EXTENDED TASKS FOR GCSE MATHEMATICS

A series of modules to support school-based assessment

The Teacher's Guide

MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

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Authors

This book is one of a series forming a support package for GCSE coursework in mathematics. It has been developed as part of a joint project by the Shell Centre for Mathematical Education and the Midland Examining Group.

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A large number of teachers and their students have contributed to this work through a continuing process of trialling and observation in their classrooms. We are grateful to them all for their help and for their comments. Among the teachers to whom we are particularly indebted for their contributions at various stages of the project are Paul Davison, Ray Downes, John Edwards, Harry Gordon, Peter Jones, Sue Marshall, Glenda Taylor, Shirley Thompson and Trevor Williamson.

The LEAs and schools in which these materials have been developed include Bradford: Bradford and Ilkley Community College; Derbyshire: Friesland School, Kirk Hallam School, St Benedict's School; Nottinghamshire: Becket RC Comprehensive School, The George Spencer Comprehensive School, Chilwell Comprehensive School, Greenwood Dale School, Fairham Community College, Haywood Comprehensive School, Farnborough Comprehensive School, Kirkby Centre Comprehensive School, Margaret Glen Bott Comprehensive School, Matthew Holland Comprehensive School, Rushcliffe Comprehensive School; Leicestershire: The Ashby Grammar School, The Burleigh Community College, Longslade College; Solihull: Alderbrook School, St Peters RC School; Wolverhampton: Heath Park High School, Our Lady and St Chad RC School, Regis School, Smestow School, Wolverhampton Girls High School; and Culford School, Bury St Edmonds.

Many others have contributed to the work of the project, notably the members of the Steering Committee and officers of the Midland Examining Group - Barbara Edmonds, Ian Evans, Geoff Gibb, Paul Lloyd, Ron McLone and Elizabeth Mills.

Jenny Payne has typed the manuscript in its development stages with help from Judith Rowlands and Mark Stocks. The final version has been prepared by Susan Hatfield.

The views expressed in The Teacher's Guide are those of the project team, based upon their experiences in schools and discussions with teachers.

Contents

Preface - A Guide

This set of materials comprises

* *The Teacher's Guide*
* *8 cluster books (4 in each pack)*
* *IMPACT - a departmental development programme*

This book is your main guide to the materials; in this preface we make some suggestions as to how you might best use them.

Investigation is a major theme of GCSE mathematics, and you may like to adopt a similar exploratory approach to these materials. People rarely read documentation thoroughly, in any field, until something goes wrong. However, experience suggests that you are likely to save time and trouble if you explore in something like the way we describe below.

However, first the basics of the challenge you face. The teacher who is used to the conventional secondary mathematics curriculum faces new challenges in the classroom of the following kinds

* *supporting students in independent work;* this less imitative mathematical activity requires a broader range of teaching strategies, emphasising general support and advice rather than detailed explanation and direction
* *mathematical discussion and explanation by the student;* this raises further management problems
* *diverse student activity;* this requires teacher support of different kinds, not simply the management of some individualised scheme.

All these are harder than conventional mathematics teaching, with its rhythm of *explanation - example - exercises,* essentially because this standard approach is *single track* with the thread of teaching, though not always of learning, firmly determined by the teacher.

Independent student activity is essentially *multi-track,* which is much more demanding in several ways

* *mathematically;* because the teacher must try to follow the students' lines of reasoning and perceive to what extent they seem fruitful

* *pedagogically;* because the teachers' interventions need to be helpful, without taking the responsibility for the approach away from the student

* *personally;* because the teacher will not always know the answers, or indeed understand the students' reasoning.

Equally, teachers must acclimatise themselves to the assessment of students' performance against specific external standards, usually expressed as criteria. This requires a shift away from content-based marking to process-based assessment.

These materials have been designed to help all teachers face these challenges effectively, and they have been developed within classrooms until they do so. They also provide a variety of rich problems suitable for extended task work and its related school-based assessment. If you can get at least one colleague to work in parallel with you, you will be able to help and support each other.

It is not intended that this guide should be read from cover to cover at the first attempt. It is designed as a 'dip-in' resource. IMPACT will help you and your colleagues as you move gradually towards the 'new' style of teaching which is necessary for this type of work. A steady and careful programme of development is offered within IMPACT - the departmental development programme.

We suggest that you may like to start in the following way

* look carefully through, say, two of the lead tasks; try thinking about them briefly yourself, discuss them with a colleague, then look at one or two of the student responses and the external moderator's comments

* read Chapters 1 and 2 of this guide to set the scene

* read the teacher's notes for the tasks you have already looked at

* read the introductory notes in IMPACT, and think about how you may wish to implement this programme

* use the remainder of this teacher's guide as and when you feel appropriate

* try one of the lead tasks with a GCSE group, or maybe a third year group

* gradually work through IMPACT

* continue to look through and discuss the cluster books

* try using other supportive resources with your younger students.

1 *Introduction and Overview*

In January 1984, Sir Keith Joseph, the then Secretary of State for Education and Science, announced his intention to introduce a new common 16+ examination in England, Wales and Northern Ireland. His announcement emerged after almost twenty years of speculation and debate about whether General Certificate of Education (GCE) O-Level should merge with the Certificate of Secondary Education (CSE). This new examination, the General Certificate of Secondary Education (GCSE) provides a single system of examining pupils at 16+. It is examined by six Examining Groups under the aegis of the School Examinations and Assessment Council (SEAC). The new examination was introduced with the minimum of delay. The first courses began in September 1986 and the first examinations were held in the Summer of 1988.

The main aims of the GCSE, as detailed in *'GCSE : A General Introduction'* (HMSO, 1985) are

To improve the quality of education

To raise standards of attainment

To produce a system that is fairer to candidates

To motivate teachers and pupils by setting clear targets and by the provision of stimulating and engaging courses

To enhance the esteem in which examinations are held and to make the results more intelligible to users

To promote improvements in the secondary school curriculum and the ways subjects are taught, particularly in the fourth and fifth years

To remove the need for schools to enter candidates for both O-level and CSE in the same subject or to prepare pupils in the same class for entirely different examinations.

The GCSE is available to candidates of all abilities and is designed to bring 80-90% of all pupils *at least* to the level associated with CSE Grade 4.

All GCSE examinations must conform to the National Criteria, which are nationally agreed rules and principles. There are General Criteria which establish guidelines across all subjects, and subject specific criteria which provide aims, assessment objectives and minimum content for each subject.

GCSE grades are awarded on a seven-point scale, A to G. The approximate relationships between GCSE grades and GCE O-Level/CSE are as shown in the following table

GCE O-Level	GCSE	CSE
A	A	
B	B	
C	C	1
D	D	2
E	E	3
	F	4
	G	5

In mathematics, the syllabuses of all examination groups have three levels of assessment, with a restricted number of grades available at each level. Throughout this material these are referred to as Foundation, Intermediate and Higher levels. This is discussed in more detail in Chapter 4.

The reasons underlying the change to a common 16+ examination system were many and various. However, the following major reasons emerged

A need to rationalise the dual system of the GCE O-level and CSE examinations. By 1984 the twenty existing Examining Boards had developed a multiplicity of different syllabuses. This created difficulties and confusion for teachers, students, parents and employers.

A need to improve the quality of education. Timed written examinations at the end of a lengthy course of study tend to emphasise the importance of the ability to memorise and organise information. What is assessed becomes important in the curriculum. Recently, we have begun to place increasing emphasis on problem solving and the application of knowledge as important objectives. Consequently, there arose a need to develop methods of measuring these aspects of student behaviour so that they could be incorporated into the assessment procedures.

A need to reward positive achievement. Many CSE syllabuses were merely diluted O-level syllabuses. Consequently, both the courses of study and the examinations associated with them provided frustrating and dispiriting experiences for many students. There was a need to identify basic levels of knowledge, skills and understanding for all students, and from this foundation, structure syllabuses and examinations for the more able.

In 1985 Sir Wilfred Cockcroft suggested that the GCSE is 'an attempt to see the best of curriculum development, and the best of examination development put together so as to represent the best of teaching practice'.

The National Criteria for each subject emphasise three areas for major reconsideration and change

* Differentiation
* Methodology
* School-based assessment.

Mathematics, unlike every other subject, was given a three year build-up period on the third aspect, school-based assessment. Although the GCSE examinations first took place in 1988, it is not until the summer of 1991 that this component of the assessment becomes compulsory. For all other subjects it was compulsory from the first examination. This delay meant that all schools receiving students at the age of eleven in September 1986 could work with their new students in the styles being advocated.

These styles include a greater need for active learning, for students to accept more responsibility for their learning, for greater understanding of mathematics to be gained, and for a more open-ended and enquiry-based approach. For some teachers this requires little change, while for others this demands significant and major changes. Teachers are being asked to adopt a different role from that which they have previously, and in most cases successfully, used. There are to be new methods of assessment, carried out in the classroom, and a greater emphasis on practical and investigative work.

These changes are highlighted in the aims and assessment objectives laid down in the National Criteria for Mathematics on pages 9 and 10.

These objectives reflect fully the spirit of the Cockcroft Report (HMSO, 1982), particularly in relation to paragraphs 243-252 which are included on pages 11-14.

Aims

To enable students

1. to develop their mathematical knowledge and oral, written and practical skills in a way which encourages confidence and provides satisfaction and enjoyment.
2. to read mathematics, and write and talk about the subject in a variety of ways.
3. to develop a feel for number, carry out calculations and understand the significance of the results obtained.
4. to apply mathematics in everyday situations and develop an understanding of the part which mathematics plays in the world around them.
5. to solve problems, present the solutions clearly, check and interpret the results.
6. to develop an understanding of mathematical principles.
7. to recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem.
8. to use mathematics as a means of communication with emphasis on the use of clear expression.
9. to develop an ability to apply mathematics in other subjects, particularly science and technology.
10. to develop the abilities to reason logically, to classify, to generalise and to prove.
11. to appreciate patterns and relationships in mathematics.
12. to produce and appreciate imaginative and creative work arising from mathematical ideas.
13. to develop their mathematical abilities by considering problems and conducting individual and cooperative enquiry and experiment, including extended pieces of a practical and investigative kind.
14. to appreciate the interpendence of different branches of mathematics.
15. to acquire a foundation appropriate to further study of mathematics and of other disciplines.

Extract from the *National Criteria for Mathematics*

Assessment Objectives

Candidates should be able

1. to recall, apply and interpret mathematical knowledge in the context of everyday situations.
2. to set out mathematical work, including the solution of problems, in a logical and clear form using appropriate symbols and terminology.
3. to organise, interpret and present information accurately in written, tabular, graphical and diagrammatic forms.
4. to perform calculations by suitable methods.
5. to use an electronic calculator.
6. to understand systems of measurement in everyday use and make use of them in the solution of problems.
7. to estimate, approximate and work to degrees of accuracy appropriate to the context.
8. to use mathematical and other instruments to measure and to draw to an acceptable degree of accuracy.
9. to recognise patterns and structures in a variety of situations, and form generalisations.
10. to interpret, transform and make appropriate use of mathematical statements expressed in words or symbols.
11. to recognise and use spatial relationships in two and three dimensions, particularly in solving problems.
12. to analyse a problem, select a suitable strategy and apply an appropriate technique to obtain its solution.
13. to apply combinations of mathematical skills and techniques in problem solving.
14. to make logical deductions from given mathematical data.
15. to respond to a problem relating to a relatively unstructured situation by translating it into an appropriately structured form.

The school based component will, in addition, provide the opportunity for testing the ability of candidates to do some or all of the following

16. to respond orally to questions about mathematics, discuss mathematical ideas and carry out mental calculations.
17. to carry out practical and investigational work, and undertake extended pieces of work.

Extract from the *National Criteria for Mathematics*

Classroom Practice

243 Mathematics teaching at all levels should include opportunities for

- exposition by the teacher;
- discussion between teacher and pupils and between pupils themselves;
- appropriate practical work;
- consolidation and practice of fundamental skills and routines;
- problem solving, including the application of mathematics to everyday situations;
- investigational work.

In setting out this list we are aware that we are not saying anything which has not already been said many times and over many years. The list which we have given has appeared, by implication if not explicitly, in official reports, DES publications, HMI discussion papers and the journals and publications of the professional mathematical associations. Yet we are aware that although there are some classrooms in which the teaching includes, as a matter of course, all the elements which we have listed, there are still many in which the mathematics teaching does not include even a majority of these elements.

244 We believe that one of the reasons for this may be that a brief statement such as "mathematics teaching should include opportunities for investigational work" does not explain sufficiently what is intended. We wish, therefore, to consider more fully each of the elements which we have listed.

Exposition

245 Exposition by the teacher has always been a fundamental ingredient of work in the classroom and we believe that this continues to be the case. We wish, though, to stress one aspect of it which seems often to be insufficiently appreciated. Questions and answers should constitute a dialogue. There is a need to take account of, and to respond to, the answers which pupils give to questions asked by the teacher as the exposition develops. Even if an answer is incorrect, or is not the one which the teacher was expecting or hoping to receive, it should not be ignored; exploration of a pupil's incorrect or unexpected response can lead to worthwhile discussion and increased awareness for both teacher and pupils

Extract from *Mathematics Counts*

Classroom Practice : continued

of specific misunderstandings or misinterpretations.

Discussion

246 By the term 'discussion' we mean more than the short questions and answers which arise during exposition by the teacher. In the National Primary Survey report we read "In some cases, particularly in the older classes, more attention could usefully have been given to more precise and unambiguous use of ordinary language to describe the properties of number, size, shape or position". The National Secondary Survey report noted that "the potential of mathematics for developing precision and sensitivity in the use of the language was underused". The ability to 'say what you mean and mean what you say' should be one of the outcomes of good mathematics teaching. This ability develops as a result of opportunities to talk about mathematics, to explain and discuss results which have been obtained, and to test hypotheses. Moreover, the many different topics which exist within mathematics at both primary and secondary level should be presented and developed in such a way that they are seen to be inter-related. Pupils need the explicit help, which can only be given by extended discussion, to establish these relationships; even pupils whose mathematical attainment is high do not easily do this for themselves.

Practical work

247 Practical work is fundamental to the development of mathematics at the primary stage; we discuss this in detail in the following chapter. It is too often assumed that the need for practical activity ceases at the secondary stage but this is not the case. Nor is it the case that practical activity is needed only by pupils whose attainment is low; pupils of all levels of attainment can benefit from the opportunity for appropriate practical experience. The type of activity, the amount of time which is spent on it and the amount of repetition which is required will, of course, vary according to the needs and attainment of pupils. The results of the practical testing carried out by the Assessment of Performance Unit and described in the reports of both primary and secondary tests illustrate clearly the need to provide opportunities for practical experience and experiment for pupils of all ages.

Extract from *Mathematics Counts*

Classroom Practice : continued

Practice

248 All pupils need opportunities to practise skills and routines which have been acquired recently, and to consolidate those which they already possess, so that these may be available for use in problem solving and investigational work. The amount of practice which is required varies from pupil to pupil, as does the level of fluency which is appropriate at any given stage. However, as we have pointed out already, practice of fundamental skills is not by itself sufficient to develop the ability to solve problems or to investigate - these are matters which need separate attention.

Problem solving

249 The ability to solve problems is at the heart of mathematics. Mathematics is only 'useful' to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name 'problem solving'. However, the solution of a mathematical problem cannot begin until the problem has been translated into the appropriate mathematical terms. This first and essential step presents very great difficulties to many pupils - a fact which is often too little appreciated. At each stage of the mathematics course the teacher needs to help pupils to understand how to apply the concepts and skills which are being learned and how to make use of them to solve problems. These problems should relate both to the application of mathematics to everyday situations within the pupils' experience, and also to situations which are unfamiliar. For many pupils this will require a great deal of discussion and oral work before even very simple problems can be tackled in written form.

Investigational work

250 The idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very many fields. We suspect that there are many teachers who think of 'mathematical investigations' as being in some way similar to 'projects' which in recent years have become common as a way of working in many areas of the curriculum; in other words, that a mathematical investigation is an extensive piece of work which will take quite a long time to complete and will probably be undertaken individually or as a member of a small group. But although this is one of the forms which mathematical investigation can take, it is by no means the only form nor need it be the most common. Investigations need be neither lengthy nor difficult. At

Extract from *Mathematics Counts*

Classroom Practice : continued

the most fundamental level, and perhaps most frequently, they should start in response to pupils' questions, perhaps during exposition by the teacher or as a result of a piece of work which is in progress or has just been completed. The essential condition for work of this kind is that the teacher must be willing to pursue the matter when a pupil asks "could we have done the same thing with three other numbers?" or "what could happen if ...?" Very often the question can be resolved by a few minutes of discussion either with the pupil or with a group of pupils; sometimes it may be appropriate to suggest that the pupil or a group of pupils, or even the whole class, should try to find the answer for themselves; sometimes it will be necessary to find time on another occasion to discuss the matter. The essential requirement is that pupils should be encouraged to think in this way and that the teacher takes the opportunities which are presented by the members of the class. There should be a willingness on the part of the teacher to follow some false trails and not to say at the outset that the trail leads nowhere. Nor should an interesting line of thought be curtailed because "there is no time" or because "it is not in the syllabus".

251 Many investigations lead to a result which will be the same for all pupils. On the other hand, there are many investigations which will produce a variety of results and pupils need to appreciate this. For example, the answer to the question "In how many different ways can you carry out this calculation on your calculator; which may require the least number of steps?" depends on the particular model of calculator used, and pupils who undertake an investigation of this kind will produce a variety of answers, all of which may be equally valid. Mathematical puzzles of various kinds also offer valuable opportunities for investigational work. Even practice in routine skills can sometimes, with benefit, be carried out in investigational form; for example, 'make up three subtraction sums which have 473 as their answer'. The successful completion of a task of this kind may well assist understanding of the fact that subtraction can be checked by means of addition.

252 It is necessary to realise that much of the value of an investigation can be lost unless the outcome of the investigation is discussed. Such discussion should include consideration not only of the method which has been used and the results which have been obtained but also of false trails which have been followed and mistakes which may have been made in the course of the investigation.

Extract from *Mathematics Counts*

This whole situation has implications for every teacher of mathematics, both in the short and longer term.

School-based assessment could have taken a variety of forms

* Practical tasks
* Pure investigations
* Oral interviews
* Short, long or extended tasks
* Informal and formal assessment
* Mental arithmetic tests.

The six new examining groups were free to put forward their own proposals for GCSE schemes within the framework of the National Criteria. Initially, this led to six very different schemes, although there is common ground, and indeed this has continued to be the case, possibly more so, as all GCSE schemes have been refined.

Clearly, such major changes in many aspects of the teaching, learning and assessment of mathematics created a need to offer teachers guidance relating to the

What, Why and How

of GCSE mathematics.

Many teachers felt that the first of these three aspects, the *What*, was reasonably clear. However, the *Why* seemed rather sketchy to say the least, and useful information on the *How* virtually non-existent at the time.

Considerable amounts of INSET and a variety of publications have tried to help teachers to overcome some of these difficulties, with varying degrees of success.

This package of material attempts to offer some help to teachers as they look at all three of these aspects when working within the GCSE framework. In particular, it looks at the most significant challenge, but potentially the most fruitful area of mathematical activity, the *extended task*.

2 *What Is Coursework And Why?*

Coursework is the term used to describe work produced by students during their course of studies, which forms part of the school-based assessed element of any GCSE scheme. Here, we shall look particularly at extended tasks within the coursework element. The Secondary Examinations Council (SEC), now succeeded by the School Examinations and Assessment Council (SEAC), has produced a series of working papers relating to GCSE. *Working Paper 2, Coursework Assessment in GCSE* (1985), provides a summary of its views on assessment by means of coursework. This paper highlights the value and features of such work, and these relate directly to extended tasks within mathematics, although it is directed more generally at coursework across all subject areas. The SEC document is included on pages 19-22.

The three main points which emerge from this and other similar discussions are

* Some educational outcomes are more appropriately measured through this type of assessment
* This type of work can enhance the curriculum
* This form of assessment can help us to assess what is important, rather than making important only that which is easy to assess.

It is clear that in the past many aspects of a student's attainment have not been assessed due, simply, to the structure of examinations. Many of these aspects are those which we, as teachers or students, and society in general, hold in high esteem. Also, it has often been true that since there has been little or no assessment of these aspects, they have often been ignored, either wholly or in part, throughout the mathematics curriculum. We have to accept that we live and work with an assessment-led curriculum. Clearly, there are implications when considering the National Curriculum.

What You Test Is What You Get.

There are many important skills which cannot be assessed within formal timed and written examinations. These include

* Research skills
* Hypothesis testing
* The ability to change direction when a dead end is met: lateral thinking
* Sustaining work over a period of time
* Working independently
* Working co-operatively with others
* Communication
* Reflecting upon results
* Making decisions and living with the consequences.
* Assuming responsibility for one's own actions.

If we value the skills listed above and wish to ensure that they are acquired by students, then we need to assess them. In so doing, we enhance the curriculum.

So far, our discussion leads naturally to the proposal that the key features of extended task work should include

* Students making a strong personal contribution to their own studies, both in relation to planning and implementation
* Communication at all stages, both oral and written
* Work which is of a problem solving, investigative and enquiry-based nature.

In order to satisfy these requirements, the approach adopted throughout this material, has been to work with broad open-ended tasks. Further discussion relating to such tasks is delayed until later in Chapter 6. Having established a model for the type of work which is to form an extended task, it is worthwhile outlining the mathematical areas within which such tasks are to be completed, as well as trying to define more closely the term extended task. All six examining groups have focussed upon three main categories of mathematics for assessment using extended tasks. These are

* Pure investigations
* Practical in method
* Practical in purpose.

While these areas of mathematical activity are common to all groups, the labels given by different examining groups vary considerably. Practical in purpose is used to refer to problems, real problem solving, applications, everyday mathematics and many more. A further area has been considered separately within several schemes, this is

* Statistics.

Statistical studies, including those based on probability, can usually be classified as either practical in method or practical in purpose. However, because of the significant role that this area of mathematics plays in modern society, there is an argument for it to be considered as an independent fourth category. In the proposals for the National Curriculum it is suggested that students need to be able to handle data; to collect, sort, represent, analyse and interpret different types of data. In addition, students should be able to make predictions, draw inferences, and use data in order to make decisions.

What period of time should be spent on an extended task makes an interesting discussion. When does a mathematical task become an extended one? The National Curriculum proposals suggest that extended tasks should occupy up to about 10 hours of class time. Within the project which has produced this handbook and the related materials, a total period of about twelve to fifteen hours, part of which is outside class time, is considered an appropriate period of time. This period of time allows students to sustain their activity over two to three weeks and to reach some valid conclusions, as well as providing opportunity to reflect upon their work and communicate their findings through oral exchanges and a written report. Periods of time shorter than this tend to be less fruitful when considering many of the key issues which we believe should take place. Longer periods of time may be suitable, but only if appropriate for the individual teacher, group of students and the GCSE scheme within which they are working.

Throughout this work a four stage approach, similar to that put forward by Polya (1945) in his book *How To Solve It* has emerged. We believe that if students are to carry out significant studies, then they will need to give a full complement of time to each of these four stages

* Understanding and exploring the problem
* Devising and planning individual studies
* Implementing plans and pursuing ideas
* Reviewing and communicating findings

The first two stages are essential elements of any study if a real personal contribution is to be made and therefore cannot be rushed. Having outlined the style of approach which we believe typifies extended task work within a GCSE scheme, we now move on to look in detail at how this work can be implemented within the classroom.

Coursework Assessment in GCSE

This paper, the second in the series of SEC Working Papers concerned with assessment, outlines the SEC's views on assessment by means of coursework in the new GCSE examination.

1. Most of the statements of subject-specific National Criteria call for 'coursework' as part of the assessment. Although detailed reasons for adopting coursework and firm proposals for the forms it may take are presented for only a few subjects, it seems likely that such reasons and proposals may be more widely applicable, both within and beyond the twenty subjects for which criteria exist. Drawing therefore largely on the criteria, this paper seeks to identify first some aspects of attainment which particularly lend themselves to assessment by means of coursework, secondly some general respects in which coursework can enhance both the assessment and the curriculum, and thirdly some of the varied forms which coursework may take. The paper may thus prove useful as a set of checklists for all those developing, proposing or approving GCSE syllabuses. The paper does not consider the various issues of moderation: it will be for the Examining Groups to address themselves to these, in accordance with the requirements of the General Criteria.

2. The pressures resisting the assessment of coursework are considerable: quite apart from reluctance to innovate and understandable anxiety about the additional demands made on pupils, teachers and examiners, there is a genuine awareness that coursework assessment requires close monitoring if reliability and comparability are to be maintained at the levels associated with timed written papers. It can reasonably be argued, therefore, that if coursework is added to a paper-based scheme it should be with the intention of increasing the examination's 'validity' (the extent to which it measures the aspects of attainment which require to be measured).

The aim in each subject, then, should be one of making what is important measurable rather than of making what is measurable important; the task might well be begun by asking, first, how the subject as taught to 15 and 16 year olds differs from the subject as taught to younger children or as pursued in the adult world, and, secondly, to what extent these differences are attributable to the need to prepare for timed written papers at 16-plus. Some aspects of attainment which may not easily or adequately be tested by such papers are listed below; their order is not intended to imply priority.

(a) The ability to use and develop techniques for making and recording accurate observations, in the contexts of, for example, fieldwork or experimental work.

(b) Research skills, including the ability to organise the systematic collection and ordering of pertinent information; familiarity with and use of a wide range of sources; the ability to distinguish sources of different status in weighing evidence - for example, primary and secondary sources.

(c) Interactive skills (responding appropriately to the consequences of an earlier action); such interaction may be with people, information sources (including information technology), tools, or concrete materials.

(d) The ability to find a rôle and cooperate with others in an activity.

(e) Motor skills including manipulation of apparatus, operation of machinery, and marking out and processing of materials.

SEC Working Paper 2

(f) Skills involving a sense of timing; the ability to 'think on one's feet'.

(g) The exercise of safety awareness.

(h) The ability to design, conduct and evaluate a simple experiment or survey to test some hypothesis or illuminate some issue.

(i) The ability to make a simple theoretical model of a 'real-life' situation and to test and refine the model by examining both it and the real-life situation further.

(j) The determination and ability to sustain a chosen study from conception to realisation.

(k) Attainment in tasks which, by their nature, require time for exploration: investigational, planning and design activities where several approaches may need to be considered before a specific solution is developed; activities where several resource constraints (such as those of cost, time and skill) have to be investigated and weighed before a solution is pursued.

(l) Attainment in areas where it is desirable to allow time for reflection, for example, in articulating a thoughtful personal response to the expressive arts or to religious experience or in reaching an objective and informed view of some current social or moral issue.

(m) Skills of adaptation and improvisation in the widest sense: the ability to restructure information or modify objects to suit immediate needs; the ability periodically to review the progress of a long-term enterprise (such as a scientific experiment, a piece of planning or a craft or agricultural project) and to change tactics if necessary; the exercise of awareness of possible sources of difficulty or error.

3. The careful use of coursework assessment can enhance both the examination and the curriculum. Some possible benefits are outlined below.

(a) A coursework component can offer a fairer treatment to the hard-working pupil whose attainment never receives proper credit in formal examinations because of the anxiety they bring about.

(b) Just as some candidates are disadvantaged by examination stress, others suffer out of all proportion across a whole range of examination subjects because of difficulty in understanding and expressing themselves in written English. Comprehending spoken instructions and questions is, in many subjects, a skill worth developing and assessing in its own right; the same is true of oral response. Furthermore, discussion may sometimes give a clearer picture of a child's understanding than a written answer can.

(c) At the same time, coursework can, in all subjects, provide a useful vehicle for communication skills in the widest sense, including reading, writing and a feeling for mathematical ideas, and thus contribute to the development of English and mathematics across the curriculum.

SEC Working Paper 2

(d) Worthwhile tasks of certain types - those which are 'hard to get into' or may lead down blind alleys, or depend heavily upon insight - are likely to cause general anxiety if set in a timed written examination, anxiety which could adversely affect candidates' performance in other parts of the paper. But tasks of these types can provide an interesting challenge, worthy of credit in the assessment, if set in the classroom with time pressure reduced or removed.

(e) Continual assessment of coursework can, because of its proximity to the task, provide reinforcement or a spur and may therefore contribute to raising the quality of pupils' work. School-based assessment also allows the possibility of giving pupils credit for initiating tasks and assuming responsibility for organising their own work.

(f) In the same way, coursework can stimulate a sense of exploration and discovery.

(g) If moderation difficulties can be overcome, coursework may increase the reliability of the assessment by providing wider evidence of candidates' achievement demonstrated on different occasions in samples of work which more fully represent the range of assessable abilities.

(h) Coursework of various kinds can provide the flexibility needed for assessment across a wide ability range through pupils being presented with tasks appropriate to their individual levels of ability. Much depends here on the sensitivity and skill of the teacher in devising or selecting tasks and in guiding pupils in choosing and carrying out their own pieces of work.

(i) Coursework offers scope for developing an understanding of the part many school subjects play in the everyday world, their connections with other areas of study and their relevance to pupils' own lives. Attempts within written examinations to assess awareness of the wider implications of a subject run the risk of being unfair because they depend on knowledge 'outside the syllabus'. But if the pupil is free to seek out the facts of a wider context this kind of understanding can be fostered and assessed. Work can gain in authenticity by stemming from the pupil's first hand enquiry based, perhaps, upon the local community, with its own particular culture and economic character, or upon up-to-date information on social or technological issues. Setting a task in a familiar and real context can help a less able pupil understand 'what it is about' and allow him or her to make a good start on the work. For pupils of all abilities, the opportunity to 'cross-question' the setter of a task, to clarify the terms of reference and discuss any assumptions which may have to be made corresponds more closely to the way action is initiated in the real world and provides scope for greater sensitivity and inventiveness on the part of the pupil. Applying a subject to a real-life problem can present challenges too: assumptions will have to be made, unforeseen factors will have to be accounted for, further skills and concepts will have to be brought to bear; various facets of the subject will have to be integrated within a unified piece of work. A transition from neat topic-by-topic, subject-by-subject learning to work of this kind is never easy and carries its own dangers, yet many young people face just such a transition when they eventually leave the classroom for the wider world.

(j) Above all, the assessment of coursework can correspond much more closely to the scale of values in this wider world, where the individual is judged as much by his or her style of working and ability to cooperate with colleagues as by the eventual product.

SEC Working Paper 2

4. No attempt has been made to distinguish the types of activity which may fall under the general heading 'coursework'. A wide variety of forms is available, and some of the 'variables' are listed below, together with commentary. In each case, the form should be chosen for its fitness for purpose in assessing the required aspects of attainment.

(a) The length of a piece of work may vary (from a few minutes' activity to work pursued over several weeks).

(b) The formality of an assessed task may vary. Formal practical tests have existed in the sciences at all levels and in craft subjects for many years; in spite of their apparent objectivity, however, they have sometimes been criticised as amounting to stereotyped exercises of doubtful use in assessing true practical skill. The formal-informal dimension applies to oral work, graphical work (preparatory sketches leading to a finished product) and many other kinds of project activity.

(c) The task may be presented in written or oral form or may arise out of discussion. The task may to a greater or lesser degree be the idea of the pupil; where this is the case it is important to realise that the pupil may 'close doors' by an inappropriate choice of topic, and for the teacher to provide suitable guidance.

(d) The end product may vary in its form.

(e) The degree of peer or teacher input may vary. Where there is such input the teacher may be the only one who can judge the pupil's own contribution and must therefore keep careful records of interaction taking place.

(f) Where 'process skills' are being assessed separately from the 'product', the teacher again will have the responsibility for observing and assessing.

(g) The task may be relatively 'closed' or relatively 'open-ended'. Closed tasks - tasks which admit of only one response or a limited range of responses - can make it difficult for the teacher to assess to what extent the pupil has worked unaided. Open-ended tasks pose an assessment and moderation challenge because of the difficulty of setting down fairly objective mark guidelines.

5. The development of good coursework schemes and the provision of satisfactory methods of moderation will not be a simple matter and it is to be expected that styles of coursework will evolve as teachers and moderators gain in assessment expertise and confidence. From the outset, however, a major concern must be to ensure that coursework is integrated into the curriculum and does not become a sudden and unwelcome burden on pupils and their teachers in the few months prior to the written examinations.

SEC Working Paper 2

3 Differentiation Within Extended Tasks

Differentiation is a key feature within GCSE schemes. Differentiation is achieved by ensuring that students work on mathematical tasks appropriate to their personal level of achievement. Within GCSE schemes students must be entered for assessment at one of three levels. The three levels are referred to throughout this material as Foundation, Intermediate and Higher. At each level of assessment a restricted range of grades is available. The relationship between levels of entry and grades available is, typically, as follows.

Level of Entry	Grades available
Higher	A, B, C, D
Intermediate	C, D, E, F
Foundation	E, F, G

Careful thought needs to be given to how differentiation may be achieved within coursework, particularly in relation to extended tasks. By allowing students to work on tasks which are appropriate to their individual levels of ability, it is hoped that they will be able to demonstrate what they 'know, understand and can do'. Several models have been designed to facilitate differentiation, each having its own strengths and weaknesses. Differentiation within extended task work may be achieved through

* task selection
* support and guidance provided
* by outcome or response.

Differentiation through task selection

There is no doubt that the idea of differentiation is to allow each student to be assessed in a way which recognises and rewards positive achievement. Clearly, one possible procedure to adopt for extended tasks is the one which is used for the timed and written component of the examination, tasks set at the three levels of entry. An example which demonstrates this within the Pure Investigations category is on the page opposite. Here we set out the tasks in 'shorthand' forms, the 'cluster books' contain more detailed statements.

Here we see three tasks, each presented in a similar way; offering just a starting point, but requiring different levels of mathematics to obtain some form of generalisation, and with the Foundation level task relating much more closely to real life.

Which tasks match particular levels, makes an interesting discussion. Often an overlap exists, and a task may be considered as suitable for two levels of study rather than just one. You might like to think how appropriate, if at all, these three starting points are for your own students.

Questions may arise such as

* If a Foundation Level student tackled *Consecutive Sums,* what would be expected?
* Could a good Higher Level student demonstrate grade A achievement through studying *Slabbing?*
* What if a student is eventually entered at a level which is different from that of the extended task problems investigated by her?

Clearly, there are many issues to consider when using this model of differentiation.

Higher Level

CONSECUTIVE SUMS

15 = 4 + 5 + 6

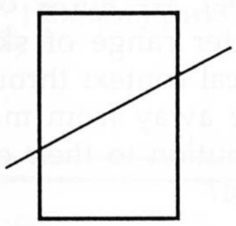
49 = 24 + 25

⋮

Investigate The Problem

Intermediate Level

CHOPPING THE RECTANGLE

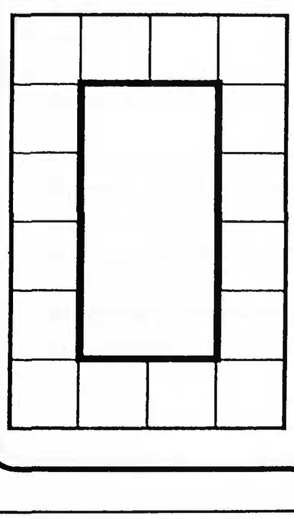


One chop

Two pieces

Investigate The Problem

Foundation Level



SLABBING

Pool length = 4m

Pool width = 2m

Number of slabs = 16

Investigate The Problem

Differentiation through support and guidance provided

Using this approach to differentiation, the task undertaken may be the same for students studying at all three levels. However, differentiation is achieved through the amount and type of support and guidance given to the student, when establishing the framework within which the task will be completed. Again, this is best demonstrated through an example. In this case we use a task which fits into the Statistics category.

In some ways this method of differentiation is comparable to the previous one, in the sense that in some respects students are tackling different tasks. However, although the level of study is clearly different, there is considerable common ground since all students investigate the same problem. The major difference is in the amount of support and guidance provided, according to the level of entry.

The first model through which differentiation could be achieved assumes that different levels of mathematics are available to students. This second model assumes different levels of planning skills and personal autonomy within students. In our view this assumption raises major questions. If extended task work is to form an integral part of the learning process, then all students need to be provided with the opportunity to plan their own course of action and to seek such advice and guidance as they require. By adopting this model, we allow only Higher Level students to acquire and demonstrate the broader range of skills which we are trying to develop and assess within a mathematical context through this type of work? By giving guidance of this type, we take away from many students the opportunity to make a significant personal contribution to their own study, and hence to some extent invalidate the whole experience?

Higher Level

SM TIPS

SM Tips is one of the best selling teas and tea bags on the market.

Cards are given away in each packet.

Look at the chances of getting swaps.

Intermediate Level

SM TIPS

SM Tips is one of the best selling teas and tea bags on the market. For many years they have given away cards for people to collect. Imagine that you get one card in each pack and that there are 30 in the full set.

How many packs on average do you need to buy before you get a swap?

What if there were more or less than 30 in the set?

How many packs would you expect to buy to collect the whole set?

Foundation Level

SM TIPS

SM Tips is one of the best selling teas and tea bags on the market.

For many years they have given away cards for people to collect.

You get one card free when you buy their smallest pack of tea. There are 30 cards in the full set.

Make a set of 30 cards to experiment with.

Use your set to decide how many cards you usually have to buy before you get a swap. Repeat your experiment 100 times using the computer software your teacher will tell you about. Record your results in a table. Draw a bar chart of your results and write down a sentence which describes what you have noticed.

Invent an experiment with your own set of cards to see how many packs are needed before you get a whole set of SM Tips cards.

Differentiation by outcome or response

Differentiation by outcome or response involves differentiation which is achieved through students' responses to the tasks set. Instead of providing tasks aimed at particular levels of ability, the achievement of students is assessed through their different levels of response to common tasks. This is often referred to as providing a 'neutral' stimulus and it is commonly advocated in English and the Humanities. An unpublished SEC feasibility study on 'Differentiation by Outcome in History' (1985) suggests

'that differentiation by outcome requires a greater awareness and understanding of the qualities in a question that make it accessible to the full ability range'

We agree with the argument that differentiation can only really be achieved by this method, since any other system could actually prevent candidates from showing what they know, understand and can do, simply because the teacher's perception and expectation has consigned the question to a particular level of entry. In conclusion, the History Report suggests that differentiation by outcome 'allows maximum flexibility in assessing candidate response and may therefore be the only real means of differentiating.'

The proposals for the National Curriculum also suggest that 'extended tasks should be as open in character as is feasible, so as to allow pupils to have some control of the directions they take and the methods they use.'

Coursework is an important situation in which differentiation can be achieved, since it provides opportunity to demonstrate and reward a much wider range of student behaviour than do written examination papers. Because we have sought maximum flexibility in assessing students' responses across a wide range of student behaviour, we have selected the *differentiation by outcome* model when writing our extended tasks for student coursework. The main tasks are designed to be as open as possible, but often a closed starting point is provided to allow easy access to the task. This establishes the model of a *low entry - high exit task* which is suitable for all students.

4 Classroom Practice And The Teacher's Role

The introduction of GCSE has many implications for classroom practice. The section of the Cockcroft Report which relates to classroom practice suggests that it is neither desirable nor possible to indicate a definitive style for the teaching of mathematics. However, it does identify six elements which need to be present in successful mathematics teaching to students of all ages. The Report advocates the use of a whole range of teaching styles; exposition, discussion, practical work, practice, problem solving and investigational work.

In many mathematics classrooms exposition and practice are well established, while the benefits which can accrue from discussion, practical work, problem solving and investigational work are comparatively unexplored. Perhaps because of an over-emphasis on exposition and practice, many adults and students dislike, and even fear, mathematics. Many adults and students are unable, or unwilling, to use and apply their mathematical skills in everyday life. At least some of these difficulties may have arisen because an over-large syllabus of abstract mathematical ideas, deemed suitable for bright students, has been diluted and is dispensed to students across the whole ability range, through the medium of exposition and practice.

The Cockcroft Report proposed a 'foundation list' of topics, to be covered by all students, a 'bottom-upwards' approach to syllabus construction, together with an examination system which rewards positive achievement. Ensuring that all students acquire sufficient confidence to make effective use of whatever mathematical skills and understanding they possess has become an educational priority.

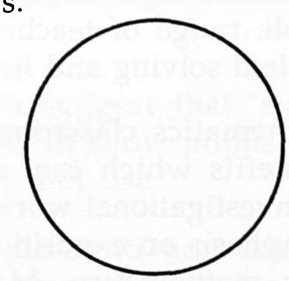
In the list of aims for all those following GCSE mathematics courses, we find reference to oral, written and practical skills. In the National Criteria for Mathematics it is suggested that GCSE mathematics courses should enable students 'to have experience of mathematics as a means of solving practical problems and to develop confidence in their use of mathematics' and that 'the power of mathematics in tackling practical problems from everyday life and the world of work justifies its position in the school curriculum.'

Traditional teaching methods rely heavily upon exposition by the teacher followed by consolidation and practice by students. Knowledge is transmitted from 'knower' to 'learners', and then each 'learner' performs tasks to consolidate or practice what has been learned. Within this model of the learning process, each student works as an individual member of the class, often on fairly abstract tasks and in a competitive situation. Recent moves towards more active forms of learning place emphasis on practical work, investigational approaches and problem solving. Such methods of working create opportunities for pupils to work cooperatively as members of a group and to exchange views and ideas as they discuss details of their collective tasks. The *Proposals for the National Curriculum* suggest: 'Mathematics is not only taught because it is useful. It should also be a source of delight and wonder, offering pupils intellectual excitement.'

The following example may provide a helpful illustration. Let us consider Circle Theorems.

The traditional approach is Theorem -> Copy -> Theorem -> Copy .... Theorem -> Copy -> Examples (Do lots of similar examples)

An alternative investigative approach is as follows.



By drawing some lines on or around this circle, find out what you can about the circle and its related lines. These lines should cut or touch the circle.

Discussion

Many teachers, mistakenly believe when they talk to students, and students respond to their questions, that discussion has taken place. However, Shuard (1980) suggests that what is meant by discussion 'is an extended joint exploration by the pupils, under the chairmanship and guidance of the teacher, but in which the teacher does not force the direction and pace of the oral exchange, and in which ideas are explored and argued out with mutual respect for each other's point of view.'

It may be helpful to consider in more detail why discussion, practical work, problem solving and investigational work are currently being advocated. The traditional belief that mathematics is a private activity, performed by an individual, using only pencil and paper is being challenged and we are now being encouraged to participate in discussion. Recent research suggests that discussion enables students to come to terms with their problems, to understand what is intended, to comprehend the relevance of available data and to decide upon the strategies which could be used to produce solutions. Discussion also encourages students to reflect upon the outcomes of their enquiries and experiences. It is unwise to imagine that because our students have lived through certain direct experiences of the world around them, that they will necessarily have learned from what has taken place. Students need to reflect upon what has happened, to examine it from different perspectives and to link it with previous experience. Talking with other students and with their teachers encourages reflection and reorganisation of students' thinking to create new insights and knowledge. The quality of students' mathematical thinking, as well as their ability to express themselves, can be considerably enhanced through discussion.

Practical work

There is a tendency to minimise the importance of practical work at the secondary stage. Nevertheless, we are firmly of the view that all students benefit from appropriate practical work, whatever their age or ability. In *Practical Work in GCSE Mathematics* (1989), a Shell Centre research report, it is stated

'Mathematics springs from our experience of the real world. Its twin aims are to gain insight into this world, and to explore the patterns and relationships which arise. Reasoning and diagrams and symbol systems are tools in this process. Thus the readings of a car speedometer and of the distance on a road sign are combined by calculation to give an estimate of a time of arrival; considerations of combining movements tell us in how many ways we can turn a mattress; and the concept of inertial motion as being with constant speed in a straight line helps us to understand why we feel thrown outward when a bus turns a corner.

In school mathematics teaching there have sometimes been tendencies to move too quickly into abstractions and symbols without allowing sufficient time to explore and understand the real situations from which they arise. The attempt to avoid this ever-present pedagogical hazard has been at the root of many important developments in teaching, including the recognition of the place of discovery, the importance of discussion, and the need for adequate concrete experience. These are some of the considerations which lie behind our current concern with practical aspects of the mathematics curriculum, which has found expression in the Cockcroft Committee's list of six types of activity which should be a normal part of mathematics lessons - one of these being "appropriate practical work" - and, most recently, in the GCSE National Criteria for Mathematics, which now require the assessment of the "ability to carry out practical and investigational work, and undertake extended pieces of work."

In addition to this direct demand for a practical assessment, the aims and assessment objectives of the National Criteria show a practical emphasis at many other points, for example:

* to apply mathematics in everyday situations and to develop an understanding of the part which mathematics plays in the world around us.
* to develop a feel for number and understand the significance of results obtained.
* to apply mathematics in other subjects, particularly science and technology.
* to recognise when and how a situation may be represented mathematically.
* to understand systems of measurement in everyday use.

Practical in method and purpose

We have found it necessary - and helpful - to distinguish different meanings of the notion of practical work. A dictionary definition of 'practical' gives

> "relating to, concerned with, well adapted to, or inclining to look to, actual practice ... utility ... actually engaged in doing something: efficient in action: workable."

but this still does not separate the two aspects.

We distinguish

* *practical in method:* hand-on mathematical activity: constructing, manipulating and using models and apparatus.

and

* *practical in purpose:* the mathematics that arises in life and work.

We also distinguish two reasons for its inclusion in the curriculum:

* as a curriculum objective in itself - to enable pupils to operate in practical ways and in practical settings,
* as a means of assisting the learning process, as caught in the phrases "I know it in my fingers" and "I do and I understand", and the use of concrete material to assist the learning of number.

Practical work is of three main types

* it can enable students to understand mathematical concepts
* it is sometimes carried out with a particular purpose in mind. For example, it we need to know the size of a room which is to be carpeted, we need to measure the room.

* it is sometimes the most suitable approach to completing a certain task for certain students. A practical in method approach to problem solving is often useful rather than using pencil and paper.'

The *Proposals for the National Curriculum* emphasise the role of discussion in practical work. 'Communication skills are important generally in mathematics, but especially so in practical work. Pupils need to understand the task; it must mean something to them. Discussion of mathematical ideas greatly aids mathematical understanding and gives pupils confidence. Group discussion is an undervalued activity, yet, when properly developed, it can enhance the mathematical performance of pupils in schools.'

Investigational approach

There is currently some debate about the difference between problem solving and an investigational approach. Initially, it may be helpful to think of problem solving as being a convergent activity, students finding a solution to a well defined problem, whereas an investigational approach can be applied to more open-ended activities, and hence is a more divergent activity. There is, however, a difference between using investigations and adopting an investigational approach. The use of investigations is a first step for many, and is a subset of the adoption of investigational approaches, which is a longer term goal. In an investigational approach students might be encouraged to think of alternative interpretations of the task; different strategies; to vary the conditions imposed, constraints; to consider what if ....?

Discussion, practical work, problem solving and investigational approaches, are all time consuming activities. However, these can be directly related to items of content. A practical, problem solving, investigational approach to the acquisition of mathematical content, using discussion techniques, has much to recommend it.

Problem solving and investigational work are not things that we become able to do because we know about them. Like many other aspects of behaviour, we need to practice how to do them. Both teachers and students will benefit greatly from participating in the relevant activities suggested in IMPACT. However, changes in behaviour are not accomplished overnight. Both teachers and students will need time to adjust to the new demands, and to develop new patterns of behaviour. Whenever we feel threatened and insecure, we tend to revert to established patterns of behaviour which have proved to be, at least partially, successful in the past. It is only when the rewards from the new behaviour patterns become well established, that we can consider long term change to have been effected.

Textbooks and worksheets often present mathematics in a 'closed' predigested form. Sometimes, the tasks set are not problems, they are merely lists of instructions to be followed. An introductory example is followed by a list of stereotyped questions and there is little opportunity for independent thinking or enquiry. In real life we often have to solve open problems. We may have too much or too little information. A method of solution may be a suitably simple mathematical model of the situation under consideration.

In the examples provided on pages 35-36, taken from the Shell Centre Blue Box, *Problems With Patterns and Numbers,* we have collected together a few textbook exercises alongside some more open imaginative tasks which cover similar content. Perhaps you might like to consider the following questions

* What will students really learn from completing each task?

* Are the tasks accessible to students? At what age or ability level?

* Which tasks would students enjoy? Why?

* Try to rewrite a 'closed' task in a more 'open' form

* Are the tasks likely to develop the following communication skills suggested in the *National Curriculum* proposals.

'Students need to be able to

present and explain results to other pupils, teachers and other adults;

make a report;

discuss the implications and accuracy of the conclusions reached;

relate the result to the world about them as appropriate;

discuss other possible interpretations of the conclusions.

The personal qualities which pupils need to develop include:

motivation and preparedness to tackle the unfamiliar and unknown - willingness to 'have a go';

flexibility and creative thinking in overcoming difficulties and developing new approaches;

perseverance, reliability and accuracy in working through a sequence of stages in an extended task;

willingness to check, monitor and control their own work;

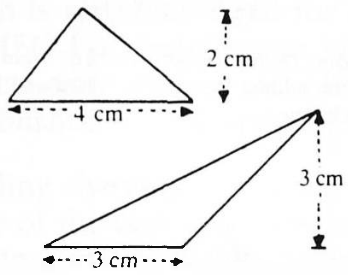
independence of thought and action as well as the ability to cooperate within a group.'

A "TEXTBOOK" VERSION	A MORE "OPEN" VERSION

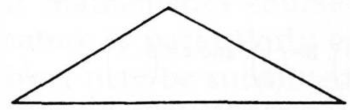
Area

A "Textbook" Version

Find the areas of these triangles:



A More "Open" Version



Find the area of this triangle, taking any measurements you consider necessary.

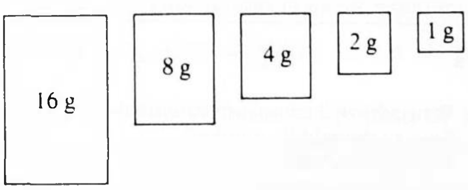
How accurate is your answer?

How can you check your answer, without repeating the same calculation?

An Introduction to Binary Numbers

A "Textbook" Version

Here is a set of five weights:



Copy and complete the table below, which shows how these weights may be used to make up every weight from 1g to 31g.

16g	8g	4g	2g	1g	Grammes
				1	1
			1	0	2
			1	1	3
		1	0	0	4

A More "Open" Version

Here is a set of five numbers:

{1, 3, 6, 11, 13}

18 can be made by adding some of them together:

$$18 = 1 + 6 + 11$$

Can you make 20? 30? 26?

(Each number in the set may only be used once).

Which other numbers can be made?

Which other numbers cannot be made?

Which numbers can be made in more than one way?

Invent a different set of numbers which can produce every number up to the highest in only one way.

Matrices and Transformations

A "Textbook" Version

Draw diagrams to show the effects on the unit square, defined by (0,0), (1,0), (1,1), (0,1), of the transformations whose matrices are

1) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 2) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 3) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

4) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ 5) $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ 6) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

A More "Open" Version

Investigate the transformations produced by the matrices in the following set:

$$\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b=c+d \right\}$$

(Initially, you may like to limit the values that a, b, c, d can take to 1, 0 or −1)

Vectors

A "TEXTBOOK" VERSION

If $\mathbf{a}=\begin{pmatrix}3\\1\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}1\\2\end{pmatrix}$, and $\mathbf{c}=\begin{pmatrix}5\\-1\end{pmatrix}$

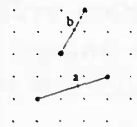
then write down answers to the following, and illustrate your answers with diagrams.

1) $\mathbf{a}+\mathbf{b}$ 2) $\mathbf{a}+2\mathbf{c}$ 3) $2(\mathbf{a}+\mathbf{b})$
4) $\mathbf{a}-\mathbf{b}$ 5) $4\mathbf{c}-\mathbf{b}$ 6) $3(\mathbf{a}-2\mathbf{b})$

Show diagrammatically, that

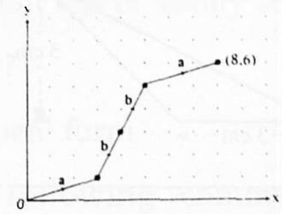
$$\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$$
$$2(\mathbf{a}+\mathbf{b})=2\mathbf{a}+2\mathbf{b}$$
$$\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$$

A MORE "OPEN" VERSION



This diagram shows two vectors, $\mathbf{a}=\begin{pmatrix}3\\1\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}1\\2\end{pmatrix}$

The point (8, 6) can be "reached" from the origin by adding the two vectors **a** and **b** as shown below:



Which other points can be reached from the origin?
In how many different ways can each of these points be reached?
Suppose we allow subtraction? . . .

Algebra

A "TEXTBOOK" VERSION

Expand the following:

1) $x(x+2)$
2) $(x+1)^2$
3) $(x+1)(x-1)$
4) $(x+2)^2-x(x+4)$
5) $(a+b)(a-b)$
6) $(p+q)^2-(p+2q)p$

A MORE "OPEN" VERSION

Write down 3 consecutive numbers:
Square the middle number:
Multiply the other two together:
What do you notice?

e.g.: 81, 82, 83
$82\times82=$
$81\times83=$

Try other groups of 3 consecutive numbers.
What happens if you use decimals?

e.g.: 79.6, 80.6, 81.6

What happens if the numbers are not consecutive but go up in

twos? e.g.: 621, 623, 625
or threes? e.g.: 381, 384, 387

. . . .

Generalise and prove your results.

Graphs and Equations

A "TEXTBOOK" VERSION

Draw the graphs of the following equations:

1) $y=4$ 2) $x=7$
3) $x+y=5$ 4) $y=3x$
5) $y=2x+3$ 6) $y=2x-7$
7) $y=-2x+3$ 8) $y=x/2$
9) $2x+3y=12$

A MORE "OPEN" VERSION

Write down a simple linear equation (e.g. $3x+2y=12$) and draw its graph.

Give only the graph to your neighbour. See if she can reconstruct the original equation.

Now make up harder examples . . .

Cross-curricular activities

One of the aims of GCSE mathematics courses, is to develop an ability to apply mathematics in other subject areas. In their mathematics coursework syllabus, MEG state that, 'Work of a cross-curricular nature is particularly encouraged and work which is also submitted for another subject may be submitted in accordance with the MEG General Course Work Regulations. The marks submitted for the mathematical examination must however be based on assessment according to the scheme published in this syllabus.'

While trialling the extended tasks which are included in these materials, we have been aware of the fact that some of the tasks could have been pursued for a longer period of time but, possibly, within other subject areas. When pursuing *Connect Four* and the *Play Fair* tasks, some students spent a considerable amount of time actually producing different games. Could these tasks also have been assessed within a different subject area?

Sorting Shapes, Anyone for Tennis, Why Are We Waiting? Findings the News, Do You Need A Water Meter?, and *The Celebration*, which are included in this pack of materials, also appear to us to offer cross-curricular potential.

We see it as entirely desirable that a topic or problem should be pursued across subject boundaries and that the student's outcome should be submitted for assessment in more than one subject area. Transferability of skills across subject boundaries is much more likely to happen if teachers in different departments are willing and able to cooperate on projects which cross traditional subject boundaries.

Modes of operation within the classroom

Examining groups offer varying types or amounts of guidance and control in relation to coursework tasks. Within the guidelines provided by their examining groups, teachers are free to decide how they should operate inside their own classrooms. Different teachers interpret their roles, in relation to supporting coursework tasks, in very different ways. Some teachers allow their students great freedom when selecting and interpreting extended tasks.

One possible course of action is to allow students total freedom. 'You have three weeks and I want you to complete a piece of coursework.'

At the opposite end of the spectrum, the teacher could adopt a completely restrictive approach. Here we refer rather bluntly to the 'Death by a thousand worksheets' method.

Sheet 1	-	Read
Sheet 2	-	Read and fill in gaps
Sheet 3	-	....
Sheet 999	-	Copy and complete
Sheet 1000	-	Do your own.

Clearly the first of these two extremes is not a realistic or acceptable approach in most classrooms. The opposite approach is unacceptable when considering any GCSE coursework scheme. However, there is vast scope for interpretation and personal style within the continuum between the two extremes outlined above.

The flow chart on page 40 is typical of the way in which some teachers approach this type of work. This indicates that at certain crucial stages the student should consult the teacher. In our view, the teacher should be closely involved during all stages of the student's work. Some guidelines for teachers working on school-based tasks are on page 41.

The tasks included in this pack, together with the relevant teacher's notes, have been written and structured in a way which provides what we perceive to be an appropriate balance between freedom and direction. We have aimed to produce *rich* classroom activities, which enable all students to become involved in enjoyable and challenging work. The attributes of *rich* classroom activities are summarised in *Better Mathematics,* (HMSO, 1987).

What makes a rich classroom activity?

* It must be accessible to everyone at the start.
* It needs to allow further challenges and be extendible.
* It should invite children to make decisions.
* It should involve children in speculating, hypothesis making and testing, proving or explaining, reflecting, interpreting.
* It should not restrict pupils from searching in other directions.
* It should promote discussion and communication.
* It should encourage originality or intervention.
* It should encourage 'what if' and 'what if not' questions.
* It should have an element of suprise.
* It should be enjoyable.

Marking and assessment

We believe it is important that students should be provided with details of the criteria which are used for the assessment of their work. It is beneficial for all concerned to discuss teacher assessment of student work alongside these criteria. In some schools we have visited, students are encouraged to attempt to assess their own and other students' work. We have been impressed by the sense of responsibility with which students perform this task and by the beneficial effects this produces on their work. This is discussed in greater detail in Chapter 6 and Chapter 7.

The roles of the teacher

It is extremely useful to observe other teachers in action and to compare their styles with your own, as suggested in Chapter 4 of IMPACT.

One way of analysing the varying roles of the teacher, suggested in the Shell Centre 'Blue Box', *Problems with Patterns and Numbers* is as follows

Explainer

Manager

Task setter

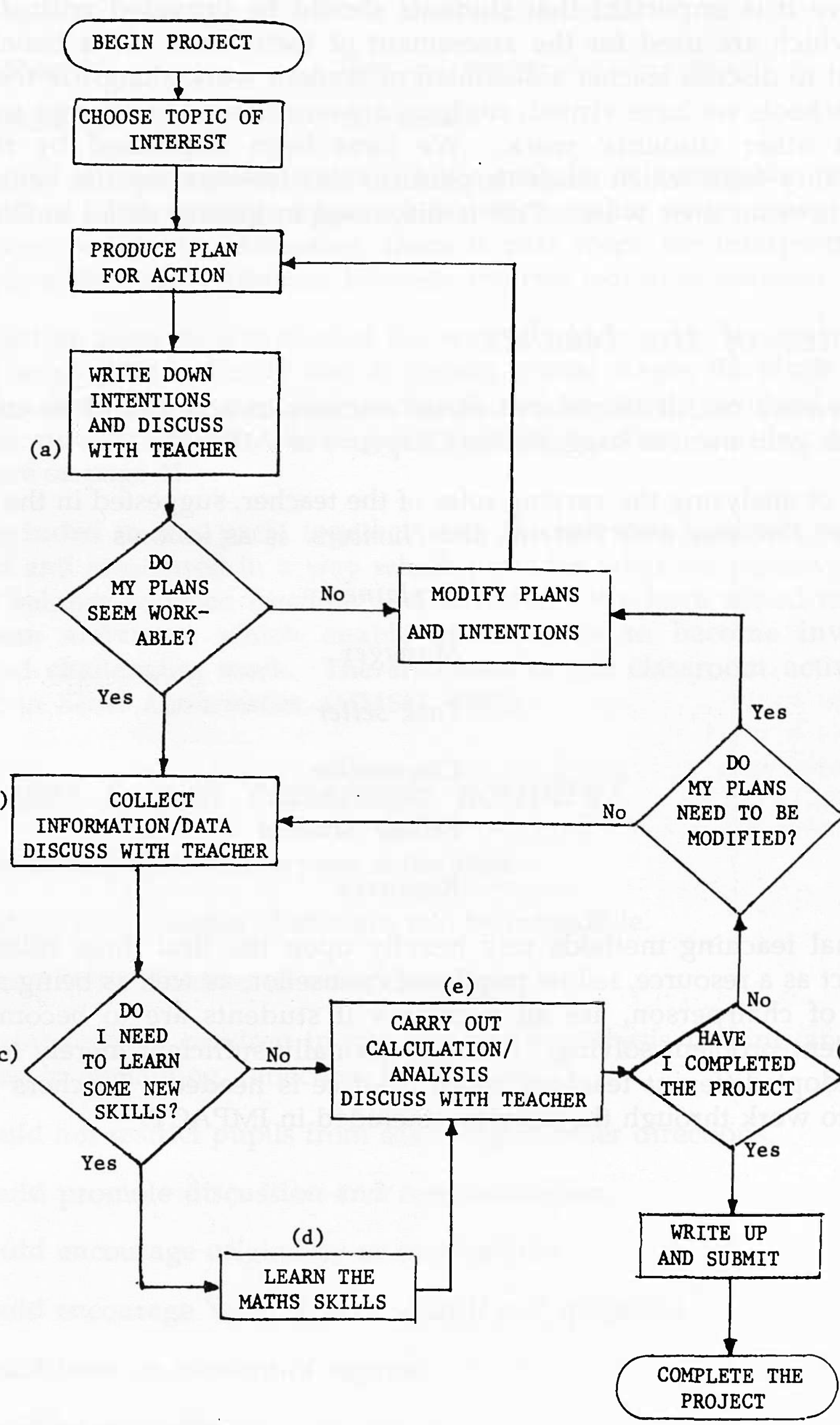
Counsellor

Fellow student

Resource.

Traditional teaching methods rely heavily upon the first three roles. Learning how to act as a resource, fellow pupil and counsellor, as well as being able to adopt the role of chairperson, are all necessary if students are to become skilled in independent problem solving. It is not normally sufficient merely to wish to be able to adopt different teaching roles; practice is needed. Teachers are strongly advised to work through the activities included in IMPACT.

A Flow diagram to guide pupils through their projects



From *Pupils' Projects. Their Use in Secondary School* (Mathematical Association 1980)

GUIDELINES FOR TEACHERS WORKING ON SCHOOL-BASED TASKS

The two main types of school-based tasks are Projects and Investigations. The former would normally require a relatively structured approach, with the pupil planning what he or she is going to do in some detail beforehand; whereas the latter is venturing into unknown territory, so that they would usually plunge in first and plan out the presentation of their conclusions considerably later.

Some comments relevant to both types of work are:

1. The tasks need not be lengthy or difficult. The important point is that the pupils achieve something positive in mathematical terms, and that they present their results intelligently.
2. Relate your expectations on the depth of the work to your knowledge of the pupils' ability, though be prepared for some surprises.
3. Decide whether the whole class should work on the same task or on a series of different ones. In either case, an initial discussion with the teacher is beneficial and, indeed, often essential. Plan for this.
4. Try working in small groups on some tasks. When you do this, manipulate the groups so that there are not some pupils doing all the work and others simply "hanging on".
5. Constantly watch for pupils running dry on ideas, leading to boredom and mischief. Step in and prompt them with new ideas, new lines to follow. Do not let them abandon an incomplete task unless it is clearly in some way inappropriate.
6. On the other hand, watch for conscientious pupils spending too much time on the task; both in class and as homework. Remember that they may well have project work in several other subjects as well, and they could become overburdened.
7. Notwithstanding 5 and 6, stand back as much as possible. Part of the value of this type of work is for the pupils to make their own mistakes and pursue ideas up blind alleys; to arrive at a conclusion by a roundabout method.
8. Keep the aims of this type of work always in mind. Some of these are:

(a) to develop mathematical thinking, including logical reasoning;
(b) to encourage discussion of mathematics;
(c) to develop initiative in pursuing their own line;
(d) to use the mathematical skills they have learnt;
(e) to identify and collect necessary data and information;
(f) to formulate and interpret problems;
(g) to write about the mathematics they have produced.

From IGCSE Training Manual Mathematics

5 *Working Together*

During the coming years, teachers will need to continue to review their approach to mathematics teaching. Teacher development is a complex process. In *Better Mathematics* (HMSO, 1987) it is suggested that

> 'Teachers are education's best resource. All teachers can become more confident and effective in the classroom. No effective far reaching curriculum development can taken place without teacher development.'

If teachers are to reach the stage where they feel confident enough to implement change and to learn from their 'failures', they need support. Working in isolation tends to inhibit sustained change. Changes in approach are often best stimulated when groups of teachers come together to share ideas, compare teaching strategies and engage in doing some mathematics. When colleagues discuss and work together, common problems appear lighter, feelings of isolation disappear and insights into how students learn mathematics are acquired. In the final analysis, each teacher must evolve her own individual strategies for coping within each of her own classrooms. However, discussion stimulates the organisation, re-organisation and reflection of teachers' thoughts and ideas about what they could do, or have done, in their classrooms.

Within schools, mathematics departments will need to come together to consider how they can implement change. A checklist for the Head of Department is included on page 46.

It is important to remember that teacher enthusiasm is a crucial factor in any curriculum development. Teacher enthusiasm for, and personal involvement in pursuing mathematical tasks of an open nature will greatly enhance the value of the mathematical experiences of students as they complete their extended coursework tasks. Some teachers may not yet have reached the stage where they find themselves able to agree with views expressed in *Better Mathematics*. 'Mathematics is effectively learned only by experimenting, questioning, reflecting, discovering, inventing and discussing.'

'The more solutions and strategies pupils see and discuss, the more likely they are to develop a real appreciation of mathematics at their own level.'

Again, teachers are strongly recommended to carry out the activities suggested in IMPACT. These activities are designed to encourage and support groups of teachers who wish to work together in broadening their teaching style.

We believe it is important that teachers should make efforts to explain to parents their new approaches to mathematics. Educational change is much more easily effected when parents are aware of, involved in and support new curriculum developments.

Many strategies for involving parents are currently used including

* Inviting parents into school to work on mathematical tasks
* Family mathematics workshops
* Mathematics exhibitions which incorporate workshop activities
* Local radio and television 'mathematics puzzles' programmes.

While it is possible to achieve some desirable effects through 'one-off' sessions, regular involvement and activity is much more likely to produce long term desirable effects.

During a research study which was designed to observe collaboration between the members of mathematics departments as they implemented extended pieces of work in their mathematics syllabuses, Mike Hawkins produced a checklist of features that facilitate collaboration which is included on pages 44-45, together with the rationale for their selection which is included in the final column.

Professional development *across schools* is encouraged within some LEAs. Groups of schools sometimes form consortia in order to exchange ideas, problems and solutions. When joint activities are well organised and meetings do not become over-involved in administrative details, they can be extremely profitable. The opportunity to visit other classrooms in other schools can be extremely useful.

CHECKLIST OF FEATURES THAT FACILITATE COLLABORATION

GLOBAL FEATURES	*SPECIFIC FEATURES*	*RATIONALE*
The geography of the school.	A subject department staff room is available.	To enable the members of the department to assemble in their leisure time and establish some rapport between them.
	The classrooms for teaching a subject are all together.	To facilitate informal discussions among the members of the department and a general awareness of the activities of others.
	All the school buildings are on the same site.	To enhance the cohesion between the members of the department.
The management of departmental meetings.	An agenda is distributed prior to meetings.	To enable the members of the department to prepare for the business and to participate more actively.
	A record of meetings is distributed to staff.	To serve as a reminder and a record and encourage all the members of the department to respondto them.
	Staff discussion to reach consensus is encouraged.	To ensure that all the members of the department have some stake in the decisions.
The arrangements of departmental meetings.	There are sufficient meetings to discuss issues.	To provide opportunities for all the members of the department to interact with each other.
	All the members of the department are able to attend meetings.	To enable all the members of the department to participate.
	Administrative issues do not dominate meetings business.	To enable specific issues of general interest and concern to be discussed.
	Time is provided in the school timetable for meetings.	To enable all the members of the department to meet and discuss specific issues during the school day.

GLOBAL FEATURES	*SPECIFIC FEATURES*	*RATIONALE*
The organisation of the department.	A plan of implementation is displayed.	To establish dead-lines and ensure that all the members of the department are adhering to the same time scale.
	The subject department is focusing on only one innovation.	To maximise attention on a single change rather than introducing other changes simultaneously which may be mutually conflicting.
	Members of the department are free of additional administrative responsibilities.	To enable them to concentrate their energy and attention on improving their teaching.
	The responsibility for implementing the innovation is entrusted to one individual.	To administer its progress and obtain the trust and confidence of colleagues.
The arrangements of in-service training meetings.	School-based INSET is purposeful and well organised.	To maximise their benefits encouraging participation from all the members of the department.
	Members of the department are encouraged to attend out of school INSET.	To provide them with additional skills the information for dissemination back to their colleagues.
	Tangible benefits result from school-based INSET.	To instil a sense of achievement, justifying their participation.
The composition of the members of the department.	There are both experienced and inexperienced teachers in department.	To provide a range of perspectives for their mutual benefit.
	There are both males and females in the department.	To enhance the potential for discussion among the group.
	Most of the members of the department only teach one subject.	To maximise their energy and focus their attention.
The attitudes of the members of the department.	Most of the members of the department approve of the innovation.	To encourage them to pursue its implementation with enthusiasm.
	Most of the members of the department consider the innovation of benefit to their students.	To establish a positive attitude towards its eventual usefulness.
	Most of the members of the department like teaching at the school.	To maintain the stability of the membership of the department.
	Most of the members of the department like departmental meetings.	To enhance their participation and the prospect of deriving some benefit from them.
The involvement of the department in the preparation of materials.	All the staff are involved with another colleague in preparing materials.	To provide a focus for their interaction.

CHECKLIST FOR THE HEAD OF THE MATHEMATICS DEPARTMENT

1. Incorporate problem solving and investigational work as an integral part of the syllabus for all years in the school.
2. Draw up a list of IGCSE issues suitable for departmental discussion.
3. Review all current record keeping methods within the department, and adapt them as necessary to the new demands.
4. Ensure that there are regular opportunities for feedback and evaluation within the department.
5. Introduce aural testing into the early years in the school if it is not done already.
6. Reconsider current teaching methods.
7. Create a teaching structure which delays as long as possible the level of entry choice for IGCSE candidates.
8. Form sub-groups in the department to investigate in detail the curriculum implications of IGCSE Mathematics.
9. Devise procedures for internal moderation of course work assessments.
10. Involve any non-specialists in the department in discussions on all new methods and procedures.

From *IGCSE Training Manual Mathematics*

6 *The Support Material*

The need to provide support material for classroom teachers was recognised immediately following the announcement that the GCSE was to be introduced. It was realised that there existed a need to provide material designed to support both the general change in classroom methodology and the introduction of school-based assessment. The material, of which this teacher's guide is part, looks particularly at the problems relating to extended tasks within the school-based assessment component of any GCSE scheme. However, it is not possible to look at this aspect of classroom work and learning without also considering the general styles of learning which are a necessary aspect of all GCSE work.

This material is the product of a two year project funded by the Midland Examining Group and carried out at the Shell Centre for Mathematical Education which is based at the University of Nottingham. The project was funded for two calendar years, 1987-1988, and thus worked within the first three academic years of the life of GCSE. The teaching material has deliberately been developed to support any GCSE scheme, or indeed to be used in its own right.

The material comprises three parts

* The Teacher's Guide
* A Departmental Development Programme (IMPACT)
* The Classroom Material.

This teacher's guide attempts to look at the *What, How* and *Why?* of GCSE coursework and extended tasks. This is achieved by considering several of the major issues that have arisen during the life of the project. This guide also includes a reference section concerning material which already exists in many schools and can be used for extended task work.

The departmental development programme is entitled IMPACT, *Improving Mathematical Practice And Classroom Teaching*. This offers easy to use do-it-yourself in-service experiences for any group of teachers who are working together and wish to broaden the range of teaching strategies they use in their classrooms. The programme looks at a range of teaching issues and suggests that each should be considered over a period of, say, a fortnight. Each issue is supported by a teaching activity to be tried out in the classroom, together with a few brief notes for teachers, to provoke some discussion both prior to and following the teaching activity. The programme attempts to take account of the workload and everyday demands which teachers face, and it has been made easy and light to work with. Issues considered within the programme include; handling discussion, group work, practical activities and student autonomy, together with looking at general change, assessment and real problem solving in the classroom. The whole programme assumes that change is a slow, gradual, but worthwhile, process and hence it spans at least a couple of school terms.

The third, and final, aspect of the material forms the major part of the project's work and output. This is classroom material. This takes the form of eight 'cluster' books each offering

* A lead task supported by detailed teacher's notes
* A set of students' responses to the lead task with moderator's comments and GCSE grades
* A case study of the lead task in action
* A set of six alternative tasks, each supported by brief teacher's notes.

The eight cluster books fall into four pairs, one for each of the following general categories

Applications	(Books A1 and A2)
Practical Geometry	(Books G1 and G2)
Pure Investigations	(Books I1 and I2)
Statistics and Probability	(Books S1 and S2)

The material is available in two parts

Part One The Teacher's Guide

IMPACT

I1 - Looking Deeper

I2 - Making The Most Of It

S1 - Take a Chance

S2 - Finding Out

Part Two G1 - Pack It In

G2 - Construct It Right

A1 - Plan It

A2 - Where There's Life, There's Maths

The lead tasks within each cluster have been developed and trialled throughout the life of the project in a large number of schools and classrooms. The teacher's notes, student's notes and case studies have been written in the light of these trials. The students' responses have been gathered from these trials and demonstrate a wide range of positive achievement within the task. The comments and grades relating to the work have been supplied by the Chief Coursework Moderator of the MEG and account has been taken of all GCSE assessment schemes. Naturally, a grade awarded to a piece of work ought to be common across all schemes. The responses within each set are arranged in reverse order and it is suggested that these are viewed and discussed, alongside the criteria listed in Chapter 7, before the moderator's comments are read.

The alternative tasks have been chosen so that a set of similar tasks appears within each cluster book. Not all of these tasks have been trialled directly through the work of the project. Some alternative tasks have been suggested by members of the project team or teachers from the related working groups or trial schools, as being suitable similar tasks and ones which they have used. The teacher's notes for the alternative tasks are brief and need to be read in conjunction for those supplied with the lead task and in addition to the general teacher's guide.

The whole philosophy behind the style of this material is to help teachers and students as they cope with this type of work, irrespective of their previous experience, and in particular with

Getting started, Keeping going and Finishing off.

Throughout the life of the project, a common model of support seems to have unfolded and this links closely with the stages of problem solving put forward by Polya over forty years ago.

We have chosen the following descriptions of our four stages

* *Understanding and exploring the problem*
* *Devising and planning individual studies*
* *Implementing plans and pursuing ideas*
* *Reviewing and communicating findings.*

The teacher's notes for each lead task outline one possible approach students could follow as they investigate the problem in the classroom. In the teacher's notes we use Sherlock Holmes logos to signal the beginning of each new stage. Since we also wish to emphasise the importance of group work and discussion, we have chosen to use a further logo, four students viewed through a magnifying glass, to serve as an aide-memoire to teachers who may not normally work in this way.

Our model emerged, naturally, from the learning experiences of the students involved in our trials. During the very early stages, it became clear that if students were to gain from the experience of completing an extended task and satisfy the requirements of a GCSE assessment scheme, then such a model provided a supportive framework within which they could work.

One advantage of working within such a framework is that it emphasises the need to allow an appropriate period of time for each stage. The project support material always offers what may be considered to be a fairly closed starting point. This provides an activity for students to work on and to discuss, both individually and in groups, as they come to terms with the first stage, *Understanding and exploring the problem.*

During the second stage, *Devising and planning individual studies,* it is often useful to incorporate some small group brainstorming and class discussion activities. This stage forms a crucial link from the initial stage through to each student's individual work on the task. Ideas may need to be shared and, perhaps, briefly discussed. It will often be found that if a teacher makes suggestions, then these become the questions that all students feel they ought to answer. A brainstorming and class reporting-back session, during which students share ideas while the teacher acts as chairperson, can open up a large number of avenues which could be explored mathematically. Using this type of approach a greater diversity of avenues to be explored often emerges from the students. During the second stage, each student needs to select one or two avenues to pursue in depth and devise a plan of attack.

During the third stage, *Implementing plans and pursuing ideas,* it is important that students realise that their teacher is a resource upon which they may draw. Extended tasks are intended to form part of the learning process for students, and it is

expected that they will receive help and advice from their teachers. The marks or grades awarded for each task are intended to reflect the personal contributions of the students, including the extent to which they are able to use the help and advice they receive as the task develops.

The final stage, *Reviewing and communicating findings* should not, merely, be the writing of a report to satisfy the teacher or the examining group. The writing of reports ought to be of great value to students as they pull together their ideas and experiences. Students need to summarise their findings and to think about what they have learnt and gained from their experiences. Students need to be encouraged to reflect upon the various facets of, as well as the totality of, their experiences. Questions such as 'How would I tackle a similar problem?', 'What did I find difficult?', 'Why?' are all worthwhile considerations at this stage. Students need to be supported through this stage of reviewing, synthesising and communicating their findings.

Examples of information sheets for students on how to 'write up' tasks are provided on pages 52-54. These examples have been devised by individual schools and are reproduced as they were originally designed. Many of them relate to tasks falling into the pure investigative category, but they can be developed to cover the other categories.

At all stages of their work students will find it helpful if they are provided with criteria for assessment or an assessment progress checklist, against which they can check their progress. Several examples are included in Chapter 7.

Suggestions for teachers undertaking investigations are included on page 55.

On pages 56-57 we provide an example of a booklet which was produced by the mathematics department of one of the schools visited during the project. A copy of this booklet is provided for each GCSE student in order to facilitate communication between parents, students and teachers. The booklet is currently being reviewed in the light of two years' use.

WRITE UP 1

1. Explain the problem

    - use sentences and diagrams.

2. Explain what you have done (this is by far the biggest part)

    - write it like a diary

    - make little notes as you do or discover things

    - use lots of diagrams (these are helpful in explaining)

    - remember to write about things that didn't work.

3. Conclusion - write down clearly and simply the most important things that you found out, for example any rules or patterns you noticed.

WRITE UP 2

1. Introduce the problem in your own words - use diagrams if needed.

2. Write down what you are going to do.

3. Try your ideas out - use - diagrams - be systematic
   - tables
   - words - ask yourself questions and answer them.

4. Look at your diagrams, tables - can you see a problem/rule
   - can you explain the rule
   - does it work all the time
   - try some simple and/or complicated cases.

5. Try to add something to the problem of your own.

   Make up your own problems.

   Change the type of paper used.

6. Conclusion - what have you found out in this piece of work?

WRITE UP 3

Some notes to help you with this type of work.

1. Introduce the problem in your own words.
2. State how you are going to try to solve the problem, or investigate the situation.
3. Try your method showing all the working.
4. Collect any results that your work has produced. Always tabulate the results if you can.
5. Look at your results and see if you can find any rules or patterns. If you think you have spotted a rule or a pattern, say what it is. Check your rule by seeing if it works on some new data.
6. State any conclusions that you think your work entitles you to make.
7. If your method doesn't work, try to find another method and start again but make sure you write something about why it doesn't work. Mathematicians often have to do this.
8. Try to add something of your own to the problem; something original by extending the problem or changing it slightly (eg, by changing the rules).

Write something about your investigation

When you produce the final version of your work, bear these things in mind:

(a) Choose the right sort of paper for the job (lined, plain, squared, dotty, graph, etc).

(b) Illustrate your work with diagrams, sketches, graphs, charts, etc, if it helps to make things clearer.

(c) Use coloured pens/pencils only if it really helps, sometimes colour confuses.

(d) Don't write essays, keep things brief. Be precise, write in short sentences as far as possible.

(e) Don't make statements or draw conclusions that you haven't checked out or that your work doesn't entitle you to make.

(f) Number the pages clearly.

(g) Fasten your report together in order and check through before handing it in.

INVESTIGATIONS

1. An investigation may be taken from an outside source, it may be the teacher's own, or it may be the pupil's own. The last of these is the ideal situation; then *we* follow up *their* ideas. In this case, you will obviously want to see the proposed investigation has some potential for Mathematics, but otherwise, encourage it all you can.

2. If you take the investigation from an outside source, work through part of it beforehand so that you have an idea where it might lead.

3. Discussion is important, both with the teacher and with other pupils, and should be encouraged.

4. Guidance is also important, but allows them to follow false trails and go along with them. It is difficult to preserve the balance between too much guidance and too little - but try.

5. Pupils often need help in systematising and recording the result of their investigations in such a way that they are likely to see some generalisations or conclusions.

6. Try to keep notes of work done - some pupils may try a series of approaches which deserve credit, but then they throw their work away if it gets nowhere.

7. Presenting their conclusions in intelligible form is probably the most difficult part of an investigation. A lot of guidance may be necessary here; but prompt and discuss, rather than tell them exactly what to do.

INTRODUCTION: Please keep this booklet in the front of your coursework folder. You will want to refer to this several times, so keep it handy. Here are answers to 22 questions about maths cswk. We hope they are helpful. Your teacher will run through this booklet with you - and you may also wish to run through this booklet with your parents too. However, if you have any further questions, do not hesitate to ask your maths teacher.

1. Q: DOES MATHS COURSEWORK MEAN THERE IS NO EXAM?
A: No, the coursework counts for only 25% and there are still TWO exam papers counting for the other 75%.

2. Q: HOW MANY PIECES OF COURSEWORK MUST I DO?
A: 5 under the headings of ..
1) Practical Geometry;
2) An Everyday Application of Mathematics;
3) Statistics and/or Probability;
4) An Investigation;
5) A Centre-Approved Topic.

3. Q: WHICH LEVEL OF MATHS COURSE AM I ON?
A: There are 3 levels - higher, intermediate, foundation - aimed at pupils of different strengths in Mathematics. The grades available are ..

Foundation	E, F, G / U
Intermediate	C, D, E, F / U
Higher	A, B, C, D / U

Your maths teacher will explain which course you are on, though some adjustments may be necessary in a few cases later on. In these cases, things will be explained both to you and your parents either personally or in writing.

4. Q: HOW LONG DOES A PIECE OF COURSEWORK LAST?
A: Each piece is designed to take 2 to 3 weeks (including homework). You will be given a deadline for handing it in, though it will be up to you to decide when the task is complete.

5. Q: WHAT IF I'M AWAY?
A: It is very important that you meet the deadlines for handing in your cousework. Exceptions will only be allowed in cases of a period of illness. But ALL candidates must submit 5 pieces of work.

6. Q: WHEN WILL WE BE DOING COURSEWORK?
A: You will do 3 pieces in the 4th year - Oct, Dec, Mar - plus 2 pieces in the 5th year - Oct, Jan.

7. Q: HOW DO I START?
A: Of course, you can't begin to set out on a problem if you don't really understand it. So don't be afraid to ask questions early on if you are unclear or unsure.
Try too to have your own ideas; don't be influenced by the others. (For further tips see answers 19-21).

8. Q: CAN MY TEACHER HELP ME?
A: I'm afraid the answer is both 'yes' and 'no'.
Yes, you can discuss any matter with your teacher and he/she will try to be helpful - but - No, they cannot steer you into answers and solutions.

9. Q: WHEN DO I WRITE IT UP?
A: Don't make the mistake of leaving everything to the end. It is important that you write things up as you go along whilst it is fresh in your mind.

10. Q: HOW DO I KNOW I'VE GOT THE ANSWER?
A: There is a misunderstanding here. In the coursework tasks you will be set, there will be no such thing as THE answer - a unique solution which you are all meant to arrive at. You can tackle the task in whatever way(s) you think suitable. Even if an approach proves unsuitable and has to be abandoned, it is best to include this idea and then explain the reasons which led you to reject any further development of it. Don't simply leave it out and make the mistake of only handing in your final "super-new-improved" version.

11. Q: HOW DO I KNOW WHEN I'VE FINISHED A PIECE OF COURSEWORK?
A: You will NOT be told, "Stop there. That's now finished". Provided you are within the given deadline, it will be up to you to decide whether you have prepared a complete response to the task you were set.

12. Q: HOW IS IT MARKED? WHAT IS MY TEACHER LOOKING FOR?
A: Each piece will be marked out of 20 by your teacher according to the same marking scheme for all pupils. This scheme is shown at the back of this booklet and your maths teacher will go through it with the whole group before the start of any coursework.
NOTE: It is important to note that your teacher is NOT looking for quantity, but is looking for the quality and thought and reasoning in your work. A piece of work 6 sides long can get more marks that a piece 56 sides long.

13. Q: WILL I BE TOLD THE MARK I GET?
A: Yes, you will be given a copy of the actual mark sheet (shown at the back of this booklet), so you will be able not only to know your total mark, but also how that mark was awarded.
So you will be able to see where you can improve next time, or see what strengths you are showing.

14. Q: WHAT'S THE CONTROLLED ELEMENT?
A: Simply, it's a test given at the end of a piece of cswk to check how much you learned and understood from your piece of work; and this mark also counts towards your 20 possible marks.

15. Q: DO I NEED ANYTHING EXTRA FOR COURSEWORK?
A: Only an A4 size folder or wallet for transporting your work to and from school - though eventually your teacher will keep your final completed submission. You may find too that colour helps your presentation, a calculator helps you handle the figures, etc.

16. Q: DO I HAVE TO WORK ON MY OWN?
A: You may work either as a small group or individually. If you choose to form a small group, you may discuss ideas with each other, but the work you produce MUST be your own. You will not be given any credit for work which is not yours.

17. Q: IF I GET A LOW MARK, CAN I DO IT AGAIN?
A: No, there will not be time. We need to use the remaining time outside of cswk time to prepare for the 2 exams - which after all do carry 75% of the marks. So, it is important to do your best in the cswk time allocated.

18. Q: IF I FAIL MATHS, DO I HAVE TO GO THROUGH THE 2 YEARS AGAIN?
A: No, a 1 year course is laid on in the sixth form and can lead either to a re-entry at the same level or entering at a higher level. Also, November re-sits are available in certain cases.

19. Q: DO YOU HAVE ANY TIPS ABOUT COURSEWORK?
A:

DO	DON'T
Record everything you do as you do it.	Rely on your memory.
Keep all the notes you make.	Expect to get a higher mark just because you have produced masses of work.
Keep all your results and comments.	
Make sure your account has: - a category (1-5) - a title - an explanation of the assignment in your intro. - put references to other books at the end. - make sure your account is written as a personal acct. - acknowledges other peoples ideas. - includes any ideas you throw out with reasons.	

20. Q: WHAT ABOUT INVESTIGATIONS?
A: Work simply, logically and systematically ..
- try some easy examples.]
- try to put them in order.
- draw up a table or list.
- can you spot a pattern?
- can you make a rule?
- test it; does it always work?
- if I think of a number, say 'n', can you write a formula for 'n'?
- can you explain why or how your rule works?

21. Q: CAN YOU SUGGEST SOME HEADINGS TO WORK UNDER?
A:
1 Category
2 Title — Outline of the problem
3 Outline of Approach
4 Description of Method
5 Readings/Results/Observations
6 Presentation of Solution — Solving the problem
7 Conclusion(s)
8 Equipment/Apparatus
9 References

22. Q: CAN I CHOOSE MY PIECE OF COURSEWORK?
A: Yes, but check with your teacher first. If you prefer to devise your own, your teacher will need to know straightaway and will tell you if it is permissable or not.

If your parents wish to comment upon this booklet, do so here please ..

PARENT'S COMMENTS:

PARENT'S SIGNATURE ____________________

7 Assessment

In this chapter, we consider one of the major concerns of the classroom teacher in relation to GCSE coursework and in particular extended tasks. The two key issues for teachers are the assessment and internal moderation of this type of work.

However, before looking specifically at these two areas of concern we include, for the sake of general background and completeness, an extract from *Practical Work in GCSE Mathematics* , (Shell Centre, 1989) which looks at general assessment issues.

General Assessment Issues

'In this chapter, we outline some of the key issues that seem to underlie the problem of designing GCSE assessment of high quality. Though we are concerned with the domain of coursework, most of the issues involved are more general.

External and internal tasks

At one end of the spectrum, all tasks on which assessment is to be based are externally provided - sent to schools by the Board, to be done at chosen times during the course, and either marked externally, or internally using a given mark scheme. Then there are courses where the assessment tasks are selected by the school, (or student), but from a bank of tasks which have mark schemes and/or criteria provided with them. Towards the other end of the spectrum are schemes in which all tasks are internally chosen, but must cover certain categories, either task by task or as a collection.

Externally provided tasks have some advantages, as long as sufficient time and resources are spent on their design and trialling:

They may

* display to the teachers and the students some fresh and interesting possibilities beyond those which they would have devised themselves

* be easier to mark and provide more reliable assessments.

However,

* the tasks cannot be 'tuned' to individual students
* neither the teacher nor the student has that sense of 'ownership' of the tasks which can increase motivation.

With internal choice, on the other hand

* tasks may arise naturally from the curriculum
* tasks may be chosen by students, or by the teacher, as of interest to them
* the teacher can 'tune' the task to the class, or to the student, perhaps through a process of negotiation.

However, there are disadvantages in that:

* it places extra burdens on teachers
* it is not possible to predict the difficulties that may arise in the classroom with an untried task, unless it has previously been tried with another class.
* it is difficult to set the level of difficulty so that the task and the marking are fair, and seen as fair within the class and between schools.
* freedom does not necessarily lead to variety and richness - indeed there are pressures, which are hard for many teachers to resist towards caution and the choice of standard easy but "acceptable" tasks.

Choice and coverage

If externally provided tasks are used, the degree of coverage which is encouraged depends on the number of such tasks, the conditions governing selection, and the degree of predictability of the tasks set in relation to the universe of possible tasks representing the curriculum. With internal tasks, coverage depends on positive action by teacher and student to review the set of tasks completed at a suitable stage, and to choose subsequent tasks to fill gaps. In this case, the teacher needs to work from a well designed specification of the critical abilities in the area. This activity may indeed have benefits for the students in helping them to become aware of the scope and structure of the subject and of its component parts. In some cases, an element of unexpectedness in the tasks offered may provide a challenge which is realistic in terms of the future situations in which mathematics may be required. It is in the nature of investigative work that it demands dealing with the fresh situations which arise when the first-posed questions are answered.

We should not leave the issue of coverage without mentioning another procedure, which is separate testing of each component which is considered important. For example, the fact that knowledge of basic addition and multiplication facts is necessary for almost all school mathematical work has not been sufficient in the past to secure adequate attention to development of fluency in these areas, and we now see in most GCSE examinations a requirement for separate testing of mental mathematics.

Assessment by criteria or by task-specific mark schemes

We now discuss the forms of guidance that may be given to examiners, who in the case of coursework will be the class teachers; this has already been mentioned in relation to task design, with which it is closely linked. Assessment criteria of some kind will always be provided, along with some instructions as to how they should be applied; the key issues relate to the level of detail of these instructions, and to the amount of training provided.

The advantages of detailed guidance are principally

* consistency between different examiners in the grades awarded, and for the same examiner over time
* the process of preparing and of learning the mark scheme represents a considered analysis of the aspects of student performance to be rewarded
* ease of communication to and between examiners and students
* speed of operation where substantial numbers of candidates are involved.

However,

* detailed mark schemes can only be supplied for specific tasks, which therefore have to be closed
* the preparation of such schemes is time consuming if it is done well, so there is a risk of simplistic schemes that focus on easily described aspects of performance rather than all the objectives, particularly those associated with higher level skills.

Less detailed guidance is commonly in the form of subheadings under which grades are to be awarded on the basis of descriptive criteria, together with rules for their aggregation. This approach has the advantage that

* it may be applied directly to any relevant tasks, and is thus an obvious approach where task freedom is involved
* no substantial preparation is involved for each task, once the criteria have been absorbed
* it usually focusses attention more closely on the assessment objectives themselves, rather than the details of the task.

However,

* since the difficulty of meeting any given criterion is usually sensitively dependent on the task, including its complexity, context and form of presentation in detail, fairness is not easily achieved

* the 'load' on the moderating procedure is thus greatly increased

* there is consequent natural pressure towards "task trivialisation" - choosing the simplest tasks that allow a given criterion to be achieved; this is directly counter to the need for students to be able to tackle effectively substantial worthwhile tasks, pure or applied.

Thus, not surprisingly, the choice of appropriate grading procedures is closely associated with the issues of task choice discussed previously.

Thus, the amount and the nature of training and support for examiners is an important factor. It is perhaps surprising that at present markers of written papers commonly undergo careful training and work under close guidance. Yet it is in schemes requiring the application of more general criteria that training and support are more urgently required. Some such training occurs incidentally during the moderation process but this needs strengthening and developing more explicity. This is an ongoing process. As teachers gain experience of these methods it will become possible to make further developments.'

Assessing GCSE Coursework

Assessment has proved to be a major worry for most teachers as they face the challenge of coping with extended task work for the first time. It is an issue that has been debated regularly during in-service sessions and visits to trial schools. It is, however, something which most teachers are more comfortable with once they become involved in this type of work.

Perhaps this major worry is generated by the thought of marking an extended piece of work in the way that we would mark, in a traditional way, an exercise from a text book. The main feature about assessing GCSE extended tasks is that we need to use a process based assessment scheme rather than one based on content. The coursework component of GCSE is intended to assess objectives which teachers may not previously have attempted to assess. We are looking at many of the aspects of the work which were put forward in Chapter 2 as being the reason for coursework. *We cannot mark an extended task in the traditional manner nor can we consider anything which resembles a tight marking scheme.*

Within this material, each lead task is supported by examples of students' work together with detailed comments and grades. But what criteria have been used in awarding these grades? In the previous chapter, a four stage model of problem solving has been suggested. It would seem appropriate to start from this point when considering assessment. We could simply allocate marks equally to each stage. If the total number of marks available is twenty we get

STAGE	DESCRIPTION	MARK
1	Understanding and exploring the problem	5
2	Devising and planning individual study	5
3	Implementing plans and pursuing ideas	5
4	Reviewing and communicating findings	5
	TOTAL	20

However, it is not always clear cut where and when the boundaries occur within a student's work. Moreover, each centre needs to follow the scheme of assessment provided by its own examining group and it would not be appropriate to consider any alternative proposals. During our school trials and within the guidelines provided by their examining group, some teachers found it useful to base their assessment on suggestions taken from the SEC *Grade Related Criteria* (1985). This model rolls our four stages into three areas for assessment, fitting broadly into our previous discussion of getting started, keeping going and finishing off. The three areas for assessment are thus

* Identification of task and selection of strategy

* Implementation, content and accuracy

* Interpretation and communication.

This model has been used by many trial schools with their lower forms when leading up to completing extended tasks of the length for GCSE.

The tables included on page 64-65 show how such an approach has been applied to two of the tasks included in these materials. The first task is one which is categorised as practical geometry; it involves students in designing and making a child's shape sorter. The second is a statistics or applications task which looks at queueing systems. Within this scheme of assessment the three areas are not equally weighted. The maximum marks available are 5, 10, 5. Guidance for allocating marks within each area is provided: there are descriptions of what we might, typically, expect of students at each of the three levels of attainment.

These two frameworks for assessment demonstrate that there is a considerable overlap in what is being looked for, even though the two examples come from such different categories of mathematics. This situation reinforces the process based assessment which is necessary with this type of work and which makes the assessment of a range of pieces of work much easier. During our school trials, teachers who were used to marking traditionally, using content-based marking schemes, very quickly found themselves able to adopt and apply the process-based scheme provided. However, this scheme does assume that the full range of marks is spread across the three levels of attainment.

After the students' scripts have been marked, it is then necessary to allocate a grade, using, perhaps, something similar as to the following table.

<table>
<tr><th>Model X</th><th>or</th><th>Model Y</th></tr>
<tr><td rowspan="2">A</td><td>20</td><td rowspan="2">A</td></tr>
<tr><td>19</td></tr>
<tr><td rowspan="3">B</td><td>18</td><td rowspan="2">B</td></tr>
<tr><td>17</td></tr>
<tr><td>16</td><td rowspan="2">C</td></tr>
<tr><td rowspan="3">C</td><td>15</td></tr>
<tr><td>14</td><td rowspan="2">D</td></tr>
<tr><td>13</td></tr>
<tr><td rowspan="3">D</td><td>12</td><td rowspan="2">E</td></tr>
<tr><td>11</td></tr>
<tr><td>10</td><td rowspan="2">F</td></tr>
<tr><td rowspan="3">E</td><td>9</td></tr>
<tr><td>8</td><td rowspan="2">G</td></tr>
<tr><td>7</td></tr>
<tr><td rowspan="3">F</td><td>6</td><td rowspan="7">U</td></tr>
<tr><td>5</td></tr>
<tr><td>4</td></tr>
<tr><td rowspan="2">G</td><td>3</td></tr>
<tr><td>2</td></tr>
<tr><td rowspan="2">U</td><td>1</td></tr>
<tr><td>0</td></tr>
</table>

Clearly, agreement has to be reached about the relationship between the number of marks, the grade and the mark descriptions. This agreement can often only be obtained after such a scheme has been carefully applied and reflected upon. Model X was the one used by the centre which produced the previous mark descriptions.

TASK: SORTING SHAPES GCSE INTERNAL ASSESSMENT SHEET

CLASSIFICATION	MAXIMUM MARKS	GUIDANCE FOR MARKING	
IDENTIFICATION OF TASK AND SELECTION OF STRATEGY	5	5	Shows clear understanding of task by demonstrating an appreciation of the properties of the shapes and selecting a suitable strategy to determine their dimensions and satisfy the constraints.
		3	Shows an understanding of task by demonstrating some knowledge of the properties of the shapes and making comparisons between them to determine dimensions to satisfy the constraints.
		1	Shows little understanding of task and gives little or no consideration to constraints in determining dimensions of shapes.
IMPLEMENTATION, CONTENT AND ACCURACY	10	10	Shows detailed knowledge of the properties of the shapes. Applies appropriate geometrical techniques to design and produce nets which construct to give models which meet given constraints. Selects and uses appropriate measuring and drawing instruments to appropriate degree of accuracy.
		6	Shows adequate knowledge of the properties of the shapes. Applies appropriate geometrical techniques to produce and design adequate nets which construct to give models which meet most of the given constraints. Selects and uses appropriate measuring and drawing instruments to appropriate degree of accuracy.
		2	Shows little knowledge of properties of shapes and applies few geometrical techniques in the design and production of nets. Produces incomplete or - unsuitable nets for some or all of the shapes through inaccurate measurements, inappropriate techniques or unsuitable designs.
INTERPRETATION AND COMMUNICATION	5	5	Gives clear explanations and descriptions of strategies and designs used, problems faced and adjustments/alterations made to overcome them. Makes effective use of mathematical language and notation, diagrams and nets.
		3	Gives adequate explanations and descriptions of strategies and designs used, problems faced and adjustment/alteration to overcome these problems or incorporate all relevant factors in developing a design. Makes adequate use of mathematical language and notation, diagrams and nets.
		1	Gives no clear explanations and descriptions of strategies or methods used. Does not evaluate models made or appreciate inadequacies in designs and makes little use of mathematical language and notation and no or limited use of diagrams and nets.

TASK: WHY ARE WE WAITING? GCSE INTERNAL ASSESSMENT SHEET

CLASSIFICATION	MAXIMUM MARKS		GUIDANCE FOR MARKING
IDENTIFICATION OF TASK AND SELECTION OF STRATEGY	5	5 -	Shows clear understanding of the principles involved by defining and classifying a broad range of queueing systems. Clearly identifying the areas for investigation and the appropriate questions to be answered.
		3 -	Shows understanding of the task by defining and classifying a selection of queueing systems and showing some appreciation of the effect of changing appointment intervals. Identifies sufficient areas for investigation and deals with some of the appropriate questions to be answered.
		1 -	Shows poor understanding of the task: few examples of queueing systems and shows little or no appreciation of the effects of changing appointment intervals. Identifies insufficient areas for investigation and does not deal with the appropriate issues and questions to be answered.
IMPLEMENTATION AND COMMUNICATION	10	10 -	Generates and processes data accurately. Applies sound reasoning in interpreting data and recommends a viable system.
		6 -	Generates and processes data with few errors. Recommends a viable system but with incomplete reasoning or explanation.
		2 -	Generates inaccurate or incomplete data and makes no recommendation or provides a recommendation which is not supported by sound reasoning or explanation.
INTERPETATION AND COMMUNICATION	5	5 -	Selects an appropriate and clear method of recording results, with effective use of mathematical language and notation, diagrams, lists and tables. States results achieved and supports results and recommendations by clear explanations and reasoning.
		3 -	Selects an appropriate method of recording results, with adequate use of mathematical language and notation, diagrams, lists and tables. States results achieved and supports results and recommendations by some explanation and reasoning.
		1 -	Makes limited or no use of appropriate methods of recording results and draws few or no valid conclusions.

An alternative method which, it is argued, allows for the assessment of positive achievement in a more fruitful way, allocates marks according to positive achievement within each level of GCSE. In order to facilitate differentiation, the full range of marks is available at each level. Such a system still based on the three areas for assessment, is included on pages 62-64 of IMPACT.

Again we can see the common approach to wording in order to simplify the assessment. Here we could simply allocate grades to a piece of work or marks based on the philosophy that an average student at each level ought to gain approximately two-thirds of the available marks. Such a conversion table is as follows

Level / Grade	HIGHER	INTERMEDIATE	FOUNDATION
A	17, 18, 19, 20		
B	13, 14, 15, 16		
C	10, 11, 12	17, 18, 19, 20	
D	7, 8, 9	13, 14, 15, 16	
E		10, 11, 12	17, 18, 19, 20
F		7, 8, 9	13, 14, 15, 16
G			10, 11, 12
U	<7	<7	<10

One purpose of the assessment of coursework is to enrich the curriculum. This suggests *incidental assessment* by teachers of students as they pursue learning activities in the classroom. Incidental assessment implies that the teacher observes and records some specific attainment by a student during a normal classroom activity. In the past, incidental assessment of students by teachers has been informal. However, the MEG scheme of assessment for 1991 which is included on page 61 of IMPACT, contains a new classification of assessment, *Oral Skills*.

The following suggestions are included in the scheme of assessment.

Oral skills are assessed through conversation between student and teacher during or after the completion of an assignment. The purpose of oral assessment is to assess students' ability to communicate mathematically as they

* respond directly to questions
* discuss mathematical ideas
* explain mathematical arguments.

It is anticipated that the assessment will be made as a result of continuing evaluation of students' performance in discussing the progress of their assignments and their conclusions with their teacher and fellow students. Formal one-to-one interviews are not required

When teachers are making assessments they may find it helpful to consider the following attributes

* discussion of the problem and how to begin
* discussion of avenues of approach, including why some are adopted and others rejected
* questions asked both of themselves and their teachers
* justification of their work and their conclusions
* explanations to other people of their problem, their work and their conclusions
* insight shown in discussion about the work of other candidates
* clarify of expression of ideas using appropriate language.

During the classroom trials of these materials, the need to assess oral skills did not exist. For many teachers this will present a considerable challenge due to lack of confidence, skills, time and resources. However, in our trial classrooms most teachers did use informal communication with their students to aid their general assessment. One particularly noticeable feature which arose from the trials was the amount of time available for teachers to carry out such work when a full group of students are genuinely accepting responsibility for their own GCSE extended task work in mathematics. Having experienced this situation, these teachers now have few worries about this type of assessment. Extended tasks ought to be submitted by students and assessed quite soon after completion. This is advantageous for students since they obtain immediate feedback on the strengths and weaknesses of the work, while it is still fresh in their minds and before completing of any further tasks. It is also advantageous to the teacher for the same reasons, and also because there is then little need to record, other than mentally, the incidental and informal assessment of oral and other skills which has taken place during the actual completing of the tasks.

Students need to be aware of their examining group's scheme of assessment, but it may be helpful to provide them with alternative statements of the criteria which will be used when assessing their coursework. Such alternative statements should be designed to enable students to check their own progress as they work through their tasks. Some examples of such checklists are included on pages 56-57 and 70-74.

Apart from using an examining group's scheme of assessment, or even an internal one, to record the achievement of any student, it is worthwhile considering how assessment procedures can be of benefit to students. Many schools we have worked with, or visited, during the project's life, have used assessment descriptions and details to report directly to their students. Often such documents are issued to students at the beginning of the GCSE course. They are then used to help them to plan and organise their own work. This has proved to be a powerful tool in helping students to understand what is required of them. Often students have been asked to make a self-assessment of their work and to hand this in with their report. It is interesting that students are often much harsher on themselves than they ought to be.

These suggestions have been put forward because they seem to encompass the range of approaches, wording and models used within the various GCSE schemes.

Internal Moderation

After individual teachers have assessed their own students' coursework, it is necessary that the scripts should be internally moderated. We recommend that each department should work through the assessment sections of IMPACT. Collaborative efforts to mark a small number of students' scripts, followed by in-depth discussion until the concensus is reached, is a valuable activity.

It is important to remember that the internal moderators cannot read through *all* students' work. Each internal moderator can only read through a representative sample of scripts. Concensus should be reached about the grading of this sample.

This representative sample should contain scripts from each teacher and across the grades. The internal moderator needs to be convinced that the notes from individual teachers provide appropriate guidance and support for the external moderator. *All* scripts should be available to the internal moderator during the moderation period.

It is essential that there should be internal feedback from the internal moderator to teachers, both individually and as members of a group, so that everyone feels comfortable about the grading of students.

SCHEME OF ASSESSMENT FOR COURSE WORK COMPONENT

CLASSIFICATION	MAXIMUM MARKS	GUIDANCE FOR MARKING
OVERALL DESIGN AND STRATEGY	4	* Do you have a definite problem that you are looking at? * Have you been following your own ideas? * Have you asked and answered your own questions? * Have you tackled your problems in a suitable way?
MATHEMATICAL CONTENT	4	* Have you used a suitable range of mathematics? * Have you developed the mathematics as you have gone through the work?
ACCURACY	4	* Is your work accurate? * This can be calculations, drawings, graphs, practical work, collecting data and conclusions.
CLARITY OF ARGUMENT AND PRESENTATION	4	* Have you explained in your report what you were doing at all times? * Have you an introduction and conclusion? * Have you explained the link from one stage to another? * Have you used mathematical tables, graphs, diagrams, language, symbols, etc.

CRITERIA FOR ASSESSING COURSEWORK

Preparation and Approach. This is about your ability to get on with an investigation on your own. To score this mark you have shown that you can:

1.

2. decide, with guidance, how to investigate situations devised by others

3.

4. *and in addition* decide, with little guidance how to investigate other situations

5.

6. *and in addition* apply reasoning to plan the approach to a variety of tasks including some devised by yourself.

7.

Use of appropriate methods. This is about your ability to carry out an investigation in an organised and systematic way. To score this mark you have shown that you can:

1.

2. explore situations experimentally and, where appropriate, collect data, tabulate information and generate diagrams

3.

4. *and in addition* explore situations systematically and organise data or information in a helpful way

5.

6. *and in addition* check the completeness of the methods chosen by considering different possibilities.

7.

Quality of mathematics. This is about your ability to use mathematical ideas and skills correctly. To score this mark you have shown that you can:

1.

2. use appropriate mathematical techniques calculations, comparison of data, generate simple plans, diagrams, and graphs and have studied an area of mathematics not included in the syllabus for Level 1

3.

4. *and in addition* interpret calculations, analyse data, generate plans, diagrams and graphs, use simple formulae and have studied an area of mathematics not included in the syllabus for Level 2

5.

6. *and in addition* apply a variety of mathematical skills, knowledge and procedures, critically analyse calculations and data, use formulae and have studied an area of mathematics not included in the syllabus for Level 3.

7.

Structure of Mathematics. This is about your ability to summarise your work and draw conclusions. To score this mark you have shown that you can:

1.

2.

3. spot a pattern and make some valid observations

4.

5.

6. *and in addition* make predictions and valid generalisations

7.

8. *and in addition* with guidance, attempt to verify and justify generalisations

9.

10. *and in addition* verify and justify generalisations

11.

12.

Level of communication. This is about your ability to explain what you have been doing and what you have found out. To score this mark you have shown that you can:

1.

2. explain the situation explored and the strategies employed, making use of visual forms of presentation including diagrams, tables, charts and models where appropriate

3.

4. *and in addition* present results in a logical sequence making use of mathematical symbols and graphs where appropriate

5.

6. *and in addition* give reasons for the strategies employed, stating assumptions or simplifications made, and select the most appropriate methods of communicating results using a variety of mathematical symbols and visual forms of presentation

7.

TOTAL OUT OF 40

ITEM OF ASSESSED COURSEWORK

NAME ....

COURSEWORK TITLE ....

ITEM NUMBER 1/2/3/4 DATE .... TEACHER ....

We will tick a box from 1 to 6, 1 being the highest, to indicate the quality of the work you have done under each heading.

A *Introduction to the task* 1 2 3 4 5 6

Quality of:

1. clear introduction to your work
2. method used and your explanation of it
3. questions written down that lead your work on
4. information recorded to start the task
5. accurate and careful use of equipment

B. *Development of the task*

Quality of:

1. accuracy of your work
2. your checking of your work
3. the use you have put your results to
4. patterns noticed and used in your work
5. predictions
6. general descriptions or algebra used in your work
7. graphs (where appropriate)
8. tables of results
9. extension of your work beyond the original question

C. *Conclusion to the task*

Quality of:

1. clear and careful conclusions
2. support your evidence given to your conclusions

D. *Communication*

Your ability to discuss the ideas of the tasks with your teacher	1	2	3	4	5	6

How much work is the original idea

OVERALL LEVEL H I F

Teacher Comment

Pupil Comment

MATHEMATICS SUMMARY PROFILE

PUPIL'S NAME .... FORM .... GROUP .... DATE ....

MODULE TITLE ....

ASSESSED COURSEWORK

(a) *Planning*

Foundation: requires assistance to write a plan to tackle the first stage of the task and to choose appropriate equipment.

Intermediate: can plan the first stage of the task, recognises further stages but requires some assistance to tackle these. Can select useful equipment and information.

Higher: can write a plan, which is capable of modification, to deal with all stages of the task.

(b) *Carrying out the task*

Foundation: can follow the plan carrying out some measurements and calculations accurately. Can recognise familiar patterns, sometimes with assistance.

Intermediate: can make reasonably accurate measurements and calculations and write rules from the patterns observed with some help.

Higher: can make accurate measurements and calculations and write algebraic rules from patterns observed. Can predict results, check their accuracy and amend if necessary.

(c) *Communication*

Foundation: can write about the work done using tables, graphs, diagrams and calculations where appropriate with some observations. Can talk about the work.

Intermediate: can summarise the work in a logical sequence using techniques including tables, graphs, diagrams and calculations. Can explain the method used and results. Can discuss ideas used in the task.

Higher: can give a detailed explanation of the reasoning behind each stage of the task and results using a wide range of mathematical techniques. Can discuss fluently the ideas used in the task and possible implications.

PUPIL'S COMMENT: to comment on my most satisfying piece of work, any difficulties experienced during the module and suggestions for improvement.

TEACHER'S COMMENT (to be discussed with pupil. May include comments about working with others, ability to complete the tasks, homework and problem solving).

8 Classroom Resources

The materials in these packs provide a fairly comprehensive set of resources for supporting GCSE coursework in mathematics, together with a considerable number of classroom tasks of high quality. Part of that support, however, is to give a view of other materials that are available, and of how teachers may usefully come to grips with them.

We distinguish two crucial aspects

* *the nature of the learning activities* that the material promotes. One of the aims of coursework is to change the mathematics curriculum from the traditional overwhelming emphasis on concepts and technical skills, mostly applied to quite short tasks, towards a balance that includes substantial amounts of coherent, extended work of a more varied and less routine kind with greater student independence and responsibilty including discussion and communication of methods and results. Within this chapter we discuss other materials that may be used to support these kinds of classroom activity.

* *the degree of support* that the materials offer to teachers and to their students. Most teachers of mathematics use textbooks, or other comprehensive schemes of work, as the basis of their mathematics teaching, sometimes enriching the variety of activity using other materials. However, the majority of material produced to support problem solving and the other investigative activities, that are the core of coursework, is very much less supportive. Often such materials simply comprise sets of good problems, with a few general suggestions as to how they should be handled in the classroom, and what may be expected.

The minimal level of support partly reflects a plausible assumption that no more can be done - "Because the activities are not imitative but 'open' to the students, with a variety of possible approaches and outcomes, how can you give the teacher more than general guidance?" However, over the last few years it has been shown that a great deal more support can be offered to the teacher without 'closing down', or otherwise stultifying the student activity. This support has proven to be helpful, even essential, for many teachers in the first few years of this kind of work, and seems also to be appreciated by some who are more experienced. You may note that, in the present materials, the lead tasks are much more fully supported than the alternative tasks, though neither is at the extreme in terms of level of support. We therefore believe that these materials offer something for every teacher of mathematics.

The balance between offering support for teachers which facilitates student independence, and support which is directive of student activity, is a delicate one. In response to this dilemma, the Shell Centre has over the last seven years, produced a series of 'modules' which aim to provide effective support for these less familiar kinds of classroom activities. These have been developed to combine high-quality mathematical activity with exceptional classroom robustness. The key to achieving both of these ambitious aims simultaneously, lies in the research based methods of development used. This involves successive rounds of revision and refinement, based on detailed classroom observation of the materials in use in typical classrooms, and discussions with the teachers and students involved. The modules have been well received and are now widely used. The GCSE Mathematics Guide for Teachers contains many examples taken from the first two modules. The first module was described in the GCSE Mathematics Guide for Teachers, as a 'gentle introduction to investigative work', and that spirit has been sustained throughout these packages of extended task material for GCSE coursework.

Seven modules have been published prior to this material; all are available from the Shell Centre:

* *Problems with Patterns and Numbers* focusses on pure mathematical investigation and is built around learning standard problem solving strategies and their use on shorter problems. Initially, close support is provided for both students and teachers; this support is gradually decreased over three units. The module finishes with a collection of problems, many of which may also form the basis of extended investigations. The module contains illustrative assessment tasks with marking schemes, and a do-it-yourself in-service package on teaching and assessment, including videotape of teachers in action, and computer software that supports investigative work. This module is widely known as The 'Blue Box'.

* *The Language of Functions and Graphs* focusses on 'translation skills' - the ability to translate information from one form to another, with an emphasis on interpreting and formulating graphs and algebraic expressions. It aims to develop student fluency in 'speaking the language of mathematics'. The teaching agenda is on handling classroom discussion. This 'Red Box' has the same structure as the 'Blue Box', including the assessment, in-service video and software elements.

These 'modules' may also be used with younger more able students. Teachers will find them a valuable complement to the present materials. Each module provides focussed support on key aspects of investigative work : coaching for problem solving, handling classroom discussion, and the various aspects of practical problem solving in the mathematics classroom.

* *Numeracy through Problem Solving* is a collection of five modules, aimed at making practical problem solving a reality in the mathematics classroom. It works well right across the ability range for ages 13-16 and involves group work in an important way. In each case the activities involved are planned, carried through, and critically evaluated by the pupils. The five modules are

  Design a Board Game — *Be a Paper Engineer*

  Produce a Quiz Show — *Be a Shrewd Chooser*

  Plan a Trip

  A do-it-yourself in-service element relating to all five modules is available separately under the title *'Handling Real Problem Solving In The Classroom'*.

By now a great variety of published material is available for GCSE, some of which contains some indication of specific aspects of its use in the classroom, including the level of challenge it provides for students but less frequency for teachers. The professional associations have been pioneers in this regard. Different teachers are looking for different things, and quite rightly. Whilst the material outlined above may well be suitable for most, if not all teachers, a list of, say, 50 starters will be of use only to the teachers with confidence and experience of this type of work.

However, all teachers need to be constantly on the lookout for new and helpful material. One possible approach, using this package of material, could involve allocating new-found tasks of a suitable type to one of our cluster books. In this way, a task outline and brief notes may be further supported by the lead task notes and general teacher's guide, in the same way as any other alternative task within the cluster. This would involve thinking clearly about what each of the four stages within the task may entail. However, teachers do need to think carefully about whether or not a task is in fact suitable for a GCSE extended task.

Clearly there is a great deal more to classroom resources than just the teaching materials. The teacher and her students are probably the most valuable resource, and the IMPACT programme is designed to help develop this. The GCSE classroom, perhaps, needs a layout which is different from the traditional 'rows of desks' model associated with teachers of mathematics. Practical apparatus and equipment, including games, models, everyday items, card, glue, scissors, compasses, mathematical equipment, calculators, computers etc, all play an essential part in the learning of mathematics and the completing of extended tasks. The local community and environment, the school itself, and your teaching colleagues, may also be valuable resources for GCSE extended task work.

Within this chapter it would be impossible to produce a comprehensive list of materials and resources, and to classify each according to the nature of the learning activities and the degree of support provided. However, what we can do, is to produce a list of lists. Such lists have been produced at regular intervals since the conception of the GCSE.

Resource Lists

are included within

Examining Group Training Manuals

GCSE : Mathematics A Guide For Teachers (SEC, 1986)

Better Mathematics (HMSO, 1987)

Practical Work In GCSE (Shell Centre, 1988)

Shell Centre Publications List (Shell Centre)

and are produced by the professional associations

The Association of Teachers of Mathematics

The Mathematical Association.

9 *Looking Forward*

The implementation of the GCSE has presented every teacher of mathematics with new challenges, particularly because of the requirements for coursework and its related school-based assessment. In these materials we analyse the nature of these challenges, and provide practical suggestions for action, in a supportive form that many typical teachers have found effective.

The first GCSE examination took place at a time when further change was already on the agenda. The National Curriculum initiative is under way, with gradual implementation in secondary mathematics classrooms from September 1989. The National Curriculum itself will develop slowly over the next five or ten years, in a way that cannot be foreseen, but certain important elements are already in place and others are in prospect. The Report of the Mathematics Working Group in August 1988 set out a reasonably coherent scheme for mathematics, within the framework of Attainment Targets, Programmes of Study, and Assessment and Testing, that was defined in the government Consultation Document of July 1977 and in the report of the Task Group of Assessment and Testing. After some consultation, parts of the mathematics report were revised by the National Curriculum Council, and some elements, following further revision, were made statutory by the Government. This is supplemented in the first instance by the non-statutory guidance issued by the National Curriculum Council.

Though the aims for mathematical education remain very much those of the Cockcroft Report and the GCSE National Criteria, the framework and the language of the National Curriculum present a rather different perspective which will take time to develop and absorb.

What changes are likely to happen? The emphasis on providing regular information on children's progress, to parents and others, on the basis of official assessments throughout the period of schooling, will probably lead to the discussion of *levels, profile components* and *attainment targets* at parents' evenings and in school reports, as well as within the required Records of Achievement. School-based assessment throughout every year, with Standard Assessment Tasks and formal reporting every few years, will ensure that young people approaching the GCSE already have a fairly clear, official picture of their levels of achievement. The Government has stated that the GCSE will remain the basis of assessment at 16; it will surely change in various ways to fit in with this overall framework. However, all these changes seem likely to give more, rather than less, emphasis to coursework and the school-based assessment of extended tasks. The range of mathematical activity required by the National Curriculum is even broader, in some respects, than that required by the GCSE. How far and how soon these requirements will be fully realised in the Standard Assessment Tasks, and in school-based assessment, remains to be seen. However, it is a fair assumption that the work of teachers facing the demands of GCSE coursework will meet essential needs for the future.

We hope that these materials will prove useful to all who use them. It is with this hope in mind that we suggest that the whole of the material within this package be treated on a regular 'dip-in' basis. Use the material in a way that allows for a balance between your own individual approach and that which is required by the GCSE National Criteria and assessment requirements. We look forward, with hope, enthusiasm and excitement, regarding what our students may learn about mathematics through working on extended tasks.

Printed in England by Burgess & Son (Abingdon) Limited.

Published by the Shell Centre for Mathematical Education.

ISBN 0 906126 43 6

First published 1989.

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