Exploration and Verification Analysis of a Linear Reluctance Accelerator

Thor Barrera[1,2], Robby Beard[1,2,3]

[1] Department of Physics and Astronomy, California State University Sacramento, Sacramento, CA 95819 USA
[2]Department of Electrical and Electronic Engineering, California State University Sacramento, Sacramento, CA 95819 USA
[3]Department of Mechanical Engineering, California State University Sacramento, Sacramento, CA 95819 USA

Abstract--Little research is done in field of Linear Reluctance Accelerators (coilguns) owing to their notorious inefficiency, difficulties in modeling their behavior, as well as little interest in them in lieu of seemingly better alternatives such as railguns, has stifled much of the advancement of the field. Here, a study and re-examination of Dr. William Slades's 2005 paper "A Simple Unified Physical Model for a Reluctance Accelerator" was performed to validate and continue further research suggestions mentioned within the paper. In our experimentation, the Slade accelerator was rebuilt using high saturation armatures, thin walled barrels, and a more powerful capacitor bank. We carried out our experiment using armatures of vanadium permendur, electrical iron, and 12L14 steel. Tektronix oscilloscope interface software was implemented to measure coil voltage, current, and armature exit velocities, and compared our results to Slade's results and his modeling equations. Results compiled establish a reasonable set of guidelines for designing these devices to achieve maximum efficiency.

***Index Terms*— Coilgun, Reluctance Accelerator, Pulsed-Power Magnetics, Bill Slade, Vanadium Permendur**

I. INTRODUCTION

Linear Reluctance Accelerators are a class of electromagnetic mass accelerators that accelerate ferromagnetic armatures via magnetic reluctance. These devices already find use in many practical applications as actuators and magnetomotive tools, but high velocity high mass applications for such devices are still not fully realized.

There is in the literature, a somewhat scattered analysis of these types of systems of which include most notably: Slade [1], He, Zabur & Birenbaum [2], Marder [3], and Elliott [4]. Of the various types of models proposed, Slade's work is special for its use of the variational method (lagrangian physics) and formulation to preserve the electromechanical interactions through energy conservation while trying to find a closed form solution to the nonlinearity of the system dynamics.

This paper will discuss the simple reluctance mass accelerator of a single accelerating solenoid. In particular, the paper will examine the Slade equations and the Slade model of the single stage linear reluctance accelerator. This greatly simplifies the various and overly complicated analyses of [2], [3], and [4]. While Slade's equations are a powerful predictive tool, they ignore several important interactions: friction, eddy current interactions, and hysteresis effects. The result of the solution to this non-linear system is a set of equations for which there are 9 variables that have to be solved in unison to provide a numerical solution to the system. This is indeed too many variables to consider all at one time as there are effectively 9 parameters that simultaneously influence our main point of interest: the exit velocity. It is hard to say if the problem is non-optimizable: not much has been done with Slade's equations to attempt to develop an optimization algorithm as far as the literature goes; but what is certain is that the picture of reluctance accelerators is still not fully developed. The objective therefore of our research and work is to further examine the Slade model by reproducing with as close to or better results, the performance characteristics that were achieved by Bill Slade put forth in his 2005 paper [1]. This paper is therefore in many ways very qualitative in nature in the theoretical aspect of the work, referencing the theory primarily to the Slade papers [1]. Our primary focus, therefore, is on the experimental methodology and verification analysis with a brief suggestion on shortcut guidelines to constructing efficient reluctance accelerators.

II. THE BRIEFEST THEORY OF THE SIMPLE RELUCTANCE ACCELERATOR

In essence, the characteristic magnetic circuit, the LRC oscillator, best represents the accelerator system as a whole. However, in this system the inductor acts as an unconstrained variable reluctance actuator. The resistance consists of internal circuit resistance of the capacitors, switching devices and the solenoid. We ideally strive to create an energy storage system which can operate as an ideal voltage source; this allows for the best possible pulse shaping in the current wave form, (which is discussed later in the paper). To get a better idea of the dynamics, we can follow the current as it makes its way through the circuit: As the capacitor bank discharges into the coil, it creates a magnetic field that interacts with and induces a field with a ferromagnetic armature by magnetizing it and thereby creating a two-magnet system of attraction. The simplest analog therefore of this phenomenon is the manner in which a magnet is pulled towards a fellow ferromagnetic material such as an iron bar-thereby inducing a field in the iron and causing dipole magnetic attraction.

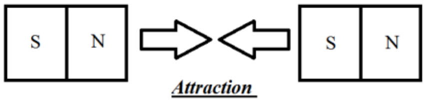

Fig. 1. A simple induced dipole magnetic attraction

The governing equations of the system are given by solving the Slade Lagrangian[1],

$$\mathcal{L} = T_m + T_e - U_e - U_m \tag{1}$$

This becomes,

$$\mathcal{L} = \frac{1}{2}m\dot{z}^2 + \frac{Q^2}{2C} - \pi a^2 N\dot{Q}B - \pi a^2\frac{(l+2\beta)B^2}{2\mu_o} + \pi a^2 Sf\begin{cases} z & \mathit{Entering\ Coil} \\ l & \mathit{Fills\ Coil} \\ l+p-z & \mathit{Exiting\ Coil} \end{cases} \tag{2}$$

Equation 2 is the general form of the ideal Slade accelerator. Solving the lagrangian gives three sets of separate equations of motion: One as the armature enters the coil, one when the armature is in the center of the coil, and one as the armature is exiting the coil. Note that the form does not deal with other problematic interactions, such as friction or eddy currents [1]. Furthermore, Slade's model utilizes a saturation model for the ferromagnetic material without coercive remittance. Adding terms to Eqn. 2 or imposing restrictions to it can make Eqn. 2 nearly impossible to solve, and certainly make a numerical analysis even trickier. To avoid these problems we simply design our system in a way to minimize the potential effects they might display or have as interactions with the system as a whole. Since, at an intermediate level the Slade theory is somewhat easy to follow, it is recommended to the reader that they review his work for further reference to the theory involved behind his model [1] and for further reference to ideas discussed in this paper.

III. DESIGN, DEVELOPMENT AND METHODOLOGY

Developing precision instrumentation and precision machine parts, of course is not easy. Access to a university machine shop has the added benefit of enabling construction of precision machined solenoids, a coil sheath of external ferromagnetic material, and armatures. To this end we believed a verification analysis of the Slade model would be possible with excellent precision.

Our methodology is divided into three parts: Instrumentation Design, Machining, and finally experimental methodology. In particular, there were several construction goals necessary to validate the Slade model from an instrumentation perspective:

1. Construct a sufficient capacitor bank and DC power supply to charge the capacitors quickly and reliably.
2. Construct a photogate sensor to measure exit projectile velocities.
3. Construct a circuit using a linear Hall effect current transducer to measure the coil current during firing.
4. Precision machine projectiles with slits cut along the length in order to reduce eddy currents.
5. Construct the coil sheath out of 12L14 steel, with slits cut for eddy currents, including a single main gap down the entire length to reduce the eddy current circulation, and to prevent the conductive coil sheath from acting like a transformer with a shorted secondary winding.
6. Construct the coils out of 20 gauge magnet wire wound in the same manner as Bill Slade's coils. However these coils will be wound on precision machined polycarbonate tubes, with a 0.38 mm wall thickness, supported on a mandrel during winding. Polycarbonate was chosen for the tube material due to its dielectric properties, impact resistance, and reasonable optical clarity.

IV. POWER SUPPLY DESIGN AND OPERATION

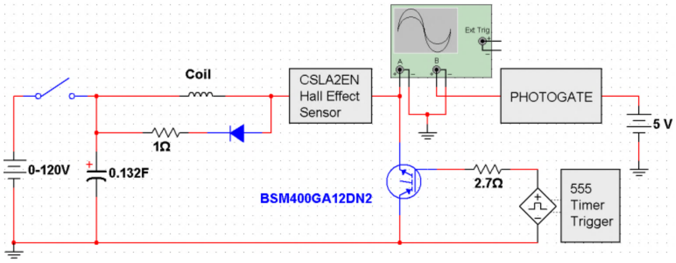

Fig. 2 Simplified Power Supply Circuit

This accelerator system was designed to initially provide the simplest circuit possible to aid in the analysis of the accelerator's mechanics.

V. PHOTOGATE DESIGN AND OPERATION

Our photogate, shown below measures the time difference between the two phototransistor/laser pairs when the projectile passes between them. A schematic of the apparatus helps to understand the reason for this choice in design methodology. In particular, this method allowed us to resolve the analog signal as two digital pulses, thereby allowing the output to be read by either analog or digital means.

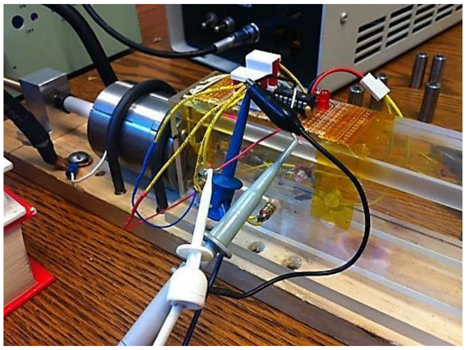

Fig. 3 Photogate Timing Apparatus

We chose to have lasers focused on the transistors to keep them in a constant state of saturation, in order to ensure that there could be no triggering from ambient light, and to preserve the TTL logic levels in the XOR gate.

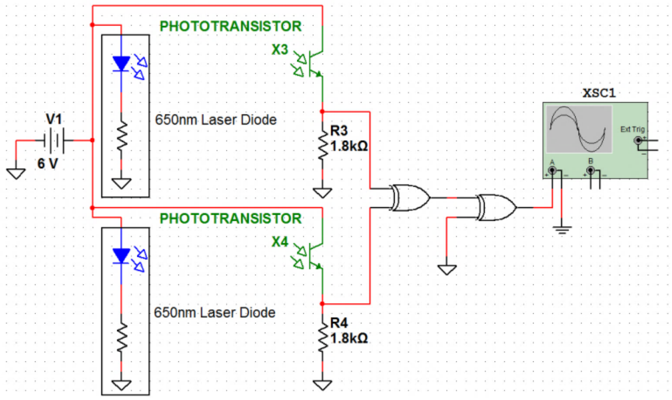

Fig. 4 Photogate Circuit Schematic

Here, the photo-transistor output is passed through an XOR gate giving outputs of either approximately 0V or 4.5V logic with an additional XOR buffer.

When the armature flies through the apparatus, the photogates send two digital pulses, from which we were able to measure the period between them on a Tektronix digital storage oscilloscope. The distance between the two sensors can be accurately determined since the photo transistor/laser pairs are press fitted into holes drilled in an acrylic box, spaced 42.8mm apart. Thus our projectile velocity can be determined as follows:

$$velocity = \frac{dx}{dt} = \frac{x_{gate\,2} - x_{gate\,1}}{period} \qquad (3)$$

VI. ARMATURE AND SHEATH MAGNETIC PROPERTIES

In order for a strong magnetic coupling to be produced between the accelerating solenoid and the armature, the armature must be made from a material with a high magnetic energy density, given by its inital magnetization curve[6],[8].

$$W_m = \int_0^B H\, dB \qquad (4)$$

Where $B = \mu H$

The flux density at which the material saturates is important. Materials that saturate at low levels (such as ferrites) will not have nearly as high energy storage as materials with a high saturation magnetization[5]. We chose the three magnetic materials since we wanted to observe and compare the effects of using materials with increasingly high saturation values. The 12L14 steel was chosen because it is often used in industry as a low cost, soft magnet. Though we found no formal literature on its magnetic abilities, it appears to have decent magnetic properties, and the coil even produces higher inductance values when measured on an inductance meter compared to a common mild steel sample (1008 steel) of the same dimensions, which is evidence at the very least that it has a higher initial permeability than a normal mild steel[6].

In order to maximize the energy transfer in the solenoid, precision machined armatures were created to fit inside the solenoid with as little air gap as possible, yet still be freely mobile. Since Dr. Slade demonstrated mathematically in his paper and mentioned that materials with high staturations are absolutely necessary for efficiency[1], we choose to buy the highest saturating commercially available materials: Hiperco 50A in hot-forged condition from Carpenter Techology, also known as vanadium permendur, which saturates at 2.4T, and commercially pure iron, in cold drawn form from CMI Specialty Products, which saturates close to 2.15T We wanted to compare all three metals to determine out how much more efficient a material such as vanadium permendur could be.

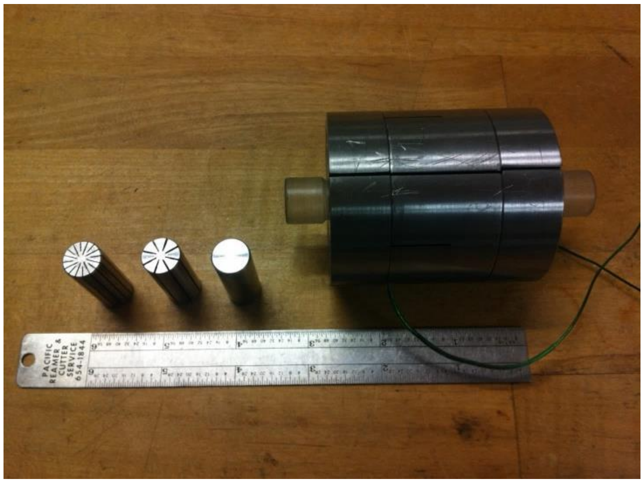

Fig. 5 Some of the machined armatures and coil sheath

Eddy currents in the armatures must be supressed in some way. Dr. Slade created his hexagonal shaped projectile using 7 identical pieces of cylindrical mild steel stock, each kept insulated from one another. However since that would leave only 77.8% of the cross sectional area of a solid projectile, a lot of space is wasted so we decided upon a different approach. We decided to machine slits in our projectiles to reduce the area of the eddy current loops. Our first slitting job consisted of 16 slits in the 12L14 projectile, with a sawblade thickness of 0.15mm. We cut 8 slits a depth of 5.08mm, spaced 45 degrees apart. The CNC machine then rotated the part 22.5 degrees and began cutting 8 more slits at a depth of only 2.54mm, making all slits exacltly 22.5 degrees apart. We had to change our design, since due to having limited experience with cutting such tiny slits, and on a machine that didn’t have flood coolant, we eventually broke all 6 of our sawblades. Since time prevented us from purchasing more 0.15mm saws, we were forced to resort to a thicker, more robust sawblade of 0.5mm thick, which we used to cut the rest of the projectiles, each one now with only 8 slits. The cross sectional area of the armature with 16 slits had roughly 92.5% of the original cross sectional area, with better eddy current reduction; while the projectiles with 8 slits each had only 85%

of original cross sectional area, with less eddy current reduction.

Measurements of maximum inductance (projectile fully inserted into the coil) help to show the potential magnetic coupling the system is capable of and allows for some kinds of early predictions to be made about the efficiency of the system with various projectiles and geometries used. Since the inductance meter has to test the coil using a signal at 1kHz, there are two factors at play when it comes to just how inductive the coil will be when a particular projectile is inserted. These factors are eddy current reduction, and permeability. If we want to compare permeability, we take two different armatures of the same size and geometry (they both must either be solid or slitted, and all their machined dimensions must be very close). The one that produces a higher inductance will have a higher inital permeability. Note that this procedure can only be used when comparing one conductive armature to another, and couldn't work properly for comparing one non conductive armature to a conductive armature such as comparing a mild steel to a ferrite. The other test we can run is to check how well an armature can reduce eddy currents by comparing a solid projectile to a slitted projectile of the same material. As long as they have the same length and diameter, the one that produces a higher inductance has done a better job at reducing the eddy currents. Note that the values for the iron were lower than 12L14, which should have been higher since the tabulated permeability values for iron are higher than most mild steels. We attritube this to poor machining on our behalf, since we somehow made the iron projectiles roughly 0.25mm shorter than all the other projectiles. More testing will be needed in the future.

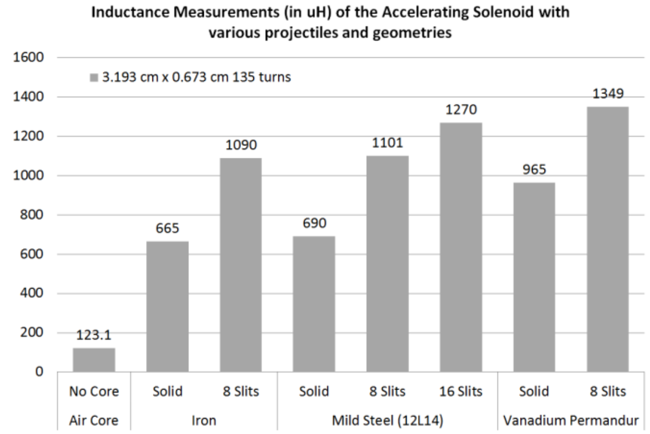

Fig. 6 Inductance Measurements of Armatures for Maximum Inductance

There is also the problem that there will still be eddy currents in the sheath. However, Dr. Slade has shown in a FEA simulation of armature and sheath losses [7], that for coilguns that switch at slow speeds, such as in the millisecond range, most induced eddy currents will dissipate quickly, resulting in minimal loss and the applied magnetic field should penetrate deep into the material.

Table 1 shows some inductance values for a coil sheath and its effect on various armatures. Note that these are two different coils, rolled with the same wire gauge and a comparable number of turns (+/- 2 turns) though the difference in inductance is quite large.

Sample Inductance Tests (uH)		
Material	Coil w/ Sheath	Regular Coil
Air Core	125.4	111.3
8 Slit Iron	669	476
16 Slit 12L14	1226	649
8 Slit V. Perm.	1353	686

Table 1. Inductance Measurements

VII. EXPERIMENTATION AND DATA ANALYSIS

To carry out our experiment we used a Tektronix 3015 MSO to collect the data. We used the oscilloscope to measure the capacitor voltage, projectile velocity, IGBT gate pulse length, and the signal from our CSLA2EN linear Hall effect current sensor. Using the Open Choice Desktop software, we are able to pull the data off all 4 channels in the oscilloscope, as well as the math function, which converts the current sensor output voltage to the actual current value in order to display the true coil current waveform at 200A/div:

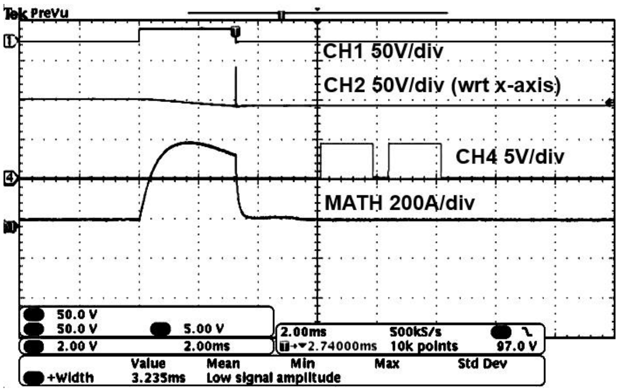

Fig.5 Oscilloscope Outputs During Vanadium Permendur 8 Slit firing

Here channel 1 is the pulse from the IGBT driver, channel 2 is the capacitor voltage, and channel 4, the pulses from the photo transistors.

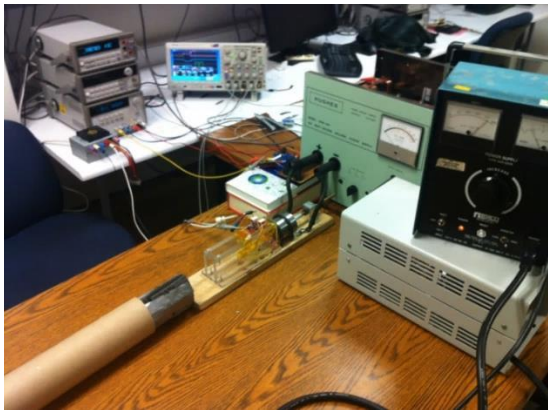

Fig 6. Experimental Apparatus

Summary of Experimentation						
Material & Typology	**Mass (g)**	**Initial Voltage (V)**	**Final Voltage (V)**	**Velocity (m/s)**	**Efficiency**	**% Error**
Van Perm #2 8 Slit	30.475	100.29	92.4	23.07	8.08	4.72
Theoretical Van Perm #2 8 Slit	30.475	100.29	91.9	24.34	8.48	
Van Perm Solid	35.004	100.58	92.7	22.13	8.53	6.23
Theoretical Van Perm Solid	35.004	100.58	91.59	22.87	8.03	
Steel #2 16 Slit	29.634	100.45	92.45	22.60	7.43	1.07
Theoretical Steel #2 16 Slit	29.634	100.45	91.83	23.55	7.51	
Steel #2 8 Slit	29.181	100.14	92.2	22.41	7.26	3.59
Theoretical Steel #2 8 Slit	29.181	100.14	91.59	23.65	7.53	
Steel Solid	33.616	100.69	92.7	19.60	6.33	11.47
Theoretical Steel Solid	33.616	100.69	91.52	22.24	7.15	
Iron 8 Slit	29.989	100.68	92.7	22.83	7.67	2.42
Theoretical Iron 8 Slit	29.989	100.68	92.15	23.86	7.86	
Iron Solid	33.996	100.37	92.38	18.59	5.78	22.83
Theoretical Iron Solid	33.996	100.37	91.3	22.48	7.49	

Table 2. Summary of Experimentation

Figure 6 and Table 2 show our experimental setup and results respectively. Most interesting to note is the very excellent agreement with our earlier statements about the connection between inductance and efficiency. Indeed our higher inductance materials/geometries performed better on average than the lower ones. One quandry is the vanadium permandur 8 slit projectile, of which had a lower effiicency on the average than our solid vanadium permandur. Since it is susceptible to aggressive collisions, which can change its magnetic properities, there is a possibility that this may have been a direct result from performing the machining operation on the material and improper set up or feed/speeds which could have altered the materials domain structure. However the results from the steel performed as expected according to the Slade model, with the higher inductance steel projectiles achieving higher efficiencies than their lower inductance unslitted counterparts. On the whole, these results seem to have a decent agreement, most within 5% difference, with many of the parameters of the model such as initial and final velocities, final voltages and max current exibiting even smaller differences between theory and experiment. In particular, it is of interest to examine the 16-slit 12L14 projectile and its results as compared with the Slade model because we expected the 16-slit 12L14 to be very effective in reducing unwanted eddy currents, thus allowing us to observe results that were closer to the the model. Plotting the coil current curves for theory and experimentation give us this:

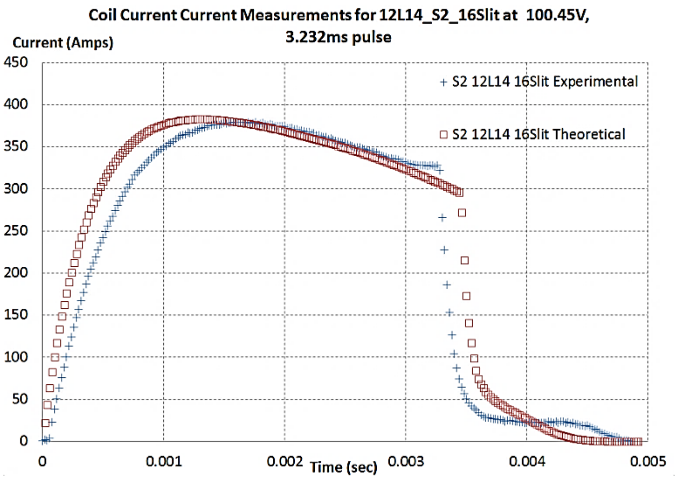

Fig. 7 Coil Current Measurements During Firing Sequence

The experimental curve for the 12L14 16-slit coil current matches with good agreement to the theoretical coil current, and what's more it even matches the characteristic features of the Slade experimentation with excellent agreement.

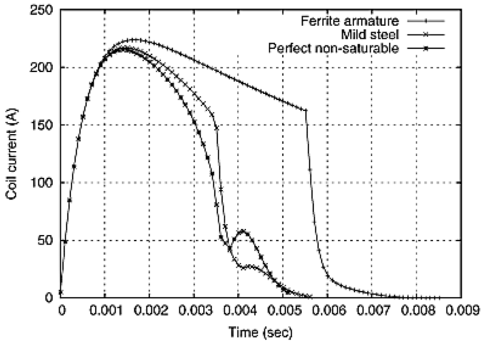

Fig. 8 Slades Experimental Coil Current Data [1]

In particular, the foot of the firing sequence for our 12L14 measurement as compared to Slade's mild steel measurement shows noticable difference in the "back-emf" introduced back into the system[1], specifically the elimination of a back emf spike.

Parameter	**Material**						
	Iron		**12L14**			**Vanadium Permandur**	
	Solid	**8 Slits**	**Solid**	**8 Slits**	**16 Slits**	**Solid**	**8 Slits**
Coil Length (m)	31.75mm						
Armature Length (m)	.0348m	.0347m	.0350m	.03495m	.03491m	.0350m	.03507m
Gap Length (m)	4.6E-4 m						
Capacitance (F)	.132 F +/- 20%						
Coil Core Radius (m)	6.3E-3 m						
# Turns	135 turns						
Relative Permeability	200		180			650	
Flux Saturation (T)	2.15T		~2.0T			2.4T	
Coil Resistance (Ohms)	.243 Ohms						
Armature Mass(g)	33.996g	29.989g	33.616g	29.181g	29.634g	35.004g	30.475g
Quench Resistor (Ohms)	.998 Ohms						
Simulation Time Step	1.00E-07						
Tolerence	1.00E-08						

Table 3. Parameters of the Experiment for use in Computation

We computed simulations for the armatures and compared the results between parameters and the important data outputs from the Slade equations with our experimental results

From table 2, our error between the results of the 16-slit 12L14 theoretical and experiemental is about 1.1% with room for improvement. With these results, Slade's model can be justified as being an excellent model of these devices, but the lack of accounting for eddy current and friction forces make one wonder if these forces are simply too difficult to implememnt into this model or if it is possible to add a "correction" factor to obtain a better model that can accurately describe the dynamics of the solid armatures, since a good slitted armature, to a great degree of accuracy, can be modeled with excellent agreement over a wide range of voltages and starting positions using the Slade model.

VIII. DESIGN PARAMETERS

Designing a reluctance accelerator is a terribly difficult process, even simple rules of thumb aren't very helpful when considering the fact that there are so many parameters that truly are working against the designer at any given moment. But there are some very clear things about the design process.

These were significantly important factors to us, but also go beyond the basic rules of thumb of linear reluctance accelerators.

1. The energy storage system should approximate as best possible a constant voltage source[8].
2. There should be as minimal a gap length as possible[1].
3. Thin-cut eddy current slitting radially about the z-axis of the projectile will improve efficiency despite losing permeable volume that can be additionally magnetized [8].
4. The solenoid core should be tightly precision rolled on the flytube itself and resin coated between each layer of windings. Adding kapton tape about the coil will additionally help insulate the solenoid windings, which is necessary for using a conducting sheath[5].
5. High saturation armatures are necessary[1].
6. Conductive coil sheaths can be used for flux concentration so long as eddy current slits are made in the sheath[7].

IX. CONCLUSIONS

This work gives the promise that there may be much experimental support for the possibility of a high efficiency electromagnetic linear reluctance accelerator. Our preliminary results are limited because of the nature of the system, high precision measurement of the armature velocities, coil current, and armature hysterisis are still necessary to develop more accurately a cohesive theory of the electromagnetic linear reluctance accelerator.

It’s especially interesting to note the very simple relationships that control a much larger more complex and non-linear dynamical system. For materials such as 12L14 with good eddy current slitting the model is an excellent fit. Our research got within ~1.1% of the theoretically simulated efficiency for the 16-slit 12L14 projectile, which is a very good result given the simplicity of the model. Most other efficiencies were somewhat close to their theoretical values with solid projectiles being the exception however, as high as ~23% off the theoretical simulated efficiency for solid iron. Slade’s simulation reduces the complexities of the problem to only a few variables, of which perhaps the most difficult thing was to find reliable values for flux saturation. Even with some error in measurements of parameters, these simulations are for most purposes, an excellent model of the important system dynamics. They produce rough guidelines to understanding more complex electrodynamic interactions between the solenoid flux and the armatures’ material properities.

Incidentally, much of the Slade theory and many others, rely upon the symmetry of the accelerating solenoid but, different geometries and winding fashions may, in fact, produce more favorable results. And in the future this might be a point of interest for further studies. Introducing a way to account for eddy currents present in the system would help to develop a much more comprehensive model. This however would create the problem of needing to describe the armature geometry and the development of the eddy currents within it mathematically in such a way that it could be represented either within the lagrangian or externally as a correction factor.

It would be a disservice to say that much of this information is ‘groundbreaking’[9]. Despite that, the research has definitively shown that though we did not achieve Dr. Slade’s results for efficiency, excellent agreement with the theory is very possible if the system is carefully designed around the limitations of the model. Thus, the model can work for a wide range of systems and materials with varying properities.

X. REFERENCES

[1] Slade, William G. *“A Simple Unified Physical Model for a Reluctance Accelerator.”* IEEE Transactions on Magnetics Vol. 41. No. 11, November 2005. 4270-4276. Web

[2] B. Marder, *“A coilgun design primer,”* IEEE Trans. Magn., vol. 29, no.1, pp. 701–705, Jan. 1993.

[3] J. L. He, Z. Zabar, E. Levi, L. Birenbaum. *“Transient Performance of Linear Induction Launchers Fed By Generators and By Capacitor Banks”* IEEE Transactions on Magnetics, Vol. 27, No.1, January 1991 585. Web

[4] Wang, Liu, Qiang Liu, Jun - Cheng, Min Zhu. *“The Design and Structural Analysis of a Coilgun for Low Acceleration of Heavy Loads”* IEEE Transactions on Magnetics, Vol.35, No.1 January 1999

[5] Colonel William T. McLyman *“Transformer and Inductor Design Handbook, Third Edition”*, CRC Press Mar 31, 2004. ISBN 0-824-75393-3

[6] Jackson, J. D. (1962, 1975, 1998). *“Classical Electrodynamic”s*. New York: John Wiley & Sons. ISBN 0-471-30932-X

[7] William Slade, *“Notes on the effects of metallic coil sheaths and armature losses in reluctance coilguns”*, July 4, 2006 Accessed November 23, 2013 (http://users.skynet.be/BillsPage/SimulationResults1.0.pdf)

[8] Branislav M. Notaros. “*Electromagnetics”* Prentice Hall 2010, ISBN 0-132-433842

[9] The Rand Corp “*A Million Random Digits with 100,000 Normal Deviates*” Free Press Publishers 1955 2001© ISBN 0-8330-3047-7