.1.3 Sketch $x(n)$, $x_1(n)$, $x_2(n)$, and $x_3(n)$ where

$$x(n) = u(n+4) - u(n), \qquad x_1(n) = x(n-3),$$

$$x_2(n) = x(5-n), \qquad x_3(n) = \sum_{k=-\infty}^{n} x(k)$$

1.1.4 Sketch $x(n)$ and $x_1(n)$ where

$$x(n) = (0.5)^n\, u(n), \qquad x_1(n) = \sum_{k=-\infty}^{n} x(k)$$

1.1.5 Sketch $x(n)$ and $x_1(n)$ where

$$x(n) = n\,[\delta(n-5) + \delta(n-3)], \qquad x_1(n) = \sum_{k=-\infty}^{n} x(k)$$

1.1.6 Make a sketch of each of the following signals

(a)

$$f(n) = \sum_{k=0}^{\infty} (-0.9)^k\ \delta(n-3\,k)$$

(b)

$$g(n) = \sum_{k=-\infty}^{\infty} (-0.9)^{|k|}\ \delta(n-3\,k)$$

(c)

$$x(n) = \cos(0.25\,\pi\,n)\,u(n)$$

(d)

$$x(n) = \cos(0.5\,\pi\,n)\,u(n)$$

1.1.7 **Plotting discrete-time signals in MATLAB.**

Use `stem` to plot the discrete-time impulse function:

```
n = -10:10;
f = (n == 0);
stem(n,f)
```

Use `stem` to plot the discrete-time step function:

```
f = (n >= 0);
stem(n,f)
```

Make stem plots of the following signals. Decide for yourself what the range of n should be.

$$f(n) = u(n) - u(n-4) \tag{1}$$
$$g(n) = r(n) - 2\,r(n-5) + r(n-10) \quad \text{where } r(n) := nu(n) \tag{2}$$
$$x(n) = \delta(n) - 2\,\delta(n-4) \tag{3}$$
$$y(n) = (0.9)^n\ (u(n) - u(n-20)) \tag{4}$$
$$v(n) = \cos(0.12\ \pi n)\ u(n) \tag{5}$$

1.2 System Properties

1.2.1 A discrete-time system may be classified as follows:

- memoryless/with memory
- causal/noncausal
- linear/nonlinear
- time-invariant/time-varying
- BIBO stable/unstable

Classify each of the following discrete-times systems.

(a)

$$y(n) = \cos(x(n)).$$

(b)

$$y(n) = 2\,n^2\,x(n) + n\,x(n+1).$$

(c)

$$y(n) = \max\,\{x(n),\ x(n+1)\}$$

Note: the notation $\max\{a,\ b\}$ means for example; $\max\{4,\ 6\} = 6$.

(d)

$$y(n) = \begin{cases} x(n) & \text{when } n \text{ is even} \\ x(n-1) & \text{when } n \text{ is odd} \end{cases}$$

(e)

$$y(n) = x(n) + 2\,x(n-1) - 3\,x(n-2).$$

(f)

$$y(n) = \sum_{k=0}^{\infty} (1/2)^k\,x(n-k).$$

That is,

$$y(n) = x(n) + (1/2)\,x(n-1) + (1/4)\,x(n-2) + \cdots$$

(g)

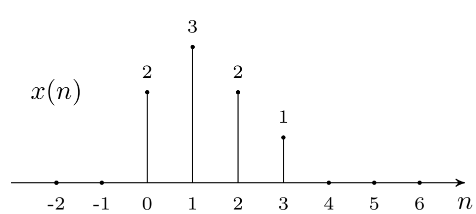
$$y(n) = x(2\,n)$$

1.2.2 A discrete-time system is described by the following rule

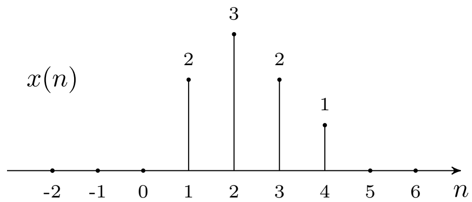
$$y(n) = 0.5\,x(2n) + 0.5\,x(2n-1)$$

where x is the input signal, and y the output signal.

(a) Sketch the output signal, $y(n)$, produced by the 4-point input signal, $x(n)$ illustrated below.



(b) Sketch the output signal, $y(n)$, produced by the 4-point input signal, $x(n)$ illustrated below.



(c) Classify the system as:

i. causal/non-causal

ii. linear/nonlinear

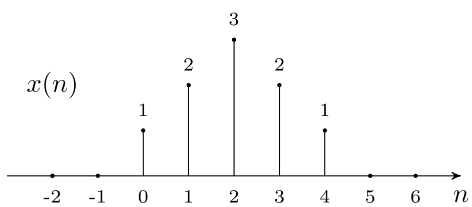
iii. time-invariant/time-varying

1.2.3 A discrete-time system is described by the following rule

$$y(n) = \begin{cases} x(n), & \text{when } n \text{ is an even integer} \\ -x(n), & \text{when } n \text{ is an odd integer} \end{cases}$$

where x is the input signal, and y the output signal.

(a) Sketch the output signal, $y(n)$, produced by the 5-point input signal, $x(n)$ illustrated below.



(b) Classify the system as:

i. linear/nonlinear

ii. time-invariant/time-varying

iii. stable/unstable

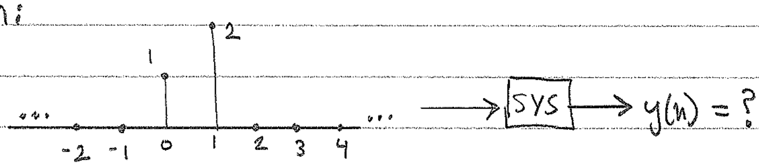
1.2.4 System classification: classNL

2) A discrete-time system is described by

$$y(n) = x(n) + \left[x(n+1)\right]^2$$

y(n) represents the output signal.
x(n) " " input " .

a) find the output signal produced by the input shown:



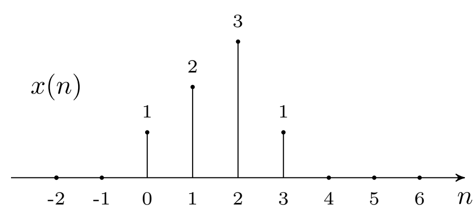
b) Is the system : linear or nonlinear?
c) " " time invariant or time-varying?
d) " " causal or non-causal?

1.2.5 A discrete-time system is described by the following rule

$$y(n) = (-1)^n \, x(n) + 2 \, x(n-1)$$

where x is the input signal, and y the output signal.

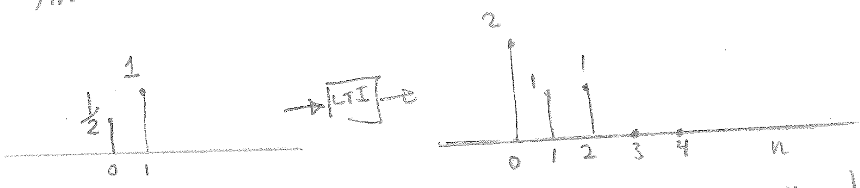
(a) Accurately sketch the output signal, $y(n)$, produced by the input signal $x(n)$ illustrated below.



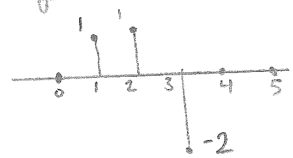
(b) Classify the system as:
   i. causal/non-causal
   ii. linear/nonlinear
   iii. time-invariant/time-varying

1.2.6 Predict the output of an LTI system:

An LTI is observed with input signal shown producind output signal shown.



Find the output signal produced by the signal



1.2.7 The impulse response of a discrete-time LTI system is

$$h(n) = 2\,\delta(n) + 3\,\delta(n-1) + \delta(n-2).$$

Find and sketch the output of this system when the input is the signal

$$x(n) = \delta(n) + 3\,\delta(n-1) + 2\,\delta(n-2).$$

1.2.8 Consider a discrete-time LTI system described by the rule

$$y(n) = x(n-5) + \frac{1}{2}\,x(n-7).$$

What is the impulse response $h(n)$ of this system?

1.2.9 The impulse response of a discrete-time LTI system is

$$h(n) = \delta(n) + 2\,\delta(n-1) + \delta(n-2).$$

Sketch the output of this system when the input is

$$x(n) = \sum_{k=0}^{\infty} \delta(n - 4\,k).$$

1.2.10 The impulse response of a discrete-time LTI system is

$$h(n) = 2\,\delta(n) - \delta(n-4).$$

Find and sketch the output of this system when the input is the step function

$$x(n) = u(n).$$

1.2.11 Consider the discrete-time LTI system with impulse response

$$h(n) = n\,u(n).$$

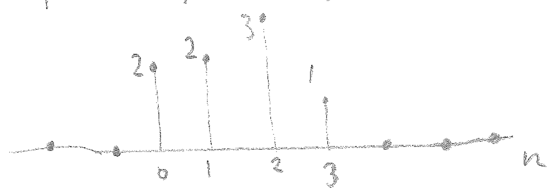
(a) Find and sketch the output $y(n)$ when the input $x(n)$ is

$$x(n) = \delta(n) - 2\,\delta(n-5) + \delta(n-10).$$

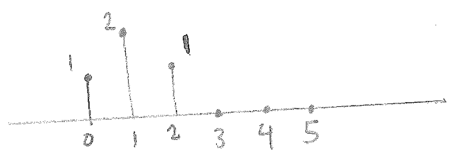
(b) Classify the system as BIBO stable/unstable.

1.2.12 Predict the output of an LTI system:

The impulse response of an LTI system is given by



Find the system output produced by input signal



1.2.13 The impulse response $h(n)$ of an LTI system is given by

$$h(n) = \left(\frac{2}{3}\right)^n u(n).$$

Find and sketch the output $y(n)$ when the input is given by

(a) $x(n) = \delta(n)$

(b) $x(n) = \delta(n-2)$

1.2.14 For the LTI system with impulse response

$$h(t) = \cos(\pi t)\, u(n),$$

find and sketch the step response $s(t)$ and classify the system as BIBO stable/unstable.

1.2.15 Consider the LTI system with impulse response

$$h(n) = \delta(n-1).$$

(a) Find and sketch the output $y(n)$ when the input $x(n)$ is the impulse train with period 6,

$$x(n) = \sum_{k=-\infty}^{\infty} \delta(n-6k).$$

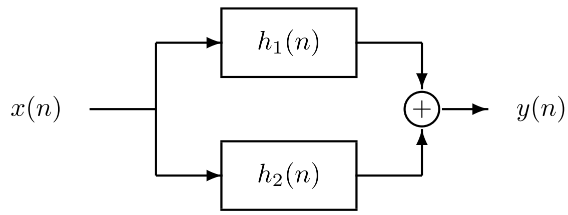
(b) Classify the system as BIBO stable/unstable.

1.2.16 An LTI system is described by the following equation

$$y(n) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k x(n-k).$$

Sketch the impulse response $h(n)$ of this system.

1.2.17 Consider the parallel combination of two LTI systems.



You are told that

$$h_1(n) = u(n) - 2\,u(n-1) + u(n-2).$$

You observe that the step response of the total system is

$$s(n) = 2\,r(n) - 3\,r(n-1) + r(n-2)$$

where $r(n) = n\,u(n)$. Find and sketch $h_2(n)$.

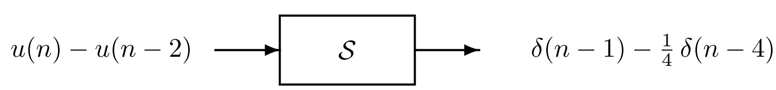
1.2.18 The impulse response of a discrete-time LTI system is given by

$$h(n) = \left\{ \begin{array}{ll} 1 & \text{if } n \text{ is a positive prime number} \\ 0 & \text{otherwise} \end{array} \right\}$$

(a) Is the system causal?

(b) Is the system BIBO stable?

1.2.19 You observe an unknown LTI system and notice that



Sketch the step response $s(n)$. The step response is the system output when the input is the step function $u(n)$.

1.2.20 For an LTI system it is known that input signal

$$x(n) = \delta(n) + 3\,\delta(n-1)$$

produces the following output signal:

$$y(n) = \left(\frac{1}{2}\right)^n u(n).$$

What is the output signal when the following input signal is applied to the system?

$$x_2(n) = 2\,\delta(n-2) + 6\,\delta(n-3)$$

1.3 More Convolution

1.3.1 Derive and sketch the convolution $x(n) = (f * g)(n)$ where

(a)

$$f(n) = 2\,\delta(n+10) + 2\,\delta(n-10)$$

$$g(n) = 3\,\delta(n+5) + 3\,\delta(n-5)$$

(b)

$$f(n) = \delta(n-4) - \delta(n-1)$$

$$g(n) = 2\,\delta(n-4) - \delta(n-1)$$

(c)

$$f(n) = -\delta(n+2) - \delta(n+1) - \delta(n)$$

$$g(n) = \delta(n) + \delta(n+1) + \delta(n+2)$$

(d)

$$f(n) = 4$$

$$g(n) = \delta(n) + 2\,\delta(n-1) + \delta(n-2).$$

(e)

$$f(n) = \delta(n) + \delta(n-1) + 2\,\delta(n-2)$$

$$g(n) = \delta(n-2) - \delta(n-3).$$

(f)

$$f(n) = (-1)^n$$

$$g(n) = \delta(n) + \delta(n-1).$$

1.3.2 The impulse response of a discrete-time LTI system is

$$h(n) = u(n) - u(n-5).$$

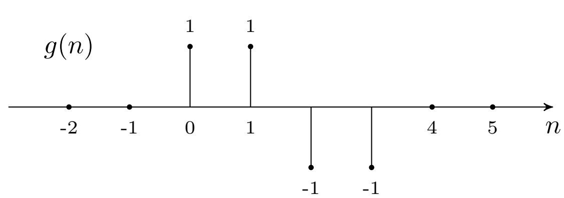
Sketch the output of this system when the input is

$$x(n) = \sum_{k=0}^{\infty} \delta(n - 5\,k).$$

1.3.3 The signal f is given by

$$f(n) = \cos\left(\frac{\pi}{2}\,n\right).$$

The signal g is illustrated.



Sketch the signal, $x(n)$, obtained by convolving $f(n)$ and $g(n)$,

$$x(n) = (f * g)(n).$$

1.3.4 The signals f and g are given by

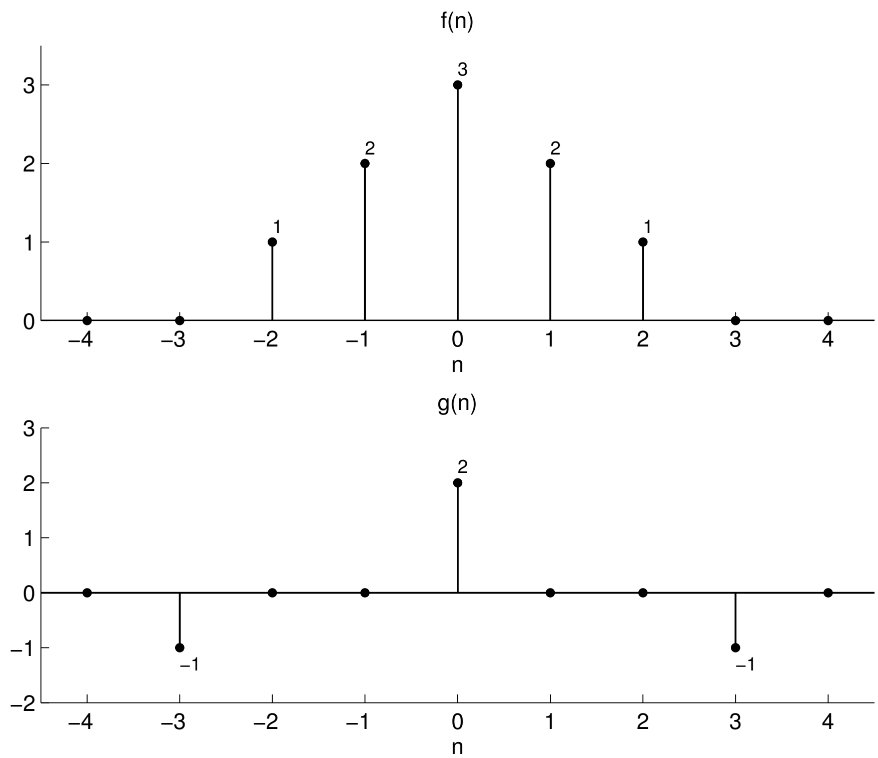
$$f(n) = 2,$$

$$g(n) = \left(\frac{1}{2}\right)^n u(n).$$

Sketch the signal, $x(n)$, obtained by convolving $f(n)$ and $g(n)$,

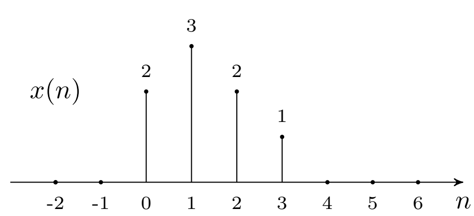
$$x(n) = (f * g)(n).$$

1.3.5 The signals $f(n)$ and $g(n)$ are shown:



Sketch the convolution $x(n) = f(n) * g(n)$.

1.3.6 Sketch the convolution of the discrete-time signal $x(n)$



with each of the following signals.

(a) $f(n) = 2\delta(n) - \delta(n-1)$

(b) $f(n) = u(n)$

(c) $f(n) = 0.5$

(d) $f(n) = \displaystyle\sum_{k=-\infty}^{\infty} \delta(n - 5k)$

1.3.7 Discrete-time signals f and g are defined as:

$$f(n) = a^n \, u(n)$$

$$g(n) = f(-n) = a^{-n} \, u(-n)$$

Find the convolution:

$$x(n) = (f * g)(n)$$

Plot f, g, and x when $a = 0.9$. You may use a computer for plotting.

1.3.8 The N-point *moving average filter* has the impulse response

$$h(n) = \begin{cases} 1/N & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

Use the Matlab `conv` command to compute

$$y(n) = h(n) * h(n)$$

for $N = 5, 10, 20$, and in each case make a stem plot of $h(n)$ and $y(n)$.

What is the general expression for $y(n)$?

1.3.9 The convolution of two finite length signals can be written as a matrix vector product. Look at the documentation for the Matlab `convmtx` command and the following Matlab code that shows the convolution of two signals by (1) a matrix vector product and (2) the `conv` command. Describe the form of the convolution matrix and why it works.

```
>> x = [1 4 2 5]; h = [1 3 -1 2];

>> convmtx(h',4)*x'

ans =

     1
     7
    13
     9
    21
    -1
    10

>> conv(h,x)'

ans =

     1
     7
    13
     9
    21
    -1
    10
```

1.3.10 The convolution $y = h * g$, where h and g are finite-length signals, can be represented as a matrix-vector product, $y = Hg$ where H is a convolution matrix. In MATLAB, a convolution matrix H can be obtained with the command `convmtx(h(:), K)`.

Given finite-length sequences h and x, define

```
H = convmtx(h(:), M)
```

where M is such that the matrix-vector product $H^T x$ is defined, where H^T denotes the transpose of H.

In terms of convolution, what does the matrix-vector product $H^T x$ represent?

Write a MATLAB function to compute $H^T x$ using the function `conv` and without creating the matrix H. The input to your function should be vectors, h and x.

1.3.11 MATLAB `conv` function

Let

$$f(n) = u(n) - u(n-5)$$
$$g(n) = r(n) - 2\,r(n-5) + r(n-10).$$

where $r(n) := n\,u(n)$.

In MATLAB, use the`conv` function to compute the following convolutions. Use the `stem` function to plot the results. Be aware about the lengths of the signals. Make sure the horizontal axes in your plots are correct.

(a) $f(n) * f(n)$

(b) $f(n) * f(n) * f(n)$

(c) $f(n) * g(n)$

(d) $g(n) * \delta(n)$

(e) $g(n) * g(n)$

Comment on your observations: Do you see any relationship between $f(n) * f(n)$ and $g(n)$? Compare $f(n)$ with $f(n) * f(n)$ and with $f(n) * f(n) * f(n)$. What happens as you repeatedly convolve this signal with itself?

Use the commands `title`, `xlabel`, `ylabel` to label the axes of your plots.

1.3.12 Convolution of non-causal signals in MATLAB

Note that both of these signals start to the left of $n = 0$.

$$f(n) = 3\,\delta(n+2) - \delta(n-1) + 2\,\delta(n-3) \tag{6}$$
$$g(n) = u(n+4) - u(n-3) \tag{7}$$

First, plot the signals f, g, and $f * g$ by hand, without using MATLAB. Note the start and end points.

Next, use MATLAB to make plots of f, g, and $f * g$. Be aware that the `conv` function increases the length of vectors.

To turn in: The plots of $f(n)$, $g(n)$, $x(n)$, and your Matlab commands to create the plots.

1.3.13 Smoothing data by N-point convolution.

Save the data file `DataEOG.txt` from the course website. Load the data into Matlab using the command `load DataEOG.txt` Type `whos` to see your variables. One of the variables will be `DataEOG`. For convenience, rename it to `x` by typing: `x = DataEOG;` This signal comes from measuring electrical signals from the brain of a human subject.

Make a stem plot of the signal $x(n)$. You will see it doesn't look good because there are so many points. Make a plot of $x(n)$ using the `plot` command. As you can see, for long signals we get a better plot using the `plot` command. Although discrete-time signals are most appropriately displayed with the `stem` command, for *long* discrete-time signals (like this one) we use the plot command for better appearance.

Create a simple impulse response for an LTI system:

```
h = ones(1,11)/11;
```

Compute the convolution of h and x:

```
y = conv(x, h);
```

Make a MATLAB plot of the output y.

(a) How does convolution change x? (Compare x and y.)

(b) How is the length of y related to the length of x and h?

(c) Plot x and y on the same graph. What problem do you see? Can you get y to "line up" with x?

(d) Use the following commands:
```
y2 = y;
y2(1:5) = [];
y2(end-4:end) = [];
```
What is the effect of these commands? What is the length of `y2`? Plot `x` and `y2` on the same graph. What do you notice now?

(e) Repeat the problem, but use a different impulse response:
```
h = ones(1,31)/31;
```
What should the parameters in part (d) be now?

(f) Repeat the problem, but use
```
h = ones(1,67)/67;
```
What should the parameters in part (d) be now?

Comment on your observations.

To turn in: The plots, your Matlab commands to create the signals and plots, and discussion.

1.4 Z Transforms

1.4.1 The Z-transform of the discrete-time signal $x(n)$ is

$$X(z) = -3\, z^2 + 2\, z^{-3}$$

Accurately sketch the signal $x(n)$.

1.4.2 Define the discrete-time signal $x(n)$ as

$$x(n) = -0.3\, \delta(n+2) + 2.0\, \delta(n) + 1.5\, \delta(n-3) - \delta(n-5)$$

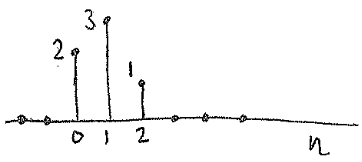
(a) Sketch $x(n)$.

(b) Write the Z-transform $X(z)$.

(c) Define $G(z) = z^{-2}\, X(z)$. Sketch $g(n)$.

1.4.3 The signal $g(n)$ is defined by the sketch.

g(n)



a) sketch the signal $x(n)$ defined thru $X(z) = z^{-2}\, G(z)$

b) " " $f(n)$ " " $F(z) = G(z^2)$

c) " " $v(n)$ " " $V(z) = G(-z)$

1.4.4 Let $x(n)$ be the length-5 signal

$$x(n) = \{\underline{1},\, 2,\, 3,\, 2,\, 1\}$$

where $x(0)$ is underlined. Sketch the signal corresponding to each of the following Z-transforms.

(a) $X(2z)$

(b) $X(z^2)$

(c) $X(z) + X(-z)$

(d) $X(1/z)$

1.4.5 Sketch the discrete-time signal $x(n)$ with the Z-transform

$$X(z) = (1 + 2\,z)\,(1 + 3\,z^{-1})\,(1 - z^{-1}).$$

1.4.6 Define three discrete-time signals:

$$a(n) = u(n) - u(n-4)$$
$$b(n) = \delta(n) + 2\,\delta(n-3)$$
$$c(n) = \delta(n) - \delta(n-1)$$

Define three new Z-transforms:

$$D(z) = A(-z), \qquad E(z) = A(1/z), \qquad F(z) = A(-1/z)$$

(a) Sketch $a(n)$, $b(n)$, $c(n)$

(b) Write the Z-transforms $A(z)$, $B(z)$, $C(z)$

(c) Write the Z-transforms $D(z)$, $E(z)$, $F(z)$

(d) Sketch $d(n)$, $e(n)$, $f(n)$

1.4.7 Find the Z-transform $X(z)$ of the signal

$$x(n) = 4\left(\frac{1}{3}\right)^n u(n) - \left(\frac{2}{3}\right)^n u(n).$$

1.4.8 The signal x is defined as

$$x(n) = a^{|n|}$$

Find $X(z)$ and the ROC. Consider separately the cases: $|a| < 1$ and $|a| \geq 1$.

1.4.9 Find the right-sided signal $x(n)$ from the Z-transform

$$X(z) = \frac{2z+1}{z^2 - \frac{5}{6}z + \frac{1}{6}}$$

1.4.10 Consider the LTI system with impulse response

$$h(n) = 3\left(\frac{2}{3}\right)^n u(n)$$

Find the output $y(n)$ when the input $x(n)$ is

$$x(n) = \left(\frac{1}{2}\right)^n u(n).$$

1.4.11 A discrete-time LTI system has impulse response

$$h(n) = -2\left(\frac{1}{5}\right)^n u(n)$$

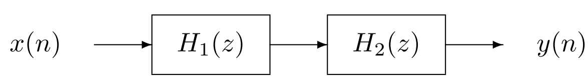
Find the output signal produced by the system when the input signal is

$$x(n) = 3\left(\frac{1}{2}\right)^n u(n)$$

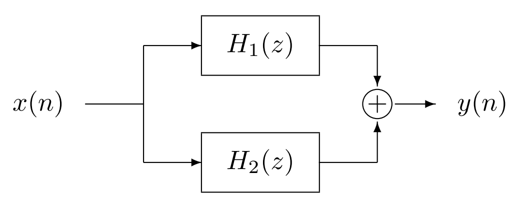
1.4.12 Consider the transfer functions of two discrete-time LTI systems,

$$H_1(z) = 1 + 2z^{-1} + z^{-2},$$
$$H_2(z) = 1 + z^{-1} + z^{-2}.$$

(a) If these two systems are cascaded in series, what is the impulse response of the total system?



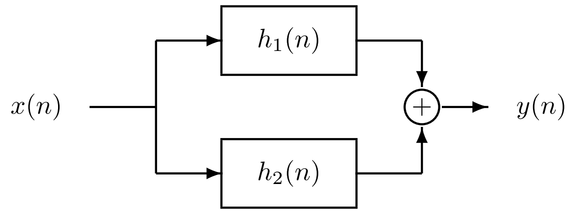
(b) If these two systems are combined in parallel, what is the impulse response of the total system?



1.4.13 Connected systems:

Two LTI systems are connected in Series $\to \boxed{H_1} \to \boxed{H_2} \to$
with transfer functions $H_1(z) = 2 + z^{-1}$ and
$H_2(z) = 3 + 2z^{-1}$. Find the impulse response
of the total system.

1.4.14 Consider the parallel combination of two LTI systems.



You are told that the impulse responses of the two systems are

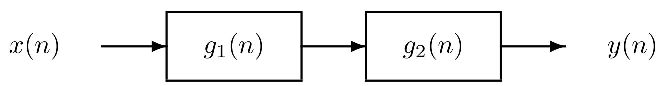
$$h_1(n) = 3\left(\frac{1}{2}\right)^n u(n)$$

and

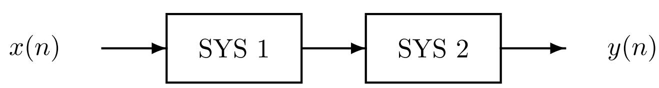
$$h_2(n) = 2\left(\frac{1}{3}\right)^n u(n)$$

(a) Find the impulse response $h(n)$ of the total system.

(b) You want to implement the total system as a *cascade* of two first order systems $g_1(n)$ and $g_2(n)$. Find $g_1(n)$ and $g_2(n)$, each with a single pole, such that when they are connected in cascade, they give the same system as $h_1(n)$ and $h_2(n)$ connected in parallel.



1.4.15 Consider the cascade combination of two LTI systems.



The impulse response of SYS 1 is

$$h_1(n) = \delta(n) + 0.5\,\delta(n-1) - 0.5\,\delta(n-2)$$

and the transfer function of SYS 2 is

$$H_2(z) = z^{-1} + 2\,z^{-2} + 2\,z^{-3}.$$

(a) Sketch the impulse response of the total system.

(b) What is the transfer function of the total system?

1.5 Inverse Systems

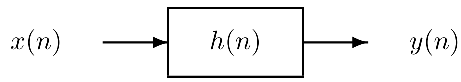
1.5.1 The impulse response of a discrete-time LTI system is

$$h(n) = -\delta(n) + 2\left(\frac{1}{2}\right)^n u(n).$$

(a) Find the impulse response of the stable inverse of this system.

(b) Use MATLAB to numerically verify the correctness of your answer by computing the convolution of $h(n)$ and the impulse response of the inverse system. You should get $\delta(n)$. Include your program and plots with your solution.

1.5.2 A discrete-time LTI system



has the impulse response

$$h(n) = \delta(n) + 3.5\,\delta(n-1) + 1.5\,\delta(n-2).$$

(a) Find the transfer function of the system $h(n)$.

(b) Find the impulse response of the **stable inverse** of this system.

(c) Use MATLAB to numerically verify the correctness of your answer by computing the convolution of $h(n)$ and the impulse response of the inverse system. You should get $\delta(n)$. Include your program and plots with your solution.

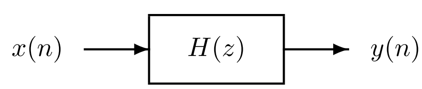
1.5.3 Consider a discrete-time LTI system with the impulse response

$$h(n) = \delta(n+1) - \frac{10}{3}\,\delta(n) + \delta(n-1).$$

(a) Find the impulse response $g(n)$ of the **stable** inverse of this system.

(b) Use MATLAB to numerically verify the correctness of your answer by computing the convolution of $h(n)$ and the impulse response of the inverse system. You should get $\delta(n)$. Include your program and plots with your solution.

1.5.4 A **causal** discrete-time LTI system



is described by the difference equation

$$y(n) - \frac{1}{3}\,y(n-1) = x(n) - 2\,x(n-1).$$

What is the impulse response of the **stable** inverse of this system?

1.6 Difference Equations

1.6.1 A causal discrete-time system is described by the difference equation,

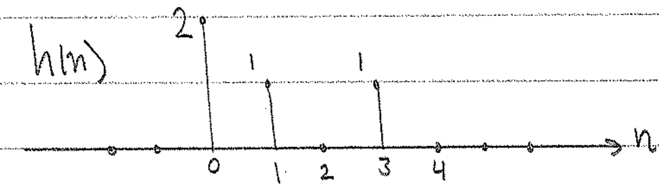
$$y(n) = x(n) + 3\,x(n-1) + 2\,x(n-4)$$

(a) What is the transfer function of the system?

(b) Sketch the impulse response of the system.

1.6.2 Given the impulse response . . .

If the impulse response of an LTI system is



then a) find the transfer function $H(z)$.

b) find a difference equation that implements the system.

1.6.3 A causal discrete-time LTI system is implemented using the difference equation

$$y(n) = x(n) + x(n-1) + 0.5\,y(n-1)$$

where x is the input signal, and y the output signal. Find and sketch the impulse response of the system.

1.6.4 Given the impulse response . . .

If an LTI system has impulse response

$$h(n) = \left(\tfrac{1}{3}\right)^n u(n)$$

then a) Find the difference equation to implement the system.

b) find the output signal when the input signal is

$$x(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

(find an explicit formula for output $y(n)$.)

1.6.5 Given two discrete-time LTI systems described by the difference equations

$$H_1: \quad r(n) + \frac{1}{3}\,r(n-1) = x(n) + 2x(n-1)$$

$$H_2: \quad y(n) + \frac{1}{3}\,y(n-1) = r(n) - 2r(n-1)$$

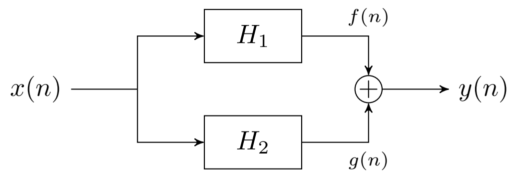
let H be the cascade of H_1 and H_2 in series.

$$x(n) \longrightarrow \boxed{H_1} \xrightarrow{r(n)} \boxed{H_2} \longrightarrow y(n)$$

Find the difference equation of the total system, H.

Suppose H_1 and H_2 are causal systems. Is H causal? Is H stable?

1.6.6 Two causal LTI systems are combined in parallel:



The two systems are implemented with difference equations:

$$H_1: \quad f(n) = x(n) + x(n-2) + 0.1\, f(n-1)$$

$$H_2: \quad g(n) = x(n) + x(n-1) + 0.1\, g(n-1)$$

Find the difference equation describing the total system between input $x(n)$ and output $y(n)$.

1.6.7 Consider a causal discrete-time LTI system described by the difference equation

$$y(n) - \frac{5}{6}\, y(n-1) + \frac{1}{6}\, y(n-2) = 2\, x(n) + \frac{2}{3}\, x(n-1).$$

(a) Find the transfer function $H(z)$.

(b) Find the impulse response $h(n)$. You may use MATLAB to do the partial fraction expansion. The MATLAB function is `residue`. Make a stem plot of $h(n)$ with MATLAB.

(c) OMIT: Plot the magnitude of the frequency response $|H(e^{j\omega})|$ of the system. Use the MATLAB function `freqz`.

1.6.8 A room where echos are present can be modeled as an LTI system that has the following rule:

$$y(n) = \sum_{k=0}^{\infty} 2^{-k}\, x(n - 10\, k)$$

The output $y(n)$ is made up of delayed versions of the input $x(n)$ of decaying amplitude.

(a) Sketch the impulse response $h(n)$.

(b) What is transfer function $H(z)$?

(c) Write the corresponding finite-order difference equation.

1.6.9 **Echo Canceler.** A recorded discrete-time signal $r(n)$ is distorted due to an echo. The echo has a lag of 10 samples and an amplitude of 2/3. That means

$$r(n) = x(n) + \frac{2}{3}\, x(n-10)$$

where $x(n)$ is the original signal. Design an LTI system with impulse response $g(n)$ that removes the echo from the recorded signal. That means, the system you design should recover the original signal $x(n)$ from the signal $r(n)$.

(a) Find the impulse response $g(n)$.

(b) Find a difference equation that can be used to implement the system.

(c) Is the system you designed both causal and stable?

1.6.10 Consider a causal discrete-time LTI system with the impulse response

$$h(n) = \frac{3}{2}\left(\frac{3}{4}\right)^n u(n) + 2\,\delta(n-4)$$

(a) Make a stem plot of $h(n)$ with MATLAB.

(b) Find the transfer function $H(z)$.

(c) Find the difference equation that describes this system.

(d) Plot the magnitude of the frequency response $|H(e^{j\omega})|$ of the system. Use the MATLAB command `freqz`.

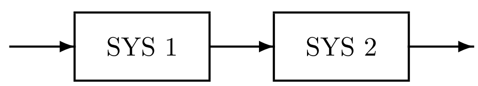
1.6.11 Consider a *stable* discrete-time LTI system described by the difference equation

$$y(n) = x(n) - x(n-1) - 2\,y(n-1).$$

(a) Find the transfer function $H(z)$ and its ROC.

(b) Find the impulse response $h(n)$.

1.6.12 Two LTI systems are connected in series:



The system SYS 1 is described by the difference equation

$$y(n) = x(n) + 2\,x(n-1) + x(n-2)$$

where $x(n)$ represents the input into SYS 1 and $y(n)$ represents the output of SYS 1. The system SYS 2 is described by the difference equation

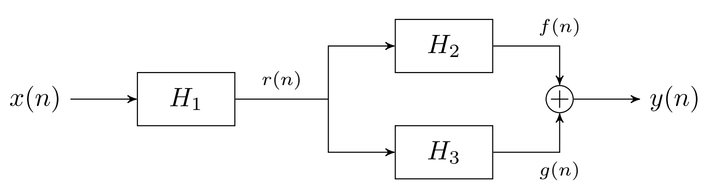
$$y(n) = x(n) + x(n-1) + x(n-2)$$

where $x(n)$ represents the input into SYS 2 and $y(n)$ represents the output of SYS 2.

(a) What difference equation describes the total system?

(b) Sketch the impulse response of the total system.

1.6.13 Three causal discrete-time LTI systems are used to create the a single LTI system.



The difference equations used to implement the systems are:

$$H_1: \quad r(n) = 2\,x(n) - \frac{1}{2}\,r(n-1)$$

$$H_2: \quad f(n) = r(n) - \frac{1}{3}\,f(n-1)$$

$$H_3: \quad g(n) = r(n) - \frac{1}{4}\,r(n-1)$$

What is the transfer function $H_{\text{tot}}(z)$ for the total system?

1.6.14 Given the difference equation...

5) If the difference equation of an CAUSAL LTI system is

$$y(n) = 2\,x(n) + x(n-2) - 0.2\,y(n-1)$$

a) what is the transfer function of the system?
b) Sketch the pole/zero diagram of the system.
c) What is the dc gain of the system?
d) What output y(n) is produced by the input x(n) = 2.
(Sketch the input & out signals.)

1.6.15 Difference equations in MATLAB

Suppose a system is implemented with the difference equation:

$$y(n) = x(n) + 2\,x(n-1) - 0.95\,y(n-1)$$

Write your own MATLAB function, `mydiffeq`, to implement this difference equation using a `for` loop. If the input signal is N-samples long ($0 \le n \le N-1$), your program should find the first N samples of the output $y(n)$ ($0 \le n \le N-1$). Remember that MATLAB indexing starts with 1, not 0, but don't let this confuse you. Use $x(-1) = 0$ and $y(-1) = 0$.

(a) Is this system linear? Use your MATLAB function to confirm your answer:
```
y1 = mydiffeq(x1)
y2 = mydiffeq(x2)
y3 = mydiffeq(x1+2*x2)
```
Use any signals `x1`, `x2` you like.

(b) Is this system time-invariant? Confirm this in MATLAB (how?).

(c) Compute and plot the impulse response of this system. Use `x = [1, zeros(1,100)];` as input.

(d) Define $x(n) = \cos(\pi\, n/8)\, [u(n) - u(n-50)]$. Compute the output of the system in two ways:
(1) $y(n) = h(n) * x(n)$ using the `conv` command.
(2) Use your function to find the output for this input signal.
Are the two computed output signals the same?

(e) Write a new MATLAB function for the system with the difference equation:

$$y(n) = x(n) + 2\,x(n-1) - 1.1\,y(n-1)$$

Find and plots the impulse response of this system. Comment on your observations.

(f) For both systems, use the MATLAB function `filter` to implement the difference equations. Do the output signals obtained using the MATLAB function `filter` agree with the output signals obtained using your function `mydiffeq`? (They should!)

To turn in: The plots, your MATLAB commands to create the signals and plots, and discussion.

1.7 Complex Poles

1.7.1 A causal discrete-time LTI system is implemented using the difference equation

$$y(n) = x(n) - y(n-2)$$

where x is the input signal, and y the output signal.

(a) Sketch the pole/zero diagram of the system.

(b) Find and sketch the impulse response of the system.

(c) Classify the system as stable/unstable.

1.7.2 The impulse response of an LTI discrete-time system is

$$h(n) = \left(\frac{1}{2}\right)^n \cos\left(\frac{2\pi}{3}\, n\right) u(n).$$

Find the difference equation that implements this system.

1.7.3 A causal discrete-time LTI system is implemented using the difference equation

$$y(n) = x(n) - 4\, y(n-2)$$

where x is the input signal, and y the output signal.

(a) Sketch the pole/zero diagram of the system.

(b) Find and sketch the impulse response of the system.

(c) Classify the system as stable/unstable.

(d) Find the form of the output signal when the input signal is

$$x(n) = 2\left(\frac{1}{3}\right)^n u(n).$$

You do not need to compute the constants produced by the partial fraction expansion procedure (PFA) — you can just leave them as constants: A, B, etc. Be as accurate as you can be in your answer without actually going through the arithmetic of the PFA.

1.7.4 A causal discrete-time LTI system is implemented using the difference equation

$$y(n) = x(n) - \frac{1}{2}\, x(n-1) + \frac{1}{2}\, y(n-1) - \frac{5}{8}\, y(n-2)$$

where x is the input signal, and y the output signal.

(a) Sketch the pole/zero diagram of the system.

(b) Find and sketch the impulse response of the system.

(c) Use Matlab to verify your answers to (a) and (b). Use the command `residue` and `zplane`. Use the command `filter` to compute the impulse response numerically and verify that it is the same as your formula in (b).

1.7.5 A causal discrete-time LTI system is implemented using the difference equation

$$y(n) = x(n) - \sqrt{2}\, x(n-1) + x(n-2) - 0.5\, y(n-2)$$

where x is the input signal, and y the output signal.

(a) Find the poles and zeros of the system.

(b) Sketch the pole/zero diagram of the system.

(c) Find the dc gain of the system.

(d) Find the value of the frequency response at $\omega = \pi$.

(e) Based on parts (a),(b),(c), roughly sketch the frequency response magnitude $|H^f(\omega)|$ of the system.

(f) Suppose the step function $u(n)$ is applied as the input signal to the system. Find the steady state behavior of the output signal.

(g) Suppose the cosine waveform $\cos(0.25\pi n)u(n)$ is applied as the input signal to the system. Find the steady state behavior of the output signal.

(h) Find the impulse response of the system. Your answer should not contain j.

1.7.6 Consider an LTI system with the difference equation

$$y(n) = x(n) - 2.5x(n-1) + y(n-1) - 0.7y(n-2) \tag{8}$$

Compute the impulse response of the system in three ways:

(a) Use the MATLAB function `filter` to numerically compute the impulse response of this system. Make a stem plot of the impulse response.

(b) Use the MATLAB function `residue` to compute the partial fraction of $\frac{1}{z}H(z)$. Write $H(z)$ as a sum of first-order terms. Then write the impulse response as

$$h(n) = C_1\,(p_1)^n\,u(n) + C_2\,(p_2)^n\,u(n). \tag{9}$$

The four values C_1, C_2, p_1, p_2 are found using the `residue` command. For this system they will be complex! Use Equation (9) to compute in Matlab the impulse response,

```
n = 0:30;
h = C1*p1.^n + C2*p2.^n;
```

Note that even though C_1, C_2, p_1, p_2 are complex, the impulse response $h(n)$ is real-valued. (The imaginary parts cancel out.) Is this what you find? Make a stem plot of the impulse response you have computed using Equation (9). Verify that the impulse response is the same as the impulse response obtained using the `filter` function in the previous part.

(c) Compute the impulse response using the formula for a damped sinusoid:

$$h(n) = A\,r^n\,\cos(\omega_o\,n + \theta_o)\,u(n). \tag{10}$$

This formula does not involve any complex numbers. This formula is obtained from Equation (9) by putting the complex values C_1, C_2, p_1, p_2 into polar form:

$$\begin{aligned}
C_1 &= R_1\,\mathrm{e}^{\mathrm{j}\,\alpha_1}\\
C_2 &= R_2\,\mathrm{e}^{\mathrm{j}\,\alpha_2}\\
p_1 &= r_1\,\mathrm{e}^{\mathrm{j}\,\beta_1}\\
p_2 &= r_2\,\mathrm{e}^{\mathrm{j}\,\beta_2}.
\end{aligned}$$

To put a complex number, c, in to polar form in MATLAB, use the functions `abs` and `angle`. Specifically $c = r\mathrm{e}^{\mathrm{j}\theta}$ where $r = $ `abs(c)` and $\theta = $ `angle(c)`.

Using MATLAB, find the real values R_1, α_1, etc. You should find that $R_2 = R_1$, $\alpha_2 = -\alpha_1$, $r_2 = r_1$, and $\beta_2 = -\beta_1$. Is this what you find? Therefore, the formula in Equation (9) becomes

$$\begin{aligned}
h(n) &= R_1\,\mathrm{e}^{\mathrm{j}\,\alpha_1}\,(r_1\,\mathrm{e}^{\mathrm{j}\,\beta_1})^n\,u(n) + R_1\,\mathrm{e}^{-\mathrm{j}\,\alpha_1}\,(r_1\,\mathrm{e}^{-\mathrm{j}\,\beta_1})^n\,u(n)\\
&= R_1\,\mathrm{e}^{\mathrm{j}\,\alpha_1}\,r_1^n\,\mathrm{e}^{\mathrm{j}\,\beta_1\,n}\,u(n) + R_1\,\mathrm{e}^{-\mathrm{j}\,\alpha_1}\,r_1^n\,\mathrm{e}^{-\mathrm{j}\,\beta_1\,n}\,u(n)\\
&= R_1\,r_1^n\,(\mathrm{e}^{\mathrm{j}(\beta_1\,n+\alpha_1)} + \mathrm{e}^{-\mathrm{j}(\beta_1\,n+\alpha_1)})\,u(n)\\
&= 2\,R_1\,r_1^n\,\cos(\beta_1\,n + \alpha_1)\,u(n).
\end{aligned}$$

This finally has the form of a damped sinusoid (10).

Using MATLAB, compute the impulse response using Equation (10)

```
n = 0:30;
h = A * r.^n .* ...
```

Verify that the impulse response is the same as the impulse response obtained using the `filter` function in (a).

1.7.7 Repeat the previous problem for the system

$$y(n) = x(n) - 2.5x(n-1) + x(n-2) + y(n-1) - 0.7y(n-2) \tag{11}$$

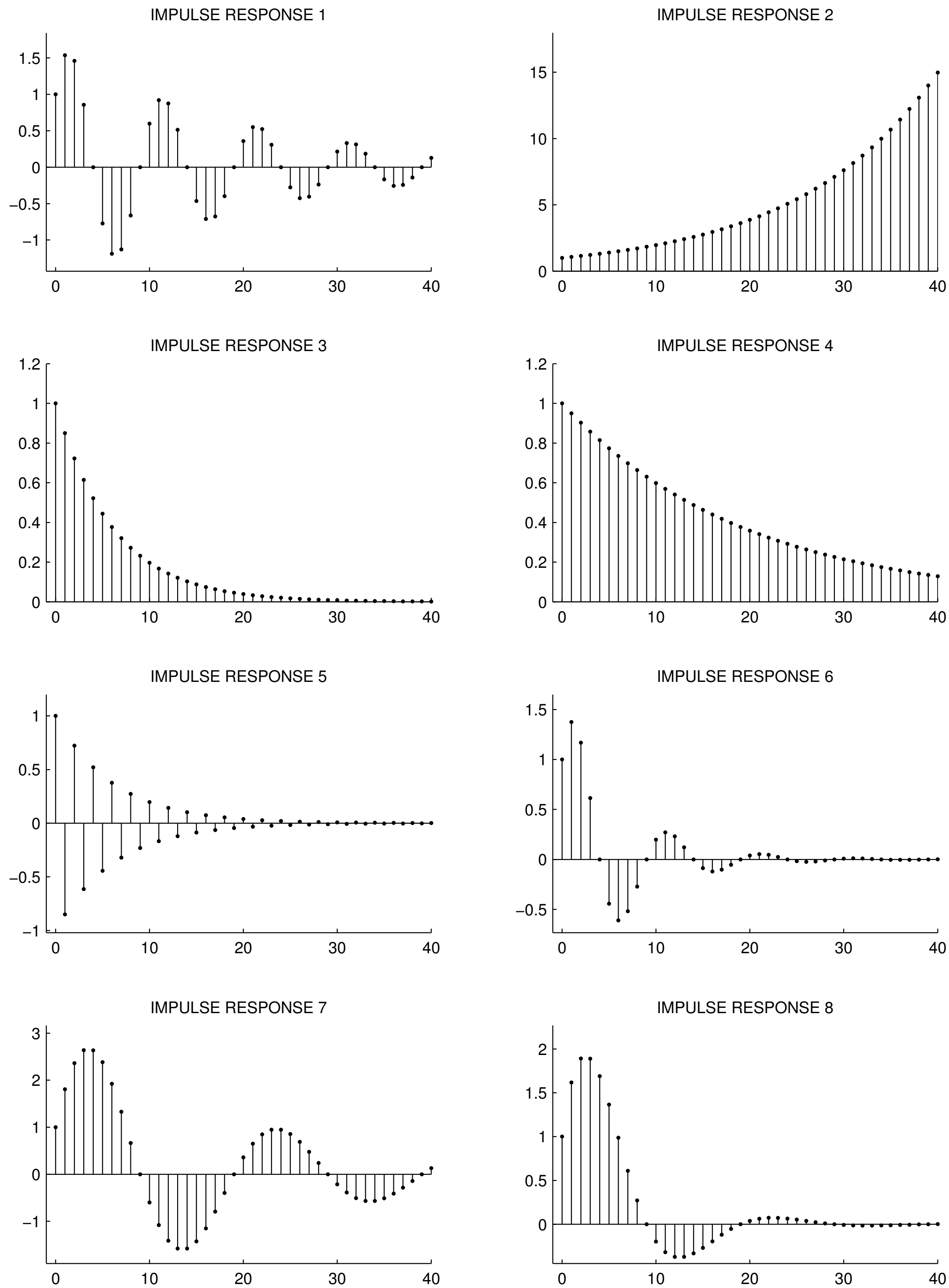
1.7.8 The diagrams on the following pages show the impulse responses and pole-zero diagrams of 8 causal discrete-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

You should do this problem *without* using MATLAB or any other computational tools.

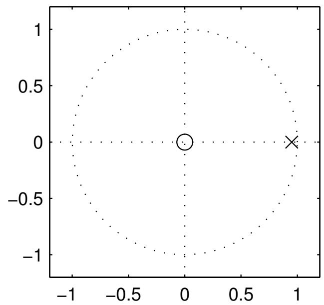
In the pole-zero diagrams, the zeros are shown with 'o' and the poles are shown by 'x'.

IMPULSE RESPONSE	POLE-ZERO DIAGRAM
1	
2	
3	
4	
5	
6	
7	
8	

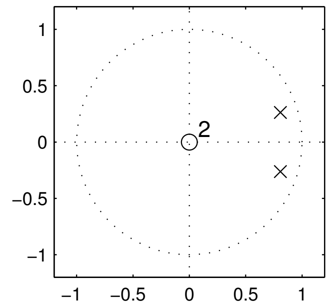
SecondOr


IMPULSE RESPONSE 1
IMPULSE RESPONSE 2
IMPULSE RESPONSE 3
IMPULSE RESPONSE 4
IMPULSE RESPONSE 5
IMPULSE RESPONSE 6
IMPULSE RESPONSE 7
IMPULSE RESPONSE 8

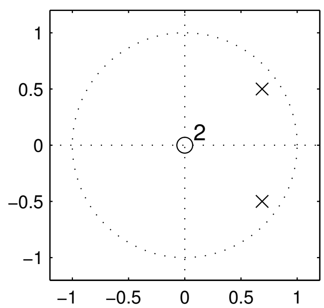
ZERO−POLE DIAGRAM 1



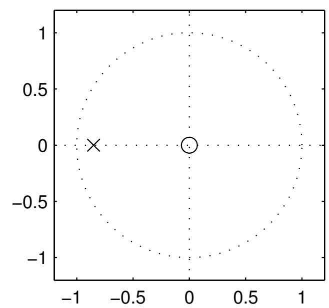
ZERO−POLE DIAGRAM 2



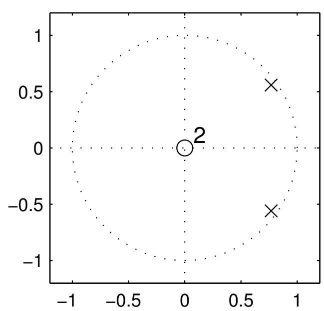
ZERO−POLE DIAGRAM 3



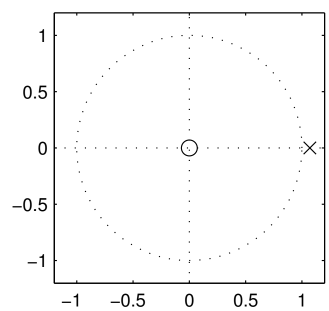
ZERO−POLE DIAGRAM 4



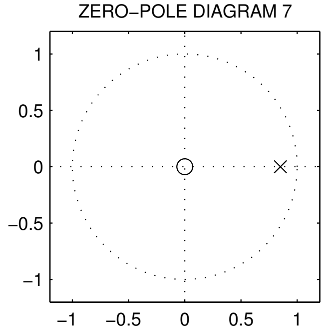
ZERO−POLE DIAGRAM 5



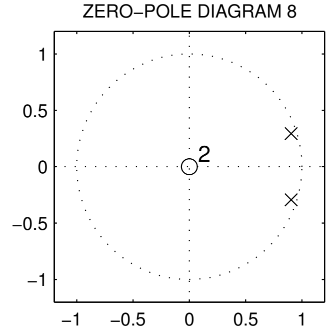
ZERO−POLE DIAGRAM 6



ZERO−POLE DIAGRAM 7



ZERO−POLE DIAGRAM 8



1.7.9 The diagrams on the following pages show the pole-zero diagrams and impulse responses of 8 causal discrete-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

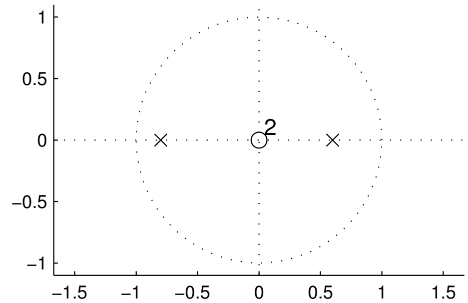
You should do this problem *without* using MATLAB or any other computational tools.

In the pole-zero diagrams, the zeros are shown with 'o' and the poles are shown by 'x'.

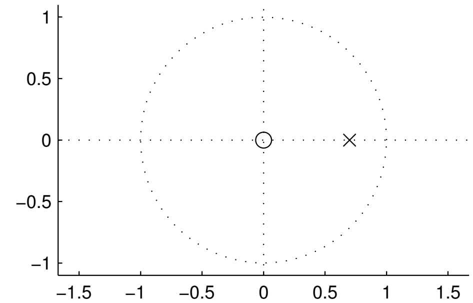
POLE-ZERO DIAGRAM	IMPULSE RESPONSE
1	
2	
3	
4	
5	
6	
7	
8	

PoleZero

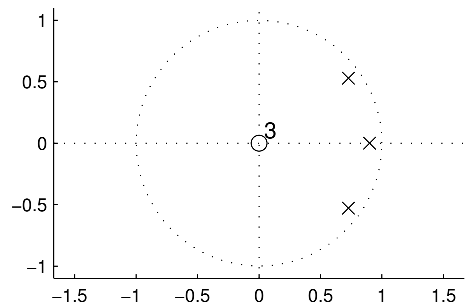
POLE−ZERO DIAGRAM 1



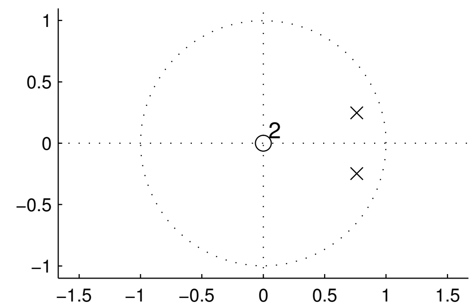
POLE−ZERO DIAGRAM 2



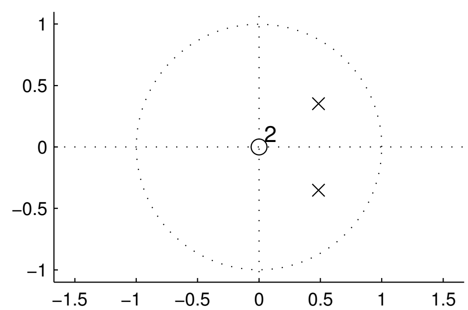
POLE−ZERO DIAGRAM 3



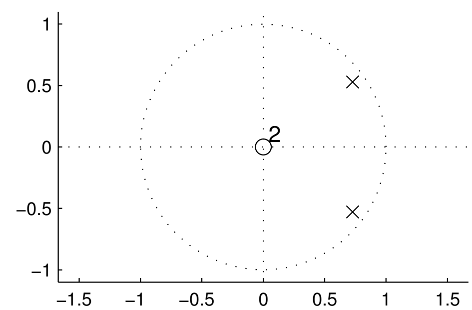
POLE−ZERO DIAGRAM 4



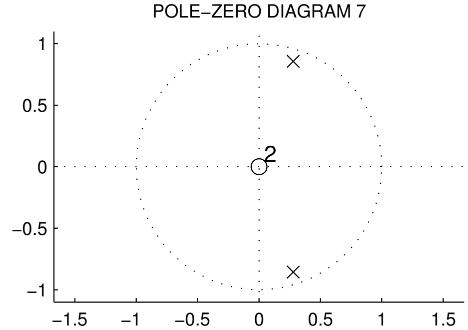
POLE−ZERO DIAGRAM 5



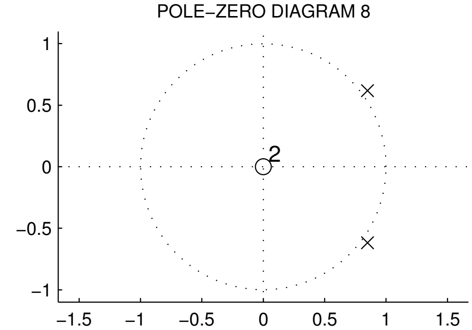
POLE−ZERO DIAGRAM 6

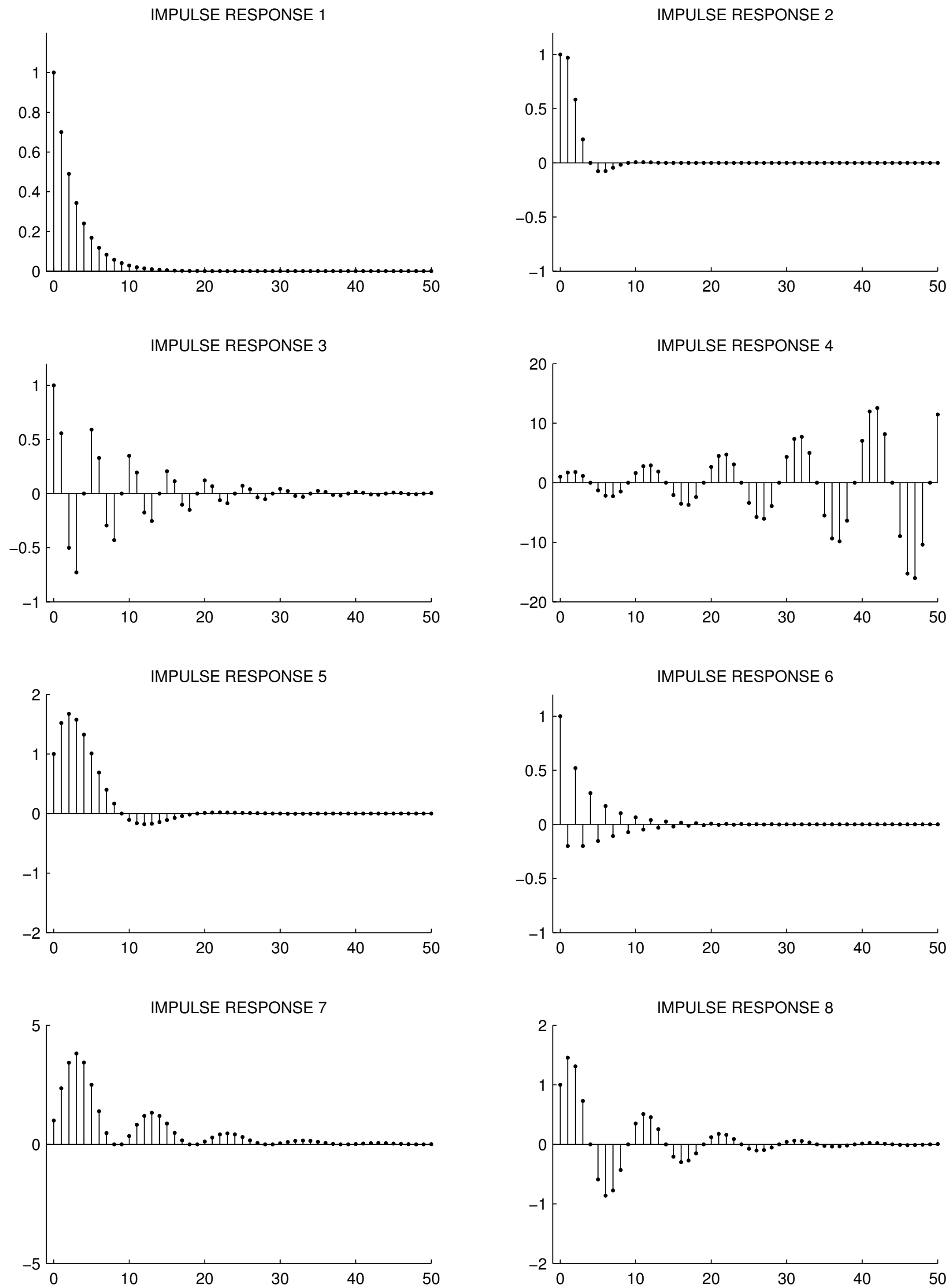


POLE−ZERO DIAGRAM 7



POLE−ZERO DIAGRAM 8

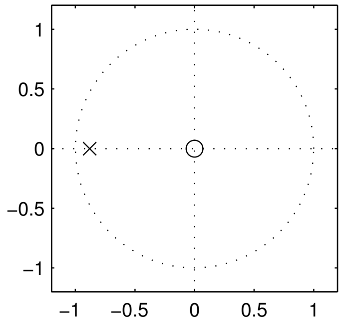



IMPULSE RESPONSE 1
IMPULSE RESPONSE 2
IMPULSE RESPONSE 3
IMPULSE RESPONSE 4
IMPULSE RESPONSE 5
IMPULSE RESPONSE 6
IMPULSE RESPONSE 7
IMPULSE RESPONSE 8

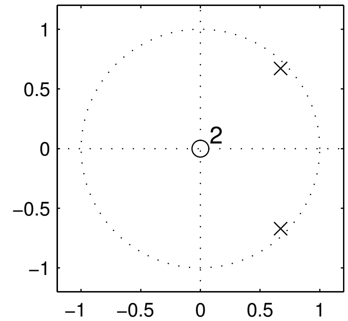
1.7.10 The pole-zero diagrams of eight discrete-time systems are illustrated below. The impulse response $h(n)$ of each system is also shown, but in a different order. Match each frequency response to its pole-zero diagram by filling out the table.

POLE-ZERO DIAGRAM	IMPULSE RESPONSE
1	
2	
3	
4	
5	
6	
7	
8	

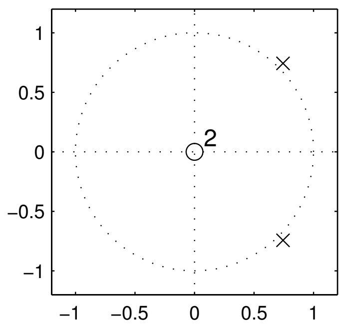
ZERO−POLE DIAGRAM 1



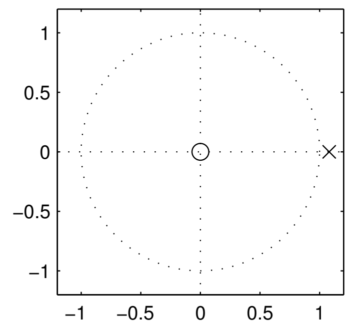
ZERO−POLE DIAGRAM 2



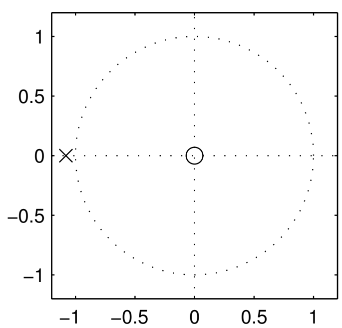
ZERO−POLE DIAGRAM 3



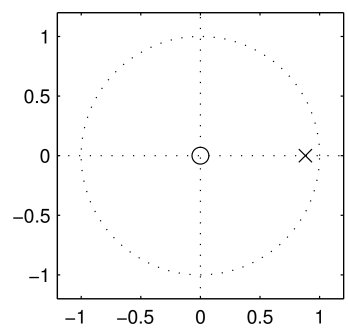
ZERO−POLE DIAGRAM 4



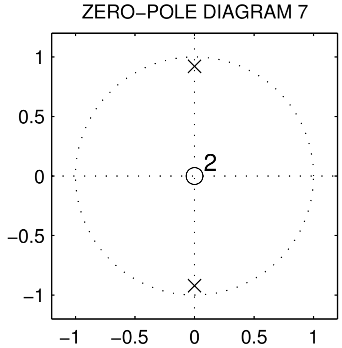
ZERO−POLE DIAGRAM 5



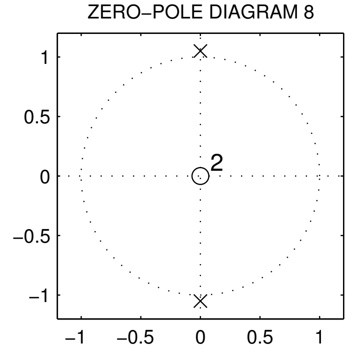
ZERO−POLE DIAGRAM 6

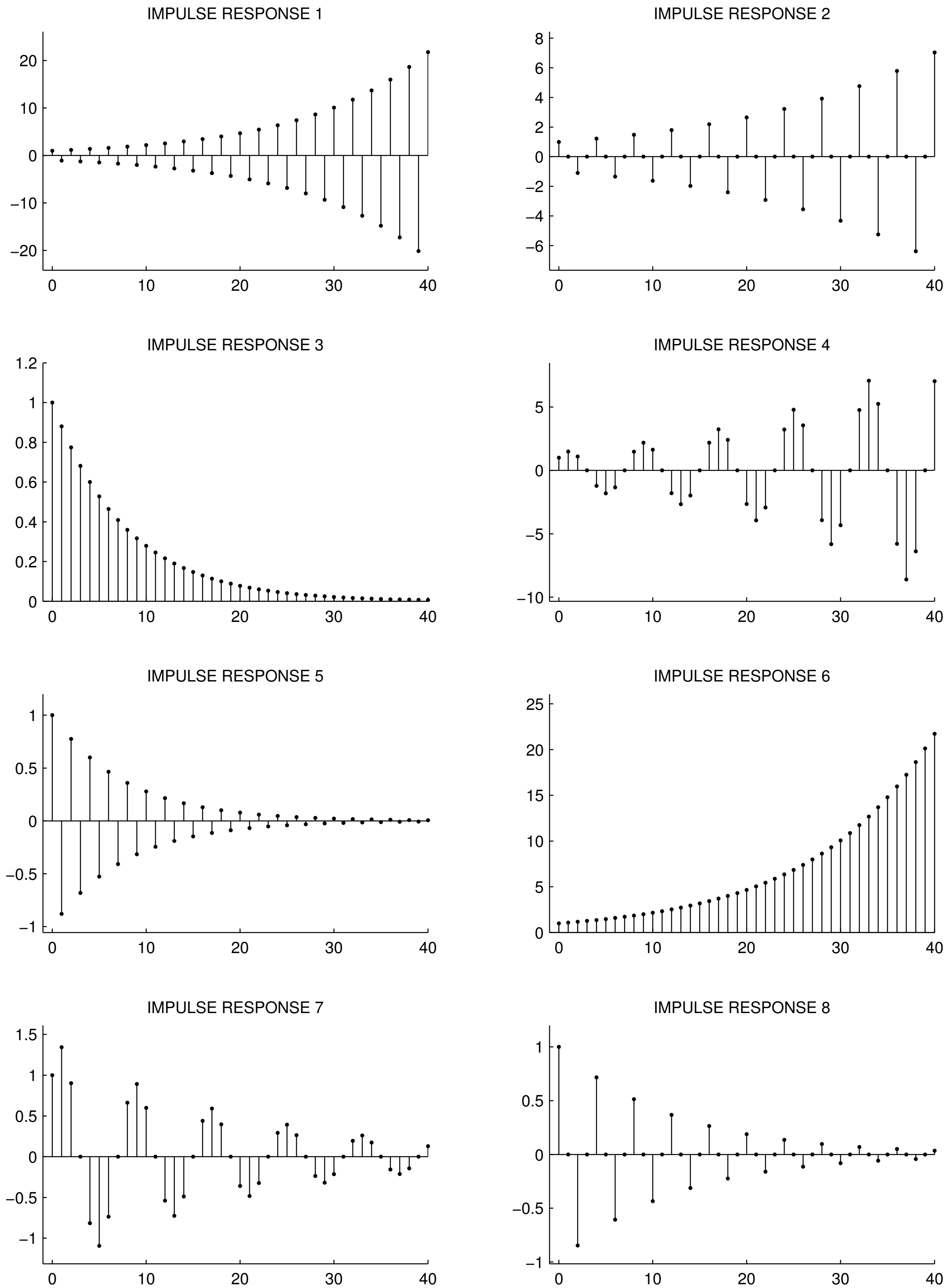


ZERO−POLE DIAGRAM 7



ZERO−POLE DIAGRAM 8





1.7.11 The impulse response of a discrete-time LTI system is given by

$$h(n) = A\,(0.7)^n\,u(n).$$

Suppose the signal

$$x(n) = B\,(0.9)^n\,u(n)$$

is input to the system. A and B are unknown constants. Which of the following could be the output signal $y(n)$? Choose all that apply and provide an explanation for your answer.

(a) $y(n) = K_1\,(1.6)^n\,u(n) + K_2\,(0.2)^n\,u(n)$
(b) $y(n) = K_1\,(0.7)^n\,u(-n) + K_2\,(0.2)^n\,u(-n)$
(c) $y(n) = K_1\,(0.7)^n\,u(n) + K_2\,(0.9)^n\,u(n)$
(d) $y(n) = K_1\,(0.7)^n\,u(n) + K_2\,(0.2)^n\,u(-n)$
(e) $y(n) = K_1\,(0.7)^n\,u(n) + K_2\,(0.2)^n\,u(n) + K_3\,(0.9)^n\,u(n)$
(f) $y(n) = K_1\,(0.7)^n\,u(n) + K_2\,(0.2)^n\,u(n) + K_3\,u(n)$

1.7.12 The impulse response of a discrete-time LTI system is given by

$$h(n) = A\,(0.7)^n\,u(n).$$

Suppose the signal

$$x(n) = B\,\cos(0.2\,\pi\,n)\,u(n)$$

is input to the system. A and B are unknown constants. Which of the following could be the output signal $y(n)$? Choose all that apply and provide an explanation for your answer.

(a) $y(n) = K_1\,(0.7)^n\,\cos(0.2\,\pi\,n + \theta)\,u(n)$
(b) $y(n) = K_1\,(0.14)^n\,\cos(0.5\,\pi\,n + \theta)\,u(n)$
(c) $y(n) = K_1\,(0.14)^n\,u(n) + K_2\,\cos(0.14\,\pi\,n + \theta)\,u(n)$
(d) $y(n) = K_1\,(0.14)^n\,u(-n) + K_2\,\cos(0.14\,\pi\,n + \theta)\,u(n)$
(e) $y(n) = K_1\,(0.7)^n\,u(-n) + K_2\,\cos(0.2\,\pi\,n + \theta)\,u(n)$
(f) $y(n) = K_1\,(0.7)^n\,u(n) + K_2\,\cos(0.2\,\pi\,n + \theta)\,u(n)$

1.7.13 A causal LTI discrete-time system is implemented using the difference equation

$$y(n) = b_0\,x(n) - a_1\,y(n-1) - a_2\,y(n-2)$$

where a_k, b_k are unknown real constants. Which of the following could be the impulse response? Choose all that apply and provide an explanation for your answer.

(a) $h(n) = K_1\,a^n\,u(n) + K_2\,b^n\,u(n)$
(b) $h(n) = K_1\,a^n\,u(n) + K_2\,r^n\,\cos(\omega_1\,n + \theta)\,u(n)$
(c) $h(n) = K_1\,r^n\,\cos(\omega_1\,n + \theta)\,u(n)$
(d) $h(n) = K_1\,r_1^n\,\cos(\omega_1\,n + \theta_1)\,u(n) + K_2\,r_2^n\,\cos(\omega_2\,n + \theta_2)\,u(n)$ (with $\omega_1 \neq \omega_2$).

1.8 Frequency Responses

1.8.1 The frequency response $H^f(\omega)$ of a discrete-time LTI system is

$$H^f(\omega) = \begin{cases} e^{-\mathrm{j}\omega} & -0.4\pi < \omega < 0.4\pi \\ 0 & 0.4\pi < |\omega| < \pi. \end{cases}$$

Find the output $y(n)$ when the input $x(n)$ is

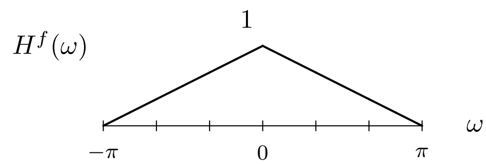
$$x(n) = 1.2\,\cos(0.3\,\pi\,n) + 1.5\,\cos(0.5\,\pi\,n).$$

Put $y(n)$ in simplest real form (your answer should not contain j).

Hint: Use Euler's formula and the relation

$$e^{\mathrm{j}\omega_o n} \longrightarrow \boxed{\text{LTI SYSTEM}} \longrightarrow H^f(\omega_o)\,e^{\mathrm{j}\omega_o n}$$

1.8.2 The frequency response $H^f(\omega)$ of a discrete-time LTI system is as shown.



$H^f(\omega)$ is real-valued so the phase is 0.

Find the output $y(n)$ when the input $x(n)$ is

$$x(n) = 1 + \cos(0.3\,\pi\,n).$$

Put $y(n)$ in simplest real form (your answer should not contain j).

1.8.3 A stable linear time invariant system has the transfer function

$$H(z) = \frac{z(z+2)}{(z-1/2)(z+4)}$$

(a) Find the frequency response $H^f(\omega)$ of this system.

(b) Calculate the value of the frequency response $H^f(\omega)$ at $\omega = 0.2\pi$.

(c) Find the output $y(n)$ produced by the input $x(n) = \cos(0.2\pi\,n)$.

1.8.4 A causal LTI system is implemented with the difference equation

$$y(n) = 0.5\,x(n) + 0.2\,x(n-1) + 0.5\,y(n-1) - 0.1\,y(n-2).$$

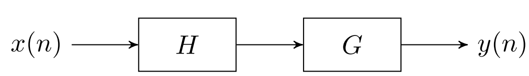
(a) Find the frequency response of this system.

(b) Compute and plot the frequency response magnitude $|H^f(\omega)|$ using the MATLAB command `freqz`.

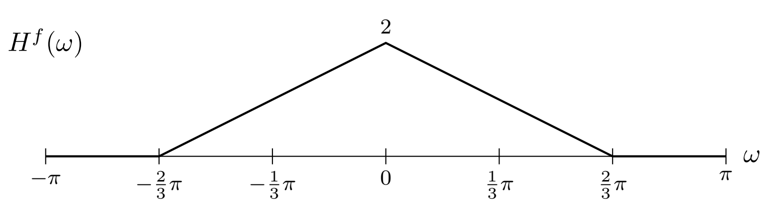
(c) Find the output produced by the input $x(n) = \cos(0.2\pi\,n)$. Compare your answer with the output signal found numerically with the MATLAB command `filter`.

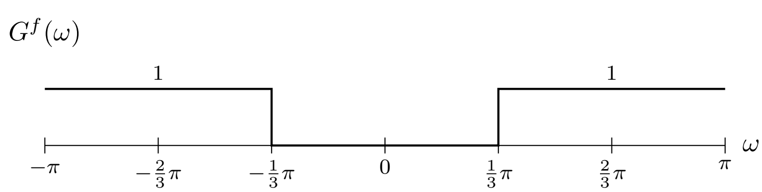
(d) Use the Matlab command `zplane` to make the pole-zero diagram.

1.8.5 Two discrete-time LTI systems are used in series.



The frequency responses are shown.



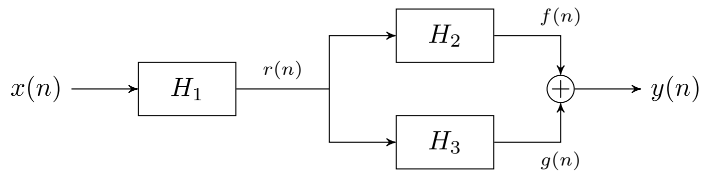


(a) Accurately sketch the frequency response of the total system.

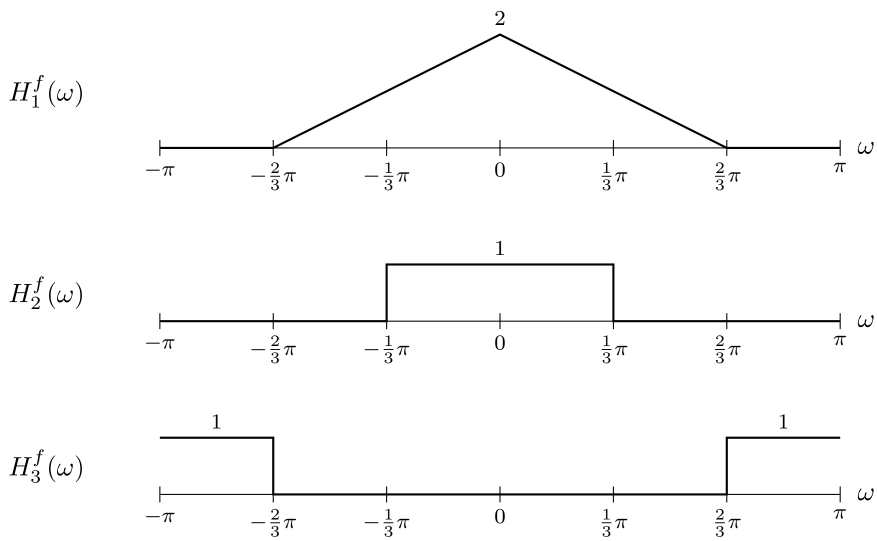
(b) Find the output signal $y(n)$ produced by the input signal

$$x(n) = 5 + 3\cos\left(\frac{\pi}{2}n\right) + 2\cos\left(\frac{2\pi}{3}n\right) + 4\,(-1)^n.$$

1.8.6 Three discrete-time LTI systems are combined as illustrated:



The frequency responses of the systems are:

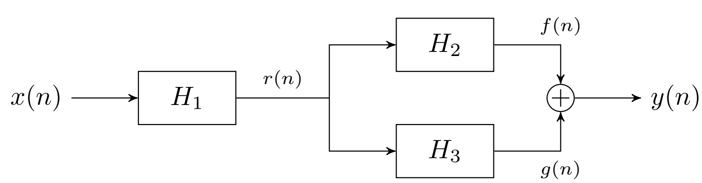


(a) Accurately sketch the frequency response of the total system.

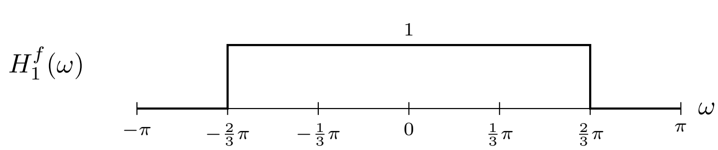
(b) Find the output signal $y(n)$ produced by the input signal

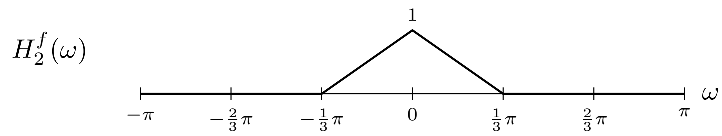
$$x(n) = 5 + 3\cos\left(\frac{\pi}{6}n\right) + 2\cos\left(\frac{\pi}{2}n\right) + 4\,(-1)^n.$$

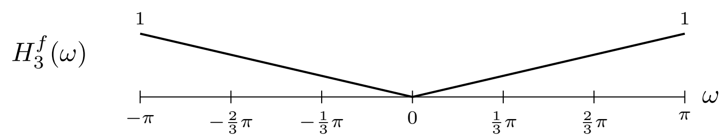
1.8.7 Three discrete-time LTI systems are combined as illustrated:



The frequency responses of the systems are:







(a) Accurately sketch the frequency response of the total system.

(b) Find the output signal $y(n)$ produced by the input signal

$$x(n) = 2 + \cos\left(\frac{\pi}{3}n\right) + 3\,\cos\left(\frac{\pi}{2}n\right) + 0.5\,(-1)^n.$$

1.8.8 The mangitude and phase of the frequency response of a discrete-time LTI system are:

$$|H^f(\omega)| = \begin{cases} 2 & \text{for } |\omega| < 0.5\,\pi \\ 1 & \text{for } 0.5\pi < |\omega| < \pi. \end{cases}$$

$$\angle H^f(\omega) = \begin{cases} 0.3\,\pi & \text{for } -\pi < \omega < 0 \\ -0.3\,\pi & \text{for } 0 < \omega < \pi. \end{cases}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$ for $|\omega| \le \pi$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$ for $|\omega| \le \pi$.

(c) Find the output signal $y(n)$ produced by the input signal

$$x(n) = 2\,\sin(0.2\,\pi\,n) + 3\,\cos(0.6\,\pi\,n + 0.2\,\pi).$$

1.8.9 The frequency response of a discrete-time LTI system is given by,

$$H^f(\omega) = \begin{cases} 1, & |\omega| \le 0.25\,\pi \\ 0, & 0.25\,\pi < |\omega| \le 0.5\pi \\ 1, & 0.5\,\pi < |\omega| \le \pi \end{cases}$$

(a) Sketch the frequency response.

(b) Find the output signal produced by the input signal

$$x(n) = 3 + 2\,\cos(0.3\,\pi\,n) + 2\,\cos(0.7\,\pi\,n) + (-1)^n.$$

(c) Classify the system as a low-pass filter, high-pass filter, band-pass filter, band-stop filter, or none of these.

1.8.10 The frequency response of a discrete-time LTI system is given by

$$H^f(\omega) = \begin{cases} -\mathrm{j}, & 0 < \omega \le 0.4\pi \\ \mathrm{j}, & -0.4\pi \le \omega < 0\pi \\ 0, & 0.4\,\pi < |\omega| \le \pi \end{cases}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$ for $|\omega| \le \pi$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$ for $|\omega| \le \pi$.

(c) Find the output signal $y(n)$ when the input signal is

$$x(n) = 2\,\cos(0.3\,\pi\,n) + 0.7\,\cos(0.7\,\pi\,n) + (-1)^n.$$

Simplify your answer so that it does not contain j.

1.8.11 The frequency response of a real discrete-time LTI system is given by

$$H^f(\omega) = \begin{cases} 0, & 0 \le |\omega| \le 0.4\pi \\ -\mathrm{j}, & 0.4\pi < \omega < \pi \\ \mathrm{j}, & -\pi < \omega < -0.4\,\pi \end{cases}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$ for $|\omega| \le \pi$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$ for $|\omega| \le \pi$.

(c) Find the output signal $y(n)$ produced by the input signal

$$x(n) = 3 + 2\,\cos(0.3\,\pi\,n) + 2\,\cos(0.7\,\pi\,n).$$

Simplify your answer so that it does not contain j.

(d) Classify the system as a low-pass filter, high-pass filter, band-pass filter, band-stop filter, or none of these.

1.8.12 The frequency response of a discrete-time LTI system is given by

$$H^f(\omega) = \begin{cases} 2\,\mathrm{e}^{-\mathrm{j}\,1.5\,\omega}, & \text{for } |\omega| \le 0.4\pi \\ 0, & \text{for } 0.4\,\pi < |\omega| \le \pi \end{cases}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$ for $|\omega| \le \pi$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$ for $|\omega| \le \pi$.

(c) Find the output signal $y(n)$ produced by the input signal

$$x(n) = 3 + 2\,\cos(0.3\,\pi\,n) + 0.7\,\cos(0.7\,\pi\,n) + (-1)^n.$$

Simplify your answer so that it does not contain j.

1.8.13 The frequency response of a discrete-time LTI system is given by

$$H^f(\omega) = \begin{cases} 0, & |\omega| \le 0.25\,\pi \\ e^{-j2.5\omega}, & 0.25\,\pi < |\omega| \le 0.5\pi \\ 0, & 0.5\,\pi < |\omega| \le \pi \end{cases}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$ for $|\omega| \le \pi$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$ for $|\omega| \le \pi$.

(c) Find the output signal produced by the input signal

$$x(n) = 3 + 2\,\cos(0.3\,\pi\,n) + 2\,\cos(0.7\,\pi\,n) + (-1)^n.$$

Simplify your answer so that it does not contain j.

(d) Classify the system as a low-pass filter, high-pass filter, band-pass filter, band-stop filter, or none of these.

1.8.14 The frequency response of a discrete-time LTI system is given by

$$H^f(\omega) = \begin{cases} 2\,\mathrm{e}^{-\mathrm{j}\omega}, & |\omega| \le 0.25\,\pi \\ \mathrm{e}^{-\mathrm{j}2\omega}, & 0.25\,\pi < |\omega| \le 0.5\pi \\ 0, & 0.5\,\pi < |\omega| \le \pi \end{cases}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$ for $|\omega| \le \pi$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$ for $|\omega| \le \pi$.

(c) Find the output signal produced by the input signal

$$x(n) = 3 + 2\,\cos(0.3\,\pi\,n) + 2\,\cos(0.7\,\pi\,n) + (-1)^n.$$

(d) Is the impulse response of the system real-valued? Explain.

1.8.15 The following figure shows the frequency response magnitudes $|H^f(\omega)|$ of four discrete-time LTI systems (Systems A, B, C, and D). The signal

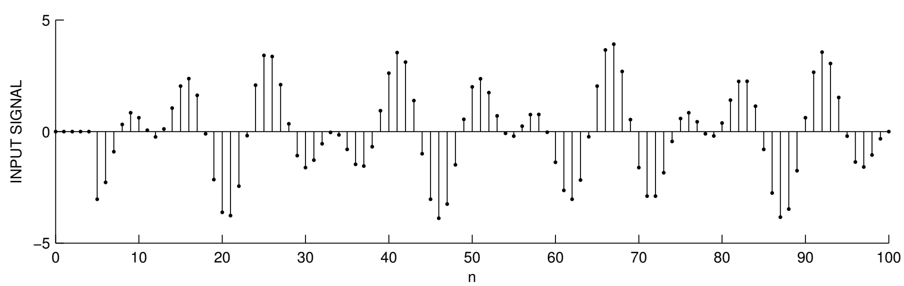
$$x(n) = 2\,\cos(0.15\,\pi\,n)\,u(n-5) + 2\,\cos(0.24\,\pi\,n)\,u(n-5)$$

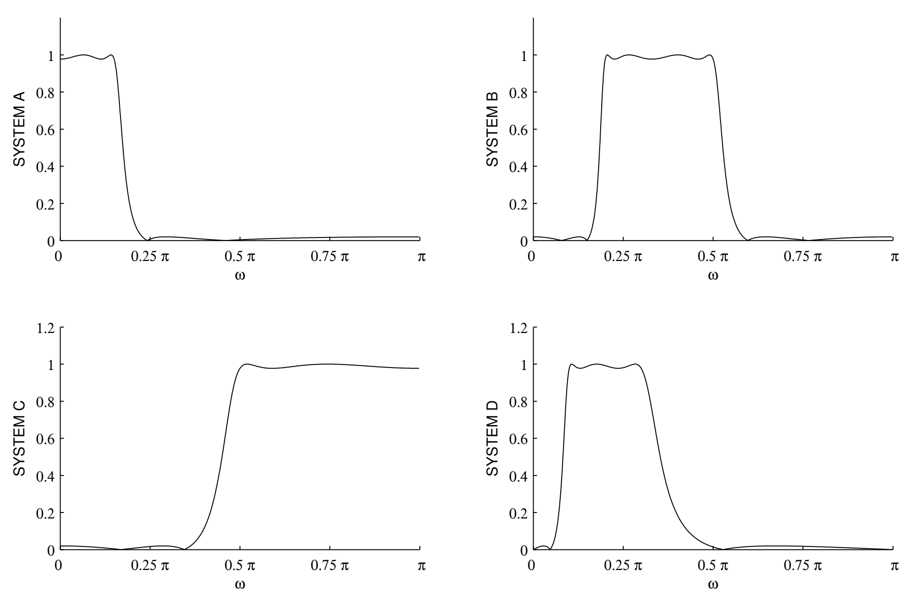
shown below is applied as the input to each of the four systems. The input signal $x(n)$ and each of the four output signals are also shown below. But the output signals are out of order. For each of the four systems, identify which signal is the output signal. **Explain** your answer.

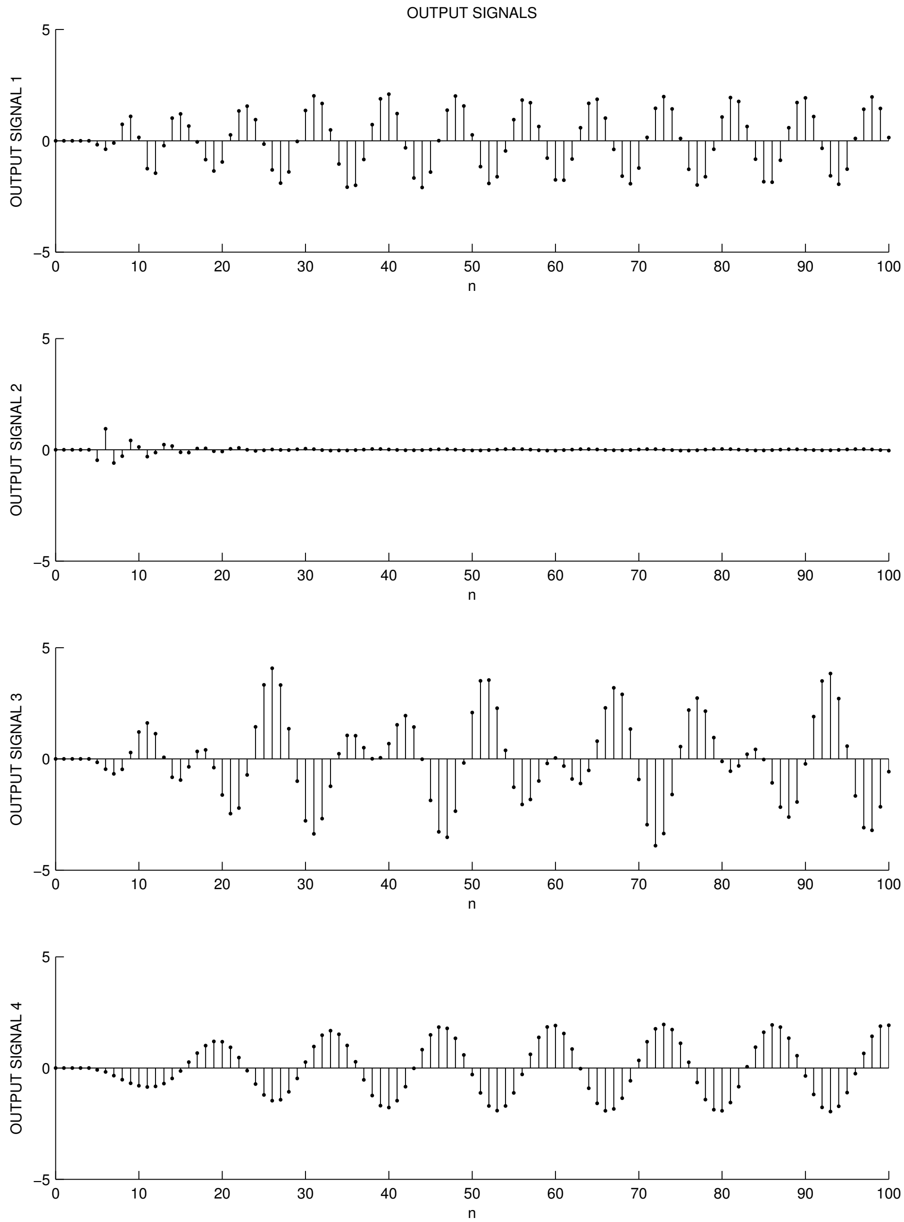
You should do this problem *without* using MATLAB or any other computational tools.

System	Output signal
A	
B	
C	
D	

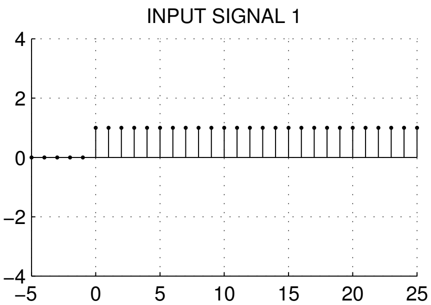
InputOut

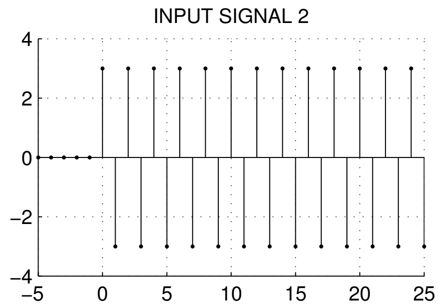


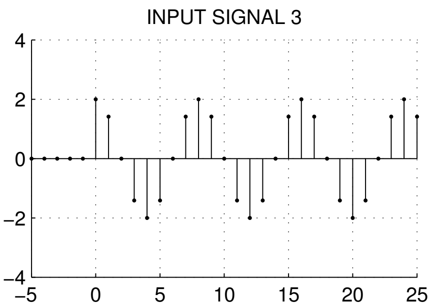


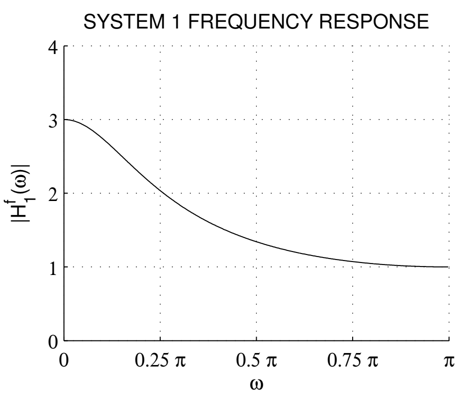

OUTPUT SIGNALS
OUTPUT SIGNAL 1
OUTPUT SIGNAL 2
OUTPUT SIGNAL 3
OUTPUT SIGNAL 4
n
5
0
−5
0
10
20
30
40
50
60
70
80
90
100

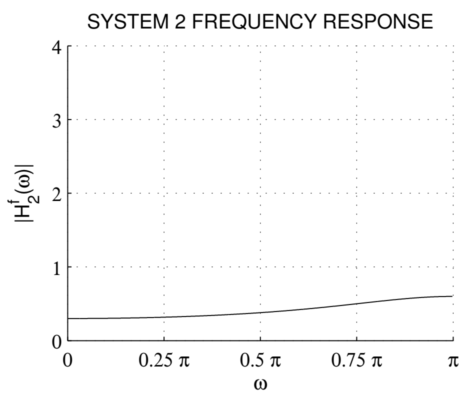
1.8.16 The three discrete-time signals below are each applied to two discrete-time systems to produce a total of six output signals. The frequency response of each system is shown below. Indicate how each of the six output signals are produced by completing the table below.



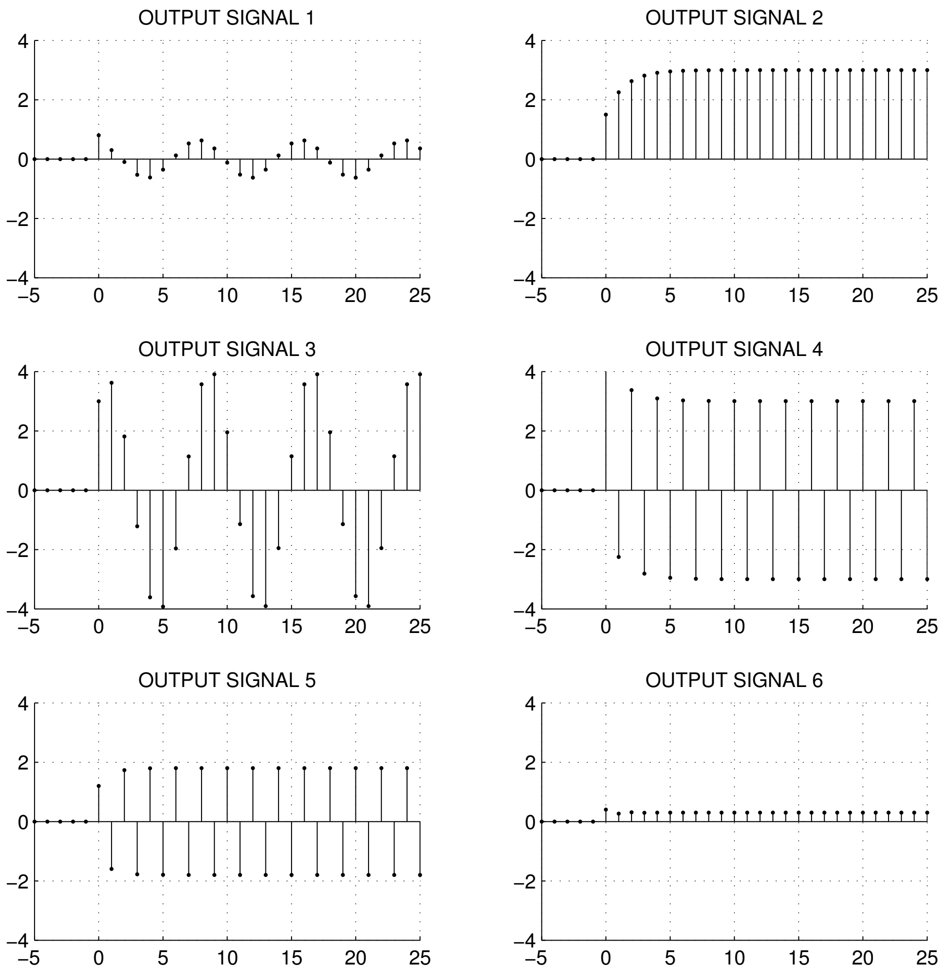






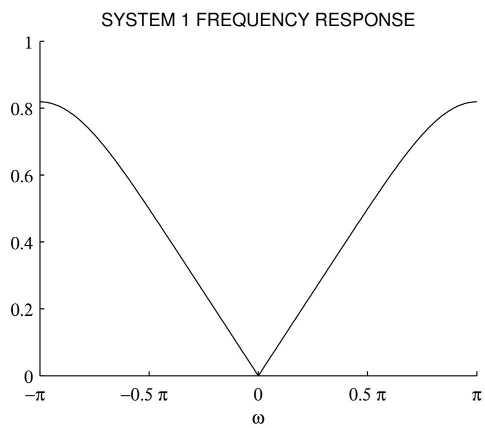


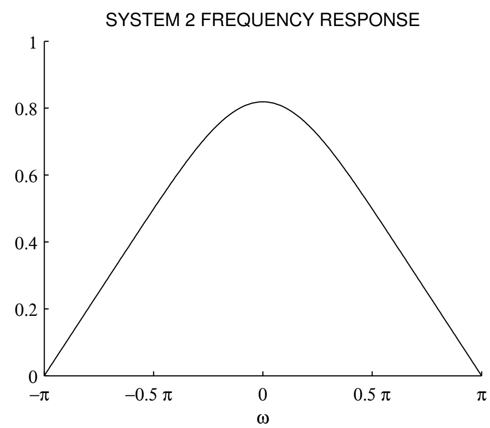
Input signal	System	Output signal
1	1	
2	1	
3	1	
1	2	
2	2	
3	2	

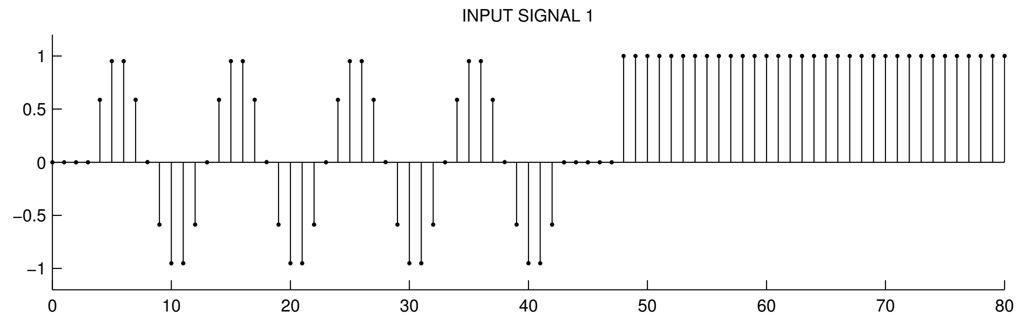

OUTPUT SIGNAL 1
OUTPUT SIGNAL 2
OUTPUT SIGNAL 3
OUTPUT SIGNAL 4
OUTPUT SIGNAL 5
OUTPUT SIGNAL 6
4
2
0
−2
−4
−5
0
5
10
15
20
25

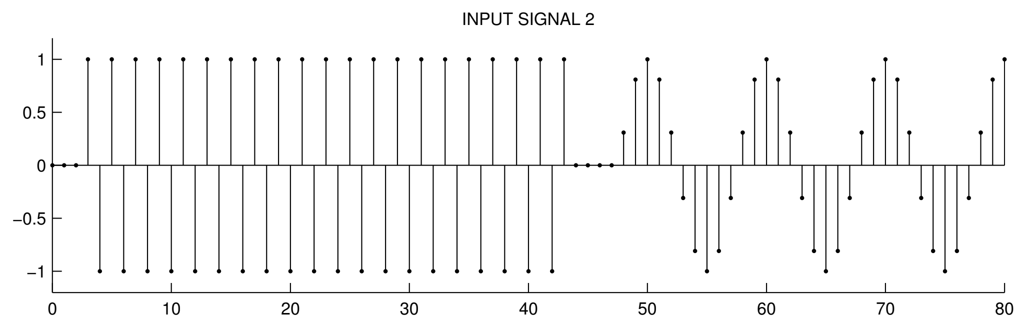
1.8.17 The three discrete-time signals below are each applied to two discrete-time LTI systems to produce a total of six output signals. The frequency response $H^f(\omega)$ of each system is shown below. Indicate how each of the six output signals are produced by completing the table below.

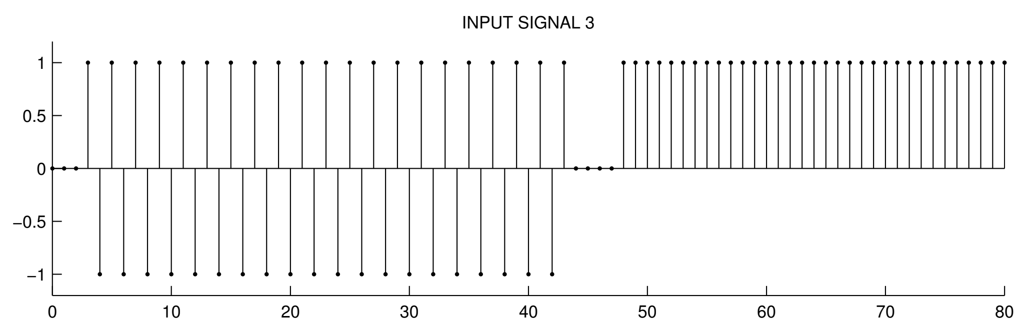
Input signal	System	Output signal
1	1	
1	2	
2	1	
2	2	
3	1	
3	2	

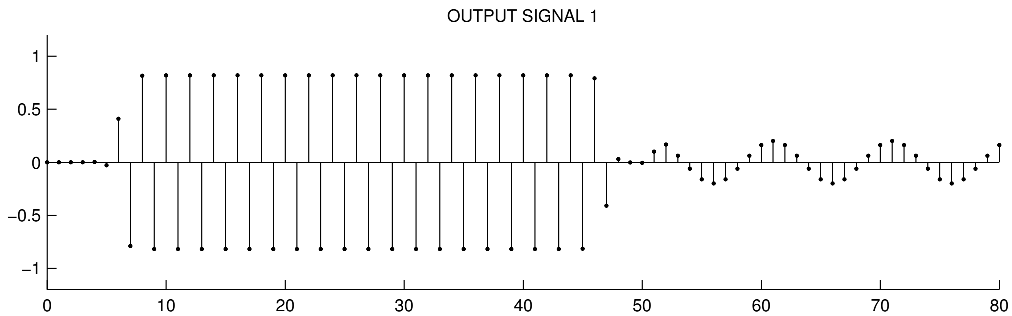


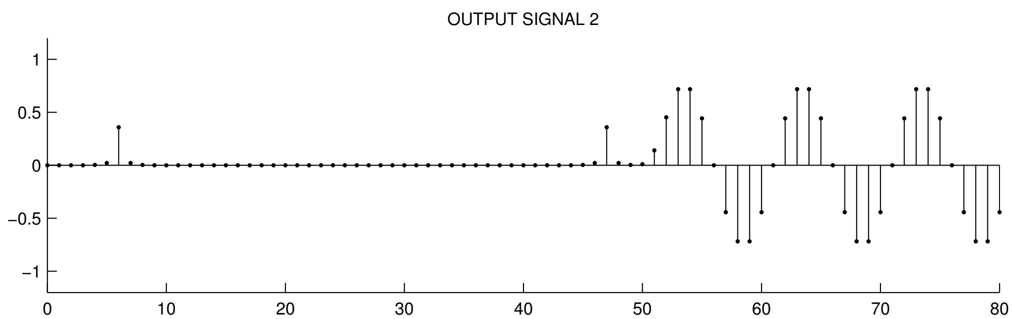


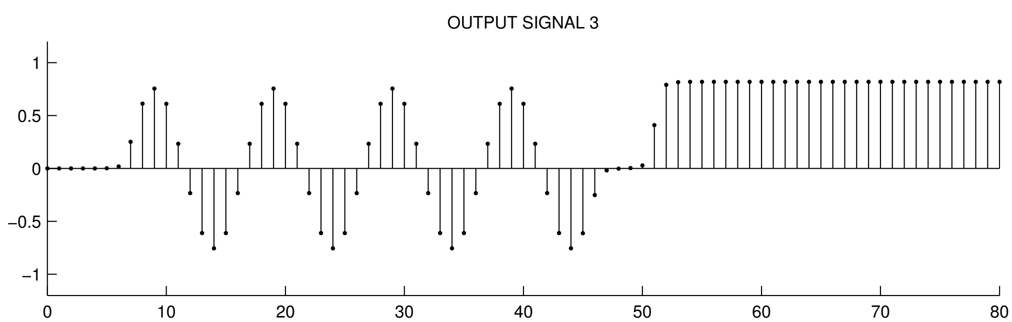

INPUT SIGNAL 1
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80

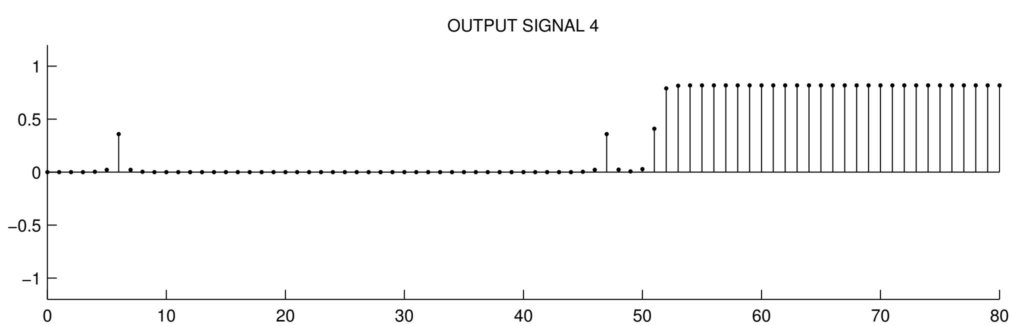

INPUT SIGNAL 2
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80

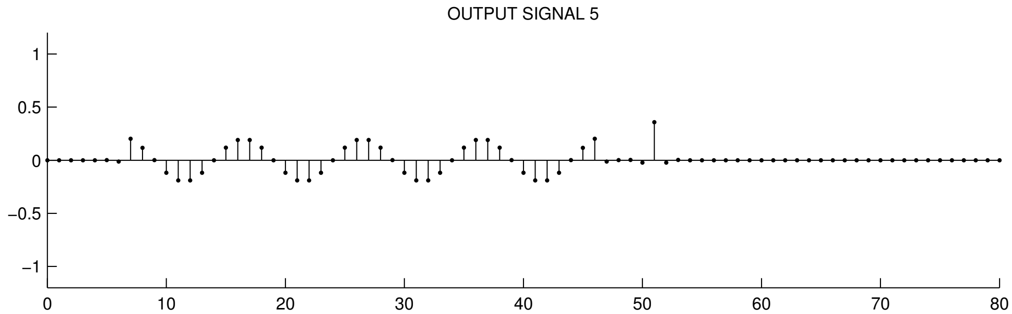

INPUT SIGNAL 3
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80

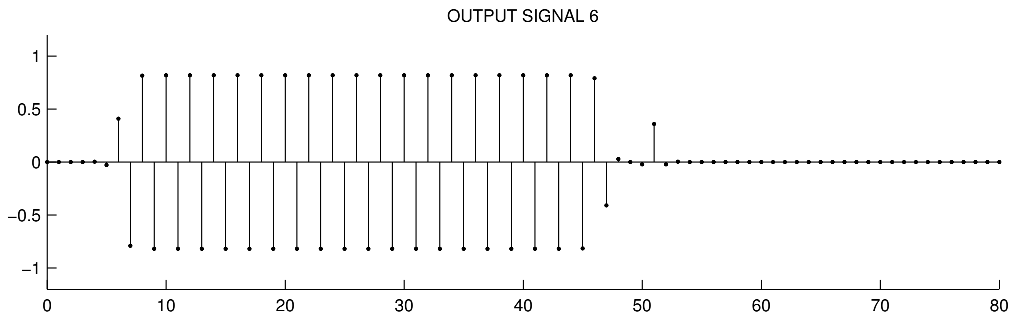

OUTPUT SIGNAL 1
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80


OUTPUT SIGNAL 2
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80


OUTPUT SIGNAL 3
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80

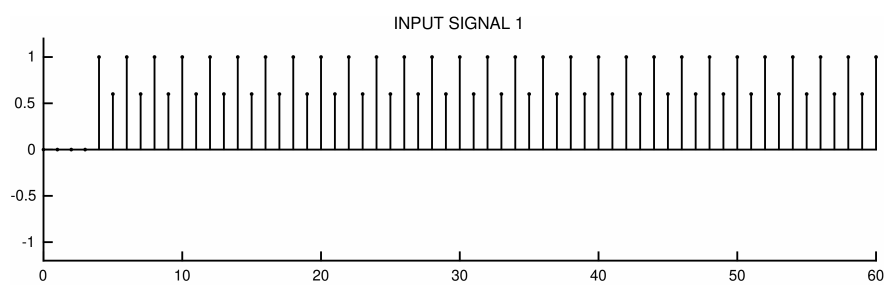

OUTPUT SIGNAL 4
1
0.5
0
−0.5
−1
0
10
20
30
40
50
60
70
80

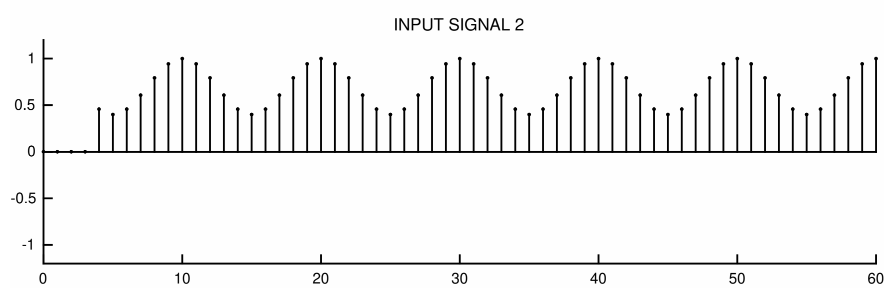


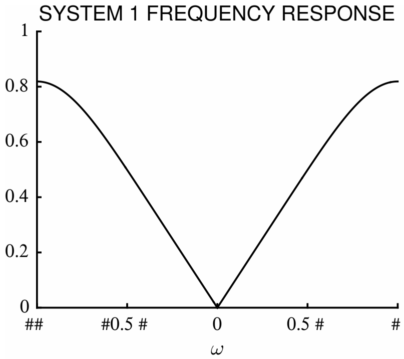


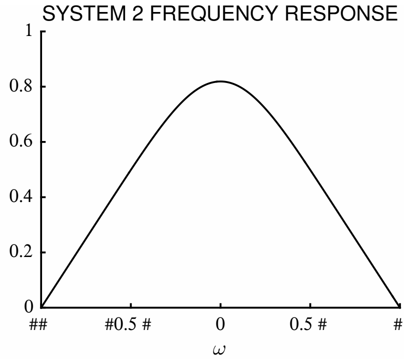
1.8.18 Each of the two discrete-time signals below are processed with each of two LTI systems. The frequency response magnitude $|H^f(\omega)|$ are shown below. Indicate how each of the four output signals are produced by completing the table below.

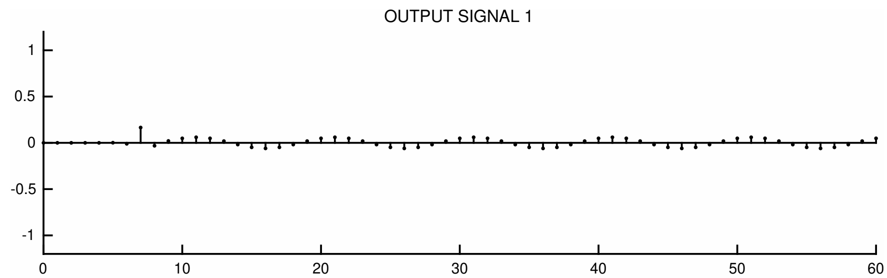
Input signal	System	Output signal
1	1	
1	2	
2	1	
2	2	

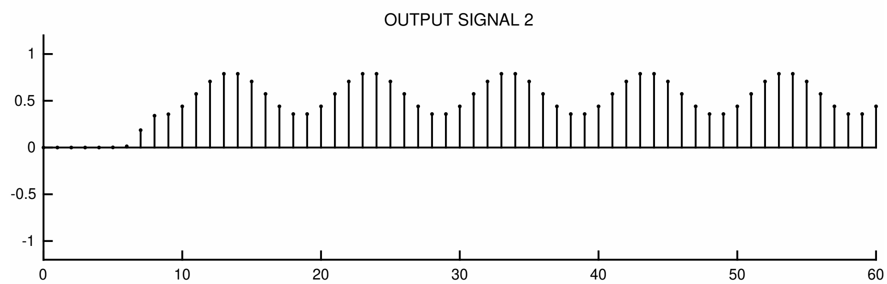


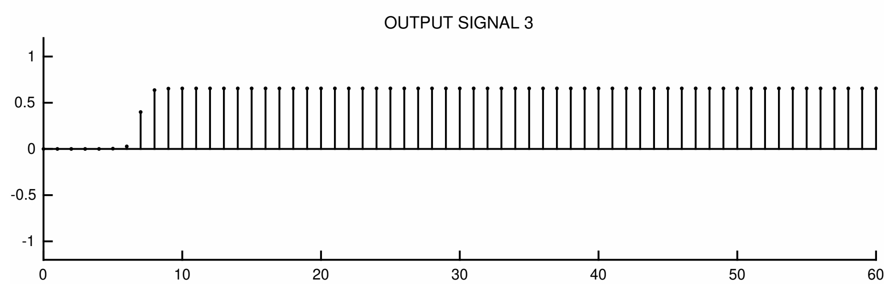


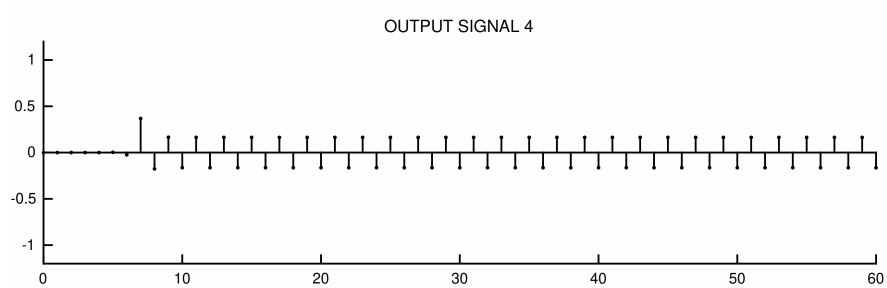

SYSTEM 1 FREQUENCY RESPONSE
1
0.8
0.6
0.4
0.2
0
##
#0.5 #
0
0.5 #
#
ω


SYSTEM 2 FREQUENCY RESPONSE
1
0.8
0.6
0.4
0.2
0
##
#0.5 #
0
0.5 #
#
ω


OUTPUT SIGNAL 1
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60


OUTPUT SIGNAL 2
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60


OUTPUT SIGNAL 3
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60

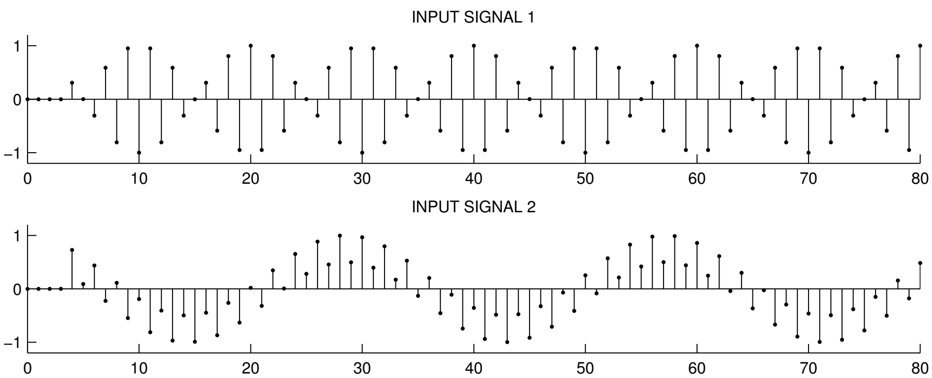

OUTPUT SIGNAL 4
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60

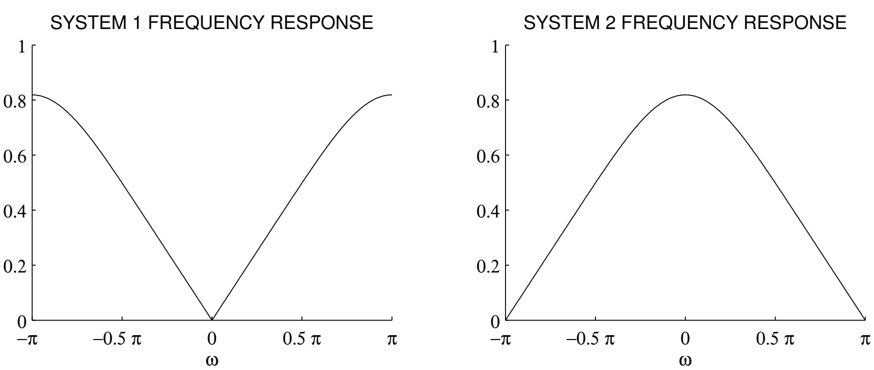
1.8.19 Each of the two discrete-time signals below are processed with each of two LTI systems. The frequency response magnitude $|H^f(\omega)|$ are shown below. Indicate how each of the four output signals are produced by completing the table below.

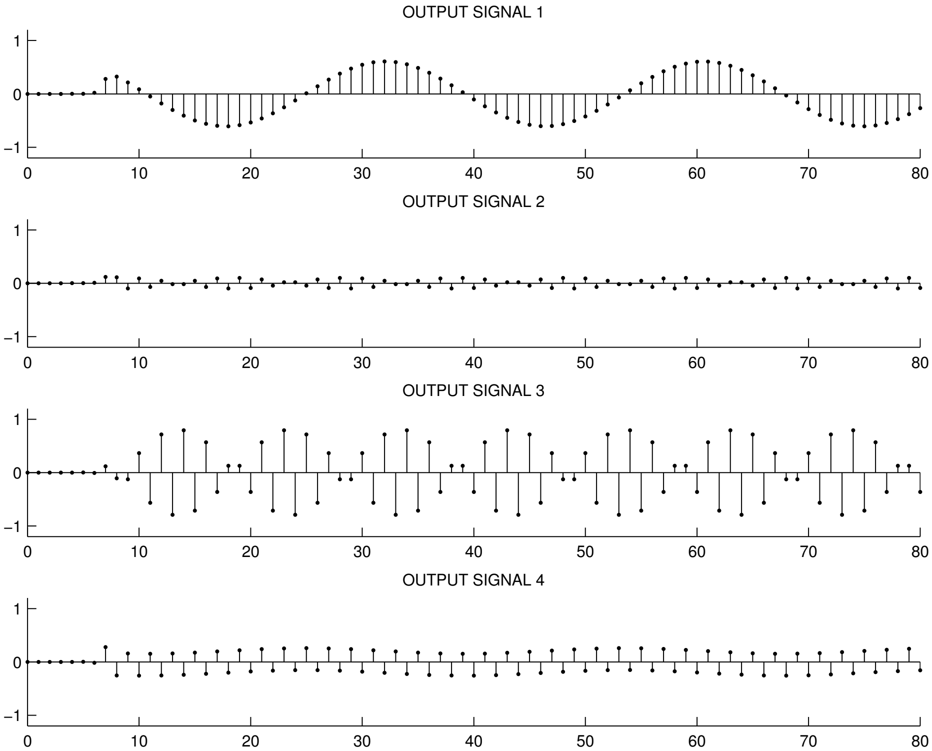
Input signal 1 is given by: $\cos(0.9\,\pi\,n)\,u(n-4)$

Input signal 2 is given by: $0.75\,\cos(0.07\,\pi\,n)\,u(n-4) + 0.25\,(-1)^n\,u(n-4)$

Input signal	System	Output signal
1	1	
1	2	
2	1	
2	2	



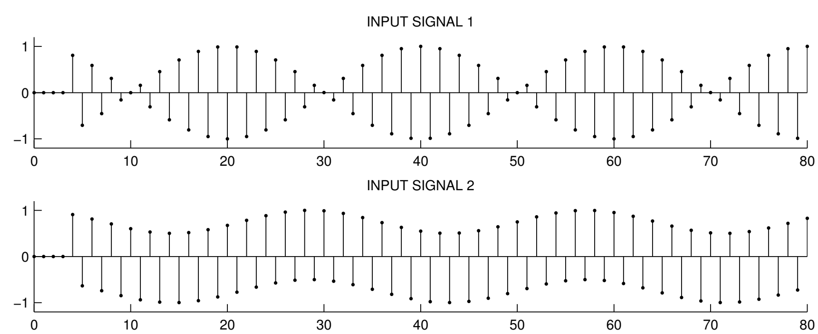


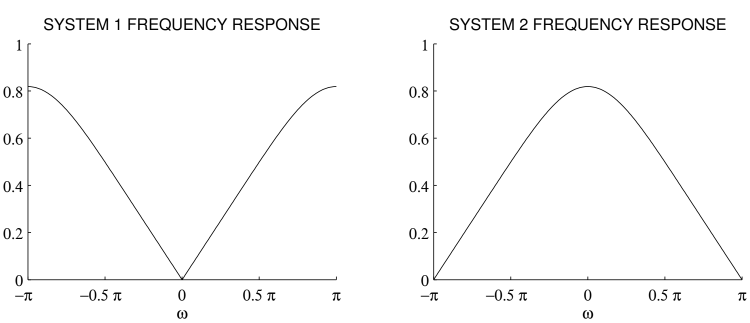


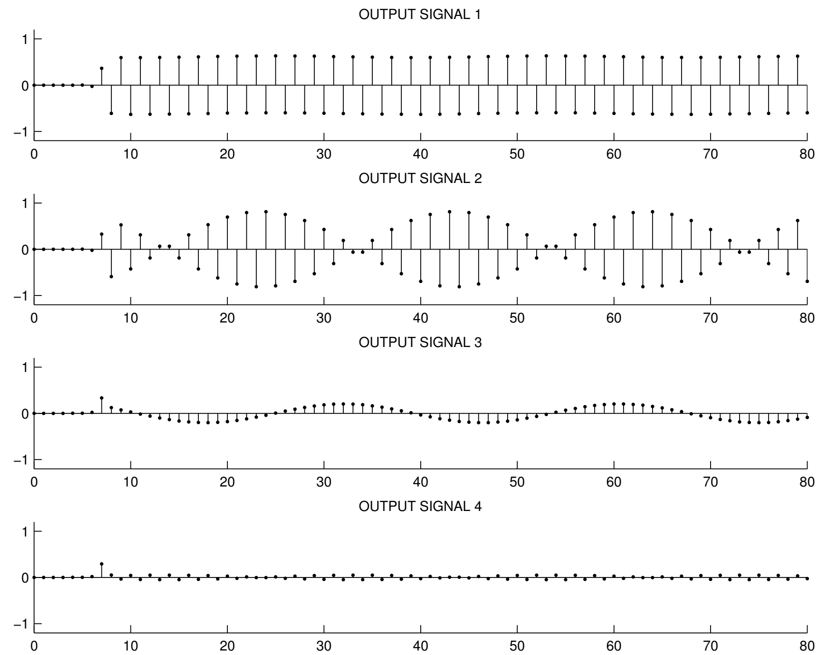
1.8.20 Each of the two discrete-time signals below are processed with each of two LTI systems. The frequency response magnitude $|H^f(\omega)|$ are shown below. Indicate how each of the four output signals are produced by completing the table below.

Input signal 1 is given by: $\cos(0.95\,\pi\,n)\,u(n-4)$

Input signal 2 is given by: $0.25\,\cos(0.07\,\pi\,n)\,u(n-4) + 0.75\,(-1)^n\,u(n-4)$







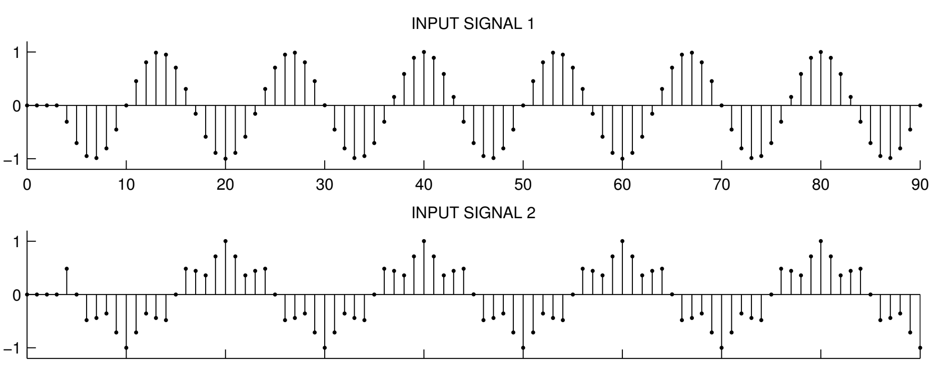
Input signal	System	Output signal
1	1	
1	2	
2	1	
2	2	

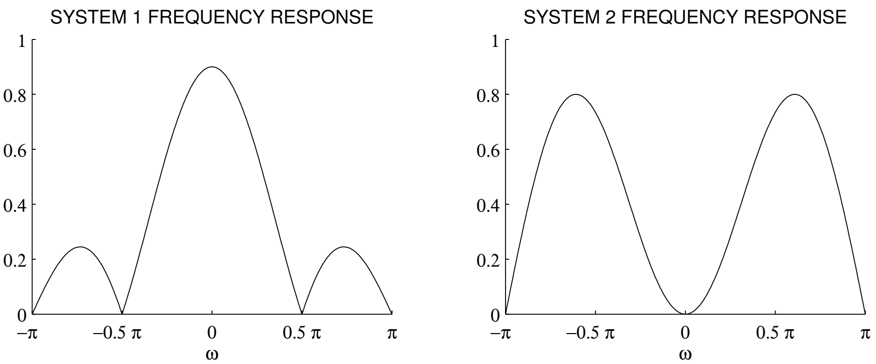
1.8.21 Each of the two discrete-time signals below are processed with each of two LTI systems. The frequency response magnitude $|H^f(\omega)|$ are shown below. Indicate how each of the four output signals are produced by completing the table below.

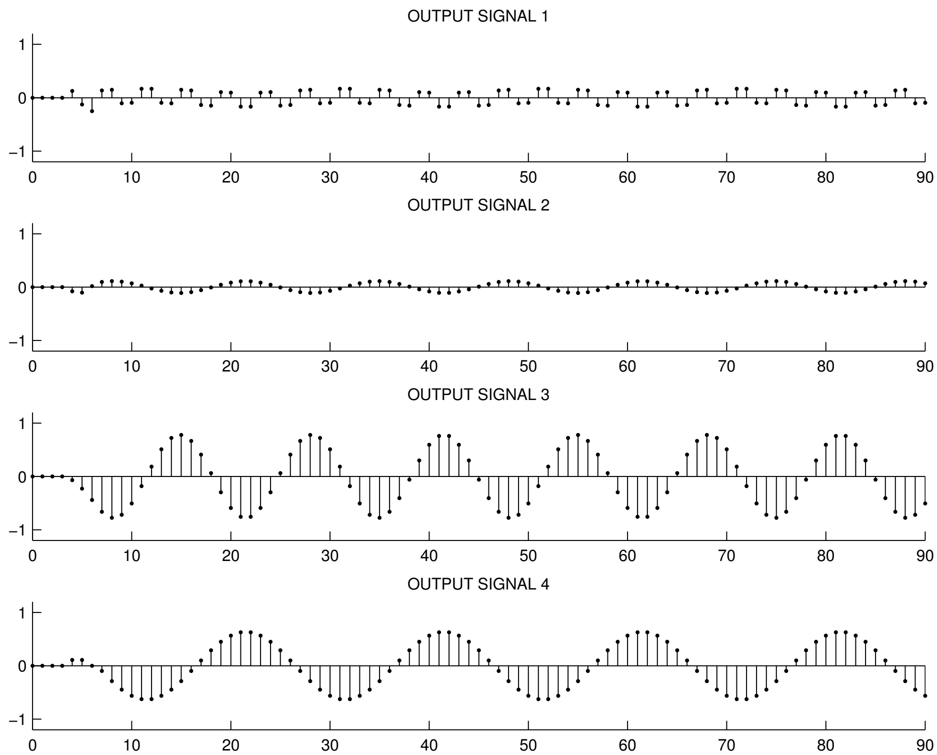
Input signal 1: $\cos(0.15\,\pi\, n)\, u(n-4)$

Input signal 2: $0.75\,\cos(0.1\,\pi\, n)\, u(n-4) + 0.25\,\cos(0.5\,\pi\, n)\, u(n-4)$

Input signal	System	Output signal
1	1	
1	2	
2	1	
2	2	





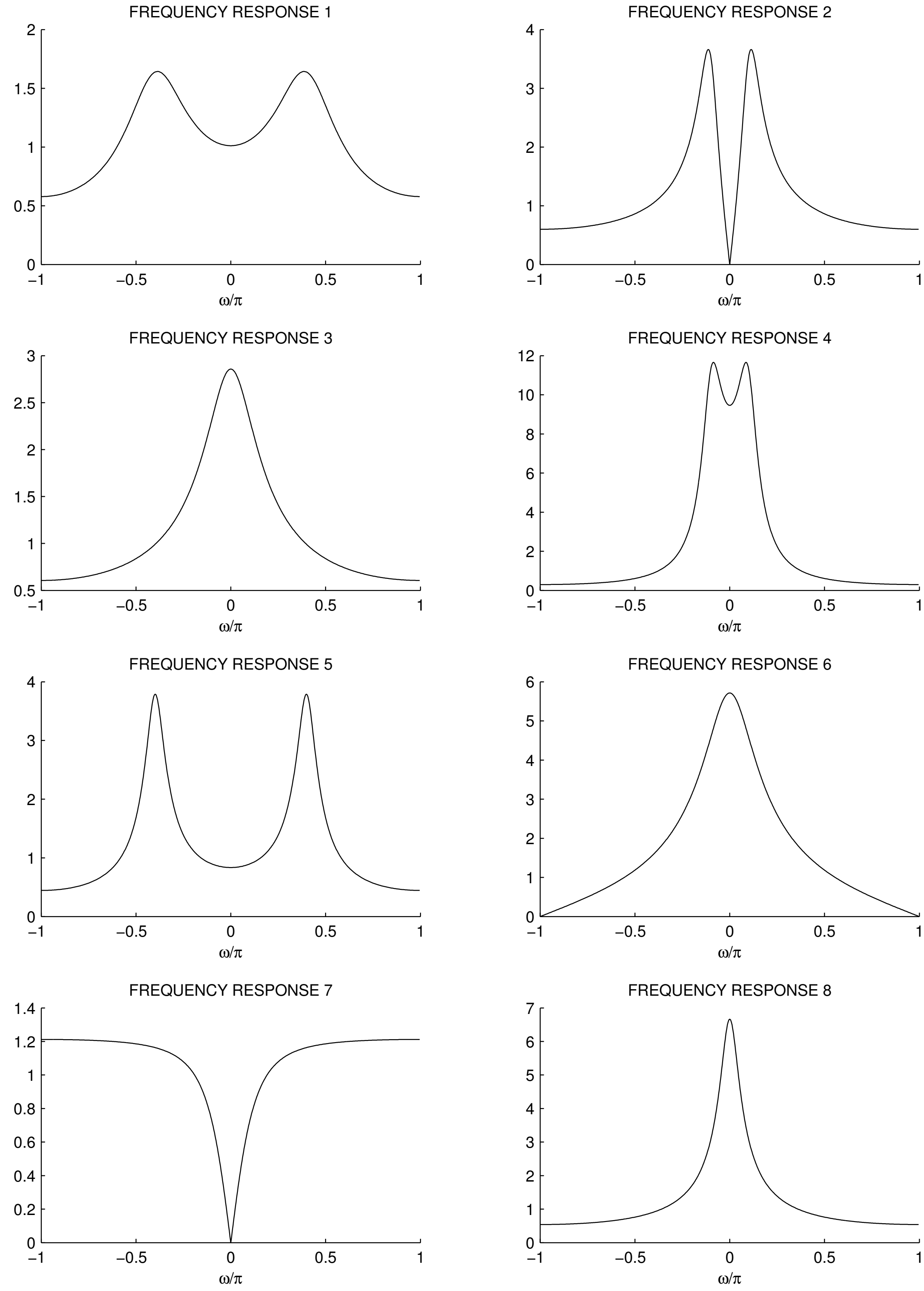


1.8.22 The diagrams on the following pages show the frequency responses magnitudes $|H^f(\omega)|$ and pole-zero diagrams of 8 causal discrete-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

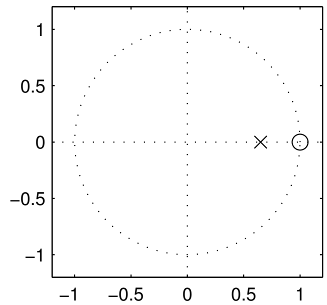
You should do this problem *without* using MATLAB or any other computational tools.

In the pole-zero diagrams, the zeros are shown with 'o' and the poles are shown by 'x'.

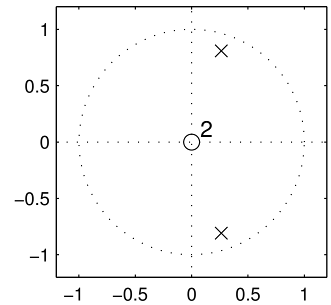
FREQUENCY RESPONSE	POLE-ZERO DIAGRAM
1	
2	
3	
4	
5	
6	
7	
8	


FREQUENCY RESPONSE 1
FREQUENCY RESPONSE 2
FREQUENCY RESPONSE 3
FREQUENCY RESPONSE 4
FREQUENCY RESPONSE 5
FREQUENCY RESPONSE 6
FREQUENCY RESPONSE 7
FREQUENCY RESPONSE 8
ω/π

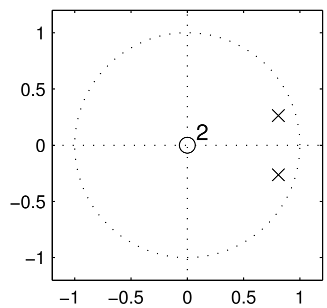
ZERO−POLE DIAGRAM 1



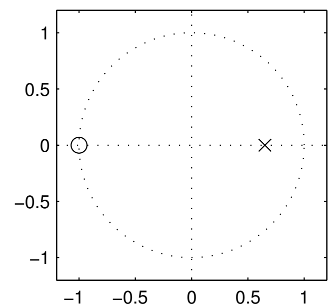
ZERO−POLE DIAGRAM 2



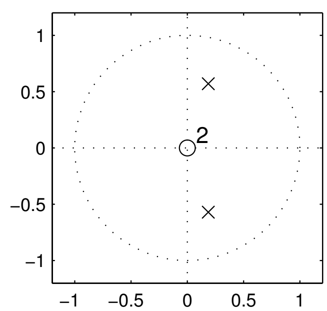
ZERO−POLE DIAGRAM 3



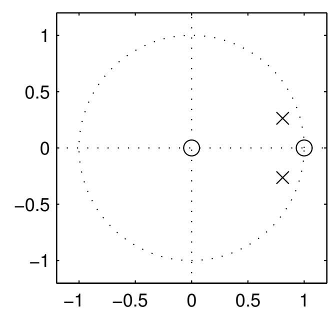
ZERO−POLE DIAGRAM 4



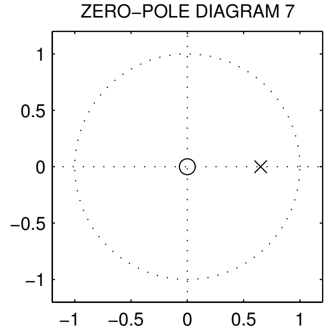
ZERO−POLE DIAGRAM 5



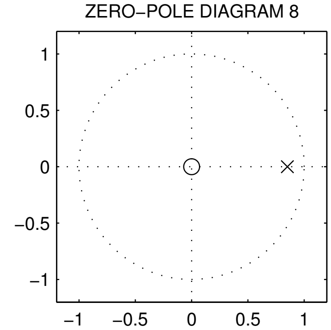
ZERO−POLE DIAGRAM 6



ZERO−POLE DIAGRAM 7



ZERO−POLE DIAGRAM 8



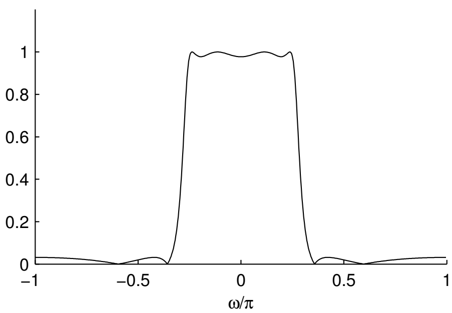
1.8.23 The diagrams on the following pages show the frequency responses and pole-zero diagrams of 6 causal discrete-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

You should do this problem *without* using MATLAB or any other computational tools.

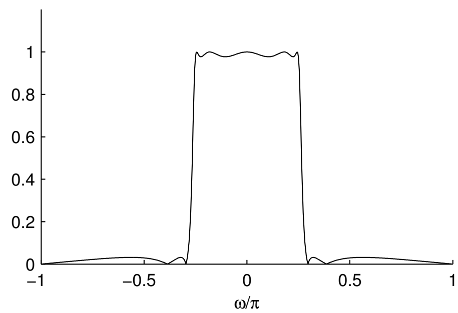
In the pole-zero diagrams, the zeros are shown with ‘o’ and the poles are shown by ‘x’.

FREQUENCY RESPONSE	POLE-ZERO DIAGRAM
1	
2	
3	
4	
5	
6	

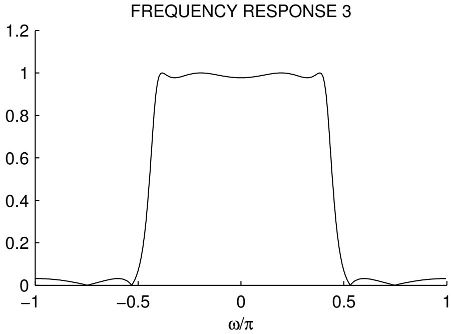
FREQUENCY RESPONSE 1



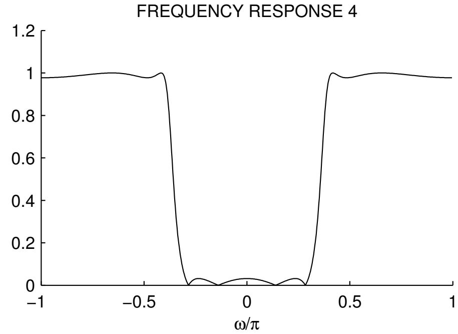
FREQUENCY RESPONSE 2



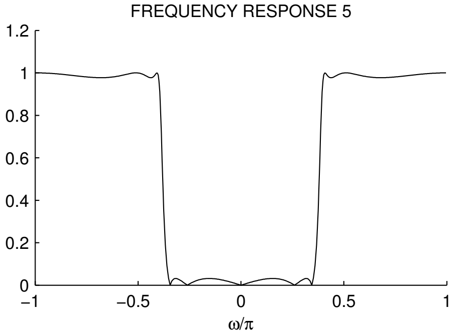
FREQUENCY RESPONSE 3



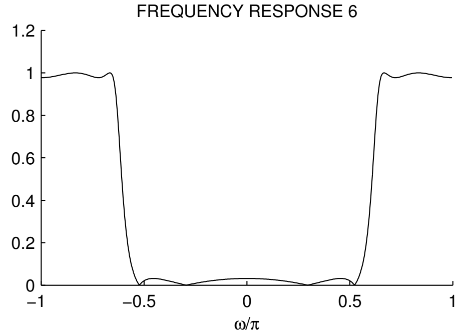
FREQUENCY RESPONSE 4



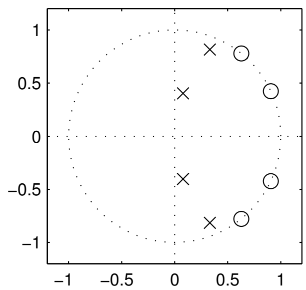
FREQUENCY RESPONSE 5



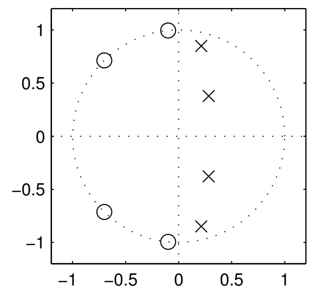
FREQUENCY RESPONSE 6



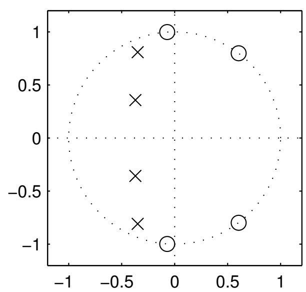
ZERO−POLE DIAGRAM 1



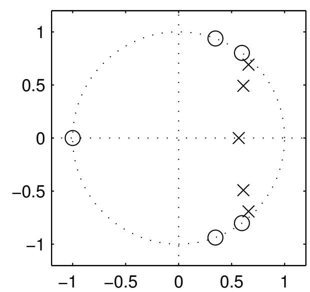
ZERO−POLE DIAGRAM 2



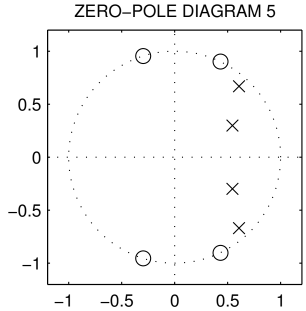
ZERO−POLE DIAGRAM 3



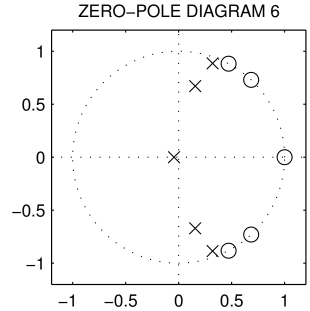
ZERO−POLE DIAGRAM 4



ZERO−POLE DIAGRAM 5

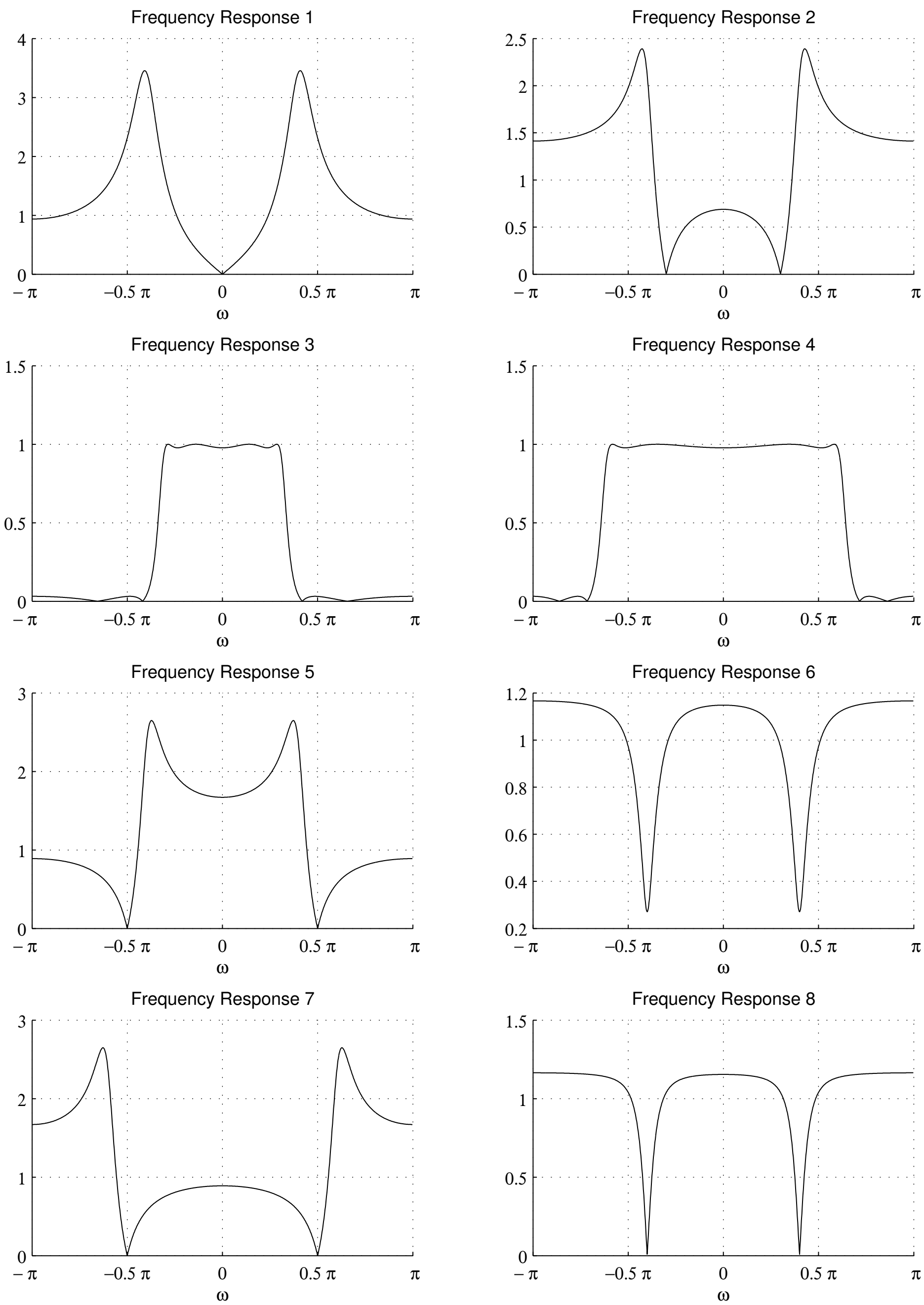


ZERO−POLE DIAGRAM 6



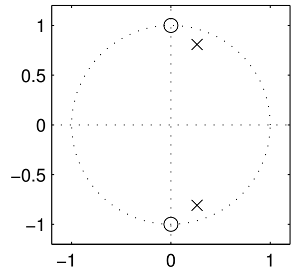
1.8.24 The frequency responses and pole-zero diagrams of eight discrete-time LTI systems are illustrated below. But they are out of order. Match them to each other by filling out the table.

FREQUENCY RESPONSE	POLE-ZERO DIAGRAM
1	
2	
3	
4	
5	
6	
7	
8	

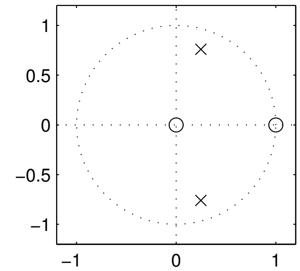


Frequency Response 1
Frequency Response 2
Frequency Response 3
Frequency Response 4
Frequency Response 5
Frequency Response 6
Frequency Response 7
Frequency Response 8
− π
−0.5 π
0
0.5 π
π
ω

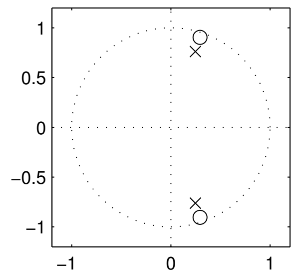
ZERO–POLE DIAGRAM 1



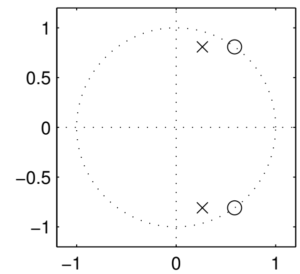
ZERO–POLE DIAGRAM 2



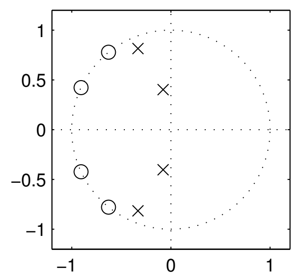
ZERO–POLE DIAGRAM 3



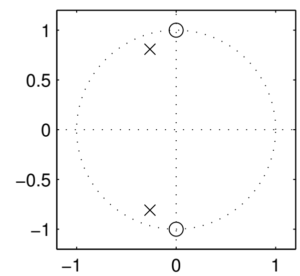
ZERO–POLE DIAGRAM 4



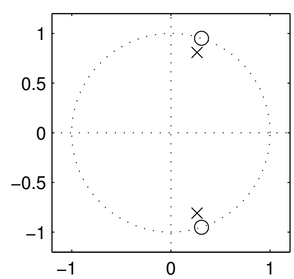
ZERO–POLE DIAGRAM 5



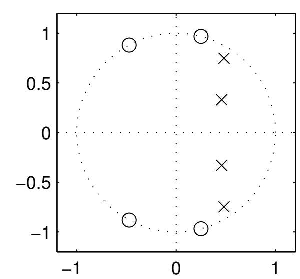
ZERO–POLE DIAGRAM 6



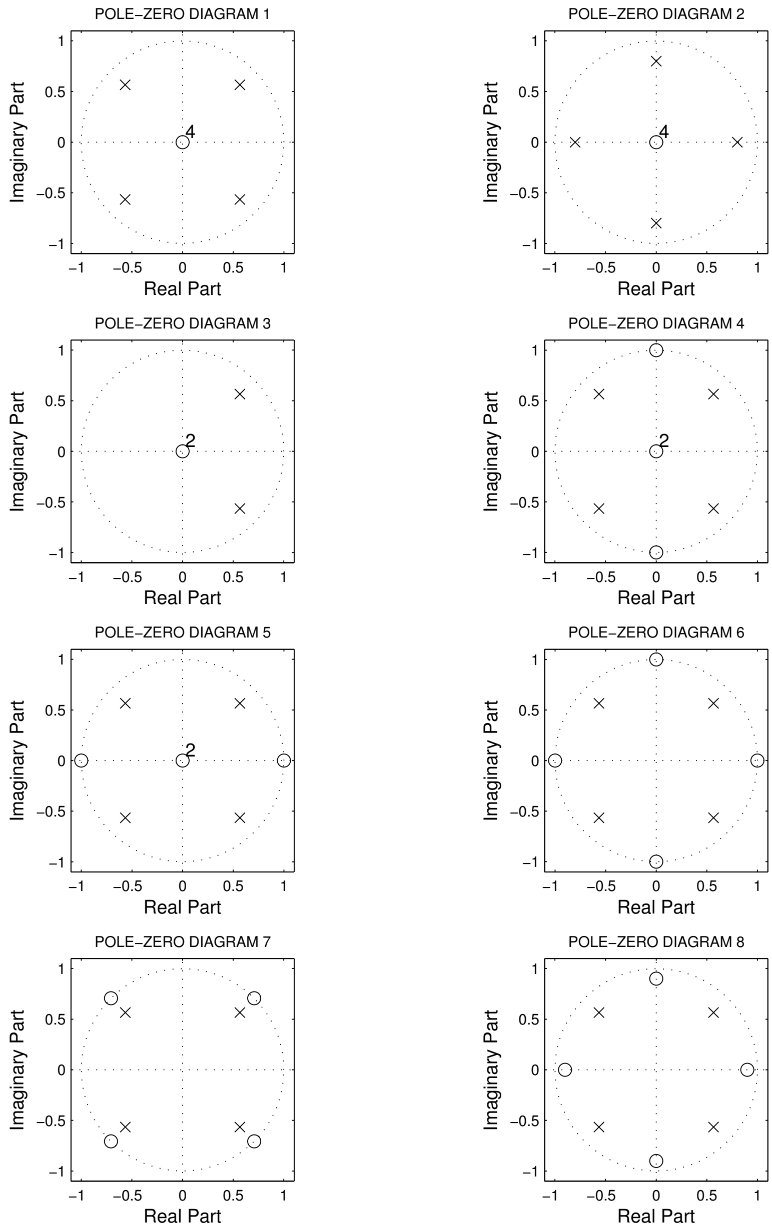
ZERO–POLE DIAGRAM 7

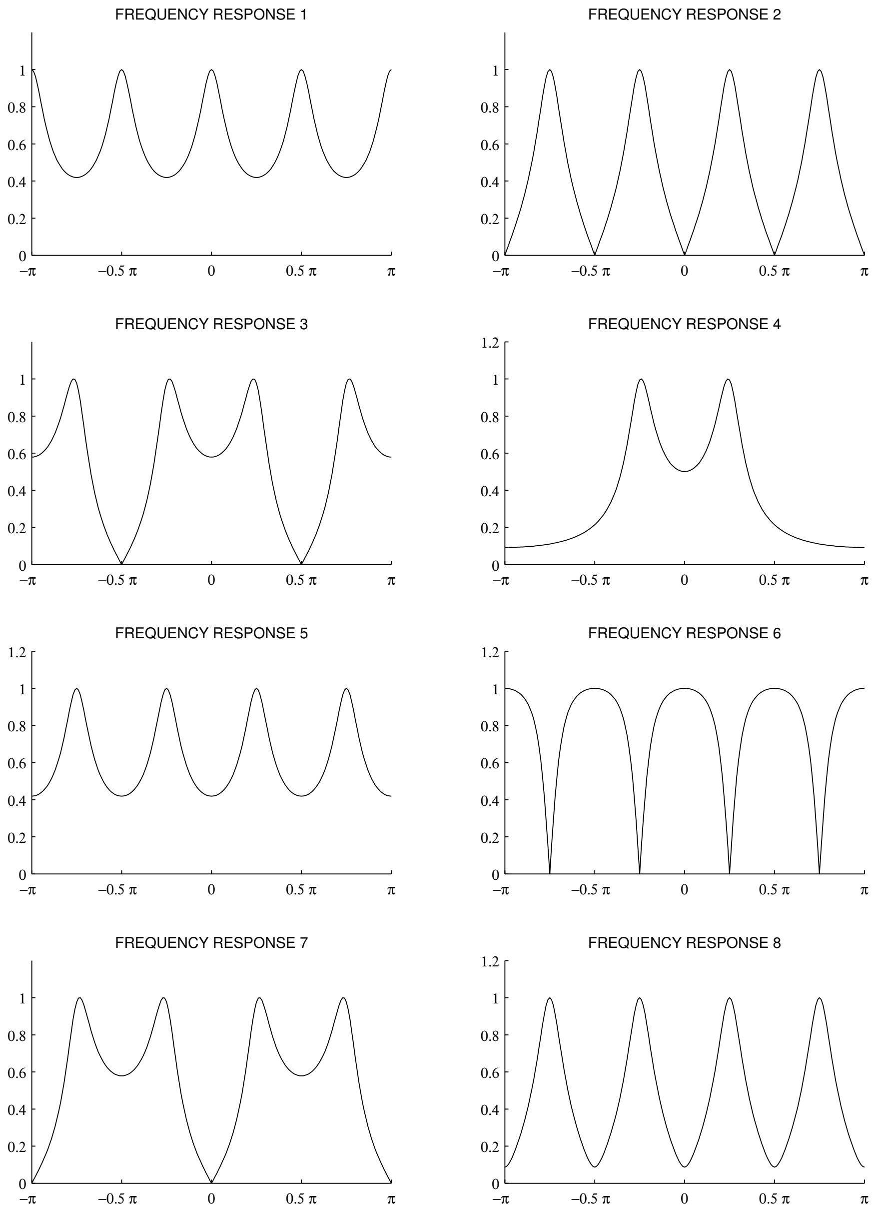


ZERO–POLE DIAGRAM 8



1.8.25 The pole-zero diagrams of eight discrete-time systems are illustrated below. The frequency response $H^f(\omega)$ of each system is also shown, but in a different order. Match each frequency response to its pole-zero diagram by filling out the table.





POLE-ZERO DIAGRAM	FREQUENCY RESPONSE
1	
2	
3	
4	
5	
6	
7	
8	

1.9 Summary Problems

1.9.1 The impulse response of an LTI discrete-time system is given by

$$h(n) = 2\,\delta(n) + \delta(n-1).$$

(a) Find the transfer function of the system.

(b) Find the difference equation with which the system can be implemented.

(c) Sketch the pole/zero diagram of the system.

(d) What is the dc gain of the system? (In other words, what is $H^f(0)$?).

(e) Based on the pole/zero diagram sketch the frequency response magnitude $|H^f(\omega)|$. Mark the value at $\omega = 0$ and $\omega = \pi$.

(f) Sketch the output of the system when the input $x(n)$ is the constant unity signal, $x(n) = 1$.

(g) Sketch the output of the system when the input $x(n)$ is the unit step signal, $x(n) = u(n)$.

(h) Find a formula for the output signal when the input signal is

$$x(n) = \left(\frac{1}{2}\right)^n u(n).$$

1.9.2 A causal LTI system is implemented using the difference equation

$$y(n) = x(n) + \frac{9}{14}\, y(n-1) - \frac{1}{14}\, y(n-2)$$

(a) What is the transfer function $H(z)$ of this system?

(b) What is the impulse response $h(n)$ of this system?

(c) Use the MATLAB command `filter` to numerically verify the correctness of your formula for $h(n)$. (Use this command to numerically compute the first few values of $h(n)$ from the difference equation, and compare with the formula.)

1.9.3 A causal discrete-time LTI system is implemented using the difference equation

$$y(n) = x(n) + 0.5\, x(n-1) + 0.2\, y(n-1)$$

where x is the input signal, and y the output signal.

(a) Sketch the pole/zero diagram of the system.

(b) Find the dc gain of the system.

(c) Find the value of the frequency response at $\omega = \pi$.

(d) Based on parts (a),(b),(c), roughly sketch the frequency response magnitude of the system.

(e) Find the form of the output signal when the input signal is

$$x(n) = 2\left(\frac{1}{3}\right)^n u(n).$$

You do not need to compute the constants produced by the partial fraction expansion procedure (PFA) — you can just leave them as constants: A, B, etc. Be as accurate as you can be in your answer without actually going through the arithmetic of the PFA.

1.9.4 For the causal discrete-time LTI system implemented using the difference equation

$$y(n) = x(n) + 0.5\, x(n-1) + 0.5\, y(n-1),$$

(a) Sketch the pole/zero diagram.

(b) Find the dc gain of the system.

(c) Find the output signal produced by the input signal $x(n) = 0.5$.

(d) Find the value of the frequency response at $\omega = \pi$.

(e) Find the *steady-state* output signal produced by the input signal $x(n) = 0.6\,(-1)^n\, u(n)$. (The steady-state output signal is the output signal after the transients have died out.)

(f) Validate your answers using Matlab.

1.9.5 A causal discrete-time LTI system is implemented using the difference equation

$$y(n) = x(n) - x(n-2) + 0.8\,y(n-1)$$

where x is the input signal, and y the output signal.

(a) Sketch the pole/zero diagram of the system.

(b) Find the dc gain of the system.

(c) Find the value of the frequency response at $\omega = \pi$.

(d) Based on parts (a),(b),(c), roughly sketch the frequency response magnitude $|H^f(\omega)|$ of the system.

1.9.6 The impulse response of a discrete-time LTI system is given by

$$h(n) = \begin{cases} 0.25 & \text{for } 0 \le n \le 3 \\ 0 & \text{for other values of } n. \end{cases}$$

Make an accurate sketch of the output of the system when the input signal is

$$x(n) = \begin{cases} 1 & \text{for } 0 \le n \le 30 \\ 0 & \text{for other values of } n. \end{cases}$$

You should do this problem *with out* using MATLAB, etc.

1.9.7 If a discrete-time LTI system has the transfer function $H(z) = 5$, then what difference equation implements this system? Classify this system as memoryless/with memory.

1.9.8 **First order difference system:** A discrete-time LTI system is implemented using the difference equation

$$y(n) = 0.5\,x(n) - 0.5\,x(n-1).$$

(a) What is the transfer function $H(z)$ of the system?

(b) What is the impulse response $h(n)$ of the system?

(c) What is the frequency response $H^f(\omega)$ of the system?

(d) Accurately sketch the frequency response magnitude $|H^f(\omega)|$.

(e) Find the output $y(n)$ when the input signal is the step signal $u(n)$.

(f) Sketch the pole-zero diagram of the system.

(g) Is the system a low-pass filter, high-pass filter, or neither?

1.9.9 A causal discrete-time LTI system is implemented with the difference equation

$$y(n) = 3\,x(n) + \frac{3}{2}\,y(n-1).$$

(a) Find the output signal when the input signal is

$$x(n) = 3\,(2)^n\,u(n).$$

Show your work.

(b) Is the system stable or unstable?

(c) Sketch the pole-zero diagram of this system.

1.9.10 A causal discrete-time LTI system is described by the equation

$$y(n) = \frac{1}{3}\,x(n) + \frac{1}{3}\,x(n-1) + \frac{1}{3}\,x(n-2)$$

where x is the input signal, and y the output signal.

(a) Sketch the impulse response of the system.

(b) What is the dc gain of the system? (Find $H^f(0)$.)

(c) Sketch the output of the system when the input $x(n)$ is the constant unity signal, $x(n) = 1$.

(d) Sketch the output of the system when the input $x(n)$ is the unit step signal, $x(n) = u(n)$.

(e) Find the value of the frequency response at $\omega = \pi$. (Find $H^f(\pi)$.)

(f) Find the output of the system produced by the input $x(n) = (-1)^n$.

(g) How many zeros does the transfer function $H(z)$ have?

(h) Find the value of the frequency response at $\omega = \frac{2}{3}\pi$. (Find $H^f(2\pi/3)$.)

(i) Find the poles and zeros of $H(z)$; and sketch the pole/zero diagram.

(j) Find the output of the system produced by the input $x(n) = \cos\left(\frac{2}{3}\pi n\right)$.

1.9.11 **Two-Point Moving Average.** A discrete-time LTI system has impulse response

$$h(n) = 0.5\,\delta(n) + 0.5\,\delta(n-1).$$

(a) Sketch the impulse response $h(n)$.

(b) What difference equation implements this system?

(c) Sketch the pole-zero diagram of this system.

(d) Find the frequency resposnse $H^f(\omega)$. Find simple expressions for $|H^f(\omega)|$ and $\angle H^f(\omega)$ and sketch them.

(e) Is this a lowpass, highpass, or bandpass filter?

(f) Find the output signal $y(n)$ when the input signal is $x(n) = u(n)$.
Also, $x(n) = \cos(\omega_o\, n)\, u(n)$ for what value of ω_o?

(g) Find the output signal $y(n)$ when the input signal is $x(n) = (-1)^n\, u(n)$.
Also, $x(n) = \cos(\omega_o\, n)\, u(n)$ for what value of ω_o?

1.9.12 A causal discrete-time LTI system is described by the equation

$$y(n) = \frac{1}{4}\sum_{k=0}^{3} x(n-k)$$

where x is the input signal, and y the output signal.

(a) Sketch the impulse response of the system.

(b) How many zeros does the transfer function $H(z)$ have?

(c) What is the dc gain of the system? (Find $H^f(0)$.)

(d) Find the value of the frequency response at $\omega = 0.5\,\pi$. (Find $H^f(0.5\,\pi)$.)

(e) Find the value of the frequency response at $\omega = \pi$. (Find $H^f(\pi)$.)

(f) Based on (b), (d) and (e), find the zeros of $H(z)$; and sketch the pole/zero diagram.

(g) Based on the pole/zero diagram, sketch the frequency response magnitude $|H^f(\omega)|$.

1.9.13 **Four-Point Moving Average.** A discrete-time LTI system has impulse response

$$h(n) = 0.25\,\delta(n) + 0.25\,\delta(n-1) + 0.25\,\delta(n-2) + 0.25\,\delta(n-3)$$

(a) Sketch $h(n)$.

(b) What difference equation implements this system?

(c) Sketch the pole-zero diagram of this system.

(d) Find the frequency resposnse $H^f(\omega)$. Find simple expressions for $|H^f(\omega)|$ and $\angle H^f(\omega)$ and sketch them.

(e) Is this a lowpass, highpass, or bandpass filter?

1.10 Simple System Design

1.10.1 In this problem you are to design a simple causal real discrete-time FIR LTI system with the following properties:

(a) The system should kill the signal $\cos(0.75\,\pi\,n)$

(b) The system should have unity dc gain. That is, $H^f(0) = 1$.

For the system you design:

(a) Find the difference equation to implement the system.

(b) Sketch the impulse response of the system.

(c) Roughly sketch the frequency response magnitude $|H^f(\omega)|$. Clearly show the nulls of the frequency response.

1.10.2 In this problem you are to design a causal discrete-time LTI system with the following properties:

(a) The transfer function should have two poles. They should be at $z = 1/2$ and at $z = 0$.

(b) The system should kill the signal $\cos(0.75\,\pi\,n)$.

(c) The system should have unity dc gain. That is, $H^f(0) = 1$.

For the system you design:

(a) Find the difference equation to implement the system.

(b) Roughly sketch the frequency response magnitude $|H^f(\omega)|$. What is the value of the frequency response at $\omega = \pi$?

(c) Find the output signal produced by the system when the input signal is $\sin(0.75\,\pi\,n)$.

1.10.3 In this problem you are to design a causal discrete-time LTI system with the following properties:

(a) The transfer function should have two poles. They should be at $z = j/2$ and at $z = -j/2$.

(b) The system should kill the signal $\cos(0.5\,\pi\,n)$.

(c) The system should have unity dc gain. That is, $H^f(0) = 1$.

For the system you design:

(a) Find the difference equation to implement the system.

(b) Roughly sketch the frequency response magnitude $|H^f(\omega)|$. What is the value of the frequency response at $\omega = \pi$?

(c) Find the output signal produced by the system when the input signal is $\sin(0.5\,\pi\,n)$.

1.10.4 In this problem you are to design a simple causal real discrete-time LTI system with the following properties:

(a) The system should kill the signals $(-1)^n$ and $\cos(0.5\,\pi\,n)$

(b) The system should have unity dc gain. That is, $H^f(0) = 1$.

For the system you design:

(a) Find the difference equation to implement the system.

(b) Sketch the impulse response of the system.

(c) Roughly sketch the frequency response magnitude $|H^f(\omega)|$. Clearly show the nulls of the frequency response.

1.10.5 Design a simple causal real discrete-time LTI system with the properties:

(a) The system should exactly preserve the signal $\cos(0.5\,\pi\,n)$.

(b) The system should annihilate constant signals. That is, the frequency response should have a null at dc.

Hint: It can be done with an impulse response of length 3.

For the system you design:

(a) Find the difference equation to implement the system.

(b) Sketch the impulse response of the system.

(c) Sketch the poles and zeros of the system.

(d) Find and sketch the frequency response magnitude $|H^f(\omega)|$.
Clearly show the nulls of the frequency response.

1.11 Matching

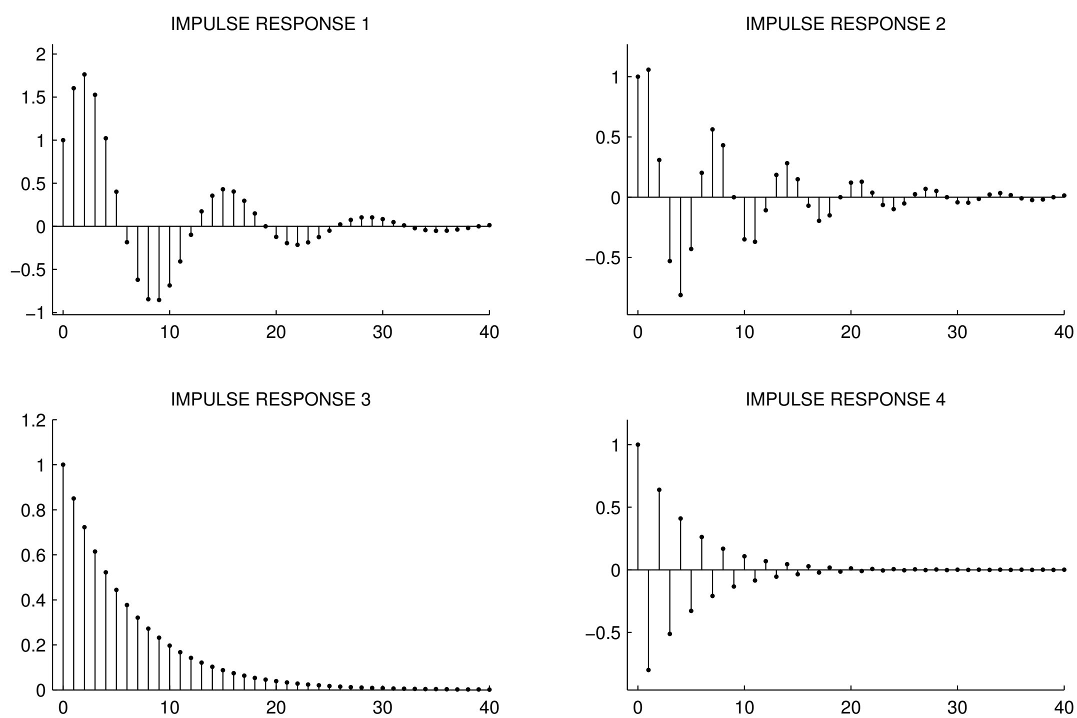
1.11.1 The diagrams on the following pages show the impulse responses, frequency responses, and pole-zero diagrams of 4 causal discrete-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

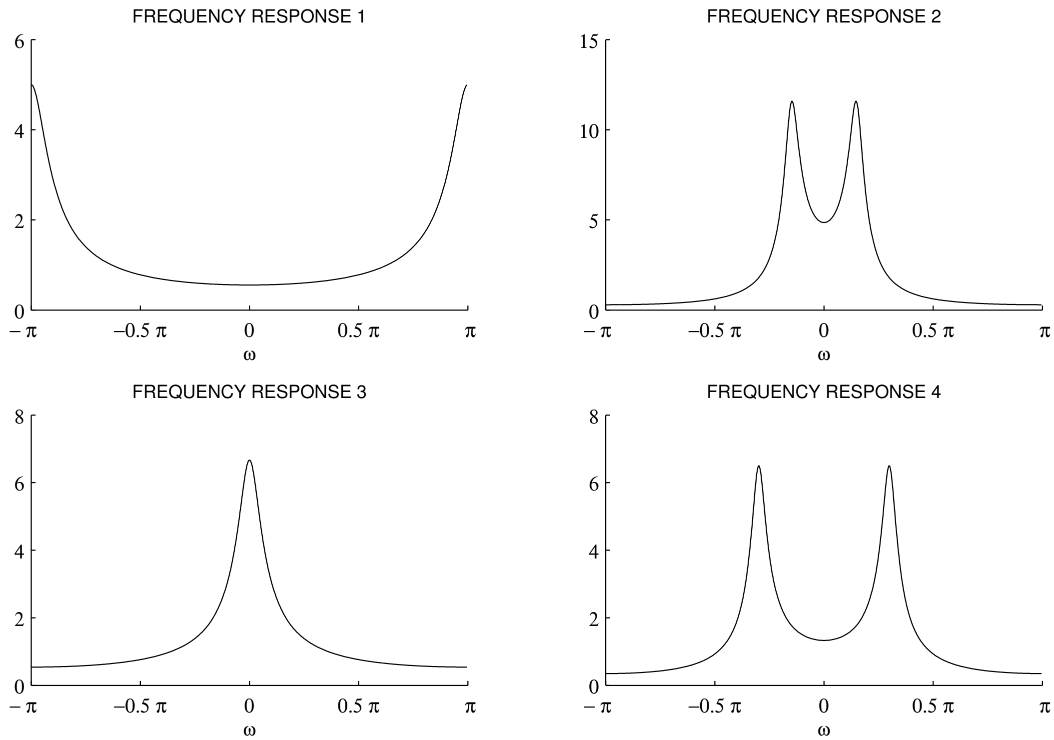
You should do this problem *without* using MATLAB or any other computational tools.

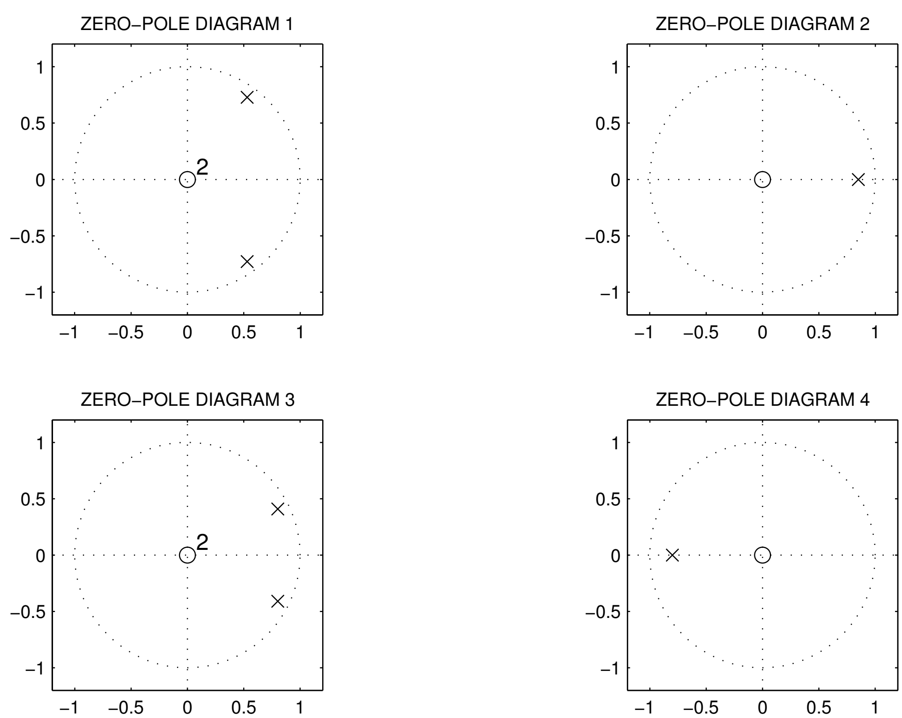
In the pole-zero diagrams, the zeros are shown with 'o' and the poles are shown by 'x'.

IMPULSE RESPONSE	FREQUENCY RESPONSE	POLE-ZERO DIAGRAM
1		
2		
3		
4		

MatchA




FREQUENCY RESPONSE 1
FREQUENCY RESPONSE 2
FREQUENCY RESPONSE 3
FREQUENCY RESPONSE 4
ω
−π
−0.5 π
0
0.5 π
π


ZERO−POLE DIAGRAM 1
ZERO−POLE DIAGRAM 2
ZERO−POLE DIAGRAM 3
ZERO−POLE DIAGRAM 4
2
2

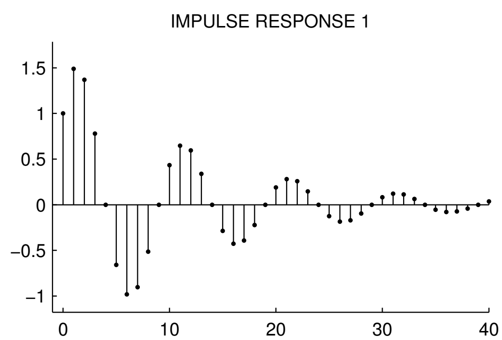
1.11.2 The diagrams on the following pages show the impulse responses, frequency responses, and pole-zero diagrams of 4 causal discrete-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

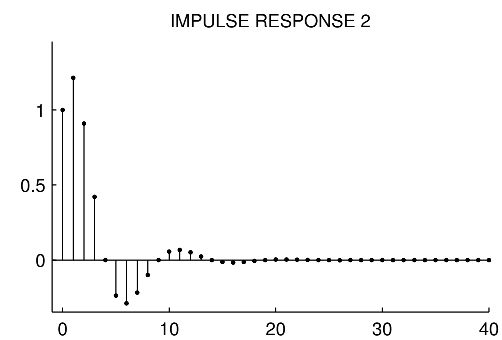
You should do this problem *without* using MATLAB or any other computational tools.

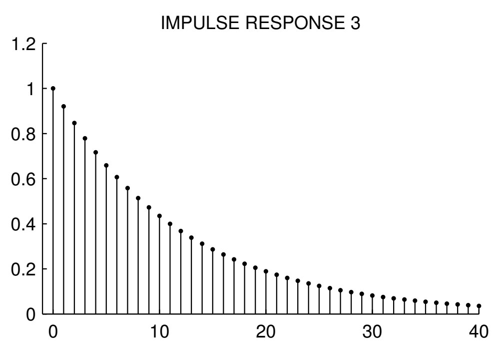
In the pole-zero diagrams, the zeros are shown with 'o' and the poles are shown by 'x'.

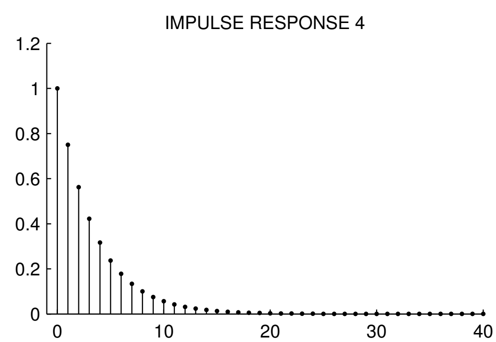
IMPULSE RESPONSE	FREQUENCY RESPONSE	POLE-ZERO DIAGRAM
1		
2		
3		
4		

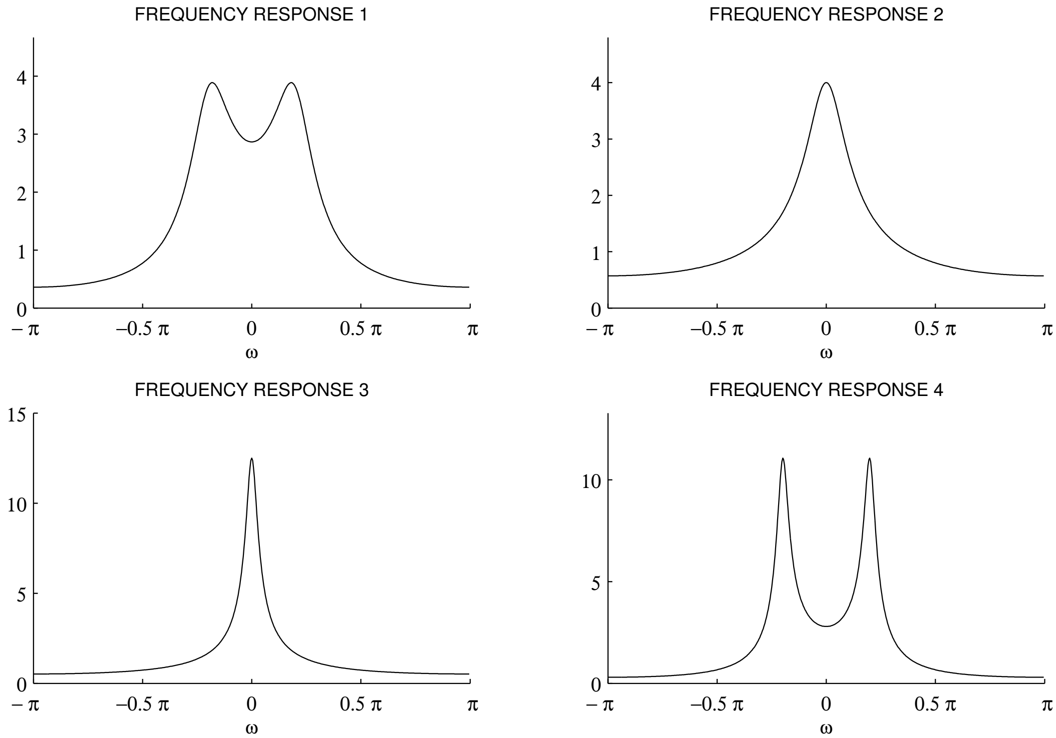
MatchB

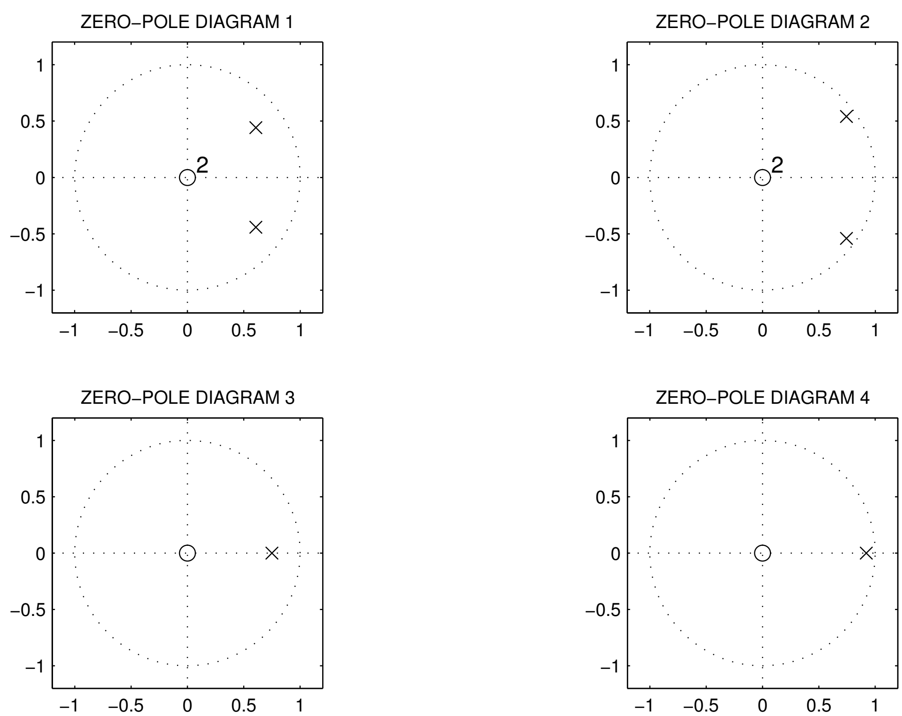









FREQUENCY RESPONSE 1
FREQUENCY RESPONSE 2
FREQUENCY RESPONSE 3
FREQUENCY RESPONSE 4
− π
−0.5 π
0
0.5 π
π
ω


ZERO−POLE DIAGRAM 1
ZERO−POLE DIAGRAM 2
ZERO−POLE DIAGRAM 3
ZERO−POLE DIAGRAM 4
2

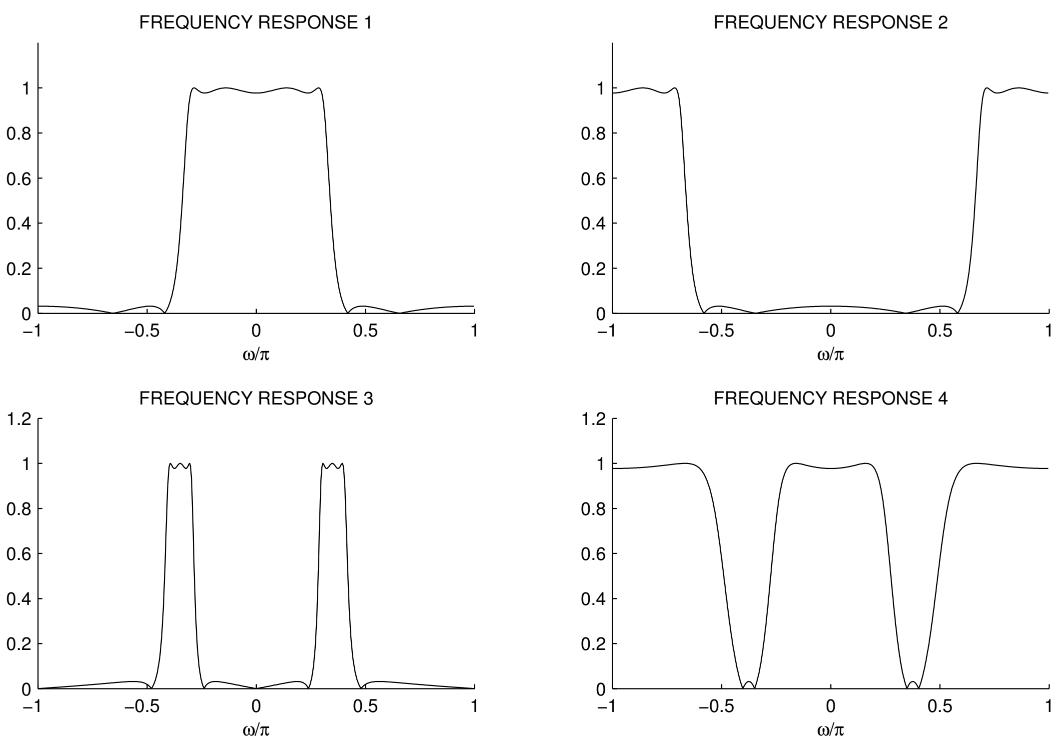
1.11.3 The diagrams on the following pages show the frequency responses, impulse responses and pole-zero diagrams of 4 causal discrete-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

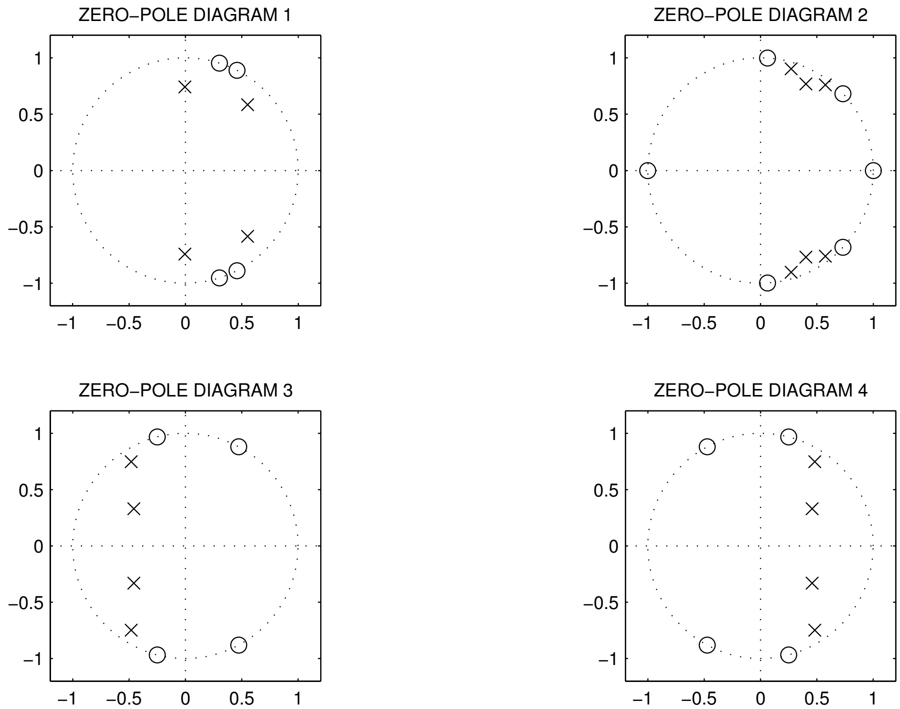
You should do this problem *without* using MATLAB or any other computational tools.

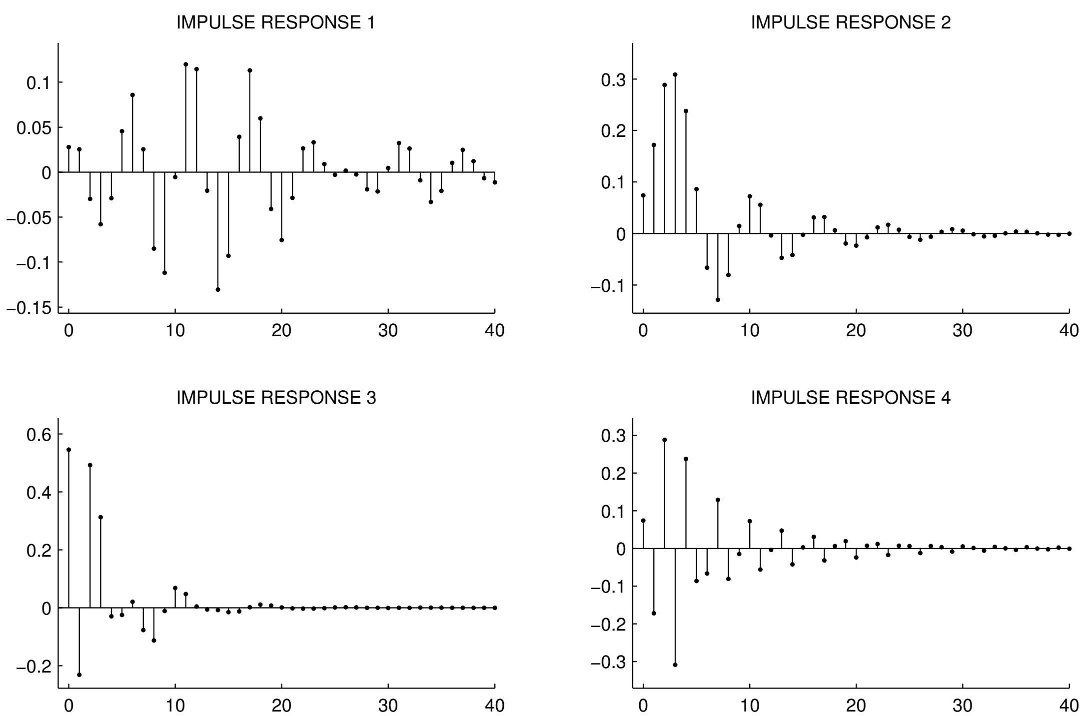
In the pole-zero diagrams, the zeros are shown with 'o' and the poles are shown by 'x'.

FREQUENCY RESPONSE	POLE-ZERO DIAGRAM	IMPULSE RESPONSE
1		
2		
3		
4		

MatchC

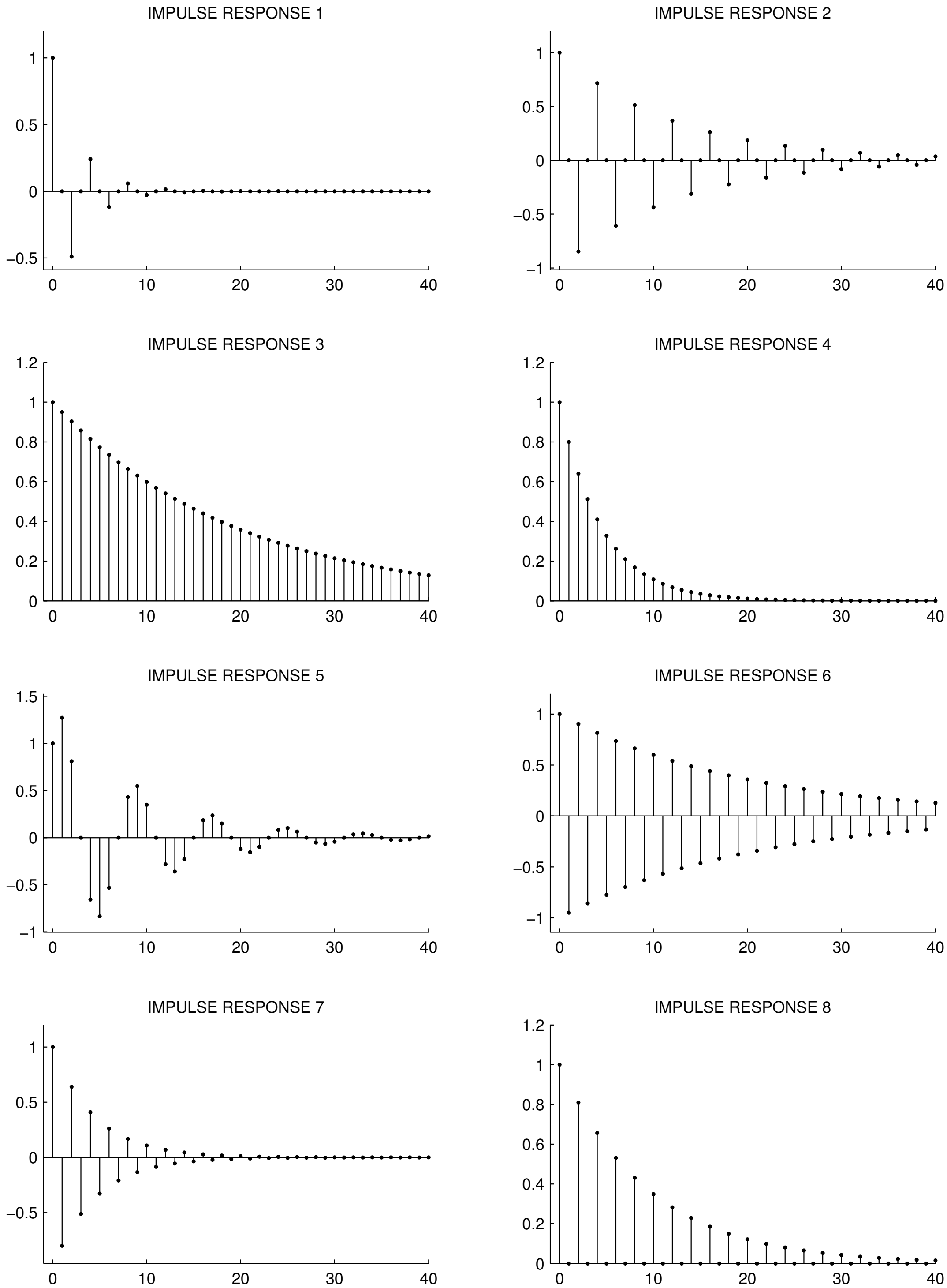


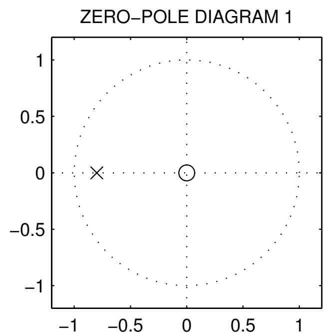

ZERO−POLE DIAGRAM 1
ZERO−POLE DIAGRAM 2
ZERO−POLE DIAGRAM 3
ZERO−POLE DIAGRAM 4
1
0.5
0
−0.5
−1
−1 −0.5 0 0.5 1

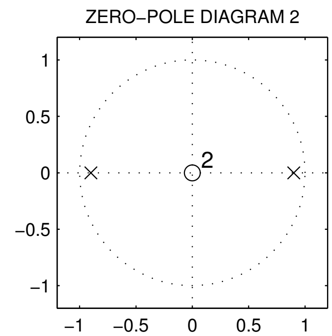

IMPULSE RESPONSE 1
IMPULSE RESPONSE 2
IMPULSE RESPONSE 3
IMPULSE RESPONSE 4
0.1
0.05
0
−0.05
−0.1
−0.15
0.3
0.2
0.1
−0.1
−0.2
−0.3
0.6
0.4
0 10 20 30 40

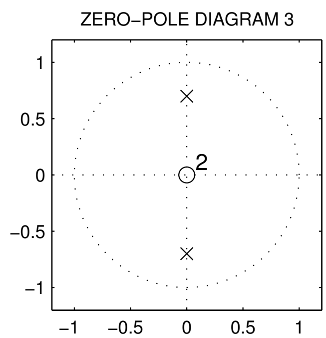
1.11.4 The impulse responses, pole-zero diagrams, and frequency responses of eight discrete-time LTI systems are illustrated below. But they are out of order. Match them to each other by filling out the table.

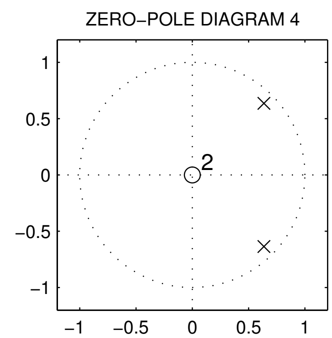
IMPULSE RESPONSE	POLE-ZERO DIAGRAM	FREQUENCY RESPONSE
1		
2		
3		
4		
5		
6		
7		
8		

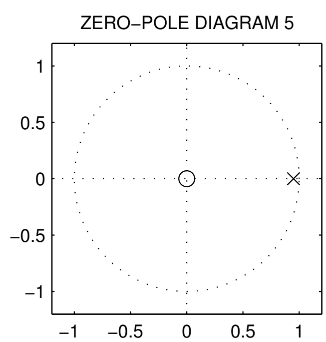

IMPULSE RESPONSE 1
IMPULSE RESPONSE 2
IMPULSE RESPONSE 3
IMPULSE RESPONSE 4
IMPULSE RESPONSE 5
IMPULSE RESPONSE 6
IMPULSE RESPONSE 7
IMPULSE RESPONSE 8

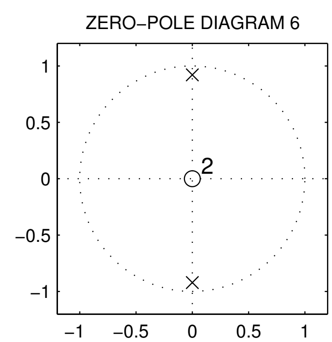

ZERO−POLE DIAGRAM 1

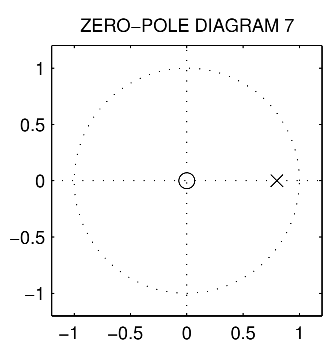

ZERO−POLE DIAGRAM 2
2

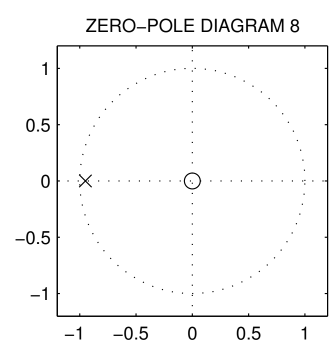

ZERO−POLE DIAGRAM 3
2

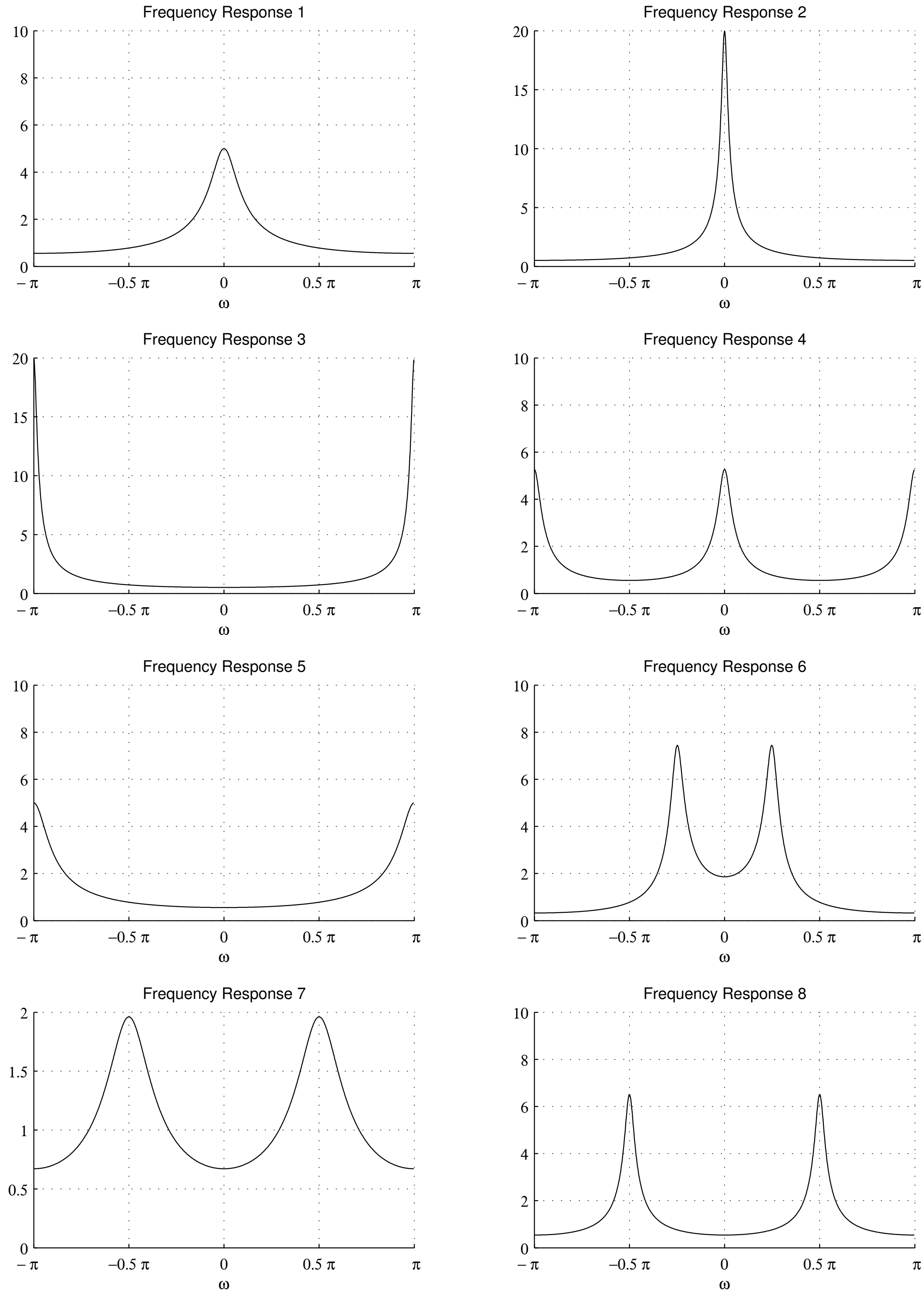

ZERO−POLE DIAGRAM 4
2


ZERO−POLE DIAGRAM 5


ZERO−POLE DIAGRAM 6
2


ZERO−POLE DIAGRAM 7


ZERO−POLE DIAGRAM 8



1.12 More Problems

1.12.1 The impulse response $h(n)$ of an LTI system is given by

$$h(n) = \left(\frac{1}{2}\right)^n u(n).$$

Find and sketch the output $y(n)$ when the input is given by

$$x(n) = u(n) - u(n-2).$$

Simplify your mathematical formula for $y(n)$ as far as you can. Show your work.

1.12.2 The impulse response $h(n)$ of an LTI system is given by

$$h(n) = \delta(n) + \delta(n-2).$$

Find and sketch the output $y(n)$ when the input is given by

$$x(n) = \sum_{k=-\infty}^{\infty} (-1)^k \, \delta(n - 2\,k).$$

Simplify your mathematical formula for $y(n)$ as far as you can. Show your work.

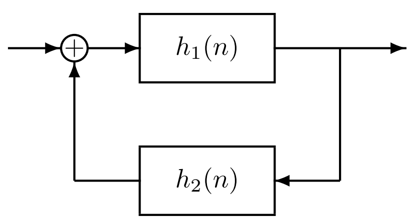
1.12.3 The impulse response $h(n)$ of an LTI system is given by

$$h(n) = u(n) - u(n-5).$$

(a) What is the transfer function $H(z)$ of this system?

(b) What difference equation implements this system?

1.12.4 **Feedback Loop** Consider the following connection of two discrete-time LTI systems.



You are told that

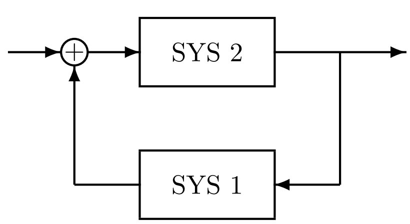
$$h_1(n) = \delta(n) + \delta(n-1), \qquad h_2(n) = \frac{1}{2}\,\delta(n-1).$$

(a) Find the impulse response $h(n)$ of the total system.

(b) Sketch $h_1(n)$, $h_2(n)$, and $h(n)$.

(c) Find the difference equation describing the total system.

1.12.5 **Feedback Loop** Consider the following interconnection of two causal discrete-time LTI systems



where SYS 1 is implemented using the difference equation

$$y(n) = 2\,x(n) - 0.5\,y(n-1)$$

and SYS 2 is implemented using the difference equation

$$y(n) = 2\,x(n) - 0.2\,y(n-1)$$

Find the impulse response of the total system, $h_{\text{tot}}(n)$.

Find the difference equation for the total system.

1.12.6 The signal $g(n)$ is given by

$$g(n) = \left(\frac{1}{2}\right)^n u(n-1)$$

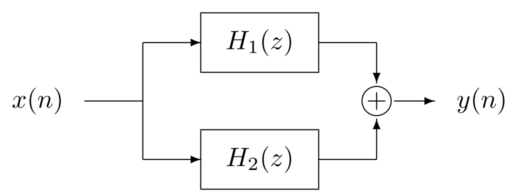
(a) Sketch $g(n)$.

(b) Find the Z-transform $G(z)$ of the signal $g(n)$ and its region of convergence.

1.12.7 Two discrete-time LTI systems have the following impulse responses

$$h_1(n) = \left(\frac{1}{2}\right)^n u(n), \qquad h_2(n) = \left(-\frac{1}{2}\right)^n u(n)$$

If the two systems are connected in parallel,



Find and make an accurate sketch of the impulse response of the total system.

2 Continuous-Time Signals and Systems

2.1 Signals

2.1.1 Make an accurate sketch of each continuous-time signal.

(a)

$$x(t) = u(t+1) - u(t), \qquad \frac{d}{dt}x(t), \qquad \int_{-\infty}^{t} x(\tau)\, d\tau$$

(b)

$$x(t) = e^{-t}\, u(t), \qquad \frac{d}{dt}x(t), \qquad \int_{-\infty}^{t} x(\tau)\, d\tau$$

Hint: use the product rule for $\frac{d}{dt}(f(t)\, g(t))$.

(c)

$$x(t) = \frac{1}{t}\, [\delta(t-1) + \delta(t+2)], \qquad \int_{-\infty}^{t} x(\tau)\, d\tau$$

(d)

$$x(t) = r(t) - 2\, r(t-1) + 2\, r(t-3) - r(t-4)$$

where $r(t) := t\, u(t)$ is the ramp function.

(e)

$$g(t) = x(3 - 2\, t), \quad \text{where } x(t) \text{ is defined as} \quad x(t) = 2^{-t}\, u(t-1).$$

2.1.2 Sketch the continuous-time signals $f(t)$ and $g(t)$ and the product signal $f(t) \cdot g(t)$.

(a)

$$f(t) = u(t+4) - u(t-4), \qquad g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 3\, k)$$

(b)

$$f(t) = \cos\left(\frac{\pi}{2}\, t\right), \qquad g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$$

Also write $f(t) \cdot g(t)$ in simple form.

(c)

$$f(t) = \sin\left(\frac{\pi}{2}\, t\right), \qquad g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2\, k)$$

(d)

$$f(t) = \sin\left(\frac{\pi}{2}\, t\right), \qquad g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k - 1)$$

Also write $f(t) \cdot g(t)$ in simple form.

(e)

$$f(t) = \left(\frac{1}{2}\right)^{|t|}, \qquad g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2\,k).$$

(f)

$$f(t) = \sum_{k=-\infty}^{\infty} (-1)^k\, \delta(t - 0.5\,k), \qquad g(t) = 2^{-|t|}$$

2.1.3 Given

$$f(t) = \delta(t) - 2\delta(t-1) + \delta(t-2), \qquad g(t) = t\,u(t)$$

Define $x(t) = f(t)\,g(t)$. Accurately sketch $x(t)$.

2.2 System Properties

2.2.1 A continuous-time system is described by the following rule

$$y(t) = \sin(\pi t)\,x(t) + \cos(\pi t)\,x(t-1)$$

where x is the input signal, and y is the output signal. Classify the system as:

(a) linear/nonlinear
(b) time-invariant/time-varying
(c) stable/unstable

2.2.2 A continuous-time system is described by the following rule

$$y(t) = \frac{x(t)}{x(t-1)}.$$

Classify the system as:

(a) memoryless/with memory
(b) causal/noncausal
(c) linear/nonlinear
(d) time-invariant/time-varying
(e) BIBO stable/unstable

2.2.3 You observe an unknown system and notice that

$$u(t) - u(t-1) \longrightarrow \boxed{\mathcal{S}} \longrightarrow e^{-t}\,u(t)$$

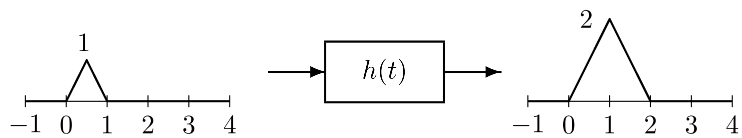
and that

$$u(t) - u(t-2) \longrightarrow \boxed{\mathcal{S}} \longrightarrow e^{-2t}\,u(t)$$

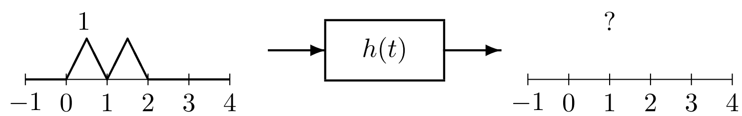
Which conclusion can you make?

(a) The system is LTI.
(b) The system is not LTI.
(c) There is not enough information to decide.

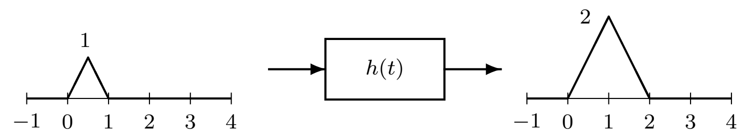
2.2.4 You observe an unknown LTI system and notice that



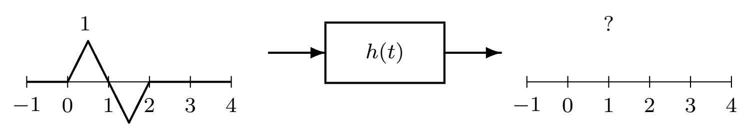
What is the output of the same LTI system when the input is as shown?



2.2.5 You observe an unknown continuous-time LTI system and notice that

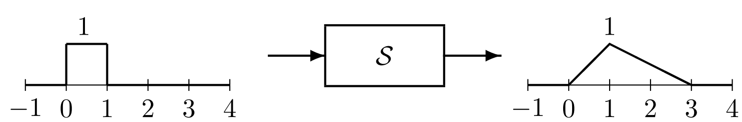


What is the output of the same LTI system when the input is as shown?

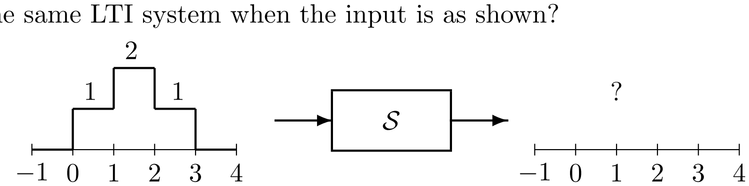


Use the LTI properties and be careful!

2.2.6 You observe an unknown LTI system and notice that

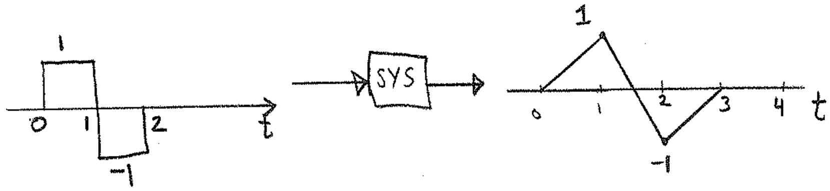


What is the output of the same LTI system when the input is as shown?

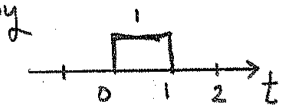


2.2.7 Predict the output

You observe of an LTI system that:



accurately sketch the output signal produced by the input:



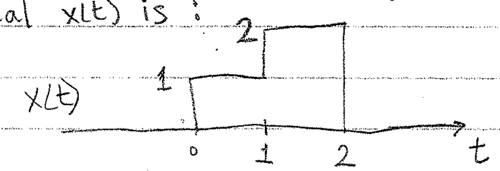
(hint - use LTI properties!)

2.2.8 Find the output.

③ A continuous-time system is desribed by the rule

$$x(t) \rightarrow \boxed{SYS} \rightarrow y(t) = \int_{-\infty}^{t} [x(\tau)]^2 \, d\tau$$

ⓐ sketch the output signal $y(t)$ when the input signal $x(t)$ is:



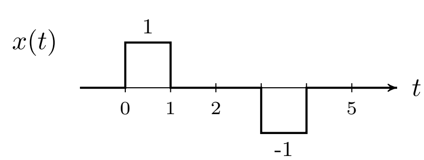
ⓑ is the system linear or non-linear?

ⓒ is " " time-invariant or time-varying?

2.2.9 (a) A continuous-time system is described by the equation,

$$y(t) = \frac{1}{2} \int_{t-2}^{t} (x(\tau))^2 \, d\tau$$

where x is the input signal, y the output signal. Find the output signal of the system when the input signal is



(b) Classify the system as:

i. linear/nonlinear

ii. time-invariant/time-varying

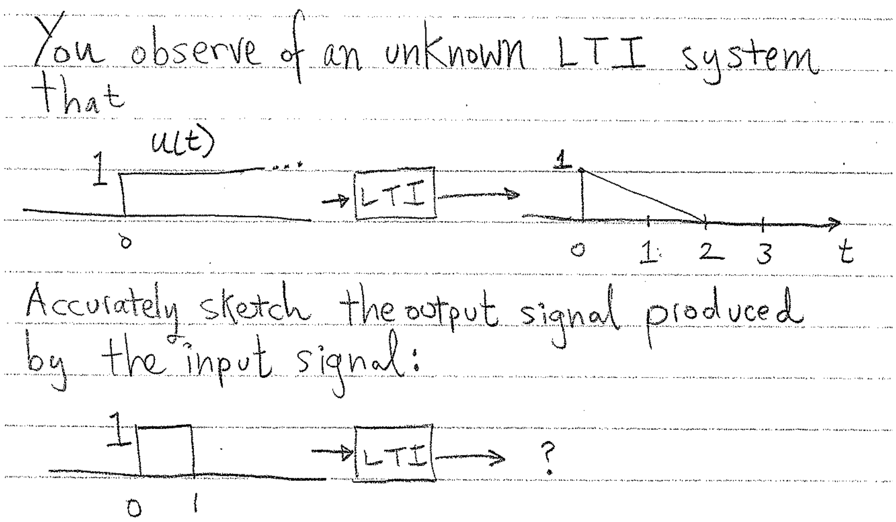
iii. stable/unstable
iv. causa/non-causal

2.2.10 You observe an unknown continuous-time LTI system and notice that

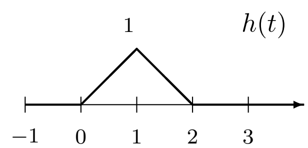
$$u(t) - u(t-1) \longrightarrow \boxed{\mathcal{S}} \longrightarrow r(t) - 2\,r(t-1) + r(t-2)$$

(a) Find and sketch the step response $s(t)$.
(b) Find and sketch the impulse response $h(t)$.
(c) Classify the system as BIBO stable/unstable.

2.2.11 Find the output.



2.2.12 The impulse response $h(t)$ of an LTI system is the triangular pulse shown.



Suppose the input $x(t)$ is the periodic impulse train

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - k\,T).$$

Sketch the output of the system $y(t)$ when

(a) $T = 3$
(b) $T = 2$
(c) $T = 1.5$

2.2.13 Consider an LTI system described by the rule

$$y(t) = x(t-5) + \frac{1}{2}\,x(t-7).$$

Find and accurately sketch the impulse response $h(t)$ of this system.

2.2.14 Consider the LTI system with impulse response

$$h(t) = t\,u(t).$$

(a) Find and sketch the output $y(t)$ when the input $x(t)$ is

$$x(t) = \delta(t) - 2\,\delta(t-1) + \delta(t-2).$$

(b) Classify the system as BIBO stable/unstable.

2.2.15 Consider the LTI system with impulse response

$$h(t) = \delta(t) - \frac{1}{4}\,\delta(t-2).$$

(a) Find and sketch the output $y(t)$ when the input $x(t)$ is

$$x(t) = 2^{-t}\,u(t).$$

(b) Classify the system as BIBO stable/unstable.

2.2.16 Consider the continuous-time LTI system with impulse response

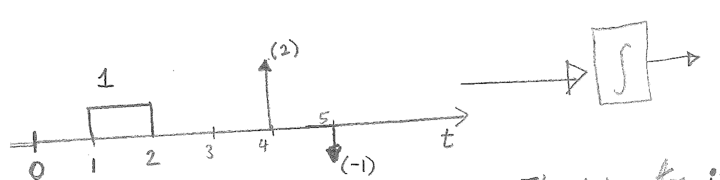
$$h(t) = \delta(t-1).$$

(a) Find and sketch the output $y(t)$ when the input $x(t)$ is the impulse train with period 2,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k).$$

(b) Classify the system as BIBO stable/unstable.

2.2.17 (a) The integrator:



? Sketch the output

The integrator is: $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

(b) Same (a), but system is defined as

$$y(t) = \int_{t-1}^{t} x(\tau)\,d\tau$$

2.2.18 A continuous-time LTI system is described by the equation,

$$y(t) = \frac{1}{2}\int_{t-2}^{t} x(\tau)\,d\tau$$

where x is the input signal, y the output signal.

(a) Write in words what this system does to the input signal.

(b) Accurately sketch the impulse response of the system.

(c) What is the dc gain of the system?

2.2.19 A continuous-time LTI system is described by the equation,

$$y(t) = \int_{t-1}^{t} x(\tau)\, d\tau + \frac{1}{2} \int_{t-3}^{t-1} x(\tau)\, d\tau$$

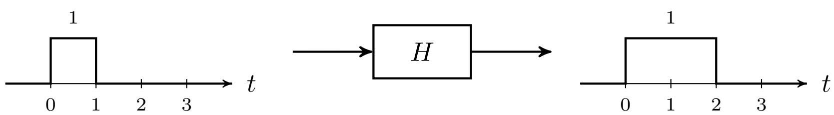
where x is the input signal, y the output signal.

(a) Accurately sketch the impulse response of the system.

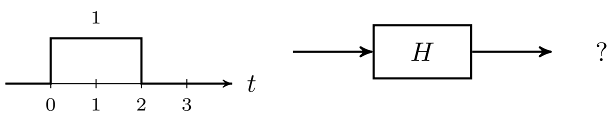
(b) Accurately sketch the step response of the system.

(c) What is the dc gain of the system?

2.2.20 You observe a continuous-time LTI system and notice that



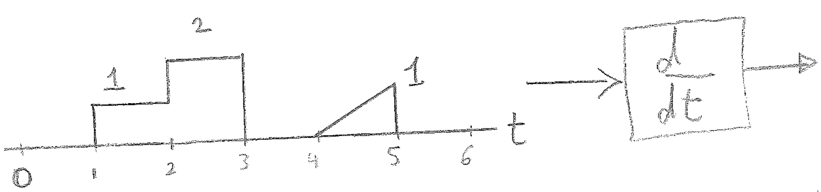
(a) Accurately sketch the output produced by the following input signal.



(b) Accurately sketch the step response of the system.

(c) Accurately sketch the impulse response of the system.

2.2.21 Differentiator.



sketch the output signal

2.2.22 A left-sided signal.

$x(t) = e^{t} u(-t)$. Sketch the output signal when $x(t)$ is the input to:

(a) integrator ($x(t) \to \boxed{\int} \to ?$)

(b) differentiator ($x(t) \to \boxed{\frac{d}{dt}} \to ?$)

2.2.23 A continuous-time LTI system is described by the equation,

$$y(t) = \frac{1}{5} \int_{t-4}^{t-2} x(\tau)\, d\tau$$

where x is the input signal, y the output signal.

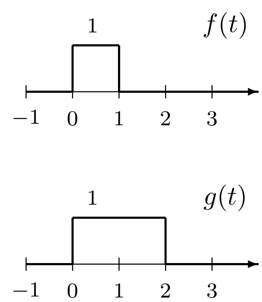
(a) Accurately sketch the impulse response of the system.
(b) Accurately sketch the step response of the system.
(c) What is the dc gain of the system?

2.3 Convolution

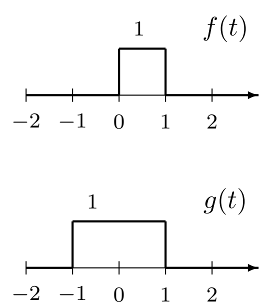
2.3.1 Find the convolution of the following signals

(a) $u(t) * u(t)$
(b) $[u(t) - u(t-1)] * u(t)$
(c) $[u(t) - u(t-1)] * [u(t) - u(t-1)]$
(d) $u(t) * \mathrm{e}^{-2t} u(t)$
(e) $\mathrm{e}^{-t} u(t) * \mathrm{e}^{-2t} u(t)$

2.3.2 Derive and sketch the convolution $v(t) = f(t) * g(t)$ where $f(t)$ and $g(t)$ are as shown.

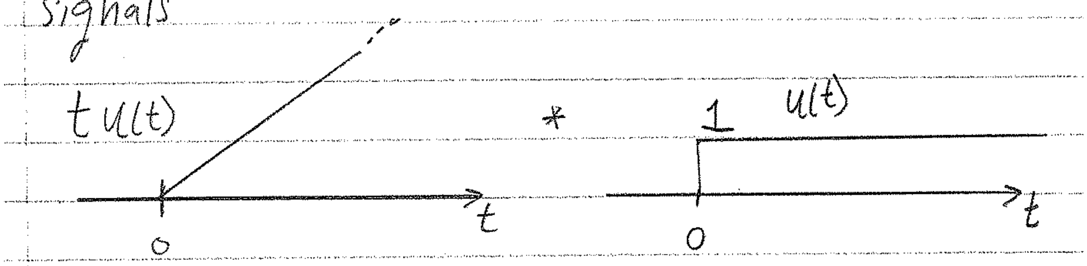


2.3.3 Derive and sketch the convolution $x(t) = f(t) * g(t)$ where $f(t)$ and $g(t)$ are as shown.

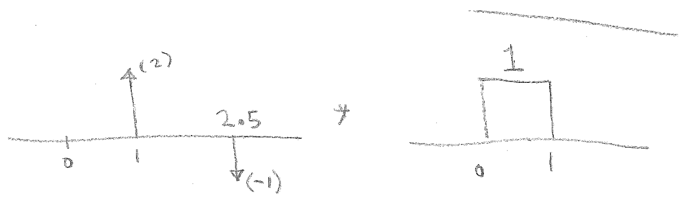


2.3.4 Convolution:

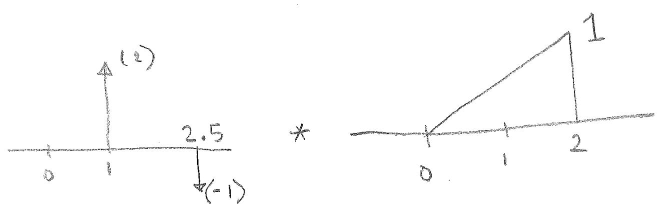
Find the convolution of the following two signals



2.3.5 (a) Convolution:



(b) Convolution:



2.3.6 Find and sketch the convolution $x(t) = f(t) * g(t)$ where

$$f(t) = u(t), \qquad g(t) = 3\,\mathrm{e}^{-2t}\,u(t)$$

2.3.7 Find and sketch the convolution $x(t) = f(t) * g(t)$ where

$$f(t) = \mathrm{e}^{t}\,u(-t), \qquad g(t) = \mathrm{e}^{-t}\,u(t)$$

2.3.8 Sketch the continuous-time signals $f(t)$, $g(t)$.
Find and sketch the convolution $y(t) = f(t) * g(t)$.

$$f(t) = e^{-t}\,u(t),$$

$$g(t) = e^{-t}\,u(t)$$

2.3.9 Using the convolution integral, find the convolution of the signal $f(t) = e^{-2t}u(t)$ with itself.

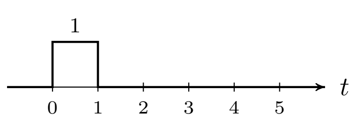
$$e^{-2t}u(t) \;*\; e^{-2t}u(t) = ?$$

2.3.10 Find and sketch the convolution of

$$g(t) = \mathrm{e}^{-2t}\,u(t)$$

and

$f(t)$

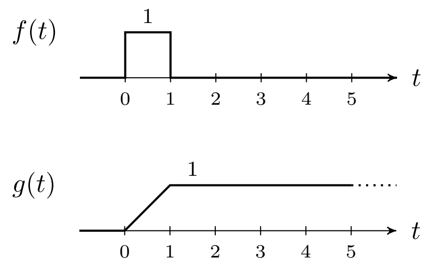


2.3.11 Sketch the continuous-time signals $f(t)$, $g(t)$.
Find and sketch the convolution $y(t) = f(t) * g(t)$.

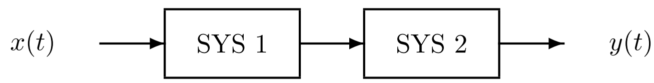
$$f(t) = u(t+2) - u(t-2)$$

$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 3\,k)$$

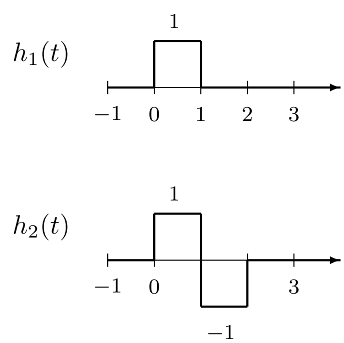
2.3.12 Sketch the convolution of the following two signals.



2.3.13 Consider the cascade connection of two continuous-time LTI systems

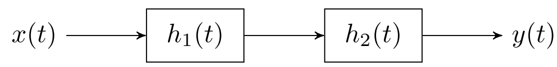


with the following impulse responses,

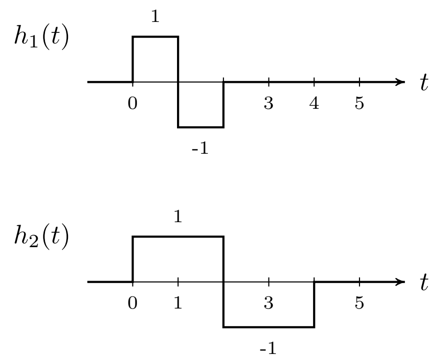


Accurately sketch the impulse response of the total system.

2.3.14 Two continuous-time LTI systems are connected in cascade.

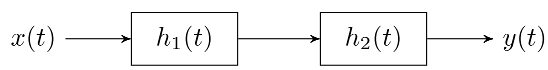


The impulse responses of the two systems are:

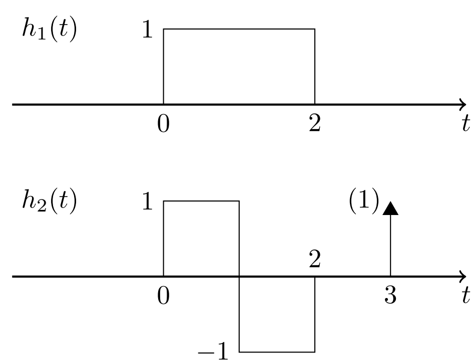


Sketch the impulse response of the total system.

2.3.15 Two continuous-time LTI systems are connected in cascade.

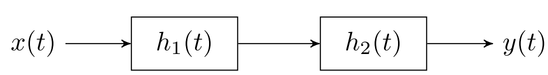


The impulse responses of the two systems are:

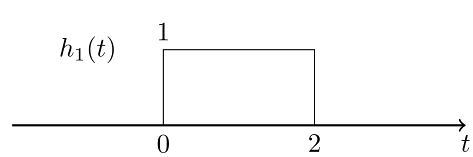


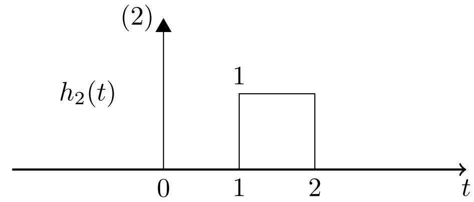
Sketch the impulse response of the total system. (The convolution of $h_1(t)$ and $h_2(t)$.)

2.3.16 Two continuous-time LTI systems are connected in cascade.



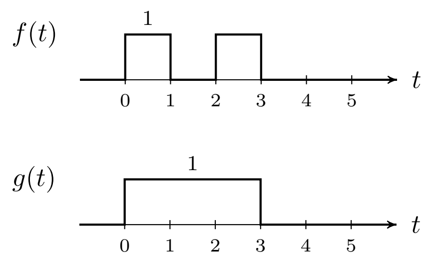
The impulse responses of the two systems are:





Sketch the impulse response of the total system.

2.3.17 Sketch the convolution of the following two signals.



2.3.18 Sketch the continuous-time signals $f(t)$, $g(t)$; find and sketch the convolution $f(t) * g(t)$.

(a)

$$f(t) = -u(t+1) + u(t)$$
$$g(t) = -u(t-1) + u(t-2).$$

(b)

$$f(t) = \delta(t+1) - \delta(t-2.5)$$

$$g(t) = 2\,\delta(t+1.5) - \delta(t-2)$$

(c)

$$f(t) = \delta(t) + \delta(t-1) + 2\,\delta(t-2)$$

$$g(t) = \delta(t-2) - \delta(t-3).$$

(d)

$$f(t) = \delta(t+1.2) - \delta(t-1)$$

$$g(t) = \delta(t+0.3) - \delta(t-1).$$

2.3.19 The impulse response of a continuous-time LTI system is given by $h(t)$. Find and sketch the output $y(t)$ when the input is given by $x(t)$. Also, classify each system as BIBO stable/unstable.

(a)

$$h(t) = 2\,e^{-3t}\,u(t).$$

$$x(t) = u(t-2) - u(t-3).$$

(b)

$$h(t) = \begin{cases} 1 - |t| & |t| \le 1 \\ 0 & |t| \ge 1 \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 4\,k).$$

(c)

$$h(t) = e^{-t}\, u(t)$$

$$x(t) = u(t) - u(t-2),$$

(d)

$$h(t) = \cos(\pi t)\, u(t).$$

$$x(t) = u(t) - u(t-3).$$

2.4 Laplace Transform

You may use MATLAB. The `residue` and `roots` commands should be useful for some of the following problems.

2.4.1 Given $h(t)$, find $H(s)$ and its region of convergence (ROC).

$$h(t) = 5\mathrm{e}^{-4t}u(t) + 2\mathrm{e}^{-3t}u(t)$$

2.4.2 Given $H(s)$, use partial fraction expansion to expand it (by hand). You may use the Matlab command `residue` to verify your result. Find the causal impulse response corresponding to $H(s)$.

(a)

$$H(s) = \frac{s+4}{s^2+5s+6}$$

(b)

$$H(s) = \frac{s-1}{2s^2+3s+1}$$

2.4.3 Let

$$f(t) = \mathrm{e}^{-t}\, u(t), \qquad g(t) = \mathrm{e}^{-2t}\, u(t)$$

Use the Laplace transform to find the convolution of these two signals, $y(t) = f(t) * g(t)$.

2.4.4 Let

$$f(t) = \mathrm{e}^{-t}\, u(t), \qquad g(t) = \mathrm{e}^{-t}\, u(t)$$

Use the Laplace transform to find and sketch the convolution of these two signals, $y(t) = f(t) * g(t)$. [The signals $f(t)$ and $g(t)$ are the same here.]

2.4.5 The impulse response of a continuous-time LTI system is given by

$$h(t) = \left(\frac{1}{2}\right)^t u(t).$$

Sketch $h(t)$ and find the transfer function $H(s)$ of the system.

2.4.6 Let $r(t)$ denote the ramp function, $r(t) = t\, u(t)$. If the signal $g(t)$ is defined as $g(t) = r(t-2)$, then what is the Laplace transform of $g(t)$?

2.5 Differential Equations

2.5.1 The impulse response $h(t)$ of a continuous-time LTI system is given by

$$h(t) = 3\,\mathrm{e}^{-2t}\,u(t) - \mathrm{e}^{-3t}\,u(t).$$

(a) Find the transfer function $H(s)$ of the system.

(b) Find the differential equation for this system.

(c) Classify the system as stable/unstable.

2.5.2 The differential equation of a causal continuous-time LTI system is given by

$$y''(t) + y'(t) - 2\,y(t) = x(t)$$

(a) Find the transfer function $H(s)$ of the system.

(b) Find the impulse response $h(t)$ of this system.

(c) Classify the system as stable/unstable.

2.5.3 The impulse response of a continuous-time LTI system is given by

$$h(t) = \delta(t) + 2\,\mathrm{e}^{-t}\,u(t) - \mathrm{e}^{-2t}\,u(t)$$

Find the differential equation that describes the system.

2.5.4 A causal continuous-time LTI system is described by the differential equation

$$2\,y'(t) + y(t) = 3\,x(t).$$

(a) Find the output signal when the input signal is

$$x(t) = 5\,\mathrm{e}^{-3\,t}\,u(t).$$

Show your work.

(b) Is the system stable or unstable?

(c) Sketch the pole-zero diagram of this system.

2.5.5 A causal continuous-time LTI system is described by the differential equation

$$y''(t) + 4\,y'(t) + 3\,y(t) = 2\,x'(t) + 3\,x(t)$$

(a) Find the output signal $y(t)$ when the input signal $x(t)$ is

$$x(t) = 3\,\mathrm{e}^{-2t}\,u(t).$$

(b) What is the *steady state* output when the input signal is

$$x(t) = 3\,u(t).$$

(The steady state output is the output signal value after the transients have died out.)

2.5.6 Consider a causal continuous-time LTI system described by the differential equation

$$y''(t) + 3\,y'(t) + 2\,y(t) = 2\,x'(t)$$

(a) Find the transfer function $H(s)$, its ROC, and its poles.

(b) Find the impulse response $h(t)$.

(c) Classify the system as stable/unstable.

(d) Find the output of the system when the input signal is

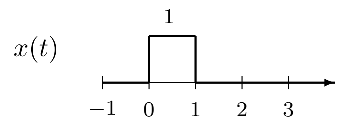
$$x(t) = 2\,u(t).$$

2.5.7 A causal continuous-time LTI system is described by the differential equation

$$y''(t) + 5\,y'(t) + 4\,y(t) = 3\,x'(t) + 6\,x(t)$$

(a) Find the impulse response $h(t)$.

(b) Find the output signal $y(t)$ when the input signal $x(t)$ is as shown:



2.5.8 Differential equation:

7) An causal LTI system is described by the diff. eq.

$$y''(t) + 5y'(t) + 4y(t) = 2x'(t) + x(t).$$

a) Find the impulse response, $h(t)$.

b) Sketch the pole/zero diagram.

2.5.9 Impulse response:

) The impulse response of an LTI system is

$$h(t) = 2u(t) + 3e^{-2t}u(t)$$

a) Find the differential equation for the system.

b) Sketch the pole/zero diagram.

c) classify the system as stable/unstable.

2.5.10 Consider a continuous-time LTI system with the impulse response

$$h(t) = \mathrm{e}^{-3t}\,u(t) + 2\,\mathrm{e}^{-t}\,u(t)$$

(a) Find the differential equation that describes the system.

(b) Sketch the pole-zero diagram.

(c) Find the output signal $y(t)$ produced by the input signal

$$x(t) = \mathrm{e}^{-2t}\,u(t).$$

You may leave the residues as unspecified constants.

2.5.11 A causal LTI system is described by the differential equation

$$y''(t) + 7\,y'(t) + 12\,y(t) = 3\,x'(t) + 2\,x(t).$$

(a) Find the transfer function $H(s)$ and the ROC of $H(s)$.

(b) List the poles of $H(s)$.

(c) Find the impulse response $h(t)$.

(d) Classify the system as stable/unstable.

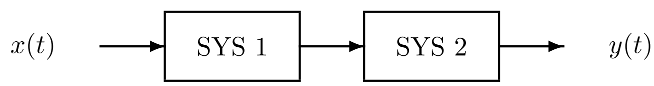
(e) Fnd the output $y(t)$ when the input is

$$x(t) = e^{-t}\,u(t) + e^{-2t}\,u(t).$$

2.5.12 Given the two LTI systems described the the differential equations:

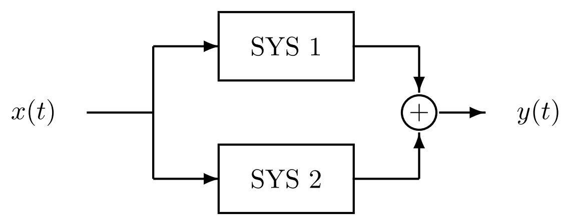
$$T_1: \quad y''(t) + 3\,y'(t) + 7\,y(t) = 2\,x'(t) + x(t)$$
$$T_2: \quad y''(t) + y'(t) + 4\,y(t) = x'(t) - 3\,x(t)$$

(a) Let T be the cascade of T_1 and T_2, $T[x(t)] = T_2[T_1[x(t)]]$, as shown in the diagram.



What are $H_1(s)$, $H_2(s)$, $H_{tot}(s)$, the transfer functions of T_1, T_2 and T? What is the differential equation describing the total system T?

(b) Let T be the sum of T_1 and T_2, $T[x(t)] = T_2[x(t)] + T_1[x(t)]$, as shown in the diagram.



What is $H_{tot}(s)$, the transfer function of the total system T? What is the differential equation describing the total system T?

2.5.13 Given a causal LTI system described by

$$y'(t) + \frac{1}{3}\,y(t) = 2\,x(t)$$

find $H(s)$. Given the input $x(t) = e^{-2t}u(t)$, find the output $y(t)$ without explicitly finding $h(t)$. (Use $Y(s) = H(s)X(s)$, and find $y(t)$ from $Y(s)$.)

2.5.14 The impulse response of an LTI continuous-time system is given by

$$h(t) = 3\,e^{-t}\,u(t) + 2\,e^{-2\,t}\,u(t) + e^{-t}\,u(t)$$

(a) Find the transfer function of the system.

(b) Find the differential equation with which the system can be implemented.

(c) List the poles of the system.

(d) What is the dc gain of the system?

(e) Sketch the output signal produced by input signal, $x(t) = 1$.

(f) Find the steady-state output produced by input signal, $x(t) = u(t)$.

2.6 Complex Poles

2.6.1 Consider a causal continuous-time LTI system described by the differential equation

$$y''(t) + y(t) = x(t).$$

(a) Find the transfer function $H(s)$, its ROC, and its poles.

(b) Find the impulse response $h(t)$.

(c) Classify the system as stable/unstable.

(d) Find the step response of the system.

2.6.2 Given the impulse response of a continuous-time LTI system, find the transfer function $H(s)$, the ROC of $H(s)$, and the poles of the system. Also find the differential equation describing each system.

(a) $h(t) = \sin(3\,t)\,u(t)$

(b) $h(t) = \mathrm{e}^{-t/2}\,\sin(3\,t)\,u(t)$

(c) $h(t) = \mathrm{e}^{-t}\,u(t) + \mathrm{e}^{-t/2}\,\cos(3\,t)\,u(t)$

2.6.3 A causal continuous-time LTI system is described by the equation

$$y''(t) + 2\,y'(t) + 5\,y(t) = x(t)$$

where x is the input signal, and y is the output signal.

(a) Find the impulse response of the system.

(b) Accurately sketch the pole-zero diagram.

(c) What is the dc gain of the system?

(d) Classify the system as either stable or unstable.

(e) Write down the form of the step response of the system, as far as it can be determined without actually calculating the resides. (You do not need to complete the partial fraction expansion).

2.6.4 Given a *causal* LTI system described by the differential equation find $H(s)$, the ROC of $H(s)$, and the impulse response $h(t)$ of the system. Classify the system as stable/unstable. List the poles of $H(s)$. You should the Matlab `residue` command for this problem.

(a) $y''' + 3\,y'' + 2\,y' = x'' + 6\,x' + 6\,x$

(b) $y''' + 8\,y'' + 46\,y' + 68y = 10\,x'' + 53\,x' + 144\,x$

2.6.5 It is observed of some continuous-time LTI system that the input signal

$$x(t) = \mathrm{e}^{-2\,t}\,u(t)$$

produces the output signal

$$y(t) = 0.5\,\mathrm{e}^{-2\,t}\,u(t) + 2\,\mathrm{e}^{-3t}\,\cos(2\pi t)\,u(t).$$

What can be concluded about the pole positions of the LTI system?

2.6.6 A causal continuous-time LTI system is described by the equation

$$y''(t) + 4\,y'(t) + 5\,y(t) = x'(t) + 2x(t)$$

where x is the input signal, and y is the output signal.

(a) Find the impulse response of the system.

(b) Accurately sketch the pole-zero diagram.

(c) What is the dc gain of the system?

(d) Classify the system as either stable or unstable.

2.6.7 Suppose the impulse response of an LTI system has the form

$$h(t) = B\,\mathrm{e}^{-3t}\,u(t).$$

Suppose a signal $x(t)$ with the form

$$x(t) = A\,\cos(10\,\pi\,t)\,u(t)$$

is applied to the system. Which of the following signal forms can the output take? (Chose all that apply.)

(a) $y(t) = C\,\mathrm{e}^{-3t}\cos(10\,\pi\,t + \theta)\,u(t)$

(b) $y(t) = C\cos(10\,\pi\,t + \theta)\,u(t) + D\,\mathrm{e}^{-3t}\,u(t)$

(c) $y(t) = C\,\mathrm{e}^{-3t}\cos(10\,\pi\,t + \theta_1)\,u(t) + D\cos(10\,\pi\,t + \theta_2)\,u(t) + E\,\mathrm{e}^{-3t}\,u(t)$

2.6.8 It is observed of some continuous-time LTI system that the input signal

$$x(t) = 3\,u(t)$$

produces the output signal

$$y(t) = 4\,u(t) + 2\,\cos(2\pi t)\,u(t).$$

(a) What can be immediately concluded about the pole positions of the LTI system?

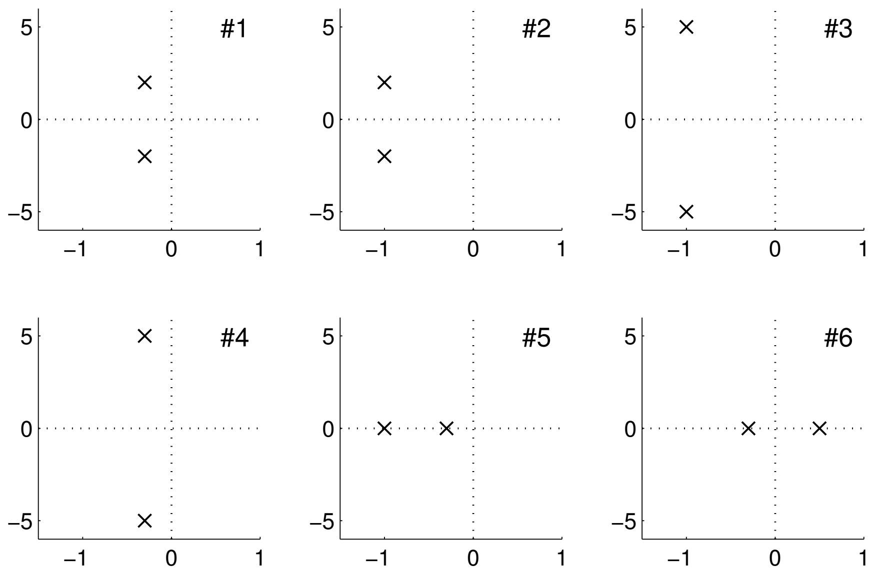
(b) What is the dc gain of the system?

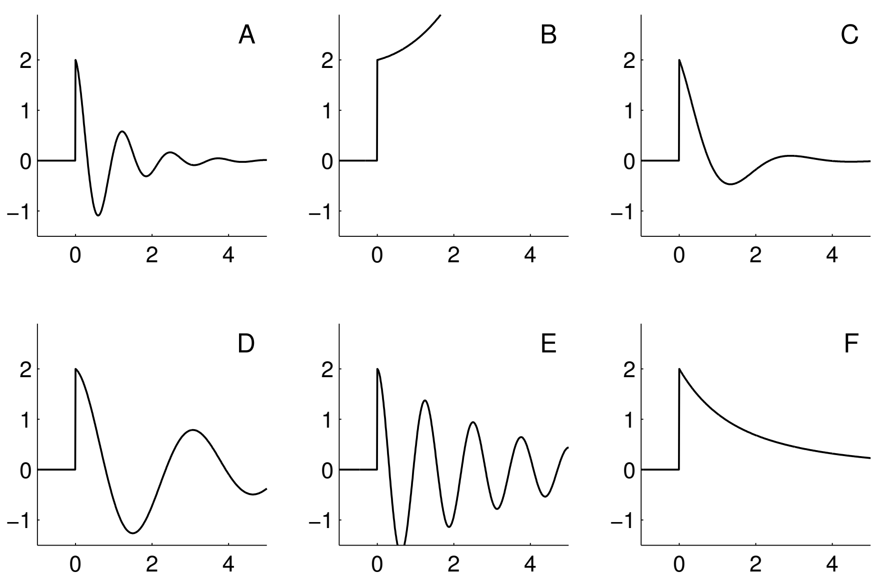
(c) Can you make any conclusion about the stability of the system?

(d) Find the impulse response $h(t)$ of the system.

2.6.9 The following diagrams indicate the pole locations of six continuous-time LTI systems. Match each with the corresponding impulse response with out actually computing the Laplace transform.

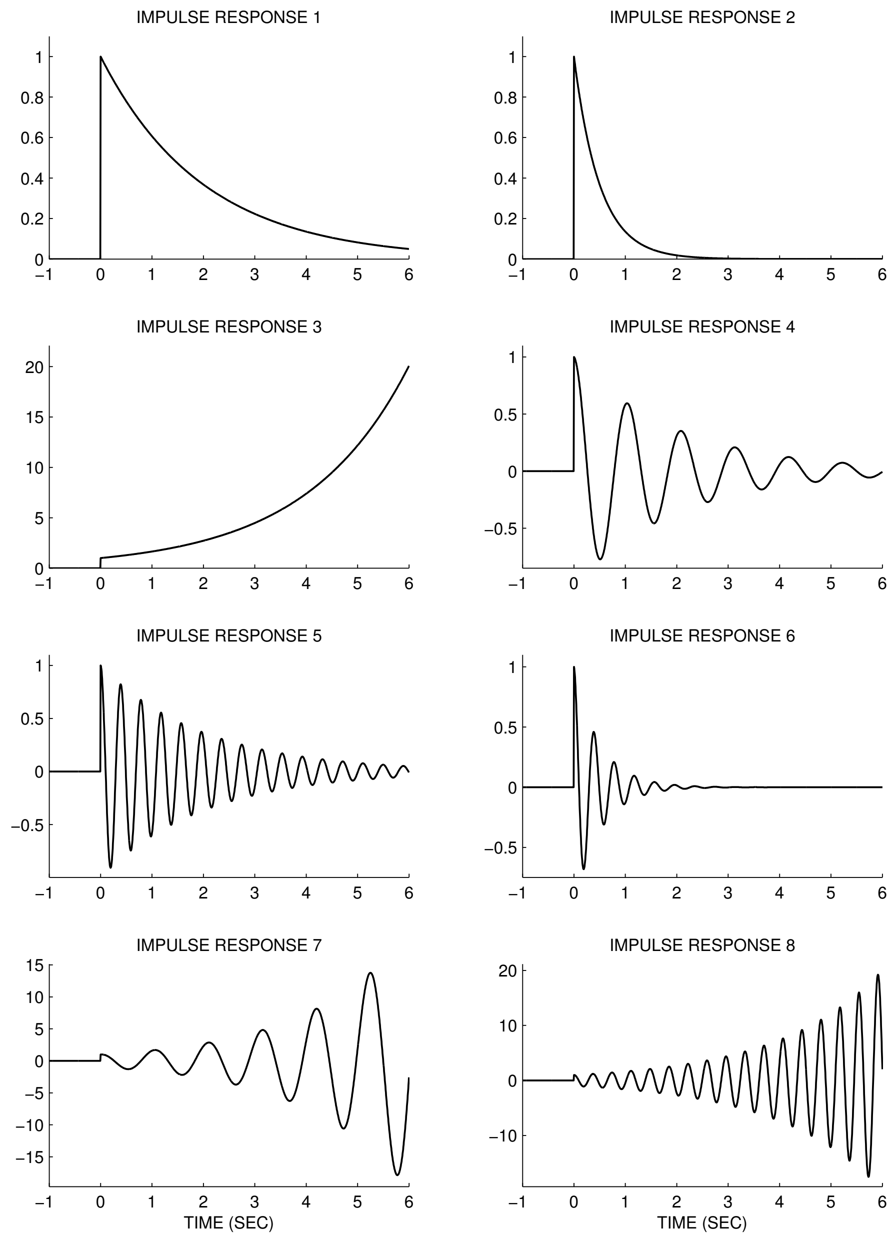
POLE-ZERO DIAGRAM	IMPULSE RESPONSE
1	
2	
3	
4	
5	
6	

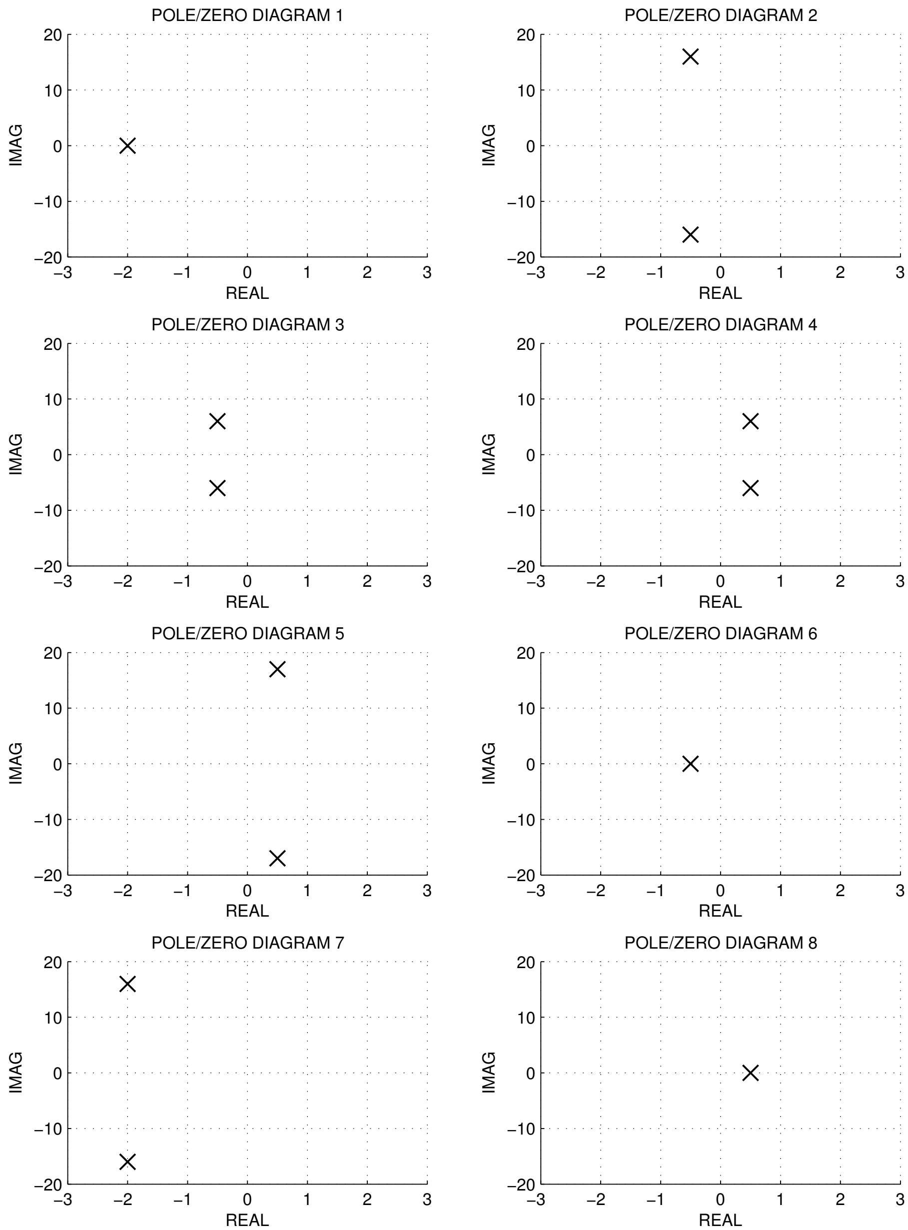




2.6.10 The impulse responses of eight causal continuous-time systems are illustrated below, along with the pole/zero diagram of each system. But they are out of order. Match the figures with each other by completing the table.

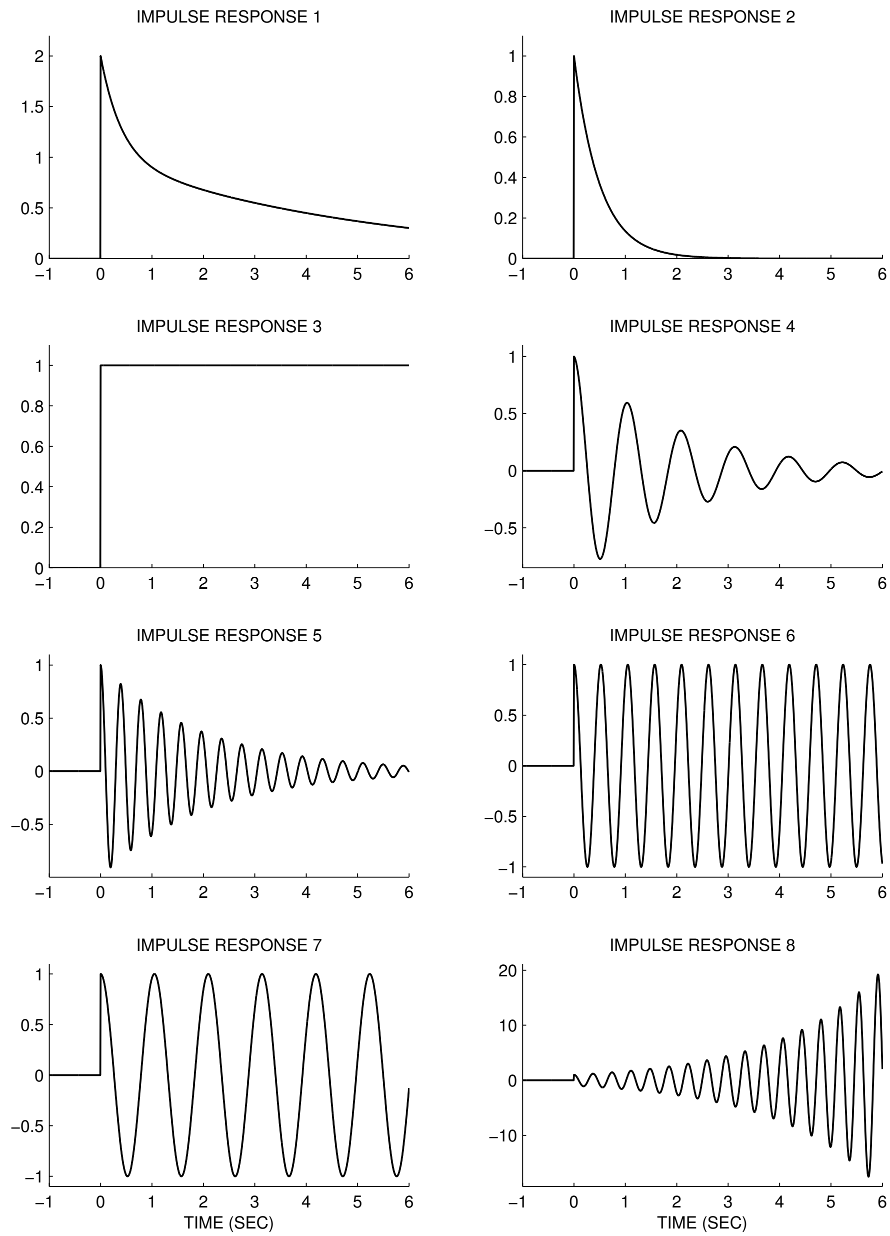
IMPULSE RESPONSE	POLE-ZERO DIAGRAM
1	
2	
⋮	
8	



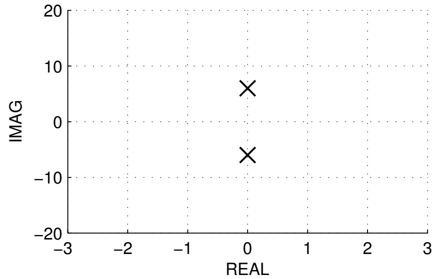

POLE/ZERO DIAGRAM 1
POLE/ZERO DIAGRAM 2
POLE/ZERO DIAGRAM 3
POLE/ZERO DIAGRAM 4
POLE/ZERO DIAGRAM 5
POLE/ZERO DIAGRAM 6
POLE/ZERO DIAGRAM 7
POLE/ZERO DIAGRAM 8
IMAG
20
10
0
−10
−20
REAL
−3
−2
−1
0
1
2
3

2.6.11 The first six seconds of the impulse responses of eight causal continuous-time systems are illustrated below, along with the pole/zero diagram of each system. But they are out of order. Match the figures with each other by completing the table.

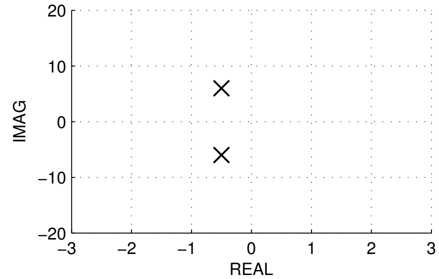
IMPULSE RESPONSE	POLE-ZERO DIAGRAM
1	
2	
⋮	
8	



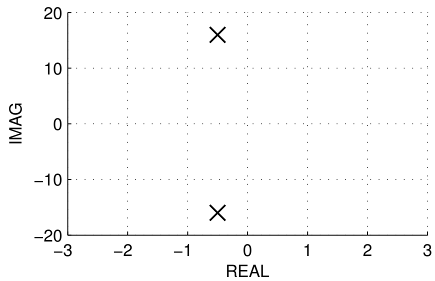
POLE/ZERO DIAGRAM 1



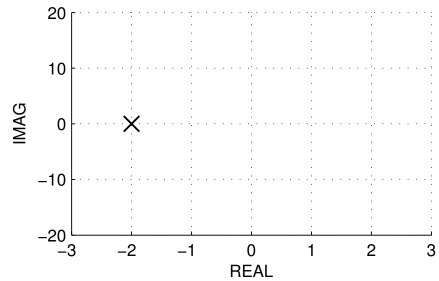
POLE/ZERO DIAGRAM 2



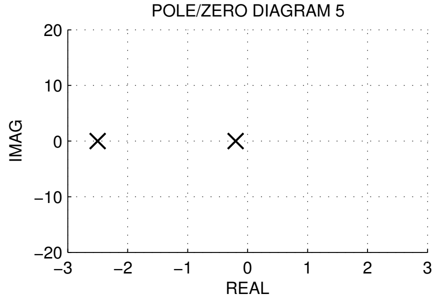
POLE/ZERO DIAGRAM 3



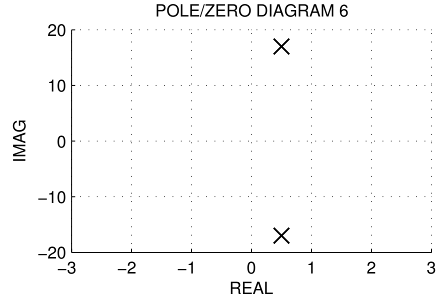
POLE/ZERO DIAGRAM 4



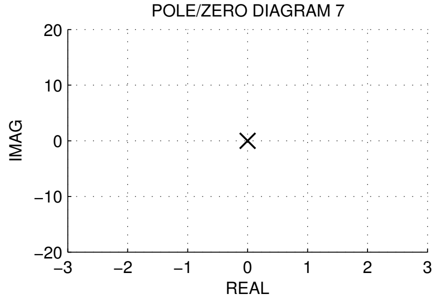
POLE/ZERO DIAGRAM 5



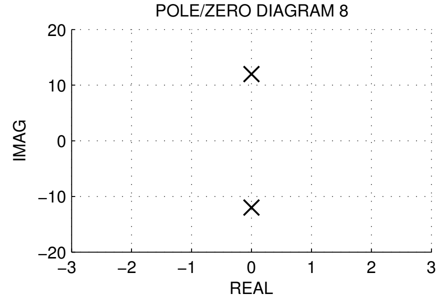
POLE/ZERO DIAGRAM 6



POLE/ZERO DIAGRAM 7



POLE/ZERO DIAGRAM 8



2.6.12 Given the impulse response

The impulse response on an LTI system is

$$h(t) = 2e^{-t}u(t) - 2e^{-t}\cos(2\pi t)u(t)$$

a) list the poles of the system.

b) find the differential equation describing the system.

c) what is the dc-gain of the system?

d) sketch the output signal produced by the input signal $x(t) = 2$.

2.6.13 A causal continuous-time LTI system is described by the equation

$$y''(t) + 2\,y'(t) + (1 + \pi^2)\,y(t) = \pi\,x(t)$$

where x is the input signal, and y is the output signal.

(a) Find the impulse response of the system.

(b) Accurately sketch the pole-zero diagram.

(c) Find the form of the step-response as far as you can without completing partial fraction expansion or integration.

2.6.14 A causal continuous-time LTI system is described by the equation

$$y''(t) + 4\,y(t) = 2\,x(t)$$

where x is the input signal, and y is the output signal.

(a) Find the impulse response of the system.

(b) Accurately sketch the pole-zero diagram.

(c) Classify the system as either stable or unstable.

2.6.15 Given the impulse response:

7 The impulse response of a C-T LTI system is

$$h(t) = 3\,\delta(t) + 2e^{-t/4}\cos(2\pi t)\,u(t)$$

a list the poles of the system.

b find the differential equation for the system.

c what is the dc gain of the system?

d sketch the pole/zero diagram of the system.

2.7 Frequency Response

2.7.1 The frequency response of a continuous-time LTI system is given by

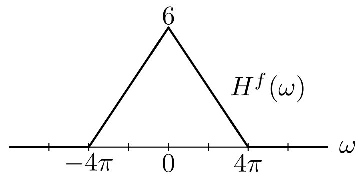
$$H^f(\omega) = \begin{cases} 1 & \text{for } |\omega| < 4\pi \\ 0 & \text{for } |\omega| \geq 4\pi. \end{cases}$$

(a) Sketch the frequency response.

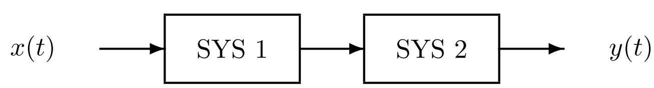
(b) Find the output $y(t)$ of the system when the input is

$$x(t) = 3\cos(2\pi t) + 6\sin(5\pi t).$$

2.7.2 The signal $x(t)$ in the previous problem is filtered with a continuous-time LTI system having the following frequency response. Find the output $y(t)$.



2.7.3 Consider the cascade combination of two continuous-time LTI systems.



The frequency response of SYS 1 is

$$H_1^f(\omega) = \begin{cases} 1 & \text{for } |\omega| < 6\pi \\ 0 & \text{for } |\omega| \geq 6\pi. \end{cases}$$

The frequency response of SYS 2 is

$$H_2^f(\omega) = \begin{cases} 0 & \text{for } |\omega| < 4\pi \\ 1 & \text{for } |\omega| \geq 4\pi. \end{cases}$$

(a) Sketch the frequency responses of each of the two systems.

(b) If the input signal is

$$x(t) = 2\cos(3\pi t) - 3\sin(5\pi t) + 4\cos(7\pi t)$$

what is the output signal $y(t)$?

(c) What is the frequency response of the total system?

2.7.4 The frequency response $H^f(\omega)$ of a continuous-time LTI system is given by

$$H^f(\omega) = \frac{1}{j\omega}.$$

(a) Find the output $y(t)$ when the input is given by

$$x(t) = \cos(2t) + \sin(4t).$$

(b) Sketch the magnitude of the frequency response, $|H^f(\omega)|$, and the phase of the frequency response, $\angle H^f(\omega)$.

(c) Classify the system as stable/unstable and give a brief explanation for your answer.

2.7.5 The impulse response of a continuous-time LTI system is given by

$$h(t) = \delta(t) - e^{-t}\, u(t).$$

(a) What is the frequency response $H^f(\omega)$ of this system?
(b) Find and sketch $|H^f(\omega)|$.
(c) Is this a lowpass, bandpass, or highpass filter, or none of those?

2.7.6 The impulse response of a continuous-time LTI system is given by

$$h(t) = \delta(t-2).$$

(This is a delay of 2.)

(a) What is the frequency response $H^f(\omega)$ of this system?
(b) Find and sketch the frequency response magnitude, $|H^f(\omega)|$.
(c) Find and sketch the frequency response phase, $\angle H^f(\omega)$.
(d) Is this a lowpass, bandpass, or highpass filter, or none of those?

2.7.7 When the continuous-time signal $x(t)$

$$x(t) = \sin(10\pi\, t) + \cos(5\pi\, t)$$

is applied as the input to an unknown continuous-time system, the observed output signal $y(t)$ is

$$y(t) = 3\, \sin(10\pi\, t) + 2\, \cos(7\pi\, t).$$

Which of the following statements is true? Given an explanation for your choice.

(a) The system is LTI.
(b) The system is not LTI.
(c) There is not enough information to decide.

2.7.8 The continuous-time signal

$$x(t) = \sin(10\pi\, t)\, u(t) + \cos(5\pi\, t)\, u(t)$$

is sent through a continuous-time LTI system with the frequency response

$$H^f(\omega) = 3 - \mathrm{e}^{-\mathrm{j}2.3\,\omega}.$$

What is the output signal $y(t)$?

2.7.9 The frequency response of a continuous-time LTI system is given by,

$$H^f(\omega) = \begin{cases} \dfrac{|\omega|}{2\,\pi}, & |\omega| \le 2\,\pi \\ 1, & |\omega| > 2\,\pi \end{cases}$$

(a) Accurately sketch the frequency response.
(b) Find the output signal produced by the input signal

$$x(t) = 1 + 2\, \cos(\pi\, t) + 4\, \cos(3\,\pi\, t).$$

2.7.10 The frequency response of a continuous-time LTI system is given by,

$$|H^f(\omega)| = \begin{cases} 1, & |\omega| \le 2\,\pi \\ 0, & |\omega| > 2\,\pi \end{cases}$$

$$\angle H^f(\omega) = -2\,\omega$$

(a) Accurately sketch the frequency response magnitude $|H^f(\omega)|$ and phase $\angle H^f(\omega)$.

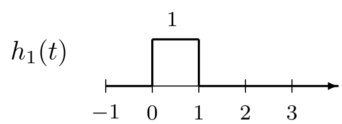
(b) Find the output signal produced by the input signal

$$x(t) = 1 + 2\,\cos(\pi\,t) + 4\,\cos(3\,\pi\,t).$$

2.7.11 Consider the cascade connection of two copies of the same continuous-time LTI system:

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_1(t)} \longrightarrow y(t)$$

where $h_1(t)$ is as shown:



(a) Find the frequency response $H_1^f(\omega)$.

(b) Find the frequency response of the total system $H_{tot}^f(\omega)$.

2.7.12 Suppose H is an ideal lowpass filter with cut-off frequency ω_o. Suppose H is connected in series with another copy of itself,

$$x(t) \quad \longrightarrow \boxed{H} \longrightarrow \boxed{H} \longrightarrow \quad y(t)$$

Find and sketch the frequency response of the total system.

2.7.13 Two continuous-time LTI systems are connected in cascade.

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

The impulse responses of the two systems are:

$$h_1(t) = 10\ \text{sinc}\,(10\,t)$$

$$h_2(t) = \delta(t) - 5\ \text{sinc}\,(5\,t)$$

(a) Accurately sketch the frequency response of the total system.

(b) Find the impulse response $h(t)$ of the total system.

2.7.14 Consider the continuous-time LTI system that delays its input by 2.5 seconds,

$$y(t) = x(t - 2.5).$$

(a) Accurately sketch the impulse response $h(t)$.

(b) Find the frequency response $H^f(\omega)$.

(c) Accurately sketch the frequency response magnitude $|H^f(\omega)|$.

(d) Accurately sketch the frequency response phase $\angle H^f(\omega)$.

(e) Find the output signal produced by the input signal

$$x(t) = 1 + 2\,\cos(\pi\,t) + 4\,\cos(3\,\pi\,t).$$

2.7.15 An echo can be modeled with a causal LTI system described by the equation

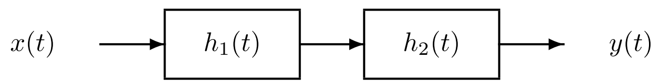
$$y(t) = x(t) - \frac{1}{2}\, x(t-10).$$

(a) Find the impulse response $h(t)$.

(b) Classify the system as stable/unstable.

(c) Find the frequency response $H^f(\omega)$ and sketch $|H^f(\omega)|^2$.

2.7.16 Two continuous-time LTI systems are connected in cascade.



The impulse responses are given by

$$h_1(t) = \delta(t - 0.5)$$

$$h_2(t) = e^{-2t}\, u(t)$$

(a) Find the impulse response $h(t)$ of the total system.

(b) Find the frequency response $H^f(\omega)$ of the total system.

(c) Find the *steady-state* output signal $y(t)$ when the input is

$$x(t) = \cos(2\,\pi\, t)\, u(t).$$

2.7.17 Consider a continuous-time LTI system with the frequency response:

$$H^f(\omega) = \begin{cases} \mathrm{j}\,\omega, & |\omega| \le 4\pi \\ 0, & |\omega| > 4\pi \end{cases}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$.

(c) Find the output signal produced by the input signal

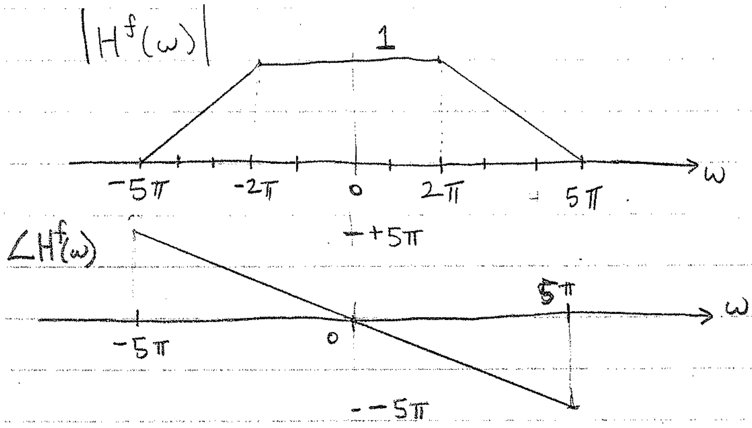
$$x(t) = 2 + \cos(\pi t) + 0.5\,\sin(2\pi t) + 3\,\cos(6\pi t).$$

Simplify your answer so that it does not contain j. Show your work.

(d) Based on (c) explain why this system may be called a low-pass differentiator.

2.7.18 Find the output:

The frequency response of an LTI system is



Find the output signal produced by the input signal

$$x(t) = 3 + \sin(\pi t) + \cos(4\pi t) + \sin(5\pi t).$$

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = ?$$

2.7.19 Sketch the frequency response:

D The freq. response of an LTI system is

$$H^f(\omega) = \begin{cases} 0 & |\omega| < 2\pi \\ j\omega & 2\pi \le |\omega| \le 4\pi \\ 0 & |\omega| \geq 4\pi \end{cases}$$

$\boxed{\text{ⓐ Sketch } |H^f(\omega)| \text{ and } \angle H^f(\omega).}$

ⓑ Find the output signal $y(t)$ produced by the input signal

$$x(t) = 3 + 2\sin(\pi t) + 3\cos(3\pi t) + \cos(5\pi t).$$

2.7.20 Given the frequency response

The freq resp. of an LTI sys is

$$H^f(\omega) = \begin{cases} j\omega e^{-j\omega} & |\omega| \leq 3\pi \\ 0 & |\omega| > 3\pi \end{cases}$$

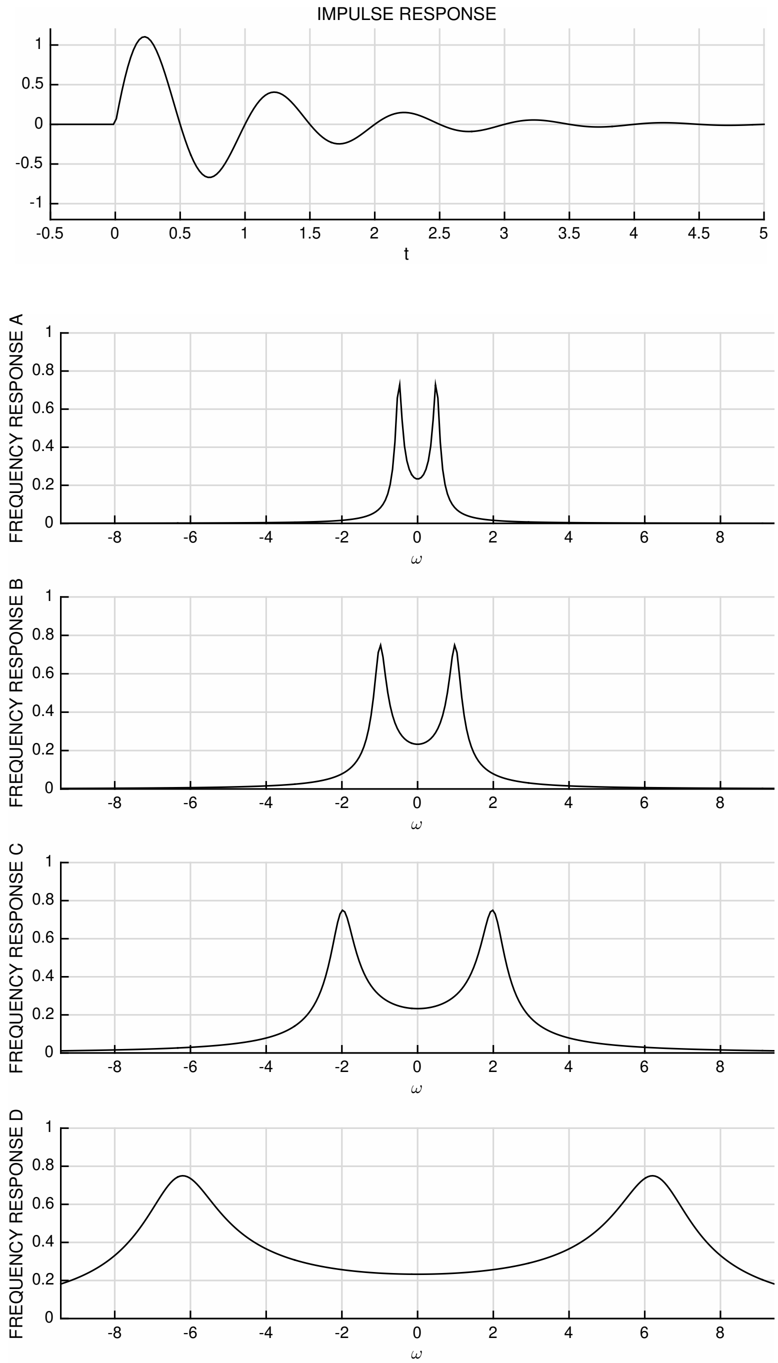
a) sketch $|H^f(\omega)|$

b) sketch $\angle H^f(\omega)$

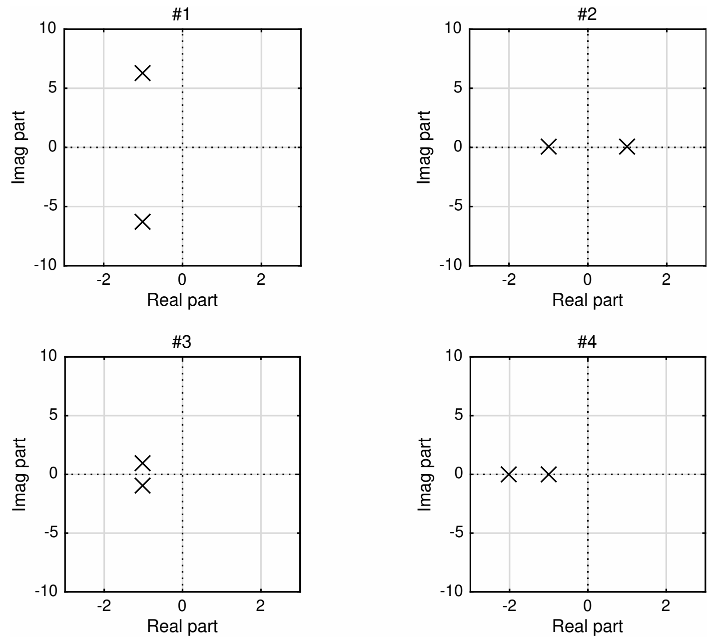
c) Find output signal produced by input signal

$$x(t) = 3 + 2\cos(2\pi t) + 5\cos(6\pi t).$$

2.7.21 Match the impulse response $h(t)$ of a continuous-time LTI system with the correct plot of its frequency response $|H^f(\omega)|$. *Explain how you obtain your answer.*

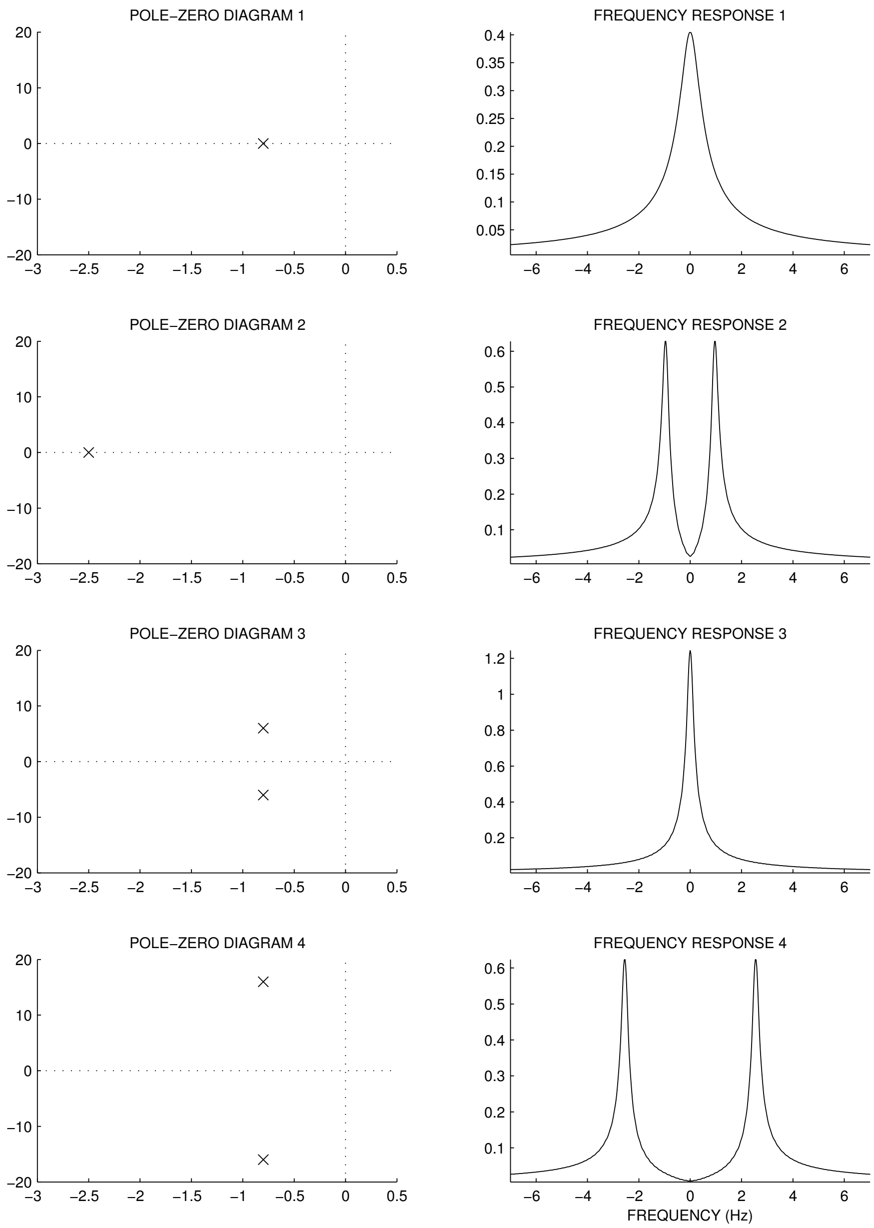


2.7.22 For the impulse response $h(t)$ illustrated in the previous problem, identify the correct diagram of the poles of $H(s)$. *Explain how you obtain your answer.*



2.7.23 The figure shows the pole diagrams and frequency responses of four continuous-time LTI systems. But they are out of order. Match each pole diagram with its frequency response.

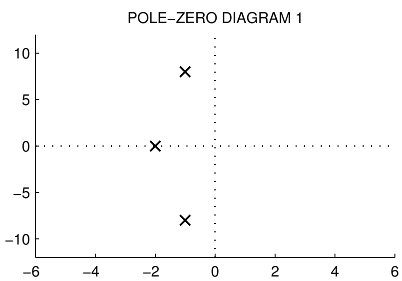
POLE-ZERO DIAGRAM	FREQUENCY RESPONSE
1	
2	
3	
4	

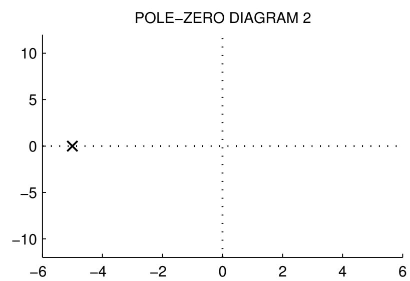


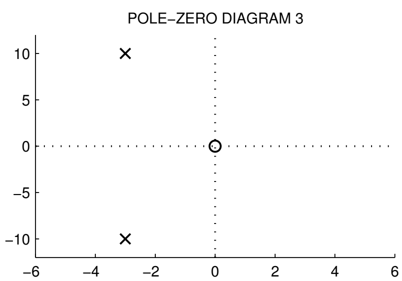
2.7.24 The diagrams on the following pages show the pole-zero diagrams and frequency responses of 8 causal continuous-time LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.

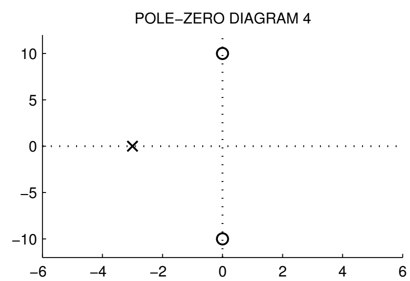
In the pole-zero diagrams, the zeros are shown with 'o' and the poles are shown by $\times$.

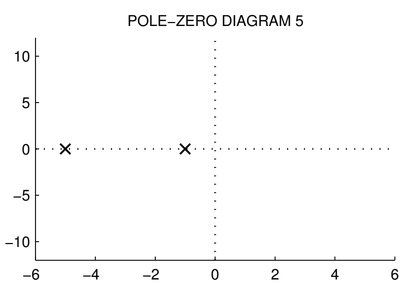
POLE-ZERO DIAGRAM	FREQUENCY RESPONSE
1	
2	
3	
4	
5	
6	
7	
8	

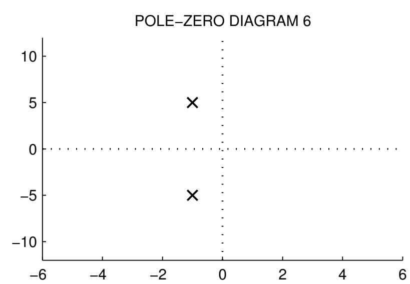


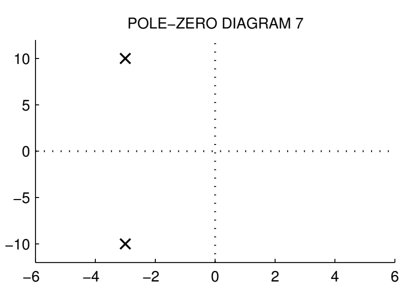


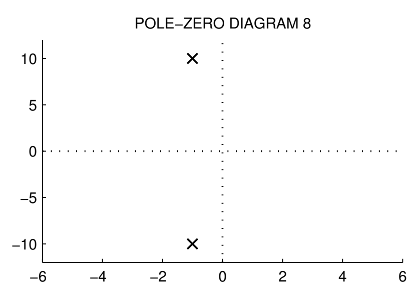


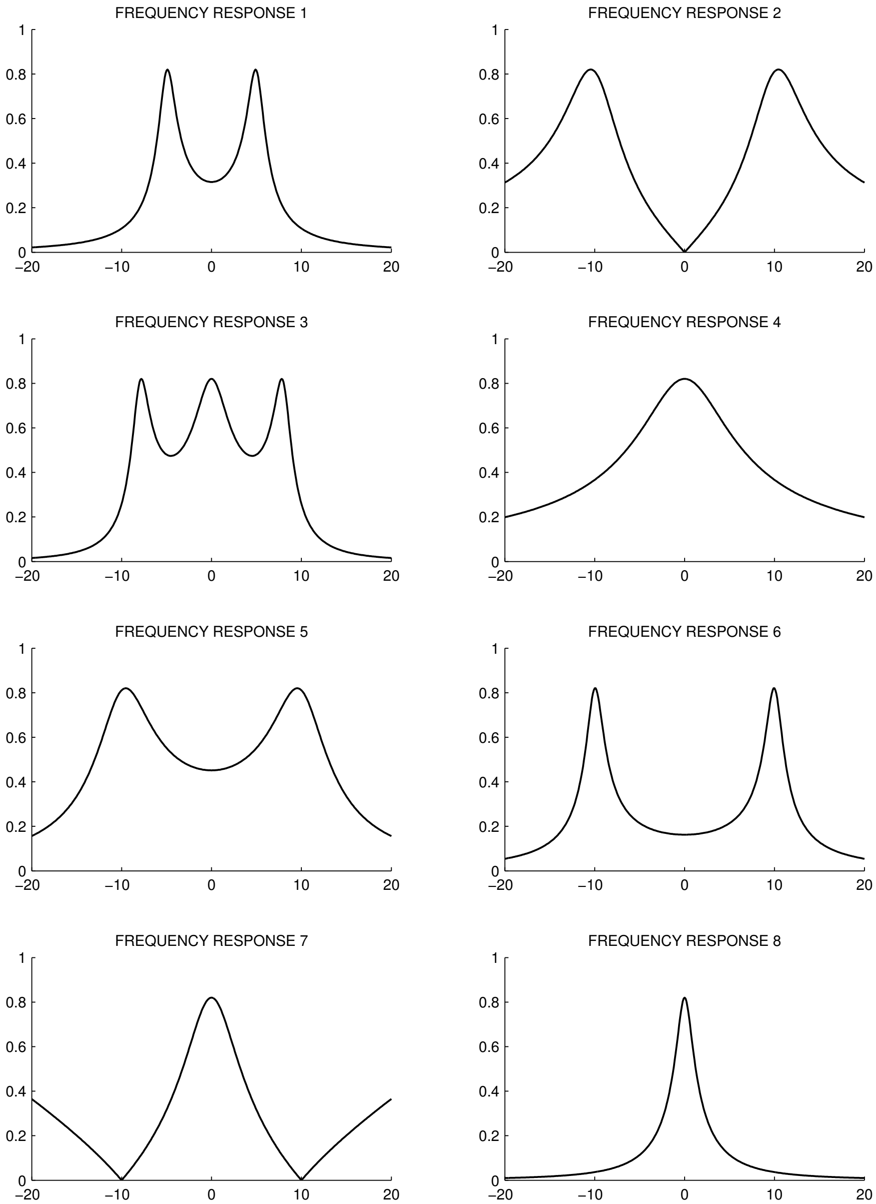








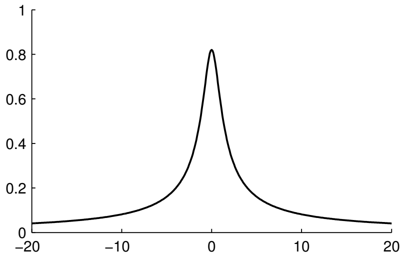



FREQUENCY RESPONSE 1
FREQUENCY RESPONSE 2
FREQUENCY RESPONSE 3
FREQUENCY RESPONSE 4
FREQUENCY RESPONSE 5
FREQUENCY RESPONSE 6
FREQUENCY RESPONSE 7
FREQUENCY RESPONSE 8
0
0.2
0.4
0.6
0.8
1
−20
−10
0
10
20

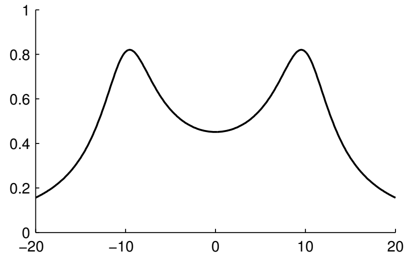
2.7.25 The frequency responses of eight causal continuous-time systems are illustrated below, along with the pole/zero diagram of each system. But they are out of order. Match the figures with each other by completing a table.

FREQUENCY RESPONSE	POLE-ZERO DIAGRAM
1	
2	
3	
4	
5	
6	
7	
8	

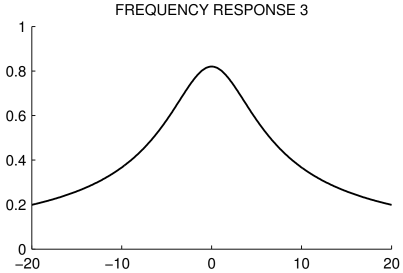
FREQUENCY RESPONSE 1



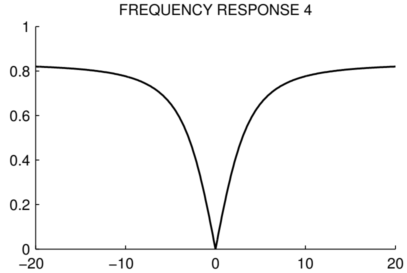
FREQUENCY RESPONSE 2



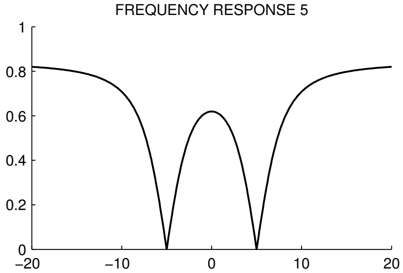
FREQUENCY RESPONSE 3



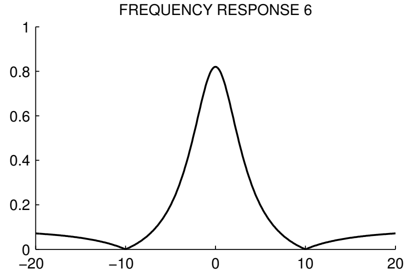
FREQUENCY RESPONSE 4



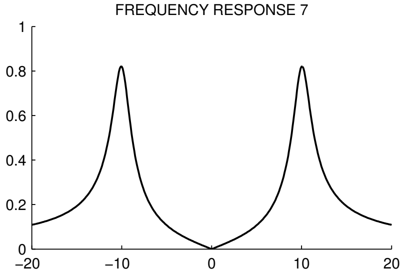
FREQUENCY RESPONSE 5



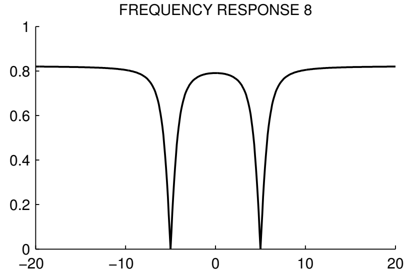
FREQUENCY RESPONSE 6



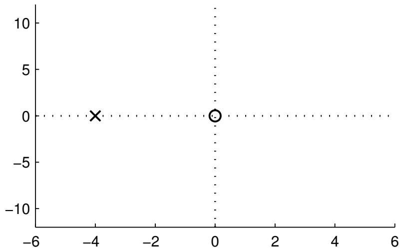
FREQUENCY RESPONSE 7



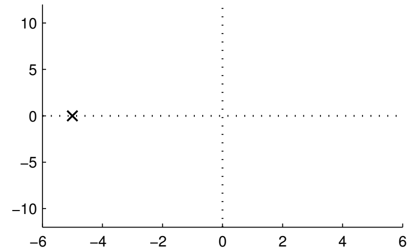
FREQUENCY RESPONSE 8



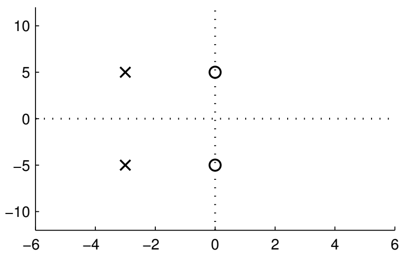
POLE−ZERO DIAGRAM 1



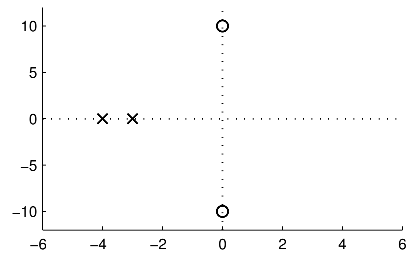
POLE−ZERO DIAGRAM 2



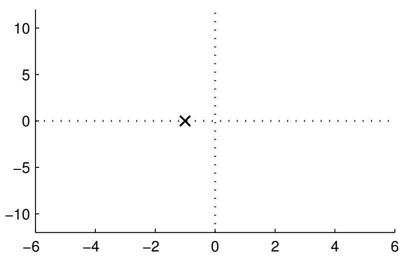
POLE−ZERO DIAGRAM 3



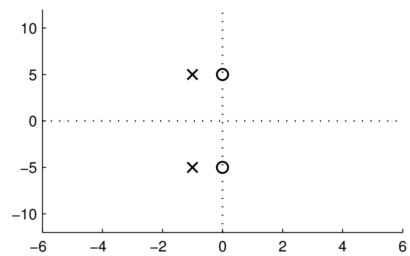
POLE−ZERO DIAGRAM 4



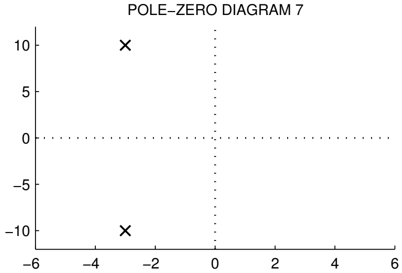
POLE−ZERO DIAGRAM 5



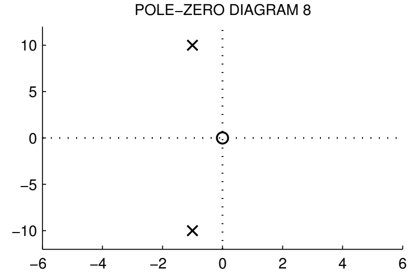
POLE−ZERO DIAGRAM 6



POLE−ZERO DIAGRAM 7



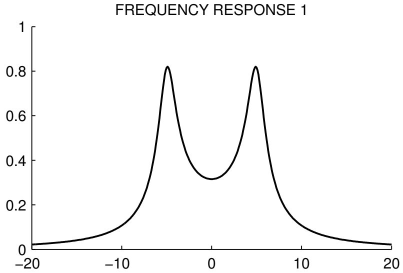
POLE−ZERO DIAGRAM 8



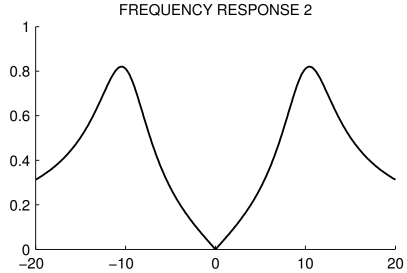
2.7.26 The frequency responses of eight causal continuous-time systems are illustrated below, along with the pole/zero diagram of each system. But they are out of order. Match the figures with each other by completing the table (copy the table into your answer book).

FREQUENCY RESPONSE	POLE-ZERO DIAGRAM
1	
2	
3	
4	
5	
6	
7	
8	

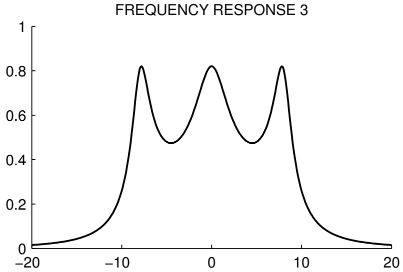
FREQUENCY RESPONSE 1



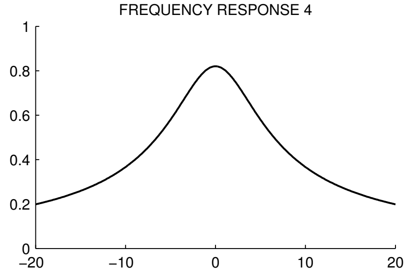
FREQUENCY RESPONSE 2



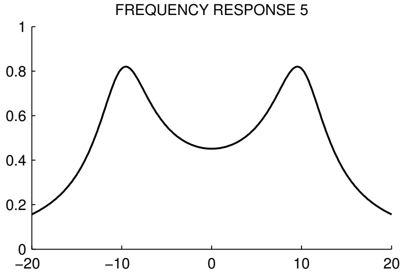
FREQUENCY RESPONSE 3



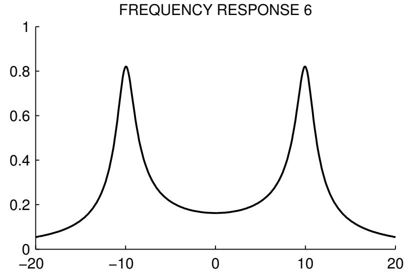
FREQUENCY RESPONSE 4



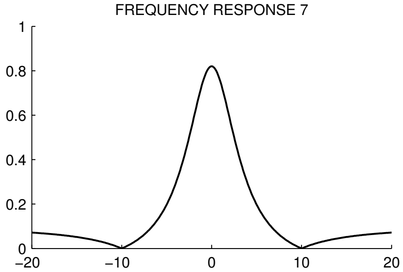
FREQUENCY RESPONSE 5



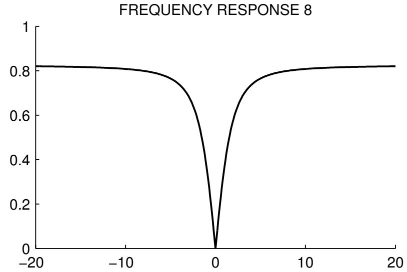
FREQUENCY RESPONSE 6

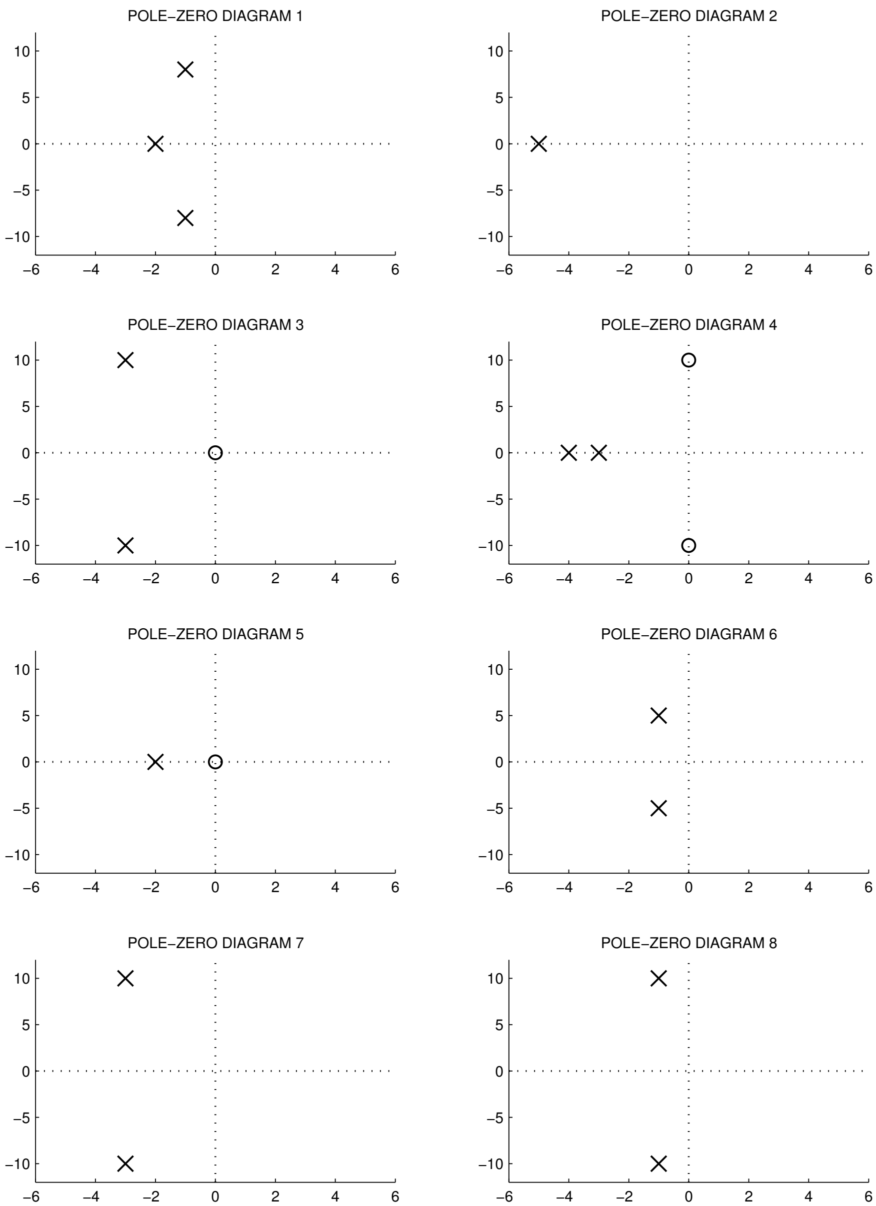


FREQUENCY RESPONSE 7



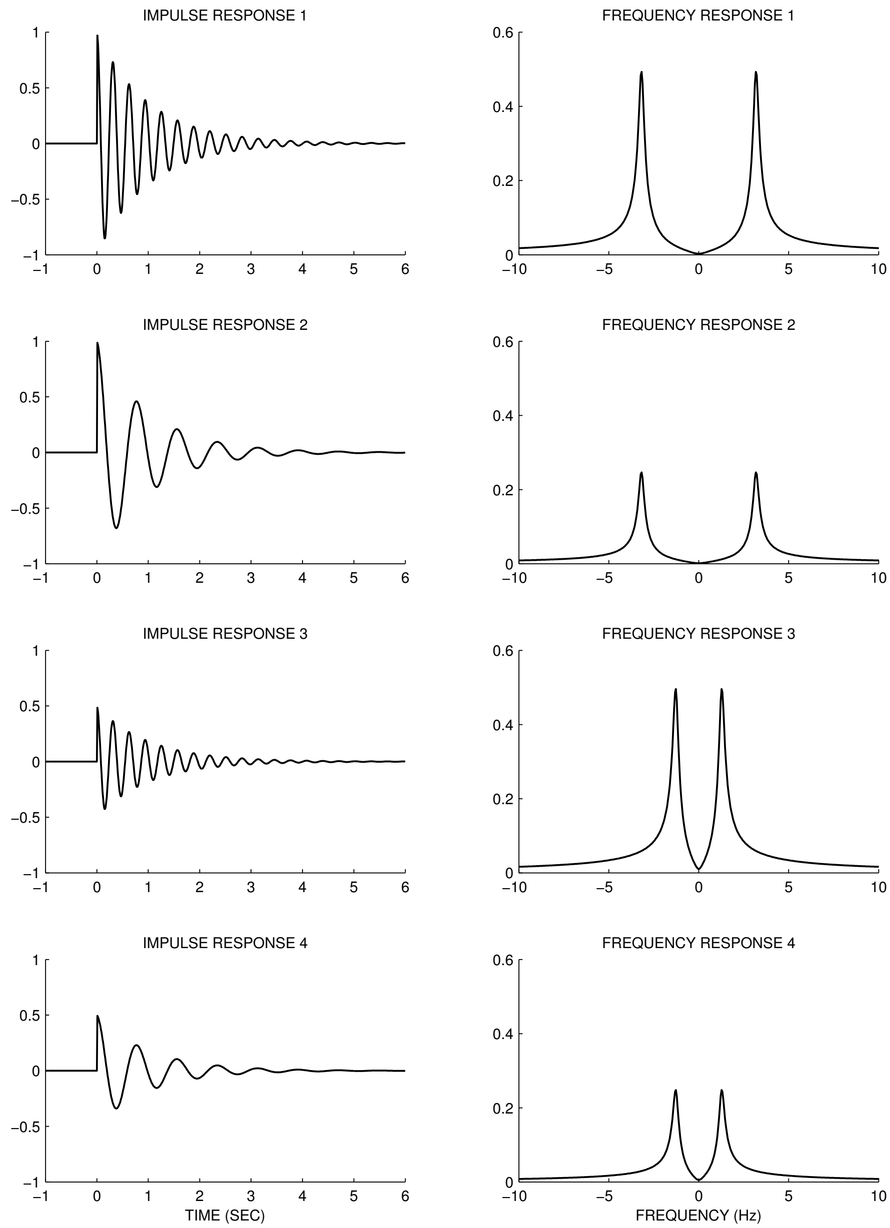
FREQUENCY RESPONSE 8




POLE−ZERO DIAGRAM 1
POLE−ZERO DIAGRAM 2
POLE−ZERO DIAGRAM 3
POLE−ZERO DIAGRAM 4
POLE−ZERO DIAGRAM 5
POLE−ZERO DIAGRAM 6
POLE−ZERO DIAGRAM 7
POLE−ZERO DIAGRAM 8

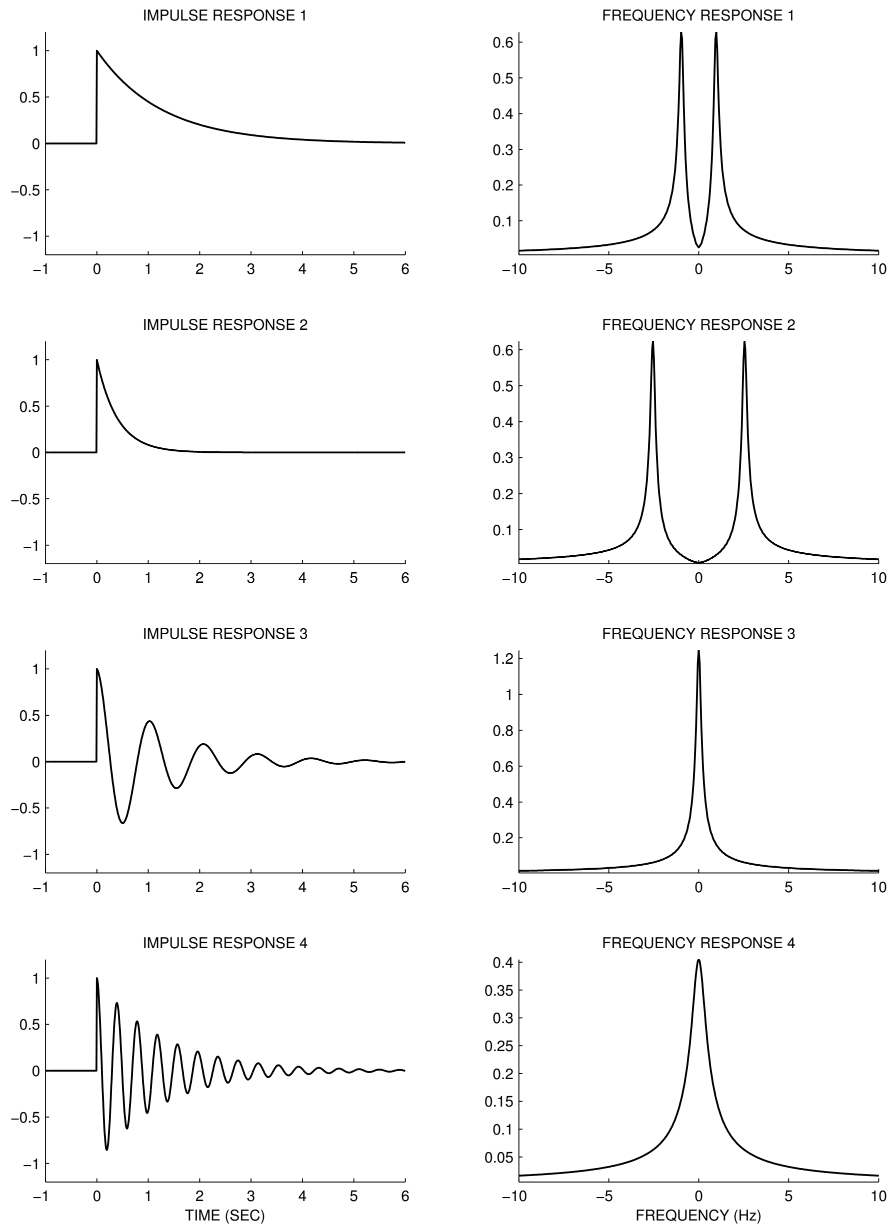
2.7.27 The figure shows the impulse responses and frequency responses of four continuous-time LTI systems. But they are out of order. Match the impulse response to its frequency response magnitude, and *explain* your answer.

IMPULSE RESPONSE	FREQUENCY RESPONSE
1	
2	
3	
4	



2.7.28 The figure shows the impulse responses and frequency responses of four continuous-time LTI systems. But they are out of order. Match each impulse response with its frequency response.

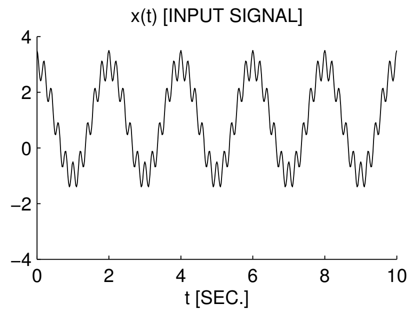
IMPULSE RESPONSE	FREQUENCY RESPONSE
1	
2	
3	
4	



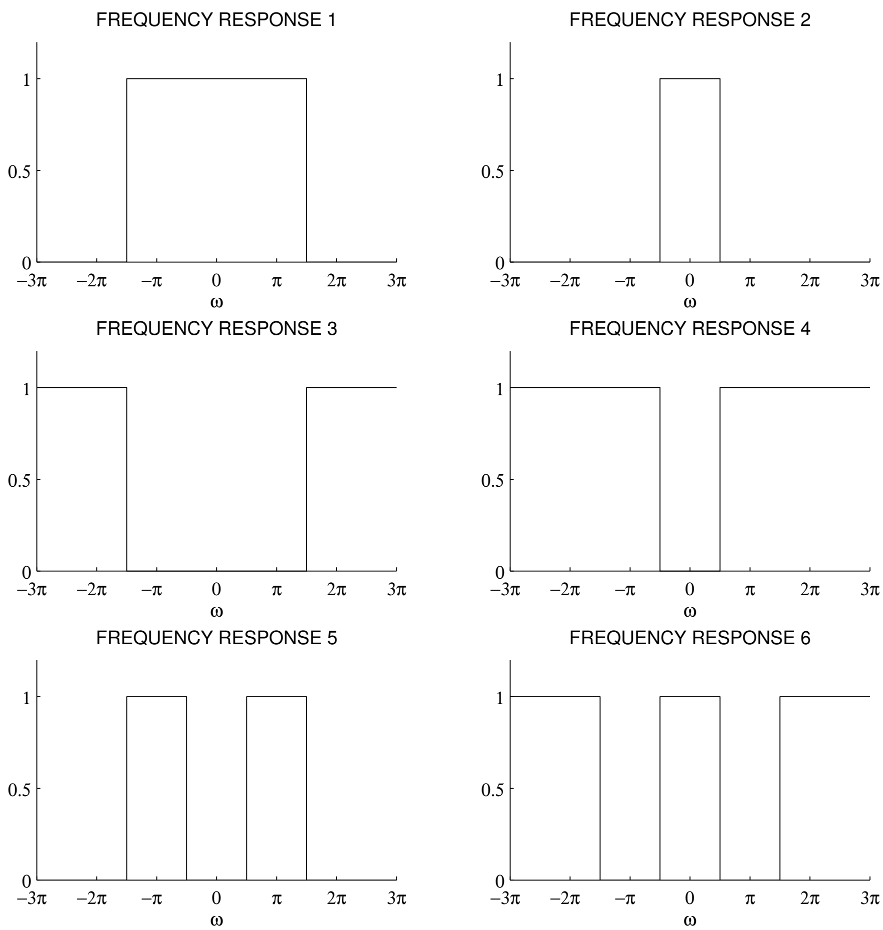
2.7.29 A signal $x(t)$, comprised of three components,

$$x(t) = 1 + 2\cos(\pi t) + 0.5\cos(10\pi t)$$

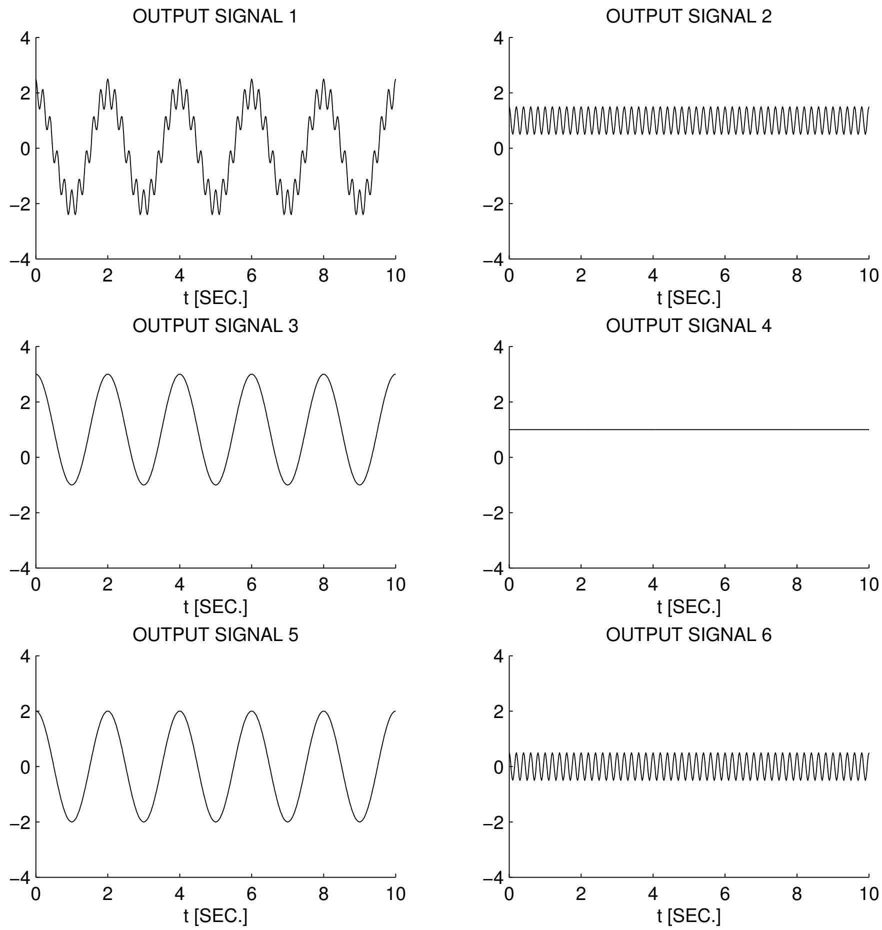
is illustrated here:



This signal, $x(t)$, is filtered with each of six different continuous-time LTI filters. The frequency response of each of the six systems are shown below. (For $|\omega| > 3\pi$, each frequency response has the value it has at $|\omega| = 3\pi$.)



The six output signals are shown below, but they are not numbered in the same order.



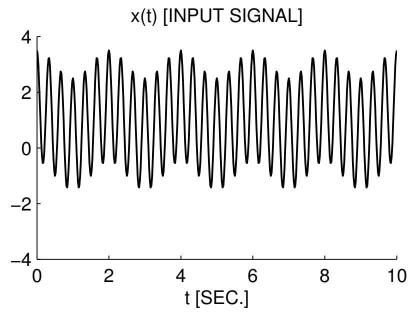
Match each output signal to the system that was used to produce it by completing the table.

System	Output signal
1	
2	
3	
4	
5	
6	

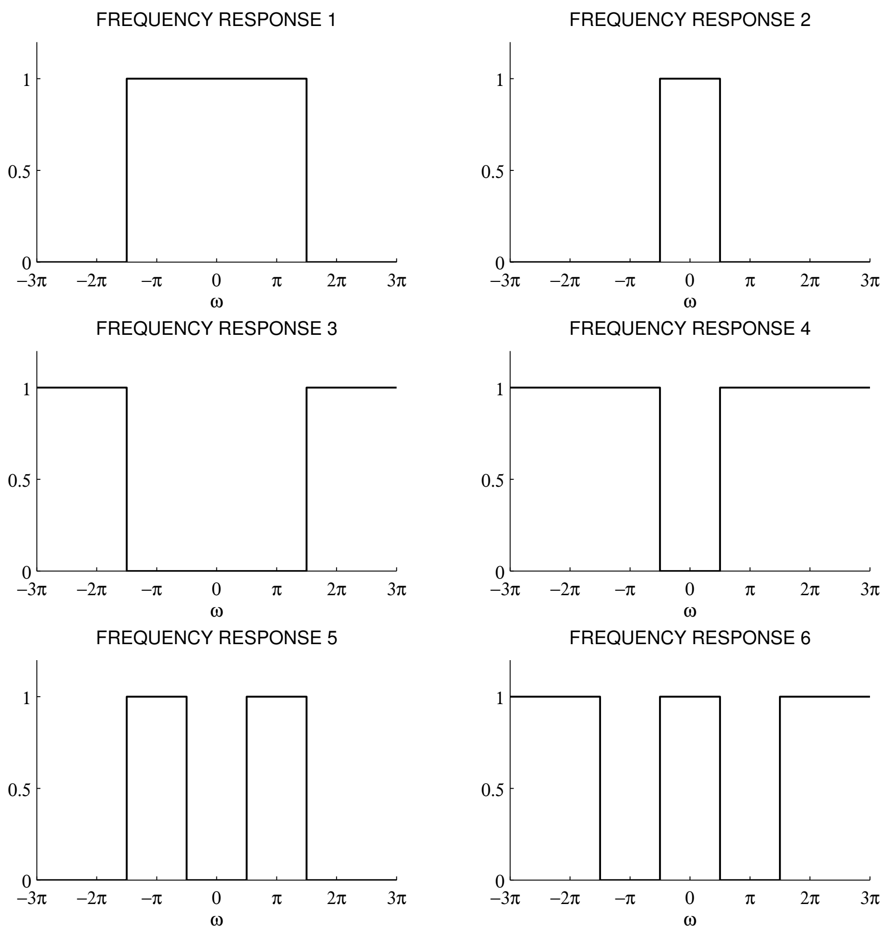
2.7.30 A signal $x(t)$, comprised of three components,

$$x(t) = 1 + 0.5\cos(\pi t) + 2\cos(6\pi t)$$

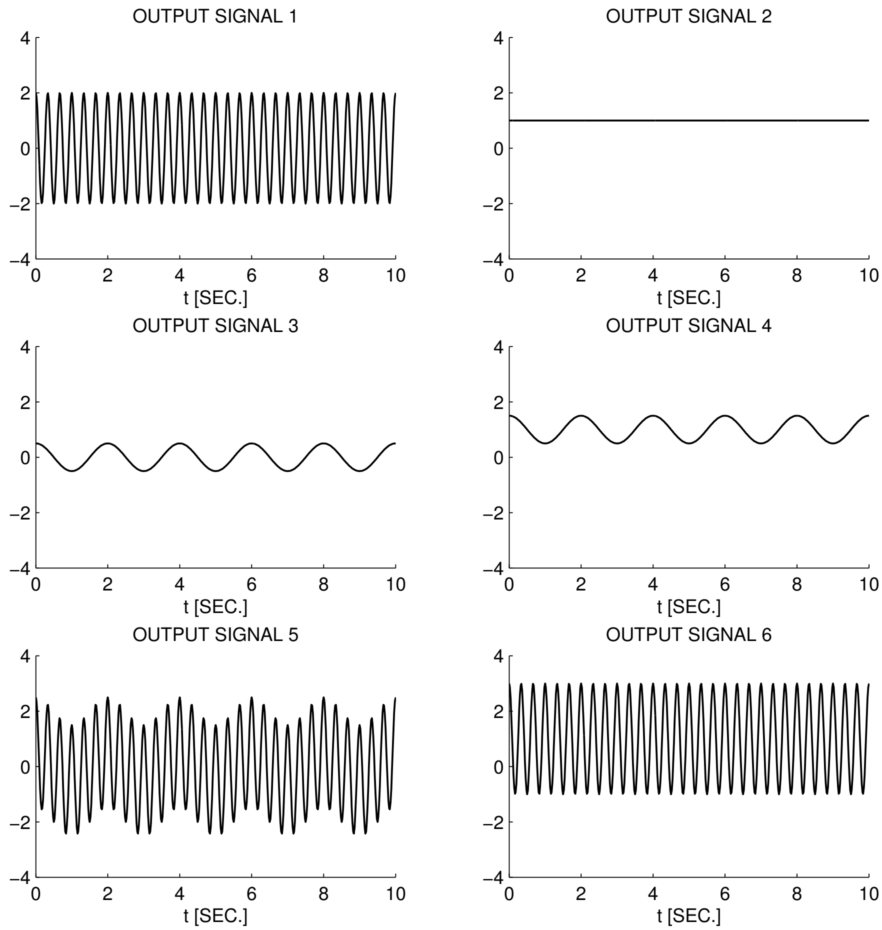
is illustrated here:



This signal, $x(t)$, is filtered with each of six different continuous-time LTI filters. The frequency response of each of the six systems are shown below. (For $|\omega| > 3\pi$, each frequency response has the value it has at $|\omega| = 3\pi$.)



The six output signals are shown below, but they are not numbered in the same order.



Match each output signal to the system that was used to produce it by completing the table.

System	Output signal
1	
2	
3	
4	
5	
6	

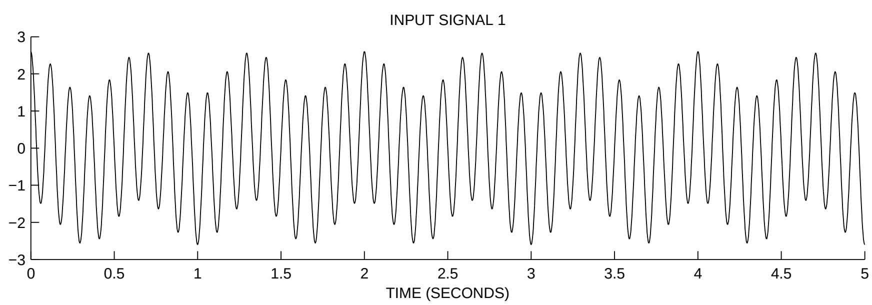
2.7.31 Each of the two continuous-time signals below are processed with each of four LTI systems. The two input signals, illustrated below, are given by:

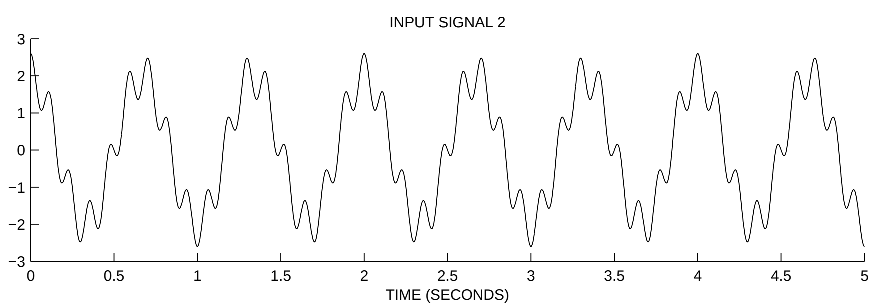
Input signal 1: $0.6\cos(3\pi t) + 2\cos(17\pi t)$

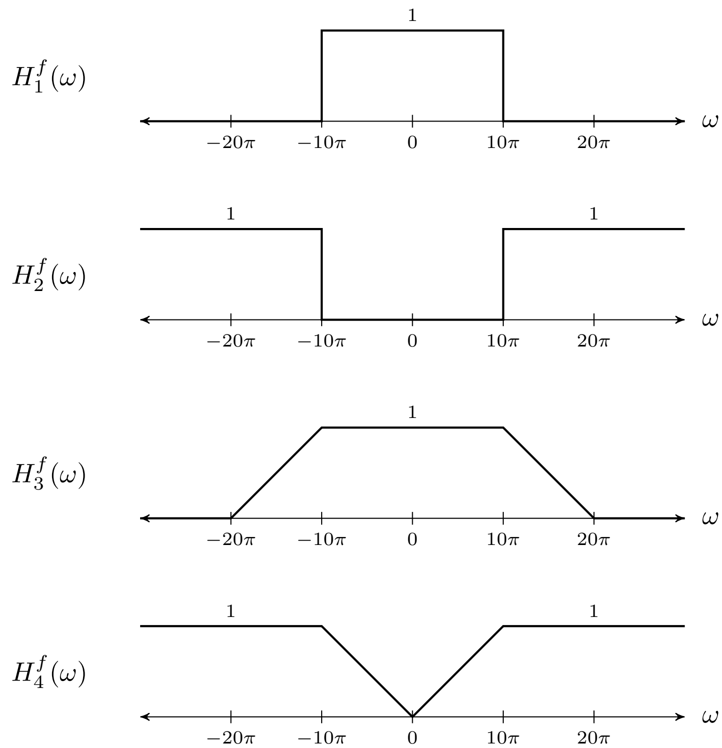
Input signal 2: $2\cos(3\pi t) + 0.6\cos(17\pi t)$

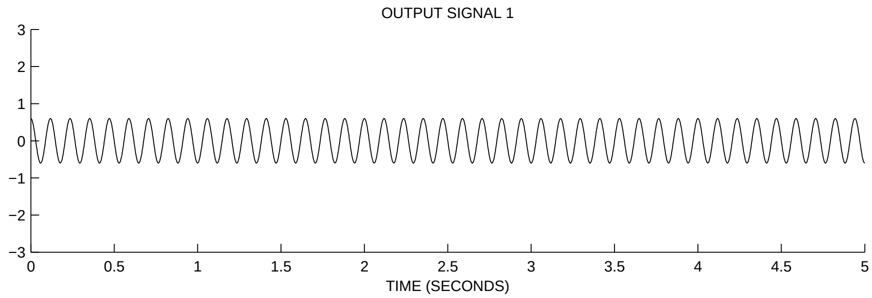
The frequency responses $H^f(\omega)$ are shown below. Indicate how each of the output signals are produced by completing the table below (copy the table onto your answer sheet). Note: one of the output signals illustrated below will appear twice in the table (there are seven distinct output signals).

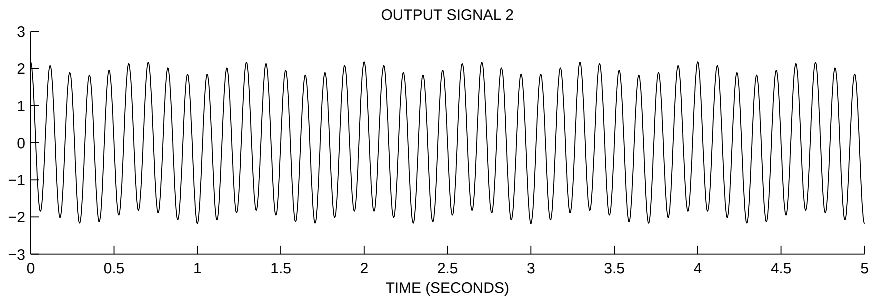
Input signal	System	Output signal
1	1	
1	2	
1	3	
1	4	
2	1	
2	2	
2	3	
2	4	

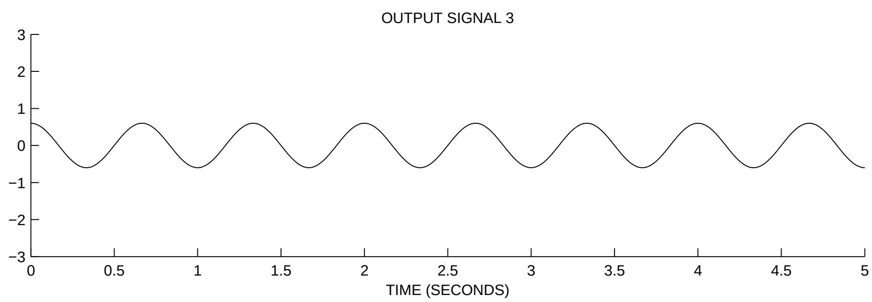


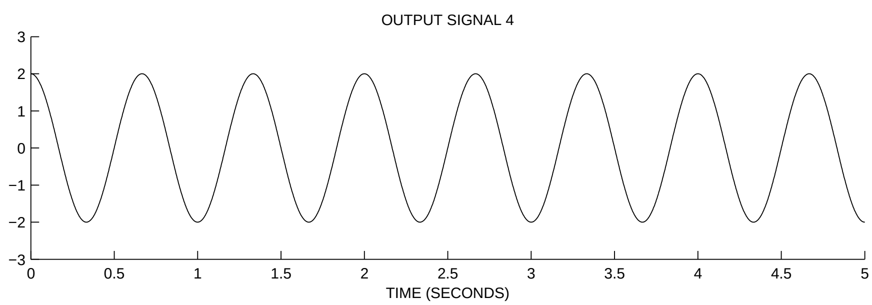


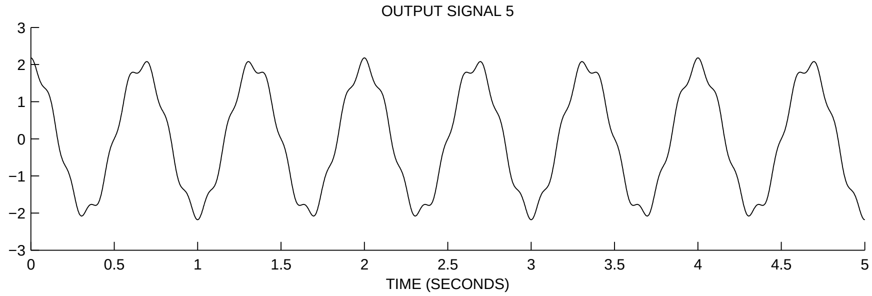

$H_1^f(\omega)$
1
ω
-20π -10π 0 10π 20π
$H_2^f(\omega)$
1 1
ω
-20π -10π 0 10π 20π
$H_3^f(\omega)$
1
ω
-20π -10π 0 10π 20π
$H_4^f(\omega)$
1 1
ω
-20π -10π 0 10π 20π

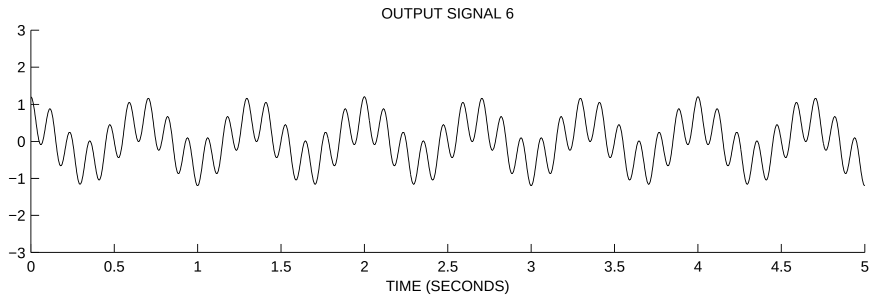

OUTPUT SIGNAL 1
TIME (SECONDS)

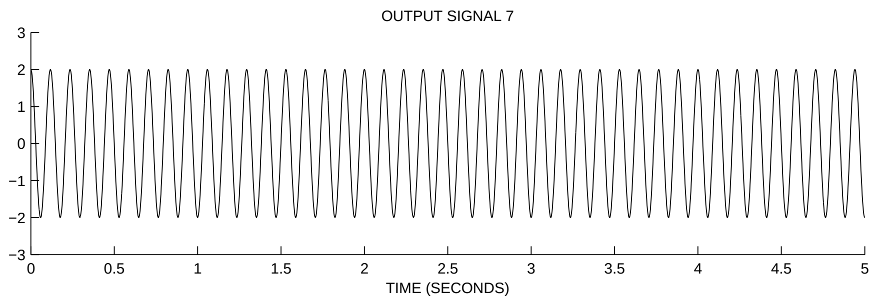

OUTPUT SIGNAL 2
TIME (SECONDS)


OUTPUT SIGNAL 3
TIME (SECONDS)


OUTPUT SIGNAL 4
TIME (SECONDS)

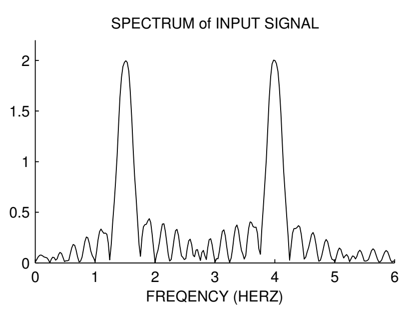


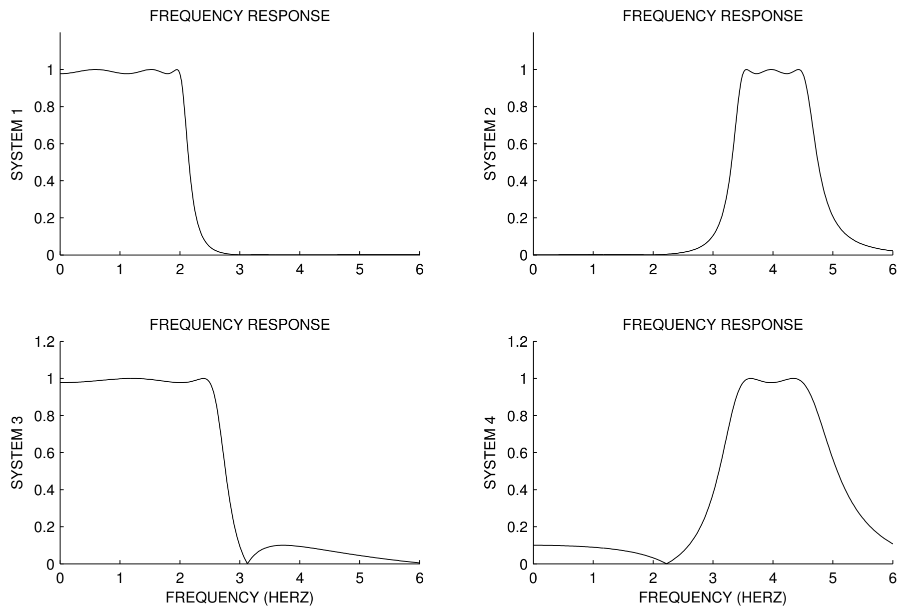


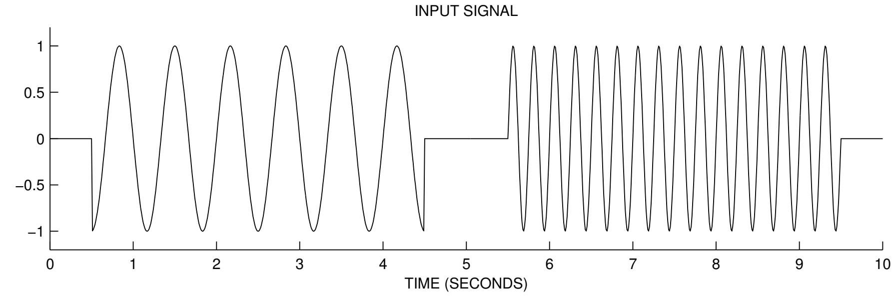


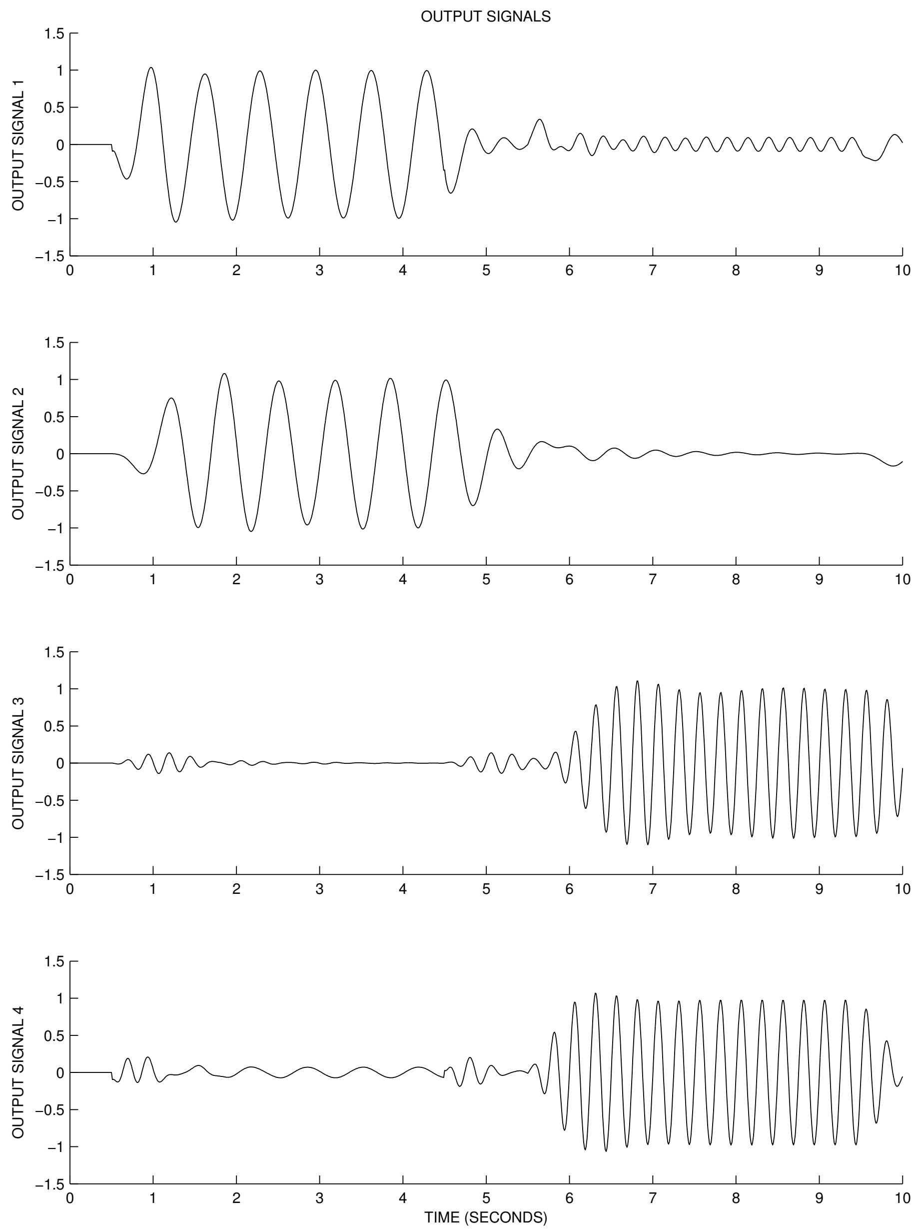
2.7.32 The following figures show a continuous-time signal $x(t)$ and its spectrum. This signal consists of one cosine pulse followed by another. This signal is used as the input to four continuous-time LTI systems (Systems A, B, C, and D) and four output signals are obtained. The frequency responses of each of these four systems are shown below. The last figure shows the four output signals. But the output signals are out of order. For each of the four systems, identify which signal is the output signal by completing the table below.

System	Output Signal
1	
2	
3	
4	

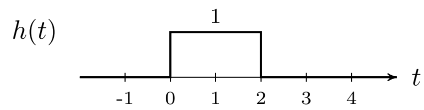



FREQUENCY RESPONSE
SYSTEM 1
FREQUENCY RESPONSE
SYSTEM 2
FREQUENCY RESPONSE
SYSTEM 3
FREQUENCY (HERZ)
FREQUENCY RESPONSE
SYSTEM 4
FREQUENCY (HERZ)





2.7.33 Consider a continuous-time LTI system with the impulse response



(a) Find the frequency response $H^f(\omega)$.

(b) Sketch the frequency response magnitude $|H^f(\omega)|$. Indicate the frequencies where $|H^f(\omega)|$ equals zero.

(c) Sketch the frequency response phase $\angle H^f(\omega)$.

(d) Find the output signal $y(t)$ produced by the input signal

$$x(t) = 2 + 5\cos(3\pi t).$$

(e) If $x(t)$ is a periodic signal with a fundamental period of 2 seconds and Fourier series coefficients c_k,

$$x(t) = \sum_k c_k e^{jk\omega_o t},$$

then what is the output signal $y(t)$ when $x(t)$ is the input signal?

2.7.34 The frequency response of a continuous-time LTI system is given by

$$H^f(\omega) = \frac{\mathrm{e}^{-\mathrm{j}\omega}}{1 + (\omega/\pi)^2}$$

(a) Sketch the frequency response magnitude $|H^f(\omega)|$.

(b) Sketch the frequency response phase $\angle H^f(\omega)$.

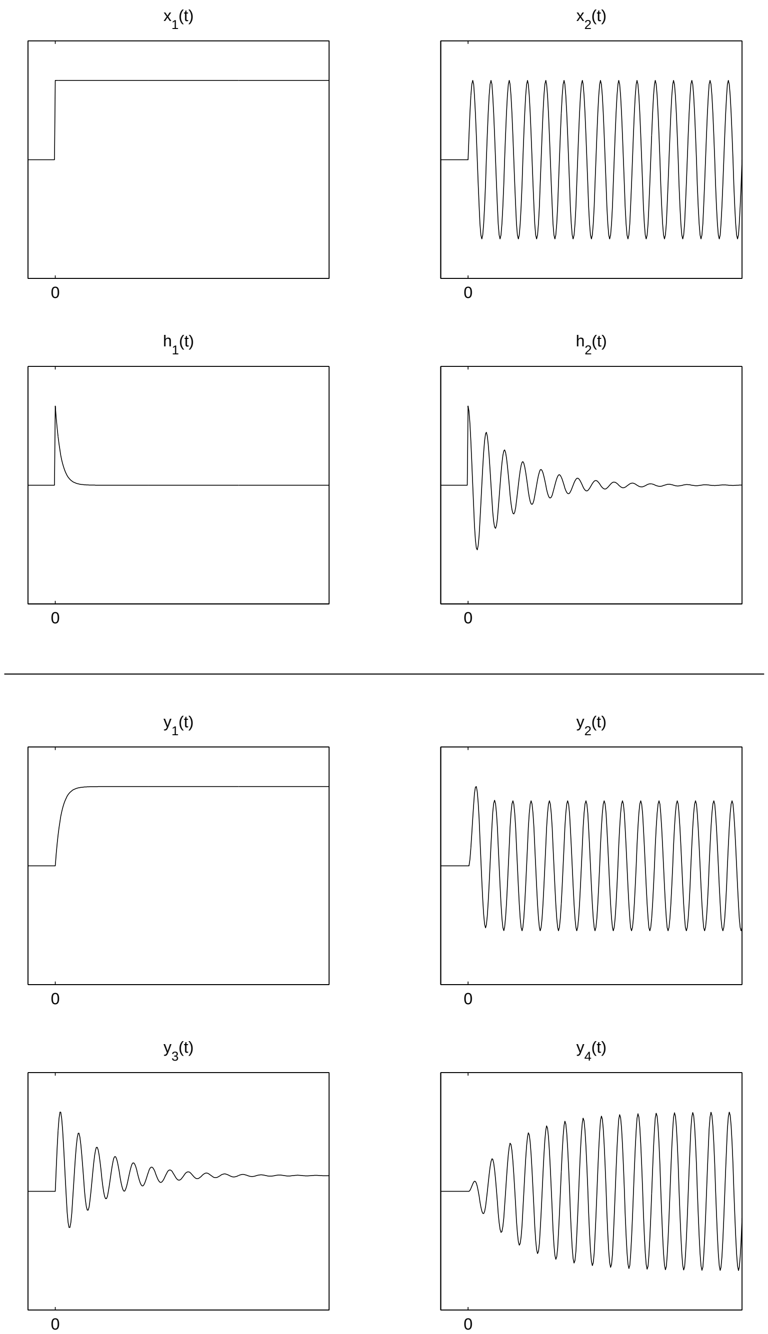
(c) Find the output signal produced by the input signal

$$x(t) = 1 + 2\cos(2\pi t).$$

2.8 Matching

2.8.1 The figure shows two input signals $x_1(t)$ and $x_2(t)$, two impulse responses $h_1(t)$ and $h_2(t)$, and four output signals $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$. Identify which input signal and which impulse response causes each of the four output signals.

(Your answer should have four parts, $y_1(t) = h_?(t) * x_?(t)$, etc.)



2.9 Simple System Design

2.9.1 Design a real causal continuous-time LTI system with poles at

$$p_1 = -1 + 2\,j, \qquad p_2 = -1 - 2\,j,$$

zeros at

$$z_1 = 2\,j, \qquad z_2 = -2\,j,$$

and a dc gain of unity.

(a) Write down the differential equation to implement the system.

(b) Sketch the pole/zero diagram.

(c) Sketch the frequency response magnitude $|H^f(\omega)|$. Mark the dc gain point and any other prominent points on the graph.

(d) Write down the form of the impulse response, as far as it can be determined without actually calculating the residues. (You do not need to complete the partial fraction expansion.)

2.9.2 Find the constants

$$y''(t) + 5y'(t) + 6y(t) = A x''(t) + B x'(t) + c\, x(t)$$

describes an LTI system.

a) find the constants A, B, C so that the system

① Kills tones with frequencies 5 Hz and

② has a dc-gain of 2.

b) sketch the pole/zero diagram.

c) write down the form of the impulse response as far as can be determined without actually completing the partial fraction expansion step.)

2.9.3 Design a simple causal continuous-time LTI system with the following properties:

- The system should remove the dc component of the input signal.
- The system should have a pole at $s = -5$.
- The frequency response $H^f(\omega)$ should approach 0.5 as ω goes to infinity.

For the system you design:

(a) Find the differential equation to implement the system.

(b) Find and sketch the impulse response of the system.

(c) Find the frequency response $H^f(\omega)$ and roughly sketch $|H^f(\omega)|$.

2.9.4 Find the constants:

$$y''(t) + 5y'(t) + 7y(t) = A x''(t) + B x'(t) + C x(t)$$

describes an LTI system.

Find constants A, B, C so that the system

ⓐ Kills tones with frequencies 10 Hz, and

ⓑ has unity dc-gain.

2.9.5 A causal continuous-time LTI system is described by the equation

$$y''(t) + 4\,y'(t) + 5\,y(t) = A\,x''(t) + B\,x'(t) + C\,x(t)$$

where x is the input signal, and y is the output signal.

Find constants A, B, C so that the system both annihilates 5 Hz tones and has unity dc gain.

2.9.6 In this problem you are to design a causal continuous-time LTI system with the following properties:

- The system should have two poles: $s = -4$ and $s = -3$.
- The system should kill 10 Hz tones.
- The system should have unity dc gain.

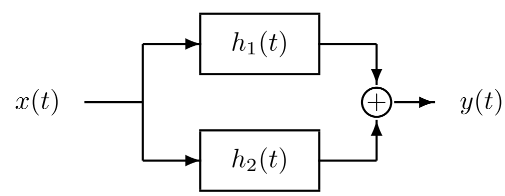
For the system you design:

(a) Find the differential equation to implement the system.

(b) Roughly sketch the frequency response magnitude $|H^f(\omega)|$.

2.10 Summary

2.10.1 Consider the parallel combination of two continuous-time LTI systems.



You are told that the step response of the upper branch is

$$s_1(t) = 2\,u(t) + u(t-1).$$

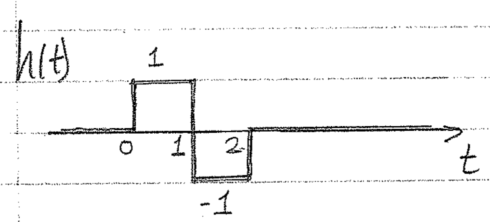
You observe that the impulse response of the total system is

$$h(t) = 2\,\delta(t) + \delta(t-2).$$

Find and sketch $h_1(t)$ and $h_2(t)$.

2.10.2 Find the step response:

③ An LTI system has the impulse response shown:

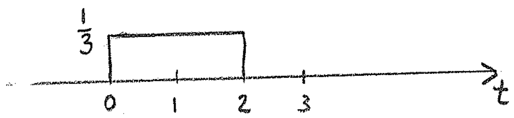


ⓐ Accurately sketch the step response of the system.

ⓑ Find the transfer function H(s).

2.10.3 An LTI system:

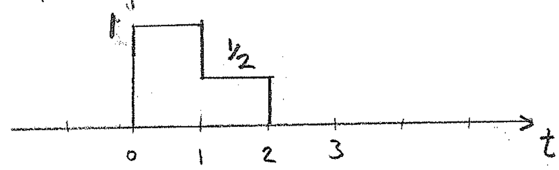
An LTI system has impulse response, $h(t)$,



a) what is the dc gain of the system?

b) what is the transfer function $H(s)$?

c) Sketch accurately the output of the system produced by the input signal:



2.10.4 It is observed of some continuous-time LTI system that the input signal

$$x(t) = 3\,u(t)$$

produces the output signal

$$y(t) = 2\,u(t) + 2\,e^{-5t}\,\sin(2\pi t)\,u(t).$$

(a) What are the poles of the LTI system?

(b) What is the dc gain of the system?

(c) Find the impulse response $h(t)$ of the system.

2.10.5 The impulse response of an LTI continuous-time system is given by

$$h(t) = 2\,e^{-t}\,u(t) + 3\,e^{-2\,t}\,u(t)$$

(a) Find the transfer function of the system.

(b) Find the differential equation with which the system can be implemented.

(c) Sketch the pole/zero diagram of the system.

(d) What is the dc gain of the system?

(e) Based on the pole/zero diagram, roughy sketch the frequency response magnitude $|H^f(\omega)|$. Mark the value at $\omega = 0$.

(f) Sketch the output signal produced by input signal, $x(t) = 1$.

(g) Find a formula for the output signal produced by the input signal, $x(t) = u(t)$.

2.10.6 A causal continuous-time LTI system is described by the equation

$$y''(t) + 6\,y'(t) + 5\,y(t) = x'(t) + 6\,x(t)$$

where x is the input signal, and y is the output signal.

(a) Find the impulse response of the system.

(b) Accurately sketch the pole-zero diagram.

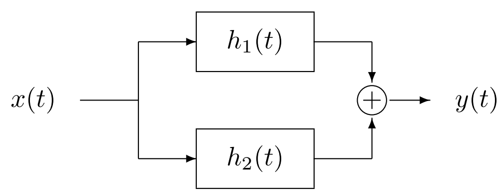
(c) Based on the pole/zero diagram, sketch the system's frequency response magnitude $|H^f(\omega)|$. Indicate the value at dc.

(d) Find the steady-state value of the system's step response.

2.10.7 Two continuous-time LTI systems have the following impulse responses

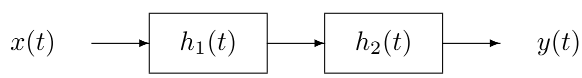
$$h_1(t) = e^{-2\,t}\,u(t) \qquad h_2(t) = e^{-3\,t}\,u(t)$$

(a) If the two systems are connected in parallel,



What is the frequency response of the total system?
What differential equation describes this system?

(b) If the two systems are connected in series,



What is the frequency response of the total system?
What differential equation describes this system?

2.10.8 The impulse response of a continuous-time LTI is given by

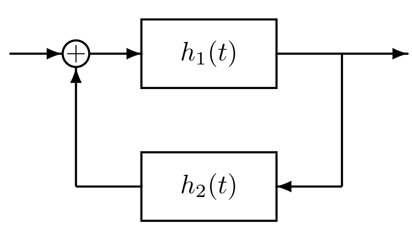
$$h(t) = u(t) - u(t-2).$$

Find the output $y(t)$ when the input $x(t)$ is given by

$$x(t) = t\,u(t).$$

Classify the system as BIBO stable/unstable.

2.10.9 Consider the following connection of two continuous-time LTI systems.



You are told that

$$h_1(t) = e^{-t}\,u(t), \qquad h_2(t) = -2\,e^{-4t}\,u(t).$$

Find the impulse response $h(t)$ of the total system.

2.10.10 The frequency response of a continuous-time LTI system is given by

$$H^f(\omega) = 2\, e^{-j\omega}.$$

(a) Find and sketch the impulse response $h(t)$.

(b) Sketch the output of this system when the input is

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 4\, k).$$

2.10.11 Suppose the impulse response of a continuous-time LTI system has the form

$$h(t) = B\, e^{-3t}\, \sin(5\, \pi\, t)\, u(t).$$

Suppose a signal $x(t)$ with the form

$$x(t) = A\, \cos(10\, \pi\, t - 0.5\, \pi)\, u(t)$$

is applied to the system. What is the generic form of the output signal?

2.10.12 Consider a causal continuous-time LTI system described by the differential equation

$$y''(t) + 5\, y'(t) + 4\, y(t) = 3\, x'(t) + 2\, x(t).$$

(a) Find the transfer function $H(s)$ and its region of convergence, and sketch the pole-zero diagram for this system.

(b) When the input signal is

$$x(t) = 3\, e^{-t}\, \cos(2\, t)\, u(t)$$

find the 'generic' form of the output signal $y(t)$. (You do not have to compute the residue of the partial fraction expansion; you may leave them as A, B, etc.)

2.10.13 The impulse response of a continuous-time LTI system is given by

$$h(t) = e^{-3\, t}\, \sin(2t)\, u(t).$$

Suppose the input signal is given by

$$x(t) = e^{-t}\, u(t).$$

(a) Without doing any arithmetic, write down the form you know the output signal $y(t)$ must take.

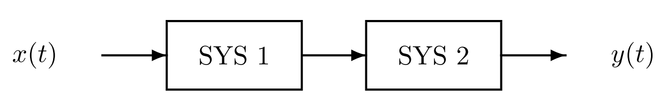
(b) Find the transfer function.

(c) Find the differential equation that represents this system.

(d) Find the frequency response $H^f(\omega)$ of this system.

(e) Sketch the pole diagram of the system.

2.10.14 Consider the cascade connection of two continuous-time LTI systems



with impulse responses

$$h_1(t) = 4\, e^{-2t}\, \cos(5\pi\, t)\, u(t)$$

$$h_2(t) = 5\, e^{-3t}\, u(t)$$

(a) Sketch the pole diagram of the two transfer functions $H_1(s)$, $H_2(s)$,

(b) Sketch the pole diagram of the transfer function of the total system $H_{tot}(s)$.

(c) Which form can the total impulse response $h_{tot}(t)$ take? (Choose all that apply.)

1) $A\,e^{-2t}\,u(t) + B\,e^{-3t}\,u(t) + C\,\cos(5\pi\,t+\theta)\,u(t)$
2) $A\,e^{-5t}\,\cos(5\pi\,t+\theta)\,u(t)$
3) $A\,e^{-3t}\,\cos(5\pi\,t+\theta)\,u(t) + B\,e^{-2t}\,u(t)$
4) $A\,e^{-2t}\,\cos(5\pi\,t+\theta)\,u(t) + B\,e^{-3t}\,u(t)$
5) $A\,e^{-2t}\,\cos(5\pi\,t+\theta_1)\,u(t) + B\,e^{-3t}\,\cos(5\pi\,t+\theta_2)u(t)$
6) $A\,e^{-2t}\,\cos(5\pi\,t+\theta_1)\,u(t) + B\,e^{-3t}\,\cos(5\pi\,t+\theta_2)u(t) + C\,e^{-2t}\,u(t) + D\,e^{-3t}\,u(t)$

2.10.15 Suppose the impulse response of a continuous-time LTI system is

$$h(t) = 3\,e^{-t}\,u(t)$$

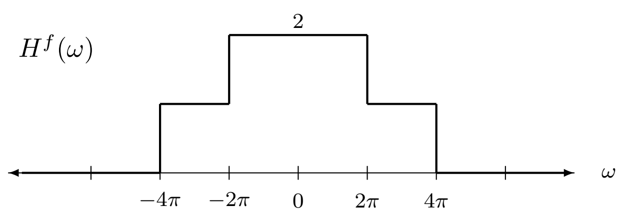
and the signal

$$x(t) = 2\,\cos(t)\,u(t)$$

is applied to the system. Find the steady-state output. (First find $H^f(\omega)$.)

2.10.16 The frequency response of a continuous-time LTI filter is given by

$$H^f(\omega) = \begin{cases} 2, & |\omega| \le 2\,\pi \\ 1, & 2\,\pi < |\omega| \le 4\,\pi \\ 0, & 4\,\pi < |\omega|. \end{cases}$$



Find the impulse response $h(t)$.

2.10.17 Suppose a tape recording is made with an inferior microphone which has the frequency response

$$H^f(\omega) = \begin{cases} 1 - \frac{|\omega|}{W}, & |\omega| < W \\ 0, & |\omega| \ge W, \end{cases}$$

where $W = 1200\pi$ (600 Hz).

(a) What is the frequency response of the system you would use to compensate for the distortion caused by the microphone? Make a sketch of your frequency response and explain your answer.

(b) Suppose you wish to digitally store a recording that was made with this microphone. What sampling rate would you use? Give your answer in Herz.

2.10.18 A continuous-time LTI system $H(s)$ is described by the differential equation

$$y'' + 2\,y' + y = x.$$

You are considering the design of an LTI system $G(s)$ that inverts $H(s)$.

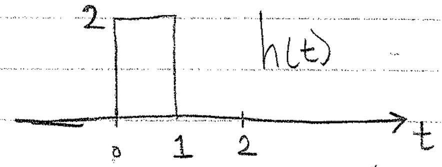
(a) Find the frequency response $G^f(\omega)$ of the inverse system.

(b) Find and sketch $|G^f(\omega)|$.

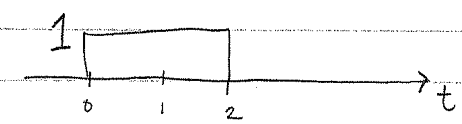
(c) Why will it be difficult to implement $G(s)$ exactly?

2.10.19 A simple system:

(5) An LTI system has the impulse response

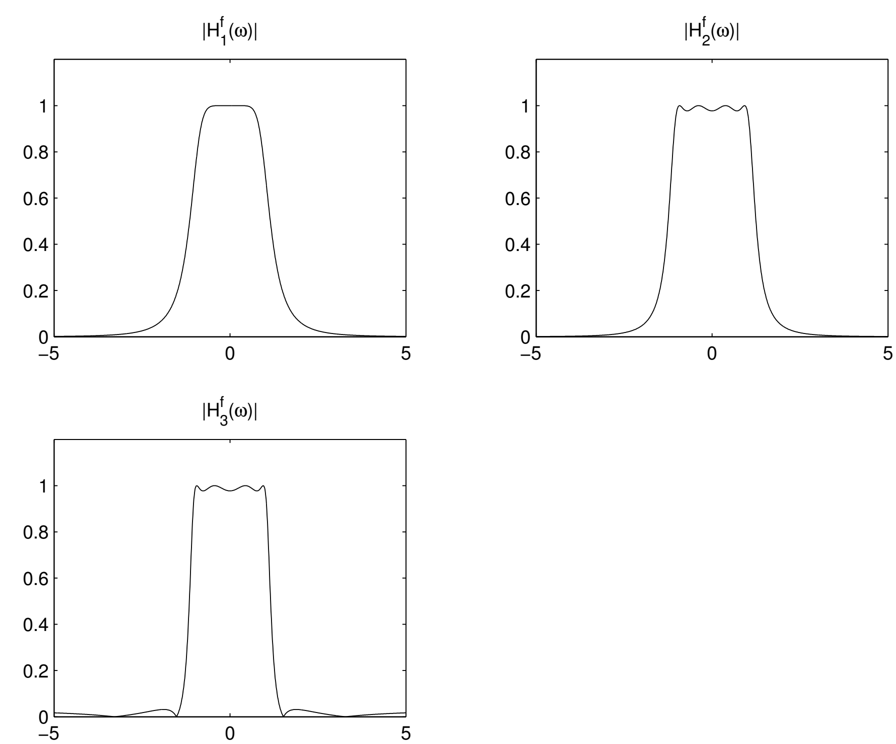


(a) What is the dc gain of the system?
(b) Accurately sketch the output signal produced by the input signal



(c) What is the transfer function H(s)?

2.10.20 Which of the following diagrams shows the frequency response of an *elliptic* filter? (Recall the Matlab exercise on filter design.)



Also identify the Chebyshev-I filter and the Butterworth filter.

3 Fourier Transform

3.1 Fourier Transform

For each of the following problems, you are encouraged to use a table of Fourier transforms and properties. The function sinc(t) is defined as

$$\text{sinc}(t) = \frac{\sin(\pi\, t)}{\pi\, t}$$

3.1.1 Sketch each of the following signals, and find its Fourier transform. You do not need to perform any integration — instead, use the properties of the Fourier transform.

(a) $x(t) = \mathrm{e}^{-(t-1)}\; u(t-1)$

(b) $x(t) = \mathrm{e}^{-(t-1)}\; u(t)$

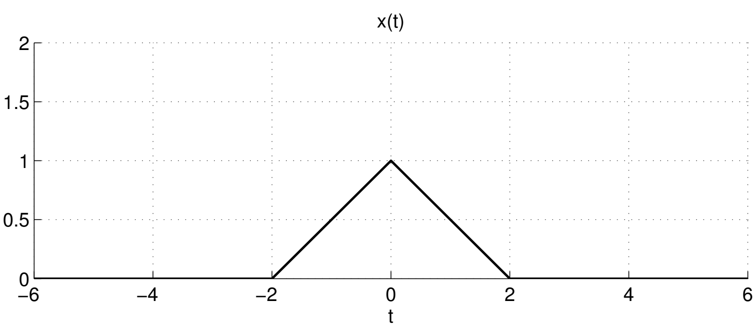
(c) $x(t) = \mathrm{e}^{-t}\; u(t-1)$

3.1.2 Sketch each of the following signals, and find its Fourier transform.

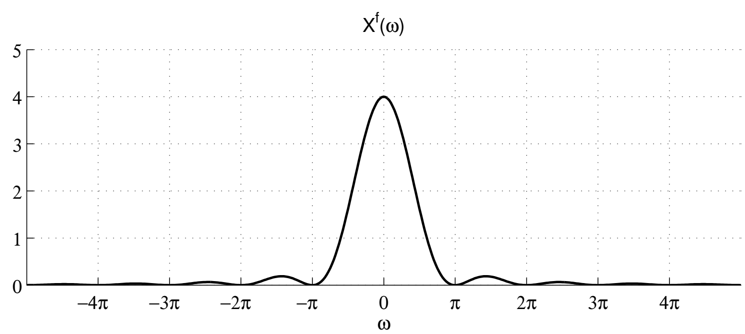
(a) $x(t) = \mathrm{e}^{-|t|}$

(b) $x(t) = \mathrm{e}^{-|t-1|}$

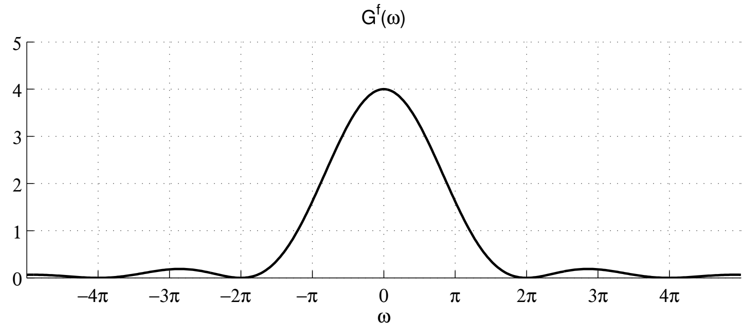
3.1.3 The signal $x(t)$:



has the Fourier transform $X^f(\omega)$:



Accurately sketch the signal $g(t)$ that has the spectrum $G^f(\omega)$:



Note that the spectrum $G^f(\omega)$ is an expanded version of $X^f(\omega)$. Specifically,

$$G^f(\omega) = X^f(0.5\,\omega).$$

3.1.4 The Fourier transform of $x(t)$ is $X^f(\omega) = \mathrm{rect}(\omega)$.

$$\mathrm{rect}(\omega) = \begin{cases} 1 & |\omega| \le 0.5 \\ 0 & |\omega| > 0.5 \end{cases}$$

Use the Fourier transform properties to sketch the magnitude and phase of the Fourier transform of each of the following signals.

(a) $f(t) = x(t-2)$

(b) $g(t) = x(2\,t)$

(c) $v(t) = x(2\,t-1)$

(d) $q(t) = x(t) * x(t)$

3.1.5 Find and sketch the Fourier transform of each of the following signals.

(a) $x(t) = \cos(6\,\pi\,t)$

(b) $x(t) = \mathrm{sinc}(3\,t)$

(c) $x(t) = \cos(6\,\pi\,t)\ \mathrm{sinc}(3\,t)$

(d) $x(t) = \cos(3\,\pi\,t)\ \cos(2\,\pi\,t)$

3.1.6 The spectrum $X^f(\omega)$ of a continuous-time signal $x(t)$ is given by

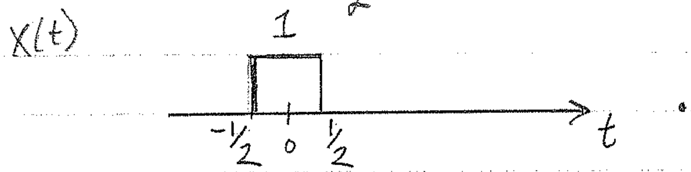
$$X^f(\omega) = \begin{cases} 1 - |\omega|/(3\,\pi) & \text{for } |\omega| \le 3\,\pi \\ 0 & \text{for other values of } \omega. \end{cases}$$

Sketch the magnitude and the phase of the Fourier transform of each of the following signals.

$$a(t) = 3\,x(2\,t)$$
$$b(t) = x(t) * x(t)$$
$$d(t) = x(t)\ \cos(5\,\pi t)$$
$$f(t) = x(t-5)$$

3.1.7 The sinc function

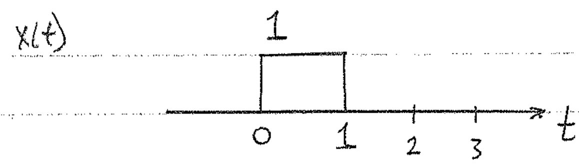
ⓐ Find the Fourier transform $X^f(\omega)$ of $x(t)$,
- write $X^f(\omega)$ using the sinc function.



ⓑ sketch $X^f(\omega)$, indicate the nulls of $X^f(\omega)$.

3.1.8 The sinc function

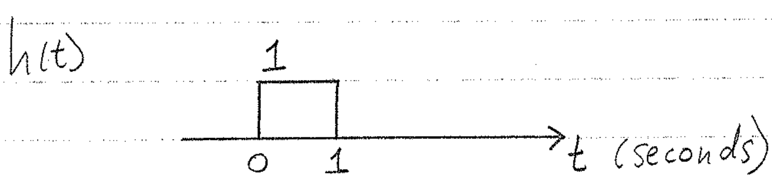
ⓐ Find $X^f(\omega)$ for $x(t)$,



ⓑ sketch $|X^f(\omega)|$, indicate the nulls of $|X^f(\omega)|$,
indicate the nulls of $|X^f(\omega)|$.

3.1.9 The sinc function

If an LTI system has the impulse response



then what frequencies are completely stopped by the system? Express the frequencies in Herz.

3.1.10 Let the signal $g(t)$ be

$$g(t) = \mathrm{sinc}(2\,t).$$

(a) Find the Fourier transform of $g(t)$.

$$\mathcal{F}\{g(t)\} = ?$$

(b) Find the Fourier transform of $g(t)$ convolved with itself.

$$\mathcal{F}\{g(t) * g(t)\} = ?$$

(c) Find the Fourier transform of $g(t-1) * g(t-2)$.

$$\mathcal{F}\{g(t-1) * g(t-2)\} = ?$$

(Use part (a) together with properties of the Fourier transform.)

(d) Find the Fourier transform of $g(t)$ multiplied with itself.

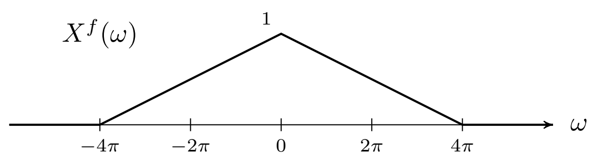
$$\mathcal{F}\{g(t) \cdot g(t)\} = ?$$

3.1.11 The signal $x(t)$ is a cosine pulse,

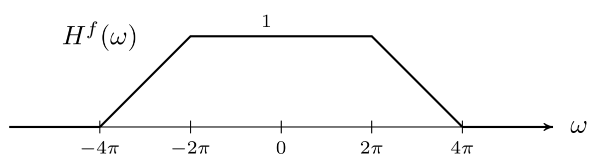
$$x(t) = \cos(10\,\pi\,t)\,[u(t+1) - u(t-1)].$$

Find and make a rough sketch of its Fourier transform $X^f(\omega)$. Also, make a sketch of $x(t)$.

3.1.12 The signal $x(t)$ has the spectrum $X^f(\omega)$,



The signal $x(t)$ is used as the input to a continuous-time LTI system having the frequency response $H^f(\omega)$,



Accurately sketch the spectrum $Y^f(\omega)$ of the output signal.

3.1.13 Consider a continuous-time LTI system with the impulse response

$$h(t) = \delta(t) - 2\,\text{sinc}(2\,t).$$

(a) Accurately sketch the frequency response $H^f(\omega)$.

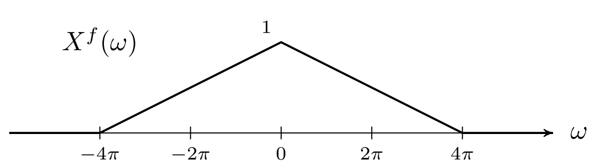
(b) Find the output signal $y(t)$ produced by the input signal

$$x(t) = 3 + 4\,\sin(\pi t) + 5\,\cos(3\pi t).$$

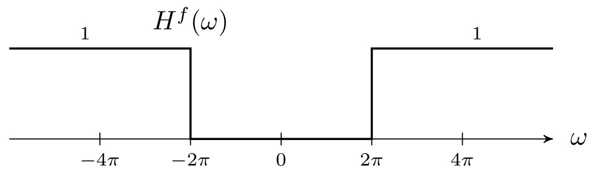
3.1.14 Find the Fourier transform $X^f(\omega)$ of the signal

$$x(t) = \cos\left(5\pi t + \frac{\pi}{4}\right).$$

3.1.15 The signal $x(t)$ has the spectrum $X^f(\omega)$ shown.



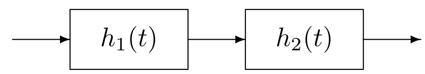
The signal $x(t)$ is used as the input to a continuous-time LTI system having the frequency response $H^f(\omega)$ shown.



Accurately sketch the spectrum $Y^f(\omega)$ of the output signal.

3.1.16 Two continuous-time LTI system are used in cascade. Their impulse responses are

$$h_1(t) = \text{sinc}(3\,t) \qquad h_2(t) = \text{sinc}(5\,t).$$



Find the impulse response and sketch the frequency response of the total system.

3.1.17 Consider a continuous-time LTI system with the impulse response

$$h(t) = 3\ \text{sinc}(3\,t).$$

(a) Accurately sketch the frequency response $H^f(\omega)$.

(b) Find the output signal $y(t)$ produced by the input signal

$$x(t) = 2 + 5\ \cos(\pi t) + 7\ \cos(4\pi t).$$

(c) Consider a second continuous-time LTI system with impulse response $g(t) = h(t-2)$ where $h(t)$ is as above. For this second system, find the output signal $y(t)$ produced by the input signal

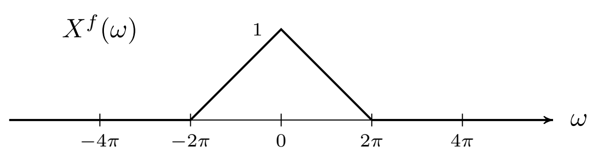
$$x(t) = 2 + 5\ \cos(\pi t) + 7\ \cos(4\pi t).$$

3.1.18 The signal $x(t)$ is given the product of two sine functions,

$$x(t) = \sin(\pi\,t) \cdot \sin(2\pi\,t).$$

Find the Fourier transform $X^f(\omega)$.

3.1.19 A continuous-time signal $x(t)$ has the spectrum $X^f(\omega)$,



(a) The signal $g(t)$ is defined as

$$g(t) = x(t)\cos(4\pi t).$$

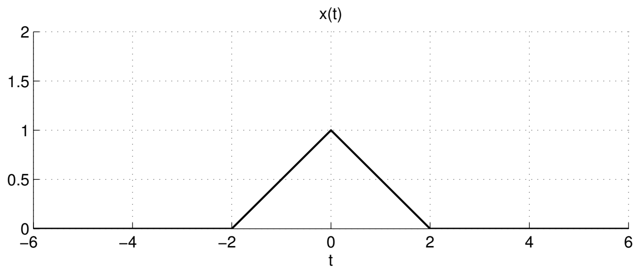
Accurately sketch the Fourier transform of $g(t)$.

(b) The signal $f(t)$ is defined as

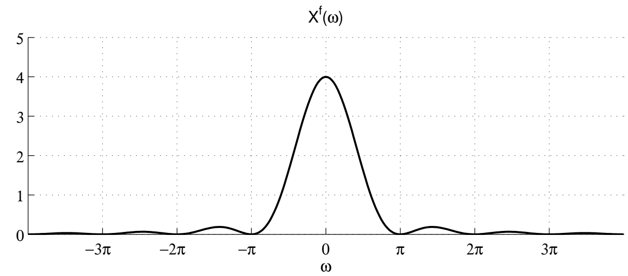
$$f(t) = x(t)\cos(\pi t).$$

Accurately sketch the Fourier transform of $f(t)$.

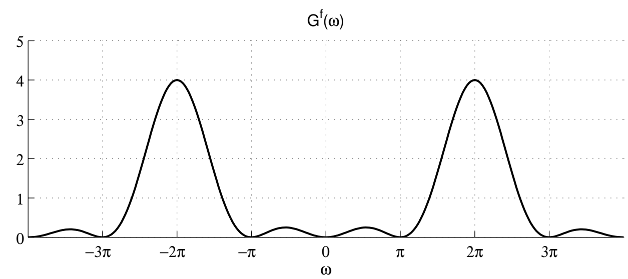
3.1.20 The signal $x(t)$:



has the Fourier transform $X^f(\omega)$:



Accurately sketch the signal $g(t)$ that has the spectrum $G^f(\omega)$:



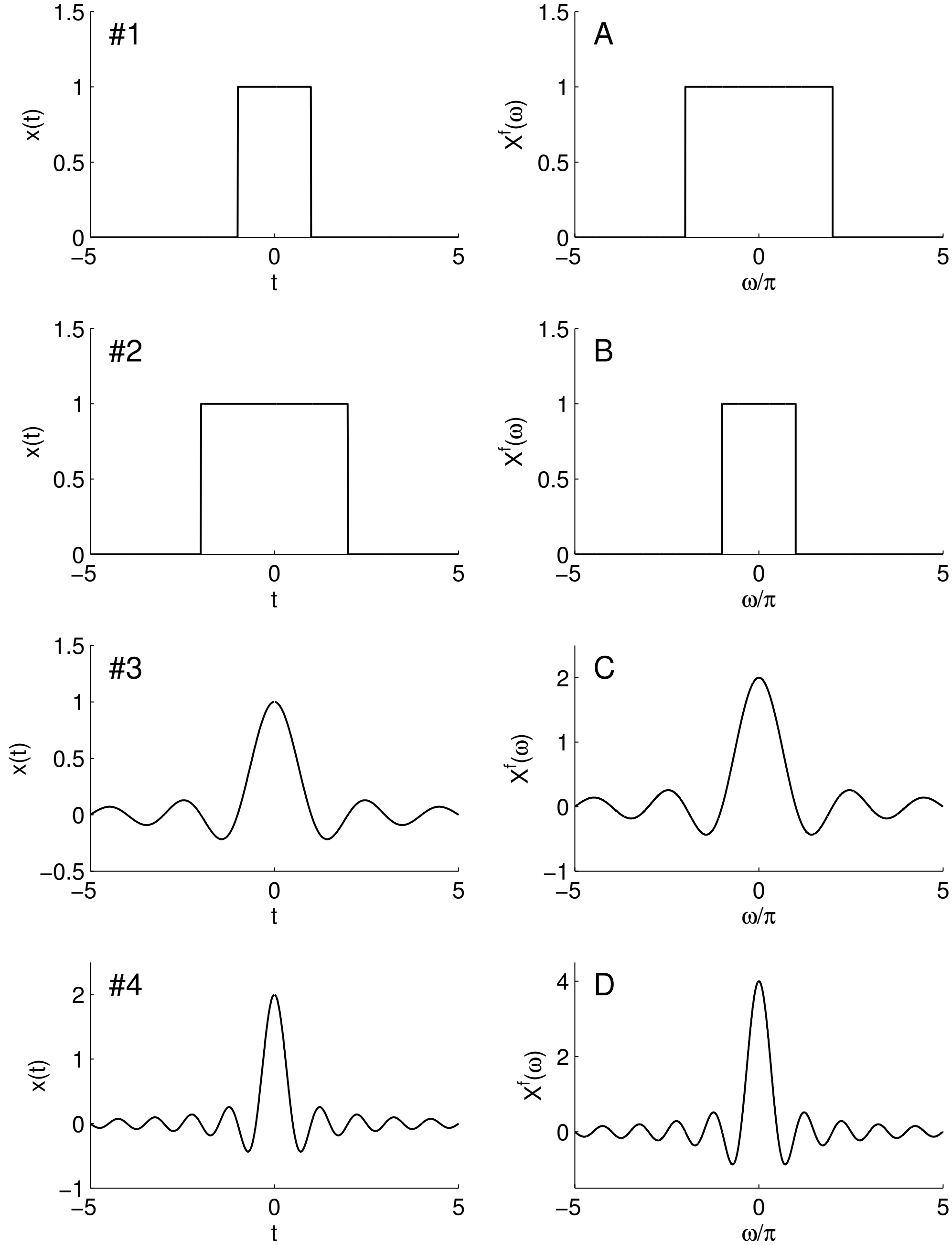
Note that the spectrum $G^f(\omega)$ is a sum of left- and right-shifted copies of $X^f(\omega)$. Specifically,

$$G^f(\omega) = X^f(\omega - 2\pi) + X^f(\omega + 2\pi).$$

In your sketch of the signal $g(t)$ indicate its zero-crossings. Show and explain your work.

3.1.21 The left-hand column below shows four continuous-time signals. The Fourier transform of each signal appears in the right-hand column in mixed-up order. Match the signal to its Fourier transform.

Signal	Fourier transform
1	
2	
3	
4	


#1
x(t)
t
A
Xf(ω)
ω/π
#2
x(t)
t
B
Xf(ω)
ω/π
#3
x(t)
t
C
Xf(ω)
ω/π
#4
x(t)
t
D
Xf(ω)
ω/π

3.1.22 The ideal continuous-time high-pass filter has the frequency response

$$H^f(\omega) = \begin{cases} 0, & |\omega| \le \omega_c \\ 1, & |\omega| > \omega_c. \end{cases}$$

Find the impulse response $h(t)$.

3.1.23 The ideal continuous-time band-stop filter has the frequency response

$$H^f(\omega) = \begin{cases} 1 & |\omega| \le \omega_1 \\ 0 & \omega_1 < |\omega| < \omega_2 \\ 1 & |\omega| \ge \omega_2. \end{cases}$$

(a) Sketch $H^f(\omega)$.

(b) What is the impulse response $h(t)$ of the ideal band-stop filter?

(c) Describe how to implement the ideal band-stop filter using only lowpass and highpass filters.

3.1.24 The ideal continuous-time band-pass filter has the frequency response

$$H^f(\omega) = \begin{cases} 0 & |\omega| \le \omega_1 \\ 1 & \omega_1 < |\omega| < \omega_2 \\ 0 & |\omega| \ge \omega_2. \end{cases}$$

Describe how to implement the ideal band-pass using two ideal low-pass filters with different cut-off frequencies. (Should a parallel or cascade structure be used?) Using that, find the impulse response $h(t)$ of the ideal band-pass filter.

3.1.25 The frequency response of a continuous-time LTI is given by

$$H^f(\omega) = \begin{cases} 0 & |\omega| < 2\,\pi \\ 1 & |\omega| \ge 2\,\pi \end{cases}$$

(This is an ideal high-pass filter.) Use the Fourier transform to find the output $y(t)$ when the input $x(t)$ is given by

(a) $x(t) = \mathrm{sinc}(t) = \frac{\sin(\pi\,t)}{\pi\,t}$

(b) $x(t) = \mathrm{sinc}(3\,t) = \frac{\sin(3\,\pi\,t)}{3\,\pi\,t}$

3.1.26 What is the Fourier transform of $x(t)$?

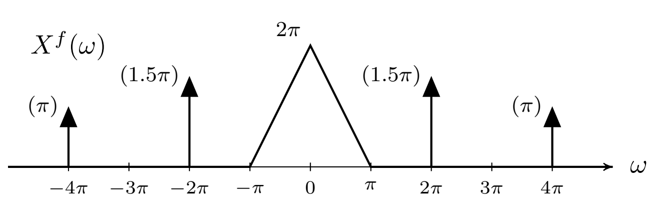
$$x(t) = \cos(0.3\,\pi\,t) \cdot \sin(0.1\,\pi\,t)$$

$$X^f(\omega) = \mathcal{F}\{x(t)\} = ?$$

3.1.27 A continuous-time LTI system has the impulse response

$$h(t) = \delta(t) - 3\ \mathrm{sinc}(3t)$$

The input signal $x(t)$ has the spectrum $X^f(\omega)$ shown,

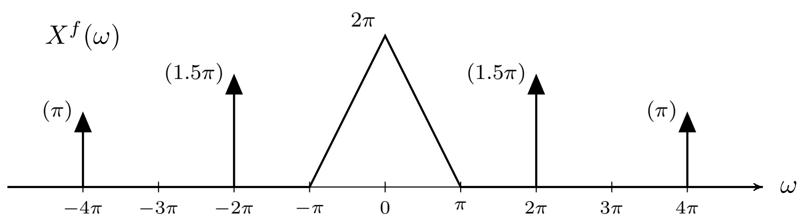


Find the output signal $y(t)$.

3.1.28 A continuous-time LTI system has the impulse response

$$h(t) = 3\,\text{sinc}(3t) - \text{sinc}(t).$$

The input signal $x(t)$ has the spectrum $X^f(\omega)$ shown,



Find the output signal $y(t)$. (Hint: first find $H^f(\omega)$.)

3.2 Fourier Series

3.2.1 Consider the signal

$$x(t) = \cos(10\,\pi\,t)$$

(a) Write the Fourier series expansion of $x(t)$.

(b) Sketch the line spectrum of $x(t)$.

3.2.2 Consider the signal

$$x(t) = \cos(12\,\pi\,t)\,\cos(8\,\pi\,t).$$

(a) Write the Fourier series expansion of $x(t)$.

(b) Sketch the line spectrum of $x(t)$.

3.2.3 Find the Fourier series coefficients $c(k)$ for the periodic signal

$$x(t) = |\cos(t)|.$$

Using Matlab, plot the truncated Fourier series:

$$x_N(t) = \sum_{k=-N}^{N} c_k\, e^{jk\omega_o t}$$

for $N = 10$. Does it resemble the signal $x(t)$?

3.2.4 The periodic signal $x(t)$ is given by

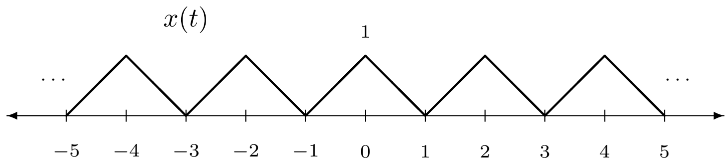
$$x(t) = |\sin(t)|.$$

(a) Sketch the signal $x(t)$. What is its period?

(b) Find the Fourier series coefficients of $x(t)$. Simplify!

(c) Is $x(t)$ band-limited?

3.2.5 Find the Fourier series coefficients $c(k)$ for the periodic triangular pulse shown below.



3.2.6 Fourier series:

) $X(t) = 2\cos(\pi t) + \cos(1.5\pi t)$

a) Find and sketch the Fourier Transform $X^f(\omega)$.

b) Find the Fourier Series coefficients $c(k)$ and the fundamental frequency ω_0,

c) Also, sketch the line spectrum of $x(t)$.

3.2.7 The signal $x(t)$ is

$$x(t) = [\cos(\pi\, t)]^2\, \sin(\pi t).$$

(a) Find its Fourier transform $X^f(\omega)$.

(b) Is $x(t)$ periodic? If so, determine the Fourier series coefficients of $x(t)$.

3.2.8 The signal $x(t)$, which is periodic with period $T = 1/4$, has the Fourier series coefficients

$$c_k = \frac{1}{1+k^2}.$$

The signal is filtered with an ideal lowpass filter with cut-off frequency of $f_c = 5.2\,\text{Hz}$. What is the output of the filter?

Simplify your answer so it does not contain any complex numbers. Note: $\omega = 2\,\pi\, f$ converts from physical frequency to angular frequency.

3.2.9 Consider the signal

$$x(t) = |0.5 + \cos(4\pi t)|$$

The signal is filtered with an ideal bandpass filter that only passes frequencies between 2.5 Hz and 3.5 Hz.

$$H^f(\omega) = \begin{cases} 0 & |\omega| < 5\,\pi \\ 1 & 5\,\pi \le |\omega| \le 7\,\pi \\ 0 & |\omega| > 7\,\pi \end{cases}$$

(a) Sketch the input signal $x(t)$.

(b) Find and sketch the output signal $y(t)$. Hint: Use the Fourier series, but do not compute the Fourier series of $x(t)$.

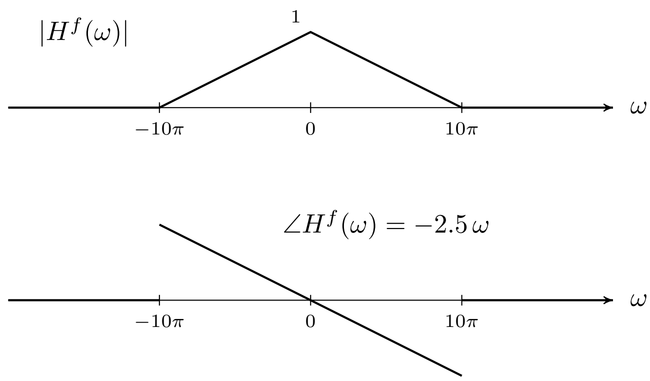
3.2.10 A continuous-time signal $x(t)$ is given by

$$x(t) = 2\cos(6\pi t) + 3\cos(8\pi t).$$

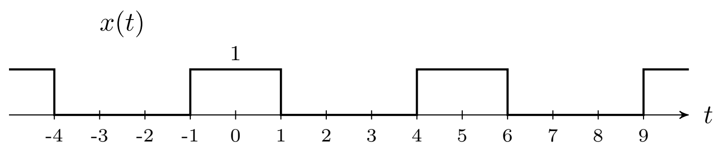
(a) Find the fundamental frequency ω_o and the Fourier series coefficients c_k of the signal $x(t)$.

(b) Sketch the line spectrum of $x(t)$.

(c) If $x(t)$ is filtered with a continuous-time LTI system with the frequency response shown, then find the output signal $y(t)$.



3.2.11 The continuous-time signal $x(t)$ is periodic with period 5 seconds.



(a) The signal $x(t)$ is filtered with an ideal low-pass filter. If the cut-off frequency of the low-pass filter is 0.1 Hz, then find and sketch the output signal $y(t)$.

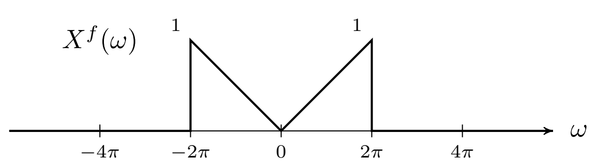
(b) The signal $x(t)$ is filtered with an ideal *high*-pass filter. If the cut-off frequency of the high-pass filter is 0.1 Hz, then accurately sketch the output signal $y(t)$.

3.3 Modulation

3.3.1 In amplitude modulation (AM) the signal $x(t)$ to be transmitted is multiplied by $\cos(\omega_o t)$. Usually, a constant is added before the multiplication by cosine. If $y(t)$ is given by

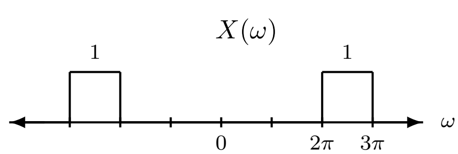
$$y(t) = (2 + x(t))\,\cos(4\pi t)$$

and the spectrum $X^f(\omega)$ is as shown,

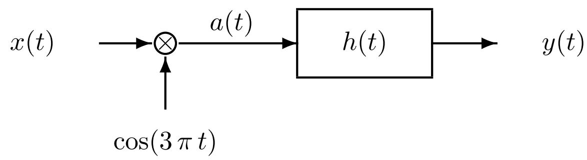


then accurately sketch the spectrum $Y^f(\omega)$.

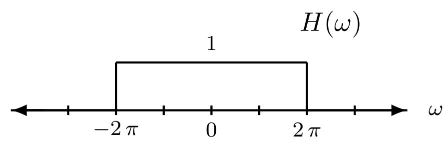
3.3.2 Suppose the signal $x(t)$ has the following spectrum $X(\omega)$.



Suppose the signal $x(t)$ is sent through the following continuous-time system.



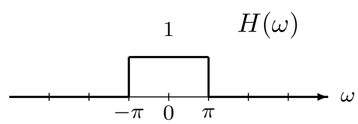
where the frequency response $H(\omega)$ is an ideal low-pass,



(a) Sketch the spectrums (Fourier transforms) of $\cos(3\,\pi\,t)$, $a(t)$, and $y(t)$.

(b) Is the total system an LTI system?

3.3.3 Suppose $h(t)$ is the impulse response of a lowpass filter with frequency response shown.



Suppose a new system is constructed so that its impulse response $h_2(t)$ is given by

$$h_2(t) = 2\,\cos(6\,\pi\,t) \cdot h(t).$$

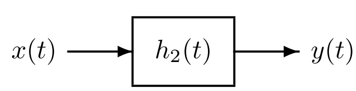
(a) Sketch the frequency response of the new system.

(b) Is the new system a low-pass, band-pass, or high-pass filter?

(c) If the signal

$$x(t) = 2\cos(3\pi\,t) + 4\sin(6\pi\,t)$$

is sent through the new system, what is the output $y(t)$?



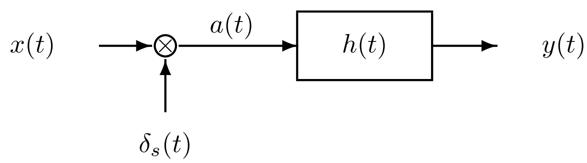
4 The Sampling Theorem

4.0.1 The Fourier transform of a signal $x(t)$ satisfies

$$X(\omega) = 0 \quad \text{for} \quad |\omega| \geq 8\,\pi.$$

What is the maximum sampling period T such that $x(t)$ can be recovered from the samples $x(nT)$?

4.0.2 For the following continuous-time system



the input to the filter $h(t)$ is the product of $x(t)$ and $\delta_s(t)$:

$$a(t) = x(t) \cdot \delta_s(t)$$

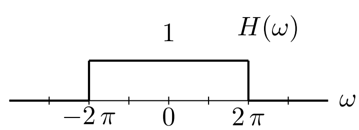
and the impulse train is given by

$$\delta_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\,T_s)$$

where the sampling period is

$$T_s = \frac{2}{3}.$$

The frequency response $H(\omega)$ is an ideal low-pass,



Suppose the input signal is

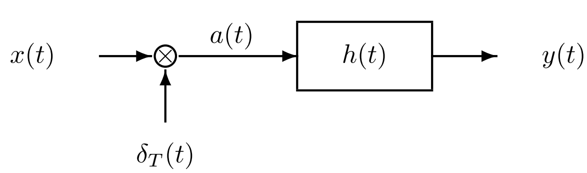
$$x(t) = 4\,\cos(6\,\pi\,t).$$

(a) Sketch the spectrums (Fourier transforms) of $x(t)$, $\delta_s(t)$, $a(t)$, and $y(t)$.

(b) Find and sketch $a(t)$ and $y(t)$.

(c) Does $y(t) = x(t)$? Does this contradict the sampling theorem? Explain.

4.0.3 For the following continuous-time system



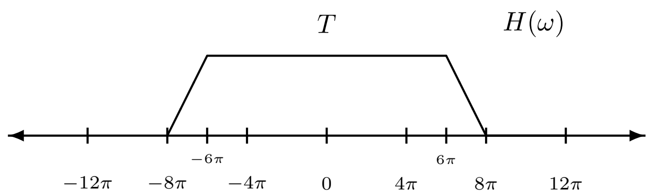
the input to the filter $h(t)$ is the product of $x(t)$ and $\delta_T(t)$:

$$a(t) = x(t) \cdot \delta_T(t)$$

and the impulse train is given by

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\,T).$$

The filter $h(t)$ is *not* an ideal low-pass filter. Instead, its frequency response $H(\omega)$ is



What is the slowest sampling rate such that $y(t) = x(t)$ when . . .

(a) the spectrum of the signal $x(t)$ satisfies

$$X(\omega) = 0 \quad \text{for} \quad |\omega| \geq 6\,\pi.$$

(b) when the spectrum of the signal $x(t)$ satisfies

$$X(\omega) = 0 \quad \text{for} \quad |\omega| \geq 4\,\pi.$$

Express the sampling rate in samples/sec.