

Experiment No. 45(a)

GEAR MEASUREMENT

Aim: To measure spur gear tooth elements.

Instruments: Flange micrometer, Gear tooth vernier caliper.

Theory:

Gears are mainly used for transmission of power and motion. It is a round wheel that has teeth, which meshes with another gear allowing force to be fully transferred without slippage. Depending on their construction and arrangement, geared devices can transmit forces at different speeds, torques, or in a different direction, from the power source. A gear can also mesh with any device having compatible teeth, such as linear moving racks. For closer control over the accuracy of manufacture of the gear, precision measurement of gear plays a vital role. A brief overview of different types of gear has been documented herewith.

Spur Gear - The edge of each tooth is straight and aligned parallel to the axis of rotation.



Fig 1 Spur gear

Helical Gear - The leading edges of the teeth are not parallel to the axis of rotation, but are set at an angle. The angled teeth engage more gradually than do spur gear teeth. This causes helical gears to run more smoothly and quietly than spur gears.



Fig 2 Helical gear

Bevel gear - The angle between the shafts of mating gears can be anything except zero or 180 degrees. Bevel gears with equal numbers of teeth and shaft axes at 90 degrees are called **miter gears**.



Fig 3 Bevel gear

Worm gear – Type of helical gear, but its helix angle is usually somewhat large and its body is usually fairly long in the axial direction; and it is these attributes which give it screw like qualities.



Fig 4 Worm gear

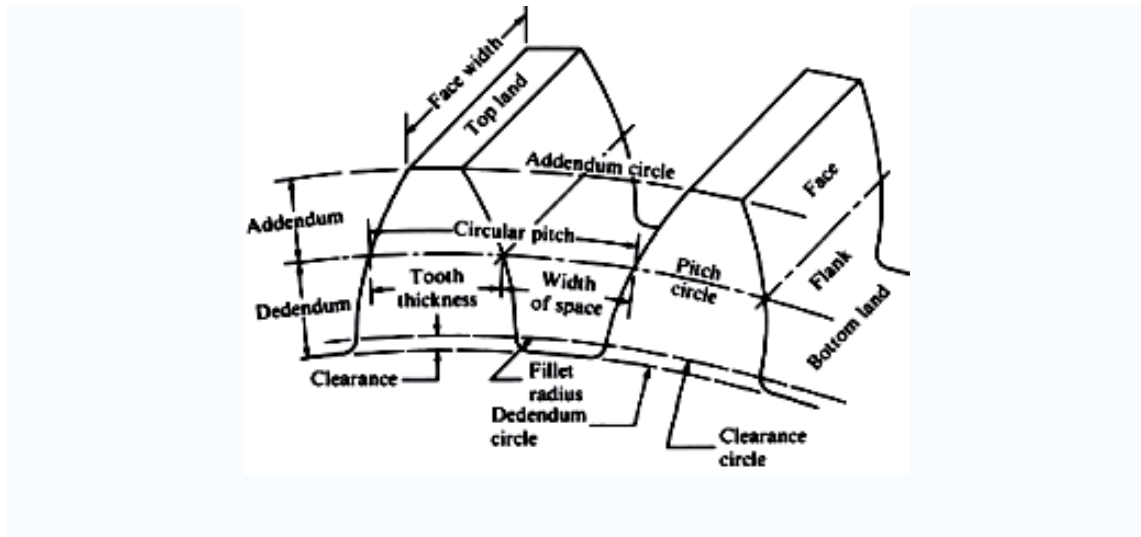


Fig.5. Gear nomenclature.

Gear geometric nomenclature:

The **tooth thickness** is generally measured at pitch circle shown in Fig 5. The curve most commonly used for gear-tooth profile is the **involute** of a circle. It may be defined as the curve traced by a point on a taut (line BC in Fig.6), inextensible string as it unwinds from another circle. The circle from which the involute is derived is called the base circle. The involute profile is shown in Fig.6. **Pressure Angle** (ψ) is defined as the angle between the line of action and the common tangent to the pitch circles shown in Fig.7. The base radius and the pitch radius are r_b and r_p respectively. Pitch circle radius is denoted by symbol R. The involute function (δ) is found from the fundamental principle of involute.

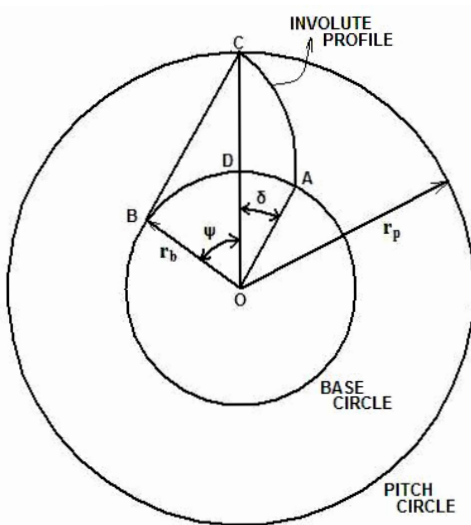


Fig.6. Involute profile

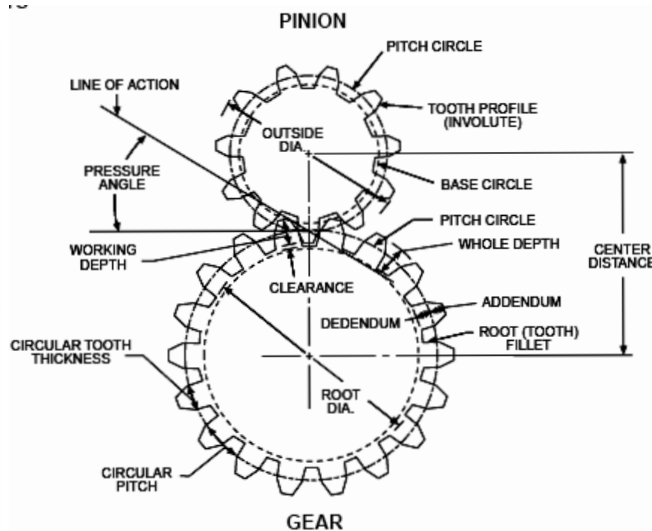


Fig.7. Gear in mesh with a

pinion

From the Fig.6

$$OB = OC \cos \psi = r_b$$

$$\text{Arc } BD = r_b \psi \text{ (radians)}$$

$$\delta = \tan \psi - \psi \text{ (radians)}$$

$$BC = \text{arc } AB = r_b \tan \psi$$

$$r_b \delta = r_b \tan \psi - r_b \psi$$

The following gear tooth elements are measured using vernier

- Outside diameter
- Diametral pitch
- Pitch circle diameter
- Module
- Addendum
- Dedendum
- Tooth thickness of all teeth

- **Outside diameter (O.D):** Using vernier caliper, measure the outer diameter of the given gear with 6 different readings and record in the following tabular columns. If **odd number** of teeth is there then consider the **highest reading** as O.D. If **even number** of teeth is there then consider the **average reading** as O.D.

Sl No.	1	2	3	4	5	6	Avg Dia, mm
Outer diameter, mm							

Outside diameter = _____ mm

- **Diametral pitch (Dp):** $\text{Diametral pitch} = (N+2) / \text{Outside diameter}$
Where, N= Number of teeth
- **Pitch circle diameter (P.C.D):** $\text{Pitch circle diameter} = N / \text{Diametral pitch}$
- **Module (m):** $\text{Module} = 1 / \text{Diametral pitch}$
- **Addendum (a):** $\text{Addendum} = \text{Module}$
- **Dedendum (d):** $\text{Dedendum} = 1.157 \times (\text{Addendum})$
- **Tooth thickness (t):**

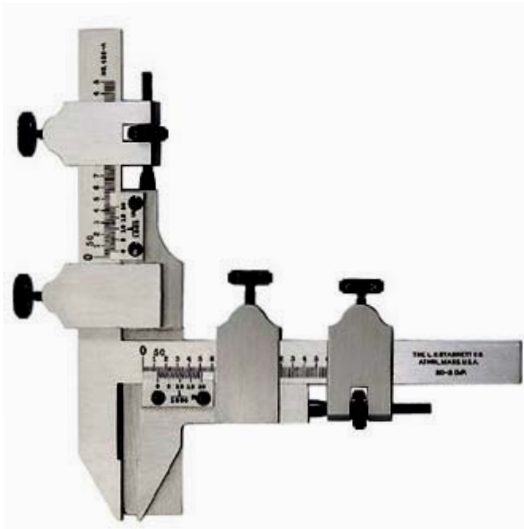


Fig.8. Gear tooth vernier caliper

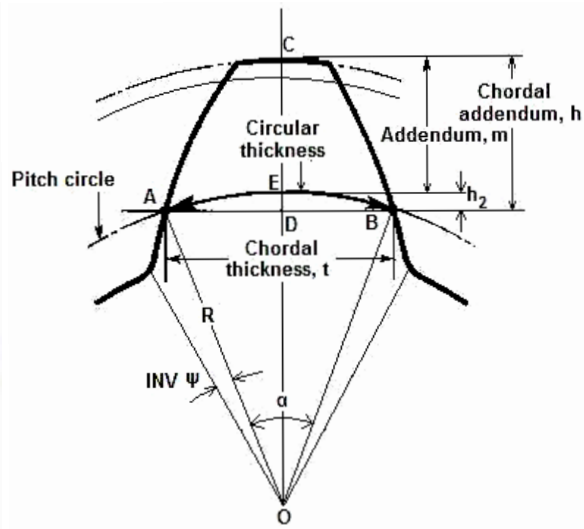


Fig.9. Gear tooth geometry

Tooth thickness is measured by the gear tooth vernier. The vernier shown in Fig.8 consists of two vernier calipers set at 90° to each other. Since the gear tooth thickness varies from the root to the tip, the vernier must be capable of measuring the tooth thickness at a specified

position on the tooth. The tooth thickness is measured at the pitch circle as shown in Fig.9. The thickness of the tooth at pitch line and the addendum is measured by an adjustable tongue, each of which is adjusted independently by adjustable screws on the graduated bars. The gear tooth vernier is set with its vertical scale at a distance equal to chordal addendum so that the thin slit will be at a height 'm' from the tip of the jaw. Hence the gear tooth slit will sit on the top land and the tip of the jaws will measure the chordal thickness, t.

$$h = (m + h_2)$$

$$t = AB = 2AD$$

$$\angle AOD = \alpha / 2 = 2\pi / 4N = \pi / 2N$$

Where, R = Pitch Circle Radius = Nm/2

$$t = 2AO \sin(\alpha / 2) = 2R \sin(\pi / 2N)$$

$$t = Nm \sin(\pi / 2N)$$

$$h_2 = DE = R(1 - \cos(\alpha / 2)) = \frac{Nm}{2}(1 - \cos \pi / 2N)$$

“t” is the chord ADB while tooth thickness is the arc AEB. Therefore the tooth thickness thus measured is called “Chordal tooth thickness”.

Procedure to measure tooth thickness: Start from any one tooth by marking it and measure its thickness as t₁. Leaving one tooth measure the next one as t₂. Leaving another tooth measure the next one as t₃. Follow the procedure till all the 30 teeth are measured.

Observation No.	t ₁	t ₂	t ₃	Mean t	T	V
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

Maximum variation of chordal tooth thickness = (Max mean t – Min mean t)

Chordal tooth thickness from theoretical formula = T

Variation between the observed and theoretical values = V = t-T

Checking the tooth thickness error with Flange Micrometer

[Span measurement/ Base tangent measurement]

From the principle of the involute profile the sum of the generators

$AF + FB = A'F + FB' = \text{arc length CD along the base circle as shown in figure 10.}$

Hence the measurement of the span AB can be taken in any position with the Flange Micrometer touching tooth flange. Any tooth thickness error will show a corresponding error in the value of AB.

Let the number of teeth in the span of AB be “n”

$$\text{Then } \beta = \frac{(n-1)2\pi}{N}$$

$$OD = \frac{Nm \cos(\gamma/2)}{2}$$

$$\gamma = \alpha + 2 \text{inv}\psi$$

where

$$\text{inv}\psi = \tan \psi - \psi (\text{radians})$$

$$CD = (\beta + \gamma)OD$$

$$AB = CD = A'B'$$

$$AB = \frac{(\beta + \gamma)}{2} Nm \cos(\gamma/2)$$

$$\alpha = \pi / N$$

where, $\beta = (n-1)2\pi / N$

$$\gamma = \alpha + 2 \text{inv}\psi$$

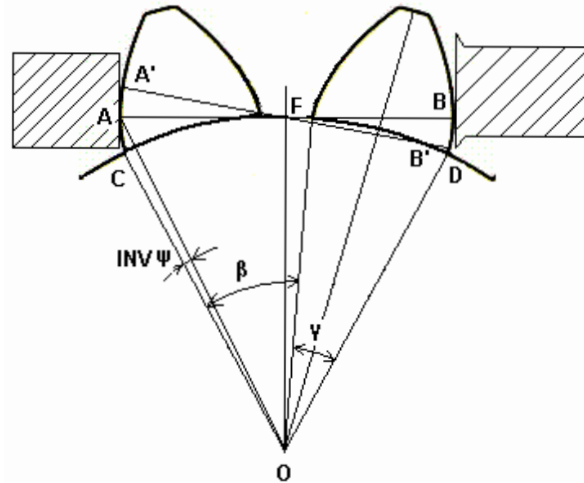


Fig 10. Measurement of tooth thickness error

The optimum number of teeth “n” for the measurement of the span can be found by taking the contact points near the pitch points.

$$AB = AF + FB = CD = 2OF(\psi + \text{inv}\psi)$$

$$n = \text{nearest integer to } \frac{AB \times N}{\pi N m \cos \psi} = \frac{N(\psi + \text{inv}\psi)}{\pi}$$

$$n = \frac{N \tan \psi}{\pi} \quad \text{here } \psi = 20^\circ$$

n = _____

Nearest integer = _____

Range of Micrometer = _____

Take 3 measurements at one place. Repeat those measurements at four places at 90° spacing.

Observation no.	Span1	Span2	Mean value of the span S, mm
1			
2			
3			
4			

Maximum variation in values of the span measured =

Theoretically the value of the span is given by

$$AB = \frac{(\beta + \gamma)Nm \cos(\gamma/2)}{2}$$

$$\text{inv}\psi = \tan \psi - \psi = \tan 20^\circ - 20 \frac{\pi}{180} = \text{_____ radians}$$

$$\alpha =$$

$$\beta =$$

$$\gamma =$$

$$AB = \text{_____ mm}$$

$$\text{Difference between the theoretical and observed values} = (AB - S) = \text{_____ mm}$$

Discussion: Discuss the results, especially the tooth thickness variation.

Experiment No. 5B

Measurement of gear tooth profile using coordinate measuring machine

Objective: To find out the profile error of an involute gear using 'optical CMM'.

Equipment required:

1. Involute gear
2. Coordinate measuring machine (having continuous contact or optical measuring capability)

Theory: Gear is used for power transmission, specially when high power is required to transmit. Pressure angle of a gear plays very important role in power transmission, since it is the angle at which the driving gear applies normal force on the driven gear. The tangential component of this normal force contributes to the torque transmission, and the radial component loads the shaft and bearings. For more power transmission and lesser pressure on the bearings, pressure angle must be kept small. Standard pressure angles are 20° and 25° , gears with 14.5° pressure angle have become almost obsolete. The pressure angle depends on gear tooth profile. If there are errors in gear tooth profile then it will affect the pressure angle and hence direction of power transmission will get affected; which will result in failure of either gear, shaft or bearings. So inspection of gear tooth profile is necessary at periodic interval.

Procedure: Following are few gear tooth profile measuring techniques.

1. Using Toolmaker's microscope connected with profile projector.
2. Using Involute tester.
3. Using Coordinate measuring machine (which has either continuous contact or optical inspection capability).

1. Toolmaker's microscope: Gear tooth profile can be inspected on the screen of toolmaker's microscope connected to a profile projector. First of all the measuring gear tooth profile is projected on the projector's screen, and then standard involute profile is also projected on the same screen. After comparing the measured profile with the standard profile, deviations of the measured tooth profile can be evaluated. But this method is good for gears having module smaller than one. When the width of the gear blank is large, it is necessary to give proper illumination so that the projected tooth should be free of shadow. In order to conduct test on successive teeth the gear blank must be properly clamped on the rotary table of toolmaker's microscope.

Advantages:

- This method can be used for small gears (module up to 1 mm)
- This method can also be used for measurement of tooth thickness, base pitch, space width etc.

Limitation:

- Gear's size limits this technique to use.

2. Involute tester: Figure below shows schematic diagram of ‘The Klingelberg involute tester’ used to test gear tooth profile (an involute curve).

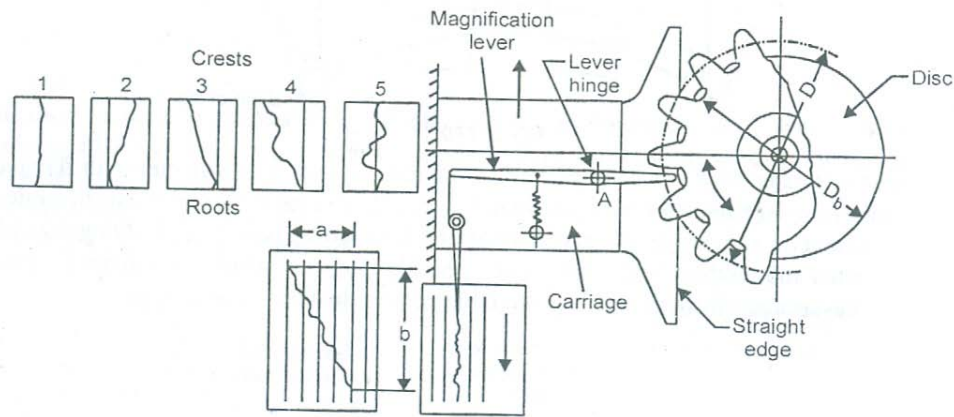


Fig. 1. Schematic diagram of ‘Involute tester’. [Ref.1]

- It consists of a disc whose diameter must be equal to the base circle diameter of the gear under test.
- The gear being tested is mounted on the same spindle on which the disc is fixed.
- A straight-edge is made to contact the disc and to move tangentially to it so that the disc will be rotated without slip.
- A magnification lever fitted with a diamond stylus that contacts the tooth flank at the base circle is hinged at A.
- The movement of the stylus is magnified (250X) through a lever and gear mechanism and then recorded.
- Due to the linear motion of the straight-edge and the rotational motion of the gear tested, the stylus, which will trace an involute curve, will slide over the tooth flank.
- If the profile of the flank is exactly involute, the lever will remain stationary. In this case, the recorded curve is a straight line parallel to the direction of the motion of the flat surface. Irregularities in the straightness of the recorded line indicate errors in the gear tooth profile while the inclination of the recorded line to the direction of motion of the recording paper will indicate the errors of the base circle, i.e. errors of the pressure angle. The actual base circle diameter D_b can be calculated by adding to or subtracting its error e_b from the theoretical base circle diameter D'_b . [Ref.1]

$$D_b = D'_b \pm e_b$$

and

$$e_b = D'_b \frac{a \times M_a}{b \times M_b}$$

Where, M_a = Magnification of the stylus lever and gear mechanism,

M_b = Tangential magnification of the recorder, and

a = Deviation of the recorded line over the recorded length b .


Advantage:

- The principle used in this method is based on the definition of an Involute.

Limitations:

- This is time consuming method.
- Accuracy is limited.

3. Coordinate measuring machine (CMM): Coordinate measuring machine of continuous contact type or optical inspection type can also be used for the gear tooth profile measurement. The machine comes with different hardware configuration like 'rotary table', 'indexing probe' etc. apart from the different software (e.g. 'PCDMIS', 'CALYPSO', 'CONTURA' etc.) which contains options for measuring gears, turbine blades, profile elements, torus, general surfaces, general curves, free form surfaces etc. Yet for measuring gears, turbine blades etc., another software (e.g. 'Gear pro' etc.) is required to be used with the CMM software. So this is the ideal method for gear measurement using CMM. But in the absence of these specific software (e.g. 'Gear pro') gear profile can also be measured using 2D & 3D curves inspection techniques of the CMM software. For that gear must properly be cleaned of any dust, and then it is placed on worktable of the CMM. After part alignment, 'VISCAN' is turned on. Now selecting proper illumination setting and suitable camera zoom, gear's teeth images are seen in 'camera window'. Then for gear tooth profile scanning following steps can be followed:

- Go to Menu bar, & select 2D curve option under 'features' menu. Now you can see one 2D curve in the measurement plan area under 'features' tab.
- Double click on that 2D curve 
- In features window, under the 'Nominal data' drop down menu select 'Additional' & then 'Digitization on'. Now put check mark on 'Stylus correction'.
- Open 'Strategy window' and click on 'Unknown cut'. Set suitable 'Clearance data'.
- Double click on 'Unknown cut' and specify suitable values for 'expected tolerance', 'speed', 'step width' etc. Also specify the 'Start point' & 'End point'.
- Specify 'End criterion' & 'Space axis' and then check the direction of travel of stylus for curve scanning. If the direction is opposite, then change it.
- Click on 'Execute'. Keep full controls on CMM speed using speed regulating knob.
- Once curve scanning will complete, click on 'OK'.
- Go to 'Form & Location' and select 'Curve form'.
- Double click on 'Curve form' and select the scanned curve. Now click on 'OK'.
- Specify the nominal value for the curve and click on the 'Plot' icon. If required, 'custom printout' can also be taken. For that go to 'View' and select 'Custom printout'.
- If it is required to look the profile error of a particular tooth face, then create a theoretical curve and pick feature points using 'Recall feature points', of a particular tooth face from the complete curve.

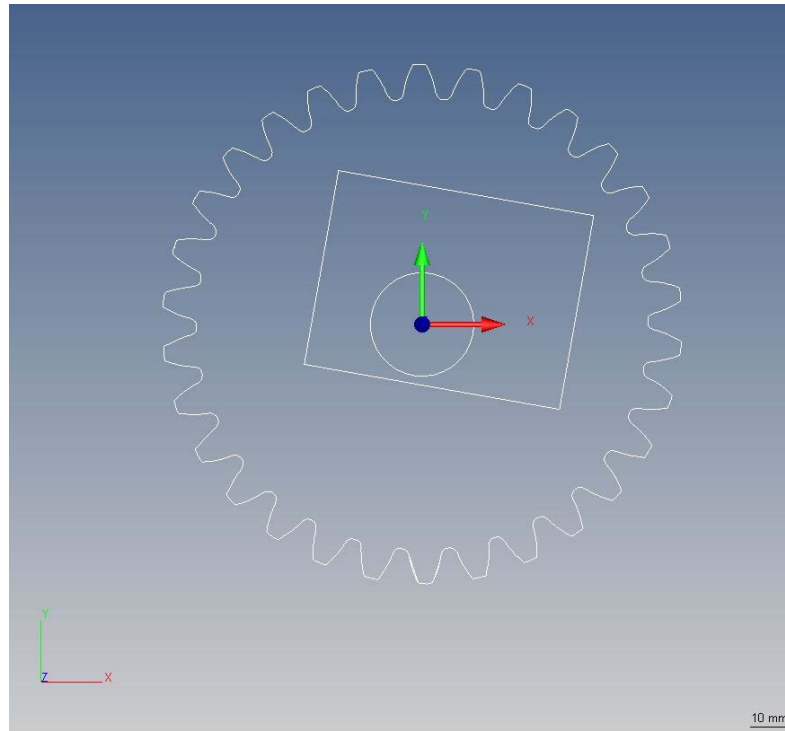


Fig.2. Measured gear at the CMM software's cad window. {Inside features (rectangle & circle) are the features of a measured gear, used just for part alignment.}

Results:

Sr. No.	Curve form	Nominal	Actual	Deviation
01.				
02.				
03.				
04.				
05.				

Conclusion:

From the measured gear tooth profile (shown in 'Appendix') it can be seen that, the wear is maximum at the place where one tooth starts touching the mating gear's tooth. This is because of slightly impact loading (due to backlash in gear) on the tooth at that zone. This wear goes on decreasing towards pitch point. At the pitch point wear is zero and after that again it goes on increasing towards the addendum.

Advantages:

- It is less time consuming method.
- It gives very accurate (accuracy ≈ 1 nm) & reliable results.
- Depending upon the workspace & load bearing capability of the machine's worktable, big size gears can also be measured.
- All dimensional elements of a gear (e.g. tooth thickness, pitch, space width etc.) can be measured.

- Using 'Gear pro' software with the existing CMM software any types of gear (e.g. helical, herringbone, bevel, worm, spiral, hypoid etc.) can be measured, if the CMM has sufficient hardware configuration.


Limitations:

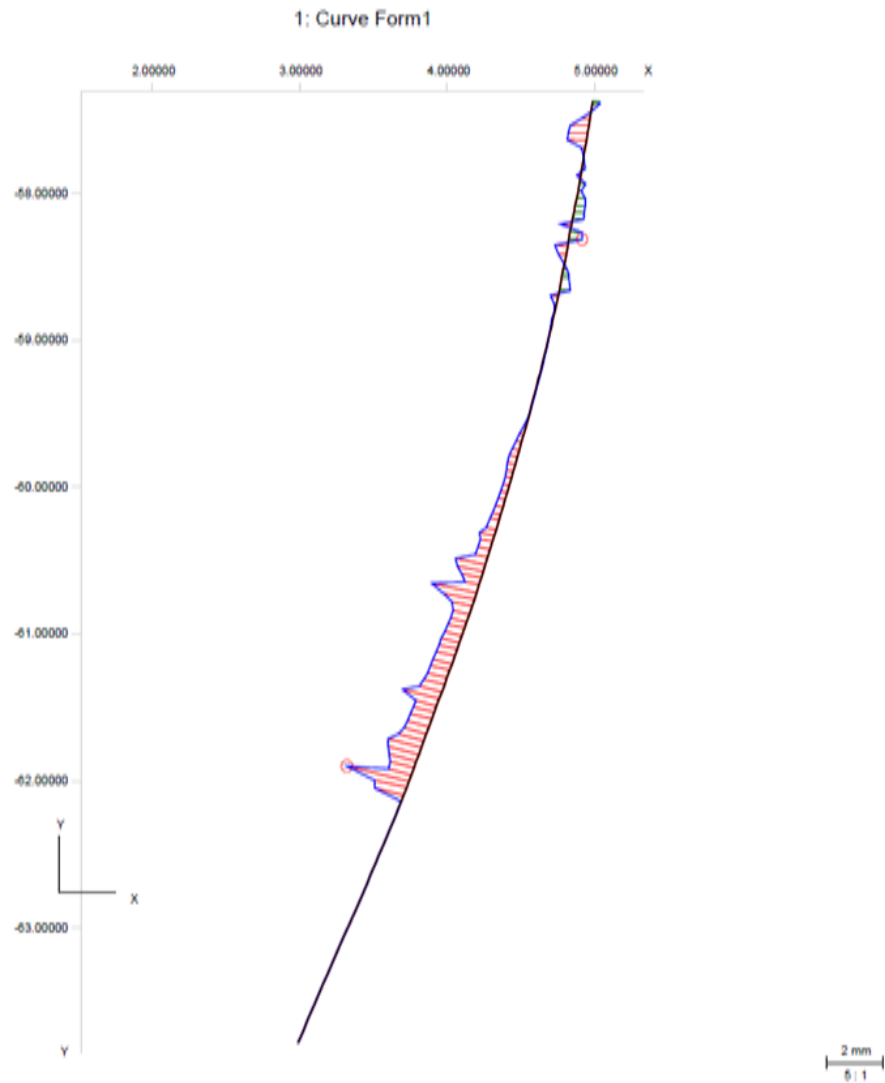
- The machine and software are very costly.
- Skilled operator is required.

Reference:

[1] Francis T. Farago, Mark A. Curtis, "Handbook of Dimensional Measurement", 'Industrial press inc', 3rd ed., 1994.

Measured gear tooth profile error at one face of a tooth

	Calypso 5.2.02	Carl Zeiss		Date Order
Part Number 1	CMM Type O_Inspect	Drawing No.		Department: Operator Signature:
Meas. Plan Name Measurement Plan 10				



No	Identifier	Sigma [mm]	Form [mm]	Number of Points	Lower Tol. [mm]	Upper Tol. [mm]	MinInd	Min Dev. [mm]	MaxInd	Max Dev. [mm]
1	Curve Form1	0.51576	2.71831	136	0.00000	0.00000	97	-2.25379	21	0.46452
	Best Fit	Translation	X [mm]-47.01...	Y [mm]15.621...	Z [mm]1.34112	Rotation	X 2.36039	Y -101.3...	Z 41.715...	