Explanatory Notes to Senior Secondary Mathematics Curriculum — Compulsory Part

Mathematics Education Section
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Foreword

The *Mathematics Curriculum and Assessment Guide (Secondary 4 – 6)* (2007) (abbreviated as "C&A Guide" in this booklet) has been prepared to support the new academic structure implemented in September 2009. The Senior Secondary Mathematics Curriculum consists of a Compulsory Part and an Extended Part. The Extended Part has two optional modules, namely Module 1 (Calculus and Statistics) and Module 2 (Algebra and Calculus).

In the C&A Guide, the Learning Objectives of the Compulsory Part are grouped under different Learning Units in the form of a table. The notes in the "Remarks" column of the table in the C&A Guide provide supplementary information about the Learning Objectives. The explanatory notes in this booklet aim at further explicating:

1. the requirements of the Learning Objectives of the Compulsory Part;
2. the strategies suggested for the teaching of the Compulsory Part;
3. the connections and structures among different Learning Units of the Compulsory Part;
4. the context of development from different key stages, such as Key Stage 3, to the Compulsory Part; and
5. the curriculum articulation between the Compulsory Part and the Extended Part.

The explanatory notes in this booklet together with the "Remarks" column and the suggested lesson time of each Learning Unit in the C&A Guide are to indicate the breadth and depth of treatment required. Teachers are advised to teach the contents of the Compulsory Part as a connected body of mathematical knowledge and develop in students the capability to use mathematics to solve problems, reason and communicate. Furthermore, it should be noted that the ordering of the Learning Units and Learning Objectives in the C&A Guide does not represent a prescribed sequence of learning and teaching. Teachers may arrange the learning content in any logical sequence which takes account of the needs of their students.

Comments and suggestions on this booklet are most welcomed. They should be sent to:

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Learning Unit	Learning Objective	Time
Number and Algebra Strand		
1. Quadratic equations in one unknown	1.1 solve quadratic equations by the factor method	19
	1.2 form quadratic equations from given roots	
	1.3 solve the equation $ax^2 + bx + c = 0$ by plotting the graph of the parabola $y = ax^2 + bx + c$ and reading the x-intercepts	
	1.4 solve quadratic equations by the quadratic formula	
	1.5 understand the relations between the discriminant of a quadratic equation and the nature of its roots	
	1.6 solve problems involving quadratic equations	
	1.7 understand the relations between the roots and coefficients and form quadratic equations using these relations	
	1.8 appreciate the development of the number systems including the system of complex numbers	
	1.9 perform addition, subtraction, multiplication and division of complex numbers	

Explanatory Notes:

At Key Stage 3 (KS3), students learnt how to formulate and solve linear equations in one unknown, and simultaneous linear equations in two unknowns by both the algebraic method and the graphical method. In the Compulsory Part, students are required to solve quadratic equations and other more complicated algebraic equations.

In this Learning Unit, students are required to use:

- the factor method
- the quadratic formula
- the graph of the parabola $y = ax^2 + bx + c$

to solve the quadratic equation $ax^2 + bx + c = 0$, and to form a quadratic equation from given roots. Students should be able to choose an appropriate strategy in solving quadratic equations.

The graphical method will be further elaborated in Learning Objective 9.2 and teachers have to pay attention to the Explanatory Notes to Learning Unit 9.

Students should be able to solve the problems involving quadratic equations. The problems should be related to their experiences as far as possible. Solving the equations which can be transformed into quadratic equations such as $\frac{6}{x} + \frac{6}{x-1} = 5$ and their related problems, solving higher degree equations by using the factor theorem or the graphs of functions will be treated in Learning Units "More about Equations", "More about Polynomials" and "More about Graphs of Functions" accordingly.

In this Learning Unit, all the coefficients of quadratic equations and the given roots in Learning Unit 1.2 "Form Quadratic Equations from Given Roots" are confined to real numbers.

Regarding the expression of the solutions to quadratic equations, it should be noted that manipulation of surds and expressing the surds in a more concise form belong to the Non-foundation Part of KS3. In this connection, when using the quadratic formula to solve quadratic equations such as $x^2 - 4x - 4 = 0$, the students who have not studied the above-mentioned topics in the Non-foundation Part are not required to simplify surds like $2 \pm \frac{\sqrt{32}}{2}$.

Students have to understand the relations between the discriminant of a quadratic equation and the nature of its roots in addition to solving quadratic equations. In the C&A Guide, the term "understand" usually implies a more demanding learning objective than the term "recognise" does. For example, "understand the relations between the discriminant of a quadratic equation and the nature of its roots" means that students should know the details of the relations, the justification of why the relations hold, and how to use these relations to perform further operations and to solve problems.

The concept of complex numbers has been introduced in Learning Objective 1.8. When students are asked to determine the nature of the roots of a quadratic equation with a negative discriminant, they must point out that "the equation has **no real roots**" or, more precisely, "the equation has **two nonreal** roots" instead of "the equation has no roots" or "the equation has two complex roots". The students who do not study Learning Objective 1.9 (Non-foundation Topics) are not required to express the nonreal roots in the form of $a \pm bi$ when using the quadratic formula to solve quadratic equations.

Students should understand the relations between the roots and the coefficients, which includes:

- $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$, where α and β are the roots of the equation $ax^2 + bx + c = 0$ and $a \neq 0$.

Teachers may discuss with students or let them explore other relations between the roots and coefficients such as $\alpha^2 + \beta^2$. Nevertheless, rote learning of these results is not recommended.

Mathematical concepts have come a long way and their development has often been influenced by cultures and other humanistic factors. The development of number systems makes no exception. By organising various activities, such as decorating display boards or reading projects, teachers may get students to appreciate how the number system was developed from the natural number system to the rational number system, the real number system and the complex number system. For example, "Why did Pythagoreans in ancient Greece not accept the existence of irrational numbers?", "Why were imaginary numbers not well-articulated until the sixteenth century?", etc. are some interesting topics for discussion. Moreover, teachers may ask students to discuss topics such as the hierarchy of the number system, the representation of terminating decimals and recurring decimals in fractional form, the proof of irrationality of $\sqrt{2}$, etc.

The students who have studied Learning Objective 1.9 (Non-foundation Topics) should be able to perform the addition, subtraction, multiplication and division of complex numbers, where the form of complex numbers is confined to the standard form $a \pm bi$. The polar form of complex numbers, Argand diagram and De Moivre's theorem are not required in the Curriculum.

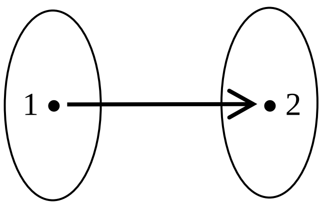
Teachers may refer to the C&A Guide (pp. 111 – 112) for using the co-construction approach to teach the quadratic formula.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
2. Functions and graphs	2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables 2.2 recognise the notation of functions and use tabular, algebraic and graphical methods to represent functions 2.3 understand the features of the graphs of quadratic functions 2.4 <u>find the maximum and minimum values of quadratic functions by the algebraic method</u>	10

Explanatory Notes:

By the end of KS3, students should have a preliminary recognition of the concept of sequences, which may be considered as an embryonic form of functions. In this Learning Unit, students have to recognise the intuitive concepts of functions, dependent and independent variables. They should also be able to distinguish between the examples of functions and non-functions. Furthermore, students have to recognise the intuitive concepts of domain and co-domain as these two concepts are indispensable for defining functions. Knowing these two concepts can help students compare different functions in detail (see the Remarks of Learning Unit 3.4), but using set language to rigorously define functions or express ranges are not required in the Curriculum. Besides, compositions of functions are not required in the Curriculum. When students encounter functions such as $f(x) = \sin x^2$, they may simply regard the calculation of the functional value as a sequence of consecutive manipulations, say in the above-mentioned example: taking the square of x and the sine of the square.

When explicating the notations of function, teachers may introduce the concept of dummy variables. There are various ways to represent functions. Under different circumstances, teachers may adopt different methods, i.e. tabular, algebraic, graphical methods, or even the following intuitive method to represent functions:



Having acquired the concept of functions, students have to consolidate the concept by studying their familiar quadratic functions, and to recognise the following features of the graphs of quadratic functions.

- The vertex
- The axis of symmetry
- The direction of opening
- Its relations with the axes

Students should be able to determine the direction of opening of the graph of a quadratic function from the coefficient of x^2, to find the y-intercept from the constant term, and to use the discriminant to determine whether the graph cuts the x-axis. On the other hand, students should also be able to read, from the graph of a quadratic function, the information about its axis of symmetry and vertex. They should also understand the relation between the maximum/minimum value of the quadratic function and the vertex of its graph, and hence be able to find the maximum/minimum value of the quadratic function by the graphical method.

The students who have studied Learning Objective 2.4 (Non-foundation Topics) should be able to use the algebraic method to find the maximum/minimum value of a quadratic function, and to solve related problems. Apart from the method of completing the square, teachers may introduce other algebraic methods. For instance, teachers may guide the more able students to recognise, from the characteristics of the graphs of quadratic functions, that the x-coordinate of the vertex of $y = x^2 - 2x$ is $\frac{\alpha + \beta}{2} = 1$. By substituting $x = 1$, they can obtain the minimum value of the function, i.e. $1^2 - 2(1) = -1$. Using differentiation to find maximum and minimum values is beyond the scope of the Compulsory Part, and belongs to a Learning Objective of the Learning Unit "Applications of Differentiation" of Module 1 or Module 2 of the Extended Part.

Teachers may lead students to further explore other relations between the coefficients of the quadratic functions and the corresponding graphs. For example, to determine, from the sign of the value of $\alpha\beta$, whether the two x-intercepts are on the same side or opposite sides of the y-axis.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
3. Exponential and logarithmic functions	3.1 understand the definitions of rational indices 3.2 understand the laws of rational indices 3.3 understand the definition and properties of logarithms (including the change of base) 3.4 understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs 3.5 solve exponential equations and logarithmic equations 3.6 appreciate the applications of logarithms in real-life situations 3.7 appreciate the development of the concepts of logarithms	16

Explanatory Notes:

Students at KS3 should learn the laws of integral indices. In this Learning Unit, the laws of indices will be extended from integral indices to rational ones.

At KS3, students should learn the definitions of a^n, a^0 and a^{-n}, where n is a positive integer. In the Compulsory Part, students have to recognise the definitions of other rational indices: $\sqrt[n]{a}$, $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$, where a is a positive real number, m is an integer and n is a positive integer. When n is odd and a is negative, students are expected to be able to evaluate simple expressions $\sqrt[n]{a}$, e.g. $\sqrt[3]{-8}$, while the written form $(-8)^{\frac{1}{3}}$ should be avoided. Furthermore, students have to recognise that the laws of indices do not hold when a is negative.

When studying the laws of rational indices and the properties of logarithms, students should clearly understand the necessary conditions for the assertion. For example, when $\log_a 1 = 0$, a must be positive and $a \neq 1$.

Once students understand the formula for change of base, they can make use of calculators to obtain the value of any logarithm (e.g. $\log_2 3$). However, the study of natural logarithm, a

Learning Objective in the Extended Part, is not required in the Compulsory Part.

Although indices in Learning Objective 3.2 are confined to rational numbers, students should recognise that real indices are an extension of rational indices. The details of extension are not required in the Curriculum, but it may be an interesting topic for further exploration. On the other hand, students should be aware that the domain of exponential functions is the set of real numbers while the domain of logarithmic functions is the set of positive real numbers. The domain of the latter is different from that of the quadratic functions with which students are familiar. Teachers may guide their students to discuss the differences between the graphs of the exponential functions (and the logarithmic functions) for $a > 1$ or $0 < a < 1$. Since the concept of inverse functions is not required in the Curriculum, the term "inverse function" need not be introduced when discussing the symmetric relation between the graphs of $y = a^x$ and $y = \log_a x$. When students have understood the relation between exponential functions and logarithmic functions, they can deduce the corresponding features of logarithmic functions from those of exponential functions. Through various examples such as $y = 2^x$, $y = x^2$ and $y = x^3$, teachers can discuss the rate of increasing/decreasing of the functions.

Learning Objective 3.5 mainly involves simple equations such as $2^x = 5$ or $\log_3(x+4) = 2$. The equations which can be transformed into quadratic equations such as $4^x - 3\cdot 2^x - 4 = 0$ or $\log_2(x+1) + \log_2(x-3) = 3$ are tackled in Learning Objective 5.3.

In this Learning Unit, students not only have to understand the concepts of exponential and logarithmic functions, but also have to appreciate the applications of logarithms in real-life situations, by discussions on topics such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels, and understand why logarithms have to be involved in these formulae. Students may try to use different formulae to calculate the earthquake intensity, but memorisation of such formulae is not required.

By organising various activities for students to experience the difficulties in performing complicated operations without any calculation tools, teachers may guide students to discuss the topics such as the history of development of the concepts of logarithm and its applications to the design of some calculation tools in the past, such as slide rules and the logarithmic table.

Students studying Module 1 or Module 2 of the Extended Part will continue the study of power functions, other properties and applications of exponential functions and logarithmic functions.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
4. More about polynomials	4.1 perform division of polynomials 4.2 understand the remainder theorem 4.3 understand the factor theorem 4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials 4.5 perform addition, subtraction, multiplication and division of rational functions	14

Explanatory Notes:

By the end of KS3, students should have learnt addition, subtraction and multiplication of polynomials, and factorisation of simple polynomials. They have also learnt the operations of algebraic fractions with linear denominators. In this Learning Unit, students have to further their study on the division of polynomials, more complicated problems on factorisation of polynomials, and manipulations on rational functions with denominators of degree higher than one. Addition, subtraction, multiplication and division of rational functions can be considered as mixed fundamental operations of polynomials.

Long division is a standard procedure for performing the division of polynomials. Teachers may introduce other procedures, like synthetic division. It should however be noted that complicated computations of polynomials are not the objective of the Curriculum.

Students have to understand the significance of the division algorithm $f(x) = g(x)Q(x) + R(x)$ and understand how to deduce the remainder theorem under the condition $g(x) = ax + b$. Furthermore, the factor theorem can be considered as a special case of the remainder theorem. When using the factor theorem to factorise polynomials, teachers may lead students to appreciate its usefulness (e.g. to solve some equations with degree greater than two) and limitations (e.g. not all higher degree equations can be solved efficiently by this method).

To articulate with the Extended Part, the term “rational function” is used in this Learning Unit to replace the term “algebraic fraction” used at KS3. Nevertheless, the study of the properties of rational functions is not required in the Compulsory Part.

The concepts of the greatest common divisor and the least common multiple play a crucial role in performing multiplication, division and simplification of rational functions. In this connection, students should have a comprehensive understanding of these two concepts. When teaching the greatest common divisor (sometimes called "the highest common divisor") and the least common multiple (sometimes called "the lowest common multiple"), teachers should feel free to choose any commonly used short form, such as "H.C.F.", "gcd" or "(a , b)", to denote the greatest common divisor of a and b. To facilitate students to read other reference books, teachers should introduce other commonly used short forms in addition to the chosen one. When performing the operations of addition, subtraction, multiplication and division of rational functions, the number of variables should not be more than two to avoid over-complicated computations. Division of rational functions includes computations like "$\frac{1}{x^2 - y^2}$ is divided by $\frac{1}{x + y}$", but it should be noted that complicated computations are not the objective of this Learning Unit.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
5. More about equations	5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y = ax^2 + bx + c$ 5.2 use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic 5.3 solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations 5.4 solve problems involving equations which can be transformed into quadratic equations	10

Explanatory Notes:

By the end of KS3, students should have learnt to formulate simultaneous linear equations in two unknowns and how to use the algebraic method and the graphical method to solve simultaneous linear equations in two unknowns. In this Learning Unit, one of the simultaneous equations will be quadratic and the problems will be extended from those in Learning Unit 1.6 "Solving Problems Involving Quadratic Equations" to the problems involving equations which can be transformed into quadratic equations. Students should note the differences between the number of solutions of simultaneous equations, one linear and one quadratic, and that of simultaneous linear equations.

Students are not required to study the graphs of quadratic equations such as $x = y^2 - 3y + 6$ or $xy + y^2 = 1$ in the Compulsory Part. Hence, when using the graphical method to solve simultaneous equations in Learning Objective 5.1, the quadratic equations in two unknowns should be confined to the form $y = ax^2 + bx + c$. However, there are no restrictions imposed on the quadratic equations when using the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic.

Students have to note whether the solutions are reasonable or not when solving equations which can be transformed into quadratic equations. For example, when solving the equation $2\sin^2\theta - 5\sin\theta + 2 = 0$, students have to note that $\sin\theta = 2$ does not give a real solution. Moreover, the solutions of equations involving trigonometric functions are confined to the interval from 0° to 360° (see Learning Objective 13.2).

In order to arouse students' interest, teachers may select the problems related to students' daily-life experience when teaching Learning Objective 5.4. During discussion, teachers may lead students to discover the diversity of methods of solving equations. Students should explore various problem solving strategies and be able to select the most appropriate one.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
6. Variations	6.1 understand direct variations (direct proportions) and inverse variations (inverse proportions), and their applications to solving real-life problems 6.2 understand the graphs of direct and inverse variations 6.3 understand joint and partial variations, and their applications to solving real-life problems	9

Explanatory Notes:

At KS3, students recognised some relations between quantities in Learning Unit "Rate and ratio" and in the Learning Unit 2 of the Compulsory Part, they should have learnt the basic concept of functions and thus recognise that relations may hold between variables. In this Learning Unit, different forms of the relations between variables will be further elaborated, including direct variation, inverse variation, joint variation and partial variation and their daily-life applications.

When discussing the graphs of the direct variation $y = kx$ and inverse variation $y = \frac{k}{x}$, teachers may remind their students that the domains of the two functions may contain negative real numbers. When explicating direct variation and inverse variation, teachers may introduce the two terms "direct proportion" and "inverse proportion", which are commonly used in daily life. Nevertheless, teachers have to clarify with students about the common misunderstandings of direct proportion and inverse proportion. For instance, if y increases (decreases) when x increases, then x and y must be in direct (inverse) proportion. Teachers may use a counterexample to disprove each of the above assertions. For example,

x	1	2	3	4
y	2	5	7	11

where x and y do not satisfy any direct variation.

On the other hand, if x and y are in direct (inverse) proportion, the proposition "y increases (decreases) as x increases" is not necessarily true. For instance, when k is negative, the proposition is false.

When solving problems involving partial variation, students usually have to solve quadratic or higher degree equations. Therefore, students should have studied Learning Units 1 and 4 before studying this Learning Unit.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
7. Arithmetic and geometric sequences and their summations	7.1 understand the concept and the properties of arithmetic sequences 7.2 understand the general term of an arithmetic sequence 7.3 understand the concept and the properties of geometric sequences 7.4 understand the general term of a geometric sequence 7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems 7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems 7.7 solve related real-life problems	17

Explanatory Notes:

At KS3, students should have a basic idea of sequences when studying number patterns. In this Learning Unit, they have to further discuss the concepts, properties, the general formulae of summations of the two common sequences (arithmetic sequences and geometric sequences) and their applications.

Teachers may guide students to investigate the following property: for any real numbers *a* and *k*, the sequences $T_1 + a,\ T_2 + a,\ T_3 + a, \ldots$ and $kT_1,\ kT_2,\ kT_3, \ldots$ must be arithmetic if $T_1,\ T_2,\ T_3,\ \ldots$ is an arithmetic sequence, and hence the sequence $kT_1 + a,\ kT_2 + a,\ kT_3 + a, \ldots$ is also arithmetic. Moreover, students should be able to discover that, when the sequence $T_1,\ T_2,\ T_3, \ldots$ is geometric and $k \neq 0$, the sequence $kT_1,\ kT_2,\ kT_3,\ \ldots$ is geometric. The discussion on whether the sequence 0, 0, 0, … is geometric is not required in the Compulsory Part.

When solving geometrical problems relating to the sum of arithmetic or geometric sequences, students may sometimes need to apply the knowledge of the Learning Unit “More about Trigonometry”. Teachers should thus note the order of learning and teaching of these related Learning Units.

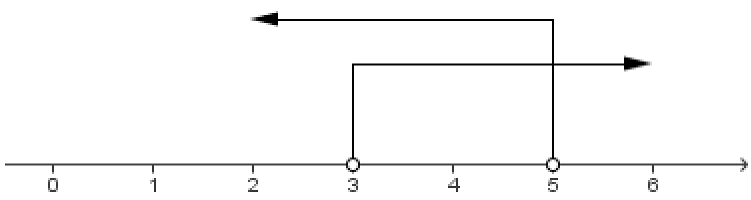
The terms “arithmetic mean” and “geometric mean” need not be introduced when the property of the arithmetic sequence $T_n = \frac{1}{2} (T_{n-1} + T_{n+1})$ and the property of the geometric sequence $T_n^2 = T_{n-1} \times T_{n+1}$ are being discussed.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
8. Inequalities and linear programming	8.1 solve compound linear inequalities in one unknown 8.2 solve quadratic inequalities in one unknown by the graphical method 8.3 solve quadratic inequalities in one unknown by the algebraic method 8.4 represent the graphs of linear inequalities in two unknowns on a plane 8.5 solve systems of linear inequalities in two unknowns 8.6 solve linear programming problems	16

Explanatory Notes:

Students at KS3 learnt how to solve linear inequalities in one unknown and to represent the solution on the number line. In this learning unit, students have to solve compound linear inequalities in one unknown involving logical connectives “and” or “or”, quadratic inequalities in one unknown by using the graphical method and algebraic method, linear inequalities in two unknowns by the graphical method, and linear programming problems.

In Learning Objective 8.1, teachers should let students discover that, in solving compound linear inequalities in one unknown, the solutions of the two linear inequalities may, in general, divide the number line into three regions. The solutions of the compound inequalities can thus be obtained. Students should note that the solutions of “$x > 3$ and $x < 5$” and “$x > 3$ or $x < 5$” are different.



When students are familiar with the features of quadratic graphs, they should be able to solve quadratic inequalities in one unknown by the graphical method. However, they also have to use the algebraic method to solve quadratic inequalities in one unknown.

Students at KS3 should learn how to draw the graphs of linear equations on the coordinate plane. When solving linear inequalities in two unknowns, they have to determine which region(s) bounded by the straight lines will correspond to the solutions of the linear inequalities. The method of testing values is one that can be easily mastered by students.

In Learning Objective 8.5, students are required to use the graphical method to solve systems of linear inequalities in two unknowns. However, using the algebraic method to solve systems of linear inequalities in two unknowns is not required in the Curriculum.

When teaching Learning Objective 8.6, teachers have to select the problems related to students' daily life experiences as far as possible and provide their students with opportunities to discuss the problem solving strategies.

Learning Unit	Learning Objective	Time
Number and Algebra Strand		
9. More about graphs of functions	9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions 9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$ 9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$ 9.4 understand the transformations of the function $f(x)$ including $f(x) + k$, $f(x + k)$, $k f(x)$ and $f(kx)$ from tabular, symbolic and graphical perspectives	11

Explanatory Notes:

After studying Learning Unit 2, students should have a preliminary recognition of the concept of function. In this Learning Unit, they have to compare the graphs of different functions, to solve equations and inequalities by using the graphical method and to understand the concept of transformations of function.

Some students may regard expressions like $y = 4$ as merely solutions of some equations and even do not know how to draw the graph of $y = 4$ on the coordinate plane. Teachers should introduce the concept of "constant function" to their students. The students who do not take Learning Unit 3 (Non-foundation Topics) are not required to discuss the graphs of exponential functions and logarithmic functions. When comparing the graphs of different functions, students should try to compare their domains, existence of maximum or minimum values, symmetry and periodicity.

In Learning Objective 1.3, students should learn how to solve the quadratic equation $ax^2 + bx + c = 0$ by reading the x-intercept(s) of the graph of $y = ax^2 + bx + c$. In Learning Objective 9.2, students are required to use the parabola $y = ax^2 + bx + c$ and the straight line $y = k$ to solve the quadratic equation $ax^2 + bx + c = k$. For example, from the graph of $y = 2x^2 - 5x - 1$, students can solve not only the quadratic equation $2x^2 - 5x - 1 = 0$, but also the quadratic equation $2x^2 - 5x - 4 = 0$ by making use of the graph of the straight line $y = 3$. However, using the graph of $y = 2x^2 - 5x - 1$ to solve quadratic equations like $2x^2 - 6x + 1 = 0$ is not required in the Curriculum.

Students have to extend the method learnt in Learning Objective 8.2 to the functions other than quadratic functions. In other words, even though the function $f(x)$ is not of the form $ax^2 + bx + c$, students still have to know how to read the solution(s) of $f(x) = k$ by using the graph of $y = f(x)$. They can then be able to make use of the graph of $y = f(x)$ and the straight line $y = k$ to obtain the solutions of inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$.

When exploring the transformations of functions, students may first observe the change of the relation between the independent and dependent variables by making use of the tabular form, and then use graphing software to compare the changes of the graphs of the functions after transformations. Teachers may encourage their students to adopt the approach of mathematical thinking from particular to general in obtaining the relations between the graphs of $y = f(x) + k$, $f(x + k)$, $k f(x)$, $f(kx)$ and the original function $y = f(x)$. During discussions, teachers should ask their students to apply the concepts and terminologies, e.g. translation, reflection and dilation, learnt at KS3 to describe the changes of the graphs. On the other hand, teachers may guide their students to discuss the effects of transformations of the graphs of functions from the algebraic forms of the functions. For example, students have to understand that the reflection of the graph of $y = f(x)$ along the *x*-axis can be represented by $y = -f(x)$. The changes of the algebraic forms of the functions should be confined to $f(x) + k$, $f(x + k)$, $k f(x)$, $f(kx)$ or their combinations. That is to say, the rotation of the graphs of functions is not required in the Curriculum.

The concept of composition of functions is not required in the Curriculum. However, the graph of functions such as $y = -x^2 + 4$ can be considered as a reflection of the graph of $y = x^2$ in the *x*-axis, and a subsequent upward translation of 4 units along the *y*-axis.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
10. Basic properties of circles	10.1 understand the properties of chords and arcs of a circle 10.2 understand the angle properties of a circle 10.3 understand the properties of a cyclic quadrilateral 10.4 <u>understand the tests for concyclic points and cyclic quadrilaterals</u> 10.5 <u>understand the properties of tangents to a circle and angles in the alternate segments</u> 10.6 <u>use the basic properties of circles to perform simple geometric proofs</u>	23

Explanatory Notes:

At KS3, students should learn geometry through both an intuitive approach and a deductive approach and the contents were mainly about rectilinear figures. In the Compulsory Part, the scope of study is extended to circles and the learning process may still follow the same sequence of approaches, i.e. from intuitive to deductive. For example, to develop students' exploratory spirit and the capability to reason logically, teachers may ask their students to explore the basic geometric properties of circles by using dynamic geometry software, and subsequently to attempt to prove their conjectures under teachers' guidance. Teachers may refer to the C&A Guide (pp. 109 – 110) for using the inquiry approach to teach the properties of a cyclic quadrilateral.

In Learning Objectives 10.1 – 10.3, students are required to understand the basic properties of circles. Students not only have to know the properties and to use the properties to perform calculations, but also have to understand the justifications or proofs for these properties. However, using these properties to perform other simple geometric proofs belongs to the Non-foundation Topics.

The property that the length of an arc of a circle is proportional to the angle subtended at the centre should have been discussed at KS3, but teachers may remind their students that the length of a chord of a circle is not proportional to the angle subtended at the centre.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
11. Locus	11.1 understand the concept of loci 11.2 describe and sketch the locus of points satisfying given conditions 11.3 describe the locus of points with algebraic equations	7

Explanatory Notes:

To introduce the concept of loci, teachers may make use of daily examples such as long exposure photographs of headlights of moving cars and star trails. Students may make use of dynamic geometry software to explore the locus of a point moving under the given condition, but they have to know that a locus does not necessarily involve the movement of points. For example, all points which are at a fixed distance from a fixed point constitute the locus of a circle.

In Learning Objective 11.2, students are required to describe a locus in words and to sketch its graph while in Learning Objective 11.3, students are required to use algebraic equations to represent locus of points. When finding the equations of loci, students have to make use of the knowledge learnt in the Unit "Coordinate Geometry of Straight Lines" at KS3, e.g. using the distance formula to find the equation of the locus of points maintaining an equal distance from two given points.

In this Learning Unit, students have to find the equations of loci of straight lines, circles, and parabolas of the form $y = ax^2 + bx + c$ under given conditions. The equations of straight lines and circles will be discussed in detail in Learning Unit 12.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
12. Equations of straight lines and circles	12.1 understand the equation of a straight line 12.2 understand the possible intersection of two straight lines 12.3 understand the equation of a circle 12.4 <u>find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle</u>	14

Explanatory Notes:

In the last Learning Unit "Locus", students should have an initial grasp of the concept of locus and have solved simple problems on loci. Teachers may guide students to understand the relation between an equation and its graph from the perspective of locus and then to formulate the equation of a given graph and understand the properties of the graph from its equation.

At KS3, students should have a preliminary understanding of the graphs of linear equations in Unit "Linear Equations in Two Unknowns". In Learning Objective 12.1 of this Learning Unit, they are required to find the equation of a straight line from given conditions. Based on the abilities and needs of their students, teachers may decide if the terms "Two-Point Form", "Point-Slope Form", "Slope-Intercept Form", etc. should be introduced or not. Nevertheless, the conversions among different forms of linear equations are not objectives of the Curriculum.

Regarding the equations of circles (Learning Objective 12.3), students should have understood why there is one and only one circle passing through three non-collinear points in Learning Objective 10.1. Students may now prove this fact again by using coordinate geometry.

Students taking Learning Objective 5.2 (Non-foundation Topics) should have a comprehensive understanding of using the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic. In Learning Objective 12.4, students may apply this knowledge to deduce that there are three possible intersections of a circle and a straight line. As a result, they may use the discriminant of a quadratic equation to determine the number of points of intersection of a straight line and a circle or to find the equations of tangents to the circle. On the other hand, teachers may guide students to use plane geometry to prove that no straight line can cut a circle at more than two points.

Learning Unit	Learning Objective	Time
Measures, Shape and Space Strand		
13. More about trigonometry	13.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity 13.2 solve the trigonometric equations $a\sin\theta = b$, $a\cos\theta = b$, $a\tan\theta = b$ (solutions in the interval from 0° to 360°) <u>and other trigonometric equations (solutions in the interval from 0° to 360°)</u> 13.3 <u>understand the formula ½ *ab* sin *C* for areas of triangles</u> 13.4 <u>understand the sine and cosine formulae</u> 13.5 <u>understand Heron's formula</u> 13.6 <u>use the above formulae to solve 2-dimensional and 3-dimensional problems</u>	21

Explanatory Notes:

Students at KS3 should have well versed in the definitions of sine, cosine and tangent ratios for an acute angle in a right-angled triangle. Teachers may now use a unit circle in the rectangular coordinate plane to define trigonometric functions and to introduce the concepts of positive angles and negative angles. Students should be able to find the maximum and minimum values of trigonometric functions. They should also be able to obtain the periodicity of trigonometric functions from their graphs, and to simplify sine, cosine and tangent expressions involving $-\theta$, 90° ± θ, 180° ± θ …etc., according to the periodicity of the functions. Radian measure, a Learning Objective of Module 2 of the Extended Part, is not required in the Compulsory Part.

In Learning Objective 13.2, solving the trigonometric equations $a\sin\theta = b$, $a\cos\theta = b$ and $a\tan\theta = b$ belongs to the Foundation Topics, while solving other trigonometric equations such as $\sin 2\theta = 0.5$, $\sin\theta = 2\cos\theta$ and $\tan\theta - \cos(90° - \theta) = 0$ belongs to the Non-foundation Topics. Moreover, solving the equations that can be transformed into quadratic equations like $6\sin^2\theta + 5\sin\theta + 1 = 0$ and $\tan\theta = \cos\theta$, belongs to the Non-foundation Topics (see Learning Objective 5.3).

When applying the formulae in Learning Objectives 13.3 – 13.5 to solve 3-dimensional problems, teachers may consolidate students' concepts about the relations between lines and planes grasped in "More about 3-D Figures" at KS3.

Learning Unit	Learning Objective	Time
Data Handling Strand		
14. Permutation and combination	14.1 understand the addition rule and multiplication rule in the counting principle 14.2 understand the concept and notation of permutation 14.3 solve problems on the permutation of distinct objects without repetition 14.4 understand the concept and notation of combination 14.5 solve problems on the combination of distinct objects without repetition	11

Explanatory Notes:

Through different key stages, students should have acquired an intuitive idea on counting. In this Learning Unit, students should have an in-depth understanding on the basic counting principle, like knowing when to apply the addition rule and the multiplication rule. Besides, students will be able to solve more complicated real-life problems through permutation and combination.

When explicating the concepts of permutation and combination, teachers should feel free to choose any common notations, such as "P_r^n", "${}_nP_r$", "${}^nP_r$" and "C_r^n", "${}_nC_r$", "${}^nC_r$", "$\binom{n}{r}$". To facilitate students to read reference books, teachers should introduce other commonly used notations in addition to the chosen ones.

In this Learning Unit, students are required to understand the differences between permutation and combination. They should also understand the relations $C_r^n = \frac{P_r^n}{r!}$ and $C_r^n = C_{n-r}^n$. More complicated properties of permutation or combination, such as $C_r^n + C_{r-1}^n = C_r^{n+1}$, are not required in the Compulsory Part.

There is a wide variety of problems on permutation and combination. Only simple problems on permutation and combination, such as "permutation of objects in which three particular objects are put next to each other", are required to solve in this Learning Unit. Solving problems involving circular permutation, permutation or combination of identical objects or distinct objects with repetitions is not required in the Curriculum.

Learning Unit	Learning Objective	Time
Data Handling Strand		
15. More about probability	15.1 recognise the notation of set language including union, intersection and complement 15.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events 15.3 understand the multiplication law of probability and the concept of independent events 15.4 recognise the concept and notation of conditional probability 15.5 use permutation and combination to solve problems relating to probability	10

Explanatory Notes:

Having learnt the basic concept of probability and how to use counting methods to calculate probabilities at KS3, students are required, in the Compulsory Part, to use the addition law and the multiplication law of probability to solve more complicated probability problems. Further problems on probability will be treated in Module 1 of the Extended Part.

In order to express the relationship between different events (including mutually exclusive, complementary and independent events), the addition law and multiplication law of probability, the events in this Learning Unit are all expressed in set notations. In this connection, before learning the contents of the Learning Objectives 15.2 – 15.5, students should have acquired the basic concepts of sets, which include expressing sets by listing the elements, using descriptions to characterise the elements and Venn diagrams. They should also recognise the concepts and notations of empty set, universal set, union, intersection and complement, which often appear in probability problems, but the rigorous definitions of these concepts and rules of set operations, such as De Morgan's Law, are not required in the Curriculum.

When applying the addition rule and multiplication rule of probability, students should understand how the rules vary under particular conditions, say when A and B are mutually exclusive events. Furthermore, when the multiplication rule is being introduced, students should recognise the concept and the notation of conditional probability and be able to solve problems related to simple conditional probability. However, Bayes' Theorem is not required in the Compulsory Part and will be treated in Module 1 of the Extended Part.

Students who have learnt Learning Unit 14 "Permutation and Combination" (Non-foundation Topics) are required to apply the techniques of counting to solve problems related to probability.

Learning Unit	Learning Objective	Time
Data Handling Strand		
16. Measures of dispersion	16.1 understand the concept of dispersion 16.2 understand the concepts of range and inter-quartile range 16.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data 16.4 understand the concept of standard deviation for both grouped and ungrouped data sets 16.5 compare the dispersions of different sets of data using appropriate measures 16.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution 16.7 explore the effect of the following operations on the dispersion of the data: (i) adding an item to the set of data (ii) removing an item from the set of data (iii) adding a common constant to each item of the set of data (iv) multiplying each item of the set of data by a common constant	14

Explanatory Notes:

At KS2, students should learn a simple way of measuring the central tendency of a set of discrete data – average (arithmetic mean). At KS3, they should learn other ways of measuring the central tendency for both ungrouped and grouped data. In the Compulsory Part, they have to know further that, in many cases, only central tendency is not enough for describing the distribution of a set of data. Students are required to understand the concepts of dispersion, range and inter-quartile range and they should be able to construct and interpret the box-and-whisker diagrams, sometimes called "boxplots". When given a set of ungrouped or grouped data, students should be able to find its standard deviation, and understand their meanings. Moreover, they should be able to choose an appropriate measure to compare the dispersions of different sets of data.

Being a very commonly used term, "variance", as well the relation between variance and standard deviation should be recognised by students in studying this Learning Unit. However, further calculations involving variance will be studied in Module 1 of the Extended Part. In the Compulsory Part, the formula for standard deviation is confined to population only, i.e. $\sigma = \sqrt{\frac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}}$. The formula for the estimate of the standard deviation of the population from which a sample has been taken will be treated in Module 1 of the Extended Part.

Students who have learnt Learning Objective 16.6 (Non-foundation Topics) should understand the simple applications of standard deviation on real-life problems involving standard scores and the normal distribution. During the calculation process, students are not required to look up the table of normal distribution or memorise the percentages of data lying within 1, 2, or 3 standard deviations from the mean. Students who have learnt Learning Objective 16.7 (Non-foundation Topics) should be led to explore how the changes of some of the data within the group will affect the central tendency and the dispersion. As a result, students are expected to have a more in-depth understanding of the properties of different statistics.

Learning Unit	Learning Objective	Time
Data Handling Strand		
17. Uses and abuses of statistics	17.1 recognise different techniques in survey sampling and the basic principles of questionnaire design 17.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations 17.3 assess statistical investigations presented in different sources such as news media, research reports, etc.	8

Explanatory Notes:

At primary and junior secondary levels, students should be familiar with statistics, in particular, data collection, presentation and interpretation of statistical diagrams and statistical charts. They should have a basic understanding of the concept of statistic and, at the senior secondary level, are expected to have a more thorough recognition of real-life statistics.

The concepts of populations and samples may be introduced by quoting several daily examples. Students have to understand why survey sampling is almost inevitable in daily statistics and recognise different techniques in survey sampling. Regarding sampling techniques, students should recognise the basic concepts of both probability sampling and non-probability sampling. However, it should be noted that no calculations involving survey sampling, such as the calculation of standard deviations of samples, are required in the Compulsory Part. Since questionnaire is a very commonly used means to collect data, students should recognise how the factors such as the types, wording and ordering of questions and response options can influence the reliability and validity of questionnaires when constructing questionnaires.

Students should have acquainted themselves at KS3 with the uses and abuses of statistical diagrams/charts and measures of central tendency. In the Compulsory Part, students should discuss further the uses and abuses of statistical methods in various daily-life activities or investigations. Discussions should include the purpose of survey, the sampling method adopted, the way of collecting data and the method of analysis. Moreover, students should be able to have a more thorough analysis of statistical investigations, presented in different sources such as news media and research reports. Analyses of the sampling method of data collection, design of questionnaires, organisation and presentation of data, statistical analysis and inferences should be included. Students are expected to be able to integrate the statistical knowledge learnt at different stages.

Learning Unit	Learning Objective	Time
Further Learning Unit		
18. Further applications	Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learnt in different areas The main focuses are: (a) to explore and solve more sophisticated real-life problems (b) to appreciate the connections between different areas of mathematics	20

Explanatory Notes:

This Learning Unit is different from the application topics in other Learning Units. It does not aim at learning some particular mathematical knowledge, but it allows students to appreciate the connections between different areas of mathematics through exploring and solving more sophisticated real-life problems and to develop their ability to intergrate and apply what they have learnt in different Learning Units. The examples suggested in the C&A Guide (pp.41 – 42) are for teachers' reference only. Teachers may, based on their students' abilities and needs, choose other more appropriate topics for their students. Moreover, teachers should allow their students to search the information for clues, explore different strategies and avoid giving too many hints to students.

Learning Unit	Learning Objective	Time
Further Learning Unit		
19. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	20

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units. In other words, this is not an independent and isolated learning unit and the activities may be conducted in different stages of a lesson, such as motivation, development, consolidation or assessment.

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