Experiment 12: Simple Harmonic Motion

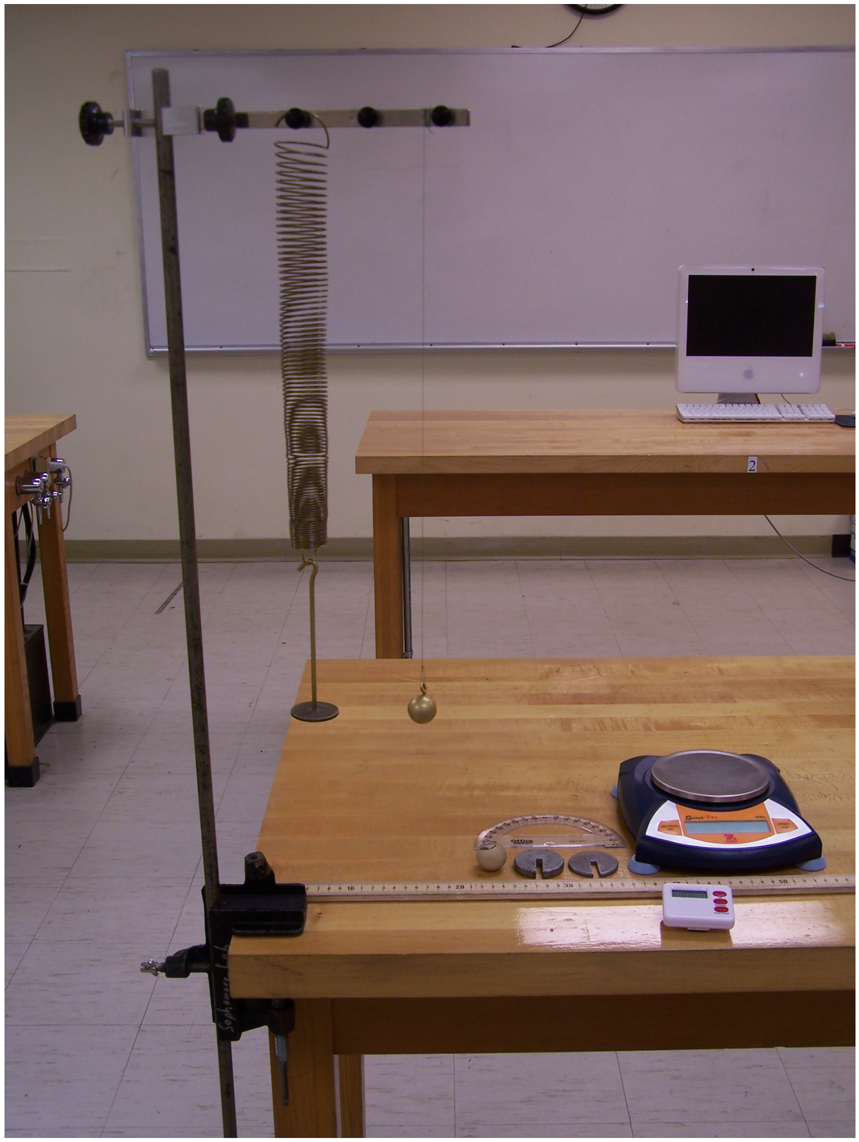


Figure 12.1

EQUIPMENT

Spring
Metal Ball
Wood Ball
(*Note: sharp hooks*)
Meter Stick
Digital Balance
Stopwatch
Pendulum Clamp and Rod
String
Masses: (2) 100g, (1) 50g
Mass Hanger
Table Clamp
Protractor

Advance Reading

Text: Simple harmonic motion, oscillations, wavelength, frequency, period, Hooke's Law.

Lab Manual: Appendix C

Objective

To investigate simple harmonic motion using a simple pendulum and an oscillating spring; to determine the spring constant of a spring.

Theory

Periodic motion is "motion of an object that regularly returns to a given position after a fixed time interval." *Simple harmonic motion* is a special kind of periodic motion in which the object oscillates sinusoidally, smoothly. Simple harmonic motion arises whenever an object is returned to the equilibrium position by a *restorative force* proportional to the object's displacement.

$$F = -kx \tag{12.1}$$

The illustrative example above is *Hooke's Law*, which describes the restorative force of an oscillating spring of stiffness k (spring constant).

For an ideal, massless spring that obeys Hooke's Law, the time required to complete an oscillation (period, T, seconds) depends on the spring constant and the mass, m, of an object suspended at one end:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{12.2}$$

The inverse of period is the frequency of oscillation. Recall that frequency, f, is the number of oscillations completed by a system every second. The standard unit for frequency is hertz, Hz (inverse second, s^{-1}).

The period of oscillation of an ideal, simple pendulum depends on the length, L, of the pendulum and the acceleration due to gravity, g:

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{12.3}$$

When setting the pendulum in motion, small displacements are required to ensure simple harmonic motion. Large displacements exhibit more complex, sometimes chaotic, motion. Simple harmonic motion governs where the *small angle approximation* is valid:

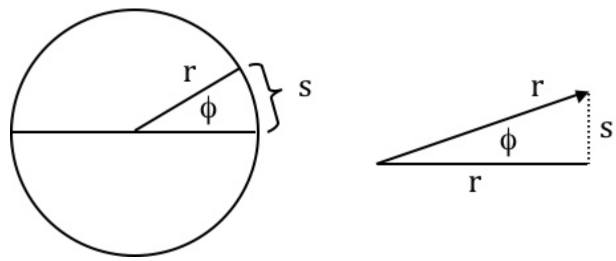


Figure 12.2: Small Angle Approximation

The arc length, s, of a circle of radius r is:

$$s = r\phi \tag{12.4}$$

When ϕ is small, the arc length is approximately equal to a straight line segment that joins the two points. Therefore, the following approximations are valid:

$$\phi \approx \sin\phi \approx \tan\phi \tag{12.5}$$

Name: ____

1. Define simple harmonic motion. What conditions must be met? (20 pts)

2. What physical phenomenon does the relationship $T = 2\pi\sqrt{\frac{m}{k}}$ describe? (20 pts)

3. What physical phenomenon does the relationship $T = 2\pi\sqrt{\frac{L}{g}}$ describe? (20 pts)

4. The following data were collected for Part 1 of the lab procedure. Complete the table. The force is due to the gravitational force. All distances are measured from the *bottom of the hanger to the top of the stool.* You should ignore the initial weight of the hanger. *Note* that Δx is the change from initial position, $x_f - x_0$, not the change from the previous position, $x_2 - x_1$. (40 pts)

Mass (g)	**Height (cm)**	Δx **(m)**	**Force (N)**
0	**57.5**	0	0
100	**46.5**		
200	**36.5**		
300	**25.5**		
400	**15.5**		
500	**4.5**		

Name: ______________________________ Section: __________ Date: ____________

Worksheet - Exp 12: Simple Harmonic Motion

Objective: To investigate simple harmonic motion using a simple pendulum and an oscillating spring; to determine the spring constant of a spring.

Theory: Simple harmonic motion describes an object that is drawn to equilibrium with a force that is proportional to its distance from equilibrium. Such a restorative force will return an object to equilibrium after a fixed time interval, regardless of where the object is placed. The time it takes for an oscillator to return the object to its original position is called the period of oscillation.

One example of a harmonic oscillator is a **spring** that obeys Hooke's Law ($F = -kx$). The period of an ideal, massless spring is related to the spring constant, k (or spring stiffness), and the mass of the object, m, that it moves:

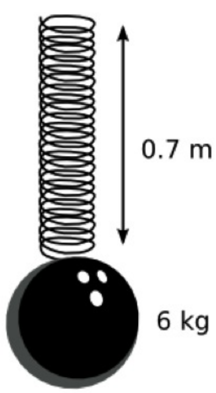
$$T = 2\pi\sqrt{\frac{m}{k}}$$

The other harmonic oscillator modeled in this experiment is the ideal simple **pendulum**, whose period is related to its length, L, and the pull of gravity:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

1. A 6 kg bowling ball is hung from a spring of unstretched length 0.5 m. It stretches the spring to 0.7 m as shown. Find the spring constant of this spring. (3 pts)

   $k =$ ____________ N/m



2. The restorative force of a pendulum is the part of gravity that acts perpendicular to the pendulum arm: $F = -mg\sin\theta$. For small angles, this force is directly proportional to displacement because $\sin\theta \approx \theta$. To verify this, convert 20° to radians, then compute the sine of the angle. (5 pts)

   $\theta =$ ______________

   $\sin\theta =$ ____________

   % Diff = ____________

   By the small angle approximation, then, the pendulum behaves as a simple harmonic oscillator: $F = -(mg)\theta$.

Procedure:

Part 1: Spring Constant: Hooke's Law

3. Hang the spring from the pendulum clamp and hang the mass hanger from the spring. Measure the initial height x_0 above the floor.

4. Add 50 g to the mass hanger and determine the change in position caused by this added weight.

5. Add 50 g masses incrementally until 250 g has been added to the mass hanger. Determine the total displacement and the total added weight with each addition

Added Mass	Force Applied	Δx
0 kg (hanger only)	0 N	0 m ($x = x_0$)

(15 pts)

6. Generate a graph of F *vs.* Δx using Graphical Analysis. Analyze the graph with a linear fit; print a copy for each partner and staple it to this datasheet. (6 pts)

7. Does force vary directly with displacement? Do we expect this system to undergo simple harmonic oscillation? (5 pts)

8. What is the slope of the graph? ____________________ What does the slope of this graph represent? (Hint: What value is obtained when two proportional quantities are related in this way?) (7 pts)

Part 2: Spring Constant: Oscillation

> *Note:* The spring used for this experiment is not ideal; its mass affects the period of oscillation. Account for this by adding 1/3 the mass of the spring to the value of suspended mass, m, in your calculations.

9. Measure the mass of the spring, mass hanger, and 100 g mass

10. Hange the spring from the pendulum clamp.
    Hang the mass hanger + 100 g from the spring.

11. Pull the mass hanger down slightly and release it to create small oscillations. Measure the time required for 20 oscillations. (This is like measuring one period twenty times over.)

12. Calculate the period for the oscillating spring. (3 pts)

    $T =$ ________________

13. Calculate the spring constant of the spring using your knowledge of the object's mass and period of oscillation. Show your work. (6 pts)

    $k =$ ________________

14. a) Compare this spring constant (*Step 13*) to the slope found in *Step 8*. Are these values similar? (3 pts)

    b) What are some uncertainties involved when using the Hooke's law method? (5 pts)

    c) What uncertainties are involved with the oscillation method? (5 pts)

Simple Pendulum

15. Measure the mass of the metal ball. $m_M =$ ________________ (2 pts)
16. Construct a simple pendulum 100.0 cm in length using the metal ball and some string. (L is measured from the center of mass of the ball)
17. Move the pendulum from equilibrium (about 10°-20°) and release it. Measure the time required for 20 oscillations.
18. Determine the period. Record it in the table provided.
$T = \frac{\text{time required}}{20\ cycles}$
19. Shorten L in increments of 20.0 cm and measure T for each length.
20. Repeat the procedure using the wooden ball. Mass of the wooden ball. m_W= ________________ (2 pts)

Period of Oscillation:

	100 cm	80 cm	60 cm	40 cm	20 cm	0 cm
Metal						
Wood						

(12 pts)

21. Produce graphs of T^2 *vs.* L for each ball. Apply a linear fit; print a copy for each partner (6 pts). Squaring both sides of the simple pendulum equation, it is apparent that the slope of a T^2 *vs.* L graph should be equal to $4\pi^2/g$. From the slopes of the graphs, calculate g. Show work. (10 pts)

	g (Graph)	% Error
Metal		
Wood		

22. Consider the pendulums pictured at the right. Describe how one would determine how much more time pendulum A takes to travel a distance $2x$ than for pendulum B to travel x, given they have the same length. Assume the pendulums are at rest at the positions in the figures, then released. (5 pts)