Exponentials & Logarithms Unit 8

Tentative TEST date_______________

Big idea/Learning Goals

This unit begins with the review of exponent laws, solving exponential equations (by matching bases method and trial & error method) and problems solving with exponential functions. You will then learn about the inverse of exponential functions – which is a logarithmic function of the same base. Just like there are several exponent laws there will be several logarithmic laws that you will have to know. These laws will help you in solving more complicated exponential equations where previous methods don't work or to avoid the use of trial & error method. The laws will also help you solve logarithmic equations. You will learn how to graph logarithmic functions by the use of transformations or by the use of key characteristics and finally you will study some real life situations that involve logarithms.

<u>Corrections for the textbook answers:</u>

Sec 8.1 #9c) 3.
Sec 8.2 #4d) iii) domain x>0 #8a) (25, -1)
Sec 8.3 #4d) 1.40 #14a) 223 mph
Sec 8.4 #10c) x=4
Sec 8.5 #6a) 9.01
Sec 8.6 #10 x=2 the only solution
Sec 8.8 #7 table is wrong for it to be exponential with same growth rate
Review #7d) log144

Success Criteria

☐ I <u>understand the new topics</u> for this unit if I can do the practice questions in the textbook/handouts

Date	pg	Topics	# of quest. done? You may be asked to show them	**Questions I had difficulty with** ask teacher before test!
	2-4	Review of Grade 11 (2 days if there's time) THREE Handouts Exponent laws, Solve exponential equations by matching bases, Solve exponential word problems by trial & error		
	5-6	What is a Logarithmic Function? Section 8.1 & 8.3 & THREE Handouts		
	7-8	Exponential & Logarithmic Functions Section 8.2 & THREE Handouts		
	9-10	Laws of Logarithms Section 8.4 & TWO Handouts		
	11-12	Solving Exponential Equations by using Logs Section 8.5 & TWO Handouts		
	13-14	Solve Logarithmic Equations Section 8.6 & TWO Handouts		
	15-16	Solve Problems Section 8.7 & Handout		
		REVIEW		

Reflect – previous TEST mark ___________, Overall mark now___________.

Review of Grade 11

1. Summarize the laws you learned in grade 9-11 (multiplication, division, power of power, zero, negative, rational, distributive properties)

2. Apply the laws to the following examples as you simplify the questions. Leave everything as exact numbers, with positive exponent answers.

a. $\dfrac{\left(-2xy^{3}\times 3x^{-3}y^{-2}\right)^{3}}{6x^{0}y^{-1}}$

b. $\left(8x^{6}y^{9}\right)^{\frac{1}{3}}\left(27x^{-12}y^{2}\right)^{-\frac{1}{3}}$

c. $\left(\dfrac{64m^{15}}{343}\right)^{-\frac{2}{3}}$

d. $\dfrac{\left(-3c^{4}\right)^{-2}}{c^{-1}\times\left(3c^{-2}\right)^{-2}}$

e. $\left(8x^{\frac{3}{4}}y^{2}\right)^{-\frac{1}{3}}$

f. $\left(\dfrac{\left(-2a^{-2}\right)^{3}a^{3}}{4a^{-4}}\right)^{-3}$

There are several useful constants that are used for math
π is called __________ and π = ___________________ is used with anything circular
e is called _________________________ and e = ___________________ is used with exponential continuous growth/decay

Read up more on the number *e:*
http://betterexplained.com/articles/an-intuitive-guide-to-exponential-functions-e/\

When the variable is **on the base**, not in the exponent, to solve it you must isolate it by using
____

When the variable is **in the exponent**, not on the base, to solve it you must
____

When the variable is **in the exponent** and bases **cannot** be matched you must use
____

3. Match the method to each given question. Then solve.
   a. $2^{2x}=8$
   b. $2x^{\frac{4}{5}}+12=174$
   c. $2\cdot e^{x}+5=22$

4. Practice matching bases method:
   a. $3^{2x-5}=1$
   b. $\left(\frac{1}{5}\right)^{-3x}\cdot 25^{x-1}=\frac{1}{125}$
   c. $5^{x-2}+5^{x-3}=150$
   d. $4^{x}=\frac{1}{256}$
   e. $2\cdot\left(\frac{1}{64}\right)^{x}=\left(\frac{1}{2}\right)^{x+2.5}$
   f. $3^{x+3}-3^{x}=234$

5. Here is the general equation for exponentials that will most often be used for exponential word problems that have horizontal asympote of y = 0. Explain the significance of EACH letter in the context of a word problems and summarize how to find the 'b' in the equation.

$$y = a\left(b\right)^{\frac{x}{p}}$$

6. Solve the following problems by using the trial & error method.
(you will later learn how to use logs to solve these without the use of trial and error)

a.
A drug's effectiveness decreases as time passes. Each hour the 250mg drug loses 5% of its effectiveness. How long will it take for the dose to reach the low level of 52mg?

b.
Carbon-14 has a half life of 5730 years.
(If no initial amount is given, assume 100% is the initial amount)
Some pre-historic cave paintings were discovered in a cave in France. If the paint contained 48% of the orinial carbon-14, estimate the age of the painting.

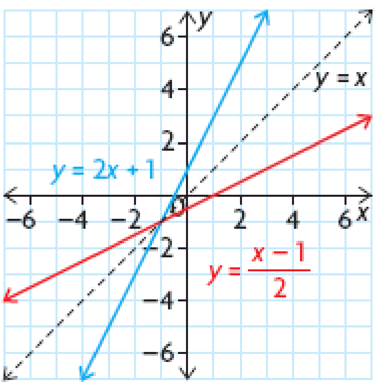
c.
A cottage is originally bought for $150 000. If the value of this cottage appreciates at the rate of 7% per year, when will the cottage be worth $200 000?

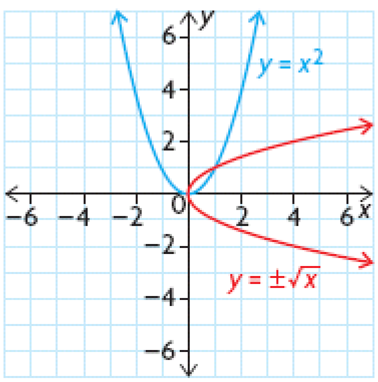
d.
The 200 fruit fly population doubles every 5 days. In how many days is the population up to 1000 flies?

What is a Logarithmic Function?

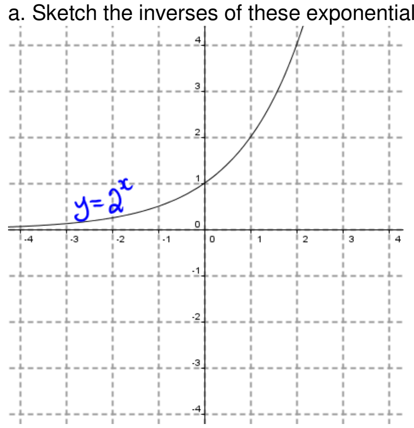
1. You've seen inverses for all the functions you've studied so far, except for exponentials. Recall that inverse graphs are just reflections in y=x lines and inverse equations have x=input and y=output switched.

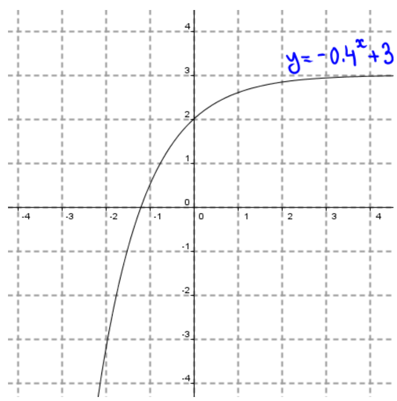
recall inverses of lines and quadratics





a. Sketch the inverses of these exponentials





b. Find the inverse equations of these functions.

2. The inverse equations you've found above don't have the output isolated and hence cannot be written in function notation. This is one of the reasons that logarithms were invented. Summarize the rule of switching exponential form to logarithmic form or vise versa, then write down the inverse functions using function notation for the above questions.

3. Practice switching the form

a. $r = \log_p q$

b. $a^b = c$

c. $\log_4 2 = \frac{1}{2}$

d. $\left(\frac{1}{5}\right)^2 = \frac{1}{25}$

4. What meaning does $\log_a x$ have?

5. Evaluate the following

a. $\log 1000000$

b. $\log_2 16$

c. $\log_5 1$

d. $\log_2 \frac{1}{64}$

e. $\log_3 \sqrt[7]{9}$

f. $\log_{\frac{1}{7}} 49^2$

6. Find the inverse functions

a. $y = 2\log_3(x-1)$

b. $y = \frac{4^x - 1}{3}$

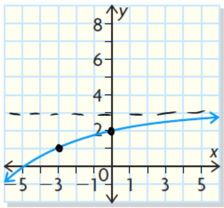
c. $y = \log_4 2x + 5$

d. $y = 2(3)^{x-9}$

Exponential & Logarithmic Functions

1. Review how to find the equation of the exponential function from a table or a graph

a.



b.

x	y
2	14.75
4	113.19
6	728.42
8	4573.64

Horizontal asymptote at y=-4.

2. Summarize the steps of sketching exponentials. $y=ab^{k(x-d)}+c$

Sketch the following functions, state domain and range

3. $y=-2(4)^{\frac{x}{2}+1}+3$

4. $y=100(0.5)^{2x-1}$

5. $y=5(3.5)^{\frac{8-x}{2}}-10$

6. Summarize the steps of sketching logarithms: $y = a\log_b[k(x-d)]+c$

Sketch the following functions, state domain and range

7. $y = -\frac{1}{6}\log_{0.2} x - 3$

8. $y = \log_{10}(6x+12)+0.5$

9. $y = 2\log_{0.5}(15-3x)+1$

10. $y = 2+\log_3(4x+1)$

Laws of Logarithms

Read and understand the following proofs to the new logarithm laws.

1. Proof for

$\boxed{\log_a a^x = x}$

let $f(x) = a^x$ then

$f^{-1}(x) = \log_a x$

you already know that

$f^{-1}(f(x)) = x,$ therefore

$\log_a(a^x) = x$

2. Proof is similar for

$\boxed{a^{\log_a x} = x}$

3. Proof for

$\boxed{\log_a(xy) = \log_a x + \log_a y}$

Use the exponent multiplication law:

$a^m \cdot a^n = a^{m+n}$

$\begin{cases} \text{let } x = a^m & \Leftrightarrow \log_a x = m \\ \text{and } y = a^n & \Leftrightarrow \log_a y = n \end{cases}$

$\log_a(x \cdot y)$

$= \log_a(a^m \cdot a^n)$

$= \log_a a^{m+n}$

$= m + n$

$= \log_a x + \log_a y$

Notice bases have to match and coefficients as well.

$\boxed{c\log_a(x \cdot y) = c\log_a x + c\log_a y}$

4. Proof is similar for

$\boxed{\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y}$

5. Proof for

$\boxed{\log_a(x)^n = n\log_a x}$

Use the exponent power of power law

$\left(a^m\right)^n = a^{mn}$

$\{\ \text{let } x = a^m \Leftrightarrow \log_a x = m$

$\log_a\left(x\right)^n$

$= \log_a\left(a^m\right)^n$

$= \log_a a^{mn}$

$= mn$

$= n\log_a x$

6. Proof for CHANGE of BASE

$\boxed{\log_b a = \frac{\log a}{\log b}}$

$\{\ b^x = a \Leftrightarrow \log_b a = x$

$b^x = a$

$\log b^x = \log a$

$x\log b = \log a$

$x = \frac{\log a}{\log b}$

$\log_b a = \frac{\log a}{\log b}$

7. Proof for

$\boxed{\log_a 1 = 0}$

Use the zero exponent law

$a^0 = 1$

$\log_a a^0 = \log_a 1$

$0 = \log_a 1$

8. Expand the following by using laws of logs

a. $\log_a\left(\frac{xy^2}{z}\right)$

b. $\log_a\left(y^3 z^{\frac{1}{3}}\right)$

c. $\log_a\sqrt{yz}$

d. $\log_a\left(\frac{x}{\sqrt{xy}}\right)^{\frac{1}{4}}$

9. Condense the following into a single logarithm

a. $4\log_a\sqrt{x}-\log_a y$

b. $-5\log_a x+2\log_a y+\frac{1}{3}\log_a z$

c. $\frac{3}{2}\log_a z+\log_a y$

d. $-2\log_a x^2-\frac{1}{2}\log_a y+\frac{1}{5}\log_a z$

10. Evaluate

a. $\log_{\frac{1}{3}} 9$

(possible without a calculator since bases can be matched)

b. $\log_8 92$

(not possible without the change of base formula and calculator)

Solving Exponential Equations by using Logs

1. An investment of $500 is invested in an account that pays 6.4% compound annually. How long will it take for the original amount of the investment to triple?

2. A culture of bacteria triples every 30 minutes. How long will it take a culture originally consisting of 40 bacteria to grow to a population of 200 000 bacteria?

Here are a few strategies to try when solving exponential equations

- Match bases (goal is to have a single base on one side of the equation and the SAME single base on the other side of the equation – no coefficients)
- If x appears once → Switch forms to log
- If x appears more than once, no plus/minus between bases → use laws of exponents to rearrange (remember if exponents are the same you can combine bases)
- If x appears more than once, WITH plus/minus between bases → Make a substitution to simplify the equation

3. Solve the following equations.

a. $5^{2x} = \sqrt{\dfrac{1}{625}}$

b. $\left(\dfrac{1}{4}\right)^{x+1} + \left(\dfrac{1}{4}\right)^{x} = 20$

c. $\left(\frac{5}{4}\right)^{10-x} = 7^{x}$

d. $4\left(3^{x}\right) + 3^{2x} = -4$

e. $3^{x-4} = 9^{x-6}$

f. $19^{\frac{x}{2}-4} = 81$

g. $4^{x} \cdot \frac{1}{16} = 2^{3x+6}$

h. $\left(\frac{1}{2}\right)^{x+1} = \left(\frac{1}{3}\right)^{x}$

Solving Logarithmic Equations

When you are are solving logarithmic equations, keep in mind the domain of logarithms and discard any solutions that make $\log(zero) = undefined$ or $\log(negative) = undefined$

Also remember that you CANNOT distribute the log over separate terms, just like you CANNOT distribute exponents over several terms
Ex.

can't

$\log(3+x)$

can't

$(3+x)^2$

Here are a few strategies to try when solving logarithmic equations

- One isolated log→ Switch forms to exp
- Two logs of same base on either side →Equate the inputs of the logs
- Many logs →Use laws of logs to condense

Solve the following equations.

a. $\log_x \frac{16}{18} = 4$

b. $\log_{2.5}(-2x) = \log_{2.5}(-6-x)$

c. $\log_4 18x - \log_4 2.5 = \log_4 39.6$

d. $\log_5(x+2\sqrt{6}) + \log_5(x-2\sqrt{6}) = 2$

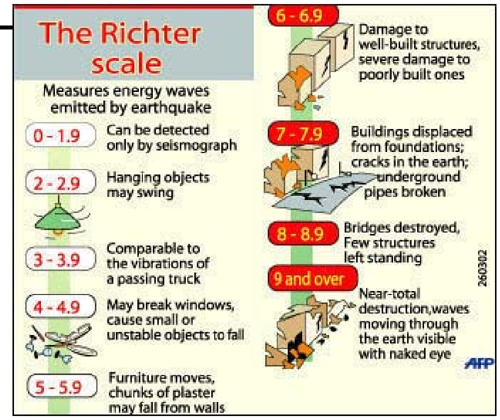
e. $\log_x 8 = \frac{1}{4}$

f. $\log_9 \frac{9}{5}x = \log_9 \frac{63}{10} + \log_2 4^{-2}$

g. $\log_3 x^5 + \log_3 x = 12$

h. $\log x + \log(x-3) = \log 10$

Solve Problems



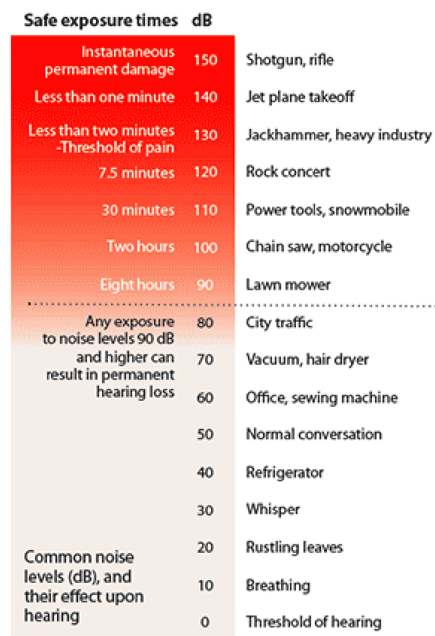
The logarithms are used in several different real life applications

- Earthquake magnitudes (developed by Charles F. Richter)
- Intensity of sound waves (how loud things are)
- pH scale

$$M = \log\left(\frac{I}{I_0}\right)$$

___ = the magnitude of the earthquake on the Richter scale
___ = the intensity/amplitude of the reference earthquake
___ = the intensity/amplitude of the wave detected by the **seismograph** of the earthquake being measured

1. In October of 2005, Pakistan experienced an earthquate of magnitude 7.6 resulting in the death of over 73 000 people. Later on that month, Owen Sound experienced an earthquake of 4.2 in magnitude. How many times more intense was the Pakistan earthquake to the quake in Owen Sound?



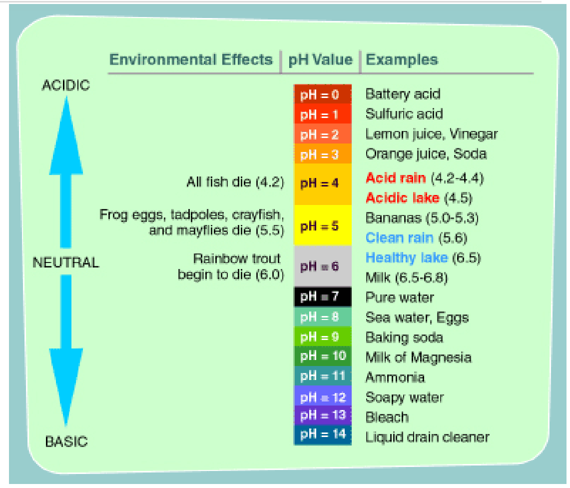
$$L = 10\log\left(\frac{I}{I_0}\right)$$

___ = the loudness of sound in decibels
___ = the intensity of sound power per unit area (watts/m^2) of the threshold of hearing
___ = the intensity of sound power per unit area (watts/m^2) being measured

2. How many times more loud is a rock concert with a sound intensity of 123 dB than the threshold of sound?

$$pH = -\log\left(H^{+}\right)$$

___ = the acidity of the substance
___ = the concetration of hydrogen ions (mol/L)



3. A fish tank's water was recently changed with distilled water of pH 7. The day after it was changed, apple juice was spilled into it which caused the pH to drop to 5.8. By what factor has $[H^{+}]$ changed?

4. A radioactive substance has a half-life of 3 days. Suppose you have 750 000 g of this substance now. In how many days will the mass be 2.3×10^{-2}?

5. A boat sells at $16 000. Each year it depreciates (decreases in value) by 15%. In how many years will the boat's value be $10 000?

6. Find the pH of a solution with hydronium ion concentration of $4.5\text{x}10^{-5}$

7. Find the decibel rating of a sound with intensity of $5000I_o$

8. If a sound has a decibel rating of 85, how much more intense is it than the threshold sound?