Name $\qquad$
Partner(s): $\qquad$

## Experiment 3 - Forces are Vectors

Objectives Understand that some quantities in physics are vectors, others are scalars. Be able to perform vector addition graphically (tip-tail rule) and with components.
Understand vector components.
Be able to apply these concepts to displacement and force problems.
Preparation You will be pressed for time during the lab. Since successful completion of all lab activities counts towards your final lab grade it will be important to be well prepared by doing Pre-Lab assignments and reading the entire lab before attending the lab.

Pre-Lab Read the Pre-Lab introduction and answer the accompanying questions and problems before this Lab.


## Pre-Lab for LAB\#3

## Intro Vectors and Trigonometry

Vectors may be used to represent anything that has both magnitude and direction: displacement, velocity, acceleration, force, etc.

## Definitions Sides of a Right Triangle

In the right triangle shown at right, the sides relative to the angle $\theta$ are designated as

$$
\begin{array}{ll}
c=\text { hypotenuse } & \\
\text { (hyp) } \\
a=\text { adjacent side } & \\
b=\text { opposite side } & \\
\text { (opp) }
\end{array}
$$


a

Side $c$ is the longest side and is opposite to the right angle.

## Relationship of Sides of a Right Triangle

The sides of a right triangle are related to each other through the Pythagorean theorem,

$$
c^{2}=a^{2}+b^{2}
$$

and through the trigonometric functions,

$$
\sin \theta=\frac{\text { opp }}{\text { hyp }}=\frac{b}{c} \quad \cos \theta=\frac{\text { adj }}{\text { hyp }}=\frac{a}{c} \quad \tan \theta=\frac{\text { opp }}{\text { adj }}=\frac{b}{a}
$$

## Relationship of Components to a Vector

In a coordinate system, a vector that is not parallel to either coordinate axis can be resolved into components that are parallel to the coordinate axes. The vector sum of the components is equivalent to the original vector.

The lengths of the components of the vector
 can be related to the length (magnitude) of the vector by the trigonometric functions. In the figure at right showing vector $A$, if the angle $\theta$ is measured with respect to the $x$ axis of the coordinate system, where $\theta$ is positive when measured counterclockwise from the $x$-axis, the components of the vector can be calculated using the trigonometric functions:

$$
A_{x}=A \cos \theta \quad A_{y}=A \sin \theta
$$

These relationships are valid for a vector in any quadrant as long as $\theta$ is measured with respect to the $x$-axis.

## Pre-Lab for LAB\#3

## Concepts Vectors and Forces

- Scalar - a quantity that is measured by magnitude only.
- Vector - a quantity defined by both magnitude and direction
- Scalar addition - the algebraic sum of two or more quantities
- Vector addition - If two vectors are parallel, being in the same (opposite) direction, their magnitudes can be added (subtracted) to obtain the magnitude of the Resultant Vector. If the two vectors are not parallel, adding them requires establishing an $x-y$ coordinate system, then breaking down each vector into its " $x$ " and " $y$ " components before algebraically adding these vector components together to yield the Resultant Vector's " $x$ " and " $y$ " components. The Resultant Vector's magnitude is then calculated as the hypotenuse of the $x-y$ vector triangle. The angle of the Resultant Vector from a designated coordinate axis uses the Tangent function of the $x-y$ Resultant Vector components.
- Weight - a force vector (magnitude $w=m g$ ) which is in the direction of gravitational acceleration ( $g$ - down, toward the center of the Earth)
- Net Force - the resultant vector that is the sum of all forces being applied to an object.
- Equilibrant Force - one that is equal in magnitude and opposite in direction to the Net Force. The Equilibrant Force balances the Net Force causing static equilibrium.


## Pre-Lab for LAB\#3

Problem 1 3-Put - An example in Vector Addition (or poor golf skills)
A golfer, putting on a green requires three strokes to "hole the ball." During the first putt, the ball rolls 5.0 m due east. For the second putt, the ball travels 2.1 m at an angle of $20^{\circ}$ north of east. The third putt is 0.50 m due north. What displacement (magnitude and direction relative to due east) would have been needed to "hole the ball" on the very first putt? Use components to solve this problem.

Solution Identify the three vectors.
Sketch the vectors and show the vector sum. Include a coordinate system.

Identify the components of the three vectors (labeled $a, b, c$ )

$$
\begin{array}{ll}
a_{\mathrm{x}}= & a_{\mathrm{y}}= \\
b_{\mathrm{x}}= & b_{\mathrm{y}}= \\
c_{\mathrm{x}}= & c_{\mathrm{y}}=
\end{array}
$$

Determine the components of the resultant vector (labeled s)

$$
s_{x}=\quad s_{y}=
$$

Convert this into the magnitude and direction of the resultant vector

```
|s|=
0=
    (measured from the positive x axis)
```


## Pre-Lab for LAB\#3

Problem 2 At a picnic, there is a contest in which hoses are used to shoot water at a beach ball from three different directions. As a result, three forces act on the ball, $\mathbf{F}_{1}, \mathbf{F}_{2}$, and $\mathbf{F}_{3}$ (see drawing). The magnitudes of $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$ are $\mathrm{F}_{1}=50.0 \mathrm{~N}$ and $\mathrm{F}_{2}=90.0 \mathrm{~N}$. $F_{1}$ acts under an angle of $60^{\circ}$ with respect to the $x$-axis and $F_{2}$ is directed along the $x$-axis. Find the magnitude and direction of $F_{3}$ such that the resultant force acting on the ball is zero.


# Laboratory 

## List of Today's Activities

Check Pre-Lab<br>Introduction Introduction to the equipment. What is expected of students.<br>Lab Activity<br>Lab Activity<br>Lab Challenge What is the "mystery" mass?<br>Find an Unknown Mass using the Force Table

## Activity 1 The Treasure Map

Equipment White boards, Markers, Protractor, Rulers
Scenario An old pirate map gives you instructions how to locate a treasure from a "Startin' Pointe" (SP); the SP is an identified landmark. Unfortunately, the "map" part of the treasure map is gone and only the instructions survived. Your instructor will write them the board and your job is it to reconstruct the treasure map.

Solve this problem using your whiteboard. Your instructor has the solution and will check your answer.

Exercise 1 Use the space below to copy the instructions from the original treasure map.

Exercise 2 Before you start to draw you first convert the old units ("paces") used by the pirates to standard units (meter).

$$
1 \text { pace }=0.4 \mathrm{~m}
$$

Then use the scale

$$
1 \mathrm{~cm}(\text { white board })=1 \mathrm{~m} \text { (real) }
$$

to convert the real vectors from the map's instructions to scaled vectors you will use in your reconstruction of the map. Mark the starting point (SP) on your whiteboard, setup a coordinate system and draw the map to locate the treasure. You need to be precise! Use a ruler and the protractor.

Call your instructor once your map is complete to check whether you found the treasure or are not even close.

Exercise 3 On your map, measure the components of the vector connecting the SP to the location of the treasure and record your results in the table below. Then convert them to standard units (meter).

| $x$-component ("scaled" units) | $x$-component (meter) |
| :--- | :--- |
| $y$-component ("scaled" units) | $y$-component (meter) |
|  |  |

Exercise 4 Instead of using the graphical tip-tail rule you can also solve this problem using trigonometry and vector components. Use the instructions given on the map to calculate the position of the treasure, i.e. the components of the vector sum; use "real" units (meter).
$x$-component (meter):

## $y$-component (meter):

## Exercise 5 Error Analysis

How well did you do? Compare your drawn and calculated components; use "real" units.

## Error on $x$-component in percent:

$$
\begin{aligned}
\text { Error } & =[(\text { map-component }- \text { trig.-component }) / \text { trig.-component }] \times 100 \\
& =
\end{aligned}
$$

## Error on $y$-component in percent:

$$
\begin{aligned}
\text { Error } & =[(\text { map-component }- \text { trig.-component }) / \text { trig.-component }] \times 100 \\
& =
\end{aligned}
$$

## Activity 2 The Force Table

Equipment Force table, Weights, Metal hangers (5-gram)
The physical quantities you will be dealing with to illustrate vector addition will be forces. The apparatus that you will be using is called the "Force Table" and is illustrated at right. It is a large metal disk ruled in degrees like a protractor. Three pulleys are clamped to the edge of the table; they can be set at any angles. Different masses hang from strings passing over the pulleys. The pulleys merely change the direction of the force exerted by the strings, from downward to outward along the surface of the table. These strings are tied to, and pull on, a


The Force Table central ring that is free to move. If the forces from the three strings balance (add vectorially to zero) the ring will remain at rest.

## PROCEDURE FOR FORCE TABLE PROBLEMS

1. The intent of the problems below is to have you experimentally verify your calculations involving vectors. When you have finished each calculation and have verified experimentally that the forces balance, have your instructor check your work before you move on to the next problem.
2. Use the force table correctly. Be certain that all the strings point toward the center post of the table so the forces on the ring are radially outward. You may have to adjust the attachment positions of the strings on the ring and you may have to adjust the pulley positions.
3. Choose the 0 degree mark on the force table to indicate the (positive) $x$ axis.
4. Make sure that the balanced ring is centered on the post. (If the ring is touching the post, your forces are not balanced.)
5. Forces will be discussed extensively in lecture (and recitation) so don't worry if you don't think you really understand them at this point. In this lab we use forces only as an example for a vector quantity. All you need to know for today's activities is that the forces on the force table are closely related to the masses you attach to the string. In fact, if you use a mass $m$, the magnitude of the force is

$$
F=m g
$$

Where $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ is the acceleration due to gravity. The mass is measured in kilograms [ kg ] and the force is measured in newtons [N].

Note The metal hangers on which the weights are hung have a mass of 5 grams each. This mass should be included in the hanging masses in your calculations.

## Exercise 1 Do weight hangers of equal weight always balance each other out?

1. Place equal masses (about 20 grams) on each hanger.
2. Experimentally determine whether or not hangers of equal weight balance each other regardless of their position. State your observations here:

## Exercise 2 Verifying Components of a Force with the Force Table

1. Setup the force table with a force $\mathbf{F}$ that has a magnitude of 2.5 N at an angle of $37^{\circ}$ above the x -axis (recall $w=m g$; use $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ ).
a. What mass do you need to attach to the string to get a force of 2.5 N ?
mass: $\qquad$ kg
b. Use trigonometry to calculate the components of this force vector.
$x$-component: $\qquad$ N
$y$-component: $\qquad$ N
2. Check your results experimentally using the force table.
a. What amount of mass (in grams) has a weight equal to the magnitude of the $x$-component of force $\mathbf{F}$ ?
b. What amount of mass (in grams) has a weight equal to the magnitude of the $y$-component of force $\mathbf{F}$ ?


To balance a force $\mathbf{F}$ with the force table you need to apply a second force in the opposite direction: -F, as is shown in the drawing at left. To confirm this attach a second mass of the value you determined above and attach it to another string on the force table. Arrange the pulley so that the two forces are back to back.

Is the ring in balance?
Yes
No

c. Back to our components. The components of force $\mathbf{F}$ acting together should be equivalent to force $\mathbf{F}$. To check this, replace force $\mathbf{F}$ (the original force) with its components by hanging the appropriate masses so that the strings pull in the positive $x$ and positive $y$ axis as shown in the drawing.

Do the components of $\mathbf{F}$ balance force $\mathbf{- F}$ ?
Yes
No

## Exercise 3 Balancing the Force Table with 3 Forces

Your instructor will tell you the masses of two of the hanging weights, and the angles at which the strings pull. These correspond to 2 of the 3 forces. Your job is to calculate the mass of the third weight, and the angle at which its string pulls, so the three forces add to zero. You then hang the calculated mass at the calculated angle and demonstrate that the system is balanced, within the experimental uncertainty.

Note In the following Example and Problem and in Activity 3, for convenience, treat the unit of grams as a force: 1 gram = 1 "gram-weight" ( $1 \mathrm{gm}-\mathrm{w}$ ).

Example The drawing below should help you to visualize this problem. Two force vectors $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$ as well as the vector sum $\mathbf{R}=\mathbf{F}_{1}+\mathbf{F}_{2}$ are shown. The vector $-\mathbf{R}$, called the equilibrant, is equal in magnitude but opposite in direction to the resultant $\mathbf{R}$ of $\mathbf{F}_{\mathbf{1}}$ and $\mathbf{F}_{\mathbf{2}} ;$ it is the vector that must be added to $\mathbf{F}_{\mathbf{1}}+\mathbf{F}_{\mathbf{2}}$ to give a total vector force of zero: $\mathbf{F}_{\mathbf{1}}+\mathbf{F}_{\mathbf{2}}+(-R)=\mathbf{0}$.


Problem Treat the two mass values as forces (in units of gm-w) and enter the magnitudes and angles of the two given forces $\mathbf{F}_{\mathbf{1}}$ and $\mathbf{F}_{\mathbf{2}}$ in the table below. Calculate and fill in the $x$ - and $y$-components of these vectors and those of the resultant vector $\mathbf{R}$.
From the components of $\mathbf{R}$, find the components of the "equilibrant" $\mathbf{- R}$.

| Table for Exercise 3 |  |  |  |  | Sketch: |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{\|l\|} \hline \begin{array}{l} \text { Magni- } \\ \text { tude } \\ \text { [gm-w] } \\ \hline \end{array} \\ \hline \end{array}$ | $\begin{aligned} & \hline \text { Angle } \\ & \text { to }+x \\ & \text { [deg] } \\ & \hline \end{aligned}$ | Comp [gm-w] | Comp [gm-w] |  |
| $\mathrm{F}_{1}$ |  |  |  |  |  |
| $\mathrm{F}_{2}$ |  |  |  |  |  |
| R |  |  |  |  |  |
| -R |  |  |  |  |  |

Hint Making a sketch like the example drawing above can be quite helpful.
The "equilibrant" $-\mathbf{R}$, represents the missing ( $3^{\text {rd }}$ ) force you need to balance $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$. Complete the table and calculate the magnitude and the direction of the vector $-\mathbf{R}$.

Magnitude: $\qquad$ gm-w

Angle with respect to the positive $x$-axis: $\qquad$ degrees

Verify $\quad$ Now setup $\mathbf{F}_{\mathbf{1}}, \mathbf{F}_{\mathbf{2}}$ and $-\mathbf{R}$ on your force table and show experimentally to your instructor that the sum of these three forces on the ring adds to zero. Remember that $\mathbf{5}$ grams is already present in the metal hangers.

## Activity 3 Lab Challenge: What is the "mystery" mass?

Note For convenience, treat the unit of grams as a force: 1 gram = 1 "gram-weight" (1 gm-w).

Your challenge today is to find the mass of an unknown object using the force table. To make it more dramatic let's call this unknown object the "mystery mass".
Here is how you set this up: The mystery mass is attached to a string and represents one of the forces on the force table. Align the mystery mass so that its string is along the positive x axis on the force table.

You will balance the weight-force of the mystery mass with two forces, $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$, similar to the previous activity.
Adjust the angles and the magnitudes of $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$ until the Force Table is balanced, but be sure that $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$ are at least $30^{\circ}$ apart.
When the force table is balanced the (vector) sum of $\mathbf{F}_{1}, \mathbf{F}_{2}$, and the weight-force of the mystery mass is 0 . Use this to determine the weight and then the mass of the mystery mass.

List the magnitudes (gm-w) and angles of the 2 forces in the table.

|  | Magnitude | Angle |
| :---: | :---: | :---: |
| $\mathrm{F}_{1}$ |  |  |
| $\mathrm{~F}_{2}$ |  |  |

Use vector components to solve for the unknown magnitude of the force from the "mystery" mass and then find the "mystery" mass itself.

[^0]$\qquad$

After your instructor has checked your force table, use the scales in the lab room to measure the "mystery" mass.
"mystery" mass scale $=$ $\qquad$

| Analysis | How well did you do? Calculate the percent error to compare the two values of the "mystery" mass: |
| :---: | :---: |
| Error | $=\left[" m y s t e r y "\right.$ mass $_{\text {force table }}$ - "mystery" mass scale $/$ "mystery" mass scale $] \times 100$ |
|  | $=\ldots$ |

## End of Lab 3


[^0]:    "mystery" mass $_{\text {force table }}=$

