

Expressions and Equations

3

Unit Overview

In this unit you will use variables to write expressions and equations. You will solve and graph equations and inequalities.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- compare
- contrast

Math Terms

- numerical expression
- order of operations
- variable
- coefficient
- algebraic expression
- term
- unit rate
- mathematical property
- equation
- solution
- inverse operations
- inequality
- rate of change
- ordered pair
- independent variable
- dependent variable

ESSENTIAL QUESTIONS



Why are tables, graphs, and equations useful for representing relationships?



How can you use equations to solve real-world problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 12 and 16, will give you an opportunity to demonstrate how you can write, graph, and solve equations and inequalities to solve mathematical and real-world problems.

Embedded Assessment 1:

Order of Operations and Expressions p. 157

Embedded Assessment 2:

Expressions and Equations p. 211

Getting Ready

Write your answers on notebook paper.
Show your work.

1. Copy and complete the table.

Input	3	7	11	14	17	21
Output	0		8			

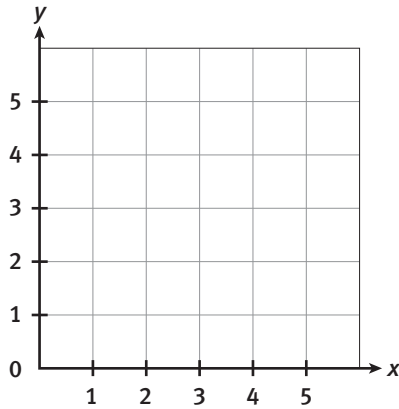
Write the rule you used.

2. Copy and complete the table.

Input	Output
3	18
7	
9	
11	
20	120
35	

Write the rule you used.

3. Make a grid like the one below. Then plot each point on it.



- a. (0, 2) b. (2, 1)
c. (1, 3) d. (4, 4)
e. (5, 0)

For Items 4–6, evaluate each expression for $n = 7$.

4. $n + 9$

5. $(n - 3) \div 4$

6. $\frac{35}{n}$

7. Tell how to undo each operation and explain why it works.

- a. adding 28
b. dividing by 17

8. Write the reciprocal of each number.

a. 7

b. $\frac{1}{2}$

c. $\frac{2}{5}$

d. $\frac{4}{3}$

e. $\frac{4}{3}$

- e. Explain how a number and its reciprocal are related.

Learning Targets:

- Use the order of operations to simplify expressions involving addition, subtraction, multiplication, and division.
- Use the order of operations to simplify expressions involving whole number exponents and parentheses.

SUGGESTED LEARNING STRATEGIES: Paraphrasing, Simplify the Problem, Critique Reasoning

Ayana and Zachary Wilson are going to the Pace County Fair.

1. Ayana loves to make lists of things to do to prepare for an activity. She made the following list for the morning of the fair.

To Do	Order
Take a shower	
Eat breakfast	
Get dressed	
Buy tickets for the fair	
Put on shoes	
Put on socks	
Ride to fair	
Get money from piggy bank	

- a. Order the steps as you think Ayana will complete them.
- b. Explain why the order of Ayana's steps is important.

At the fair, general admission is \$8.00 per person. Tickets for rides and games must be bought separately. Food and drinks are purchased at the concession stands. A ride ticket costs \$3.00.

2. Ayana plans to buy tickets for five rides. She wrote the expression $8 + 5 \cdot 3$ to represent the cost of her rides and admission to the fair. Zachary intends to go on 8 rides in the morning and on 5 rides in the afternoon. He also wrote the expression $8 + 5 \cdot 3$ to represent the total cost of the rides he wants to go on.
 - a. How much does Ayana expect to pay for admission to the fair and the rides she wants to go on?
 - b. How much does Zachary expect to pay for the total cost of his rides, not including his admission cost?

My Notes

MATH TERMS

A **numerical expression** is a mathematical phrase that uses numbers or numbers with operations. For example: 6 or $4 + 3$.

My Notes

In solving math problems, there would be many miscalculations if one math problem could have two or more answers. To prevent this from occurring, mathematicians have agreed that when evaluating an expression containing both addition and multiplication, multiplication should be performed first.

3. The expression $8 + 5 \cdot 3$ does not accurately represent both Ayana's and Zachary's costs. Explain why.

When evaluating a numerical expression with addition and subtraction, the operations should be performed in the order they appear from left to right.

4. **Make sense of problems.** Ayana has \$100 saved. Her dad gave her \$5. She took \$60 to the fair and left the \$5 home. She wrote the expression $100 - 60 + 5$ to represent the amount she has left at home. Is this expression correct? Explain.
5. **Construct viable arguments.** Zachary has \$100 saved. He takes \$60 of it for rides, food, and admission to the fair and \$5 of it for a souvenir. He wrote the expression $100 - 60 + 5$ to represent the amount of money he will have left after the fair. Does this expression correctly represent the amount he will have left? Explain.
6. Ayana and Zachary took some snacks to the fair. Ayana took a box of six granola bars and divided them evenly into three snack bags. She took three more boxes of six granola bars and shared them evenly into the same three snack bags. Zachary wrote the expression $6 \div 3 \times 4$ to represent the number of granola bars in each bag. Evaluate Zachary's expression. Does his expression work? Explain why your answer makes sense for this situation.

Lesson 11-1

Order of Operations

ACTIVITY 11

continued

To simplify a numerical expression containing addition, subtraction, multiplication, and division, follow these rules:

Step 1: Multiplication and division are performed from left to right.

Step 2: Addition and subtraction are performed from left to right.

Example A

Simplify each of the following expressions.

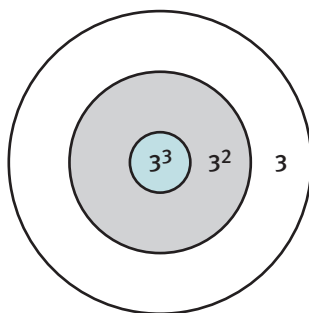
- a. $4 \cdot 5 \div 2$ Multiplication comes first when moving from left to right.
 $(4 \cdot 5) \div 2$
 $20 \div 2 = 10$
- b. $4 + 5 \cdot 2$ Multiplication is done before addition.
 $4 + (5 \cdot 2)$
 $4 + 10 = 14$
- c. $4 + 5 - 2 \cdot 3$ Multiplication is done before addition or subtraction.
 $4 + 5 - (2 \cdot 3)$ Addition and subtraction are done from left to right.
 $(4 + 5) - 6$
 $9 - 6 = 3$

Try These A

Simplify each of the following expressions.

- a. $3 \cdot 6 + 7$
b. $3 + 6 \cdot 7$
c. $3 + 6 - 2 \cdot 2$
d. $3 + 6 - 7$

7. When Ayana and Zachary got to the fair, they played a game of darts. The target had three rings labeled 3 , 3^2 , and 3^3 . Zachary threw two darts at the target, which both landed in the 3^2 circle.



- a. Ayana expressed Zachary's score as $3^2 + 3^2$. According to Ayana's expression, what is Zachary's score?
b. Zachary expressed his own score as $2 \cdot 3^2$. Could he evaluate his expression to give the same score as generated by Ayana's expression? Explain how he could, or why it is not possible.

My Notes

CONNECT TO PHYSICAL EDUCATION

Darts is a throwing game. Players throw darts at a circular target known as a dartboard.

My Notes

When an expression involves exponents, it should be evaluated before doing addition, subtraction, multiplication, or division.

8. Simplify each expression.

a. $2^4 \cdot 3^2$

b. $19 + 32 \div 2^3$

c. $9 \cdot (2 + 3)^2 - 14$

When an expression involves grouping symbols, such as parentheses or brackets, the operations inside the grouping symbols should be evaluated first.

9. Make use of structure. Add parentheses to the expression $8 + 5 \cdot 3$, so the expression will give the cost of buying 8 rides in the morning and 5 rides in the afternoon at \$3.00 per ride. Explain your reasoning.

MATH TERMS

The **order of operations** is as follows:

- Do calculations inside grouping symbols first, beginning with the innermost set.
- Evaluate expressions with exponents.
- Do multiplication and division from left to right.
- Do addition and subtraction from left to right.

10. The Wilsons stop at a restaurant on the way home from the fair. They order one hamburger value meal, four apples, two juices, and two pizza value meals.

SNACKS	
Pizza	\$3.00
Salad	\$5.00
Juice	\$2.00
Apples	\$2.00
Yogurt	\$2.50
Homemade Soup	\$4.00
VALUE MEALS	
Pizza, Yogurt, Juice	\$6.00
Hamburger, Soup, Juice	\$8.00

a. Write an expression that uses addition, multiplication, and a set of grouping symbols to represent the total cost of the meal.

b. Evaluate your expression to find the total cost of the meal.

My Notes

Learning Targets:

- Use variables to represent numbers and write expressions to solve problems.
- Evaluate expressions containing variables.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Look for a Pattern, Paraphrasing, Sharing and Responding

1. Ayana paid \$3.00 for each ride she went on at the fair. Complete this table to show the cost of the given number of rides.

Number of Rides	Total Cost of Rides
1	
2	
3	
4	
5	
6	
7	

2. Describe any patterns you notice in the table.
3. How much would it cost Ayana to go on 6 rides?
4. Write a numeric expression for the cost of 8 rides.
5. Write a numeric expression for the cost of 12 rides.

Often **variables** are used to represent parts of an expression that may change or are unknown.

6. Let r represent the number of rides Ayana goes on. Write an expression using the variable r to represent the cost of r rides.

A **coefficient** is a number multiplied by a variable in an **algebraic expression** or equation.

In the expression $2a$, a is the variable, and 2 is the coefficient.

7. Identify the coefficient and the variable in the expression you wrote in Item 6.

MATH TERMS

A **variable** is a letter or symbol used in place of unknown numbers or quantities in expressions and equations. For example, in the expression $x + 5$, x is the variable.

A **coefficient** is a number that multiplies a variable. For example, 5 is the coefficient of $5x$.

An **algebraic expression** is a mathematical phrase that uses numbers, variables, or both with operations. For example: $x + 5$ or $5x$.

MATH TIP

You cannot *solve* an expression. You *evaluate* it for a specific value by substituting that value for the variable and simplifying.

Lesson 11-2

Evaluating Algebraic Expressions

ACTIVITY 11

continued

8. Evaluate the expression you wrote in Item 6 when r is 12.

Example C

Evaluate each expression.

a. $2x + 3$ when $x = 4$

$$2(4) + 3$$

$$8 + 3 = 11$$

b. $x - 9$ when $x = 15$

$$15 - 9 = 6$$

Substitute 4 in place of x and simplify

c. $\frac{a}{12}$ when $a = 24$

$$\frac{24}{12} = 2$$

Try These C

Evaluate each expression.

a. $c + 11$ when $c = 5$

b. $15b$ when $b = 2$

c. $4a - 5$ when $a = 3$

d. $\frac{d}{6}$ when $d = 54$

9. General admission to the fair is \$8.00, and rides cost \$3.00 each.
- Write an expression to represent the cost for Zachary to attend the fair and go on r rides.
 - Evaluate the expression to find his total cost if he goes on 6 rides.
10. Ayana attends the fair, buys r ride tickets, and buys g tickets to play games. Games cost \$2.00 each.
- Reason abstractly.** Write an expression to represent the cost for Ayana to attend the fair, go on r rides, and play g games.
 - Evaluate the expression to find her total cost if she goes on 4 rides and plays 3 games.

A **term** is part of an expression containing a number, a variable, or both. Terms are separated in an expression by addition and subtraction symbols.

- In the expression $2a + 3$, there are two terms.
- The terms are $2a$ and 3.

11. Identify the terms in the expression you wrote in Item 10a.

My Notes

MATH TIP

When evaluating an expression, use the order of operations. First, do the operations inside grouping symbols. Then evaluate expressions with exponents. Next, do multiplication and division from left to right. Finally, do addition and subtraction from left to right.

My Notes

Operations in an expression are identified by certain words.

Sum means “addition.”

- The sum of 2 and 3 would be written $2 + 3$.

Product means “multiplication.”

- The product of 5 and x would be written $5 \cdot x$, or just $5x$.

Difference means “subtraction.”

- The difference of x and 3 would be written $x - 3$.

Quotient means “division.”

- The quotient of x and 4 would be written $x \div 4$ or $\frac{x}{4}$.

Check Your Understanding

12. Identify the coefficient of the variable in each expression. Then evaluate the expression for the given value of the variable.
- $7x$ when $x = 3$
 - $2b^2 + 4$ when $b = 5$
 - $80 - 5y$ when $y = 9$
13. Write expressions for the following:
- Admission to a fair of \$5.00 and going on r rides that cost 2 dollars each. Evaluate this expression when $r = 6$.
 - The sum of 12 and b . Evaluate this expression when $b = 4$.
 - The product of 9 and q . Evaluate this expression when $q = 3$.
14. Write the expression $6a$ in words.
15. Write the expression $3x + 7$ in words.

LESSON 11-2 PRACTICE

16. Evaluate each expression for the given value of the variable.
- $8a$ when $a = 2$
 - $\frac{x}{7}$ when $x = 21$
 - $c^3 - 1$ when $c = 4$
 - $2x + 9$ when $x = 5$
 - $4z - 2$ when $z = 3$
17. Write an expression representing the product of 17 and c . Evaluate this expression for $c = 4$.
18. Write the expression $12a$ in words.
19. **Reason abstractly.** Write an expression to represent the sum of $5x$ and 9, and evaluate this expression for $x = 5$.

My Notes

- d. The quotient of 12 and a number
(*Quotient* means “division,” so 12 is being divided by a number.)
 $12 \div n$ or $\frac{12}{n}$

Try These D

Tell which operation is being used, and write an algebraic expression for each verbal expression.

- a. 4 increased by a number
- b. A number divided by 3
- c. 9 more than a number squared
- d. 12 less than twice a number
-
1. The area of a rectangle is found by multiplying the base and the height or the length times the width.
- a. Write an algebraic expression for the area of a rectangle.
- b. Use your expression to find the area of a rectangle with a length of 17 inches and a width of 13 inches.
2. The perimeter of a square is determined by finding the sum of the lengths of all four sides.
- a. Write two algebraic expressions to determine the perimeter of a square.
- b. Confirm that both expressions are equivalent by using both expressions to find the perimeter of a square with side lengths of 2.6 inches.
3. Use concrete or pictorial models to determine if the expressions $3x$ and $x + x + x$ are equivalent.
4. Use algebra tiles or other concrete or pictorial models to determine if the expressions $n \cdot n$ and n^2 are equivalent.

My Notes

Check Your Understanding

9. Write an algebraic expression for each verbal expression.
 - a. Six more than a number
 - b. Three times a number
 - c. The quotient of a number and 7
 - d. Four less than five times a number
 - e. A number squared decreased by 2
 - f. Twice a number increased by 16
10. An electrician charges a \$50 house call fee and \$65 per hour for the work.
 - a. Write an algebraic expression to represent the situation.
 - b. Is it less expensive to have the electrician work for 6 hours or have him come back for two 3-hour jobs? Explain.
11. A store sells peanut butter in 28-ounce jars for \$4.29 and in 16 ounce jars for \$2.49. Find the price per ounce for each size jar and determine which jar is the better buy. Explain.

LESSON 11-3 PRACTICE

12. Write an algebraic expression for each verbal expression.
 - a. Five fewer than twice a number
 - b. A number of coins split into 4 equal groups
 - c. A number to the third power
 - d. The product of 1.5 and a number
 - e. Four times a number increased by 11
13. **Model with mathematics.** Three friends went to lunch. They all ordered the same meal. At the end of lunch, they gave the waiter a \$12 tip.
 - a. Write an algebraic expression to represent the situation.
 - b. How much total money was spent if each meal cost \$7.50?
14. Find the unit cost if a store sells a dozen eggs for \$1.99.

My Notes

MATH TIP

Multiplication can be represented using several different symbols.

For example, the expressions $3 \cdot 5$ and 3×5 both mean "3 times 5."

Likewise, "4 times a " can be written $4 \cdot a$, $4 \times a$, or just $4a$.

Usually in algebra the symbol \times is not used for multiplication, because it could be easily confused with the variable x .

7. Are subtraction and division associative? Explain your reasoning. Provide examples to support your response.
8. Describe the similarities and differences between the Commutative and Associative Properties.
9.
 - a. Apply the Commutative Property of Multiplication to create an expression equivalent to $5(6)$.
 - b. Apply the Associative Property of Addition to create an expression equivalent to $(5 + 9) + x$.
10. Identify the property illustrated in each equation.
 - a. $(2 \cdot 8) \cdot 9 = 2 \cdot (8 \cdot 9)$
 - b. $3.6 + 5.7 = 5.7 + 3.6$
 - c. $\left(\frac{1}{3} + \frac{1}{5}\right) + \left(\frac{1}{7} + 1\right) = \frac{1}{3} + \left(\frac{1}{5} + \frac{1}{7}\right) + 1$

The Additive Identity is the number that can be added to any number without changing its value.

11.
 - a. What number will make $3 + \underline{\hspace{2cm}} = 3$ a true sentence?
 - b. What number is the Additive Identity?

The Multiplicative Identity is the number that can be multiplied by any number without changing its value.

12.
 - a. What number will make $3 \cdot \underline{\hspace{2cm}} = 3$ a true sentence?
 - b. What number is the Multiplicative Identity?

13. **Make sense of problems.** Drew and Seth were in charge of collecting the money for their class field trip to the fair. They collected the estimated cost of \$36.00 from each of the 16 girls and 14 boys in their class. Then they found out that the fair would give them the special price of \$32.00 per person. To determine the total amount of money due back to the class, Drew wrote the expression $(16 + 14) \cdot 4$. Seth wrote the expression $16 \cdot 4 + 14 \cdot 4$. Explain why both boys are correct.

The expressions written by the boys illustrate the **Distributive Property**. In Seth's expression, the 4 has been distributed to both the 16 and the 14.

Distributive Property: $a(b + c) = ab + ac$ or $(b + c)a = ba + ca$

14. The Distributive Property can be used to simplify problems.
- Use the Distributive Property to rewrite the expression $19 \cdot 25 + 19 \cdot 75$ using parentheses.
 - Evaluate the expression.
 - How does rewriting the expression make it easier to evaluate?
15. Drew and Seth sold greeting cards to raise money for the school band. They sold 21 boxes for \$2.50 per box and need to find the total amount of money they collected. They don't have their calculators with them and need to find the total instead using the Distributive Property and mental math. Examine each method below.

$$\begin{aligned}\text{Seth: } 21 \cdot 2.50 &= (20 + 1) \cdot 2.50 \\ &= 20 \cdot 2.50 + 1 \cdot 2.50 \\ &= \$50.00 + \$2.50 \\ &= \$52.50\end{aligned}$$

$$\begin{aligned}\text{Drew: } 21 \cdot 2.50 &= (10 + 10 + 1) \cdot 2.50 \\ &= 10 \cdot 2.50 + 10 \cdot 2.50 + 1 \cdot 2.50 \\ &= \$25.00 + \$25.00 + \$2.50 \\ &= \$52.50\end{aligned}$$

Compare and **contrast** Seth's and Drew's methods.

16. Seth remembered that he sold several boxes of greeting cards to his math teacher, but he was not exactly sure how many boxes he sold her. Drew wrote the expression $\$2.50(x + 21)$, which he could use to determine the total amount of money they collected.
- Explain what each number and variable in Drew's expression represents.
 - Use the Distributive Property to write an equivalent expression without parentheses.

My Notes

MATH TIP

If two numbers have a common factor, you can use the Distributive Property to write their sum as a product.

$$28 + 12$$

$$4 \cdot 7 + 4 \cdot 3$$

$$4(7 + 3)$$

The product $4(7 + 3)$ consists of the two factors 4 and $(7 + 3)$.

ACADEMIC VOCABULARY

To **compare** and **contrast** means to point out similarities and differences between two or more things.

My Notes

CONNECT TO ALGEBRA

In algebra you will use the Distributive Property to simplify algebraic expressions.

For example:

$$\begin{aligned} n + n &= 1 \cdot n + 1 \cdot n \\ &= (1 + 1)n \\ &= 2n \end{aligned}$$

$$\begin{aligned} 3(n + 5) &= 3 \cdot n + 3 \cdot 5 \\ &= 3n + 15 \end{aligned}$$

Check Your Understanding

17. Use the Distributive Property to determine whether the following expressions are equivalent.
- $53 \cdot 4 + 53 \cdot 6 = 53(4 + 6)$
 - $3 \cdot (4x - 5) = 12x - 5$
 - $(y - 4) \cdot 2 = 2y - 8$
18. Use the Distributive Property to write equivalent expressions for each of the following.
- $3 \cdot (2 + x)$
 - $6 \cdot 4 + 6 \cdot 3$
 - $(4x + 3y + 2z) \cdot 7$
 - $4(4x + 3y)$
19. Identify each property.
- $2 + 0 = 2$
 - $15(1) = 15$
 - $6(3 + 2) = 6(3) + 6(2)$

LESSON 11-4 PRACTICE

20. Use the Distributive Property to determine whether the following expressions are equivalent.
- $7(3x + 2) = 21x + 2$
 - $(9y - 4) \cdot 3 = 27y - 12$
 - $8 \cdot 9 + 8 \cdot 11 = 8(9 + 11)$
21. Use the Distributive Property to write equivalent expressions for each of the following.
- $4(5 + x)$
 - $11 \cdot 3 + 11 \cdot 5$
 - $(a + 5b + 6c) \cdot 8$
 - $9(3x - 2y)$
22. **Make use of structure.** Identify each property.
- $12(10 + 2) = 12 \cdot 10 + 12 \cdot 2$
 - $20(1) = 20$
 - $0 + 20 = 20$
23. Use the Distributive Property to write each sum as a product. Identify the factors in the product.
- $26 + 10$
 - $56 + 49$
 - $42 + 18$

ACTIVITY 11 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 11-1

For Items 1–10, evaluate each expression.

- $2 \cdot 6 + 9$
- $10 - 3 \cdot 2$
- $8^2 + 19$
- $3 \cdot 4 - 5 \cdot 1$
- $36 - 20 \div 4 \cdot 3$
- $8 \cdot 6 + 15 \div 5$
- $(4 + 2)^2 \cdot 2$
- $3 \cdot 36 \div (30 \div 5 \cdot 2)$
- $8(2 + 9) \div 44$
- $54 - 6 \cdot 3^2$
- When Ayana arrived home from the fair she wanted to determine how much she and Zachary had left of the money they took to the fair.

This expression represents the total amount of money Ayana and Zachary have left of the \$60 each that they took to the fair.

$$60 \cdot 2 - [2 \cdot 8 + ((8 + 5) + 5) \cdot 3 + (1 + 2) \cdot 3 + 5]$$

Evaluate the expression to find how much money they returned home with. Show all work to justify your response.

Lesson 11-2

For Items 12–14, identify the terms in each expression. Then identify the coefficient of the variable and evaluate each expression when $x = 8$, $y = 4$, and $z = 6$.

- $2y + 3$
- $3x$
- $4z - 7$

For Items 15–18, identify each expression as a sum, difference, product, or quotient. Then evaluate each expression for $x = 8$, $y = 4$, and $z = 6$.

- $12 - x$
- $y + z$
- y^4
- $\frac{2z}{3}$
- Write an expression to represent the product of 15 and y . Evaluate your expression when $y = 3$.
- Write an expression to represent the sum of x and $2y$. Evaluate your expression for $x = 4$ and $y = 7$.
- Write expression $5a + 12$ in words.
- Write the expression $2.6b$ in words.
- Write an expression to represent admission to a zoo of \$10.00 and the cost of special exhibits s at \$4 each. Evaluate your expression when $s = 2$.
- List all the factors of 15.

Lesson 11-3

For Items 25–33, write an algebraic expression for each verbal expression.

25. Eleven more than a number
26. A number decreased by 19
27. Two more than a number squared
28. A number divided by 6
29. A number less 22
30. Eight more than twice a number
31. The product of a number and 26
32. 19 less than three times a number
33. Ten more than the product of a and b
34. A number to the third power increased by 9

A. $3n + 9$	B. $n^3 + 9$
C. $(3n)9$	D. $9n^3$
35. A repairman charges \$40 for a house call and \$50 per hour. Write an algebraic expression to represent the situation.
36. A store sells grape jelly in 15-ounce jars for \$3.59. Find the price per ounce.

Lesson 11-4

For Items 37–42, identify each property.

37. $5(a + 3b) = 5a + 15b$
38. $4x + 3 = 3 + 4x$
39. $7(8 \cdot 3) = (7 \cdot 8) \cdot 3$
40. $92 + 0 = 92$
41. $7(9) = 9(7)$
42. $66(1) = 66$

For Items 43–44, use the Distributive Property to determine whether the following expressions are equivalent.

43. $26(x + 3) = 26x + 3$
44. $a + a + a = 3a$

For Items 45–51, use the indicated property to write an expression equivalent to the given expression.

45. $6 + (9 + 7)$; Commutative Property of Addition
46. $6 + (9 + 7)$; Associative Property of Addition
47. $8(1)$; Multiplicative Identity Property
48. $12 \cdot 3 + 12 \cdot 7$; Distributive Property
49. $4(5)$; Commutative Property of Multiplication
50. $2(5 \cdot 10)$; Associative Property of Multiplication
51. $16 + 0$; Additive Identity Property
52. Write each sum as a product. Identify the factors in the product.

a. $18 + 63$	b. $84 + 35$
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MATHEMATICAL PRACTICES**Look For and Express Regularity in Reasoning**

53. The length of a rectangle is three times the width. Which algebraic expression represents the perimeter of the rectangle?

A. $3x + x$	B. $4x$
C. $x + x + x + 3x$	D. $x + 3x + x + 3x$

Learning Targets:

- Write one-variable, one-step equations to represent situations.
- Distinguish between expressions and equations.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Shared Reading, Create Representations, Discussion Groups, Sharing and Responding

Brynn has been volunteering at the animal shelter and has decided that she would like to adopt one of the puppies. Her parents have said that before she can have a puppy, they must fence in a portion of the backyard.

At the local home improvement store, her parents have determined that they can afford 360 feet of fencing materials. Brynn's parents have agreed to let her choose how she wants to build the fence as long as she takes into consideration the trees, storage building, and deck already in the back yard.

Brynn decides it will be easiest to fence in a rectangular area and remembers that the formula for finding the perimeter of a rectangle is $P = 2l + 2w$, where P represents the perimeter, l represents the length, and w represents the width. The formula $P = 2l + 2w$ is an example of an **equation**. In algebra, we use equations to determine solutions to problems. The value or values of a variable that make an equation true are the **solutions** to the equation.

1. Use substitution to rewrite the formula $P = 2l + 2w$ to represent the amount of fencing materials that Brynn's family can afford.
2. Brynn decides she can create an entrance from the existing deck to the fenced portion of the yard if she makes the enclosure 30 feet wide. Use substitution to rewrite the formula $P = 2l + 2w$ to represent this new information.

It is important to represent real-world situations with algebraic expressions and equations. Solving the resulting equations helps determine answers to real-life problems.

My Notes

MATH TERMS

An **equation** is a statement showing that two numbers or expressions are equal, such as $4 + 3 = 7$. An equation has an equal sign while an expression does not.

The **solution** of an equation is the numeric value of a variable that makes the equation a true statement.

My Notes

MATH TIP

To define a variable, choose a variable to represent an unknown number or quantity and tell what that variable represents.

CONNECT TO AP

Throughout your study of mathematics you will model written descriptions of physical situations with algebraic expressions and equations.

Example A

Write an equation to represent this situation.

What number do you add to 15 to get 23?

Step 1: Write a verbal model.

$$\text{A number} + 15 = 23$$

Step 2: Define variables for unknown quantities.

$$\text{Let } n = \text{the number}$$

Step 3: Write an equation using a variable for any unknown quantity.

$$n + 15 = 23$$

Try These A

Write an equation to represent each situation.

a. What number do you add to 86 to get 95?

b. What number do you multiply 7 by to get 56?

3. Reason abstractly. Madison and Tanisha went out to lunch. The bill for their lunches came to \$18.94. Madison knows that her lunch cost \$9.72. How much did Tanisha's lunch cost?

a. Write a verbal model for the situation.

b. Define a variable for the unknown quantity.

c. Write an equation using a variable for any unknown quantity to represent the situation.

Expressions consist of variables, numbers, and operation symbols, while **equations** also contain an equal sign.

4. Determine below whether each is an equation or an expression.

a. $5x - 9$

b. $2x + 6 = 50$

c. $8x^2 - 64 = 0$

d. $90(3a + 2b - c)$

My Notes

MATH TIP

You cannot solve an expression. You evaluate an expression for a specific value by substituting that value into the variable and simplifying.

Learning Targets:

- Understand what it means to solve an equation.
- Use substitution to determine which values from a specified set make an equation true.

SUGGESTED LEARNING STRATEGIES: Paraphrasing, Think-Pair-Share, Guess and Check, Simplify the Problem, Share and Respond

Equations can be solved using many methods. These include using mental math, using guess and check, solving algebraically, or substituting in values for the variable to see which value makes the equation true.

Brynn wrote the equation $2l + 2(30) = 360$ to represent the length of the enclosure she could build for her new puppy with 360 feet of fencing materials, if the width of the enclosure was 30 feet.

- 1. Attend to precision.** Brynn decided to find the length of the fence by substituting to test possible lengths. Substitute the following values into the equation for l and see if they result in a true statement. Show your work.
 - a. 100 feet
 - b. 200 feet
 - c. 150 feet
 - d. What length should Brynn use for her fenced area? Justify your answer.
- 2. Mental math can also be used to solve some equations.**
 - a. Explain how you could use mental math to solve $2l + 60 = 360$.
 - b. Use mental math to determine a value for l that makes the equation a true statement. This is the solution of the equation.

My Notes

Example B

Use this set of possible solutions to determine the solution to each equation.

$$\{0, 6, 15, 18\}$$

a. $2x - 4 = 32$

Step 1: Substitute in for x each value from the set of possible solutions.

Step 2: Determine which value produces a true equation.

$$2(0) - 4 = 0 - 4 = -4 \neq 32$$

$$2(6) - 4 = 12 - 4 = 8 \neq 32$$

$$2(15) - 4 = 30 - 4 = 26 \neq 32$$

$$2(18) - 4 = 36 - 4 = 32$$

Solution: 18 is the solution of the equation $2x - 4 = 32$

b. $\frac{y}{3} + 2 = 7$

$$\frac{0}{3} + 2 = 0 + 2 = 2 \neq 7$$

$$\frac{6}{3} + 2 = 2 + 2 = 4 \neq 7$$

$$\frac{15}{3} + 2 = 5 + 2 = 7$$

$$\frac{18}{3} + 2 \neq 7$$

15 is the solution of the equation $\frac{y}{3} + 2 = 7$

Try These B

Use this set of possible solutions to determine the solution to each equation.

$$\{0, 11, 13, 15\}$$

a. $61 - m = 50$

b. $3k = 39$

My Notes

Check Your Understanding

Use this set of possible solutions to determine a solution to each equation using substitution.

$$\{3, 8, 10, 14, 15, 16\}$$

3. $4 = 0.5z - 4$

4. $11 = x - 3$

5. $\frac{63}{x} = 21$

6. $21 + a = 29$

7. $\frac{y}{3} + 1 = 6$

8. $8x - 2 = 78$

LESSON 12-2 PRACTICE

Use this set of possible solutions to determine a solution to each equation using substitution.

$$\{1, 2, 5, 6, 12, 20\}$$

9. $13x = 26$

10. $6x + 9 = 45$

11. $3a - 12 = 48$

12. $\frac{y}{4} - 2 = 1$

13. $4 = z - 1$

14. $x^2 + 3 = 4$

15. **Attend to precision.** In Lesson 12-1, Item 8, you wrote an equation for this situation: Brynn needs to save \$125 to build a doghouse for her new puppy. She has saved \$68. How much more does she need to save? Now use this set of possible solutions and substitution to solve the equation.

$$\{\$193, \$67, \$57, \$125\}$$

ACTIVITY 12 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 12-1

For Items 1–8, write an equation to represent each situation. Include a verbal model and define any variables you use.

1. What number do you add to 17 to get 51?
2. What number do you multiply 6 by to get 66?
3. Morris and Luther went shopping. Their total bill was \$52.96. Morris knew he spent \$31.24. How much did Luther spend?
4. Gigi has 64 baseball cards. How many should she give to Carla so she will have only 49 left?
5. What number do you divide by 9 to get 8?
6. The local hockey team needs to win 11 games to reach the playoffs. They have already won 8 games. How many more games must they win to reach the playoffs?
7. Sam is making 100 cookies. He can fit 20 cookies on each cookie sheet. How many cookie sheets will be need?
8. The perimeter of a rectangle is 400 feet and the width is 80 feet. What is the length of the rectangle?

For Items 9–12, tell if each is an equation or an expression.

9. $6x - 17$
10. $3x + 9 = 12$
11. $58 - a = 17$
12. $x^2 + 4$

13. Sigfried is calculating the percent of tax he paid if the tax on his \$10.00 dinner was \$0.60. Which equation could he use to represent this situation, where x represents the percent of tax?
 - a. $10 + x = 60$
 - b. $10.00x = 0.60$
 - c. $0.60 + x = 10$
 - d. $0.60x = 10.00$
14. Eight less than a number is 15. Which equation represents this situation, where n represents the number?
 - a. $n - 8 = 15$
 - b. $8 - n = 15$
 - c. $8 < n = 15$
 - d. $\frac{8}{n} = 15$
15. Explain the difference between an equation and an expression.
16. Ross and Kristen disagreed about how to write an equation to represent the following situation.

Ross had twice as many DVDs as Kristen. Together they had 93 DVDs. How many did Kristen have?

They agreed that x should represent the number of DVDs that Kristen had. Ross wrote the equation $x + 2x = 93$, while Kristen wrote the equation $2x = 93$. Whose equation is correct? Explain your choice.

Lesson 12-2

17. Explain what it means to solve an equation.

For Items 18–27, use substitution to determine which number in the given set is a solution of each equation.

{0, 1, 3, 4, 5, 6, 7, 8, 9, 12, 20}

18. $4x - 10 = 14$

19. $\frac{y}{2} + 3 = 5$

20. $19 - a = 14$

21. $6b = 48$

22. $3y + 11 = 47$

23. $9 - x^2 = 9$

24. $c + 15 = 27$

25. $\frac{7}{r} + 1 = 2$

26. $p - 14 = 6$

27. $42 - 5y = 7$

For Items 28–32, use substitution to determine which number in the given set is a solution of the each equation.

{1, 2, 3, 4, 5, 6, 10}

28. $2x + 3 = x + 7$

29. $x^2 - 8x + 15 = 0$

30. $a^2 = 25$

31. $3y + 6 = 7y - 34$

32. $x^2 - 2x = 8$

For Items 33–35, Harley is fencing off a rectangular garden in his backyard. To have room for all the vegetables that he wants to grow, the area of the garden must be 100 square feet. He remembers that the formula $A = lw$ gives the area of a rectangle, where A represents the area, l represents the length, and w represents the width.

33. Write an equation to represent this situation.

34. Harley finds that the area in his backyard that gets enough sun for the garden is only 5 feet wide. Substitute this value to write a new equation representing this situation.

35. Use mental math to find the length of Harley's garden. Explain your reasoning.

MATHEMATICAL PRACTICES**Model with Mathematics**

36. Lupe is measuring the dimensions of her triangular lot. She knows that the lot is in the shape of an isosceles triangle (two sides are equal), and the perimeter can be calculated by adding the lengths of the three sides. She knows that the perimeter is 450 feet and that the side that is different in length from the other two is 100 feet. She wants to calculate the length of the other two sides.

a. Draw a diagram of this situation.

b. Write an equation that Lupe can use to find the length of the other two sides of her lot, where x represents the length of one side.

c. Lupe is not sure if the length of the other two sides is 100 feet, 125 feet, 150 feet, 175 feet, or 200 feet. Use substitution to find the solution to your equation and the length of the sides of Lupe's lot.

THE COST OF AFTER-SCHOOL ACTIVITIES

Write your answers on notebook paper. Show your work.

- Twins Terry and Tony attend Riverdale Middle School. They are on a student council committee that is deciding what the ticket price will be for a school dance. Terry and Tony know the following:
 - The decorations for the gym will cost \$189.26.
 - The cost of a security guard is \$52.50 per hour
 - The dance will be 3 hours long.
 - The deejay charges \$50.98 plus \$23.50 per hour.
 - The refreshments will cost \$2.25 per student.
 - The student council has raised \$349.49 to help pay for the dance.
 - Ninety-five students will attend and share the cost of the dance.
 - Use the Distributive Property to write an expression to find the cost of the security guard and the deejay.
 - Simplify the expression you wrote for part a to find the cost of the security guard and the deejay.
 - How much will the refreshments cost?
 - Write an expression for the total cost of the dance.
 - How much money will the student council need to collect from ticket sales to break even?
- Write a letter to the twins telling them what you think the ticket price for the dance should be. Use the information provided and your work in Item 1 to justify your recommendation.
- Terry and Tony had 50 balloons to use for decorations. They put 30 on the tables and 4 at each end of the refreshment table. Their advisor thought they needed balloons at the entrance and brought 10 more. The expression $50 - 30 - 4 \times 2 + 10$ shows the number of balloons they have for the entrance. Tony thinks they will have 42 balloons for the entrance but Terry says there will only be 22. Who is correct? Explain your reasoning.
- To help pay for the next school event, Terry and Tony want to make a profit from ticket sales for this dance. They determine that they would like to bring in a total of \$475 from ticket sales.
 - If x represents the price per ticket, write an equation to represent this situation.
 - Some student council dance committee members believe they should charge \$4.00 per ticket, while others think they should charge \$4.50 per ticket. Terry and Tony believe the price per ticket should be \$5.00. Use substitution to find the solution to your equation from part a and to determine how much each ticket should cost.

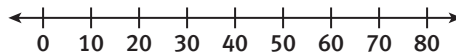
Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
Mathematics Knowledge and Thinking (Items 1a-e, 2, 3, 4a-b)	The solution demonstrates these characteristics:			
	<ul style="list-style-type: none"> A clear understanding of the order of operations and the Distributive Property. Effective understanding of and accuracy in writing, evaluating, and solving expressions and equations. 	<ul style="list-style-type: none"> A functional understanding of the order of operations and the Distributive Property. Writing, evaluating, and solving expressions and equations that usually result in correct answers. 	<ul style="list-style-type: none"> Partial understanding of the order of operations and the Distributive Property. Difficulty with writing, evaluating, and solving expressions and equations. 	<ul style="list-style-type: none"> Little or no understanding of the order of operations and the Distributive Property. Little or no understanding of writing, evaluating, and solving expressions and equations.
Problem Solving (Items 1e, 2)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer. 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but results in a correct answer. 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers. 	<ul style="list-style-type: none"> No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a, 1d, 4a)	<ul style="list-style-type: none"> Clear and accurate representation of problems as expressions and equations. 	<ul style="list-style-type: none"> Some difficulty in representing problems as expressions and equations. 	<ul style="list-style-type: none"> Difficulty in writing expressions and equations leading to errors. 	<ul style="list-style-type: none"> No understanding of representing problems as expressions and equations.
Reasoning and Communication (Items 2, 3)	<ul style="list-style-type: none"> Precise use of appropriate math terms and language to explain solutions to expressions and equations and the role of the order of operations. 	<ul style="list-style-type: none"> An adequate explanation of solutions to expressions and equations and the role of the order of operations. 	<ul style="list-style-type: none"> A misleading or confusing explanation of solutions to expressions and equations and the role of the order of operations. 	<ul style="list-style-type: none"> An incomplete or inaccurate explanation of solutions to expressions and equations and the role of the order of operations.

My Notes

The equations you wrote in the previous problems are addition equations. An addition equation can be solved using several methods that include mental math, guess and check, a balance scale, or an algebraic method. The solution can be graphed on a number line.

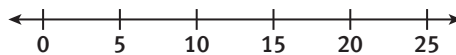
1. Make sense of problems. Samantha has 29 songs downloaded to her new tablet. She wants to have a total of 100 songs on the tablet. How many more songs does she need to download?

- a. Define a variable and write a word phrase to model the situation.
- b. Write an equation and use mental math to solve the equation.
- c. Check the solution using substitution.
- d. Graph the solution on a number line.



2. Quin has 28 state quarters collected. He wants to have a complete set of the 50 state quarters. How many more quarters does he need to collect?

- a. Define a variable and write a verbal model.
- b. Write an equation and use mental math to solve the equation.
- c. Check the solution using substitution.
- d. Graph the solution on a number line.



My Notes

Learning Targets:

- Write addition equations to represent situations.
- Solve one-step addition equations of the form $x + a = b$, where a , b , and x are all nonnegative rational numbers.
- Given an equation of the form $x + a = b$, where a , b , and x are all nonnegative rational numbers, write a corresponding real-world problem.

SUGGESTED LEARNING STRATEGIES: Sharing and Responding, Create a Plan, Create Representations

Example B

Mario wants 5 books to take on vacation. He has 2 books that he has not read yet. How many more books does he need to buy?

Step 1: Define a variable and write a verbal model.

Let x represent the number of books he needs to buy.

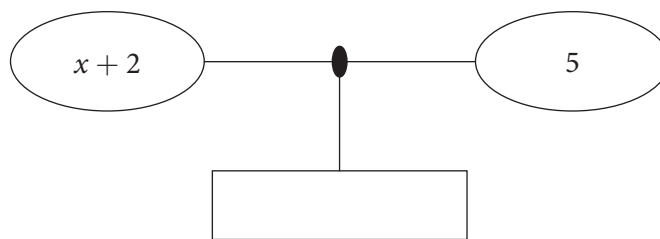
$$2 + \text{number of books he needs} = 5$$

Step 2: Write an equation.

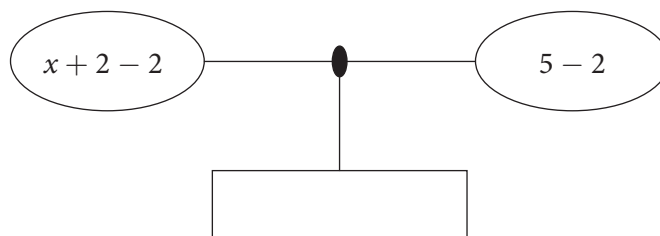
$$x + 2 = 5$$

Step 3: Solve the equation.

In addition to mental math, another method to solve an equation is using a balance scale. When using a balance scale, the goal is to get the quantity being determined, or x , on one side of the scale alone. This is called “isolating the variable.”



Subtract 2 from the left side to isolate the x , because $+2$ and -2 are a zero pair. To keep the scale balanced, you must also subtract 2 from the right side.



READING MATH

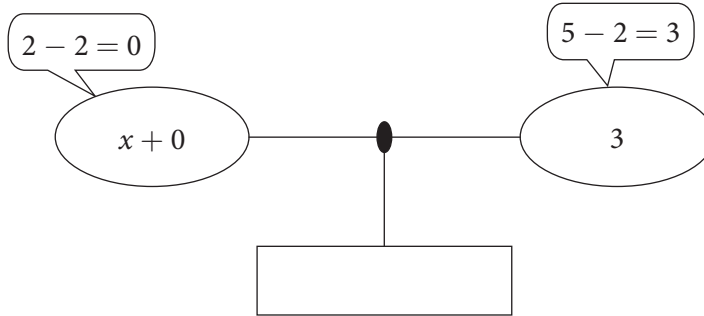
The phrase *isolate the variable* means “to get the variable alone on one side of the equation by using operations such as addition, subtraction, multiplication, and division.”

Lesson 13-2

Solving Addition Equations

ACTIVITY 13

continued



Use the Additive Identity Property to simplify $x + 0$.
The solution is $x = 3$.

Check by substitution.

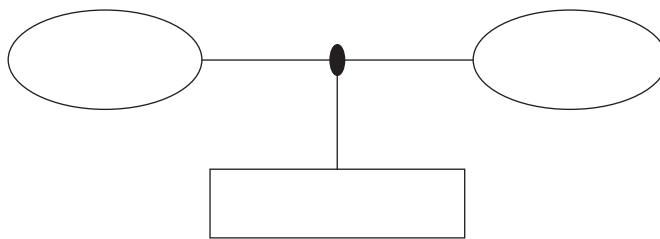
$$x + 2 = 5$$

$$3 + 2 = 5$$

$$5 = 5$$

Try These B

Solve $x + 4 = 11$ using a balance scale. Check your solution.



Addition equations can also be solved algebraically.

Example C

Solve the equation $x + 15 = 25$.

Step 1: Since this is an addition equation, use the inverse operation of subtraction. Subtract 15 from both sides.

$$x + 15 - 15 = 25 - 15$$

Step 2: Simplify both sides of the equation.

$$x + 0 = 10$$

Step 3: Use the Additive Identity Property to isolate the variable.

$$x = 10$$

Step 4: Check the solution by substitution.

$$10 + 15 = 25$$

$$25 = 25$$

Solution: $x = 10$

Try These C

Solve the equation $x + 9 = 34$ algebraically.

My Notes

MATH TERMS

Inverse operations are operations that “undo” each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

CONNECT TO AP

In AP mathematics, understanding inverse operations is important in understanding the relationship between the processes called differentiation and integration.

My Notes

An equation can also be solved by writing the steps vertically.

Example D

Solve $x + 22 = 36$

$$x + 22 = 36$$

$$\begin{array}{r} x + 22 = 36 \\ -22 \quad -22 \\ \hline x + 0 = 14 \end{array}$$
 Subtract 22 from both sides of the equation to

isolate x .

$$x = 14$$

Try These D

Solve $x + 34 = 52$.

- Samantha has scored 2,160 points in a video game on her tablet. To win the current level she needs to have 8,500 points. How many more points must she score to win the level?
 - Define a variable and write an equation.
 - Solve the equation algebraically.
 - Check the solution.
- Make sense of problems.** Alfred has made 8 cookies to give to his friends. Since he has 15 friends that he wants to give cookies to, how many more cookies does he need to make? Define a variable, write an equation, and solve your equation algebraically. Check your solution.
- Olivia has 6 of the CDs by her favorite group. The group has recorded 22 CDs. How many more of this group's CDs must she buy to have the complete collection? Define a variable, write an equation, and solve your equation algebraically. Check your solution.
- Write a real-world problem that could be represented by the equation $y + 11 = 19$.
- Write a real-world problem that could be represented by the equation $a + 68 = 79$.

My Notes

Learning Targets:

- Write a subtraction equation to represent a situation.
- Solve a subtraction equation of the form $x - a = b$, where a , b , and x are all nonnegative rational numbers.

SUGGESTED LEARNING STRATEGIES: Summarizing, Think-Pair-Share, RAFT, Self Revision/Peer Revision, Work Backward

Many real-world problems can be represented and solved using a subtraction equation very much like those represented and solved using an addition equation.

Example E

Samantha has apps on her tablet. If she deletes 7 of them she will have 17 left. How many apps did she have to begin with?

- Step 1:** Define a variable.
Let x represent the number of applications she had to begin with.
- Step 2:** Write a verbal model.
number of apps she had to begin with $- 7 = 17$
- Step 3:** Write an equation.
 $x - 7 = 17$
- Step 4:** Use mental math to solve the equation.
From what number can you subtract 7 and get 17?
 $x = 24$
- Step 5:** Use substitution to check the solution.
 $24 - 7 = 17$
 $17 = 17$

Solution: Samantha had 24 apps to begin with.

Try These E

Shiro is driving from Houston, Texas, to San Antonio, Texas. After driving 50 miles, he still has 150 miles to go. How many miles is the trip from Houston to San Antonio?

- Define a variable, write a verbal model and an equation.
- Solve the equation and check using substitution.

My Notes

Check Your Understanding

- Enzo still owes \$750 on his new furniture. He has already paid \$600 on it. What was the original cost of the furniture? Define a variable and write a verbal model and equation for this situation. Then solve the equation and check the solution.
- Yusra's dog is overweight. The vet says the dog needs to lose 7 pounds to get down to the desirable weight of 45 pounds. How much does Yusra's dog weigh? Define a variable and write a verbal model and equation for this situation. Then solve the equation and check the solution.

LESSON 13-3 PRACTICE

- You see a coat on sale for \$78. The sign says that the price is \$15 off the original price. What was the original price? Define a variable and write a verbal model and equation for this situation. Then solve the equation and check the solution.
- Model with mathematics.** Payat put \$420 from his savings account into a certificate of deposit to earn more interest. He then had \$80 left in his savings account. How much did Payat have in his savings account before he opened the certificate? Define a variable and write a verbal model and equation for this situation. Then solve the equation and check the solution.
- Critique the reasoning of others.** Marsha said she could solve $x - 3 = 7$ using the balance scale method by adding 3 to each side of the scale. Do you think her reasoning is correct? Explain.

Learning Targets:

- Write subtraction equations to represent situations.
- Solve subtraction equations by adding the same number to both sides of the equation.
- Given an equation of the form $x - a = b$, where a , b , and x are all nonnegative rational numbers, write a corresponding real-world problem.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think Aloud, Sharing and Responding, Create a Plan, Create Representations, Discussion Groups

A subtraction equation can be solved algebraically much like an addition problem.

Example F

Solve $x - 3 = 17$.

Step 1: Add 3 to each side of the equation.

$$x - 3 + 3 = 17 + 3$$

Step 2: Simplify both sides.

$$x + 0 = 20$$

Step 3: Use Additive Identity Property to isolate the variable.

$$x = 20$$

Step 4: Substitute to check the solution.

$$x - 3 = 17, x = 20$$

$$20 - 3 = 17$$

$$17 = 17$$

Solution: $x = 20$

Try These F

Solve $x - 11 = 5$ algebraically.

1. Wayne is playing a card game. He has dealt 17 cards out and has 35 left to deal. How many cards did he have to start?
 - a. Define a variable and write a subtraction equation.
 - b. Solve the equation algebraically.
 - c. Use substitution to check the solution.

My Notes

MATH TIP

The inverse operation of subtraction is addition.

My Notes

2. The Tigers lost 7 yards on their first play. After that play they were on their own 33 yard line. What yard line were they on before that play?
Define a variable and write an equation. Solve the equation algebraically and check the solution.

3. Write a real-world problem that could be represented by the equation $x - 9 = 40$.

4. Write a real-world problem that could be represented by the equation $x - 2 = 15$.

5. **Construct viable arguments.** Would $13 - x = 9$ be solved the same way as $x - 13 = 9$? Explain.

6. Glenn is writing a term paper. He has written 2,000 words and still has 1,700 words left to write. How long will the term paper be?
Define a variable and write an equation. Solve the equation algebraically and check the solution.

My Notes

Check Your Understanding

10. Solve each equation algebraically.
- $a - 7.29 = 55.64$
 - $w - \frac{2}{3} = \frac{1}{2}$
 - $a - 9 = 11$
 - $b - 28 = 2$
 - $c - 49 = 96$
11. Manny is putting his DVDs away. He put 11 of them away and still has 43 left to put away. How many DVDs does Manny have? Define a variable and write an equation. Solve the equation algebraically and check the solution.
12. Write a real-world problem that could be represented by the equation $c - 4 = 26$.

LESSON 13-4 PRACTICE

13. Solve each equation algebraically.
- $a - 1.23 = 4.72$
 - $x - 4 = 7$
 - $z - 17 = 28$
 - $c - 71 = 59$
 - $w - \frac{3}{4} = \frac{7}{12}$
14. Samantha was scanning photos. She has scanned 123 but still has 296 left to scan. How many pictures did she start with? Define a variable and write an equation. Solve the equation algebraically and check the solution.
15. Write a real-world problem that could be represented by the equation $a - 50 = 9$.
16. **Construct viable arguments.** Explain why $x + 8 = 12$ and $12 - x = 8$ have the same solution.

ACTIVITY 13 PRACTICE

Lesson 13-1

For Items 1–4, define a variable and write a verbal model and an equation. Then solve the equation and check the solution using substitution.

1. Lee is 32 years younger than his mother and his mother is 67 years old. How old is Lee?
2. Sumatra bought a loaf of bread 24 inches long. She cut it into two pieces. If one piece was 9 inches long, how long was the other piece?
3. Mt. McKinley in Alaska is 20,320 feet high. It is 9,081 feet higher than Mt. Hood in Oregon. How tall is Mt. Hood?
4. Emma has 11 pairs of shoes. She has three more pairs of shoes than Louisa has. How many pairs of shoes does Louisa have?

For Items 5–9, solve each equation using mental math or guess and check. Then check the solution.

5. $x + 11 = 26$
6. $12 + a = 19$
7. $51 = 47 + b$
8. $y + 81 = 152$
9. $890 + x = 1,359$

Lesson 13-2

For Items 10–12, define a variable and write a verbal model and an equation. Then solve the equation and check the solution using substitution.

10. Joella makes \$12.75 per hour. This is \$2.50 more than her younger sister makes per hour. How much does Joella's sister make per hour?
11. The enrollment at the University of Texas, Austin, in 2011 was 51,112. This was 11,245 more than the enrollment at Texas A&M University. What was the enrollment at Texas A&M University?
12. During the first hour on opening day 2,120 people entered the amusement park. By the end of that day 8,596 people had entered the park. How many people entered the park after the first hour?

For Items 13–14, write a real-world problem that could be represented by the equation.

13. $z + 2 = 9$
14. $15 + x = 40$

For Items 15–17, solve each equation, and then check your solution.

15. $a + 97 = 125$
16. $12.95 = y + 3.19$
17. $\frac{2}{3} + x = \frac{3}{4}$

Lesson 13-3

For Items 18–21, define a variable, write a verbal model and an equation. Then solve the equation and check the solution using substitution.

18. Devery is going on a rafting trip. She has rafted 12 miles and has 16 miles left to go. How many miles long is the trip?
19. Terry is hiking on the trail to Mt. LeConte in the Great Smoky Mountains. He has hiked 2.1 miles and has 4.5 left before he reaches the summit. How many miles long is the hike?
20. McKenzie is putting away holiday decorations. She has put 36 decorations away and still has 97 left to put away. How many holiday decorations did she have out?
21. There are seven tuba players in the high school band and 51 band members who play other instruments. How many band members are there?

For Items 22–25, use mental math or guess and check to solve each equation, then check your solution.

22. $x - 4 = 10$
23. $a - 15 = 3$
24. $b - 17 = 18$
25. $4 = c - 26$

Lesson 13-4

For Items 26–28, define a variable, write a verbal model and an equation. Then solve the equation and check the solution using substitution.

26. Grandma is knitting a scarf. She has knit 10 inches of the scarf and still has 38 inches left to finish. How long will the scarf be when it is completed?
27. Elroy is taking a train to downtown. The train has made 11 stops already and will make another 23 stops before it reaches its destination. How many total stops are there on the train route?
28. The football team is losing the game in the fourth quarter. There are still 6,700 fans in attendance but 2,500 fans have already left. What was the total attendance at the game before the fans began to leave?

For Items 29–30, write a real-world problem that could be represented by the equation.

29. $x - 12 = 20$
30. $a - 15 = 3$

For Items 31–32, choose the correct solution of the equation.

31. $x - 57.6 = 24.3$
 a. 81.9 b. 57.6 c. 33.3 d. 24.3
32. $x - \frac{1}{5} = \frac{1}{2}$
 a. $\frac{1}{5}$ b. $\frac{3}{10}$ c. $\frac{1}{2}$ d. $\frac{7}{10}$

MATHEMATICAL PRACTICES**Reason Abstractly and Quantitatively**

33. Describe the strategies for solving equations. Explain when and why you would use each strategy.

My Notes

7. The amount of trash the average American sends to the landfill each month is the difference between the total amount of trash they collect and the amount of trash they recycle.
 - a. Write an algebraic expression to represent the amount of trash the average American sends to the landfill in d days.
 - b. If an average American sends 93 pounds of trash to the landfill, write an algebraic equation that could be used to find the number of days, d .
 - c. Use guess and check or mental math to solve the equation you wrote in part b .
 - d. Check your solution from part c , using substitution.

The equations you wrote and solved above are multiplication equations. A multiplication equation is one in the form $ax = b$, where a and b are numbers. Multiplication equations can be solved by guess and check, by mental math, or by using algebra.

Example A

Gabrielle has 8 apples that weigh a total of 56 ounces. How much does each apple weigh?

- Step 1:** Define a variable.
Let x = weight of one apple.
- Step 2:** Write an algebraic equation to represent this situation.
 $8x = 56$
- Step 3:** Use mental math to solve the equation.
What number times 8 gives 56? $x = 7$
- Step 4:** Use substitution to check your solution.
 $8(7) = 56$; $56 = 56$

Solution: Each apple weight 7 ounces.

Try These A

A box of 150 paper clips costs \$3.00. How much does one paper clip cost?

- a. Define a variable and write an equation for this situation.
- b. Use mental math to solve the equation. Then check the solution.

My Notes

MATH TERMS

A **coefficient** is a number that multiplies a variable.

Learning Targets:

- Write multiplication equations to represent situations.
- Solve multiplication equations of the form $ax = b$, where a , b , and x are all positive rational numbers.
- Given an equation of the form $ax = b$, where a , b , and x are all positive rational numbers, write a corresponding real-world problem.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create a Plan, Note Taking, Think-Pair-Share, Discussion Groups, Activating Prior Knowledge

In the previous activities, you learned about isolating the variable. To solve a multiplication equation, you must also isolate the variable.

The inverse operation of multiplication is division, so to isolate the variable in a multiplication equation, you divide both sides of the equation by the **coefficient** of x .

Example B

Solve $5x = 30$

Step 1: $\frac{5x}{5} = \frac{30}{5}$ Divide both sides by 5 (coefficient of x)

Step 2: $1x = 6$ Simplify both sides

Step 3: $x = 6$ Apply the Multiplicative Identity Property:

Step 4: Check your solution: $5(6) = 30$
 $30 = 30$

Solution: $x = 6$

Try These B

Solve $9x = 108$. Show work. Check your solution.

- Pencils cost \$0.88 each. How many pencils can you buy for \$13.20? Define a variable and write an equation. Solve the equation and check your solution.
- Reason abstractly.** Barkley averaged 75 yards per game. How many games will it take him to run 600 yards? Define a variable and write an equation. Solve the equation and check your solution.

Lesson 14-2

Solving Multiplication Equations

ACTIVITY 14

continued

- Write a real-world problem that could be represented by the equation $40x = 200$.
 - Solve the equation in part *a*.
- Myrna is buying a car that costs \$19,500. If she wants to pay for it in 48 months, how much will each payment be? Define a variable and write an equation. Solve the equation and check your solution.
- If an average American composted 1.5 pounds of trash every day, how many people would it take to compost 114 pounds of trash in a day? Define a variable and write an equation. Solve the equation and check your solution.
- Solve each equation algebraically.
 - $5b = 160$
 - $2.6z = 10.4$

My Notes

MATH TIP

Instead of dividing both sides of an equation, you could also multiply each side by the reciprocal of the coefficient of the variable. For example, to solve $3x = 6$ you could multiply each side by $\frac{1}{3}$ instead of dividing by 3.

Check Your Understanding

- The sixth grade class, which consists of 130 students, is going on a field trip by bus. Each bus can hold 26 students. How many busses will they need? Define a variable and write an equation. Solve the equation and check your solution.
- One-third of the girls in the sixth-grade class have long hair. Eleven girls have long hair. How many girls are in the sixth-grade class? Define a variable and write an equation. Solve the equation and check your solution.

My Notes

A multiplication equation that has a fraction as a coefficient can still be solved by using the inverse operation of division.

Example C

Solve $\frac{2}{3}x = 6$.

Method 1

Step 1: Divide each side by $\frac{2}{3}$:

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{6}{\frac{2}{3}}$$

Step 2: Simplify each side:

$$x = 6 \cdot \frac{3}{2}$$

Solution:

$$x = 9$$

Method 2

Step 1: Multiply each side by 3:

$$3 \cdot \frac{2}{3}x = 3 \cdot 6$$

Step 2: Simplify each side:

$$2x = 18$$

Step 3: Divide each side by 2:

$$\frac{2x}{2} = \frac{18}{2}$$

Solution:

$$x = 9$$

Try These C

Solve each equation.

a. $\frac{3}{4}x = 9$

b. $\frac{1}{2}a = 12$

7. Solve each equation and check your solutions.

a. $3x = 210$

b. $11a = 22$

c. $\frac{4}{7}k = 20$

8. Write a real-world problem that can be represented by the equation $8x = 48$.

My Notes

MATH TIP

Note that the equation $\frac{1}{2}a = 12$ is the same as $\frac{a}{2} = 12$.

Learning Targets:

- Write a division equation to represent a situation.
- Solve a division equation by multiplying both sides of the equation by the same number.

A division equation is an equation of the form $\frac{x}{a} = b$, where a and b are numbers and $a \neq 0$. To solve a division equation you multiply both sides of the equation by the denominator in order to isolate the variable.

Example D

Solve $\frac{x}{4} = 9$

Step 1: Multiply both sides by 4: $4 \cdot \frac{x}{4} = 4 \cdot 9$

Notice that $4 \cdot \frac{1}{4} = 1$ and $1 \cdot x = x$.

Step 2: Simplify both sides: $1x = 36$

Solution: $x = 36$

Try These D

Solve $\frac{a}{6} = 8$.

1. Emil was dividing his candy among 15 friends and himself. If each person received 4 pieces, how many pieces of candy did Emil have to start with? Define a variable and write an equation. Solve the equation and check your solution.
2. A candy bar costs \$0.89. How much would it cost to buy 24 candy bars? Define a variable and write an equation. Solve the equation and check your solution.

My Notes

Check Your Understanding

7. Write a division equation for each situation.
- Twenty-two books fit on a shelf in the library. How many books can be displayed on 6 shelves?
 - Marguerite can save 130 pictures on a DVD. How many pictures can be stored on 7 DVDs?
 - A pet store has 19 goldfish tanks. The store can place 12 fish in each tank. How many goldfish can it keep?
8. Solve each equation.
- $\frac{x}{12} = 5$
 - $16 = \frac{a}{0.3}$

LESSON 14-3 PRACTICE

9. Write a division equation for each situation.
- The choir is lining up on stage in three rows. There are 24 choir members in each row. How many choir members are there?
 - Little league baseball has 12 teams with 15 children on each team. How many children play little league baseball?
10. Solve each equation.
- $\frac{x}{3} = 26$
 - $14 = \frac{a}{9.6}$
11. **Make sense of problems.** Write a real-world problem that can be represented by the equation $\frac{x}{6} = 16$.

ACTIVITY 14 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 14-1

For Items 1–7,

- Define a variable.
 - Write an equation.
 - Solve the equation.
 - Check the solution.
- The Houston Rockets scored 66 points in the second half. There are 24 minutes in a half. What was their average number of points per minute?
 - The temperature rose 20°F from 10 A.M. to 3 P.M. On average, how many degrees did the temperature rise per hour?
 - Sandy was playing a word game. Her total score for the last 7 turns was 154 points. What was her average score per turn?
 - A 1,000-pound horse eats about 25 pounds of hay a day. How many days would it take the horse to eat 400 pounds of hay?
 - Gold costs \$1,654 per ounce. How many ounces are needed to be worth \$16,540?
 - Three-fourths of the students in honors algebra are eighth graders. There are 21 eighth graders in honors algebra. How many total students are in honors algebra?
 - It takes two-thirds of a yard of fabric to make a pillowcase. How many pillowcases can be made from 16 yards of fabric?

For Items 8–11, use guess and check or mental math to solve the equation.

- $7x = 42$
- $12 = 3a$
- $200p = 1,400$
- $25z = 225$

Lesson 14-2

For Items 12–15,

- Define a variable.
 - Write an equation.
 - Solve the equation.
 - Check the solution.
- Kiesha has \$250 in two-dollar bills in her teller drawer at the bank. How many bills does she have?
 - There are 16 windmills along a six-mile stretch of road. If they are equally spaced, how many feet apart are the windmills? (1 mile = 5,280 feet)
 - Clayton left a 15% tip for the waitress. If the amount of the tip was \$3.30, how much was the bill before the tip?
 - A donut shop sells glazed donuts for \$5.88 a dozen. How much would one donut cost?

For Items 16–20, solve each equation and check your solutions.

- $6x = 36$
- $0.75a = 14.25$
- $350 = 140r$
- $7.8s = 499.2$
- $\frac{1}{5}k = 15$
- $\frac{5}{6}x = \frac{25}{36}$
- $\frac{4}{9}x = \frac{8}{27}$

For Items 23–25, write a real-world problem that could be represented by the equation.

- $8x = 96$
- $10n = 420$
- $\frac{4}{5}y = 16$

Lesson 14-3

For Items 26–30, define a variable and write a division equation. Solve the equation algebraically and check the solution.

26. If \$1 is equivalent to 0.77 Euros, find the number of Euros equivalent to \$20.
27. Zita wants to buy an MP3 player that is on sale for 25% off. The original price of the MP3 player was \$200. What is the amount of the discount?
28. The population of Australia in January, 2013, was about 23 million. If the population of the United States then, divided by 13.7, was approximately equal to the population of Australia, what was the population of the United States?
29. The average American composted about 1.5 pounds of trash per day. How many pounds would the average American compost in 90 days?
30. The area of Texas in square miles is approximately 5 times the area of Arkansas. If the area of Arkansas is 53,178 square miles, what is the area of Texas?

For Items 31–35, solve each equation and check your solutions.

31. $\frac{x}{17} = 3$
32. $\frac{r}{6} = 9$
33. $\frac{y}{3} = 7$
34. $\frac{x}{3.8} = 4$
35. $12.9 = \frac{n}{10}$

For Items 36–39, write a real-world problem that can be represented by each equation.

36. $\frac{x}{10} = 15$
37. $\frac{n}{21} = 6$
38. $32 = \frac{k}{4}$
39. Would the equations $\frac{x}{8} = 2$ and $\frac{8}{x} = 2$ have the same solution? Explain.

MATHEMATICAL PRACTICES**Critique the Reasoning of Others**

40. Pam said the equations $\frac{x}{16} = 4$ and $\frac{x}{4} = 16$ have the same solution. Is she correct? Explain.

Learning Targets:

- Write inequalities to represent constraints or conditions within problems.
- Use substitution to determine whether a given number makes an inequality true.
- Graph solution sets of inequalities.
- Given an inequality, write a corresponding real-world problem.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Marking the Text, Summarizing, Note Taking, Discussion Groups, Activating Prior Knowledge

Geri wants to become a commercial airline pilot someday. She found the following information while doing research on this career.

- The airplane's captain must be at least 23 years old.
- The captain must have a minimum of 1500 hours of flying experience.
- By law, pilots can fly a maximum of 100 hours in a month.
- By law, pilots may not fly more than 32 hours during any consecutive 7 days.

Each piece of information that Geri found can be modeled by using an **inequality**. Phrases like *at least*, *more than*, and *a maximum* express a quantity that is greater than another.

1. List some phrases that could be used to express a quantity that is less than another.

The phrases above, and others like them, are clues to help solve the problem. They help to explain the rules of the situation.

Let's look at one of the rules about being an airline pilot.

A captain must be at least 23 years old. The words *at least* tell you that someone who is 23 years old can be a pilot. If the person is not 23, then he or she must be older than 23 to be a pilot.

The inequality modeling this situation is written as follows:

$$x \geq 23, \text{ where } x \text{ represents the age of the person.}$$

My Notes

MATH TERMS

An **inequality** is a mathematical statement showing that one quantity is greater than or less than another. Inequalities use these symbols:

$>$ is greater than

$<$ is less than

\geq is greater than or equal to

\leq is less than or equal to

My Notes

Example A

Linda was told she had to spend less than \$15 on flight snacks. Write the inequality that represents this statement.

Step 1: Define the variable:

Let x = the amount of money Linda can spend.

Step 2: Determine which symbol should be used in the inequality.

Linda needs to spend *less than* \$15, so use the symbol $<$.

Solution: Write the inequality. $x < 15$

Try These A

Write inequalities for the following statements.

- The temperature was less than 20° F on the morning of the test.
- More than 40 students were in her flight school class.
- Training uniforms cost at least \$50.
- No more than 25 students in the class will get a job with the airline.

Much like the solution to an equation, a **solution of an inequality** is a number that makes a statement true when it is substituted for the variable in the inequality.

18 is one possible solution to the inequality $p < 25$ because $18 < 25$ is a true statement.

Commercial airplanes are required to fly at least 150 feet above the highest fixed object in a residential area. The highest building in Geri's town is 240 feet tall.

- Define a variable and write an inequality to describe the situation.
- State three possible solutions to the inequality.

Lesson 15-1

Representing Situations with Inequalities

ACTIVITY 15

continued

4. Give three values for the variable a that are not solutions to the inequality.

Check Your Understanding

Determine two possible solutions for each of the following inequalities.

5. $x \leq 3.5$

6. $m > \frac{5}{4}$

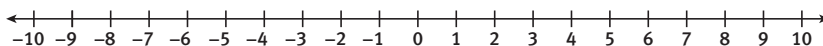
7. Daria did not want to spend more than \$200 for a flight from San Francisco to San Diego. Write the inequality that models this situation and determine two possible solutions.

The graph of an inequality shows all its possible solutions. Inequalities with one variable are graphed on a number line.

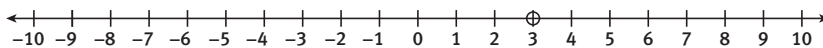
Example B

Graph all possible solutions of the inequality, $a > 3$.

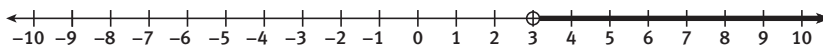
- a. Draw a number line.



- b. Draw an open dot at 3.



- c. The solution is all values greater than 3, so draw a line with an arrow to the right where the values are larger than 3.



My Notes

MATH TIP

When graphing inequalities use an open point to show an endpoint that is not included in the graph, $<$ or $>$.

Use a filled point to show an endpoint that is included as part of the solution to the inequality, \leq or \geq .

My Notes

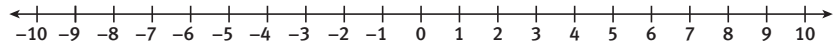
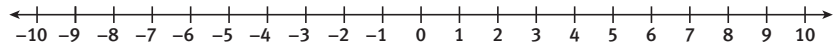
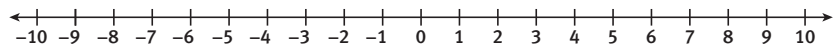
MATH TIP

When drawing a number line, the scale may be altered to best describe the situation. It is not always necessary to scale by ones.

Try These B

a. Model with mathematics.

Graph all possible solutions of the inequality $y \leq 1$.

b. Graph all possible solutions of the inequality $6 \geq x$.c. Graph all possible solutions of the inequality $x > 5$.

8. Look at the inequalities in Try These B above.

- Name two solutions for each inequality.
- How many more solutions do you think you can find to each inequality?

An inequality such as $y \leq 1$ or $y > 5$ has infinitely many solutions. That means that the number of solutions has no limit.

In the previous items, inequalities were written from the real-world scenario about becoming a pilot. Real-world problems can also be written from an inequality.

Example C

Write a real-world problem this inequality could represent.

$$x \leq 10$$

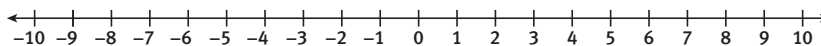
- Step 1:** Think about what the number 10 could represent.
It could be the number of people going on the same flight.
It could be the height of a room in feet.
It could be the number of seats remaining on a flight.
- Step 2:** Pick one of your choices.
The height of a room in feet.
- Step 3:** Make up a problem that could occur in the real world.
- Solution:** Lenora wants a new dresser and mirror for her bedroom. She needs to know how high the top of the dresser could be, so she measures the height of her room. The height of her room is 10 feet. The inequality above shows that Lenora's dresser and mirror can be no taller than 10 feet.

My Notes

Check Your Understanding

Define a variable and write an inequality to represent this situation.

- 9. No one under 23 is allowed to captain an airplane.
- 10. Write two possible solutions to the inequality $x < \frac{9}{5}$.
- 11. Graph all possible solutions to the inequality $x \leq 2$.



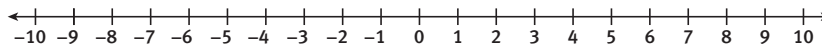
- 12. Write a problem about the number of pieces of luggage for the inequality $7 < x$.

LESSON 15-1 PRACTICE

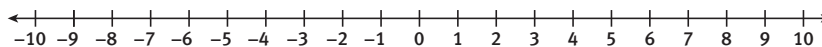
Define a variable and write an inequality to represent these situations.

- 13. She finished the license test in no more than 30 minutes.
- 14. The captain must have a minimum of 1500 hours of flying experience.
- 15. **Reason quantitatively.** What are two values that are not solutions for $11.4 > y$?
- 16. What are three possible solutions for $x > \frac{10}{8}$?

- 17. Graph all possible solutions for $x > \frac{3}{2}$.



- 18. Graph all possible solutions for $x > 9$.



- 19. Use the inequality $x \leq 20$ to write a problem about people waiting for a flight.

My Notes

- b. A hot-air balloon needs to be at least 100 feet in the air to fly safely. It is already 37 feet in the air.
- c. A block is 6 inches high. The tower must be over 100 inches high.

The person registering students for the pilot training class does not know that there is a maximum number of students that can sign up, or that some students have already signed up. She signs up 15 more students. Is there a way to know if she signed up too many people?

MATH TIP

Show a question mark over the inequality symbol to show that you are not sure if it is true or not.

Example F

Use substitution to determine if 15 additional people are too many for the class.

- Step 1:** Write the inequality using the information that is given. Let x represent the additional students that can sign up for the class.

$$x + 12 \leq 25$$

- Step 2:** Substitute the value 15 for x and solve.

$$15 + 12 \stackrel{?}{\leq} 25$$

$$27 \stackrel{?}{\leq} 25$$

Solution: You know that 27 is not less than 25, so 15 additional students cannot sign up for the class.

Try These F

Determine if the given value of x makes the inequality true.

- a. $x - 5 > 17$, $x = 12$
- b. $x + 9 > 21$, $x = 15$
- c. $4x \leq 50$, $x = 13$

To determine the values that make an inequality true, an inequality can be solved like an equation is solved.

My Notes

Example G

You know that 15 students are too many to add to the class, but how many more can sign up without going over the limit?

Step 1: Write the inequality.

$$x + 12 \leq 25$$

Step 2: Subtract 12 from both sides of the inequality to isolate the variable.

$$\begin{aligned} x + 12 - 12 &\leq 25 - 12 \\ x &\leq 13 \end{aligned}$$

Solution: $x \leq 13$.

This solution tells you that any number of students, less than or equal to 13, can sign up for the class without exceeding the limit of 25.

Try These G

Solve each inequality.

a. $x + 21 \leq 46$

b. $2x > 11$

c. $x - 1.2 < 4.8$

d. $x + \frac{3}{2} \geq 6$

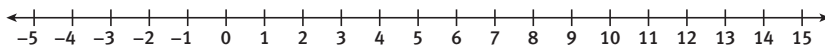
Graph the solution of an inequality on a number line.

Example H

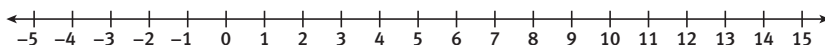
Graph the solution to the inequality $x + 12 \leq 25$.

a. From Example G you know the solution is $x \leq 13$.

b. The inequality includes an equal sign, showing that 13 is included in the solution, so the point at 13 will be solid.



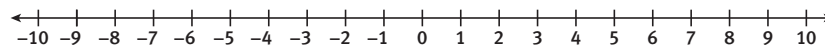
c. The solution is all numbers less than 13, so the arrow goes to the left.



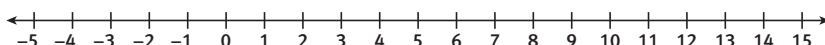
Try These H

Solve and graph each inequality.

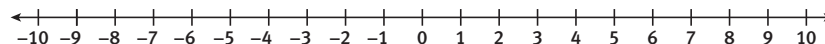
a. $x + 5 < 13$



b. $x - 3 \geq 9$



c. $3x \leq 24$

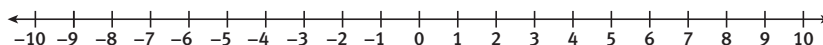


My Notes

Check Your Understanding

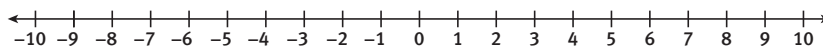
Write an inequality to represent this situation.

1. The parachute needs at least six people to hold it. There are two people holding it now.
2. Determine if $\frac{3}{4}$ is a solution for $x + \frac{1}{3} > 2$.
3. Determine if 4.7 is a solution for $x - 2.4 < 5$.
4. Solve $2.3 + x < 7.7$.
5. Solve $3x < 8$.
6. Solve this inequality and graph the solution $x + 23 < 31$.

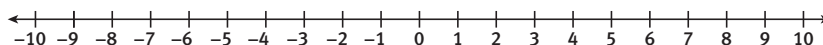


LESSON 15-2 PRACTICE

7. A captain can fly a maximum of 100 hours a month. He has flown 52 hours. Write the inequality that represents this situation.
8. **Model with mathematics.** A paper airplane contest needs at least 65 people to enter. So far, 43 people have entered. Write the inequality that represents this situation.
9. Determine if 4.4 is a solution for $x + 1.9 > 7$.
10. Determine if $\frac{5}{2}$ is a solution for $\frac{5}{4} + x \leq 3$.
11. Solve $x - 15 < 2$.
12. Solve $2.5x \geq 12.5$.
13. Solve $1.5 + x < 6.5$ and graph the solution.



14. Solve $\frac{4}{3} + x > 3$ and graph the solution.



ACTIVITY 15 PRACTICE

Write your answers on notebook paper.

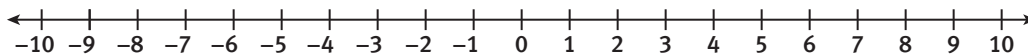
Show your work.

Lesson 15-1

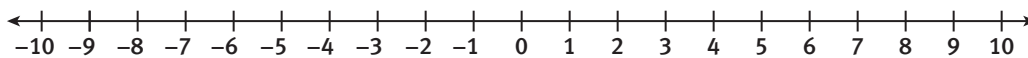
Write an inequality to represent each situation in Items 1–4.

1. A number is at least 6.
2. 7 is less than a number.
3. No more than 150 people will fit on the plane.
4. The maximum score on the pilot's test is 95 points.
5. Is $\frac{4}{5}$ a solution of $x \leq 2$?
6. Which of the following is a solution of $x > 10.4$?
a. 5.2 b. 7.4
c. 9.6 d. 11.2
7. Consider the inequality $x \geq 10.2$.
a. Is 7 a solution? Explain.
b. How many solutions does the inequality have? How can you show this?
8. Explain why 5 is a solution to both $x < 10$ and $10 > x$.

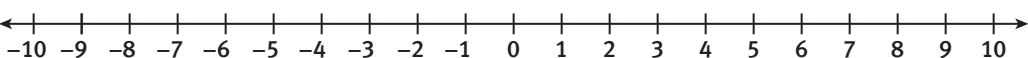
9. Graph $x < 8.5$.



10. Graph $\frac{3}{2} \leq x$.



11. Graph $x \geq 3$.



12. Use the inequality $x < 5.5$ to write a problem about the distance a paper airplane flew.
13. Use the inequality $x \geq 50$ to write a problem about the number of hours of flight training required.
14. Use the inequality $x > 15$ to write a problem about the number of balloons it takes to lift a small box off the ground.
15. Use the inequality $x > 5$ to write a problem about the wind speed it takes to keep a kite in the air.

Lesson 15-2

For Items 16–18, write an inequality to represent each situation.

16. A helium balloon can stay in the air for up to 6 hours. It has been in the air 2.5 hours.
17. An airplane can go 3,500 miles on a single filling of fuel. It has gone 1,250 miles.
18. An airplane captain must have a minimum of 1500 hours of flying experience. Tom has 715.
19. Is $\frac{4}{5}$ a solution of $\frac{5}{4} + x \leq 2$?
20. Is 10.3 a solution of $x - 2.5 > 8$?
21. Which of the following is a solution for $x + 3.2 \geq 10.2$?
- | | |
|--------|--------|
| A. 5.7 | B. 4.0 |
| C. 7 | D. 6.5 |

22. Solve $\frac{6}{4} + x \leq \frac{9}{2}$.

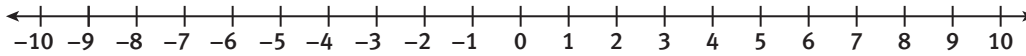
23. Solve $x + 11.3 > 15.2$.

24. Solve $x - 9.25 < 3.5$.

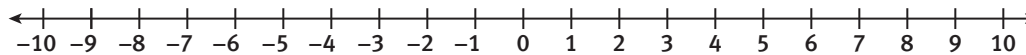
25. Solve $8x > 112$.

26. Solve $7x \leq 139$.

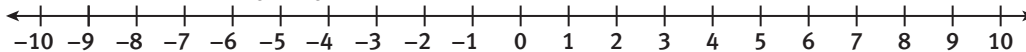
27. Solve and graph $x + 3.5 < 9.5$.



28. Solve and graph $4x \geq 14$.



29. Solve and graph $x - \frac{3}{5} \leq \frac{12}{5}$.

**MATHEMATICAL PRACTICES****Reason Abstractly**

30. How is an inequality similar to an equation? How are they different? Provide examples to support your answers.

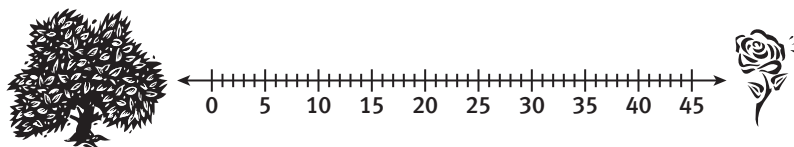
Learning Targets:

- Create a table representing a relationship given a verbal description.
- Write an equation to represent a relationship given a verbal description or a table.
- Investigate rate of change.
- Graph equations of the form $y = ax$.

SUGGESTED LEARNING STRATEGIES: Shared Reading, Role Play, Visualization, Quickwrite, Critique Reasoning, Think-Pair-Share, Create Representations, Look for a Pattern

One sunny day Sen T. Pede and his friend Lady Bug start out from the elm tree and move toward a rose bush that is 45 feet away. Sen crawls at 5 feet per minute and Lady crawls at 3 feet per minute.

- 1. Model with mathematics.** Use the diagram below to show where each critter is exactly three minutes after they start their journey. Place the letter *S* at Sen's location and the letter *L* at Lady's location.



- 2.** Complete this table to show how far Sen and Lady are from the elm tree for each time value.

Time Since Leaving Elm Tree (min.)	0	1	2	3	4	5
Sen's Distance from Elm Tree (ft)						
Lady's Distance from Elm Tree (ft)						

- 3.** Describe a pattern you see in Sen's row of the table. Use words and mathematical symbols.
- 4.** Sen crawls d feet in m minutes. Write an equation using d and m to represent the relationship between how long Sen has been traveling and how far he has travelled from the elm tree.

My Notes

My Notes

5. How far will Sen be from the elm tree after seven minutes? Use your equation to justify your response.
6. How long will it take Sen to crawl 40 feet? Use your equation to justify your response.
7. **Make use of structure.** Describe a pattern you see in Lady's row in the table. You may use words and mathematical symbols.
8. Lady crawls d feet in m minutes. Write an equation to represent the relationship between how long Lady has been traveling and how far she has travelled from the elm tree.
9. How far will Lady be from the elm tree after eight minutes? Use your equation to help justify your response.
10.
 - a. How long will it take Lady to crawl 36 feet? Use your equation to justify your response.
 - b. After Sen reaches the rose bush, how long will he wait for Lady to arrive? Explain how you determined your answer.

Check Your Understanding

11.
 - a. Lee walks his dog every morning before school. He travels 4 feet per second. Make a table to show how far Lee travels during the first 5 minutes of his walk.
 - b. Using the table created for Lee walking his dog, write an equation for this relationship if Lee travels d feet in m minutes.
 - c. How far will Lee travel in the first 15 minutes of his walk?
12.
 - a. Joey can ride his skateboard 50 feet every minute. Write an equation that determines his distance d in m minutes.
 - b. How far will Joey ride his skateboard in 1 hour?
13.
 - a. Sam can read 3 books each month. Write an equation that determines how many books b he reads every month m .
 - b. **Make sense of problems.** How many books can Sam read in 2 years?

Lesson 16-1

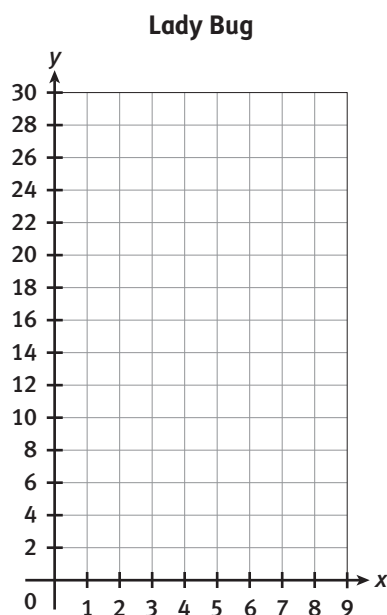
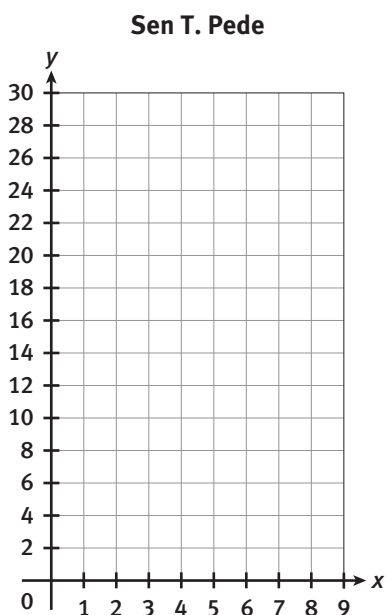
Representing Relationships

ACTIVITY 16

continued

Work with your group to answer Items 14–18.

14. Plot the data points from Item 2. Label the axes. Let the vertical axis show distance and the horizontal axis show time.



15. Write your equations from Items 4 and 8 under the corresponding graphs. What connections, if any, can you make between the graphs of the data points and the equations you wrote from the data?
16. Use Sen's graph to determine how far he would travel in 8 minutes. Explain how you used the graph to determine your response. Verify your response using Sen's equation.
17. Use Lady's graph to determine how long it would take Lady to travel 30 feet. Explain how you used the graph to determine your response. Verify your response using Lady's equation.
18. If Sen's graph were to be extended, it would contain the point $(12,60)$.
- Explain what the coordinates in this ordered pair tell you about Sen.
 - Explain how Sen's equation can be used to justify the fact that the point $(12,60)$ is on Sen's graph.

My Notes

WRITING MATH

Ordered pairs are written as (x, y) .
For example: $(5,7)$ is an ordered pair where $x = 5$ and $y = 7$.

My Notes

MATH TERMS

Rate of Change is a relationship that compares the change in one variable to the corresponding change in a related variable.

For example, for each hour you study for a test your grade increases by 5 points. The rate of change is 5 points per 1 hour.

If the rate of change remains the same in a problem situation, it is called a **constant rate of change**.

On another day, Sen and Lady want to go to the lake for a picnic. However, the lake is much farther away than the rose bush. They decide they will use their equations to determine how long it will take them to reach the lake.

19. Use Sen's equation to determine how far he can crawl in one hour.
20. Think about the distances Sen crawls.
 - a. How far does he crawl between time $m = 20$ minutes and $m = 21$ minutes?
 - b. How far does he crawl between time $m = 25$ minutes and $m = 26$ minutes?
 - c. Describe how far Sen crawls as a **rate of change**.
 - d. How is Sen's rate of change represented in his equation?
21. Use Lady's equation to determine how far she can crawl in one hour.
22. Describe how far Lady crawls as a rate of change between time:
 - a. $m = 32$ minutes and $m = 33$ minutes
 - b. $m = 32$ minutes and $m = 34$ minutes
 - c. How is Lady's rate of change represented in her equation?
23. If the lake is 540 feet from the elm tree, how long will it take each critter to reach the lake? Show your work or explain how you determined your answer.

My Notes

Learning Targets:

- Graph equations of the form $y = kx$ or $y = x + b$.
- Create a table and graph a relationship given a verbal description.
- Explain how one variable depends on another variable.
- Describe a relationship given a graph.

SUGGESTED LEARNING STRATEGIES: Marking the text, Summarizing, Visualization, Create Representations, Look for a Pattern

Sen and Lady have a turtle friend, Archimedes, who sometimes goes along on their adventures.

1. Archimedes crawls d feet in minutes. The equation that shows the relationship between d and m for his pace is $d = 7.5m$.
 - a. Use appropriate units to describe what information the coefficient 7.5 gives about how Archimedes moves.
 - b. Create a table of values and plot the data.

For Sen, Lady, and Archimedes, the **independent variable** is m , the time in minutes each crawls. The **dependent variable** is distance, because it depends on how long each critter crawls.

2. The data points appear to be **linear**. What do you think this means?
3. If all the points on Archimedes' graph are connected with a line, the line contains the **ordered pair** $(2.5, 18.75)$. An ordered pair locates points in the coordinate plane.
 - a. What does this ordered pair mean in this context?
 - b. Should the data points on the graphs in Item 14 in Lesson 16-1 be connected with a line?
4. Solve the equation $540 = 7.5m$. What question is answered by the solution to the equation?
5. Solve the equation $d = 7.5(540)$. What question is answered by the solution to the equation?

MATH TERMS

An **independent variable** is the variable for which input values are substituted in an equation.

A **dependent variable** is the variable whose value is determined by the input or value of the independent variable.

MATH TERMS

In an **ordered pair**, the first number describes the horizontal position and the second number describes the vertical position. The numbers are the coordinates of the point.

My Notes

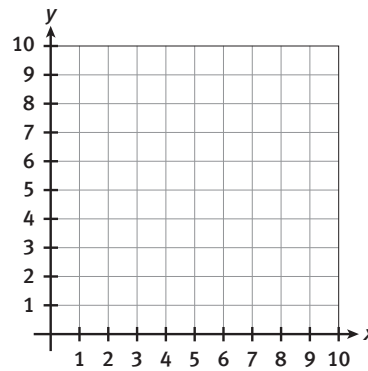
Archimedes takes a box of crackers to share at a picnic with Sen, Lady, and other friends. Archimedes did not have breakfast so he knows he will eat 3 crackers and the other critters will each eat one cracker.

The equation $y = x + 3$ relates the amount crackers, y , Archimedes brings with the number of critters, x , who will eat the crackers.

14. Complete the following table.

Input, x	$x + 3$	Output, y
1		
2		
4		
7		

15. Plot the points on the graph below.



16. How is the rate of change represented in the equation, table, and graph?

17. Interpret the meaning of the rate of change in this context.

18. State the dependent and independent variables. Justify your answers.

19. Compare and contrast the equations $y = x + 3$ and $y = 3x$.

20. Suppose the equation $y = 3x$ relates the number of crackers, y , with the number of critters, x , eating them.

a. Explain how the table of values would differ from the table in Item 14.

b. Describe the similarities and differences between the graph of $y = 3x$ and the graph in Item 15.

My Notes

Check Your Understanding

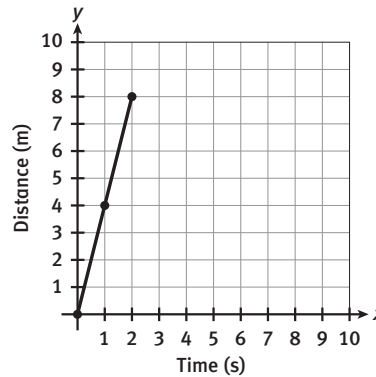
25. Brianna is three years older than her sister Lindsay.
- Use this information to complete the table below to show the relationship between their ages.

Lindsay's Age	1	2	3	4
Brianna's Age				

- Plot the data. Place Lindsay's age on the horizontal (x) axis and Brianna's age on the vertical (y) axis.
- Use the variables x and y to write an equation to represent Brianna's age if Lindsay's age is x .
- Describe each of the variables as either independent or dependent.

LESSON 16-2 PRACTICE

26. Use the equation $y = 2x + 2$ for this problem.
- Create an input/output table.
 - Graph the ordered pairs from the table.
 - Describe each of the variables as the independent or the dependent variable.
27. Use the graph below to describe the relationship between the variables.



ACTIVITY 16 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 16-1

1. A car can travel 60 miles per hour.
 - a. Create a table showing how far the car travels in 5 hours.
 - b. Write an equation to determine the distance d that the car can travel in t hours.
 - c. How long will it take the car to travel 540 miles? Explain your reasoning.
 - d. How far will the car travel in 12 hours? Explain your reasoning.

2. Use the equation $d = 5m$ for Sen and $d = 3m$ for Lady.

One day, Sen and Lady wanted to go to the lake for a picnic. However, the lake is much farther away than the rose bush is. They decide to use their equations to determine how long it will take them to reach the lake.

- a. Use Sen's equation to determine how far he can crawl in one hour.
- b. Use Lady's equation to determine how far she can crawl in one hour.
- c. If the lake is 540 feet from the elm tree, how long will it take Sen and Lady to reach the lake?

3. Sen and Lady have a friend Archie who moves according to the equation $d = 7.5m$.
 - a. Solve the equation $540 = 7.5m$. Explain what the solution to the equation means in this context.
 - b. Solve the equation $d = 7.5(540)$. Explain what the solution to the equation means in this context.
4. Emma decided to start a snow globe collection. She started with 3, and every year she added one more.
 - a. Create a table showing the growth in Emma's snow globe collection over 5 years.
 - b. Write an equation to determine the number of snow globes she has after y years.
 - c. Use the equation to determine how many snow globes she will have after 15 years.
 - d. How many years will it take her to get 25 snow globes?

Lesson 16-2

5. Niall paints three paintings a week.
 - a. Create a table showing the number of paintings Niall completes over 4 weeks.
 - b. Graph the data. Place weeks on the horizontal axis, and paintings completed on the vertical axis.
 - c. Write an equation representing the relationship between the number of weeks and the number of paintings completed.
 - d. Describe each of the variables as independent or dependent.
 - e. How many weeks will it take Niall to paint 24 paintings?
6. a. Copy and complete the table for the equation $y = 3x - 2$.

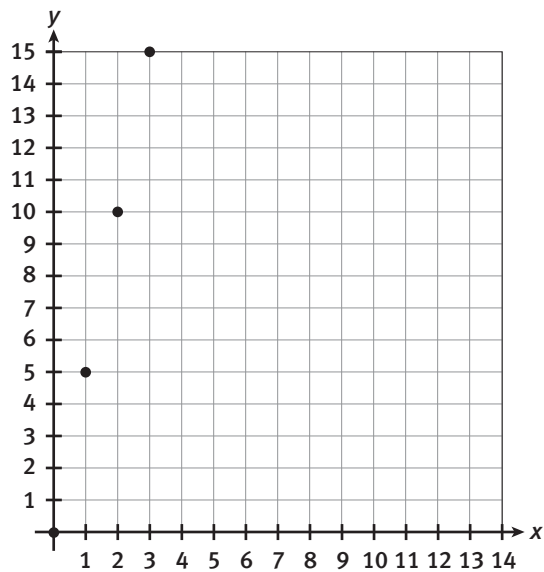
Input, x	$3x - 2$	Output, y
2		
3		
1.5		
2.5		

- b. Graph the data.
7. After buying three paintings from Niall (from Item 5), Naeem was inspired to paint. He paints one painting a week. Who will have more paintings over 4 weeks? Explain your reasoning.

8. Harry got a job assembling birdhouses. There were 2 birdhouses already completed as examples for him. Each week, he assembled 2 more.
 - a. Create a table showing the total number of birdhouses Harry completed over 5 weeks. Show the input and the output.
 - b. Plot the data.
 - c. Write an equation that shows the relationship between the number of weeks and the total number of birdhouses Harry completed.
 - d. How many birdhouses will he build after 10 weeks? Explain your reasoning.
 - e. When will he complete 28 birdhouses? Explain your reasoning.
9. Compare and contrast the graphs of parts 5b and 8b.

MATHEMATICAL PRACTICES
Make Sense of Problems

10. Describe the relationship represented in the graph below.



Write your answers on notebook paper. Show your work.

- The Middle School is holding a book fair. The costs of different types of books are in the table.

Book Type	Cost (\$)
paperback	7.50
hardcover	12.50
books on tape	8.75
reference book	10.25

- Paige brought \$5.00 to the book fair. She wants to buy three paperback books. Write and solve an equation to determine how much more money she will need to make this purchase.
 - Lesley has \$17 to spend, which she uses to buy a reference book. Write and solve an equation to show how much Lesley will receive in change.
 - Write and graph an inequality to represent the *maximum* amount of money Lesley has to spend on another book.
 - Use the inequality and its graph to explain whether Lesley has enough money to buy a paperback book.
- Darius's parents sent money to the book fair to be split among the four children in his family. After they gave the money to the cashier, she told each child they had \$11.50 to spend. Write and solve an equation to show the total amount of money Darius's parents sent to the book fair.
 - Marcus wants to buy several paperback books.
 - Create a table of values to show the cost of buying up to five paperback books.
 - Plot the data on a graph.
 - Write an equation that represents the cost to Marcus of buying the books and identify the independent and dependent variables. Explain your reasoning in making your choices.
 - If Marcus wanted to buy eight paperback books, what would be the cost?
 - Marcus's friend has \$97.50 to spend on paperback books. How many paperbacks can he buy? Explain which representation you used in determining your answer.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
Mathematics Knowledge and Thinking (Items 1a-d, 2, 3a-e)	The solution demonstrates these characteristics:			
	<ul style="list-style-type: none"> A clear understanding of tables, graphs, independent and dependent variables. Effective understanding of and accuracy in writing and solving equations; writing and graphing inequalities. 	<ul style="list-style-type: none"> A functional understanding of tables, graphs, independent and dependent variables. Writing and solving equations, and writing and graphing inequalities that usually result in correct answers. 	<ul style="list-style-type: none"> Partial understanding of tables, graphs, independent and dependent variables. Difficulty with writing and solving equations, and writing and graphing inequalities. 	<ul style="list-style-type: none"> An inaccurate understanding of tables, graphs, independent and dependent variables. Little or no understanding of writing and solving equations, and writing and graphing inequalities.
Problem Solving (Items 1a-b, 2, 3e)	<ul style="list-style-type: none"> An appropriate and efficient strategy that results in a correct answer. 	<ul style="list-style-type: none"> A strategy that may include unnecessary steps but results in a correct answer. 	<ul style="list-style-type: none"> A strategy that results in some incorrect answers. 	<ul style="list-style-type: none"> No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-d, 2, 3a-c)	<ul style="list-style-type: none"> Clear and accurate modeling using tables and graphs. Clear and accurate representation of problems as equations and inequalities. 	<ul style="list-style-type: none"> Graphing data and creating tables of data with little difficulty. Some difficulty in representing problems as equations and inequalities. 	<ul style="list-style-type: none"> Partially accurate graphing of data and use of tables. Difficulty in writing equations and inequalities leading to errors. 	<ul style="list-style-type: none"> Inaccurate or incomplete graphs and tables. No understanding of representing problems as equations and inequalities.
Reasoning and Communication (Items 1d, 3c, 3e)	<ul style="list-style-type: none"> Precise use of appropriate math terms and language to explain independent and dependent variables. Accurately explain a situation using an inequality. 	<ul style="list-style-type: none"> An adequate explanation of independent and dependent variables. Relating an inequality to a situation with little difficulty. 	<ul style="list-style-type: none"> A misleading or confusing explanation of independent and dependent variables. Poor understanding of relating an inequality to a situation. 	<ul style="list-style-type: none"> An incomplete or inaccurate description of independent and dependent variables. No understanding of how to relate an inequality to a situation.