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Experimental, analytical, and finite element vibration analyses of delaminated composite plates

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Abstract. The vibration of delaminated composites concerns the safety and dynamic behavior of composite structures since it is vital to the presence of delamination. In this research paper, finite element simulations, analytical simulations, and experimental work were combined to analyze the vibration behavior with different delamination sizes, stacking sequences, and boundary conditions. The finite element analysis software packages, such as Ansys and Abaqus, were used to determine the vibration response of carbon fiber-reinforced polymer composite plates for different boundary conditions, stacking sequences, and delamination sizes. Rayleigh-Ritz method was employed to derive the governing equations and determine the natural frequencies, the results of which were computed via MATLAB. The natural frequency values for carbon fiber-reinforced polymer decreased upon an increase in delamination size, depending on all boundary conditions. The higher natural frequency values were obtained for all-side clamped boundary conditions. The stacking sequence (0/90/45/90) exhibited higher natural frequency values for all-side clamped boundary conditions. The results can be regarded as a base point for a safe and reliable design of composite structures with and without delamination.

1. Introduction

The application of composites to different fields of sporting equipment, aerospace, marine, and agricultural products [1] has increased tremendously due to their multi-dimensional, attractive, and novel properties [2–4] and low maintenance of composite materials as well.

Composite materials are more lightweight, eco-resistant, cheaper, stiffer, and stronger than the conventional ones [5]. Carbon fiber-reinforced polymers represent a well-known example of a composite material. To form a carbon fiber-reinforced polymer, two components are required: reinforcement and matrix. The required reinforcement is achieved using carbon fiber. Carbon fiber reinforcement gives the required strength to the composite material to be formed. The matrix provides a bond between the reinforced materials. Epoxy is a good example of a matrix made of polymer resin. Usually, the formation of carbon fiber-reinforced polymers depends on the reinforcement used and its particular matrix. For instance, reinforcement components such as elasticity and stress determine the rigidity and strength of carbon fiber reinforced polymers. This is the reason why the property strength of carbon fiber reinforced polymers is usually described as directional and distinct from isotropic structures

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such as aluminum. Of note, the carbon fiber layout can influence the properties of carbon fiber-reinforced polymers. Composite laminates are preferred over others due to their numerous advantages such as their high strength, bending stiffness, and resistance to expansion [5–10]. Usually, composite laminates are formed employing a hybrid of fiber-matrix layers. For example, fiber usually consists of some materials including boron or glass, while the matrix comprises aluminum or epoxy. Undoubtedly, fiber-reinforced composites are widely applicable to various engineering fields such as shipbuilding, mechanics, aerospace, etc. These materials are preferred due to their superior mechanical characteristics including high strength and high stiffness-to-weight ratio. Generated accidentally either during the service period or manufacturing of the composites, delamination is one of the crucial parameters for composite laminates [11–13].

Knowing the vibration characteristics of investigated structures before application is of significance in improving the design parameters [14,15]. One of the most critical defects in composites is delamination and it can negatively affect the behavior of composite structures [16,17]; thus, the effects of ply layups, boundary constraints, and delaminated region on the vibration characteristics of composite structures should be carefully studied [18].

Although a notable number of studies have investigated the behavior of composite structures such as beams, shells, and plates so far, only a few have looked into the impact of different parameters on the delaminated composite plates. Delamination is a comparatively complex problem that involves material and geometry discontinuations. In the following, an aerial exploration of the studies already done on the composite structures is presented, considering the presence/absence of delamination and its effects on the vibration characteristics and the methodologies used for analysis, as well.

Jadhav and Bhoomkar [19] carried out experimental and finite element analyses of cracked laminated composite beams. They concluded that cracking reduced the natural frequencies of the laminated beam. Lee et al. [20] conducted a vibration analysis and investigated the effect of multi-delamination on the vibration properties of the multi-delaminated composite beam-columns. In their study, axial compressive loading was considered and the effects of multi-delamination were more significant than those of single delamination.

Luo et al. [21] attempted to determine the nonlinear vibration properties of composite beams subjected to variations in sizes and positions of the delamination. Since the amplitude was small, a larger frequency value was observed upon the greater length of the delamination. Frequency increased slowly with an increase in the length of the delamination. Following an increase in the amplitude value, the effect of the positions or locations of delamination became clearer. The effect of transverse shear deformation is of high importance and cannot be taken for granted for nonlinear vibration characteristics of composite beams. Hamilton's principle was employed to determine the equations of motion and the analytical results were compared with those obtained by Wang et al. [22]. Both theories predicted a decrease in the frequency values as the delamination length increased. Nanda and Sahu [23] found the vibration responses of delaminated composite shells using the first-order shear deformation theory, subjected to cylindrical, spherical, and hyperboloid shells. The value of linear frequency experienced an increasing trend with a decrease in the number of layers. This frequency difference continues to increase after six layers. Moreover, higher modes were influenced by the delamination to a great degree. Yam et al. [24] analyzed the dynamic behavior of a composite plate with a multi-layered structure and internal delamination. An eight-node rectangular thin element was used for finite element formulation. It was concluded that the local internal delamination had a slight or negligible influence on the natural frequencies of multi-layer laminated composite plates. Moreover, the natural frequency decreased with an increase in delamination. This proposed model that facilitated determining the internal delamination in multi-layer composite plates was validated and incorporated by Wei et al. [25]. Kim et al. [26] found the effect of the presence/absence of delamination on the cross-ply laminates with different degrees of delamination in multiple places. A layer-wise composite laminate theory was also used. It was observed that natural frequencies decreased with delamination and there was a significant difference between the structures with and without delamination. Luo et al. [27] carried out an analytical and experimental investigation into the dynamic behavior of delaminated composite beams. Piecewise linear model was employed to obtain the analytical results at delamination lengths of 3 cm, 10 cm, and 18 cm. At small delamination lengths, natural frequencies were less affected. The piecewise model analyzed the results using straight line graphs to predict the behavior or trend of the results.

Yashavanta Kumar and Sathish Kumar [28] carried out a modal analysis to assess the behavior of smart cantilever composite beams using Ansys. Higher modes exhibited higher natural frequency values than the lower ones. Mohammed [29] carried out a vibration analysis of the cantilever composite beam produced by the hand-lay-up method using MATLAB and SOLIDWORKS and conducted experiments to find the effect of fiber angles on the mode shapes and natural frequencies. Mallik and Rao [30] studied the vibration of composite beams with and without piezoelectric patches using Ansys. Shukla and Harsha [31] carried out a vibration response analysis of the turbine blade subjected to crack in the root. The finite element analysis software, Ansys, was used for modeling the crack and simulations. Yurddaskal et al. [32] carried out a numerical and experimental analysis to determine the effect of foam properties on vibration responses of the curved sandwich composite panels. Sandwich panels were analyzed using Ansys for clamped square composite panels. Natural frequencies increased with an increase in foam density and curvature. Juhász et al. [33] developed a model using finite element analysis, predicted the modal analysis of through-width delaminated composite plates, and validated the results experimentally. Hirwani et al. [34] studied the effect of delamination on the vibration characteristics of spherical, cylindrical, elliptical, hyperboloid, and flat structures of the laminated curved composites. Higher-order shear deformation theory was employed to develop the analytical model. Finite element software, Ansys, was also used to simulate and determine the natural frequencies. Sadeghpour et al. [35] carried out a free vibration analysis of curved sandwich beams with deboned configurations. Lagrange principle and Rayleigh-Ritz methods were used as a basis for solving the governing equations. Simulations were done in Ansys Workbench. It was concluded that curved and flat deboned composite beams had similar vibration responses. Zhang et al. [36] conducted finite element and experimental analyses to find the vibration properties of carbon fiber- and glass fiber-reinforced polymer composite plates. The effects of the delamination size and location on the natural frequencies were also analyzed. Hirwani et al. [37] carried out experiments to find the effect of delamination on the woven glass/epoxy composite plate. Then, the results were compared using finite element simulation software Ansys.

Vo et al. [38] carried out a free vibration analysis using Ansys and Abaqus and compared the obtained results with the numerical results of shear and normal deformation theory on the composite beams with axial load. Zhu et al. [39] conducted a finite element analysis to determine the free vibration properties of carbon nanotube-reinforced composite plates and compared the results using Ansys software package. According to the literature, the vibration analysis of the carbon fiber reinforced polymer composite plates subjected to CCCC, FFFF, CFFF, and SSSS boundary conditions for (0/90/45/90), (0/45), and (0/90) was quite limited and the availability of the vibration behavior for this specific structure was poor. Therefore, it is of significance to investigate the vibration characteristics of carbon fiber-reinforced polymer composite plates. To this end, the effects of boundary conditions, delamination size, and stacking sequences were extensively studied using experimental techniques, analytical results, and finite element simulations.

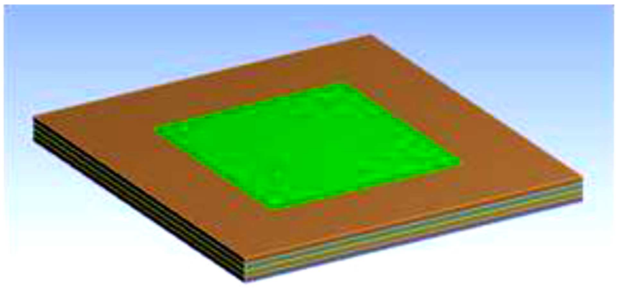

Figure 1. Schematic diagram of delaminated area using Ansys.

2. Finite element approach using Ansys

Figure 1 shows a schematic diagram of the delaminated composite plate with mid-plane delamination. The delaminated region was modeled by merging the nodes in contact, while other nodes of the area were kept unmerged in the non-delaminated area. This methodology has been extensively adopted to model delamination in composite structures. Cohesive zone model or virtual crack closure techniques were not used since the proposed delamination was not bonded. Instead, triangular elements were used to mesh the plate for the vibration properties of the delaminated composite plate. All modeling, meshing, and simulation were done using Ansys APDL version 17 software package. For simulations, solid elements were used in the thickness directions for each prepreg.

3. Finite element approach using Abaqus

Figure 2 shows the delaminated region modeled using

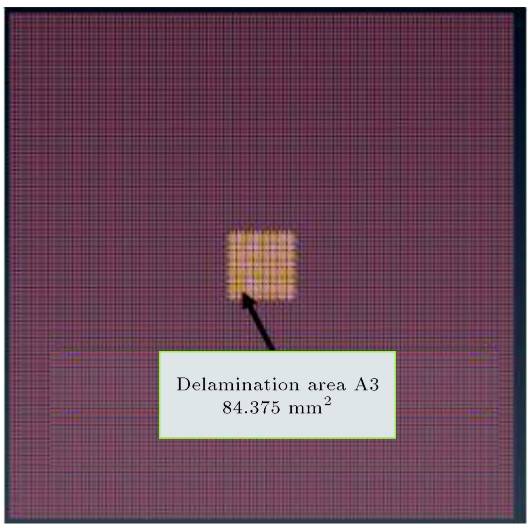

Figure 2. Representation of delamination area.

Abaqus software package version 16. The delaminated region was modeled as a discontinuous region between the two adjacent layers. In order to find the dynamic characteristics of the delaminated carbon fiber-reinforced composite plate, SOLID (eight-node brick element) was used as an element type and the average number of elements per model was 180000. The orientation of the layers was modeled using the coordinate system.

4. Analytical analysis

To conduct a vibration analysis of the composite plate, the finite element model was developed using first-order shear deformation theory and Hamilton's principle derived from the Rayleigh-Ritz method. With some modifications, the Rayleigh-Ritz method has been extensively used by numerous researchers to find the natural frequencies and mode shapes of the beams, shells, and plates [40].

The differential equation for the transverse bending of a rectangular orthotropic plate is depicted in Eq. (1):

$$D_x\frac{\partial^4\omega}{\partial x^4}+2D_{xy}\frac{\partial^4\omega}{\partial x^2\partial y^2}+D_y\frac{\partial^4\omega}{\partial y^4}+\rho\frac{\partial^2\omega}{\partial t^2}=0, \quad (1)$$

where D_x, D_y, and D_{xy} are the stiffness terms computed for the homogenous plate. Since they are computed for such a plate, they should be replaced by the terms that describe a laminate composite plate. D_x, D_y, and D_{xy} are replaced by α_{11}, α_{12}, and α_{22}, respectively. Thus:

$$D_{11}=\alpha_{11}, \quad (2)$$

$$D_{12}=\alpha_{12}, \quad (3)$$

$$D_{13}=\alpha_{13}. \quad (4)$$

The terms D_{11}, D_{12}, and D_{66} were calculated using the following equations:

$$D_{11}=\frac{E_x h^3}{12(1-v_{xy}v_{yx})}, \quad (5)$$

$$D_{12}=D_{21}=\frac{v_{yx}E_x h^3}{12(1-v_{xy}v_{yx})}=\frac{v_{xy}E_y h^3}{12(1-v_{xy}v_{yx})}, \quad (6)$$

$$D_{22}=\frac{E_y h^3}{12(1-v_{xy}v_{yx})}, \quad (7)$$

$$D_{66}=\frac{G_{xy}h^3}{6}. \quad (8)$$

The moment curvature relations were used for the governing equations:

$$M_x=-D_x\left(\frac{\partial^2\omega}{\partial x^2}+v_y\frac{\partial^2\omega}{\partial y^2}\right), \quad (9)$$

$$M_y=-D_y\left(\frac{\partial^2\omega}{\partial y^2}+v_x\frac{\partial^2\omega}{\partial x^2}\right), \quad (10)$$

$$M_{xy}=-2D_x\frac{\partial^2\omega}{\partial_x\partial_y}. \quad (11)$$

From the above equations, the Rayleigh Ritz method [41–44] was used to determine the frequencies in Eq. (12):

$$\omega^2=\frac{1}{\rho}\left(\frac{A^4D_x}{a^4}+\frac{B^4D_y}{b^4}+\frac{2CD_{xy}}{a^2b^2}\right), \quad (12)$$

where A, B, C, and D are the frequency coefficients that are dependent on the applied boundary conditions and are calculated using Eqs. (13) and (14) in a simply supported plate.

$$A=\gamma_0, \quad (13)$$

$$B=\epsilon_0. \quad (14)$$

After putting the value for all-side clamped boundary conditions in Eq. (15), we will solve it using MATLAB tool to determine the frequencies.

$$\omega_{m,n}=\frac{\pi^2}{a^2\sqrt{\rho}}\sqrt{D_x m^4+2D_{x,y}m^2n^2\left(\frac{a}{b}\right)^2+D_y n^4\left(\frac{a}{b}\right)^4}, \quad (15)$$

where a, b, and ρ are the width, height, and density of the plate, respectively.

The following equations are also used to determine the values of constants.

$$\gamma_1=\left(m+\frac{1}{4}\right)\pi, \quad (16)$$

$$\gamma_2=\left(m+\frac{1}{2}\right)\pi, \quad (17)$$

$$\epsilon_1=\left(n+\frac{1}{4}\right)\pi, \quad (18)$$

$$\epsilon_2=\left(n+\frac{1}{2}\right)\pi. \quad (19)$$

Then, using the MATLAB file provided, the values of D_{11}, D_{12}, and D_{66} were computed, as shown in the following:

$$D_{11}=4.35\times10^4 \text{ N/mm}^2,$$

$$D_{12}=8.34\times10^3 \text{ N/mm}^2,$$

$$D_{66}=6.96\times10^3 \text{ N/mm}^2.$$

At this point, although we have all the data necessary for calculating the first natural frequency for the

CCCC, no delamination and (0/90) plate are available. By substituting the values for D_{11}, D_{12}, and D_{66} as well as A, B, and C into Eq. (12), we get: $\omega 1 =$ 5571.89 rad/s, through which the natural frequency can be calculated via Eq. (15) and $f_1 = 886.7943$ Hz be obtained. All of the natural frequencies were computed for 12 modes using MATLAB tool.

5. Experimental setup

The composite plate models used in the present work are made of carbon fibers. A direct roving at 0° and 90° was used for interweaving the fabrics with the epoxy matrix. A ratio of 50:50 in the fiber weight and matrix was used for sample preparation. The individual materials used for preparing the samples are epoxy as resin, hardener as a catalyst, carbon fiber roving as reinforcement, and polyvinyl alcohol as the releasing agent. Samples were prepared using a handy layup method and cured at room temperature. During fabrication, Teflon tape was used to generate artificial delamination with the area sizes equal to 6.25%, 25%, and 56.25% of the rectangular plate. Delamination was incorporated in the mid-plane of the plate after each of four layers with the total layers being eight in number. All the models were exposed to free vibrations. The natural frequencies for the first twelve modes of the eight-layered carbon fiber reinforced polymer composite plates were experimentally determined with and without delamination. The experiments were performed using analyzers, transducers, and modal hammer. The software was also utilized during the vibration measurement. The plates were taken away using impact hammer at random points and the required data were received through data acquisition component. Signals obtained from data acquisition system were then processed using LABVIEW software. The acceleration signals were then processed through power spectrum module to determine the natural frequencies along with their mode shapes.

6. Results and discussion

The results obtained from finite element, analytical, and experimental analyses were presented for free vibration of the composite plate with mid-plane delamination. According to Table 1, the results of the three above-mentioned approaches were in good agreement.

7. Validation of approaches

Based on the four above-mentioned analysis techniques of finite element using Abaqus, Ansys, numerical, and experimental methods, the obtained results were validated on comparison for free vibration of laminated composite plates without delamination. It was shown that the results of all four techniques were in very good agreement, implying that all the adopted techniques were validated for the carbon fiber reinforced polymer composite plates.

8. Numerical, Abaqus, Ansys, and experimental results

In the present paper, the experimental, numerical, and finite element studies were carried out for eight-layered (0/90/45/90)2, (0/45)4, and (0/90)4 carbon fiber reinforced polymer composite plates. The geometrical dimensions considered for carbon fiber-reinforced polymer composites are 0.15 m for both length and width and 0.02 m for thickness of the plate. Square size delamination was incorporated in the mid-plane of the plate. The present study investigates the effect of delamination size, boundary conditions, and stacking sequences on the natural frequencies.

8.1. Effect of delamination size

To study the effect of delamination size on the vibration properties of carbon fiber-reinforced polymer composite plates, delamination of three different sizes like 6.25%, 25%, and 56.25% of the total plate areas was incorporated in the middle of the rectangular plate. Tables 2, 3, and 4 show that an increase in delamination reduces the natural frequencies of carbon fiber-reinforced composite plates for (0/90/45/90) in C-C-C-C boundary conditions. It has been observed that there is a negligible delamination effect in the lowest mode with its effect significantly increasing in higher modes. This behavior has been validated using both of the finite element methods, Abaqus and Ansys. The results reveal that for all delamination sizes, the natural frequencies are less affected in the lower modes than those in the higher modes.

The same study was extended to all-side simply supported (S-S-S-S) boundary conditions, the results of which are presented in Tables 5–7. It has been observed that all-side clamped boundary conditions show higher natural frequency values than all-side simply supported boundary conditions subjected to all delaminated lengths. Natural frequencies are less affected in the lower modes at all delaminated sizes. However, they are significantly affected at large delamination sizes in all cases.

8.2. Effect of boundary constraints

To study the effect of boundary conditions on the natural frequencies of carbon fiber-reinforced polymer composite rectangular plates, two types of boundary conditions, C-C-C-C and S-S-S-S, were considered. The results for all boundary conditions versus delam-

Table 1. Effects of all-side clamped and simply supported conditions and the effect of stacking sequence for the first four modes.

Boundary condition	Layers sequence	Method	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)	Mode 4 (Hz)
All sides clamped	0/90/45/90	Abaqus	883.41	1804.1	1804.1	2538
		Ansys	879.65	1795.6	1795.6	2547.6
		Analytical	886.790	1822.9	1822.9	2522.2
		Experimental	872	1781	1801	2508
	0/45	Abaqus	876.31	1776.6	1776.6	2601.4
		Ansys	873.84	1773.3	1773.3	2602.5
		Analytical	882.411	1802	1802	2659.03
		Experimental	871	1765	1781	2649
	0/90	Abaqus	887.42	1820	1820	2489.8
		Ansys	884.65	1815.7	1815.7	2487.6
		Analytical	886.794	1837.03	1837.03	2522.2
		Experimental	878	1828	1831	2499
All sides simply supported	0/90/45/90	Abaqus	444.78	1175.8	1175.8	1769.6
		Ansys	442	1110	1180	1800
		Analytical	447.641	1182.3	1182.3	1790.6
		Experimental	425	1168	1179	1777
	0/45	Abaqus	475.96	1191.9	1191.9	1892.9
		Ansys	473	1199	1197	1900
		Analytical	482.410	1206.02	1206.02	1929.6
		Experimental	468	1189	1185	1895
	0/90	Abaqus	421.66	1163.1	1163.1	1678.2
		Ansys	416	1169	1170	1695
		Analytical	422.205	1165.8	1165.8	1688.8
		Experimental	409	1151	1160	1674

Table 2. Effect of delamination for C-C-C-C boundary condition of the stacking sequence (0/90/45/90).

Delamination area	1		2		3		4	
	Abaqus	Ansys	Abaqus	Ansys	Abaqus	Ansys	Abaqus	Ansys
6.25%	883.41	889.36	1803.8	889.36	1803.8	1889.36	2538	2601
25%	883.41	880.68	1801.7	1803.7	1801.7	1835	2538	2567.1
56.25%	883.38	877.85	1795.2	1801.1	1795.2	1819.8	2537.4	2527.8

Table 3. Effect of delamination for C-C-C-C boundary condition of the stacking sequence (0/45).

Delamination area	1		2		3		4	
	Abaqus	Ansys	Abaqus	Ansys	Abaqus	Ansys	Abaqus	Ansys
6.25%	876.31	883.93	1776.4	1799.4	1776.4	1804.2	2601.4	2657.3
25%	876.31	874.77	1774.5	1780.5	1774.5	1782	2601.3	2627.5
56.25%	876.28	872.36	1768.7	1755.5	1768.7	1768.4	2600.8	2622

Table 4. Effect of delamination for C-C-C-C boundary condition of the stacking sequence (0/90).

Delamination area	1		2		3		4	
	Abaqus	Ansys	Abaqus	Ansys	Abaqus	Ansys	Abaqus	Ansys
6.25%	887.42	894.06	1819.7	1840	1819.7	1845	2489.8	2542.1
25%	887.42	885.73	1817.4	1824.6	1817.4	1825.8	2489.7	2501.2
56.25%	887.39	872.61	1810	1816.8	1810	1819.3	2489.1	2484.1

Table 5. Effect of delamination for S-S-S-S boundary condition of the stacking sequence (0/90/45/90).

Delamination area	1	2	3	4
6.25%	444.78	1175.7	1175.7	1769.6
25%	444.78	1174.9	1174.9	1769.6
56.25%	444.78	1172.4	1172.4	1769.4

Table 6. Effect of delamination for S-S-S-S boundary condition of the stacking sequence (0/90).

Delamination area	1	2	3	4
6.25%	421.66	1163	1163	1678.2
25%	421.66	1162.1	1162.1	1678.2
56.25%	421.65	1159.3	1159.3	1678

Table 7. Effect of delamination for S-S-S-S boundary condition of the stacking sequence (0/45).

Delamination area	1	2	3	4
6.25%	475.96	1191.7	1191.7	1892.9
25%	475.96	1191	1191	1892.8
56.25%	475.95	1188.7	1188.7	1892.7

ination of 6.25%, 25%, and 56.25% are presented in Table 8. The samples taken for this study are of eight-layered composite plates with stacking sequences of (0/90/45/90), (0/90), and (0/45).

For 6.25% delamination, the stacking sequence (0/90) with a CCCC boundary condition exhibited the highest natural frequency in the first mode, while, interestingly, the same stacking sequence had the lowest frequency for the SSSS boundary condition. This same behavior was observed at the delamination sizes of 25% and 56.25%.

8.3. Effect of stacking sequence

To study the effect of stacking sequences on the natural frequencies at 6.25%, 25%, and 56.25% sizes of the delaminated area, three 8-layered types of stacking sequences (0/90/45/90), (0/90), and (0/45) are considered. The changes in the natural frequencies corresponding to their stacking sequences are presented in Table 8.

Table 8 shows that the highest value of the natural frequency is observed for the sequence (0/90/45/90) on all-side clamped boundary conditions; in addition, the second highest and third values are observed on all-side clamped boundary conditions for (0/90) and (0/45), respectively.

9. Conclusion

In the present study, single delamination in the middle of the plate was analyzed for four types of models, i.e., finite element using Ansys and Abaqus as well as analytical and experimental methods. The results were compared in the case of all-side simply supported and clamped boundary conditions and they were in very good agreement. The effects of different parameters such as delamination size, boundary constraints, and ply orientations on the natural frequencies of carbon fiber-reinforced composite plates were also investigated. Based on the experimental, analytical, and finite element methods, the following conclusions can be drawn:

- The analytical and finite element results were in good agreement;
- The natural frequencies decreased with an increase in the delamination size;
- The natural frequencies were not only reliant on the delamination size but also influenced by the boundary conditions; in addition, they experienced significant variations in higher modes;
- At 6.25% delamination size, the stacking sequence (0/90) exhibited the highest natural frequency on all-side clamped boundary conditions;
- At 6.25% delamination size, the stacking sequence

Table 8. Natural frequencies versus CCCC and SSSS boundary conditions.

Delamination area	Boundary condition	Stacking	Mode 1	Mode 2	Mode 3	Mode 4
6.25%	CCCC	0/90/45/90	883.41	1803.8	1803.8	2538
		0/45	876.31	1776.4	1776.4	2601.4
		0/90	887.42	1819.7	1819.7	2489.8
	SSSS	0/90/45/90	444.78	1175.7	1175.7	1769.6
		0/45	475.96	1191.7	1191.7	1892.9
		0/90	421.66	1163	1163	1678.2
25%	CCCC	0/90/45/90	883.41	1801.7	1801.7	2538
		0/45	876.31	1774.5	1774.5	2601.3
		0/90	887.42	1817.4	1817.4	2489.7
	SSSS	0/90/45/90	444.78	1174.9	1174.9	1769.6
		0/45	475.96	1191	1191	1892.8
		0/90	421.66	1162.1	1162.1	1678.2
56.25%	CCCC	0/90/45/90	883.38	1795.2	1795.2	2537.4
		0/45	876.28	1768.7	1768.7	2600.8
		0/90	887.39	1810	1810	2489.1
	SSSS	0/90/45/90	444.78	1172.4	1172.4	1769.4
		0/45	475.95	1188.7	1188.7	1892.7
		0/90	421.65	1159.3	1159.3	1678

(0/90) had the lowest natural frequency value on all-side simply supported boundary conditions.

Nomenclature

a, b, h	The length, breadth, and thickness of the sheet or plate
E_1, E_2	Young's moduli in the given material or element's specific directions
G_{12}, G_{23}, G_{13}	Shear moduli on their specific planes (i.e., xy, yz, and xz planes, respectively)
P	The number of delamination
U	The total strain energy
u, ν, w	Displacements of any point in x, y, z directions
u_0, ν_0, ω_0	Displacements of a point on the mid-plane of the panel along x, y, and z directions
V	Total kinetic energy
W	Work-Done
x, y, z	Cartesian co-ordinate axes
$\theta x, \theta y$	Rotations corresponding to y and x directions
$\upsilon_{12}, \upsilon_{23}, \upsilon_{13}$	Poisson's ratios of objects in their specific directions (i.e. xy, yz, and xz planes, respectively)
ρ	Density
ϕx, ϕy, λx, λy, θz	Higher-order factors or terms of expansion of Taylor's series

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