# EXTRA HIGH VOLTAGE AC TRANSMISSION SYSTEM (EHV-AC)

## 1.COURSE OBJECTIVES

### COURSE OBJECTIVES:

1. To Provide In-depth understanding of different aspects of Extra High Voltage AC transmission system design and Analysis.
2. To Calculate the Value of Line Inductance and Capacitance of EHV transmission Line
3. To understand the concept of Voltage gradients of conductors.
4. To develop the empirical formula to determine the Corona loss occurring in EHV AC transmission Line.
5. To determine the interference caused by Corona and to measure its magnitude.
6. To calculate the Electrostatic field and to understand its effects over humans, animal and plants.
7. To derive the expression and possible solution for travelling wave and its source of excitation.
8. To develop Power circle diagram and understand various Line Compensating systems

## 2.OUTCOMES

### COURSE OUTCOMES:

1. Students learn about the trends in EHV AC Transmission.
2. Student can calculate Line inductance and capacitances of bundled conductors.
3. Students can calculate voltage gradient of bundled conductors
4. Students will understand the effects of corona like Audible noise.

5. Students understand the effect of Radio Interference
6. Students can calculate electrostatic field of EHV AC lines
7. Students can analyze travelling waves
8. Students can analyze compensated devices for voltage control.

## 3. Importance of the course

Modern power transmission is utilizing voltages between 345 kV and 1150 kV, A.C. Distances of transmission and bulk powers handled have increased to such an extent that extra high voltages and ultra high voltages (EHV and UHV) are necessary. The problems encountered with such high voltage transmission lines exposed to nature are electrostatic fields near the lines, audible noise, radio interference, corona losses, carrier and TV interference, high voltage gradients, heavy bundled conductors, control of voltages at power frequency using shunt reactors of the switched type which inject harmonics into the system, switched capacitors, overvoltages caused by lightning and switching operations, long air gaps with weak insulating properties for witching surges, ground-return effects, and many more. This course covers all topics that are considered essential for understanding the operation and design of EHV ac overhead lines and underground cables. Theoretical analysis of all problems combined with practical application are dealt in this course.

## 4. University Question papers of previous years

**B.Tech IV Year I Semester Examinations, May/June-2012 EHV AC TRANSMISSION**

**(Electrical and Electronics Engineering)**

**Time: 3 hours Max. Marks: 80**

**Answer any five questions**

**All questions carry equal marks**

**---**

1.a) `Explain the effect of resistance of conductor in EHV AC transmission system.

b) A power of 1200 MW is required to be transmitted over a distance of 1000 km. At voltage levels of 400 KV, 750 KV, 1000 KV and 1200 KV, determine:

i) Possible number of circuits required with equal magnitudes for sending and receiving end voltages with 30o phase difference.

ii) The current transmitted and

iii) Total line losses. **[6+10]**

2. Explain in detail capacitances and inductances of ground return and derive necessary expressions.

**[16]**

3.a) Determine the field of sphere gap in EHV AC system.

b) A single conductor EHV line strung above ground is used for experimental purposes to investigate high voltage effects. The conductors are of expanded ACSR with diameter of 0.06 cm and the line height is 21 m above ground.

i) Find the charging current and MVAR of the single phase transformer for exciting 1Km length of the experimental line. Assume any, if necessary. **[8+8]**

4.a) Derive the expression for energy loss from the charge- voltage diagram with corona.

b) The following is the data for a 750 KV line. Calculate the corona loss per Km and the corona loss current.Rate of rainfall ρ =5 mm/hr, K=5.35×10-10, PFW=5 KW/km V=750 KV line line, H=18 m, S=15 m phase spacing, N= 4 sub conductors each of r=0.017m with bundle spacing B=0.457 m. Use surface voltage gradient on center phase for calculation. **[8+8]**

5. Explain the lateral profile of RI and modes of propagation in EHV lines. **[16]**

6.a) Obtain the electrostatic fields of double circuit 3-phase EHV AC line.

b) Describe the difference between primary shock current and secondary shock current. **[10+6]**

7. Discuss the line energization with tapped charge voltage of traveling waves in EHV AC lines. **[16]**

8.a) List the dangers resulting from series capacitor compensation on long lines and the remedies taken to control them.

b) A 420 kV line is 750 km long. Its inductance and capacitance per km are L=1.5 mH/km and C=10.5 nF/km. The voltage at the two ends are to be held 420 kV at no load. Neglect resistance. Calculate:

i) MVAR of shunt reactors to be provided at the two ends and at intermediate station midway with all four reactors having equal resistance.
ii) The A, B, C, D constants for the entire line with shunt reactors connected.

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**

**B.Tech (CCC) IV Year Supplementary Examinations July/August - 2010**

**EHV AC TRANSMISSION**

**(Electrical and Electronics Engineering)**

**Time: 3 Hours Max.Marks:100**

**Answer Any Five Questions**

**All Questions Carry Equal Marks**

1. a) What is a bundled conductor? What are the advantages of bundled conductors? b) Write short notes on positive, Negative and zero sequence impedances corresponding to E.H.V. lines. **[20]**

2. For a 400 KV line, calculate the maximum surface voltage gradients on the centre and outer phases in horizontal configuration at the max. operating voltage of 420 KV r.m.s (line to line). The other dimensions are: H=14 m, S=12 m N=2 r=0.016 m B=0.46 m. **[20]**

3. Explain the procedure of evaluation of voltage gradients for the phase single and double circuit lines**. [20]**

4. What are the causes of over voltages in EHV A.C. lines? How do you suppress them? Explain in detail. **[20]**

5. Explain the voltage control in EHV A.C. lines by using shunt and series compensation method. **[20]**

6. Explain about audio noise and radio interference due to Corona in EHV lines. **[20]**

7. Explain the procedure of design of EHVA.C. line based on steady state limits. **[20]**

8. Explain the design procedure of EHV cables on transient limits. **[20]**

# 5. QUESTION BANK

## CHAPTER 1

1. Give ten levels of transmission voltages that are used in the world.
2. What is the necessity of EHV AC Transmission? Explain its advantages.
3. Explain the Inductance effect on: i) Round conductor with internal and external flux linkages. ii) Flux linkage calculation of 2-conductor line.
4. A 345-kV line has an ACSR Bluebird conductor 1.762 inches (0.04477 m) in diameter with an equivalent radius for inductance calculation of 0.0179 m. The line height is 12 m. Calculate the inductance per km length of conductor and the error caused by neglecting the internal flux linkage.
5. Explain in detail power-handling capacity of a.c. transmission lines and line losses?
6. A power of 2000 MW is to be transmitted from a super thermal power station in Central India over 800 km to Delhi. Use 400 kV and 750 kV alternatives. Suggest the number of circuits required with 50% series capacitor compensation, and calculate the total power loss and loss per km.
7. Explain the following: i) Single-phase line for capacitance calculation ii) Multi-conductor line for calculation of Maxwell's potential coefficients. iii) Potential coefficients for Bundled-Conductor Lines.
8. What are the different mechanical considerations in line performance and explain in detail?
9. What are the properties of Bundled conductors and explain with neat sketches?
10. What is the use of Symmetrical Components for analyzing 3-phase problems andexplain Inductance & Capacitance Transformation to Sequence Quantities.
11. Explain the following: i) The effect of conductor resistance of e.h.v lines. ii) Power Loss in Transmission. iii) Skin Effect Resistance in Round Conductors.
12. The configurations of some e.h.v lines for 400 kV to 1200 kV are given. Calculate req. for each. i) 400 kV : N = 2, d = 2r = 3.18 cm, B = 45 cm ii) 750 kV : N = 4, d = 3.46 cm, B = 45 cm iii) 1000 kV : N = 6, d = 4.6 cm, B = 12 d iv) 1200 kV : N = 8, d = 4.6 cm, R = 0.6 m.

# CHAPTER 2

1 A sphere gap with the spheres having radii R = 0.5 m has a gap of 0.5 m between their surfaces. i) Calculate the required charges and their locations to make the potentials 100 and 0. ii) Then calculate the voltage gradient on the surface of the high-voltage sphere. iii) If the partial breakdown of air occurs at 30 kV/cm peak, calculate the disruptive voltage between the spheres. [16]

2 Explain the surface voltage gradient on: a) Single conductors b) 2-conductor bundle above ground c) Distribution of voltage gradient on 2-conductor bundle illustrating the cosine law

3 Explain the voltage gradient distribution on Six-conductor bundle and gradient on sub-conductor.

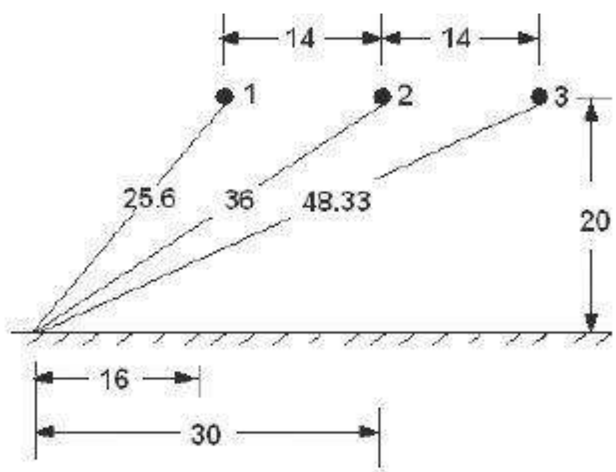
4 b) A point charge Q = 10-6 coulomb (1 μ C) is kept on the surface of a conducting sphere of radius r = 1 cm, which can be considered as a point charge located at the centre of the sphere. Calculate the field strength and potential at a distance of 0.5 cm from the surface of the sphere. Also find the capacitance of the sphere, 1 r ε = .

5 Derive general expression for the charge-potential relations for multi conductor lines: i) Maximum Charge Condition on a 3-Phase Line. ii) Numerical values of Potential Coefficients and charge of Lines.

# CHAPTER 3

1. What is Corona-loss formulae and explain the available formulae classification?
2. What are the different formulas for the corona current explain in detail?
3. Describe the mechanism of formation of a positive corona pulse train.
4. The positive and negative corona pulses can be assumed to be square pulses of amplitudes 100 mA and 10 mA respectively. Their widths are 200 ns and 100 ns respectively. Their repetition rates are 1000 pps and 10,000 pps. The bandwidth of a filter is 5 kHz. Calculate the ratio of output of the filter for the two pulse trains at a tuned frequency $f_0$ = 1 MHz.
5. Explain charge-Voltage(q-V) diagram and Corona Loss for: i) Increase in Effective Radius of Conductor and Coupling Factors ii) Charge-Voltage Diagram with Corona.
6. An overhead conductor of 1.6 cmradius is 10 m above ground. The normal voltage is 133 kV r.m.s to ground (230 kV, line-to-line). The switching surge experienced is 3.5 p.u. Taking K = 0.7, calculate the energy loss per km of line. Assume smooth conductor.
7. How Corona Pulses are going to generate and explain their properties?
8. State and explain different formulae used to calculate the power loss due to coronaon E. H.V. lines.

9. How Audible Noise frequency spectra affects ac and dc transmission lines, and what are the limits for audible noise?
10. Explain different types of Audible Noise measurement and meters.
11. What is the relation between Single-Phase and 3-Phase Audible Noise levels?
12. A 735 kV line has the following details: N = 4, d = 3.05 cm, B = bundle spacing = 45.72 cm, height H = 20 m, phase separation S = 14 m in horizontal configuration. By the Mangoldt formula, the maximum conductor surface voltage gradients are 20 kV/cm and 18.4 kV/cm for the centre and outer phases, respectively. Calculate the SPL or AN in dB (A) at a distance of 30 m along ground from the centre phase (line centre). Assume that the microphone is kept at ground level.



735 kV line configuration

13. Write short notes on frequency spectrum of the RI field of line in E.H.V. lines.
14 Draw the circuit diagram for measuring Radio Influence Voltage (RIV) with respectto E.H.V. lines.

## CHAPTER 4

1. Explain the classification of shock currents?
2. Explain the effect of Electrostatic fields to human life, plants and animals?
3. Explain the clear difference between Traveling and Standing wave theory?
4. Derive the differential expression and their solutions for a transmission line with distributed Inductance and capacitance.

5. What is the importance of Bewley Lattice diagram and explain with neat example.
6. How does the electric field at ground level influence tower design? Also explain significance of Electric field stress (voltage gradient) Potential at ground level?
7. Explain the effect of electric field intensity nearer to conductor surface and nearer to ground surface with respect to E.H.V. lines.
8. A 750-kV transmission line has a surge impedance of 275 ohms and the transformer to be connected to it has a surge impedance of 1100 ohms for its h.v. winding. The length of winding of 5 km and its far end is connected to a zero resistance ground. A surge of 2400 kV is coming in the line which is to be limited to 1725 kV at the transformer bushing by using a short cable as shown in figure.
9. Calculate the surge impedance and voltage rating of the cable to be interposed between line and transformer.
10. Calculate the voltage at the h.v. terminal of the winding as soon as the first reflection arrives from the grounded end.
11. Explain the traveling wave concept for step response of transmission line: i) Losses neglected ii) Losses and attenuation included.

## CHAPTER 5

1. What is the purpose and significance of power circle diagram and its uses and also explain in detail the receiving end circle diagram for calculating reactive compensation for voltage control buses?
2. Define compensation and explain Cascade connection of components of shunt series compensation with generalized equations and chain rule?
3. Explain the voltage control in E.H.V.A.C. lines by using shunt and seriescompensation method.
3. Explain how Harmonics are injected into Network by TCR under: a) Harmonic Injection by TCR in to high voltage system b) Connection of TCR to Δ and Y connected transformer windings c) Voltage and current wave forms for , for calculations of harmonics?
4. Explain Shunt Reactor Compensation ofVery Long Line with Intermediate Switching Station and give the Voltage and current expression at Intermediate station.
5. Find the generalized constants for transmission line with series-Capacitor Compensation at middle of line.

## 7. OBJECTIVE QUESTIONS

1. By which of the following systems electric power may be transmitted ?

(a) Overhead system
(b) Underground system
(c) Both (a) and (b)
(d) None of the above
Ans: c

2 are the conductors, which connect the consumer's terminals to the distribution

(a) Distributors
(b) Service mains
(c) Feeders
(d) None of the above
Ans: b

3. The underground system cannot be operated above
(a) 440 V
(b) 11 kV
(c) 33 kV
(d) 66 kV
Ans: d

4. Overhead system can be designed for operation up to
(a) 11 kV
(b) 33 kV
(c) 66 kV

(d) 400 kV
Ans: c

5. Which of the following materials is not used for transmission and distribution of electrical power ?

(a) Copper
(b) Aluminium
(c) Steel

(d) Tungsten
Ans: d

(d) all of the above
Ans: d

6. The corona is considerably affected by which of the following ?

(a) Size of the conductor

(b) Shape of the conductor

(c) Surface condition of the conductor

(d) All of the
above Ans: d

10. Which of the following are the constants of the transmission lines ?
(a) Resistance

(b) Inductance

(c) Capacitance

(d) All of the
above Ans: d

12. The phenomenon qf rise in voltage at the receiving end of the open-circuited or lightly

loaded line is called the

(a) Seeback effect
(b) Ferranti effect
(c) Raman effect

(d) none of the
above Ans: b

13. The square root of the ratio of line impedance and shunt admittance is called the

(a) surge impedance of the line
(b) conductance of the line
(c) regulation of the line
(d) none of the above
Ans: a

14. Which of the following is the demerit of a 'constant voltage transmission system' ?

(a) Increase of short-circuit current of the system

(b) Availability of steady voltage at all loads at the line terminals
(c) Possibility of better protection for the line due to possible use of higher terminal reactants
(d) Improvement of power factor at times of moderate and heavy loads

(e) Possibility of carrying increased power for a given conductor size in case of long-distance heavy power transmission

Ans: a

15. Low voltage cables are meant for use up to

(a) l.lkV

(b) 3.3kV

(c) 6.6kV

(d) llkV

Ans: e

17. A booster is a

(a) series wound generator
(b) shunt wound generator
(c) synchronous generator
(d) none of the above

Ans: a

26. Which of the following faults is most likely to occur in cables ?
(a) Cross or short-circuit fault

(b) Open circuit fault

(c) Breakdown of cable insulation

(d) All of the above Ans:

28. The voltage of the single phase supply to residential consumers is
(a) 110 V

(b) 210 V

(c) 230 V

(d) 400 V Ans: c

29. Most of the high voltage transmission lines in India are
(a) underground
(b) overhead
(c) either of the above

(d) none of the
above Ans: b
30. The distributors for residential areas are

(a) single phase
(b) three-phase three wire
(c) three-phase four wire
(d) none of the above
Ans: c

31. The conductors of the overhead lines are
(a) solid
(b) stranded
(c) both solid and stranded

(d) none of the
above Ans:

32. High voltage transmission lines use
(a) suspension insulators

(b) pin insulators

(c) both (a) and (b)

(d) none of the
above Ans: a

34. Distribution lines in India generally use
(a) wooden poles

(b) R.C.C. poles

(c) steel towers

(d) none of the
above Ans: b

35. The material commonly used for insulation in high voltage cables is
(a) lead

(b) paper

(c) rubber

(d) none of the
above Ans: b

1 The loads on distributors systems are generally
(a) balanced

(b) unbalanced

(c) either of the above

(d) none of the
above Ans: b

2. The power factor of industrial loads is generally

(a) unity
(b) lagging
(c) leading
(d) zero
Ans: b

3. Overhead lines generally use

(a) copper conductors
(b) all aluminium conductors

(c) A.C.S.R. conductors
(d) none of these

Ans: c

4. In transmission lines the cross-arms are made of

(a) copper
(b) wood
(c) R.C.C.
(d) steel
Ans: d

5. The material generally used for armour of high voltage cables is

(a) aluminium
(b) steel
(c) brass

(d) copper
(e) Ans: b

6. Transmission line insulators are made of

(a) glass
(b) porcelain
(c) iron
(d) P.V.C.
Ans:

7. The material commonly used for sheaths of underground cables is

(a) lead
(b) rubber
(c) copper
(d) iron
Ans: a

8. The minimum clearance between the ground and a 220 kV line is about

(a) 4.3 m
(b) 5.5 m
(c) 7.0 m

(d) 10.5 m
(e) Ans: c

9. The spacing between phase conductors of a 220 kV line is approximately equal to

(a) 2 m
(b) 3.5 m
(c) 6 m
(d) 8.5 m

# CHAPTER-1

# INTERODUCTION OF EHV-AV TRANSMISSION

## 1.1 STANDARD TRANSMISSION VOLTAGES

Voltages adopted for transmission of bulk power have to conform to standard specifications formulated in all countries and internationally. They are necessary in view of import, export, and domestic manufacture and use. The following voltage levels are recognized in India as per IS-2026 for line-to-line voltages of 132 kV and higher. Nominal System Voltage kV 132 220 275 345 400 500 750 Maximum Operating Voltage, kV 145 245 300 362 420 525 765 There exist two further voltage classes which have found use in the world but have not been accepted as standard. They are: 1000 kV (1050 kV maximum) and 1150 kV (1200 Kv maximum). The maximum operating voltages specified above should in no case be exceeded in any part of the system, since insulation levels of all equipment are based upon them. It is therefore the primary responsibility of a design engineer to provide sufficient and proper type of reactive power at suitable places in the system. For voltage rises, inductive compensation and for voltage drops, capacitive compensation must usually be provided. As example, consider the following cases.

**Example** 1.1. A single-circuit 3-phase 50 Hz 400 kV line has a series reactance per phase of 0.327 ohm/km. Neglect line resistance. The line is 400 km long and the receiving-end load is 600 MW at 0.9 p.f. lag. The positive-sequence line capacitance is 7.27 nF/km. In the absence of any compensating equipment connected to ends of line, calculate the sending-end voltage. Work with and without considering line capacitance. The base quantities for calculation are 400 kV, 1000 MVA.

**Solution.** Load voltage V = 1.0 per unit. Load current I = 0.6 (1 – j0.483) = 0.6 – j0.29 p.u. Base impedance Zb = 4002/1000 = 160 ohms. Base admittance Yb = 1/160 mho. Total series reactance of line X = j0.327 × 400 = j130.8 ohms = j 0.8175 p.u. Total shunt admittance of line Y = j 314 × 7.27 × 10–9 × 400 = j 0.9136 × 10– 3 mho = j 0.146 p.u.

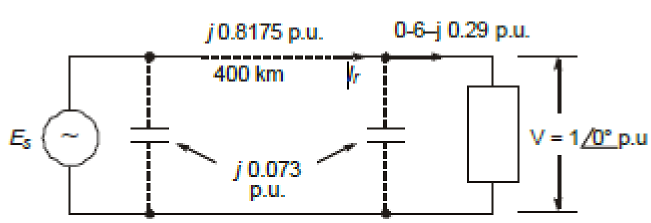



**Fig.1.1** *(a)*

When considering the line capacitance, one half will be concentrated at load end across the load and the other half at the entrance to the line at the sending end, as shown in Figure 1.1. Then, the receiving-end current is Ir = 0.6 – j0.29 + j0.073 = 0.6 – j0.217 p.u. The sending-end voltage will be Es = 1 + j (0.6 – j0.217) 0.8175 = 1.1774

+ j0.49 = 1.2753 Ð22.6° = 510 Ð22.6°, kV. When line capacitance is omitted, the sending-end voltage is Es = 1 + j (0.6 – j0.29) 0.8175 = 1.33 Ð21.6° = 532 Ð21.6°, kV. Note that in both cases, the sending-end voltage, that is, the generating station h.v. bus voltage exceeds the IS limit of 420 kV.

## 1.2 AVERAGE VALUES OF LINE PARAMETERS

Detailed calculation of line parameters will be described in Chapter 3. In order to be able to estimate how much power a single-circuit at a given voltage can handle, we need to know the value of positive-sequence line inductance and its reactance at power frequency. Furthermore, in modern practice, line losses caused by *I*2*R* heating of the conductors is gaining in importance because of the need to conserve energy. Therefore, the use of higher voltages than may be dictated by purely economic consideration might be found in order not only to lower the current *I* to be transmitted but also the conductor resistance *R* by using bundled conductors comprising of several sub-conductors in parallel. When line resistance is neglected, the power that can be transmitted depends upon (*a*) the magnitudes of voltages at the ends (*Es, Er*), (*b*) their phase difference $\delta$, and (*c*) the total positivesequence reactance*X* per phase, when the shunt caspacitive admittance is neglected.

Thus, *P* = *EsEr*sin□□/(*L*.*x*) ... (1.1)

where*P*= power in MW, 3-phase, *Es, Er*= voltages at the sending-end and receiving end, respectively, in kV line-line, □□= phase difference between *Es*and*Er*, *x* = positive-sequence From consideration of stability, d is limited to about 30°, and for a preliminary estimate
of P, we will take Es = Er = E.

## 1.3 POWER-HANDLING CAPACITY AND LINE LOSS

According to the above criteria, the power-handling capacity of a single circuit is *P* = E2 sin □□/*Lx*. At unity power factor, at the load *P*, the current flowing is*I* = *E* sin □□/ 3 *Lx*and the total power loss in the 3-phases will amount to

*p* = 3*I2rL* = *E2*. sin2□□.*r*/*Lx*2 ... (1.2)

Therefore, the percentage power loss is

*%p* = 100 *p*/*P* = 100. sin□□.(*r*/*x*) ... (1.3)

The following important and useful conclusions can be drawn for preliminary understandingof trends relating to power-handling capacity of a.c. transmission lines and line losses.

(1) One 750-kV line can normally carry as much power as four 400-kV circuits for equal distance of transmission.

(2) One 1200-kV circuit can carry the power of three 750-kV circuits and twelve 400-kV circuits for the same transmission distance.

(3) Similar such relations can be found from the table.

(4) The power-handling capacity of line at a given voltage level decreases with line length, being inversely proportional to line length *L*. From equation (2.2) the same holds for current to be carried.

(5) From the above property, we observe that if the conductor size is based on current rating, as line length increases, smaller sizes of conductor will be necessary. This will increase the danger of high voltage effects caused by smaller diameter of conductor giving rise to corona on the conductors and intensifying radio interference levels and audible noise as well as corona loss.

(6) However, the *percentage* power loss in transmission remains independent of line length since it depends on the *ratio* of conductor resistance to the positive-sequence reactance per unit length, and the phase difference □□between *Es*and*Er*.

(7) From the values of % *p* given in Table 2.2, it is evident that it decreases as the system voltage is increased. This is very strongly in favour of using higher voltages if energy is to be conserved. With the enormous increase in world oil prices and the need for conserving natural resources, this could sometimes become the governing criterion for selection of voltage for transmission. The Bonneville Power Administration (B.P.A.) in the U.S.A. has based the choice of 1150 kV for transmission over only 280 km
length of line since the power is enormous (10,000 MW over one circuit).

(8) In comparison to the % power loss at 400 kV, we observe that if the same power is transmitted at 750 kV, the line loss is reduced to (2.5/4.76) = 0.525, at 1000 kV it is 0.78/4.76 = 0.165, and at 1200 kV it is reduced further to 0.124.

## 1.4 MECHANICAL CONSIDERATIONS IN LINE PERFORMANCE

### 1.4.1 Types of Vibrations and Oscillations

Three types of vibration are recognized as being important for e.h.v. conductors, theirdegree of severity depending on many factors, chief among which are: (*a*) conductor tension, (*b*) span length, (*c*) conductor size, (*d*) type of conductor, (*e*) terrain of line, (*f*) direction of prevailing winds, (*g*) type of supporting clamp of conductor-insulator assemblies from the tower, (*h*) tower type, (*i*) height of tower, (*j*) type of spacers and dampers, and (*k*) the vegetation in the vicinity of In general, the most severe vibration conditions are created by winds without turbulence so that hills, buildings, and trees help in reducing the severity. The types of vibration are: (1) Aeolian Vibration, (2) Galloping, and (3) Wake-Induced Oscillations. The first two are present for both single-and multi-conductor bundles, while the wake-induced oscillation is confined to a bundle only. Standard forms of bundle conductors have sub-conductors ranging from 2.54 to 5 cm diameters with bundle spacing of 40 to 50 cm between adjacent conductors.For e.h.v. transmission, the number ranges from 2 to 8 sub-conductors for transmission voltages from

400 kV to 1200 kV, and up to 12 or even 18 for higher voltages which are not yet commercially in operation. We will briefly describe the mechanism causing these types of vibrations and the problems created by them.

#### 1.4.1.1 Aeolian Vibration

When a conductor is under tension and a comparatively steady wind blows across it, small vortices are formed on the leeward side called Karman Vortices (which were first observed on aircraft wings). These vortices detach themselves and when they do alternately from the top and bottom they cause a minute vertical force on the conductor. The frequency of the forces is given by the accepted formula

$$F=20065(v/d) \text{ Hz} \quad (1.4)$$

Where*v* = component of wind velocity normal to the conductor in km/ hour, and *d* = diameter of conductor in centimetres. [The constant factor of equation (2.5) becomes 3.26 when *v* is in mph and *d* in inches.] The resulting oscillation or vibrational forces cause fatigue of conductor and supporting structure and are known as aeolian vibrations. The frequency of detachment of the Karmanvortices might correspond to one of the natural mechanical frequencies of the span, which if not damped properly, can build up and destroy individual strands of the conductor at points of restraint such as at supports or at bundle spacers. They also give rise to wave effects in which the vibration travels along the conductor suffering reflection at discontinuities at points of different mechanical characteristics. Thus, there is associated with them a mechanical impedance. Dampers are designed on this property and provide suitable points of negative reflection to reduce the wave amplitudes. Aeolian vibrations are not observed at wind velocities in excess of 25 km/hour. They occur principally in terrains which do not disturb the wind so that turbulence helps to reduce aeolian vibrations. Since the aeolian vibration depends upon the power imparted by the wind to the conductor, measurements under controlled conditions in the laboratory are carried out in wind tunnels. The frequency of vibration is usually limited to 20 Hz and the amplitudes less than 2.5 cm.

#### 1.4.1.2 Galloping :

Galloping of a conductor is a very high amplitude, low-frequency type of conductor motion and occurs mainly in areas of relatively flat terrain under freezing rain and icing of conductors. The flat terrain provides winds that are uniform and of a low turbulence. When a conductor is iced, it presents an unsymmetrical corss-section with the windward side having less ice accumulation than the leeward side of the conductor. When the wind blows across such a surface, there is an aerodynamic lift as well as a drag force due to the direct pressure of the wind. the two forces give rise to torsional modes of oscillation and they combine to oscillate the conductor with very large amplitudes sufficient to cause contact of two adjacent phases, which may be 10 to 15 metres apart in the rest position. Galloping is induced by winds ranging from 15 to 50 km/hour, which may normally be higher than that required for aeolian vibrations but there could be an overlap. The conductor oscillates at frequencies between 0.1 and 1 Hz. Galloping is controlled by using "detuning pendulums" which take the form of weights applied at different locations on the span.

Galloping may not be a problem in a hot country like India where temperatures arenormally above freezing in winter. But in hilly tracts in the North, the temperatures may dipto below the freezing point. When the ice loosens from the conductor, it brings another oscillatory motion called Whipping but is not present like galloping during only winds.

### 1.4,1.3 Wake-Induced Oscillation

The wake-induced oscillation is peculiar to a bundle conductor, and similar to aeolian vibration and galloping occurring principally in flat terrain with winds of steady velocity and low turbulence. The frequency of the oscillation does not exceed 3 Hz but may be of sufficient amplitude to cause clashing of adjacent sub-conductors, which are separated by about 50 cm. Wind speeds for causing wake-induced oscillation must be normally in the range 25 to 65 km/hour. As compared to this,
aeolian vibration occurs at wind speeds less than 25 km/hour, has frequencies less than 20 Hz and amplitudes less than 2.5 cm. Galloping occurs at wind speeds between 15 and 50 km/hour, has a low frequency of less than 1 Hz, but amplitudes exceeding 10 metres. Fatigue failure to spacers is one of the chief causes for damage to insulators and conductors.

Wake-induced oscillation, also called "flutter instability", is caused when one conductor on the windward side aerodynamically shields the leeward conductor. To cause this type of oscillation, the leeward conductor must be positioned at rest towards the limits of the wake or windshadow of the windward conductor. The oscillation occurs when the bundle tilts 5 to 15° with respect to a flat ground surface. Therefore, a gently sloping ground with this angle can create conditions favourable to wake-induced oscillations. The conductor spacing to diameter ratio in the bundle is also critical. If the spacing $B$ is less than $15d$, $d$ being the conductor diameter, a tendency to oscillate is created while for $B/d > 15$ the bundle is found to be more stable. As mentioned earlier, the electrical design, such as calculating the surface voltage gradient on the conductors, will depend upon these mechanical considerations.

### 1.4.1.4 Dampers and Spacers :

When the wind energy imparted to the conductor achieves a balance with the energy dissipated by the vibrating conductor, steady amplitudes for the oscillations occur. A damping device helps to achieve this balance at smaller amplitudes of aeolian vibrations than an undamped conductor. The damper controls the intensity of the wave-like properties of travel of the oscillation and provides an equivalent heavy mass which absorbs the energy in the wave. A sketch of a Stockbridge damper is shown in Fig. 1.2.
A simpler form of damper is called the Armour Rod, which is a set of wires twisted around the line conductor at the insulator supporting conductor and hardware, and extending for about 5 metres on either side. This is used for small conductors to provide a change in mechanical impedance. But for heavier conductors, weights must be used, such as the Stockbridge, which range from 5 kg for conductors of 2.5 cm diameter to 14 kg for 4.5 cm. Because of the steel strands inside them ACSR conductors have better built-in property against oscillations than ACAR conductors.

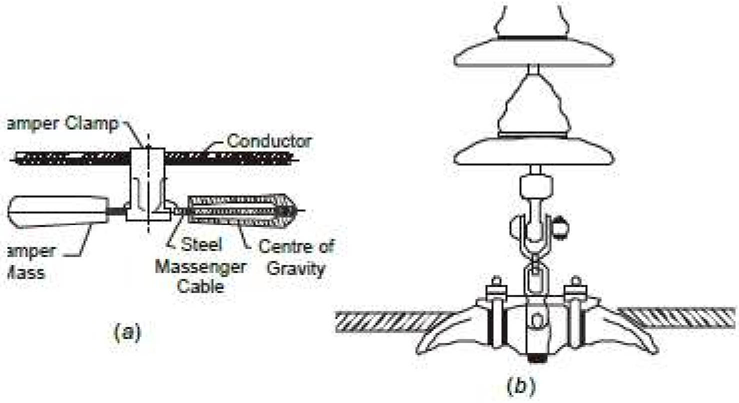



Fig.1.2. (a) Stockbridge Damper; (b) Suspension Clamp (Courtesy: Electrical Manufacturing Co., Calcutta).

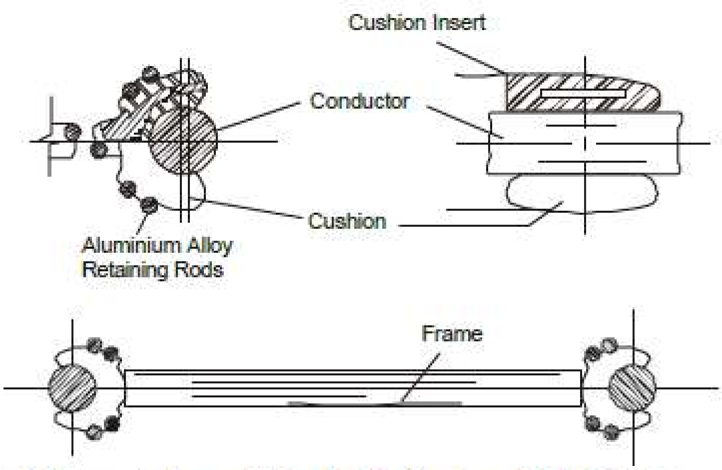



Fig.1.3 Spacer for two-conductor bundle (Courtesy: EMC, Calcutta).

## 1.5 Calculation of Line and Ground Parameters

### 1.5.1 RESISTANCE OF CONDUCTORS

Conductors used for e.h.v. transmission lines are always stranded. Most common conductors use a steel core for reinforcement of the strength of aluminium, but recently high tensile strengthaluminium is being increasingly used, replacing the steel. The former is known as ACSR (Aluminium Conductor Steel Reinforced) and the latter ACAR (Aluminium Conductor Alloy Reinforced). A recent development is the AAAC (All Aluminium Alloy Conductor) which consists of alloys of Al, Mg, Si. This has 10 to 15% less loss than ACSR. When a steel core is used, because of its high permeability and inductance, power-frequency current flows only inthealuminium strands. In ACAR and AAAC conductors, the cross-section is better utilized. Fig. 1.1 shows an example of a stranded conductor.

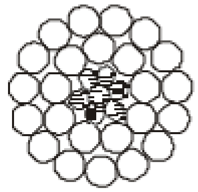


Fig.1.4 Cross-section of typical ACSR conductor

If $n_s$ = number of strands of aluminium, $d_s$ = diameter of each strand in metre and $\rho_a$ = specific resistance of Al, ohm-m, at temperature $t$, the resistance of the stranded conductor per km is

$R = \rho_a\, 1.05 \times 10^3/(\pi/4\, d_s^2 n_s) = 1337\, \rho_a/d_s^2 n_s$, ohms

The factor 1.05 accounts for the twist or lay whereby the strand length is increased by 5%.

## 1.6 Effect of Resistance of Conductor:

(1) Power loss in transmission caused by $I^2R$ heating;

(2) Reduced current-carrying capacity of conductor in high ambient temperature regions. This problem is particularly severe in Northern India where summer temperatures in the plains reach 50°C. The combination of intense solar irradiation of conductor combined with the $I^2R$ heating raises the temperature of Aluminium beyond the maximum allowable temperature which stands at 65°C as per Indian Standards. At an ambient of 48°C, even the solar irradiation is sufficient to raise the temperature to 65°C for 400 kV line, so that no current can be carried. If there is improvement in material and the maximum temperature raised to 75°C, it is estimated that a current of 600 amperes can be transmitted for the same ambient temperature of 48°C.

(3) The conductor resistance affects the attenuation of travelling waves due to lightning and switching operations, as well as radio-frequency energy generated by corona. In these cases, the resistance is computed at the following range of frequencies: Lightning—100 to 200 kHz; Switching—1000-5000 Hz; Radio frequency—0.5 to 2 MHz.

## 1.7 Power Loss in Transmission

For various amounts of power transmitted at e.h.v. voltage levels, the $I^2R$ heating loss in MW are shown in Table 3.1 below.The power factor is taken as unity. In every case the phase angle difference $\delta$ = 30° between $E_s$ and $E_r$.

The above calculations are based on the following equations:

(1) Current: $I = P/\sqrt{3}V$ ... (1.5)

(2) Loss: $p = 3I^2R = P^2R/V^2$ ... (1.6)

(3) Total resistance: $R = L.r$, ... (1.7)

where $L$ = line length in km,

and $r$ = resistance per phase in ohm/km.

(4) Total above holds for $\delta$ = 30°. For any other power-angle the loss is

$p = 3I^2rL = E^2r \sin^2\delta/(L.x^2)$ ... (1.8) where $x$ = positive-sequence reactance of line per phase.

## 1.8 Skin Effect Resistance in Round Conductors

It was mentioned earlier that the resistance of overhead line conductors must be evaluated at frequencies ranging from power frequency (50/60 Hz) to radio frequencies up to 2 MHz or more. With increase in frequency, the current tends to flow nearer the surface resulting in a decrease in area for current conduction. This gives rise to increase in effective resistance due to the 'Skin Effect'. The physical mechanism for this effect is based on the fact that the inner filaments of the conductors link larger amounts of flux as the centre is approached which causes an increase in reactance. The reactance is proportional to frequency so that the impedance to current flow is larger in the inside, thus preventing flow of current easily. The result is a crowding of current at the outer filaments of the conductor. The increase in resistance of a stranded conductor is more difficult to calculate than that of a single round solid conductor because of the close proximity of the strands which distort the magnetic field still further. It is easier to determine the resistance of a stranded conductor by experiment at the manufacturer's premises for all conductor sizes manufactured and at various frequencies.

In this section, a method of estimating the ratio $R_{ac}(f)/R_{dc}$ will be described. The rigorous formulas involve the use of Bessel Functions and the resistance ratio has been tabulated or given in the form of curves by the National Bureau of Standards, Washington, several decades ago. Figure 3.2($a$) shows some results where the ordinate is $R_{ac}/R_{dc}$ at any frequency $f$ and the abscissa is $X = mr = 0.0636 \sqrt{f/R_0}$ , where $R_0$ is the dc resistance of conductor in ohms/mile. W hen using $SI$ units, $X = 1.59 \times 10^{-3} \sqrt{f/R_m}$, where $R_m$ = dc resistance in ohm/metre.

## 1.9 PROPERTIES OF BUNDLED CONDUCTORS

Bundled conductors are exclusively used for e.h.v. transmission lines. Only one line in the world, that of the Bonneville Power Administration in the U.S.A., has used a special expanded ACSR conductor of 1.5 inch diameter for their 525 kV line. Fig. 1.5 shows examples of conductor configurations used for each phase of ac lines or each pole of a dc line.

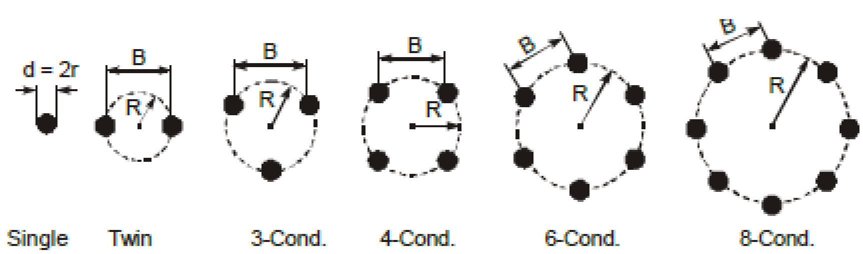



Fig :1.5Conductor configurations used for bundles in e.h.v. lines.

As of now a maximum of 18 sub-conductors have been tried on experimental lines but forcommercial lines the largest number is 8 for 1150-1200 kV lines.

## 1.10 Bundle Spacing and Bundle Radius (or Diameter)

In almost all cases, the sub-conductors of a bundle are uniformly distributed on a circle of radius*R*. There are proposals to space them non-uniformly to lower the audible noise generated by the bundle conductor, but we will develop the relevant geometrical properties of an *N*conductor bundle on the assumption of uniform spacing of the sub-conductors (Fig. 1.5). It is also reported that the flashover voltage of a long airgap is increased when a non-uniform spacing for sub-conductors is used for the phase conductor.

Calculation of Line and Ground Parameters

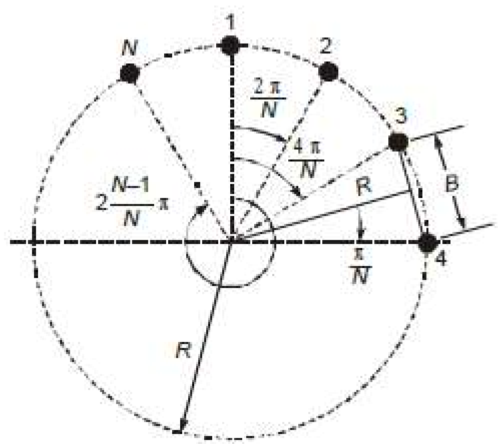



Fig :1.6 Bundle spacing *B*, and bundle radius *R*.

The spacing between adjacent sub-conductors is termed 'Bundle Spacing' and denoted byB. The radius of the pitch circle on which the sub-conductors are located will be called the'Bundle Radius', denoted as *R*. The radius of each sub-conductor is *r* with diameter *d*. The anglesub-tended at the centre by adjacent sub-conductors is (2□/N) radians, and it is readily seen that

$$\frac{B}{2} = R \sin(\pi/N) \text{ giving } R = B/2 \sin(\pi/N) \qquad (1.9)$$

For $N = 2$ to 18, the following table gives $(R/B)$ and $(B/R)$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $N =$ | 2 | 3 | 4 | 6 | 8 | 12 | 18 |
| $R/B =$ | 0.5 | 0.578 | 0.7071 | 1 | 1.308 | 1.874 | 2.884 |
| $B/R =$ | 2 | $\sqrt{3}$ | $\sqrt{2}$ | 1 | 0.7654 | 0.5344 | 0.3472 |

### 1.11 Geometric Mean Radius of Bundle (Equivalent Radius)

Except for calculating the surface voltage gradient from the charge of each sub-conductor, for most other calculations the bundle of *N*-sub-conductors can be replaced by a single conductor having an equivalent radius. This is called the 'Geometric Mean Radius' or simply the 'Equivalent Radius.' It will be shown below that its value is

*req*= (*N*.*r*.*RN*–1)1/N = *r*[*N*.(*R*/*r*)*N*–1]1/*N* = *R*(*N*.*r*/*R*)1/*N* ... (1.10)

It is the *N*th root of the product of the sub-conductor radius *r'*, and the distance of this subconductor from all the other (*N*– 1) companions in the bundle. Equation (1.11) is derived as follows: Referring to Fig. 1.5, the product of (*N* – 1) mutual distances is

$$\left(2R\sin\frac{\pi}{N}\right)\left(2R\sin\frac{2\pi}{N}\right)\left(2R\sin\frac{3\pi}{N}\right)\ldots\left(2R\sin\frac{N-1}{N}\pi\right)$$

$$= (2R)^{N-1}\left(\sin\frac{\pi}{N}\right)\left(\sin\frac{2\pi}{N}\right)\ldots\left(\sin\frac{N-1}{N}\pi\right)$$

$$r_{eq} = \left[r.(2R)^{N-1}\sin\frac{\pi}{N}.\sin\frac{2\pi}{N}\cdots\sin\frac{N-1}{N}\pi\right]^{1/N} \quad (1.11)$$

For $N = 2,\ r_{eq} = (2rR)^{1/2}$

For $N = 3,\ r_{eq} = \left(2^2.R^2.r.\sin\frac{\pi}{3}.\sin\frac{2\pi}{3}\right)^{1/3} = (3rR^2)^{1/3}$

For $N = 4,\ r_{eq} = \left(2^3.R^3.r.\sin\frac{\pi}{4}.\sin\frac{2\pi}{4}.\sin\frac{3\pi}{4}\right)^{1/4} = (4rR^3)^{1/4}$

For $N = 6,\ r_{eq} = \left(2^5.R^5.r.\sin\frac{\pi}{6}.\sin\frac{2\pi}{6}\cdots\sin\frac{5\pi}{6}\right)^{1/6} = (6.r.R^5)^{1/6}$

This is equation (3.12) where the general formula is $r_{eq} = (N.r.R^{N-1})^{1/N}$.

The reader should verify the result for $N = 8, 12, 18$.

### 1.12 INDUCTANCE OF E.H.V. LINE CONFIGURATIONS

Fig. 1.7 shows several examples of line configuration used in various parts of the world. They range from single-circuit (S/C) 400 kV lines to proposed 1200 kV lines. Double-circuit (D/C) lines are not very common, but will come into practice to save land for the line corridor. As pointed out in chapter 2, one 750 kV circuit can transmit as much power as 4-400 kV circuits and in those countries where technology for 400 kV level exists there is a tendency to favour the four-circuit 400 kV line instead of using the higher voltage level. This will save on import of equipment from other countries and utilize the know-how of one's own country. This is a National Policy and will not be discussed further.

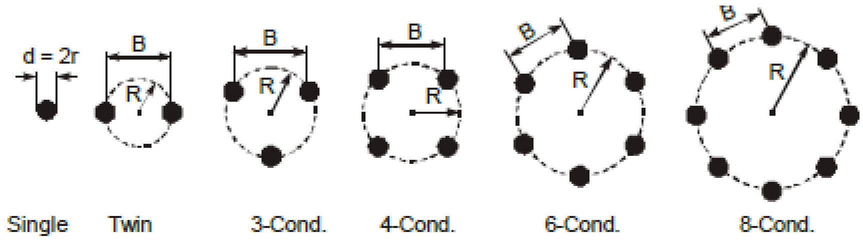



Fig.1.7 EHV Line Configuration

## 1.13 Inductance of Two Conductors :

We shall very quickly consider the method of handling the calculation of inductance of two conductors each of external radius *r* and separated by a distance *D* which forms the basis for the calculation of the matrix of inductance of multi-conductor configurations.

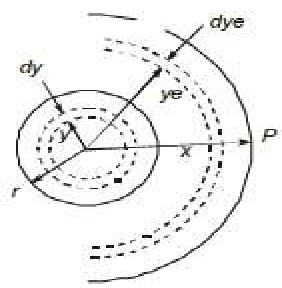



**Fig. 1.8 Round conductor with internal and external flux linkages.**

Figure 1.8 shows a round conductor carrying a current *I*. We first investigate the flux linkage experienced by it due, up to a distance *x*, to its own current, and then extend it to two conductors. The conductor for the present is assumed round and solid, and the current is also assumed to be uniform ly distributed with a constant value for current density $J = I/\pi r^2$. There are two components to the flux linkage: (1) flux internal to the conductor up to *r*; and (2) flux external to the conductor from *r* up to *x*.

***Inductance Due to Internal Flux***

At a radius *y* inside the conductor, Ampere's circuital law gives $H.dl$= current enclosed. With a uniform current density *J*, the current enclosed up to radius *y* is $I_y = y^2 I/r^2$. This gives,

$$H_y.2\pi y = Iy^2/r^2 \text{ or, } H_y = \frac{I}{2\pi r^2}.y \qquad (1.12)$$

Now, the energy stored in a magnetic field per unit volume is

$$w_y = \frac{1}{2}\mu_0\mu_r H_y^2 = \frac{I^2\mu_0\mu_r}{8\pi^2 r^4}y^2, \text{ Joules/m}^3 \qquad (1.13)$$

Consider an annular volume at $y$, thickness $dy$, and one metre length of conductor. Its volume is $(2\pi y.dy.1)$ and the energy stored is

$$dW = 2\pi y.w_y.dy = \frac{I^2\mu_0\mu_r}{4\pi r^4}y^3.dy$$

Consequently, the total energy stored up to radius $r$ in the conductor can be calculated. But this is equal to $\frac{1}{2}L_i I^2$, where $= L_i =$ inductance of the conductor per metre due to the internal flux linkage.

Therefore,

$$\frac{1}{2}L_i I^2 = \int_0^r dW = \frac{I^2\mu_0\mu_r}{4\pi r^4}\int_0^r y^3.dy = \frac{\mu_0\mu_r}{16\pi}I^2 \qquad (1.14)$$

Consequently,

$$L_i = \mu_0\mu_r/8\pi, \text{ Henry/metre} \qquad (1.15)$$

For a non-magnetic material, $\mu_r = 1$. With $\mu_0 = 4\pi \times 10^{-7}$ H/m, we obtain the interesting result that irrespective of the size of the conductor, the inductance due to internal flux linkage is

$$L_i = 0.05\ \mu \text{ Henry/metre for } \mu_r = 1$$

The effect of non-uniform current distribution at high frequencies is handled in a manner similar to the resistance. Due to skin effect, the internal flux linkage decreases with frequency, contrary to the behaviour of resistance. The equation for the inductive reactance is (W.D. Stevenson, 2nd Ed.)

$$X_i(f) = R_0.(X/2).\frac{\text{Ber}(X).\text{B}'\text{er}(X) + \text{Bei}(X).\text{B}'\text{ei}(X)}{[\text{B}'\text{er}(X)]^2 + [\text{B}'\text{ei}(X)]^2} \qquad (1.17)$$

where$X_i(f)$ = reactance due to internal flux linkage at any frequency $f$, $R_0$ = dc resistance of conductor per mile in ohms, and $X = 0.0636\ f/R_0$ . [If $R_m$= resistance per metre, then $X = 1.59 \times 10^{-3}\ f/R_m$. ]. Figure 3.2 (b) shows the ratio $L_i/L_0$ plotted against $X$, where $L_i = X_i/2\pi f$ and $L_0 = \mu_0/8\pi$ derived before.

***Inductance Due to External Flux***

Referring to Figure 3.6 and applying Ampere's circuital law around a circle of radius $y_e$ on which the field strength $H$ is same everywhere, the magnetic field strength is given as

$H_e = I/2\pi y_e$ giving $B_e = \mu_0\mu_r I/2\pi y_e$

Since e.h.v. line conductors are always located in air, $\mu_r = 1$. In a differential distance $dy_e$, the magnetic flux is $d\phi = B_e.dy_e$permetre length of conductor. Consequently, the flux linkage of conductor due to external flux up to a distance $x$ is

$$\psi_e = \int_r^x B_e . dy_e = \frac{\mu_0 \mu_r}{2\pi} I . \ln(x/r) \quad (1.18)$$

The inductance is $L_e = \Psi_e / I = \frac{\mu_0 \mu_r}{2\pi} \ln(x/r)$

In air, and with $\mu_0 = 4\pi \times 10^{-7}$ H/m,

$$L_e = 0.2 \ln (x/r) \ \mu\text{H/m}. \quad (1.19)$$

For a round conductor with uniform current density, the combined inductance due to internal and external flux linkage up to distance *x* from the centre of conductor is

*L* = 0.2[0.25 + ln (*x*/*r*)] = 0.2[in 1.284 + ln (*x*/*r*)]= 0.2 ln (*x*/0.7788*r*), □H/m or mH/km

This expression can be interpreted as though the effective radius of conductor becomes *re* = 0.7788 × actual radius. We emphasize here that this applies only to a solid round conductorwith uniform current density distribution inside. It does not apply to stranded conductors norat alternating currents where the current density is not uniformly distributed. For strandedconductors and at power frequency, conductor manufacturers provide data of the effective radius to be used for inductance calculation. This is known as the 'Geometric Mean Radius' andthe reader should consult catalogues of conductor details. Its average value lies between 0.8and 0.85 times the conductor radius.

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### The Two-Conductor Line

Figure 1.9 depicts two conductors each of radius *r*, separated by a centre-to-centre distance *D*, and carrying currents *I* and – *I*. We will derive expression for the flux linkage and inductance of 1 metre length of the 2-conductor system which will enable us to translate the result to the case of a single conductor located at a height *H* = *D*/2 above a ground plane. First consider a flux line □□*e* flowing external to both conductors. It is clear that this line links zero current and so the magnetic field strength is zero. Therefore, all the flux must flow in between the conductors from *r* to (*D* – *r*). The flux linkage of conductor 1 shown on the left has two parts: Due to its own current *I*, and (*ii*) due to the current – *I* in conductor 2. Neglecting internal flux linkage, the flux linkage due to its own current in the absence of current inconductor 2 up to the distance (*x*) is

$$\psi_{11} = \int_r^x d\psi_{11} = \frac{\mu_0 \mu_r}{2\pi} I \int_r^x dx/x = \frac{\mu_0 \mu_r}{2\pi} I \ln \frac{x}{r} \quad (1.20)$$

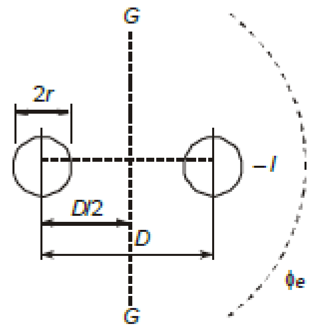


Fig. 1.9 Flux linkage calculation of 2-conductor line.

Consider the effect of current in conductor 2. Fleming's rule shows that the flux is in the same direction as that produced by current in conductor 1. The flux linkage of conductor 1 dueto current in conductor 2 is

$$\psi_{12} = \int_{D-r}^{x} d\psi_{12} = \frac{\mu_0\mu_r}{2\pi} I \ln\frac{D-r}{x}, x \to \infty. \quad (1.21)$$

Hence, the total flux linkage of conductor 1 due to both currents is

$$\psi_1 = \psi_{11} + \psi_{12} = \frac{\mu_0\mu_r}{\pi} I \ln\frac{D-r}{r} \approx \frac{\mu_0\mu_r}{\pi} I . \ln(D/r) \quad (1.22)$$

(when D r). The centre line $G – G$ between the two conductors is a flux line in the field of two equal but opposite currents. The inductance of any one of the conductors due to flux flowing up to the plane $G – G$ will be one half that obtained from equation (1.22). This is

$$L = \frac{\mu_0\mu_r}{2\pi} \ln(D/r) \quad (1.22)$$

## 1.14 Inductance of Multi-Conductor Lines—Maxwell's Coefficients

In the expression for the inductance $L = 0.2 \ln(2H/r)$ of a single conductor located above a ground plane, the factor $P = \ln(2H/r)$ is called Maxwell's coefficient. When several conductors are present above a ground at different heights each with its own current, the system of $n$conductors can be assumed to consist of the actual conductors in air and their images below ground carrying equal currents but in the opposite direction which will preserve the ground plane as a flux line. This is shown in Fig. 1.10.

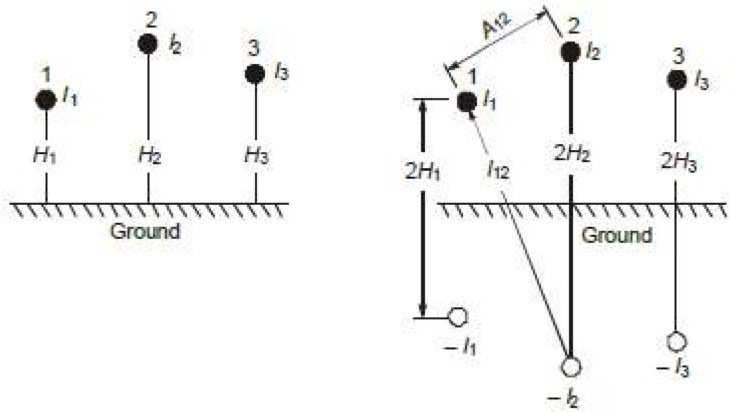


Fig. 1.10 Multi-conductor line above ground with image conductors below ground.

The flux linkage of any conductor, say 1, consists of 3 parts in a 3-phase line, due its own current and the contribution from other conductors. The self flux linkage is □11 = (□0/2□) *I*1 ln (2*H*/*r*). We may use the geometric mean radius instead of *r* to account for internal flux linkage so that we write □11 = (□02□) *I*1 ln (2H/*Ds*), where *Ds* = self-distance or GMR. For a bundleconductor, we will observe that an equivalent radius of the bundle, equation (3.12), has to be used. Now consider the current in conductor 2 only and the flux linkage of conductor 1 due to this and the image of conductor 2 located below ground. For the present neglect the presence of all other currents. Then, the flux lines will be concentric about conductor 2 and only those lines beyond the aerial distance *A*12 from conductor 1 to conductor 2 will link conductor 1. Similarly, considering only the current–*I*2 in the image of conductor 2, only those flux lines flowing beyond the distance *I*12 will link the aerial conductor 1.

Consequently, the total flux linkage of phase conductor 1 due to current in phase 2 will be

$$\psi_{12} = \frac{\mu_0\mu_r}{2\pi}\left[I_2\int_{A_{12}}^{\infty} dx/x - I_2\int_{I_{12}}^{\infty} dx/x\right] = \frac{\mu_0\mu_r}{2\pi} I_2 \ln(I_{12}/A_{12}) \qquad (1.23)$$

The mutual Maxwell's coefficient between conductors 1 and 2 will be

$$P_{12} = \ln (I_{12}/A_{12})$$

In general, it is evident that the mutual Maxwell's coefficient for the flux linkage of conductor $i$ with conductor $j$ (and vice-versa) will be with $i, j = 1, 2, \ldots n$,

$$P_{ij} = \ln (I_{ij}/A_{ij}),\ i \neq j. \qquad (1.24)$$

Thus, for a system of *n* conductors (phases or poles) shown in Fig. 3.8, the flux-linkage matrix is

$$[\psi]_n = \frac{\mu_0\mu_r}{2\pi}[P]_{nn}[I]_n = [L]_{nn}[I]_n \qquad (1.25)$$

where $[\psi]_n = [\psi_1, \psi_2, \ldots, \psi_n]_t$,

$[I_n] = [I_1, I_2, \ldots, I_n]_t$,

and the elements of Maxwell's coefficient matrix are

$$P_{ii} = \ln (2H/r_{eq}) \text{ and } P_{ji} = P_{ij} = \ln (I_{ij}/A_{ij}),\ i \neq j \qquad (1.26)$$

## 1.14 Bundled Conductor Lines: Use of Equivalent Radius

In this section we will show that for a bundle conductor consisting of *N* sub-conductors, the denominator in the self Maxwell's coefficient is to be taken as $r_{eq}$of equation (1.11). This is doneunder the following basic assumptions:

(1) The bundle spacing *B* between adjacent sub-conductors or the bundle radius *R* is very small compared to the height*H*of the phase conductor above ground. This allows the use of 2*H* as the distance between any sub-conductor of the bundle and the image of all the other (*N*–1) sub-conductors below ground, as shown in Fig. 3.9. This means that

*I*11= *I*12 = *I*13 = ... = *I*1*N* = 2*H*

(2) The total current carried by the bundle is *I* and that of each sub-conductor is $i = I/N$.

(3) Internal flux linkages are omitted, but can be included if the problem warrants it. Consider the flux linkage of conductor 1, which is

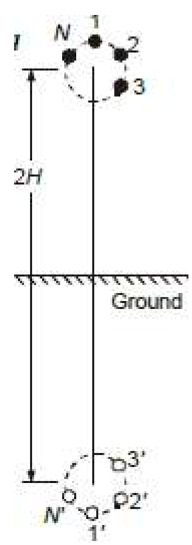


Fig :1.11Calculation of equivalent radius of bundle.

$$\psi_1 = \frac{\mu_0\mu_r}{2\pi}\frac{I}{N}\left[\ln\frac{2H}{r} + \ln\frac{2H}{2R\sin\frac{\pi}{N}} + \ln\frac{2H}{2R\sin\frac{2\pi}{N}} + \cdots + \ln\frac{2H}{2R\sin\frac{N-1}{N}}\right]$$

$$= \frac{\mu_0\mu_r}{2\pi}\frac{I}{N}\ln\frac{(2H)^N}{r(2R)^{N-1}.\sin\frac{\pi}{N}.\sin\frac{2\pi}{N}\cdots\sin\frac{N-1}{N}\pi}$$

$$= \frac{\mu_0\mu_r}{2\pi}I\ln\frac{2H}{r_{eq}} \quad (1.27)$$

Where*req*is precisely what is given in equation (1.12). The self-inductance of the entire bundle is

$$L = \psi_1/I = \frac{\mu_0\mu_r}{2\pi}\ln(2H/r_{eq}) \quad (1.28)$$

while the inductance of each sub-conductor

$$L_c = \psi_1/i = \frac{\mu_0\mu_r}{2\pi}N\ln(2H/r_{eq}) \quad (1.29)$$

which is also *N* times the bundle inductance since all the sub-conductors are in parallel. The Maxwell's coefficient for the bundle is *Pb*= ln (2*H*/*req*), as for a single conductor with equivalent radius *req*.

## 1.14 LINE PARAMETERS FOR MODES OF PROPAGATION

The sequence parameters given in the previous section apply to steady-state conditions and usephasor algebra. The quantity *a* = 1 □120° = – 0.5 + *j*0.866 is a complex number. This is not always convenient when solving equations encountered with wave propagation on the phase or pole conductors which are characterized by (*a*) velocity of propagation, (*b*) attenuation, and (*c*) surge impedance. Following the ideas propounded by Dr. Fortescue, the waves on multiconductor lines can also be resolved into 'modes of propagation'. The transformation matrix [ *T* ] and its inverse [ *T* ]–1 have to be evaluated for a given problem through standard set of rules which eventually diagonalize the given matrix of inductances, capacitances, resistances, surge impedances, and other parameters which govern the propagation characteristics. For a fullytransposed line, analytical expressions in closed form can be obtained for the transformation matrix and its inverse using real numbers. But for untransposed lines the evaluation of [ *T*] and [ *T* ]–1 can also be carried case by case when numerical values for the inductances, etc. are given. These will be discussed in detail below.

### DiagonalizationProcedure :

The resolution of mutually-interacting components of voltage, current, charge, or energy in waves propagating on the multi-conductors depends upon diagonalization of the *n* × *n* impedance matrix. A general procedure is given here while their application to Radio Noise, Switching Surges, etc, will be discussed in later chapters when we consider these problems individually. First consider the diagonalization of the inductance matrix of a transposed line

$$[L] = \begin{bmatrix} L_s, & L_m, & L_m \\ L_m, & L_s, & L_m \\ L_m, & L_m, & L_s \end{bmatrix} \quad (1.30)$$

***Step 1.***We evaluate the 'characteristic roots' or 'eigenvalues' (□) of the given matrix according to the determinantal equation

$$|\lambda[U]-[L]| = 0, \text{ which gives } \begin{vmatrix} \lambda - L_s, & -L_m, & -L_m \\ -L_m, & \lambda - L_s, & -L_m \\ -L_m, & -L_m, & \lambda - L_s \end{vmatrix} = 0$$

This gives $\lambda^3 - 3L_s\lambda^2 + 3(L_s^2 - L_m^2)\lambda - (L_s^3 - 3L_sL_m^2 + 2L_m^3) = 0$

or, $(\lambda - L_s - 2L_m)(\lambda - L_s + L_m)^2 = 0$

The three eigenvalues are

$$\lambda_1 = L_s + 2L_m,\ \lambda_2 = L_s - L_m,\ \lambda_3 = L_s - L_m$$

***Step 2.***For each of these eigenvalues in turn, we evaluate the 'eigenvector' [*x*], which is a column matrix, according to the equation.

$\{[U]\lambda_n - [L]\}[x] = [0]$

Considering $\lambda_1 = L_s + 2L_m$, there results the explicit equation

$$\{\lambda_1[U]-[L]\}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = L_m \begin{bmatrix} 2, & -1, & -1 \\ -1, & 2, & -1 \\ -1, & -1, & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

which in turn yields the three equations for $x_1, x_2, x_3$ to be

$$\left.\begin{aligned} 2x_1 - x_2 - x_3 &= 0 \\ -x_1 + 2x_2 - x_3 &= 0 \\ -x_1 - x_2 + 2x_3 &= 0 \end{aligned}\right\}$$

By choosing *x*1 = 1, there results *x*2 = *x*3 = *x*1 = 1. These corresponding eigenvector is [1, 1,1]*t*with the normalized form [1, 1, 1]*t*(1/□3). By following a similar procedure for □2 = *Ls*– *Lm*, there results

$$\{\lambda_2[U]-[L]\}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = -L_m \begin{bmatrix} 1, & 1, & 1 \\ 1, & 1, & 1 \\ 1, & 1, & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The three equations for *y*1, *y*2, *y*3 are all equal to *y*1 + *y*2 + *y*3 = 0. Once again letting *y*1 = 1, we have *y*2 + *y*3 = – 1. Now we have an infinite number of choices for the values of *y*2 and *y*3, and we make a judicial choice for them based on practical engineering considerations and utility. As a first choice, let *y*2 = 0. Then *y*3 = – 1. The resulting eigen-vector and its normalized form are

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \text{and } \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Since the third eigenvalue □3 is also the same as □2, we obtain the same equations for the components of the eigenvector which we can designate as[*z*1, *z*2, *z*3]*t*. By choosing *z*1 = 1 and *z*3 = 1, we obtain *z*2 = – 2. The third eigenvector and its normalized form are [1, –2, 1]*t*and (1/□6) [1, – 2, 1]*t*.

***Step 3.***Formulate the complete 3 × 3 eigenvector matrix or in general, *n* × *n* for the *n* eigenvalues and call it the inverse of the transformation matrix [*T*]–1. For the problem underconsideration.

$$[T]^{-1} = \begin{bmatrix} \sqrt{2}, & \sqrt{2}, & \sqrt{2} \\ \sqrt{3}, & 0, & -\sqrt{3} \\ 1, & -2, & 1 \end{bmatrix} 1/\sqrt{6}$$

*Step 4.* The transformation matrix [*T*] will be

$$[T] = \begin{bmatrix} \sqrt{2}, & \sqrt{3}, & 1 \\ \sqrt{2}, & 0, & -2 \\ \sqrt{2}, & -\sqrt{3}, & 1 \end{bmatrix} 1/\sqrt{6}$$

which turns out to be the transpose of $[T]^{-1}$.Their determinant is – 6.

*Step 5.* The given inductance matrix is diagonalized by the relation

$$[T]^{-1}\,[L]\,[T] = \begin{bmatrix} L_s + 2L_m, & & \\ & L_s - L_m, & \\ & & L_s - L_m \end{bmatrix} = [\lambda]$$

This is a diagonal matrix whose elements are equal to the three eigenvalues, which might be seen to be the same as the sequence inductances presented to the voltage and current. But now they will be called the inductances for the three modes of propagation of electromagnetic energy of the waves generating them.

## 1.5 RESISTANCE AND INDUCTANCE OF GROUND RETURN

Under balanced operating conditions of a transmission line, ground-return currents do not flow. However, many situations occur in practice when ground currents have important effect on system performance. Some of these are:

(*a*) Flow of current during short circuits involving ground. These are confined to single line to ground and double line to ground faults. During three phase to ground faults the system is still balanced;

(*b*) Switching operations and lightning phenomena;

(*c*) Propagation of waves on conductors;

(*d*) Radio Noise studies.

The ground-return resistance increases with frequency of the current while the inductance decreases with frequency paralleling that of the resistance and inductance of a conductor. In all cases involving ground, the soil is inhomogeneous and stratified in several layers with different values of electrical conductivity. In this section, the famous formulas of J.R. Carson (B.S.T.J. 1926) will be given for calculation of ground resistance and inductance at any frequency in a homogeneous single-layer soil. The problem was first applied to telephone transmission but we will restrict its use to apply to e.h.v. transmission lines. The conductivity of soils has the following orders of magnitude: 10° mho/metre for moist soil, 10–1 for loose soil, 10–2 for clay, and 10–3 for bed rock.

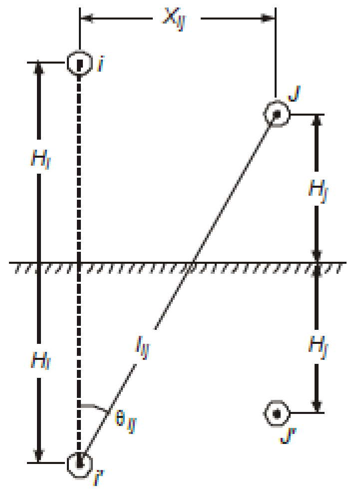


Fig1.12 Geometrical parameters for calculation of ground-return resistance and inductance.

the important parameters involved in the calculation showing two conductors *i* and *j* above ground together with their images. We take

*cs*= soil conductivity in mho/m,

*f* = frequency of current, Hz,

*G* = 1.7811, Euler's number, □0 = 4□□× 10–7 H/m,

*Iij*= distance of conductor *j* from image of conductor *i*, metre,

□*ij*= arc tan [*Xij*/(*Hi* + *Hj*)], radians

The most important parameter involved in the calculation is

*Fij*= *Iij*2□□0 *fcs* ...(3.70)

For usual e.h.v. configurations, *Fij*< 1.

[When c.g.s. units are used, *Iij*is in cm, and *cs*is also in c.g.s. e.m. units. To convert ohm/

m into c.g.s. units, the multiplying factor is 10-–11. Then, *Fij*= 8 2 ]

*Iij*□□*fcs*

Having calculated *Fij*, the ground resistance and inductance are

*Rg*= 8□*f Jr*. 10–4, ohm/km

and $L_g = 4.J_i.10^{-4}$, Henry/km

where $J_r$ and $J_i$ are calculated as follows:

$$J_r = (1 - S_4)\frac{\pi}{8} + 0.5.S_2.\ln(2/G\,F_{ij}) + 0.5\,\theta_{ij}T_2 - \frac{1}{2}W_1 + \frac{1}{2\sqrt{2}}W_2$$

$$J_i = 0.25 + 0.5\,(1 - S_4).\ln(2/G\,F_{ij}) - 0.5\,\theta_{ij}\,T_4 - \frac{\pi}{8}S_2 + \frac{1}{\sqrt{2}}(W_1 + W_3) - 0.5W_4$$

There are several quantities above, (S, T, W), which are given by Carson by the following infinite series when *Fij*< 1. For most calculations, only two or three leading terms will be sufficient as will be shown by an example of a horizontal 400-kV line.

$$S_2 = \sum_{k=0}^{\infty}(-1)^k.(F_{ij}/2)^{2(2k+1)}\frac{1}{(2k+1)!(2k+2)!}\cos(2k+1)2\theta_{ij}$$

$T_2$ = same as $S_2$ with cosine changed to sine

$$S_4 = \sum_{k=1}^{\infty}(-1)^{k-1}\left(\frac{F_{ij}}{2}\right)^{4k}\frac{1}{(k+1)!(k+2)!}\cos(4k)\theta_{ij}$$

$T_4$ = same as $S_4$ with cosine changed to sine

$$W_1 = \sum_{k=1}^{\infty}(-1)^{k-1}F_{ij}^{(4k-1)}\frac{1}{1^2.3^2.5^2\ldots(4k-1)}\cos(4k-3)\theta_{ij}$$

$$W_2 = 1.25\ S_2,\ W_4 = \frac{5S_4}{3}$$

$$W_3 = \sum_{k=1}^{\infty}(-1)^{k-1}F_{ij}^{(4k-1)}\frac{1}{3^2 5^2\ldots(4k+1)}\cos(4k-1)\theta_{ij}$$

The important and interesting properties of *Rg*and *Lg*for a- 3-phase line are illustrated by taking an example of the horizontal 400-kV line. These properties come out to be

$$[R_g] \approx R_g\begin{bmatrix}1, & 1, & 1\\ 1, & 1, & 1\\ 1, & 1, & 1\end{bmatrix}\text{and}[L_g] \approx L_g\begin{bmatrix}1, & 1, & 1\\ 1, & 1, & 1\\ 1, & 1, & 1\end{bmatrix}$$

# CHAPTER 2

# VOLTAGE GRADIENTS OF CONDUCTORS

## 2.1 ELECTROSTATICS

Conductors used for e.h.v. transmission lines are thin long cylinders which are known as 'linecharges'. Their charge is described in coulombs/unit length which was used for evaluating thecapacitance matrix of a multiconductor line in Chapter 3. The problems created by charges onthe conductors manifest themselves as high electrostatic field in the line vicinity from powerfrequency to *TV* frequencies through audio frequency, carrier frequency and radio frequency(*PF, AF, CF, TVF*). The attenuation of travelling waves is also governed in some measure bythe increase in capacitance due to corona discharges which surround the space near theconductor with charges. When the macroscopic properties of the electric field are studied, theconductor charge is assumed to be concentrated at its centre, even though the charge isdistributed on the surface. In certain problems where proximity of several conductors affectsthe field distribution, or where conducting surfaces have to be forced to become equipotentialsurfaces (in two dimensions) in the field of several charges it is important to replace the givenset of charges on the conductors with an infinite set of charges. This method is known as theMethod of Successive Images. In addition to the electric-field properties of long cylinders, thereare other types of important electrode configurations useful for extra high voltage practice inthe field and in laboratories. Examples of this type are sphere-plane gaps, sphere-to-spheregaps, point-to-plane gaps, rod-to-plane gaps, rod-rod gaps, conductor-to-tower gaps, conductorto-conductor gap above a ground plane, etc. Some of these types of gaps will also be dealt within this chapter which may be used for e.h.v. measurement, protection, and other functions.The coaxial-cylindrical electrode will also be discussed in great detail because of its importancein corona studies where the bundle of *N* sub-conductors is strung inside a 'cage' to simulate thesurface voltage gradient on the conductors in a setup which is smaller in dimensions than anactual outdoor transmission line.

### 2.1.1 Field of a Point Charge and Its Properties

The laws governing the behaviour of this field will form the basis for extendingthem to other geometries. Consider Figure 4.1 which shows the source point $S_1$ where a pointcharge + $Q$ coulombs is located. A second point charge $q$ coulomb is located at $S_2$ at a distance$r$ metre from $S_1$. From Coulomb's Law, the force acting on either charge is

$F = Q.q/4\pi e_0 e\ r^2$ , Newton` [$e = (\mu\ g\ )_{-} = 1000 / 36\pi\mu\mu F/m = 8.84194\ \mu\mu F/m$

$e_r$= relative permittivity of the medium = 1 in air]

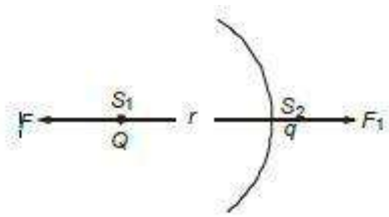


**Fig. 2.1** Point charge $Q$ and force on test charge $q$.

Where $q$ is very small ($q \to 0$) , we define the electric field produced by $Q$ at the location of $q$ as

$$E = \lim_{q \to 0} (F/q) = Q / 4\pi e_0 e_r r^2 \text{, Newton/Coulomb} \quad (2.1)$$

The condition $q \to 0$ is necessary in order that $q$ might not disturb the electric field of $Q$.
Equation (4.2) may be re-written as

$$(4\pi r^2)(e_0 e_r E) = 4\pi r^2 D = Q \quad (2.2)$$

Here we note that $4\pi r_2$ = surface are of a sphere of radius $r$ drawn with centre at $S_1$. Thequantity $D = e_0 e_r E$ is the dielectric flux density. Thus, we obtain Gauss's Law which states that 'the surface integral of the normal component of dielectric flux density over a closedsurface equals the total charge in the volume enclosed by the surface'. This is a general relationand is valid for all types of electrode geometries.

Some important properties of the field of a point charge can be noted:

($a$) The electric field intensity decreases rapidly with distance from the point charge inversely as the square of the distance, ($Q \propto 1/ r_2$) .

($b$) $E$ is inversely proportional to $e_r$.($E \propto 1/e_r$ ).

($c$) The potential of any point in the field of the point charge $Q$, defined as the work done against the force field of $Q$ in bringing $q$ from $\infty$ to $S_2$, is

$$\psi = \int_{\infty}^{r} -E dr = \frac{Q}{4\pi e_0 e_r} \int_{\infty}^{r} -\frac{dr}{r^2} = \frac{Q}{4\pi e_0 e_r} \frac{1}{r} \text{, volt} \quad (2.3)$$

($d$ ) The potential difference between two points at distances $r_1$ and $r_2$ from $S_1$ will be

$$\psi_{12} = \frac{Q}{4\pi e_0 e_r} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \text{, volt} \quad (2.4)$$

For a positive point charge, points closer to the charge will be at a higher positivepotential than a further point.

(*e*) The capacitance of an isolated sphere is

$$C = Q/\psi = 4\pi e_0 e_r r, \text{ Farad} \tag{2.5}$$

This is based on the assumption that the negative charge of – $Q$ is at infinity.These properties and concepts can be extended in a straightforward manner to apply tothe field of a line charge and other electrode configurations.

## 2.2 FIELD OF SPHERE GAP

A sphere-sphere gap is used in h.v. laboratories for measurement of extra high voltages and forcalibrating other measuring apparatus. If the gap spacing is less than the sphere radius, thefield is quite well determined and the sphere-gap breaks down consistently at the same voltagewith a dispersion not exceeding ±3%. This is the accuracy of such a measuring gap, if otherprecautions are taken suitably such as no collection of dust or proximity of other groundedobjects close by. The sphere-gap problem also illustrates the method of successive images usedin electrostatics.Figure 4.2 shows two spheres of radii $R$ separated by a centre-centre distance of $S$, withone sphere at zero potential (usually grounded) and the other held at a potential $V$. Since bothspheres are metallic, their surfaces are equipotentials. In order to achieve this, it requires aset of infinite number of charges, positive inside the left sphere at potential $V$ and negativeinside the right which is held at zero potential. The magnitude and position of these chargeswill be determined from which the voltage gradient resulting on the surfaces of the sphere ona line joining the centres can be determined. If this exceeds the critical disruptive voltage, aspark break-down will occur. The voltage required is the breakdown voltage.

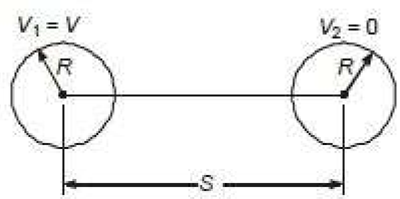


Fig. 2.2 The sphere gap.

Consider two point charges $Q_1$ and $Q_2$ located with a separation $D$, Figure 4.3. At a point$P(x, y)$ with coordinates measured from $Q_1$, the potentials are as follows:Potential at $P$ due to $Q_1$

$$= \frac{Q_1}{4\pi e_0}\frac{1}{r_1} \text{ with } r_1 = \sqrt{x^2 + y^2} \tag{2.6}$$

Potential at $P$ due to $Q_2$ $= \frac{Q_2}{4\pi e_0}\frac{1}{r_2} \text{ with } r_2 = \sqrt{(D-x)^2 + y^2}$ (2.7)

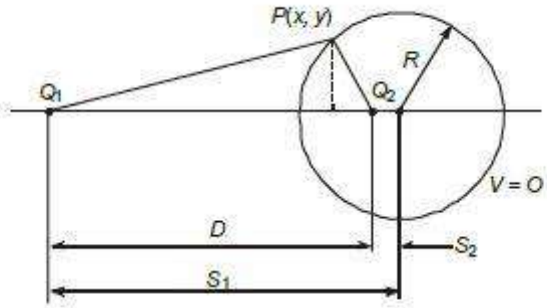


Fig. 2.3 Point charge $Q_1$ and sphere of radius $R$.

The total potential at $P$ is

$$V_P = \frac{1}{4\pi e_0}(Q_1/r_1 + Q_2/r_2). \qquad (2.8)$$

If this is to be zero, then $Q_2/Q_1 = -r_2/r_1$ (2.9)

This clearly shows that $Q_1$ and $Q_2$ must be of opposite polarity.

From (4.7),

$$r_2^2/r_1^2 = \frac{(D-x)^2 + y^2}{x^2 + y^2} = Q_2^2/Q_1^2, \text{ giving} \qquad (2.10)$$

$$\left\{x - \frac{D}{1-(Q_2/Q_1)^2}\right\}^2 + y^2 = D^2(Q_2/Q_1)^2/\{1-(Q_2/Q_1)^2\}^2 \qquad (2.11)$$

This is an equation to a circle in the two-dimensional plane and is a sphere in threedimensionalspace.

$$R = D(Q_2/Q_1)/\{1-(Q_2/Q_1)^2\} \qquad (2.12)$$

This requires $Q_2$ to be less than $Q_1$ if the denominator is to be positive. The centre of thezero-potential surface is located at ($S_1$, 0), where

$$S_1 = D/\{1-(Q_2/Q_1)^2\} = \frac{Q_1}{Q_2}R \qquad (2.13)$$

This makes $S_2 = S_1 - D = \frac{D(Q_2/Q_1)^2}{1-(Q_2/Q_1)^2} = \frac{Q_2}{Q_1}R$ (2.14)

Therefore, the magnitude of $Q_2$ in relation to $Q_1$ is

$$Q_2 = Q_1 \frac{S_2}{R} = Q_1 \frac{R}{S_1} \quad (2.15)$$

Also, $S_1 S_2 = R^2$ (2.16)

## 2.3 FIELD OF LINE CHARGES AND THEIR PROPERTIES

Figure 4.6 shows a line charge of $q$ coulomb/metre and we will calculate the electric fieldstrength, potential, etc., in the vicinity of the conductor. First, enclose the line charge by aGaussian cylinder, a cylinder of radius $r$ and length 1 metre. On the flat surfaces the field willnot have an outward normal component since for an element of charge $dq$located at $S$, therecan be found a corresponding charge located at $S'$ whose fields (force exerted on a positive testcharge) on the flat surface $F$ will yield only a radial component. The components parallel to theline charge will cancel each other out. Then, by Gauss's Law, if $E_p$= field strength normal tothe curved surface at distance $r$ from the conductor,

$(2\pi r)(e_0 e_r E_p) = q$

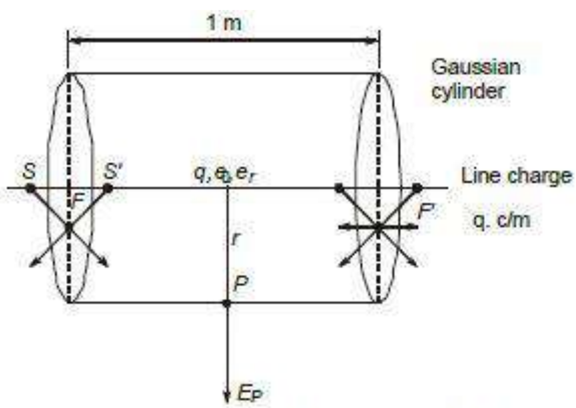


**Fig. 2.4 Line charge with Gaussian cylinder.**

The field strength at a distance $r$ from the conductor is

$$E_p = (q / 2\pi e_0 e_r)(1/r), \text{ Volts/metre} \quad (2.17)$$

This is called the $(1/r)$-field as compared to the $(1/r_2)$-field of a point charge.

Let a reference distance $r_0$ be chosen in the field. Then the potential difference betweenany point at distance $r$ and the reference is the work done on a unit test charge from $r_0$ to $r$.

Thus, $$V_r = \frac{q}{2\pi e_0 e_r} \int_{r_0}^{r} -\frac{1}{\rho} d\rho = \frac{q}{2\pi e_0 e_r} (\ln r_0 - \ln r) \quad (2.18)$$

In the case of a line charge, the potential of a point in the field with respect to infinity cannot be defined as was done for a point charge because of logarithmic term. However, we can find the p.d. between two points at distances $r_1$ and $r_2$, since(p.d. between $r_1$ and $r_2$) = (p.d. between $r_1$ and $r_0$) – (p.d. between $r_2$ and $r_0$)

$$V_{12} = \frac{q}{2\pi e_0 e_r}(\ln r_2 - \ln r_1) = \frac{q}{2\pi e_0 e_r}\ln\frac{r_2}{r_1} \qquad (2.19)$$

In the field of a positive line charge, points nearer the charge will be at a higher positivepotential than points farther away ($r_2 > r_1$).The potential (p.d. between two points, one of them being taken as reference $r_0$) in thefield of a line charge is logarithmic. Equipotential lines are circles. In a practical situation, thecharge distribution of a transmission line is closed, there being as much positive charge asnegative.

## 2.4 CHARGE-POTENTIAL RELATIONS FOR MULTI-CONDUCTOR LINES

Section 3.5 in the last Chapter 3, equations (3.38) to (3.40) describe the charge-potential relationsof a transmission line with *n* conductors on a tower. The effect of a ground plane considered asan equipotential surface gave rise to Maxwell's Potential coefficients and the general equations
are

$$[V] = [P][Q/2\pi e_0] \qquad (2.20)$$

where the elements of the three matrices are, for *i*, *j* = 1, 2, ..., *n*

$$\left.\begin{array}{l} [V] = [V_1, V_2, V_3, \ldots, V_n]_t \\ [Q/2\pi e_0] = (1/2\pi e_0)[Q_1, Q_2, \ldots, Q_n]_t \\ P_{ii} = \ln(2H_i / r_{eq}(i)), P_{ij} = \ln(I_{ij}/A_{ij}), i \neq j \end{array}\right\} \qquad (2.21)$$

The equivalent radius or geometric mean radius of a bundled conductor has already beendiscussed and is

$$r_{eq} = R(N.\ r/R)^{1/N} \qquad (2.22)$$

where$R$ = bundle radius = $B/2 \sin(\pi/N)$ ,$B$ = bundle spacing (spacing between adjacent conductors) $r$ = radius of each sub-conductor,and $N$ = number of conductors in the bundle the eliments of Maxwells potentials coefficients are all known since they depend only on the given dimensions of the line-conductor configuration on the tower. In all problems of interestin e.h.v. transmission, it is required to find the charge matrix from the voltage since this is also known. The charge-coefficient matrix is evaluated as

$$[Q/2\pi e_0] = [P]^{-1}[V] = [M][V] \tag{2.23}$$

or, if the charges themselves are necessary,

$$[Q] = 2\pi e_0[M][V] \tag{2.24}$$

In normal transmission work, the quantity $Q/2\pi e_0$ occurs most of the time and henceequation (4.32) is more useful than (4.33). The quantity $Q/2\pi e_0$ has units of volts and theelements of both [$P$] and [$M$] = [$P$]$^{-1}$ are dimensionless numbers.

### 2.4.1 Maximum Charge Condition on a 3-Phase Line

E.H.V. transmission lines are mostly single-circuit lines on a tower with one or two groundwires. For preliminary consolidation of ideas, we will restrict our attention here to 3 conductorsexcited by a balanced set of positive-sequence voltages under steady state. This can be extendedto other line configurations and other types of excitation later on. The equation for the chargesis,

$$\frac{1}{2\pi e_0}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} M_{11}, & M_{12}, & M_{13} \\ M_{21}, & M_{22}, & M_{23} \\ M_{31}, & M_{32}, & M_{33} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{2.25}$$

For a 3-phase ac line, we have

$V_1 = 2V\sin(wt+\phi)$,

$V_2 = 2V\sin(wt+\phi-120°)$

and$V_3 = 2V\sin(wt+\phi+120°)$with

$V$ = r.m.s. value of line-to-ground voltage and $w = 2\pi f$,

$f$ = power frequency in Hz. The angle $\phi$ denotes the instant on $V_1$ where

$t = 0$. If $wt+\phi$ is denoted as $\theta$, there results

$$\begin{aligned} Q_1/2\pi e_0 &= \sqrt{2}V[M_{11}\sin\theta + M_{12}\sin(\theta-120°) + M_{13}\sin(\theta+120°)] \\ &= \sqrt{2V}[\{M_{11} - 0.5(M_{12}+M_{13})\}\sin\theta + 0.866(M_{13}-M_{12})\cos\theta] \end{aligned} \tag{2.26}$$

Differencating with respect to θ and equating to zero to get

$$\frac{d}{d\theta}(Q_1/2\pi e_0) = 0$$

$$= \sqrt{2}V[(M_{11} - 0.5M_{12} - 0.5M_{13})\cos\theta - \frac{\sqrt{3}}{2}(M_{13} - M_{12})\sin\theta] \quad (2.27)$$

This gives the value of $\theta = \theta_m$ at which $Q_1$ reaches its maximum or peak value. Thus,

$$\theta_m = \arctan[\sqrt{3}(M_{13} - M_{12})/(2M_{11} - M_{12} - M_{13})]^{-1} \quad (2.28)$$

Substituting this value of $\theta$ in equation (4.35) yields the maximum value of $Q_1$.An alternative procedure using phasor algebra can be devised. Expand equation (4.35) as

$$Q_1/2\pi e_0 = \sqrt{2}V\sqrt{(M_{11} - .5M_{12} - .5M_{13})^2 + 0.75(M_{13} - M_{12})^2}\sin(\theta + \psi) \quad (2.29)$$

The amplitude or peak value of 1 0 $Q / 2\pi e$ is

$$(Q_1/2\pi e_0)_{\max} = \sqrt{2}V[M_{11}^2 + M_{12}^2 + M_{13}^2 - (M_{11}M_{12} + M_{12}M_{13} + M_{13}M_{11})]^{1/2} \quad (2.30)$$

It is left as an exercise to the reader to prove that substituting $\theta_m$ from equation (4.36) inequation (4.35) gives the amplitude of 1 0 max $(Q / 2\pi e)$ in equation (4.38).Similarly for $Q_2$ we take the elements of 2nd row of [*M*].

$$(Q_2/2\pi e_0)_{\max} = \sqrt{2}V[M_{22}^2 + M_{21}^2 + M_{23}^2 - (M_{21}M_{22} + M_{22}M_{23} + M_{23}M_{21})]^{1/2} \quad (2.31)$$

For $Q_3$ we take the elements of the 3rd row of [*M*].

$$(Q_3/2\pi e_0)_{\max} = \sqrt{2}V[M_{33}^2 + M_{31}^2 + M_{32}^2 - (M_{31}M_{32} + M_{31}M_{33} + M_{33}M_{31})]^{1/2} \quad (2.32)$$

The general expression for any conductor is, for $i$ = 1, 2, 3,

$$(Q_i/2\pi e_0)_{\max} = \sqrt{2}V[M_{i1}^2 + M_{i2}^2 + M_{i3}^2 - (M_{i1}M_{i2} + M_{i2}M_{i3} + M_{i3}M_{i1})]^{1/2} \quad (2.33)$$

### 2.4.2 Numerical Values of Potential Coefficients and Charge of Lines

In this section, we discuss results of numerical computation of potential coefficients and chargespresent on conductors of typical dimensions from 400 kV to 1200 kV whose configurations are given in Chapter 3, Figure 3.5. For one line, the effect of considering or neglecting the presenceof ground wires on the charge coefficient will be discussed, but in a digital-computer programmethe ground wires can be easily accommodated without difficulty. In making all calculations we must remember that the height $H_i$ of conductor $i$ is to be taken as the average height. It will be quite adequate to use the relation, as proved later,

$$H_{av} = H_{min} + \text{Sag}/3 \quad (2.34)$$

***Average Line Height for Inductance Calculation****

The shape assumed by a freely hanging cable of length $L_c$ over a horizontal span $S$ betweensupports is a catenary. We will approximate the shape to a parabola for deriving the averageheight which holds for small sags. Figure 4.10 shows the dimensions required. In this figure,

$H$ = minimum height of conductor at midspan

$S$ = horizontal span,

d = sag at midspan

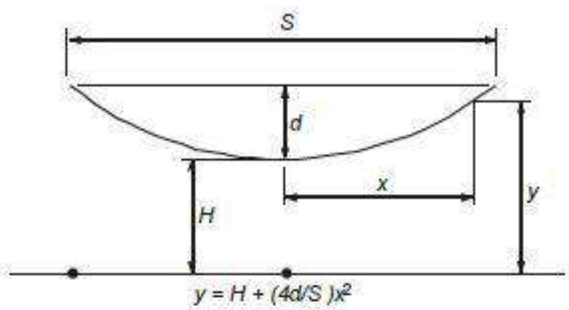


**Fig. 2.5** Calculation of average height over a span $S$ with sag $d$.

The equation to the parabolic shape assumed is

$$y = H + (4d/S^2)\, x^2 \quad (2.35)$$

The inductance per unit length at distance $x$ from the point of minimum height is

$$L = 0.2 \ln (2y/r) = 0.2 [\ln 2y - \ln r] \quad (2.36)$$

Since the height of conductor is varying, the inductance also varies with it.The average inductance over the span is

$$L_{av} = \frac{1}{S}\int_{-S/2}^{S/2} 0.2(\ln 2y - \ln r)dx \quad (2.37)$$

Voltage Gradients of Conductors

$$= \frac{0.4}{S}\int_{0}^{S/2} (\ln 2y - \ln r)dx \quad (2.38)$$

Now, $\ln 2y = \ln(2H + 8dx^2/S^2) = \ln(8d/S^2) + \ln(x^2 + S^2H/4d)$

Let $a^2 = S^2H/4d$. Then,

$$\ln(8d/S^2) + \frac{2}{S}\int_0^{S/2} \ln(x^2 + a^2)dx = \ln[(1 + H/d)\, S^2/4]$$

$$- 2 + 2\sqrt{H/d}\tan^{-1}\sqrt{d/H} + \ln(8d/S^2)$$

$$= \ln 2d + \ln(1 + H/d) - 2 + 2\sqrt{H/d}\tan^{-1}\sqrt{d/H} \quad (2.39)$$

If the right-hand side can be expressed as $\ln(2H_{av})$, then this gives the average height forinductance calculation. We now use some numerical values to show that $H_{av}$is approximatelyequal to

$$\left(H + \frac{1}{3}d\right) = H_{min} + \frac{1}{3}\text{Sag} \quad (2.40)$$

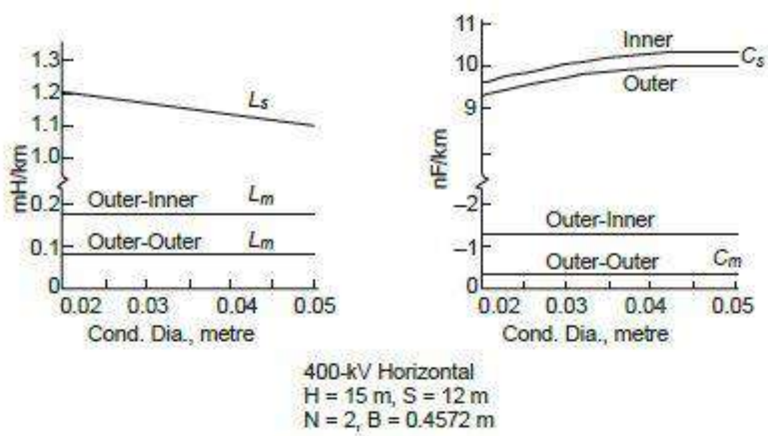


**Fig. 2.6 Inductances and capacitances of 400 kV horizontal line.**

(*a*) Consider $H = 10$, $d = 10$. Using these in equation (4.47) give

$\ln 20 + \ln(1 + 1) - 2 + 2\tan^{-1} 1 = 3 + 0.6931 - 2 + \pi/2 = 3.259 = \ln 26 = \ln(2H_{av})$

$H_{av} = 26/2 = 13 = 10 + 3 = H + 0.3d$

(*b*) $H = 10$, $d = 8$. $\ln 16 + \ln 2.25 - 2 + 2\ 1.25\tan^{-1} 0.8 = \ln 24.9$

$H_{av} = 12.45 = H + 2.45 = H + 0.306d$

(*c*) $H = 14$, $d = 10$. $H_{av} = 17.1 = 14 + 3.1 = H + 0.3d$

These examples appear to show that a reasonable value for average height is $H_{av} = H_{min}$+sag/3. A rigorous formulation of the problem is not attempted here.Figure 4.11, 4.12 and 4.13 show inductances and capacitances of conductors for typical 400kV, 750 kV and 1200 kV lines.

## 2.5 SURFACE VOLTAGE GRADIENT ON CONDUCTORS

The surface voltage gradient on conductors in a bundle governs generation of corona on theline which have serious consequences causing audible noise and radio interference. They alsoaffect carrier communication and signalling on the line and cause interference to televisionreception. The designer of a line must eliminate these nuisances or reduce them to tolerablelimits specified by standards, if any exist. These limits will be discussed at appropriate placeswhere AN, RI and other interferring fields are discussed in the next two chapters. Since coronageneration depends on the voltage gradient on conductor surfaces, this will be taken up now fore.h.v. conductors with number of sub-conductors in a bundle ranging from 1 to *N*. The maximumvalue of *N* is 8 at present but a general derivation is not difficult

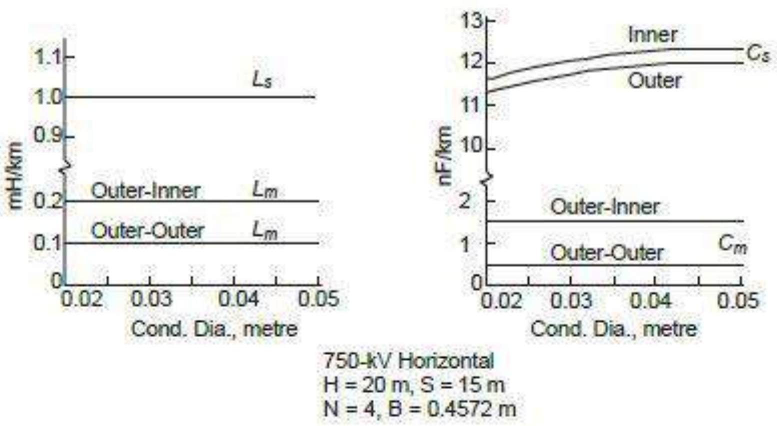


**Fig. 2.7** *L* **and** *C* **of 750 kV horizontal line.**

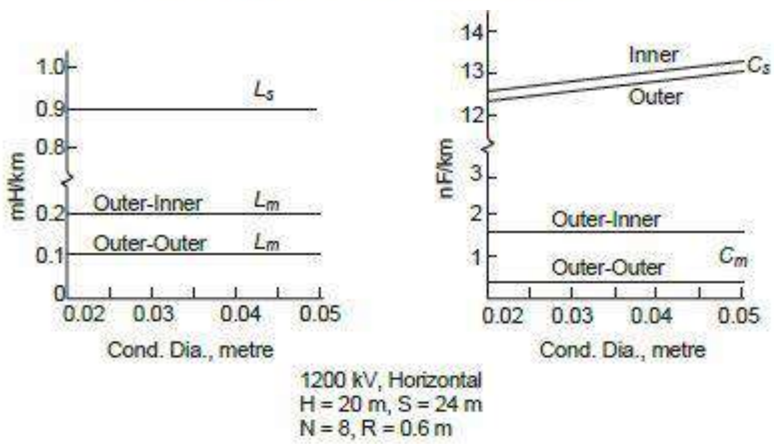


**Fig. 2.8** *L* **and** *C* **of 1200 kV horizontal line.**

## 2.5.1 Single Conductor

Figure 4.9 can be used for a single conductor whose charge is *q* coulomb/metre. We havealready found the line charges or the term s($Q_i/2\pi e_0$) in terms of the voltages $V_i$ and the Maxwell's

Potential Coefficient matrix [*P*] and its inverse [*M*], where $i = 1, 2, ..., n$, the number of conductors on a tower. For the single conductor per phase or pole, the surface voltage gradient is

$$E_c = \frac{q}{2\pi e_0} \frac{1}{r} \text{ volts/metre} \tag{2.41}$$

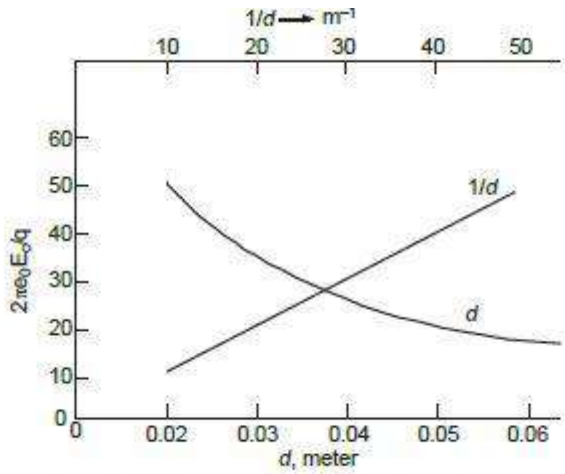


**Fig. 2.9** Voltage gradient of single conductor.

This is plotted in Figure 4.14 as a function of conductor diameters ranging from 0.02 to 0.065 m. The factor $E_c/(q/2\pi e_0)$ is also plotted against the reciprocal of diameter and yields approximately a straight line.

### 2.5.2 2-Conductor Bundle

In this case, the charge $Q$ obtained from equation (4.33) is that of the total bundle so that thecharge of each sub-conductor per unit length is $q = Q/2$. This will form one phase of an ac lineor a pole of a dc line. In calculating the voltage gradient on the surface of a sub-conductor, wewill make the following assumptions:

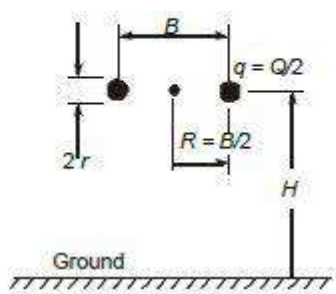


**Fig. 2.10** 2-conductor bundle above ground for voltage gradient calculation.

(1) The conductors of the other phases or poles are very far from the bundled conductor under examination, *i.e.* $S$ $B$ or $2R$.

(2) The image conductors are also very far, i.e. $2H$ $B$ or $2R$.

This allows us to ignore all other charges except that of the conductors in the bundle.Now, by definition, the electric field intensity is the force exerted on a unit positive test chargeplaced at the point where the field intensity is to be evaluated, which in this case is a point onthe sub-conductor surface. Consider point $P_i$ on the inside of the bundle, Figure 4.16. The forceon a test charge is$E_i$= Force due to conductor charge – Force due to the charge on second conductor ofbundle.At the point $P_0$ on the outside of the bundle, the two forces are directed in the same sense.It is clear that it is here that the maximum surface voltage gradient occurs.

Now, the force due to conductor charge = $e\ rq12\ \pi\ 0$. In computing the force due to thecharge of the other sub-conductor there is the important point that the conductors are metallic.When a conducting cylinder is located in the field of a charge, it distorts the field and the fieldintensity is higher than when it is absent. If the conducting cylinder is placed in a uniform fieldof a charge $q$, electrostatic theory shows that stress-doubling occurs on the surface of themetallic cylinder. In the present case, the left cylinder is placed at a distance $B$ from its companionat right. Unless $B\ r$, the field is non-uniform. However, for the sake of calculation of surfacevoltage gradients on sub-conductors in a multi-conductor bundle, we will assume that the fieldis uniform and stress-doubling takes place. Once again, this is a problem in successive imagesbut will not be pursued here.

### 2.5.3 Maximum Surface Voltage Gradients for N>3

The method described before for calculating voltage gradients for a twin-bundle conductor, $N$ =2, can now be extended for bundles with more than 2 sub-conductors. A general formula will beobtained under the assumption that the surface voltage gradients are only due to the chargesof the $N$ sub-conductors of the bundle, ignoring the charges of other phases or poles and thoseon the image conductors. Also, the sub-conductors are taken to be spaced far enough from eachother so as to yield a uniform field at the location of the sub-conductor and hence the concept ofstress-doubling will be used.

Figure 4.18 shows bundles with $N$ = 3, 4, 6, 8 sub-conductors and the point $P$ where themaximum surface voltage gradient occurs. The forces exerted on a unit positive test charge at$P$ due to all $N$ conductor-charges $q$ are also shown as vectors. The components of these forcesalong the vector force due to conductor charge will yield the maximum surface voltage gradient.Due to symmetry, the components at right angles to this force will cancel each other, as shownon the figures.

$N$ = 3. $B$ = 3$R$ for equilateral spacing.

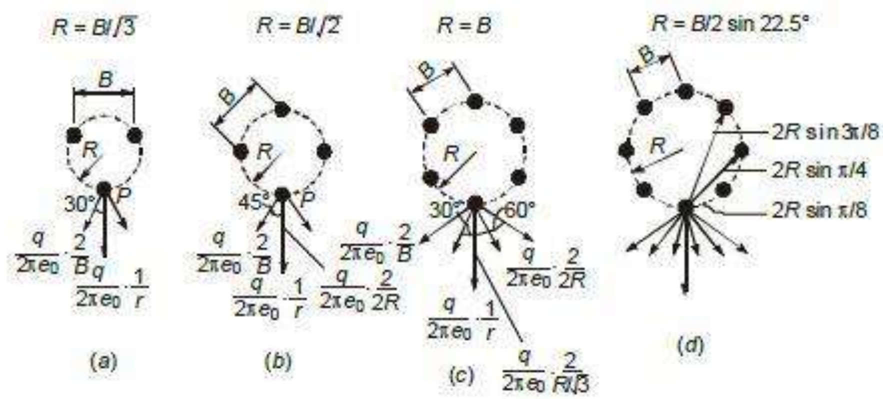


**Fig. 2.11** Distribution of surface voltage gradients on 3-, 4-, 6-, and 8-conductor bundles.

$$E_P = \frac{q}{2\pi e_0}\frac{1}{r}\left(1 + 2\times r.\frac{2}{B}\cos 30°\right) = \frac{q}{2\pi e_0}\frac{1}{r}(1 + 2r/R) \tag{2.42}$$

The sub-conductor charge is $q = Q/3$ where $Q$ is obtained from equation (4.32),[ / 2 ] [ ][ ] $Q$ $\pi e_0 = M\ V$ as discussed earlier.$N = 4.B = 2R$ for quadrilateral spacing.

$$E_P = \frac{q}{2\pi e_0}\left(\frac{1}{r} + \frac{2}{2R} + 2\times\frac{2}{R\sqrt{2}}\cos 45°\right)$$

$$= \frac{q}{2\pi e_0}\frac{1}{r}(1 + 3r/R) \tag{2.43}$$

Note the emergence of a general formula

$$E_P = \frac{q}{2\pi e_0}\frac{1}{r}[1 + (N-1)r/R] \tag{2.44}$$

$N = 6$. $B = R$ for hexagonal spacing.

$$E_P = \frac{q}{2\pi e_0}\left[\frac{1}{r} + \frac{2}{2R} + 2\times\frac{2}{R}\cos 60° + 2\times\frac{2}{R\sqrt{3}}\cos 30°\right]$$

$$= \frac{q}{2\pi e_0}\frac{1}{r}(1 + 5r/R) = \frac{q}{2\pi e_0}\frac{1}{r}[1 + (N-1)r/R] \tag{2.45}$$

$$E_P = \frac{q}{2\pi e_0}\left[\frac{1}{r} + \frac{2}{2R} + 2\times\frac{2}{2R\sin\pi/8}\cos\frac{3\pi}{8} + 2\times\frac{2}{2R\sin\pi/4}\times\cos\frac{\pi}{4} + 2\frac{2}{2R\sin 3\pi/8}\cos\frac{\pi}{8}\right]$$
$$= \frac{q}{2\pi e_0}\left(\frac{1}{r} + \frac{1}{R} + \frac{2}{R} + \frac{2}{R} + \frac{2}{R}\right) = \frac{q}{2\pi e_0}\frac{1}{r}(1 + 7r/R)$$
$$= \frac{q}{2\pi e_0}\frac{1}{r}[1 + (N-1)r/R] \quad (2.46)$$

From the above analysis, we observe that the contributions to the gradient at $P$ from eachof the $(N-1)$ sub-conductors are all equal to $e\ Rq12\ \pi\ 0$. In general, for an $N$-conductor bundle,

$$E_P = \frac{q}{2\pi e_0}\left[\frac{1}{r} + \frac{2}{2R} + 2\times\frac{2}{2R\sin\pi/N}\cos\left(\frac{\pi}{2} - \frac{\pi}{N}\right) + 2\times\frac{2}{2R\sin 2\pi/N}\cos\left(\frac{\pi}{2} - \frac{2\pi}{N}\right) + \ldots + 2\times\frac{2}{2R\sin(N-1)\pi/N}\cos\left(\frac{\pi}{2} - \frac{N-1}{N}\pi\right)\right]$$
$$= \frac{q}{2\pi e_0}\left[\frac{1}{r} + \frac{1}{R} + \frac{(N-2)}{R}\right] = \frac{q}{2\pi e_0}\frac{1}{r}[1 + (N-1)r/R] \quad (2.47)$$

## 2.6 DISTRIBUTION OF VOLTAGE GRADIENT ON SUB-CONDUCTORS OF BUNDLE

While discussing the variation of surface voltage gradient on a 2-conductor bundle in section4.5.2, it was pointed out that the gradient distribution follows nearly a cosine law, equation(4.52). We will derive rigorous expressions for the gradient distribution and discuss theapproximations to be made which yields the cosine law. The cosine law has been verified tohold for bundled conductors with up to 8 sub-conductors. Only the guiding principles will beindicated here through an example of a 2-conductor bundle and a general outline for $N \geq 3$ willbe given which can be incorporated in a digital-computer programme.Figure 4.24 shows detailed view of a 2-conductor bundle where the charges $q$ on the twosub-conductors are assumed to be concentrated at the conductor centres. At a point $P$ on thesurface of a conductor at angle θ from the reference direction, the field intensities due to the
two conductor charges are, using stress-doubling effect,

$$E_1 = \frac{q}{2\pi e_0}\frac{1}{r} \text{ and } E_2 = \frac{q}{2\pi e_0}\frac{2}{B'} \quad (2.48)$$

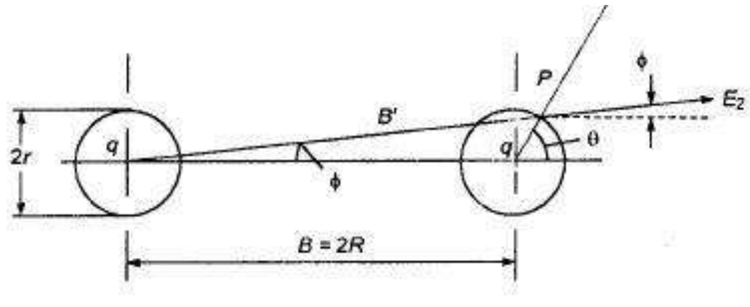


**Fig. 2.12** Gradient distribution calculation on 2-conductor bundle

# CHAPTER 3

# CORONA EFFECTS-I: POWER LOSS AND AUDIBLE NOISE

## 3.1 $I^2R$ LOSS AND CORONA LOSS

When compared to the *I*2*R* heating loss, the average corona losses on several lines from345 kV to 750 kV gave 1 to 20 kW/km in fair weather, the higher values referring to highervoltages. In foul-weather, the losses can go up to 300 kW/km. Since, however, rain does not fallall through the year (an average is 3 months of precipitation in any given locality) andprecipitation does not cover the entire line length, the corona loss in kW/km cannot be comparedto *I*2*R* loss directly. A reasonable estimate is the yearly average loss which amounts to roughly2 kW/km to 10 kW/km for 400 km lines, and 20-40 kW/km for 800 km range since usuallyhigher voltages are necessary for the longer lines. Therefore, cumulative annual average coronaloss amounts only to 10% of *I*2*R* loss, on the assumption of continuous full load carried. Withload factors of 60 to 70%, the corona loss will be a slightly higher percentage. Nonetheless,during rainy months, the generating station has to supply the heavy corona loss and in somecases it has been the experience that generating stations have been unable to supply full ratedload to the transmission line. Thus, corona loss is a very serious aspect to be considered in linedesWhen a line is energized and no corona is present, the current is a pure sine wave andcapacitive. It leads the voltage by 90°, as shown in Figure 3.1. It is still a current at power frequency,but only the fundamental component of this distorted current can result in power loss.

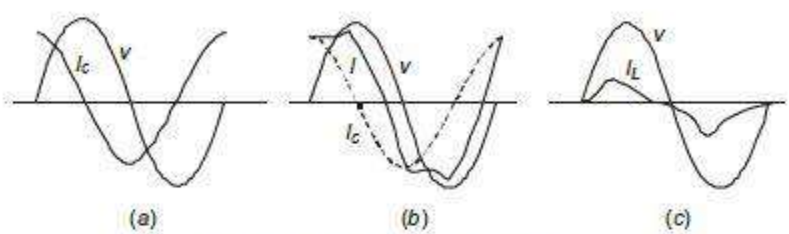


Fig. 3.1 Corona current waveform.

## 3.2 CORONA-LOSS FORMULAE

### 3.2.1 List of Formula

Corona-loss formulae were initiated by F.W. Peek Jr. in 1911. They all yield the power loss as afunction of (*a*) the corona-inception voltage, *Vo*; (*b*) the actual voltage of conductor, *V*; (*c*) theexcess voltage (*V* — *Vo*) above *Vo*; (*d*) conductor surface voltage gradient, *E*; (*e*) corona-inceptiongradient, *E*o; (*f*) frequency, *f*; (*g*) conductor size, *d*, and number of conductors in bundle, *N*, as well as line configuration; (*h*) atmospheric condition, chiefly rate of rainfall, ρ, and (*i*) conductorsurface condition.

A ***Those Based on Voltages***

(*i*) *Linear relationship* : Skilling's formula (1931):

$$Pc \propto V - Vo \quad (3.1)$$

(*ii*) *Quadratic relationship*

(*a*) Peek's formula (1911):

$$Pc \propto (V - Vo)2 \ldots \quad (3.2)$$

(*b*) Ryan and Henline formula (1924):

$$Pc \propto V(V - Vo) \ldots \quad (3.3)$$

(*c*) Peterson's formula (1933) :

$$Pc \propto V2.F\,(V/Vo) \ldots \quad (3.4)$$

where *F* is an experimental factor.

(*iii*) *Cubic Relationship*

(*a*) Foust and Menger formula (1928):

$$Pc \propto V3 \ldots \quad (3.5)$$

(*b*) Prinz's formula (1940):

$$Pc \propto V2\,(V - Vo) \quad (3.6)$$

B ***Those Based on Voltage Gradients***

(*a*) Nigol and Cassan formula (1961):

$$Pc \propto E2 \text{ In } (E/Eo) \ldots \quad (3.7)$$

(*b*) Project *EHV* formula (1966):

$$P_c \propto V.\, E_m,\ \text{m} = 5 \quad (3.8)$$

In order to obtain corona-loss figures from e.h.v. conductor configurations, outdoorexperimental projects are established in countries where such lines will be strung. The resultingmeasured values pertain to individual cases which depend on local climatic conditions existingat the projects. It is therefore difficult to make a general statement concerning which formulaor loss figures fit coronal losses universally. In addition to equations (3.1) to (3.8), the reader isreferred to the work carried out in Germany at the Rheinau 400 kV Research Project, inFrance at the Les Renardieres Laboratory, in Russia published in the

CIGRE Proceedings from1956–1966, in Japan at the CRIEPI, in Sweden at Uppsala, and in Canada by the IREQ andOntario Hydro.We will here quote some formulas useful for evaluating 3-phase corona loss in kW/km,which are particularly adopted for e.h.v. lines, and some which are classic but cannot be usedfor e.h.v. lines since they apply only to single conductors and not to bundles. There is noconvincing evidence that the total corona loss of a bundled conductor with *N* conductors is *N*times that of a single conductor.

**(1) *Nigol and Cassan Formula* (Ontario Hydro, Canada).**

$$Pc = K.f.r3.\ \theta\ .E2.\ \ln (E/E0),\ \text{kW/km, 3-ph} \ldots \quad (3.9)$$

where *f* = frequency in Hz, *r* = conductor radius in cm.,θ = angular portion in radians of conductor surface where the voltagegradient exceeds the critical corona-inception gradient,*E* = effective surface gradient at operating voltage *V*, kV/cm, r.m.s.*E*0 = corona-inception gradient for given weather and conductor surfacecondition, kV/cm, r.m.s.and *K* = a constant which depends upon weather and conductor surfacecondition.Many factors are not taken into account in this formula such as the number of subconductorsin bundle, etc.

**(2) *Anderson, Baretsky, McCarthy Formula* (Project EHV, USA)**

An equation for corona loss in rain giving the excess loss above the fair-weather loss in kW/3-
phase km is:

$$P_c = P_{FW} + 0.3606\ K.V.r^2.\ln(1 + 10\rho).\ \sum_{1}^{3N} E^5 \quad (3.10)$$

Here, *PFW* = total fair-weather loss in kW/km,= 1 to 5 kW/km for 500 kV, and 3 to 20 kW/km for 700 kV,*K* = 5.35 × 10–10 for 500 to 700 kV lines,= 7.04 × 10–10 for 400 kV lines (based on Rheinau results),*V* = conductor voltage in kV, line-line, r.m.s.,*E* = surface voltage gradient on the underside of the conductor, kV/cm,peak,ρ = rain rate in mm/hour,*r* = conductor radius in cm,and *N* = number of conductors in bundle of each phase [The factor 0.3606 =1/1.609 3 ].Formulae are not available for (*a*) snow, and (*b*) hoar frost which are typical of Canadianand Russian latitudes. The EHV Project suggests *K* = 1.27 × 10–9 for snow, but this is a highlyvariable weather condition ranging from heavy to light snow. Also, the conductor temperaturegoverns in a large measure the condition immediately local to it and it will be vastly differentfrom ground-level observation of snow.

The value of ρ to convert snowfall into equivalent rainfall rate is given as follows :Heavy snow : ρ = 10% of snowfall rate;Medium snow : ρ = 2.5% of snowfall rate;Light snow : ρ = 0.5% of snowfall rate.

The chief disadvantage in using formulae based upon voltage gradients is the lack ofdifinition by the authors of the formulae regarding the type of gradient to be used. As pointedout in Chapter 2, there are several types of voltage gradients on conductor surfaces in a bundle, such as nominal smooth conductor gradient present on a conductor of the same outer radius asthe line conductor but with a smooth surface, or the gradient with surface roughness takeninto account, or the average gradient, or the average maximum gradient, and so on. It istherefore evident that for Indian conditions, an outdoor e.h.v. project is the only way of obtainingmeaningful formulae or corona-loss figures applicable to local conditions.

## 3.3 The Corona Current

The corona loss $P_c$ is expressed as$P_c$ = 3 × line-to-ground voltage × in-phase component of current.From the previously mentioned expressions for $P_c$, we observe that different investigatorshave different formulas for the corona current. But in reality the current is generated by themovement of charge carriers inside the envelope of partial discharge around the conductor. Itshould therefore be very surprising to a discerning reader that the basic mechanism, being thesame all over the world, has not been unified into one formula for this phenomenon. We canobserve the expressions for current according to different investigators below.

**1. *Peek's Law.* F.W. Peek, Jr.**, was the forerunner in setting an example for others tofollow by giving an empirical formula relating the loss in watts per unit length of conductor Corona Effects-I: Power Loss and Audible Noise 117with nearly all variables affecting the loss. For a conductor of radius $r$ at a height $H$ aboveground,

$$P_c = 5.16 \times 10^{-3} f \, r / 2H \; V^2 (1 - V_o / V)^2, \text{kW/km} \ldots \quad (3.11)$$

where$V$, $V_o$ are in kV, r.m.s., and $r$ and $H$ are in metres. The voltage gradients are, at an air density of δ, for a smooth conductor,

$$E = V/r \ln (2H/r) \text{ and } E_o = 21.4\, \delta(1 + 0.0301/ r\delta) \ldots \quad (3.12)$$

**PROBLEMS**

**1.** For $r$ = 1 cm, $H$ = 5m, $f$ = 50 Hz, calculate corona loss $P_c$ according toPeek's formula when $E$ = 1.1 $E_o$, and δ = 1.

**Sol.** $E_0$ = 21.4(1+ 0.0301 / 0.01)= 27.84kV/cm,r.ms.∴ $E$ = 1.1 30.624 kV/cm. / 2 0.0316 $E_0 = r\ H =$
$V$ = 30.624 ln (10/0.01) = 211.4 kV. (line-to-line voltage= 211.4 3 = 366 kV)$P_c$ = 5.16 × $10^{-3}$ × 50 × 0.0316 × $211.4^2(1 - 1/1.1)^2$ = 2.954 kW/km≅ 3 kW/km.The expression for the corona-loss current is
$i_c$= $P_c/V$ = 5.16 × $10^{-2}$ $f\ r$ / 2$H$ $V(1 - V_o/V)^2$, Amp/km ... (3.13)
For this example, $i_c$= 3000 watts/211.4 kV = 14 mA/km.

2. *Ryan-Henline Formula*

$P_c = 4\ fCV(V - V_0).10^6$, kW/km ... (3.14)
Here $C$ = capacitance of conductor to ground, Farad/m= $_0$ 2$\pi e$ /ln (2$H$/$r$)and $V$, $V_0$ are in kV, r.ms.
We can observe that the quantity ($CV$) is the charge of conductor per unit length. Thecorona-loss current is$i_c$= 4 $fC(V - V_0)$. $10^6$, Amp/km ... (15)

**2.** For the previous example 3.1, compute the corona loss $P_c$ and current $i_c$using Ryan-Henline formulae, equations (3.14) and (3.15).

**Sol.** $P_c$ = 4 × 50 × $211.4^2$ (1 – 1/1.1) × $10^{-3}$/18 ln(1000)= 6.47 kW/km$i_c$= 6.47/211.4 = 3.06 × $10^{-2}$ Amp/km = 30.6 mA/km3. *Project EHV Formula.* Equation (10).

**3.** The following data for a 750 kV line are given. Calculate the corona lossper kilometre and the corona loss current.Rate of rainfall ρ = 5 mm/hr. $K$ = 5.35 × $10^{-10}$, $P_{FW}$ = 5 kW/km$V$ = 750 kV, line-to-line. $H$ = 18 m, $S$ = 15 m phase spacing $N$ = 4 sub-conductors each of $r$ = 0.0175 m with bundle spacing$B$ = 0.4572 m. (Bundle radius $R$ = $B$/ 2 = 0.3182 m). Use surface voltagegradient on centre phase for calculation.

**Sol.** From Mangoldt Formula, the gradient on the centre phase conductor will be$E$ = $V$.[1 + ($N$ – 1) $r/R$]/[N.r. ln {2$H/r_{eq}$(2$H/S)^2$ +1 }]where $r_{eq}$=$R$ ($N.r/R)^{1/N}$. Using the values given,$E$ = 18.1 kV/cm, r.m.s., = 18.1 2 peak = 25.456$P_c$ = 5 + 0.3606 × 5.35 × $10^{-10}$ × 750 × $0.175^2$ × ln(1 + 50) × 12 × $(25.456)^5$= 5 + 229 = 234 kW/km, 3-phaseThe current is$i_c$= $P_c$ / 3$V$ = 234 /750 3 = 0.54 A/km.

## 3.4 CHARGE-VOLTAGE ($q - V$) DIAGRAM AND CORONA LOSS

### *3.4.1 Increase in Effective Radius of Conductor and Coupling Factors*

The partial discharge of air around a line conductor is the process of creation and movement ofcharged particles and ions in the vicinity of a conductor under the applied voltage and field. Weshall consider a simplified picture for conditions occurring when first the voltage is passingthrough the negative half-cycle and next the positive half-cycle, as shown in Figure 3.2.

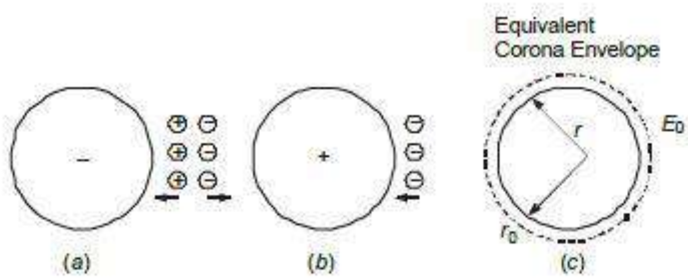


**Fig. 3.2 Space-charge distribution in corona and increase in effective radius of conductor**

In Figure 3.2(*a*), free electrons near the negative conductor when repelled can acquiresufficient energy to form an electron avalanche. The positive ions (a neutral molecule whichhas lost an electron) are attracted towards the negative conductor while the electrons drift intolower fields to attach themselves to neutral atoms or molecules of Nitrogen and Oxygen toform negative ions. Some recombination could also take place. The energy imparted for causinginitial ionization by collision is supplied by the electric field. During the positive half cycle, thenegative ions are attracted towards the conductor, but because of local conditions not all ionsdrift back to the conductor. A space charge is left behind and the hysteresis effect gives rise tothe energy loss. Furthermore, because of the presence of charged particles, the effective charge of the conductor ground electrode system is increased giving rise to an increase in effective

capacitance. This can be interpreted in an alternative manner by assuming that the conductordiameter is effectively increased by the conducting channel up to a certain extent where theelectric field intensity decreases to a value equal to that required for further ionization, namely,the corona-inception gradient, Figure 3.2(*c*).

**PROBLEMS**

1 single smooth conductor 1 cm in radius is strung 5 metres above ground;using Peek's formula for corona-inception gradient, find

(*a*) the corona-inception voltage,

(*b*) the equivalent radius of conductor to the outside of the corona envelope at 20% overvoltage. Take $\delta$ =1.

**Sol.**

(*a*) $E_0$ = 21.4(1 + 0.0301 / *r* )= 27.84 kV / cm=2784 kV /m.∴ $V_0$ = . . ln (2 / ) 2784 0.01 ln( 10 /0.01 $E_0 r$ $H$ $r$ = × × )= 192.3 kV, r.m.s.

(*b*) Let $r_0$ = effective radius of the corona envelopeThen, $E_{0c}$ = 2140(1 + 0.0301) / $r_0$ ) , and 1.2 × 192.3 = $E_{0c}.r_o$.ln (2*H*/$r_o$) giving230.8 = 2140 (1 + 0.0301/ $r_0$ ).$r_0$. ln(10 /$r_0$ ).

**5.** A single conductor 2.5 inch in diameter of a 525-kV line (line-to-line voltage)is strung 13 m above ground. Calculate (*a*) the corona-inception voltage and (*b*) the effectiveradius of conductor at an overvoltage of 2.5 p.u. Consider a stranding factor *m* = 1.25 forroughness. (*c*) Calculate the capacitance of conductor to ground with and without corona. (*d*) Ifa second conductor is strung 10 m away at the same height, calculate the coupling factors inthe two cases. Take $\delta$ =1.

**Sol.** *r* = 0.03176 m, *H* = 13.(*a*) $E_{0r}$ = 2140 × 0.8 (1 + 0.0301/ 0.03176)= 2001 kV/m = 20 kV/cm.$V_0$ = $E_0$.r. ln (2*H*/*r*) = 532.73 kV r.m.s., line-to-ground,= 753.4 kV peak.

At 525 kV, r.m.s., line-to-line, there is no corona present.

(*b*) 2.5 p.u. voltage = 2.5×525 2 /3 = 1071.65 kV, crest.

Therefore, corona is present since the corona-onset voltage is 753.4 kV, crest.When considering the effective radius, we assume a smooth surface for the envelopeso that1071.65 = 3000 (1+ 0.0301 / $r_0$ ).$r_0$. ln(26 /$r_0$ ).A trial and error solution yields $r_0$ = 0.05 metre. This is an increasein radius of 0.05 – 0.03176 = 0.01824 metre or 57.43%.

(*c*) *C* = 2 / ln(2 / ). $\pi e_0$ *H* *r*

Without corona, *C* = 8.282 nF/km;

With corona, *C* = 8.88 nF/km,

(*d*) The potential coefficient matrix is

$$[P] = \begin{bmatrix} P_{11}, & P_{12} \\ P_{21}, & P_{22} \end{bmatrix} \text{and} [M] = [P]^{-1} = \begin{bmatrix} P_{22}, & -P_{12} \\ -P_{21}, & P_{11} \end{bmatrix} \frac{1}{\Delta}$$

whereΔ = |[P]|, the determinant.

The self-capacitance is $2\pi /\Delta\, e_0 P_{11}$ while the mutual capacitance is $-2 / .\pi e_o P_{12}\, \Delta$ Thecoupling factor is $K_{12} = -P_{12}/P_{11}$.

Without corona $K_{12} = -\ln( 26_2+10_2 /10) / \ln(26 / 0.03176 )= -1.0245/6.7076 = -0.15274$

With corona, $K_{12} = -1.0245/ \ln (26/.05) = -0.1638$.This is an increase of 7.135%.

For bundled conductors, coupling factors between 15% to 25% are found in practice. Notethat with a switching surge of 1000 kV crest, the second conductor experiences a voltage ofnearly 152 kV crest to ground so that the voltage between the conductors could reach 850 Kvcrest.

## 3.5 Charge-Voltage Diagram with Corona

When corona is absent the capacitance of a conductor is based on the physical radius of themetallic conductor. The charge-voltage relation is a straight line *OA* as shown in Figure 3.3and *C* = $q_0/V_0$, where $V_0$ = the corona-inception voltage and $q_0$ the corresponding charge. However,beyond this voltage there is an increase in charge which is more rapid than given by the slope*C* of the straight-line $q_0 — V_0$ relation. This is shown as the portion *AB* which is nearly straight.When the voltage is decreased after reaching a maximum $V_m$there is a hysteresis effect andthe q – *V* relation follows the path *BD*. The slope of *BD* almost equals *C* showing that the spacecharge cloud near the conductor has been absorbed into the conductor and charges far enoughaway from the conductor are not entirely pulled back. The essential properties of the *q* –*V*diagram for one half-cycle of an ac voltage or the unipolar lightning and switching impulses canbe obtained from the trapezoidal area *OABD* which represents the energy loss.

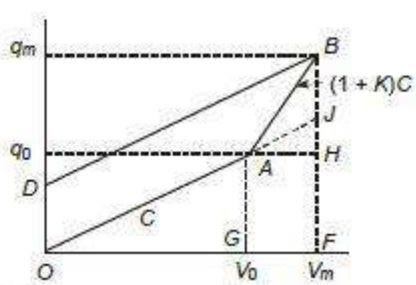


**Fig. 3.3 Charge-Voltage diagram of corona.**

Let the slope of *AB* equal (1 + *K*) *C* where *K* is an experimental factor which lies between0.6 and 0.8 having an average value of 0.7. The maximum charge correspondihg to $V_m$is denoted as$q_m$. The area of *OABD* equals ( )21 20*KC V* 2 *V m* – as shown below:

Area *OABD* = (Area *DOFB*) – (Area *OAG*) – Area (*GAHF*) – Area (*AHB*)= (Area *DOFB*) – 0.5 $q_0V_0 - q_0\,(V_m - V)_0 - 0.5(q_m - q_0)(V_m - V_0)$= Area DOFB – 0.5 $q_0\{V_m + q_m(V_m - V_0)\}$... (5.15)

Now, $BH = q_m - q_0 = (1+K)\, q_0\,(V_m - V_0)/V_0$

$JH = q_0\,(V_m - V_0)/V_0$and

$BF = q_m = q_0 + (1+K)\, q_0\,(V_m - V_0)/V_0$

$\therefore\ DO = BJ = BH - JH = (1+K)\, q_0\,(V_m - V_0)/V_0$ ... (5.16)

$$\text{Area}\quad DOFB = \frac{1}{2}(DO + BF)V_m = Kq_0(V_m - V_0)V_m/V_0 + \frac{1}{2}q_0V_m^2/V_0$$

$$\text{Area}\quad OAG = \frac{1}{2}q_0V_0;\ \text{Area}\ AGFH = q_0(V_m - V_0).$$

$$\text{Area}\quad AHB = \frac{1}{2}(q_m - q_0)(V_m - V_0) = \frac{1}{2}q_0(V_m^2 - V_0^2)/V_0 + \frac{1}{2}Kq_0(V_m - V_0)^2/V_0$$

(3.17)

$$\text{Area}\quad OAG = \frac{1}{2}q_0V_0;\ \text{Area}\ AGFH = q_0(V_m - V_0).$$

$$\text{Area}\quad AHB = \frac{1}{2}(q_m - q_0)(V_m - V_0) = \frac{1}{2}q_0(V_m^2 - V_0^2)/V_0 + \frac{1}{2}Kq_0(V_m - V_0)^2/V_0$$

$$\therefore\quad \text{Areas}\ (OAG + AGFH + AHB) = \frac{1}{2}(q_0/V_0)[V_m^2 + K(V_m - V_0)^2].$$

For a unipolar waveform of voltage the energy loss is equal to equation (3.21). For an acvoltage for one cycle, the energy loss is twice this value.

$$\therefore\ W_{ac} = KC(V_m^2 - V_0^2)\ \ldots \qquad (3.18)$$

The corresponding power loss will be

$$P_c = f\,W_{ac} = fKC(V_m^2 - V_0^2) \qquad (3.19)$$

If the maximum voltage is very close to the corona-inception voltage $V_0$, we can write

$$V_m^2 - V_0^2 = (V_m + V_0)(V_m - V_0) = 2V_m(V_m - V_0),\ \text{so that}$$

$$P_c = 2f\,KCV_m(V_m - V_0) \qquad (3.20)$$

**6.** An overhead conductor of 1.6 cm radius is 10 m above ground. The normalvoltage is 133 kV r.m.s. to ground (230 kV, line-to-line). The switching surge experienced is 3.5p.u. Taking *K*= 0.7, calculate the energy loss per km of line. Assume smooth conductor.

**Solution.** $C = 2 / \ln (2 / )\pi e_0 H r$ = 7.79 nF/km $E_0 = 30(1 + 0.0301 / 0.016)$ = 37.14 kV/cm, crest
$\therefore V_0 = E_0.r. \ln (2H/r)$ = 423.8 kV, crest $V_m$= 3.5× 133 2 = 658.3 kV, crest $\therefore W = 0.5 \times 0.7 \times 7.79 \times 10_{-9}$ $(658.3_2 - 423.8_2) \times 10_6$ Joule/km= 0.7 kJ/km.

## 3.6 ATTENUATION OF TRAVELLING WAVES DUE TO CORONA LOSS

A voltage wave incident on a transmission line at an initial point $x = 0$ will travel with a velocity $v$ such that at a later time $t$ the voltage reaches a point $x = vt$ from the point of incidence, asshown in Figure 3.4 In so doing if the crest value of voltage is higher than the corona-disruptivevoltage for the conductor, it loses energy while it travels and its amplitude decreasescorresponding to the lower energy content. In addition to the attenuation or decrease inamplitude, the waveshape also shows distortion. In this section, we will discuss only attenuationsince distortion must include complete equations of travelling waves caused by inductance andcapacitance as well as conductor and ground-return resistance.

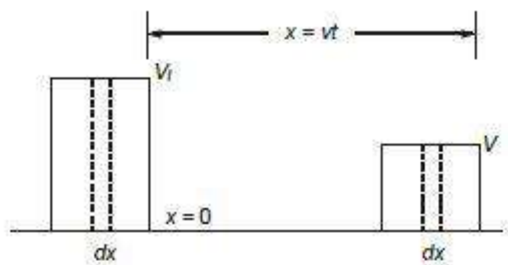

Fig. 3.4 Attenuation of voltage on a transmission line.

The energy of the wave is stored in both electromagnetic form and electrostatic form. Thetime rate of loss of stored energy is equal to the power loss due to corona, whose functionalrelationship with voltage has been given in Section 3.2. The total energy in a differential length $dx$ of the wave will be.

$$dw = \frac{1}{2}(C.dx)V^2 + \frac{1}{2}(L.dx)I^2 \quad (3.21)$$

where $L$ and $C$ are inductance and capacitance per unit length of line. For a travelling wave, the voltage $V$ and current $I$ are related by the surge impedance $Z = L / C$, and the wavevelocity is $v = 1 / LC$ Consequently, $I_2 = V_2/Z_2 = V_2C/L$. Thus, equation (3.26) becomes.

$dw = C.dx.V_2$. ... (3.22)

The rate of dissipation of energy, assuming the capacitance does not change with voltage forthe present analysis, is

$dw/dt = d\,(CV^2.dx)/dt = 2CV.dx.dV/dt$. ... (3.23)

Now, the power loss over the differential length $dx$ is

$P_c = f(V).dx$, so that $2CV\,.dV/dt = -P_c = -f(V)$. ... (3.24)

For different functional relations $P_c = f(V)$, equation (29) can be solved and the magnitude of voltage after a time of travel $t$ (or distance $x = vt$) can be determined.

(*a*) *Linear Relationship*

Let $f(V) = K_s(V - V_0)$. Then, with $V_i$ = initial voltage, $2\,CV\,dV/dt = -K_s(V - V_0)$. By separating variables and using the initial condition $V = V_i$ at $t = 0$ yields

$$(V - V_0).e^{V/V_0} = (V_i - V_0).e^{(V_i - \alpha t)/V_0}, \qquad (3.25)$$

where $\alpha = K_s/2C$ and $V_0$ = corona-inception voltage. Also, the voltages in excess of the corona-inception voltage at any time $t$ or distance $x = vt$
will be

$$(V - V_0)/V_i - V_0) = e^{(V_i - V - \alpha t)/V_0}. \qquad (3.26)$$

This expression yields an indirect method of determining $\alpha = K_s/2C$ by experiment, ifdistortion is not too great. It requires measuring the incident wave magnitude $V_i$and themagnitude $V$ after a time lapse of $t$ or distance $x = vt$at a different point on the line, whosecorona-inception voltage $V_0$ is known. When $C$ is also known, the constant $K_s$ is calculated.

## 3.7 AUDIBLE NOISE: GENERATION AND CHARACTERISTICS

When corona is present on the conductors, e.h.v. lines generate audible noise which is especiallyhigh during foul weather. The noise is broadband, which extends from very low frequency toabout 20 kHz. Corona discharges generate positive and negative ions which are alternatelyattracted and repelled by the periodic reversal of polarity of the ac excitation. Their movementgives rise to sound-pressure waves at frequencies of twice the power frequency and its multiples,in addition to the broadband spectrum which is the result of random motions of the ions, asshown in Figure 3.5. The noise has a pure tone superimposed on the broadband noise. Due todifferences in ionic motion between ac and dc excitations, dc lines exhibit only a broadbandnoise, and furthermore, unlike for ac lines, the noise generated from a dc line is nearly equalin both fair and foul weather conditions. Since audible noise (AN) is man-made, it is measuredin the same manner as other types of man-made noise such as aircraft noise, automobileignition noise, transformer hum, etc.

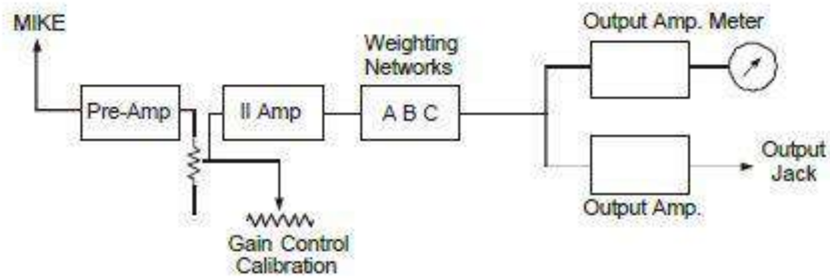


**Fig. 3.6 Block diagram of AN Measuring Circuit.**

Audible noise can become a serious problem from 'psycho-acoustics' point of view, leadingto insanity due to loss of sleep at night to inhabitants residing close to an e.h.v. line. Thisproblem came into focus in the 1960's with the energization of 500 kV lines in the USA.Regulatory bodies have not as yet fixed limits to AN from power transmission lines since suchregulations do not exist for other man-made sources of noise. The problem is left as a social onewhich has to be settled by public opinion. The proposed limits for AN are discussed in the nextsection.

## 3.8 LIMITS FOR AUDIBLE NOISE

Since no legislation exists setting limits for AN for man-made sources, power companies andenvironmentalists have fixed limits from public-relations point of view which power companieshave accepted from a moral point of view. In doing so, like other kinds of interference, humanbeings must be subjected to listening tests. Such objective tests are performed by every civicmindedpower utility organization. The first such series of tests performed from a 500-kV lineof the Bonneville Power Administration in the U.S.A. is known as Perry Criterion. The ANlimits are as follows:

No complaints :Less than 52.5 dB (*A*),

Few complaints : 52.5 dB (A) to 59 dB (*A*),

Many complaints : Greater than 59 dB (*A*),

The reference level for audible noise and the dB relation will be explained later. Thenotation (*A*) denotes that the noise is measured on a meter on a filter designated as *A*-weightingnetwork. There are several such networks in a meter.Design of line dimensions at e.h.v. levels is now governed more by the need to limit ANlevels to the above values. The selection of width of line corridor or right-of-way (R-O-W),where the nearest house can be permitted to be located, if fixed

from AN limit of 52.5 dB(*A*),will be found adequate from other points of view at 1000 to 1200 kV levels. The design aspect
will be considered in Section 5.8. The audible noise generated by a line is a function of the following factors:

(*a*) the surface voltage gradient on conductors,

(*b*) the number of sub-conductors in the bundle,

(*c*) conductor diameter,

(*d*) atmospheric conditions, and

(*e*) the lateral distance (or aerial distance) from the line conductors to the point where noise is to be evaluated.

The entire phenomenon is statistical in nature, as in all problems related to e.h.v. linedesigns, because of atmospheric conditions.While the Perry criterion is based upon actual listening experiences on test groups ofhuman beings, and guidelines are given for limits for AN from an e.h.v. line at the location ofinhabited places, other man-made sources of noise do not follow such limits. A second criterionfor setting limits and which evaluates the nuisance value from man-made sources of AN iscalled the 'Day-Night Equivalent' level of noise. This is based not only upon the variation of ANwith atmospheric conditions but also with the hours of the day and night during a 24-hourperiod. The reason is that a certain noise level which can be tolerated during the waking hoursof the day, when ambient noise is high, cannot be tolerated during sleeping hours of the nightwhen little or no ambient noises are present. This will be elaborated upon in Section 3.9.

According to the Day-Night Criterion, a noise level of 55 dB(*A*) can be taken as the limit instead of 52.5 dB(A) according to the Perry Criterion. From a statistical point of view, these levels areconsidered to exist for 50% of the time during precipitation hours. These are designated as $L_{50}$levels.

## 3.9 AN MEASUREMENT AND METERS

### 3.9.1 Decibel Values in AN and Addition of Sources

Audible noise is caused by changes in air pressure or other transmission medium so that it isdescribed by Sound Pressure Level (SPL). Alexander Graham Bell established the basic unit for SPL as $20 \times 10^{-6}$ Newton/$m^2$ or 20 micro Pascals [$2 \times 10^{-5}$ micro bar]. All decibel values arereferred to this basic unit. In telephone work there is a flow of current in a set of head-phonesor receiver. Here the basic units are 1 milliwatt across 600 ohms yielding a voltage of 775 mV and a current of 1.29 mA. For any other SPL, the decibel value isSPL(dB) = 10 $\text{Log}_{10}$ (SPL/$20 \times 10^{-6}$) Pascals ... (37)

This is also termed the 'Acoustic Power Level', denoted by PWL, or simply the audiblenoise level, AN.

### *Microphones*

Instruments for measurement of audible noise are very simple in construction in so far as theirprinciples are concerned. They would conform to standard specifications of each country, as forexample, ANSI, ISI or I.E.C., etc. The input end of the AN measuring system consists of amicrophone as shown in the block diagram, Figure 3.6. There are three types of microphones used in AN measurement from e.h.v. lines and equipment. They are (*i*) air-condenser type;(*ii*) ceramic; and (*iii*) electret microphones. Air condenser microphones are very stable andexhibit highest frequency response. Ceramic ones are the most rugged of the three types.Since AN is primarily a foul-weather phenomenon, adequate protection of microphone fromweather is necessary. In addition, the electret microphone requires a polarization voltage sothat a power supply (usually battery) will also be exposed to rain and must be protected suitably.

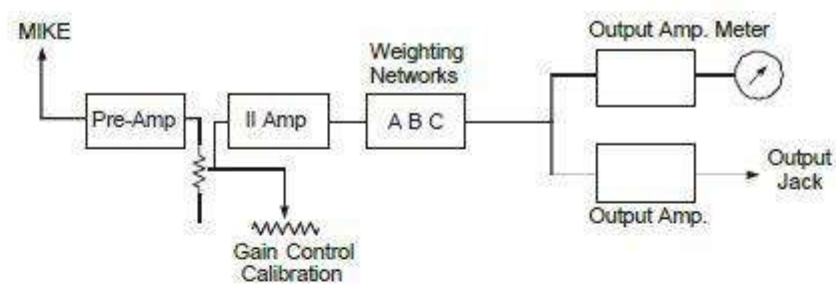


**Fig. 3.6 Block diagram of AN Measuring Circuit.**

Some of the microphones used in AN measurement from e.h.v. lines are General RadioType 1560-P, or 1971-9601, or Bruel and Kjor type 4145 or 4165, and so on. The GR type has aweather protection. Since AN level form a transmission line is much lower than, say, aircraftor ignition noise, 1″ (2.54 cm) diameter microphones are used although some have used ″ones, since these have more sensitivity than 1" microphones. Therefore, size is not thedetermining factor.The most important characteristic of a microphone is its frequency response. In makingAN measurements, it is evident that the angle between the microphone and the source is notalways 90° so that the grazing angle determines the frequency response. Some typicalcharacteristics are as shown in Figure 3.7.

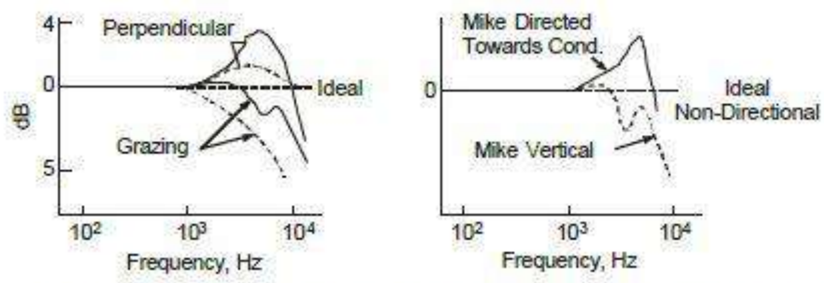


**Fig. 3.7 Response of microphone for grazing and perpendicular incidence.**

AN levels are statistical in nature and long-term measurements are carried out byprotecting the microphone from rain, wind, animals, and birds. Some types of shelters usewind-screens with a coating of silicone grease. Foam rubber wind-covers have also been usedwhich have negligible attenuation effect on the sound, particularly on the *A*-weighted networkwhich will be described below. Every wind-cover must also be calibrated and manufacturerssupply this data. Foam-rubber soaks up rain and must be squeezed out periodically and siliconegrease applied.

## 3.10 RELATION BETWEEN SINGLE-PHASE AND 3-PHASE AN LEVELS

Obtaining data of AN and other quantities from e.h.v. lines involves great expense in setting upfull-scale outdoor 3-phase experimental lines. Most of the design data can be obtained at less cost from a single-phase outdoor line or from cage experiments. The quantities of interest in so far as interference from e.h.v. lines are concerned are AN, Radio Interferecne and ElectrostaticField at 50 Hz. It is therefore worth the effort to consider what relation, if any, exists betweenexperimental results obtained from 1-phase lines and an actual 3-phase line. If such a relation can be found, then 1-phase lines can be used for gathering data which can then be extrapolated to apply to 3-phase lines. We first consider a horizontal line.

The AN level from any phase at the measuring point *M* consists of a constant part and avariable part which can be seen from equations (3.25) to (3.26). They are written as, for N ≥ 3,

$$AN_1 = (55 \log d + 26.4 \log N - 128.4) + 120 \log E_1 - 11.4 \log D_1$$

$$= K + 120 \log E_1 - 11.4 \log D_1$$

Similarly, $$AN_2 = K + 120 \log E_2 - 11.4 \log D_2$$

and $$AN_3 = K + 120 \log E_1 - 11.4 \log D_3 \quad (3.25)$$

Let the centre-phase gradient be written as $E_2 = (1 + m) E_1$ and the ratios $k_2 = D_2/D_1$ and $k_3 = D_3/D_1$. Then, total AN level of the 3-phases obtained after combining the AN levels of the

3 individual phases is

$$\begin{aligned}\mathrm{AN}_T &= 10\log_{10}\sum_{i=1}^{3}10^{0.1\,\mathrm{AN}(i)} \\ &= 10\log_{10}[10^{0.1K}.10^{12\log E_1}\{10^{-1.14\log D_1}+10^{-1.14\log D_3} \\ &\quad +10^{-1.14\log D_2+12\log(1+m)}\}] \\ &= K+120\log E_1-11.4\log D_1+10\log[1+k_3^{-1.14} \\ &\quad +(1+m)^{12}.k_2^{-1.14}]\end{aligned} \tag{3.26}$$

For a single-phase line with the same surface voltage gradient $E_2$ as the centre-phaseconductor of the 3-phase configuration, and at a distance $D_2$, the noise level is

$\mathrm{AN}_s = K + 120 \log E_1 - 11.4 \log D_1 + 10\log [ \; _{1.14}(1 \;)_{12} \;]_{10} \; k_2 + m \; - ...$ (3.27)

Therefore, the difference in AN levels of equation (3.28) and (3.29) is

$$\begin{aligned}\mathrm{AN}_s &= K+120\log E_1-11.4\log D_1 \\ &\quad +10\log_{10}[k_2^{-1.14}(1+m)^{12}]\end{aligned} \tag{3.28}$$

This is the decibel adder which will convert the single-phase AN level to that of a 3-phaseline.

**12.** Using equation (3.28) compute $\mathrm{AN}_T$ for the 735 kV line of example 3.11.

**Solution.** $K = -85.87$, $1 + m = 20/18.4 = 1.087.11.4 \log D_1 = 16.05$, $120 \log E_1 = 151.8$, $k_2 = 36/25.6 = 1.406$, $k_3 = 48.33/25.6 = 1.888$

$\therefore$ 1 $_{1.14}$ (1 )$_{12}$ 3.33$_{21.143}$ + $k_-$ + $k_-$ +$m$ =$\mathrm{AN}_T = -85.87 + 151.8 - 16.05 + 10 \log 3.33 = 55.1$ dB($A$).

## 3.11 Frequency Spectrum

The frequency spectrum of radio noise measured from long lines usually corresponds to the Fourier Amplitude Spectrum (Bode Amplitude Plot) of single pulses. These are shown in Figure 6.3. The Fourier integral for a single double-exponential pulse is

$$\begin{aligned}F(jw) &= \int_{-\infty}^{\infty} f(t).e^{-jwt}.dt = \int_0^{\infty} Ki_p(e^{-\alpha t}-e^{-\beta t}).e^{-jwt}.dt \\ &= Ki_p[1/(\alpha+jw)-1/(\beta+jw)] \\ &= Ki_p(\beta-\alpha)/(\alpha+jw)(\beta+jw)\end{aligned} \tag{3.29}$$

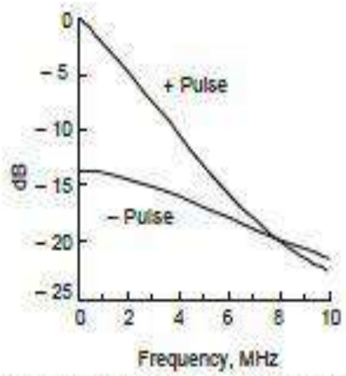

Fig 3.8 Bode frequency plot of positive and negative corona pulses.

The amplitude is

$$A(w) = K.i_p.(\beta - \alpha)/\sqrt{(\alpha^2 + w^2)(\beta^2 + w^2)} \quad (3.30)$$

## 3.12 The RI Excitation Function

With the advent of voltages higher than 750 kV, the number of subconductors used in a bundlehas become more than 4 so that the CIGRE formula does not apply. Moreover, very littleexperience of RI levels of 750 kV lines were available when the CIGRE formula was evolved, ascompared to the vast experience with lines for 230 kV, 345 kV, 400 kV and 500 kV. Severalattempts were made since the 1950's to evolve a rational method for predicting the RI level ofa line at the design stages before it is actually built when all the important line parameters arevaried. These are the conductor diameter, number of sub-conductors, bundle spacing or bundleradius, phase spacing, line height, line configuration (horizontal or delta), and the weathervariables. The most important concept resulting from such an attempt in recent years is the"Excitation Function" or the "Generating Function" of corona current injected at a given radiofrequency in unit bandwidth into the conductor.

Consider Fig. 3.9 which shows a source of corona at $S$ located at a distance $x$ from one end of a line of length $L$. According to the method using the Excitation Function to predict the RI level with given dimensions and conductor geometry, the corona source at $S$ on the conductor generates an excitation function $I$ measured in μA/ m. The line has a surge impedance $Z_0$ so that r-f power generated per unit length of line is

$$E = I_2 Z_0$$

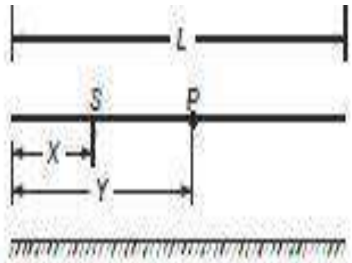


Fig. 3.9 The excitation function and its propagation on line for RI calculation.

## 3.13 MEASUREMENT OF RI, RIV, AND EXCITATION FUNCTION

The intereference to AM broadcast in the frequency range 0.5 MHz to 1.6 MHz is measured interms of the three quantities : Radio Interference Field Intensity (RIFI or RI), the Radio InfluenceVoltage (RIV), and more recently through the Excitation Function. Their units are mV/m, mV,and mA/ m or the decibel values above their reference values of 1 unit (μV /m,μV,μA / m).The nuisance value for radio reception is governed by a quantity or level which is nearly equalto the peak value of the quantity and termed the Quasi Peak. A block diagram of a radio noisemeter is shown in Fig. 3.10. The input to the meter is at radio frequency (r-f) which is amplifiedand fed to a mixer. The rest of the circuit works exactly the same as a highly sensitive superheterodyneradio receiver, However, at the IF output stage, a filter with 5 kHz or 9 kHzbandwidth is present whose output is detected by the diode *D*. Its output charges a capacitance*C* through a low resistance $R_c$such that the charging time constant $T_c = R_c C$ = 1 ms. A secondresistance $R_d$ is in parallel with *C* which is arranged to give a time constant $T_d = R_d C$ = 600 msin ANSI meters and 160 ms in CISPR or European standard meters. Field tests have shownthat there is not considerable difference in the output when comparing both time constants forline-generated corona noise. The voltage across the capacitor can either be read as a currentthrough the discharge resistor $R_d$ or a micro-voltmeter connected across it.

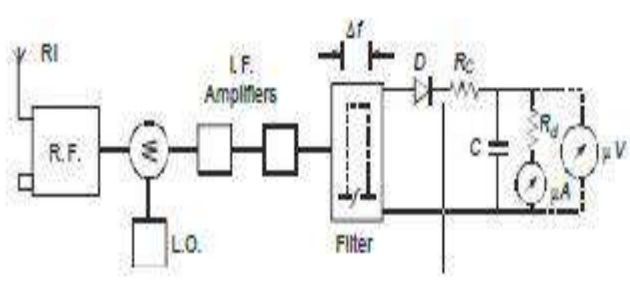


Fig. 3.10 Block diagram of Radio Noise Meter.

Conducted RIV is measured by a circuit shown schematically in Fig. 3.11. The objectunder test, which could be an insulator string with guard rings, is energized by a high voltagesource at power frequency or impulse. A filter is interposed such that any r-f energy produced by partial discharge in the test object is prevented from flowing into the source and all r-fenergy goes to the measuring circuit. This consists of a discharge-free h.v. coupling capacitor of about 500 to 2000 pF in series at the ground level with a small inductance $L$. At 50 Hz, thecoupling capacitor has a reactance of 3.11Megohms to 1.59 Megohms. The value of $L$ is chosensuch that the voltage drop is not more than 5 volts so that the measuring equipment does notexperience a high power-frequency voltage.

Let $V$ = applied power frequency voltage from line to ground,
$V_L$ = voltage across $L$,
$X_c$= reactance of coupling capacitor
and$X_L = 2\pi fL$ = reactance of inductor.

Then, $$V_L = V.X_L/(X_c - X_L) \simeq VX_L/X_c = 4\pi^2 f^2 LC_c.V \quad (3.31)$$

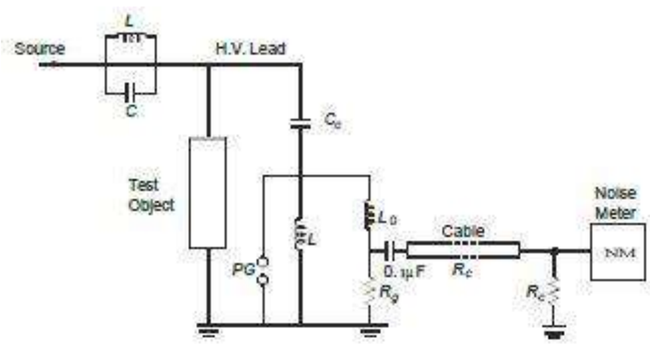


**Fig. 3.11 Circuit for measuring Radio Influence Voltage (RIV).**

## 3.14 MEASUREMENT OF EXCITATION FUNCTION

The corona generating function or the excitation function caused by injected current at radiofrequencies from a corona discharge is measured on short lengths of conductor strung inside"cages" as discussed earlier. The design of cages has been covered in great detail in Chapter 2,Some examples of measuring radio noise and injected current are shown in Fig. 3.12. In everycase the measured quantity is RIV at a fixed frequency and the excitation function calculated asdescribed later. The filter provides an attenuation of at least 25 dB so that the RI current issolely due to corona on conductor. The conductor is terminated in a capacitance $C_c$ at one endin series with resistances $R_1$ and $R_c$, while the other end is left open. The conductor is strungwith strain insulator at both ends which can be considered to offer a very high impedance at 1MHz so thatMthere is an open-termination. But this must be checked experimentally in situ.The coupling capacitor has negligible reactance at r-f so that the termination at the measuringend is nearly equal to ($R_1 + R_c$), where $R_c$=

surge impedance of the cable to the noise meter.The resistance $R_c$is also equal to the input impedance of the noise meter.

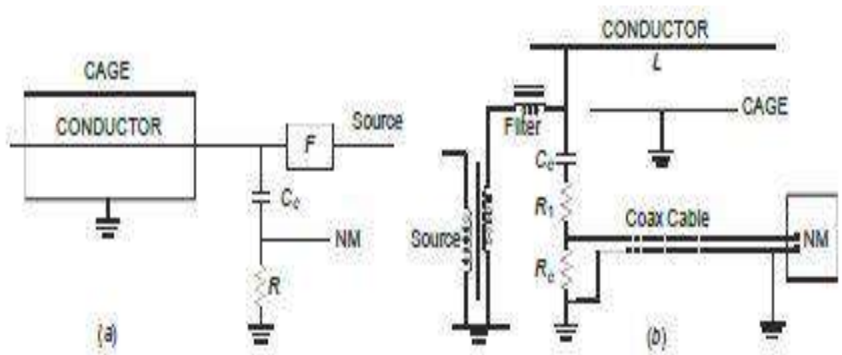


Fig. 3.12 Cage setups for measuring excitation function with measuring circuit.

The excitation function is calculated as follows:

Let $J$ = RI current injected in μA/ m,

$C$ = capacitance of conductor in cage, Farad/metre,

$R$ = outer radius of cage,$r_{eq}$= equivalent radius of bundle.

Then, the excitation function is

$$I = 2\pi\varepsilon_0 . J / C, \mu A / \sqrt{m} \qquad (3.32)$$

The injected current in terms of measured RIV is

$$J = 2(R_c + R_m)(RIV) / R_c R_m G \qquad (3.33)$$

where $/( + ) = R_cR_mR_cR_m$resistance of $R_c$and meter in parallel,

RIV =measured noise reading on meter in μ$V$, and $G$ = an amplification factor caused by addition

# UNIT-IV

# Electrostatic and Magnetic Fieldsof EHV Lines

## ELECTRIC SHOCK AND THRESHOLD CURRENTS

Electrostatic effects from overhead e.h.v. lines are caused by the extremely high voltage while electromagnetic effects are due to line loading current and short-circuit currents. Hazards exist due to both causes of various degree. These are, for example, potential drop in the earth's surface due to high fault currents, direct flashover from line conductors to human beings or animals. Electrostatic fields cause damage to human life, plants, animals, and metallic objects such as fences and buried pipe lines. Under certain adverse circumstances these give rise to shock currents of various intensities.

Shock currents can be classified as follows:

(*a*) *Primary Shock Currents*. These cause direct physiological harm when the current exceeds about 6-10 mA. The normal resistance of the human body is about 2-3 kilohms so that about 25 volts may be necessary to produce primary shock currents. The danger here arises due to ventricular fibrillation which affects the main pumping chambers of the heart. This results in immediate arrest of blood circulation. Loss of life may be due to (*a*) arrest of blood circulation when current flows through the heart, (*b*) permanent respiratory arrest when current flows in the brain, and (*c*) asphyxia due to flow of current across the chest preventing muscle contraction.

The 'electrocution equation' is $i^2t = K^2$, where $K = 165$ for a body weight of 50 kg, $i$ is in mA and $t$ is in seconds. On a probability basis death due to fibrillation condition occurs in 0.5% of cases. The primary shock current required varies directly as the body weight. For $i = 10$ mA, the current must flow for a time interval of 272 seconds before death occurs in a 50 kg human being.

(*b*) *Secondary Shock Currents*. These cannot cause direct physiological harm but may produce adverse reactions. They can be steady state 50 Hz or its harmonics or transient in nature. The latter occur when a human being comes into contact with a capacitively charged body such as a parked vehicle under a line. Steady state currents up to 1 mA cause a slight tingle on the fingers. Currents from 1 to 6 mA are classed as, 'let go' currents. At this level, a human being has control of muscles to let the conductor go as soon as a tingling sensation occurs. For a 50% probability that the let-to current may increase to primary shock current, the limit for men is 16 mA and for women 10 mA. At 0.5% probability, the currents are 9 mA for men, 6 mA for women, and 4.5 mA for children.

A human body has an average capacitance of 250 pF when standing on an insulated platform of 0.3 m above ground (1 ft.). In order to reach the let-go current value, this will require 1000 to 2000 volts. Human beings touching parked vehicles under the line may experience these transient currents, the larger the vehicle the more charge it will acquire and greater is the danger. Construction crews are subject to hazards of electrostatic induction when erecting new lines adjacent to energized lines. An ungrounded conductor of about 100 metres in length can produce shock currents when a man touches it. But grounding both ends of the conductor brings the hazard of large current flow. A movable ground mat is generally necessary to protect men and machines. When stringing one circuit on a double-circuit tower which already has an energized circuit is another hazard and the men must use a proper ground. Accidents occur when placing or removing grounds and gloves must be worn. Hot-line techniques are not discussed here.

## CALCULATION OF ELECTROSTATIC FIELD OF A.C. LINES

### Power-Frequency Charge of Conductors

In Chapter 4, we described the method of calculating the electrostatic charges on the phase conductors from line dimensions and voltage.
For $n$ phases, this is, see Fig. 7.2, with $q$ = total bundle charge and $V$ = line to ground voltage

$$\frac{1}{2\pi e_0}[q] = [P]^{-1}[V] = [M][V]$$

where $[q] = [q_1, q_2, q_3, \ldots, q_n]_t$,
$[V] = [V_1, V_2, V_3, \ldots, V_n]_t$
$[P] = n \times n$ matrix of Maxwell's Potential coefficients with
$P_{ii} = \ln(2H_i / r_{eq})$ and $P_{ij} = \ln(I_{ij} / A_{ij}), i \neq j$

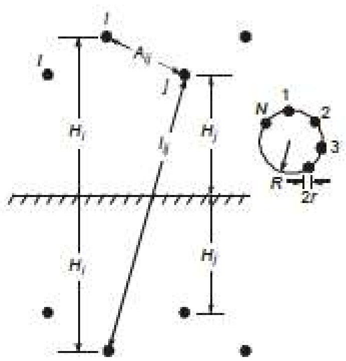


Fig. 7.2 n-phase line configuration for charge calculation.

Here, $H_i$ = height of conductor $i$ above ground = $H_{min} + \frac{1}{3}$ sag,
$I_{ij}$ = distance between conductor $i$ above ground and the image of conductor $j$ below ground, $i \neq j$,
$A_{ij}$ = aerial distance between conductors $i$ and $j$, $i \neq j$,
$r_{eq} = R(N.r/R)^{1/N}$ = equivalent bundle radius,
$R$ = bundle radius = $B/2\sin(\pi/N)$,
$N$ = number of sub-conductors in bundle,
$r$ = radius of each sub-conductor,
and $i, j = 1, 2, 3, \ldots, n$.

Since the line voltages are sinusoidally varying with time at power frequency, the bundle charges $q_1$ to $q_n$ will also vary sinusoidally. Consequently, the induced electrostatic field in the vicinity of the line also varies at power frequency and phasor algebra can be used to combine several components in order to yield the amplitude of the required field, namely, the horizontal, vertical or total vectors

## Electrostatic Field of Single-Circuit 3-Phase Line

Let us consider first a 3-phase line with 3 bundles on a tower and excited by the voltages.
$[V] = V_m[\sin(wt+\varphi), \sin(wt+\varphi-120°), \sin(wt+\varphi+120)°]$ ...(7.7)
Select an origin $O$ for a coordinate system at any convenient location. In general, this may be located on ground under the middle phase in a symmetrical arrangement. The coordinates of the line conductors are ( , ) $x_i\ y_i$. A point $A(x, y)$
is shown where the horizontal, vertical, and total e.s. field components are required to be evaluated, as shown in Fig. 7.3.

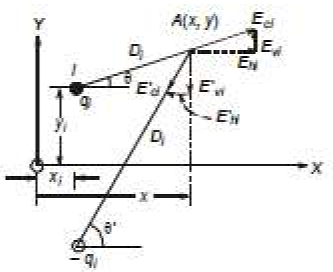


Fig. 7.3 Calculation of e.s. field components near the line.

The field vector at $A$ due to the charge of the aerial conductor is with

$$D_i^2 = (x-x_i)^2+(y-y_i)^2,$$
$$E_c = (q_i/2\pi e_0)(1/D_i) \quad ...(7.8)$$

Its horizontal and vertical components are

$$E_h = E_c \cos\theta = (q_i / 2\pi e_0)(x - x_i) / D_i^2 \quad ...(7.9)$$

and

$$E_v = E_c \sin\theta = (q_i / 2\pi e_0)(y - y_i) / D_i^2 \quad ...(7.10)$$

Similarly, due to image charge of conductor

$$E_c' = (q_i / 2\pi e_0)(1 / D_i')$$

where

$$(D_i')^2 = (x - x_i)^2 + (y + y_i)^2,$$

$$\left.\begin{aligned} E_h' &= (q_i / 2\pi e_0)(x - x_i)/(D_i')^2 \\ E_v' &= (q_i / 2\pi e_0)(y + y_i)/(D_i')^2 \end{aligned}\right\} \quad ...(7.11)$$

We observe that the field components of $E_c$ and $E_c'$ are in opposite directions. Therefore, the total horizontal and vertical components at $A$ due to both charges are

$$E_{hi} = (q_i / 2\pi e_0)(x - x_i)[1 / D_i^2 - 1/(D_i')^2] \quad ...(7.12)$$

$$E_{vi} = (q_i / 2\pi e_0)[(y - y_i) / D_i^2 - (y + y_i)/(D_i')^2] \quad ...(7.13)$$

Consequently, due to all $n$ phases, the sum of horizontal and vertical components of e.s. field at the point $A(x, y)$ will be

$$E_{hn} = \sum_{i=1}^{n} E_{hi}, \text{ and } E_{vn} = \sum_{i=1}^{n} E_{vi} \quad ...(7.14)$$

The total electric field at $A$ is

$$E_{in} = (E_{hn}^2 + E_{vn}^2)^{1/2} \quad ...(7.15)$$

We can write these out explicitly for a 3-phase line.

Let

$$J_i = (x - x_i)[1 / D_i^2 - 1/(D_i')^2] \quad ...(7.16)$$

and

$$K_i = (y - y_i) / D_i^2 - (y + y_i)/(D_i')^2 \quad ...(7.17)$$

The bundle charges are calculated from equations (7.4), (7.5), and (7.6), so that from equations (7.12), (7.13), (7.16) and (7.17) there results.

$$E_{h1} = (q_1/2\pi e_0)J_1 = V_m.J_1[M_{11}\sin(\omega t+\varphi)+M_{12}\sin(\omega t+\varphi-120°) + M_{13}\sin(\omega t+\varphi+120°)]$$

$$E_{h2} = (q_2/2\pi e_0)J_2 = V_m.J_2[M_{21}\sin(\omega t+\varphi)+M_{22}\sin(\omega t+\varphi-120°) + M_{23}\sin(\omega t+\varphi+120°)]$$

$$E_{h3} = (q_3/2\pi e_0)J_3 = V_m.J_3[M_{31}\sin(\omega t+\varphi)+M_{32}\sin(\omega t+\varphi-120°)] + M_{33}\sin(\omega t+\varphi+120°)]$$

∴ The total horizontal component is, adding vertically,

$$E_{hn} = V_m[(J_1.M_{11}+J_2.M_{21}+J_3.M_{31})\sin(\omega t+\varphi) + (J_1.M_{12}+J_2.M_{22}+J_3.M_{32})\sin(\omega t+\varphi-120°) + (J_1.M_{13}+J_2.M_{23}+J_3.M_{33})\sin(\omega t+\varphi+120°)]$$

$$= V_m[J_{h1}.\sin(\omega t+\varphi)+J_{h2}.\sin(\omega t+\varphi-120°) + J_{h3}\sin(\omega t+\varphi+120°)]$$

and in phasor form,

$$E_{hn} = V_m[J_{h1}\angle\varphi + J_{h2}\angle\varphi-120^0 + J_{h3}\angle\varphi+120^0] \quad ...(7.18)$$

This is a simple addition of three phasors of amplitudes 1 2 3 , , $J_h J_h J_h$ inclined at 120° to each other. Resolving them into horizontal and vertical components (real and $j$ parts with $\varphi$ = 0), we obtain

$$\left.\begin{aligned}\text{real part} &= J_{h1}-0.5J_{h2}-0.5J_{h3}\\ \text{and imaginary part} &= 0-0.866J_{h2}+0.866J_{h3}\end{aligned}\right\} \quad ...(7.19)$$

Consequently, the amplitude of electric field is

$$\hat{E}_{hn} = [(J_{h1}-0.5J_{h2}-0.5J_{h3})^2+0.75(J_{h3}-J_{h2})^2]^{1/2}V_m$$
$$= (J_{h1}^2+J_{h2}^2+J_{h3}^2-J_{h1}J_{h2}-J_{h2}J_{h3}-J_{h3}J_{h1})^{1/2}.C_m$$
$$= J_h.V_m.$$

The r.m.s. value of the total horizontal component at $A$ $(x, y)$ due to all 3 phases will be

$$E_{hn} = \hat{E}_{hn}/\sqrt{2} = J_h.V \quad ...(7.20)$$

where $V$ = r.m.s. value of line to ground voltage.

In a similar manner, the r.m.s. value of toal vertical component of field at $A$ due to all 3 phases is

$$E_{vn} = K_v.V = V(K_{v1}^2+K_{v2}^2+K_{v3}^2-K_{v1}K_{v2}-K_{v2}K_{v3}-K_{v3}K_{v1})^{1/2} \quad ...(7.21)$$

where
$$\left.\begin{aligned}K_{v1} &= K_1M_{11}+K_2M_{21}+K_3.M_{31}\\ K_{v2} &= K_1M_{12}+K_2.M_{22}+K_3M_{32}\\ K_{v3} &= K_1.M_{13}+K_2.M_{23}+K_3.M_{33}\end{aligned}\right\} \quad ...(7.22)$$
and (last line)

where the values of $K_1, K_2, K_3$ are obtained from equation (7.17) for $K_i$ with $i$ = 1, 2, 3.

## EFFECT OF HIGH E.S. FIELD ON HUMANS, ANIMALS, AND PLANTS

In section 7.1, a discussion of electric shock was sketched. The use of e.h.v. lines is increasing danger of the high e.s. field to (*a*) human beings, (*b*) animals, (*c*) plant life, (*d*) vehicles, (*e*) fences, and (*f*) buried pipe lines under and near these lines. It is clear from section 7.2 that when an object is located under or near a line, the field is disturbed, the degree of distortion depending upon the size of the object. It is a matter of some diffficulty to calculate the characteristics of the distorted field, but measurements and experience indicate that the effect of the distorted field can be related to the magnitude of the undistorted field. A case-by-case study must be made if great accuracy is needed to observe the effect of the distorted field. The limits for the undistorted field will be discussed here in relation to the danger it poses.

(*a*) *Human Beings*

The effect of high e.s. field on human beings has been studied to a much greater extent than on any other animals or objects because of its grave and shocking effects which has resulted in loss of life. A farmer ploughing his field by a tractor and having an umbrella over his head for shade will be charged by corona resulting from pointed spikes. The vehicle is also charged when it is stopped under a transmission line traversing his field. When he gets off the vehicle and touches a grounded object, he will discharge himself through his body which is a pure resistance of about 2000 ohms. The discharge current when more than the let-go current can cause a shock and damage to brain.

It has been ascertained experimentally that the limit for the undisturbed field is 15 kV/m, r.m.s., for human beings to experience possible shock. An e.h.v. or u.h.v. line must be designed such that this limit is not exceeded. The minimum clearance of a line is the most important governing factor. As an example, the B.P.A. of the U.S.A. have selected the maximum e.s. field gradient to be 9 kV/m at 1200 kV for their 1150 kV line and in order to do so used a minimum clearance at midspan of 23.2 m whereas they could have selected 17.2 m based on clearance required for switching-surge insulation recommended by the National Electrical Safety Council.

(*b*) *Animals*

Experiments carried out in cages under e.h.v. lines have shown that pigeons and hens are affected by high e.s. field at about 30 kV/m. They are unable to pick up grain because of chattering of their beaks which will affect their growth. Other animals get a charge on their bodies and when they proceed to a water trough to drink water, a spark usually jumps from their nose to the grounded pipe or trough.

(*c*) *Plant Life*

Plants such as wheat, rice, sugarcane, etc., suffer the following types of damage. At a field strength of 20 kV/m (r.m.s.), the sharp edges of the stalk give corona discharges so that damage occurs to the upper portion of the grain-bearing parts. However, the entire plant does not suffer dam age. At 30 kV/m , the by-products of corona, namely ozone and $N_2O$ become intense.

The resistance heating due to increased current prevents full growth of the plant and grain. Thus, 20 kV/m can be considered as the limit and again the safe value for a human being governs line design.

(*d*) *Vehicles*
Vehicles parked under a line or driving through acquire electrostatic charge if their tyres are made of insulating material. If parking lots are located under a line, the minimum recommended safe clearance is 17 m for 345 kV and 20 m for 400 kV lines. Trucks and lorries will require an extra 3 m clearance. The danger lies in a human being attempting to open the door and getting a shock thereby.
(*e*) *Others*
Fences, buried cables, and pipe lines are important pieces of equipment to require careful layout. Metallic fences parallel to a line must be grounded preferably every 75 m. Pipelines longer than 3 km and larger than 15 cm in diameter are recommended to be buried at least 30 m laterally from the line centre to avoid dangerous eddy currents that could cause corrosion. Sail boats, rain gutters and insulated walls of nearby houses are also subjects of potential danger. The danger of ozone emanation and harm done to sensitive tissues of a human being at high electric fields can also be included in the category of damage to human beings living near e.h.v. line

# ELECTROSTATIC INDUCTION ON UNENERGIZED CIRCUIT OF A D/CLINE

We shall end this chapter with some discussion of electrostatic and electromagnetic induction from energized lines into other circuits. This is a very specialized topic useful for line crew, telephone line interference etc. and cannot be discussed at very great length. EHV lines must be provided with wide enough right-of-way so that other low-voltage lines are located far enough, or when they cross the crossing must be at right angles.
Consider Fig. 7.12 in which a double-circuit line configuration is shown with 3 conductors energized by a three-phase system of voltages sin , sin( 120 ) $V_1 = V_m wt$ $V_2 = V_m wt -$ ° and $V_3 = V_m \sin(wt + 120°)$. The other circuit consisting of conductors 4, 5 and 6 is not energized. We will calculate the voltage on these conductors due to electrostatic induction which a line man may experience. Now,

$$V_4 = \frac{q_1}{2\pi e_0}\ln(I_{14}/A_{14}) + \frac{q_2}{2\pi e_0}\ln(I_{24}/A_{24}) + \frac{q_3}{2\pi e_0}\ln(I_{34}/A_{34}) \qquad ...(7.37)$$

$$= \frac{q_1}{2\pi e_0}P_{14} + \frac{q_2}{2\pi e_0}P_{24} + \frac{q_3}{2\pi e_0}P_{34}$$

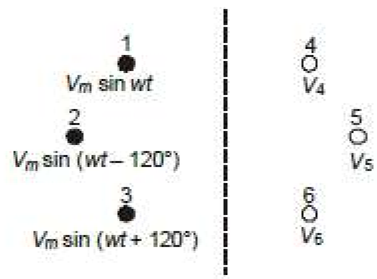


**Fig. 7.12 D/C line: One line energized and the other unenergized to illustrate induction.**

The charge coefficients $q_1, q_2, q_3$ are obtained from the applied voltages.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} P_{11}, & P_{12}, & P_{13} \\ P_{21}, & P_{22}, & P_{23} \\ P_{31}, & P_{32}, & P_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} (1/2\pi e_0) = [P][q](1/2\pi e_0) \qquad ...(7.38)$$

so that $\qquad [q/2\pi e_0] = [M][V] \qquad ...(7.39)$

$$V_4 = V_m.P_{14}[M_{11}\sin wt + M_{12}\sin(wt - 120°) + M_{13}\sin(wt + 120°)]$$
$$+ V_m P_{24}[M_{21}\sin wt + M_{22}\sin(wt - 120°) + M_{23}\sin(wt + 120°)]$$
$$+ V_m P_{34}[M_{31}\sin wt + M_{32}\sin(wt - 120°) + M_{33}\sin(wt + 120°)]$$
$$= V_m[(P_{14}M_{11} + P_{24}M_{21} + P_{34}M_{31})\sin wt$$
$$+ (P_{14}M_{12} + P_{24}M_{22} + P_{34}M_{32})\sin(wt - 120°)$$
$$+ (P_{14}M_{13} + P_{24}M_{23} + P_{34}M_{33})\sin(wt + 120°)]$$
$$= V_m[\lambda_1 \sin wt + \lambda_2 \sin(wt - 120°) + \lambda_3 \sin(wt + 120°)] \qquad ...(7.40)$$

In phasor form,

$$V_4 = V[\lambda_1\angle 0° + \lambda_2\angle -120° + \lambda_3\angle 120°], \text{r.m.s.} \qquad ...(7.41)$$

$$V_4 = V(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - \lambda_1\lambda_2 - \lambda_2\lambda_3 - \lambda_3\lambda_1)^{1/2} \qquad ...(7.42)$$

Similarly, $\qquad V_5 = V(\lambda_4^2 + \lambda_5^2 + \lambda_6^2 - \lambda_4\lambda_5 - \lambda_5\lambda_6 - \lambda_6\lambda_4)^{1/2} \qquad ...(7.43)$

and $\qquad V_6 = V(\lambda_7^2 + \lambda_8^2 + \lambda_9^2 - \lambda_7\lambda_8 - \lambda_8\lambda_9 - \lambda_9\lambda_7)^{1/2} \qquad ...(7.44)$

where

and

$$\left.\begin{aligned}
\lambda_1 &= P_{14}M_{11} + P_{24}M_{21} + P_{34}M_{31} \\
\lambda_2 &= P_{14}M_{12} + P_{24}M_{22} + P_{34}M_{32} \\
\lambda_3 &= P_{14}M_{13} + P_{24}M_{23} + P_{34}M_{33} \\
\lambda_4 &= P_{15}M_{11} + P_{25}M_{21} + P_{35}M_{31} \\
\lambda_5 &= P_{15}M_{12} + P_{25}M_{22} + P_{35}M_{32} \\
\lambda_6 &= P_{15}M_{13} + P_{25}M_{23} + P_{35}M_{33} \\
\lambda_7 &= P_{16}M_{11} + P_{26}M_{21} + P_{36}M_{31} \\
\lambda_8 &= P_{16}M_{12} + P_{26}M_{22} + P_{36}M_{32} \\
\lambda_9 &= P_{16}M_{13} + P_{26}M_{23} + P_{36}M_{33}
\end{aligned}\right\}$$

## TRAVELLING WAVES AND STANDING WAVES AT POWER FREQUENCY

On an electrical transmission line, the voltages, currents, power and energy flow from the source to a load located at a distance $L$, propagating as electro-magnetic waves with a finite velocity. Hence, it takes a short time for the load to receive the power. This gives rise to the concept of a wave travelling on the line which has distributed line parameters *r, l, g, c* per unit length. The current flow is governed mainly by the load impedance, the line-charging current at power frequency and the voltage. If the load impedance is not matched with the line impedance, which will be explained later on, some of the energy transmitted by the source is not absorbed by the load and is reflected back to the source which is a wasteful procedure. However, since the load can vary from no load (infinite impedance) to rated value, the load impedance is not equal to the line impedance always; therefore, there always exist transmitted waves from the source and reflected waves from the load end. At every point on the intervening line, these two waves are present and the resulting voltage or current is equal to the sum of the transmitted and reflected quantities. The polarity of voltage is the same for both but the directions of current are opposite so that the ratio of voltage to current will be positive for the transmitted wave and negative for the reflected wave. These can be explained mathematically and have great significance for determining the characteristics of load flow along a distributedparameter line.

The same phenomenon can be visualized through standing waves. For example, consider an open-ended line on which the voltage must exist with maximum amplitude at the open end while it must equal the source voltage at the sending end which may have a different amplitude and phase. For 50 Hz, at light velocity of $300 \times 10^3$ km/sec, the wavelength is 6000 km, so that a line of length $L$ corresponds to an angle of ($L \times 360°/6000$). With a load current present, an additional voltage caused by the voltage drop in the characteristic impedance is also present which will stand on the line. These concepts will be explained in detail by first considering a loss-less line ($r = g = 0$) and then for a general case of a line with losses present.

### Differential Equations and Their Solutions

Consider a section of line $\Delta x$ in length situated at a distance $x$ from the load end. The line length is $L$ and has distributed series inductance $l$ and shunt capacitance $c$ per unit length, which are calculated as discussed in Chapter 3. The inductance is ( $l.\Delta x$) and capacitance ( $c.\Delta x$) of the differential length $\Delta x$ . The voltages on the two sides of $l$ are ( ) $V_x + \Delta V_x$ and $V_x$, while the currents on the two sides of $c$ are ($I_x + \Delta I_x$) and $I_x$. From Figure 8.1 the following equations can be written down:

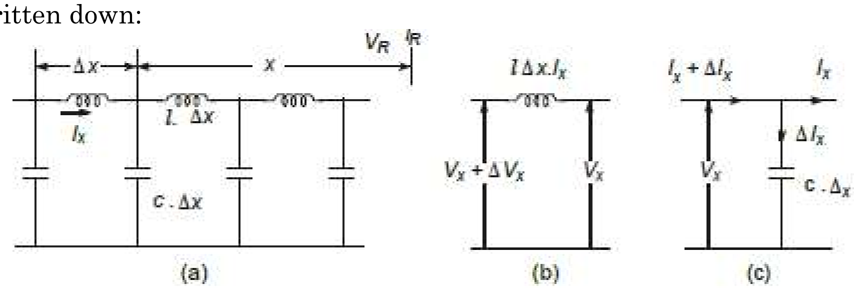


Fig. 8.1 Transmission line with distributed inductance and capacitance.

$$(V_x + \Delta V_x) - V_x = \Delta V_x = (jwl.\Delta x)I_x \quad ...(8.1)$$

and

$$(I_x + \Delta I_x) - I_x = \Delta I_x = (jwc.\Delta x)V_x \quad ...(8.2)$$

By making $\Delta x$ infinitesimal, the changes in voltage and current along the line are expressed as differential equations thus

$$dV_x/dx = z.\ I_x \text{ and } dI_x/dx = y.\ V_x \qquad ...(8.3)$$

where $z = jwl$ and $y = jwc$, the series impedance and shunt capacitive admittance at power frequency. Since all quantities are varying sinusoidally in time at frequency $f = w/2\pi$, the timedependence
is not written, but is implicit.
By differentiating (8.3) with respect to $x$ and substituting the expressions for $dV_x/dx$ and $dI_x/dx$, we obtain independent differential equations for $V_x$ and $I_x$ as follows:

$$d^2V_x/dx^2 = z.\ y.\ V_x = p^2V_x \qquad ...(8.4)$$

and $$d^2I_x/dx^2 = z.y I_x = p^2 I_x \qquad ...(8.5)$$

where $$p = \sqrt{z.y} = jw\sqrt{lc} = jw/v = j2\pi/\lambda \qquad ...(8.6)$$

which is the propagation constant, $v$ = velocity of propagation, and $\lambda$ = wavelength.
Equation (8.4) and (8.5) are wave equations with solutions

$$V_x = Ae^{px} + Be^{-px} \qquad ...(8.7)$$

$$I_x = (1/z)\,dV_x/dx = \left(\frac{p}{z}\right).(Ae^{px} - Be^{-px})$$

$$= \sqrt{\frac{y}{z}}(Ae^{px} - Be^{-px}) \qquad ...(8.8)$$

The two constants $A$ and $B$ are not functions of $x$ but could be possible functions of $t$ since all voltages and currents are varying sinusoidally in time. Two boundary conditions are now necessary to determine $A$ and $B$. We will assume that at $x = 0$, the voltage and current are $V_R$ and $I_R$. Then, $A + B = V_R$ and $A - B = Z_0 I_R$, where $Z_0 = z / y = l/c$ = the characteristic impedance of the line.

Then, $$V_x = \frac{1}{2}(V_R + Z_0 I_R)e^{j2\pi x/\lambda} + \frac{1}{2}(V_R - Z_0 I_R)e^{-j2\pi x/\lambda} \qquad ...(8.9)$$

and $$I_x = \frac{1}{2}(V_R/Z_0 + I_R)e^{j2\pi x/\lambda} - \frac{1}{2}(V_R/Z_0 - I_R)e^{-j2\pi x/\lambda} \qquad ...(8.10)$$

We can also write them as

$$V_x = V_R.\cos(2\pi x/\lambda) + jZ_0 I_R \sin(2\pi x/\lambda) \qquad ...(8.11)$$

and $$I_x = I_R.\cos(2\pi x/\lambda) + j(V_R/Z_0).\sin(2\pi x/\lambda) \qquad ...(8.12)$$

Now, both $V_R$ and $I_R$ are phasors at power frequency and can be written as $V_R$ = .( $jwt+\phi$)
$_R V e$ ,
and .exp[( )] $I_R = I_R jwt + \phi - \phi_L$ where $\phi_L$ = internal angle of the load impedance $Z_L \angle \phi_L$.
We can interpret the above equations (8.11) and (8.12) in terms of standing waves as follows:

(1) The voltage $V_x$ at any point $x$ on the line from the load end consists of two parts: $V_R \cos(2\pi x/\lambda)$ and $jZ_0 I_R \sin(2\pi x/\lambda)$. The term $V_R \cos(2\pi x/\lambda)$ has the value $V_R$ at $x = 0$ (the load end), and stands on the line as a cosine wave of decreasing amplitude as $x$ increases towards the sending or source end. At $x = L$, it has the value $V_R .\cos(2\pi L/\lambda)$ . This is also equal to the no-load voltage when $I_R = 0$. Figure 8.2(*a*) shows these voltages. The second term in equation (8.11) is a voltage contributed by the load current which is zero at $x = 0$ and $Z_0 I_R \sin(2\pi L/\lambda)$ at the source end $x = L$, and adds vectorially at right angles to $I_R$ as shown in Figure 8.2 (*c*).

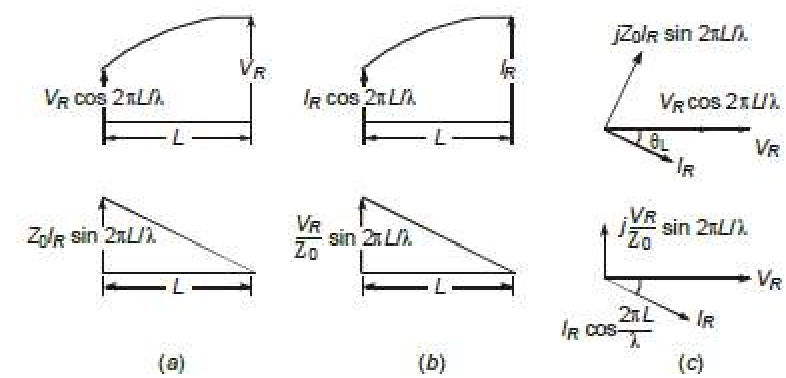


**Fig. 8.2. Standing waves of (*a*) voltage, and (*b*) current at power frequency.**

(2) The current in equation (8.12) also consists of two parts. $I_R \cos(2\pi x/\lambda)$ and
( / )sin(2 / ) $j\ V_R\ Z_0\ \pi x\ \lambda$ . At no load, $I_R = 0$ and the current supplied by the source is
( / ) $j\ V_R\ Z_0 \sin(2\pi L/\lambda)$ which is a pure charging current leading $V_R$ by 90°. These are shown in Fig. 8.2(b) and the vector diagram of 8.2(c).

A second interpretation of equations (8.9) and (8.10) which are equivalent to (8.11) and (8.12) is through the travelling wave concept. The first term in (8.9) is ( / )
( 0)
2
1 $jw\ t\ x\ v$
$V_R\ Z\ I_R\ e$ + + after
introducing the time variation. At $x = 0$, the voltage is $(V_R + Z_0 I_R)/2$ which increases as $x$ does, i.e., as we move towards the source. The phase of the wave is $e^{jw(t+x/v)}$ . For a constant value of $(t + x/v)$, the velocity is $dx/dt = -v$. This is a wave that travels from the source to the load and therefore is called the forward travelling wave. The second term is
2
1 ( / )
0 ( ) $jw\ t\ x\ v$
$_R\ _R V - Z\ I\ e$ – with
the phase velocity $dx/dt = +v$, which is a forward wave from the load to the source or a backward wave from the source to the load.

The current also consists of forward and backward travelling components. However, for the backward waves, the ratio of voltage component and current component is ($-Z_0$) is seen by the equations (8.9) and (8.10) in which the backward current has a negative sign before it.

## DIFFERENTIAL EQUATIONS AND SOLUTIONS FOR GENERAL CASE

In Section 8.1, the behaviour of electrical quantities at power frequencies on a distributedparameter line was described through travelling-wave and standing-wave concepts. The time variation of all voltages and currents was sinusoidal at a fixed frequency. In the remaining portions of this chapter, the properties and behaviour of transmission lines under any type of excitation will be discussed. These can be applied specifically for lightning impulses and switching surges. Also, different types of lines will be analyzed which are all categorized by the four fundamental distributed parameters, namely, series resistance, series inductance, shunt capacitance and shunt conductance. Depending upon the nature of the transmitting medium (line and ground) and the nature of the engineering results required, some of these four parameters can be used in different combinations with due regard to their importance for the problem at hand. For example, for an overhead line, omission of shunt conductance $g$ is permissible when corona losses are neglected. We shall develop the general differential equations for voltage and current by first considering all four quantities ($r$, $l$, $g$, $c$) through the method of Laplace Transforms, studying the solution and interpreting them for different cases when one or the other parameter out of the four loses its significance. As with all differential equations and their solutions, boundary conditions in space and initial or final conditions in time play a vital role.

### General Method of Laplace Transforms

According to the method of Laplace Transform, the general series impedance operator per unit length of line is $z(s) = r + ls$, and the shunt admittance operator per unit length is $y(s) = g + cs$, where$s$ = the Laplace-Transform operator.

Consider a line of length $L$ energized by a source whose time function is $e(t)$ and Laplace Transform $E(s)$, as shown in Fig. 8.3. Let the line be terminated in a general impedance $Z_t(s)$.

We will neglect any lumped series impedance of the source for the present but include it later on. Also, $x = 0$ at the terminal end as in Section 8.1.2. Let the Laplace Transforms of voltage and current at any point $x$ be $V(x, s)$ and $I(x, s)$.

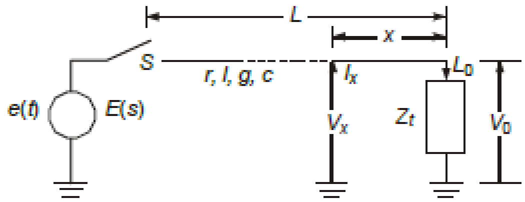


**Fig. 8.3** Distributed-parameter transmission line with source $E(s)$ and terminating impedance $Z_t(s)$.

Then, the following two basic differential equations will hold as for the steady-state excitation discussed earlier:

$$\partial V(x,s)/\partial x = z(s).I(x,s) \text{ and } \partial I(x,s)/\partial x. = y(s).V(x,s) \quad ...(8.13)$$

For simplicity in writing, we may omit $s$ in all terms but only remember that we are discussing the properties of the Laplace-Transforms of all quantities. The solutions for voltage and current in equation (8.13) will be

$$V(x) = A.e^{px} + B.e^{-px} \quad ...(8.14)$$

and

$$I(x) = (p/z).(Ae^{px} - Be^{-px}) \quad ...(8.15)$$

where again, $p$ = the propagation constant = $\sqrt{z.y}$ ...(8.16)

Also, $(p/z) = \sqrt{y/z} = Y_0$ and $(z/p) = \sqrt{z/y} = Z_0$, the characteristic or surge impedance.

For this problem, the boundary conditions are:

(1) At $x = L$, $V(L)$ = source voltage = $E(s)$; and

(2) At $x = 0$, $V(0) = Z_t.\, I(0)$. Using them in (8.14) and (8.15) yields

$$A = (Z_t + Z_0)E(s)/[(Z_t + Z_0)e^{pL} + (Z_t - Z_0)e^{-pL}] \quad ...(8.17)$$

and

$$B = (Z_t - Z_0)A/(Z_t + Z_0) \quad ...(8.18)$$

$$\therefore \quad V(x) = \frac{\cosh px + (Z_0/Z_t)\sinh px}{\cosh pL + (Z_0/Z_t)\sinh pL}E(s) \quad ...(8.19)$$

and

$$I(x) = \frac{1}{z}\frac{\partial V}{\partial x} = \sqrt{\frac{y}{z}}.\frac{\sinh px + (Z_0/Z_t)\cosh px}{\cosh pL + (Z_0/Z_t)\sinh pL}E(s) \quad ...(8.20)$$

These are the general equations for voltage $V$ and current $I$ at any point $x$ on the line in operational forms which can be applied to particular cases as discussed below.

*Line Terminations* For three important cases of termination of an open-circuit, a short circuit and $Z_t = Z_0$, the special expressions of equations (8.19) and (8.20) can be written.

*Case 1.* *Open Circuit.* $Z_t = \infty$

$$\left.\begin{aligned} V_{oc}(x,s) &= \cosh px.E(s)/\cosh pL \\ I_{oc}(x,s) &= \sqrt{y/z}.\sinh px.E(s)/\cosh pL \end{aligned}\right\} \quad ...(8.21)$$

*Case 2. Short-Circuit*. $Z_t = 0$

$$\left.\begin{aligned} V_{sc}(x,s) &= \sinh px.E(s)/\sinh pL \\ I_{sc}(x,s) &= \sqrt{y/z}\cosh px.E(s)/\sinh pL \end{aligned}\right\} \quad ...(8.22)$$

*Case 3. Matched Line*. $Z_t = Z_0$

$$\left.\begin{aligned} V_m(x,s) &= (\cosh px + \sinh px).E(s)/(\cosh pL + \sinh pL) \\ I_m(x,s) &= \sqrt{y/z}(\cosh px + \sinh px).E(s)/(\cosh pL + \sinh pL) \end{aligned}\right\} \quad ...(8.23)$$

The ratio of voltage to current at every point on the line is

$$Z_0 = \sqrt{z/y}\ .$$

$$[\sqrt{y/z} = Y_0 = \sqrt{(g+cs)/(r+ls)} = 1/Z_0.]$$

*Source of Excitation*

The time-domain solution of all these operational expressions are obtained through their Inverse Laplace Transforms. This is possible only if the nature of source of excitation and its Laplace Transform are known. Three types of excitation are important:

(1) *Step Function.* $e(t) = V$ ; $E(s) = V/s$, where $V$ = magnitude of step

(2) *Double-Exponential Function*. Standard waveshapes of lightning impulse and switching impulse used for testing line and equipment have a shape which is the difference between two exponentials. Thus $e(t) = E_0 (e^{-\alpha t} - e^{-\beta t})$ where $E_0$, α and β depend on the timings of important quantities of the wave and the peak or crest value. The Laplace Transform is

$E(s)$ = ( ) /( )( ) $E_0$ β − α $s$ + α $s$+ β ...(8.24)

(3) *Sinusoidal Excitation*. When a source at power frequency suddenly energises a transmission line through a circuit breaker, considering only a single-phase at present, at any point on the voltage wave, the time function is $e(t) = V_m \sin(wt + \phi)$ , where $w = 2\pi f$ , $f$ = the power frequency, and $\phi$ = angle from a zero of the wave at which the circuit breaker closes on the positively-growing portion of the sine wave. Its Laplace Transform is

$E(s)$ = $V$ ($w$.cos$s$.sin )/($s^2$ $w^2$ ) $_m$ ϕ+ ϕ + ...(8.25)

*Propagation Factor*. For the general case, the propagation factor is

$p = z.y = (r + ls)(g + cs)$ ...(8.16)

In the sections to follow, we will consider particular cases for the value of $p$, by omitting one or the other of the four quantities, or considering all four of them.

*Voltage at Open-End*. Before taking up a detailed discussion of the theory and properties of the voltage and current, we might remark here that for the voltage at the open end, equation (8.21) gives

$V(0, s) = E(s) / \cosh pL = 2.E(s) /(e^{pL} + e^{-pL})$

$= 2E(s)(e^{-pL} - e^{-3pL} + e^{-5pL} - ...)$ ...(8.26)

Under a proper choice of $p$ this turns out to be a train of travelling waves reflecting from the open end and the source, as will be discussed later. Equation (8.26) also gives standing waves consisting of an infinite number of terms of fundamental frequency and all its harmonics

## The Open-Circuited Line: Open-End Voltage

This is a simple case to start with to illustrate the procedure for obtaining travelling waves and standing waves. At the same time, it is a very important case from the standpoint of designing insulation required for the line and equipment since it gives the worst or highest magnitude of over-voltage under switching-surge conditions when a long line is energized by a sinusoidal source at its peak value. It also applies to energizing a line suddenly by lightning. The step response is first considered since by the Method of Convolution, it is sometimes convenient to obtain the response to other types of excitation by using the Digital Computer (see any book on Operational Calculus).

*Travelling-Wave Concept: Step Response*

*Case* 1. First omit all losses so that $r = g = 0$. Then,

$p = s\sqrt{lc} = s/v$ ...(8.27)

where $v$ = the velocity of e.m. wave propagation.

$\therefore\ V_0(s) = 2V / s(e^{sL/v} + e^{-sL/v}) = 2V(e^{-sL/v} - e^{-3sL/v} + \ldots)/s$ ...(8.28)

Now, by the time-shifting theorem, the inverse transform of

$2V.e^{-ksL/v} / s = 2V.\ U(t - kL/v)$ ...(8.29)

where $U(t - kL.v) = 0$ for $t < kL/v$

$= 1$ for $t > kL/v$.

We observe that the time function of open-end voltage obtained from equations (8.28) and (8.29) will be

$V_0(t) = 2V[U(t - L/v) - U(t - 3L/v) + U(t - 5L/v) \ldots]$ ...(8.30)

This represents an infinite train of travelling waves. Let $L/v = T$, the time of travel of the wave from the source to the open end. Then, the following sequence of voltages is obtained at the open end.

| $t$ | $T$ | $2T$ | $3T$ | $4T$ | $5T$ | $6T$ | $7T$ | $8T$ | $9T$... |
|---|---|---|---|---|---|---|---|---|---|
| $V_0$ | $2V$ | $2V$ | 0 | 0 | $2V$ | $2V$ | 0 | 0 | $2V$... |

A plot of the open-end voltage is shown in Fig. 8.4(*a*) from which it is observed that

(1) the voltage reaches a maximum value of twice the magnitude of the input step,

(2) it alternates between $2V$ and 0, and

(3) the periodic time is $4T$, giving a frequency of $f_0 = 1/4T$.

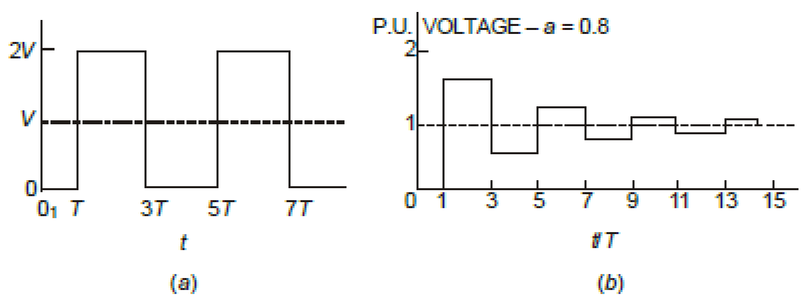


Fig. 8.4 Step response of transmission line. (*a*) Losses neglected. (*b*) Losses and attenuation included

Since all losses have been omitted, attenuation is absent and the amplitude between successive maxima does not decrease. The open-end voltage can never stabilize itself to a value equal to the excitation voltage.

*Case 2.Omit g*

For this case, $p = \sqrt{(r + ls)cs}$ or $p^2 = lcs^2 + rcs$.

Then, $p^2 = (\sqrt{lc})^2 (s + r/2l)^2 - r^2c/4l$ ...(8.31)

The last term ($r^2c/4l$) is usually negligible compared to the first. For example, consider $r$ = 1 ohm/km, $l$ = 1.1 mH/km and $c$ = 10 nF/km. Then, $r^2c/4l = 2.25 \times 10^{-6}$. Therefore, we take $p = s/v + r/2lv = s/v + \alpha$, where $\alpha = r/2lv$ = attenuation constant in Nepers per unit length.

Now, equation (8.26) becomes, with $E(s) = V/s$ for step,

$$V_0(s) = 2V/(e^{pL} + e^{-pL})s = 2V(e^{-pL} - e^{-3pL} + e^{-5pL} - \ldots)/s$$

$$= (2V/s)(e^{-\alpha L}.e^{-sL/v} - e^{-3\alpha L}.e^{-3sL/v} + e^{-5\alpha L}.e^{-5sL/v} - \ldots) \quad \ldots(8.32)$$

The time response contains the attenuation factor exp $(-k\alpha L)$ and the time shift factor exp $(-ksL/v)$ = exp $(-kTs)$. The inverse transform of any general term is $2V.\exp(-k\alpha L).U(t - kL/v)$. Let $a = \exp(-\alpha L)$, the attenuation over one line length which the wave suffers. Then the open end voltage of (8.32) becomes, in the time domain,

$$V_0(t) = 2V[a.U(t-T) - a^3.U(t-3T) + a^5.U(t-5T)\ldots] \quad \ldots(8.33)$$

With the arrival of each successive wave at intervals of $2T = 2L/v$, the maximum value also decreases while the minimum value increases, as sketched in Fig. 8.4 (*b*) for $a$ = 0.96 and 0.8. The voltage finally settles down to the value $V$, the step, in practice after a finite time. But theoretically, it takes infinite time, and because we have neglected the term ($r2c/4l$), the final value is a little less than $V$ as shown below. The values of attenuation factor chosen above (0.96 and 0.8) are typical for a line when only the conductor resistance is responsible for energy loss of the wave and when a value of 1 ohm/km for ground-return resistance is also taken into account. (For $a$ = 0.96, see next page).

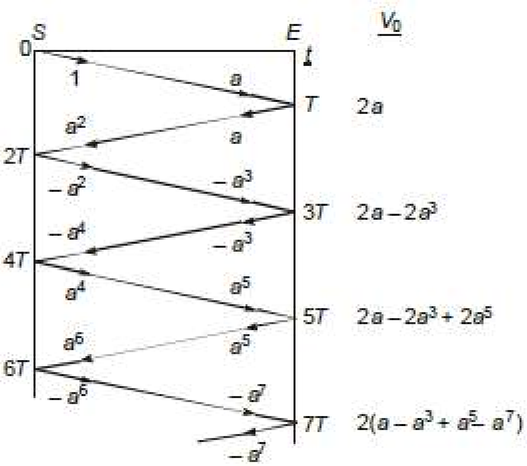


Fig. 8.5 The Bewley Lattice Diagram.

The following tabular form gives a convenient way of keeping track of the open-end voltage and Fig. 8.5 shows a graphical sketch which is due to Bewley. It is known as the Bewley Lattice Diagram. Note the systematic method employed in finding the voltage after any number of reflections.

| Time $t/T$ | $V_0(t)$ | $a = 0.96$ | $a = 08$ |
|---|---|---|---|
| 1 | $V_1 = 2V.a$ | 1.92 V | 1.6 V |
| 2 | $V_2 = 2V.a$ | | |
| 3 | $V_3 = 2V(a - a^3) = V_1 - a^2 V_1$ | 0.15 V | 0.576 V |
| 4 | $V_4 = V_3$ | | |
| 5 | $V_5 = 2V(a - a^3 + a^5) = V_1 - a^2 V_3$ | 1.78 V | 1.23 V |
| 6 | $V_6 = V_5$ | | |
| 7 | $V_7 = 2V(a - a^3 + a^5 - a^7) = V_1 - a^2 V_5$ | 0.28 V | 0.81 V |
| 8 | $V_8 = V_7$ | | |
| 9 | $V_9 = 2V(a - a^3 + a^5 - a^7 + a^9) = V_1 - a^2 V_7$ | 1.66 V | 1.08 V |
| 10 | $V_{10} = V_9$ | | |
| ⋮ | | | |
| $N$ | $V_N = V_1 - a^2 V_{N-2}$ | | |
| $N+1$ | $V_{N+1} = V_N$ | | |

According to this procedure, the final value is

$$V_\infty = 2Va(1 - a^2 + a^4 - a^6 + \ldots) = 2Va/(1 + a^2) \quad \ldots(8.34)$$

# CHAPTER 5
# VOLTAGE CONTROL

## 5.1 THE POWER CIRCLE DIAGRAM AND ITS USE

Equation (12.10) can also be written for $E_r$ and $I_r$ in terms of $E_s$ and $I_s$ as follows, with $AD - BC = 1$, and $A = D$:

$$\begin{bmatrix} E_r \\ I_r \end{bmatrix} = \begin{bmatrix} A, & B \\ C, & D \end{bmatrix}^{-1} \begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} A, & -B \\ -C, & D \end{bmatrix} \begin{bmatrix} E_s \\ I_s \end{bmatrix} \quad (5.1)$$

Also,

$$I_r = (E_s - AE_r)/B \quad (5.2)$$

All these quantities are complex numbers. The receiving-end power is

$$W_r = P_r + jQr = E_r I_r^* = E_r (E_s^* - A^* E_r^*)/B^* \quad (5.3)$$

If we consider $E_r$ as reference giving $= \angle 0°\ E_r E_r$, then the sending-end voltage $E_s = |E_s| \angle \delta$, where the angle $\delta$ is called the power angle. Also let $A = |A|\ \angle\theta_a$ and $B = |B|\ \angle\theta_b$. Then

$$\begin{aligned} W_r = P_r + jQ_r &= \frac{E_r \angle\theta_b}{|B|}(|E_s| \angle -\delta - E_r . |A| \angle -\theta_a) \\ &= \frac{E_r |E_s|}{|B|}\angle(\theta_b - \delta) - \frac{E_r^2 |A|}{|B|}\angle(\theta_b - \theta_a) \end{aligned} \quad (5.4)$$

Separating the real and $j$-parts, there result

$$P_r = \frac{E_r |E_s|}{|B|}\cos(\theta_b - \delta) - \frac{E_r^2 |A|}{|B|}\cos(\theta_b - \theta_a) \quad (5.5)$$

And $$Q_r = \frac{E_r |E_s|}{|B|}\sin(\theta_b - \delta) - \frac{E_r^2 |A|}{|B|}\sin(\theta_b - \theta_a) \quad (5.6)$$

These can be re-written as

$$\left.\begin{aligned} P_r + \frac{E_r^2 |A|}{|B|}\cos(\theta_b - \theta_a) &= \frac{E_r |E_s|}{|B|}\cos(\theta_b - \delta) \\ Q_r + \frac{E_r^2 |A|}{|B|}\sin(\theta_b - \theta_a) &= \frac{E_r |E_s|}{|B|}\sin(\theta_b - \delta) \end{aligned}\right\} \tag{5.7}$$

Then, eliminating δ by squaring and adding the two equations, we obtain the locus of $P_r$ against $Q_r$ to be a circle with given values of $A$ and $B$, and for assumed values of $E_r$ and $|E_s|$ as follows:

$$\left\{P_r + \frac{E_r^2 |A|}{|B|}\cos(\theta_b - \theta_a)\right\}^2 + \left\{Q_r + \frac{E_r^2 |A|}{|B|}\sin(\theta_b - \theta_a)\right\}^2 = \left(\frac{E_r |E_s|}{|B|}\right)^2 \tag{5.8}$$

The coordinates of the centre of the receiving-end power-circle diagram are

$$\left.\begin{aligned} x_c &= -\frac{E_r^2 |A|}{|B|}\cos(\theta_b - \theta_a), \text{MW, and} \\ y_c &= -\frac{E_r^2 |A|}{|B|}\sin(\theta_b - \theta_a), \text{MVAR} \end{aligned}\right\} \tag{5.9}$$

The radius of the circle is

$$R = E_r . |E_s| / |B|, \text{MVA} \tag{5.10}$$

Figure 12.2 shows the receiving-end circle diagram. If the receiving-end voltage is heldconstant, as is usually dictated by the load, the centre of the circle is fixed, but the radius willdepend on the value of the sending-end voltage, $E_s$. Therefore, for a chosen variation in $|E_s|$,a system of circles with centre at $C$ and proper radius given by equation (12.40) can be drawn.The power-circle diagram and the geometrical relations resulting from it are extremely usefulto a design engineer and an operating engineer to determine the status of power flow, reactivepower flow, compensation requirements for voltage control and many properties concerningthe system. These will be illustrated through typical examples.Referring to Figure 12.2, the angle $\angle OCP_m = (\theta_b - \theta_a)$ and from equations (12.35) and(12.36), when $P_r = 0$, that is when the circle intersects the vertical axis at $O'$.

$$R\cos(\theta_b - \delta_0) = |x_c|, \text{ giving } \angle O'CP_m = (\theta_b - \delta_0) \tag{5.11}$$

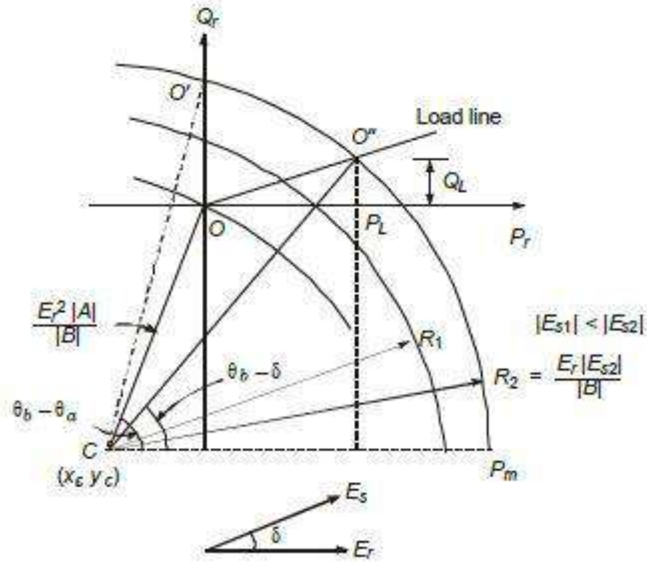


**Fig. 5.1 Receiving-end circle diagram for calculating reactive compensation for voltage control at buses.**

It is therefore a simple matter to find the angle $\delta_0$ at no-load. For any other load, the loadline is drawn with given MW and MVR along the horizontal and vertical axes shown as $P_L$ and $Q_L$. Let the circle intersect the load line at $O''$. Then, $\angle O''CP_m = \theta_b - \delta$, from equation (12.35).Because of geometrical construction using instruments, there will be slight inherent errorinvolved in the results but will serve engineering purposes. However, with the aid of DigitalComputers, great accuracy can be achieved from the properties relating to the circle diagram.

## 5.2 VOLTAGE CONTROL USING SYNCHRONOUS CONDENSERS

From the generalized constants ($A$, $B$, $C$, $D$) of a given input port and output port, the powercirclediagram or the corresponding geometrical relations can be utilized for deciding the propercompensating MVAR's to be provided at the receiving end when a set of magnitudes for $E_r$ and $E_s$ at the two ends of the line are specified. When the load has a lagging power factor or even aunity power factor, generally the control of voltage is achieved by providing leading powerfactor or capacitive compensation at the receiving end. This can take the form of switchedcapacitors, usually connected to the low-voltage tertiary of the substation transformer at theload end. At no-load, usually inductive reactive compensation is required if the sending-endvoltage is to be raised to stay within the standard specifications. This is provided by either theswitched type (regulated) or constant type (unregulated).The synchronous condenser provides both types of MVAR's, lagging or leading, that isinductive or capacitive. The synchronous motor runs without a shaft load and is enclosed in anexplosion-proof casing which is filled with hydrogen at above atmospheric pressure in order tominimize rotational losses, and to ensure that any leakage of gas will be from the inside to external air, thus preventing an explosive mixture to be formed. The reactive powers generated by the motor are controlled by varying the dc field excitation of the rotor, under-excitationproviding inductive MVAR's and over-excitation yielding capacitive MVAR's. Because oflimitations imposed on excitation, the synchronous phase modifier can provide only 60 to 70%of its rated capacity at lagging power factor (under-excited condition) and full rated leadingreactive power (over-excited condition).The design of the rating of the synchronous phase modifier (or condenser for short) andthe voltage conditions are illustrated below. The general equation satisfied by the real andreactive powers at the load end at any point on the circle is

$$(P_r + |x_c|)^2 + (Q_r + |y_c|)^2 = R^2 \tag{5.12}$$

At no load, the point $O'$ in Figure 12.2, obviously $P_r = 0$. The reactive power is

$$Q_0 = \sqrt{R^2 - x_c^2} - |y_c| \tag{5.13}$$

At full load, point $O''$ in Figure 12.2 or 12.3,

$$(P_L + |x_c|)^2 + (|y_c| - Q_r)^2 = R^2 \tag{5.14}$$

∴ The total reactive power required at the receiving end is

$$Q_r = |y_c| - \sqrt{R^2 - (P_L + |x_c|)^2} \tag{5.15}$$

The capacitive MVAR required in the synchronous condenser is than

$$Q_c = Q_L + Q_r = Q_L + |y_c| - \sqrt{R^2 - (P_L + |x_c|)^2} \tag{5.16}$$

If we use the relation ,( 1) $Q_0 = mQ_c m <$ , then the equation to be satisfied by the radius ofthe circle, which is the only quantity involving the sending-end voltage $E_s$will be, when valuesfor $E_r$, $A$, $B$, $P_L$, $Q_L$ are specified,

$$m\sqrt{R^2 - (P_L + |x_c|)^2} + \sqrt{R^2 - x_c^2} = (1+m)|y_c| + mQ_L \tag{5.17}$$

When all quantities except $R$ are specified, equation (12.50) will yield the value of$R = E_r|E_s|/|B|$, from which the sending-end voltage is determined. This is shown by thefollowing example.

## 5.3 CASCADE CONNECTION OF COMPONENTS—SHUNT AND SERIES COMPENSATION

In the previous sections, the (*A*, *B*, *C*, *D*) constants of only the line were considered. It becomesevident that through the example of the 750 kV line parameters, it is impossible to control thevoltages within limits specified by IS and IEC by providing compensation at one end only bysynchronous condensers, or by switched capacitors if the voltages are to vary over wider limitsthan discussed. In practice, shunt-compensating reactors are provided for no-load conditionswhich are controlled by the line-charging current entirely, and by switched capacitors for fullloadconditions when the load has a lagging power factor.

### Generalized Equations

For no-load conditions, $Z_t = \infty$, and the equations for sending-end and receiving-end voltagesare, Figure 12.4,

$$E_r = E_s/[\cosh pL + (Z_0/Z_{sh})\sinh pL] \quad (5.18)$$

For simplicity, let $r = 0$. Then,

$$p = jw/v_0 = j2\pi/\lambda, Z_0 = Z_{00} = \sqrt{l/c}, Z_{sh} = jX_{sh}, \quad (5.19)$$

so that

$$E_s/E_r = \cos 2\pi L/\lambda + (Z_{00}/X_{sh})\sin 2\pi L/\lambda. \quad (5.20)$$

When the ratio $E_s/E_r$is given for a system, the value of $X_{sh}$is easily determined. Inparticular, when $E_s = E_r$,

$$Z_{sh} = Z_{00}/(\operatorname{cosec} 2\pi L/\lambda - \cot 2\pi L/\lambda). \quad (5.21)$$

**Example 12.11.**For the 750-kV line of previous examples, $L$ = 500 km, $\lambda$ = 6000 km at 50*Hz* and $Z_{00}$ = 260 ohms. Assuming $E_s = E_r$= 750 kV, calculate the reactance and 3-phase MVARrequired at load end in the shunt-compensating reactor. Neglect line resistance.

**Solution.**$2\pi L/\lambda = 30°$ at 6° per 100 km of length of line.

$\therefore X_{sh}$= 260/(cosec 30° – cot 30°) = 3.73 × 260 = 970 ohms.

This is necessary at load end connected between line and ground so that there will be 3such reactors for the 3-phases.Current through each reactor $I_{sh}$= 750 /970 3 = 0.4464 kA .

∴ MVAR of each reactor per phase = 750 × 0.4464/ 3 = 193.3.

Total 3-phase MVAR at load end = 580 MVAR.

## 5.4 Shunt Reactor Compensation of Very Long Line with IntermediateSwitching Station

For very long lines, longer than 400 km at 400 kV, or at higher voltages, an intermediate station is sometimes preferable in lieu of series-capacitor compensation which will be discussedin the next section. Figure 12.5 shows the arrangement where each line section has thegeneralized constants $(A, B, C, D)_{Line}$, and each of the four shunt reactors has an admittance of$(-jB_{sh})$ and reactance $(jX_{sh})$. Then, by using the chain rule of multiplication, the total generalizedconstants ($A_T$, $B_T$, $C_T$, $D_T$) relating ($E_s$, $I_s$) with ($E_r$, $I_r$) will be as follows, upon using equation(12.55).

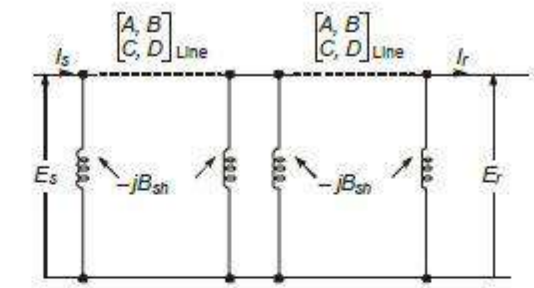


**Fig. 5.2** Extra-long line with shunt reactors at ends and at an intermediate station.

$$\begin{bmatrix} A_T, & B_T \\ C_T, & D_T \end{bmatrix} = \begin{bmatrix} A - jB_{sh}B, & B \\ C - j2B_{sh}A - B_{sh}^2 B, & D - jB_{sh}B \end{bmatrix}^2$$

$$= \begin{bmatrix} A^2 + BC, & 2AB \\ 2AC, & A^2 + BC \end{bmatrix} - B_{sh}^2 B \begin{bmatrix} 2B, & 0 \\ 6A, & 2B \end{bmatrix}$$

$$- jB_{sh} \begin{bmatrix} 4AB & B^2 \\ 2BC + 4A^2 + 2B_{sh}^2 B^2, & 4AB \end{bmatrix} \qquad (5.22)$$

For $B_{sh}$= 0, this reduces to

$$\begin{bmatrix} A_T, & B_T \\ C_T, & D_T \end{bmatrix}_{B_{sh}=0} = \begin{bmatrix} A^2 + BC, & 2AB \\ 2AC, & A^2 + BC \end{bmatrix} = \begin{bmatrix} A, & B \\ C, & D \end{bmatrix}^2 \qquad (5.23)$$

The voltage and current at the intermediate station are

$$\begin{bmatrix} E_r' \\ I_r' \end{bmatrix} = \begin{bmatrix} A - jB_{sh}B, & B \\ C - j2AB_{sh} - B_{sh}^2 B, & D - jB_{sh}B \end{bmatrix} \begin{bmatrix} E_r \\ I_r \end{bmatrix} \qquad (5.24)$$

## 5.5 SERIES CAPACITOR COMPENSATION AT THE LINE CENTRE

In order to increase the power-handling capacity of a line, the magnitude of $B$ must be reduced,as shown in equations (12.35) and (12.44). In normal practice, $\theta \approx 90^{\circ}{}_{b}$ and $\theta_a \approx 0$ for very lowseries line resistance. Therefore, $P = E_r E_s.\sin\delta / B$ . We also observe that the value of $B$ is verynearly equal to the series inductive reactance of the line so that by employing capacitors connectedin series with the line, the power-handling capacity of a line can be increased for chosen valuesof $E_s$, $E_r$and $\delta$ . All these three quantities are limited from considerations of highest equipmentvoltages and stability limits. Usually, the series capacitor is located at the line centre when onecapacitor only is used, or at the one-third points if two installations are used. On a very longline with intermediate station, the series capacitor can be located here. In this section we willonly consider the generalized constants when a series capacitor is located at the line centrewithout intermediate shunt-reactor compensation. Thus, the system considered consists ofequal shunt reactor admittances $B_{sh}$at the two ends and a capacitor with reactance $x_c$at theline centre, as shown in Figure 12.7.For each half of the line section of length $L/2$,

$$A' = D' = \cosh\sqrt{ZY/4}, B' = Z_0 \sinh\sqrt{ZY/4},$$

$$C' = \frac{1}{Z_0}\sinh\sqrt{ZY/4} \qquad (5.25)$$

where$Z$ and $Y$ refer to the total line of length $L$. The surge impedance is not altered eventhough the line length is halved.For the series capacitor, Figure 12.6($b$), the voltages and currents on the two sides arerelated by

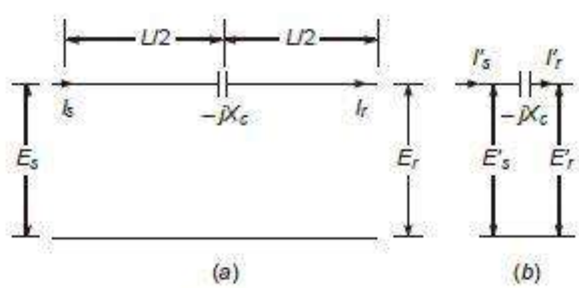


**Fig. 5.3 Transmission line with series-capacitor compensation in middle of line**

$$\begin{bmatrix} E'_s \\ I'_s \end{bmatrix} = \begin{bmatrix} 1, & -jx_c \\ 0, & 1 \end{bmatrix} \begin{bmatrix} E'_r \\ I'_r \end{bmatrix} \qquad (5.26)$$

Thus, for the two half-sections of line with the capacitor separating them, the totalgeneralized constants will be, by the chain rule of multiplication, (Figure 12.6$a$),

$$\begin{bmatrix} A_T, & B_T \\ C_T, & D_T \end{bmatrix} = \begin{bmatrix} \cosh\sqrt{ZY/4}, Z_0 \sinh\sqrt{ZY/4} \\ \frac{1}{Z_0}\sinh\sqrt{ZY/4}, \cosh\sqrt{ZY/4} \end{bmatrix} \begin{bmatrix} 1, & -jx_c \\ 0, & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} \cosh\sqrt{ZY/4}, Z_0 \sinh\sqrt{ZY/4} \\ \frac{1}{Z_0}\sinh\sqrt{ZY/4}, \cosh\sqrt{ZY/4} \end{bmatrix} = \begin{bmatrix} \cosh\sqrt{ZY}, Z_0 \sinh\sqrt{ZY} \\ \frac{1}{Z_0}\sinh\sqrt{ZY}, \cosh\sqrt{ZY} \end{bmatrix} \tag{5.27}$$

$$-j\frac{x_c}{2Z_0}\begin{bmatrix} \sinh\sqrt{ZY}, Z_0(\cosh\sqrt{ZY}+1) \\ \frac{1}{Z_0}(\cosh\sqrt{ZY}-1), \sinh\sqrt{ZY} \end{bmatrix}$$

$$= \begin{bmatrix} A, & B \\ C, & D \end{bmatrix}_{\text{Line}} - j\frac{x_c}{2Z_0}\begin{bmatrix} \sinh pL, Z_0(\cosh pL+1) \\ \frac{1}{Z_0}(\cosh pL-1), \sinh pL \end{bmatrix} \tag{5.28}$$

With no series capacitor, $x_c$= 0, equation (12.63) reduces to the generalized constants ofthe line.

## 5.6 Shunt Reactors at Both Ends and Series Capacitor in Middle of Line

If shunt-compensating reactors of admittance $B_{sh}$are located at both ends of line, the totalgeneralized constants with series capacitor located in the line centre can be obtained in theusual way by using the chain rule, from Figure 12.7. It is left to the reader to verify the result.

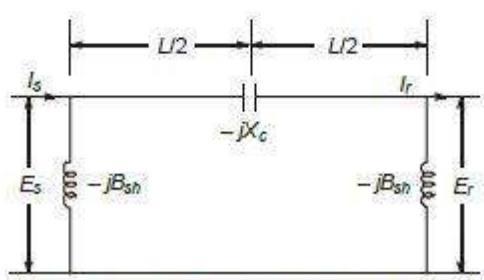


Fig. 5.4 Transmission line with series capacitor in middle and shunt reactors at ends.

$$\begin{bmatrix} A_T, & B_T \\ C_T, & D_T \end{bmatrix} = \begin{bmatrix} A, & B \\ C, & D \end{bmatrix}_{\text{Line}} - jB_{sh}\begin{bmatrix} B,0 \\ 2A - jB_{sh}B, B \end{bmatrix}$$

$$- j\frac{x_c}{Z_0}\begin{bmatrix} \sinh\ pL,\ Z_0(\cosh\ pL+1) \\ \frac{1}{Z_0}(\cosh\ pL-1), \sinh\ pL \end{bmatrix}$$

$$-\frac{x_c B_{sh}}{2}\begin{bmatrix} \cosh pL+1, & 0 \\ \frac{2}{Z_0}\sinh pL - jB_{sh}(\cosh pL+1), & \cosh pL+1 \end{bmatrix} \quad (5.29)$$

We must point out that all quantities except $x_c$ and $B_{sh}$ are complex numbers with amagnitude and phase angle.If shunt reactors are not used, $B_{sh}$= 0, then equation (12.64) reduces to (12.63). If no seriescapacitor is used, equation (12.55) results.The value of capacitance is chosen such that its reactance amounts to a chosen percentageof the series inductive reactance of the line. For example, if 50% series capacitance is to beprovided for a 800 km 400 kV line having a series inductive reactance of *j* 244 ohms, thecapacitive reactance is – *j* 122 ohms. The required capacitance will be $C = 1/wx_c$= 26 μF at 50Hz. The power-handling capacity of a line with 50% series compensation will increase to doublethat without series capacitor.While the power-handling capacity can be increased, series capacitor compensation resultscertain harmful properties in the system, chief among them are:(*a*) Increased short-circuit current. Note that for a short-circuit beyond the capacitorlocation, the reactance is very low. At the load-side terminal of the capacitor, a shortcircuitwill result in infinite current for 50% compensation.(*b*) Sub-harmonic or sub-synchronous resonance conditions during load changes and shortcircuits.This has resulted in unexpected failures to long shafts used in steam-turbinedrivenalternators and exciters when one or more of the resulting sub-harmoniccurrents due to series compensation can produce shaft torques that correspond toone of the several resonance frequencies of the shaft, called torsional modes ofoscillation or critical speeds. This aspect will be discussed in some detail and counter
measures used against possible failure will be described in the next section.

## 5.7 SUB-SYNCHRONOUS RESONANCE IN SERIES-CAPACITOR COMPENSATED LINES

### *5.6.1 Natural Frequency and Short-Circuit Current*

With series compensation used, the power-handling capacity of a single circuit is approximately

$$P = E_r E_s . \sin\delta / X_s \quad (5.30)$$

Where

$$X_s = X_L - X_c = X_L(1-m) \quad (5.31)$$

with $m = X_c/X_L$ = degree of compensation, and the reactances are at power frequency $f_0$.The approximation occurs because we have considered the series inductive reactance $X_L$ to belumped. At any other frequency $f$,

$X_L(f) = 2\pi f L_L$ = total inductive series reactance of line

and $X_c(f) = 1/2\pi f C$ = series capacitive reactance of capacitor.

∴ The resonance frequency occurs when $X_L(f_e) = X_c(f_e)$ giving

$$2\pi f_e L_L = 1/2\pi f_e C \quad (5.32)$$

We can introduce the power frequency $f_0$ by re-writing equation (12.67)

As

$$(2\pi f_0 L_L)(f_e/f_0) = 1/[2\pi f_0 C.(f_e/f_0)] \text{ at resonance.} \quad (5.33)$$

at resonance.

Consequently,

$$(f_e/f_0)^2 = 1/(2\pi f_0 L_L).(2\pi f_0 C) = X_c/X_L = m \quad (5.34)$$

Thus, the electrical resonance frequency is $f_e = f_0\sqrt{m}$ ...(12.69) .The reduction of series reactance also reflects as decrease in effective length to

$$L_e = (1-m)L \quad (5.35)$$

## 5.8 STATIC REACTIVE COMPENSATING SYSTEMS (STATIC VAR)

In the previous section, one type of reactor compensation for countering SSR was mentioned as a Dynamic Filter which uses thyristors to modulate the current through a parallel-connectedreactor in response to rotor speed variation. The advent of high-speed high-current switchingmade possible by thyristors (silicon-controlled-rectifiers) has brought a new concept in providing reactive compensation for optimum system performance. Improvements obtained by the use ofthese static var compensators (SVC) or generators (SVG) or simply static var systems (SVS) arenumerous, some of which are listed below:

1. When used at intermediate buses on long lines, the steady-state power-handlingcapacity is improved.
2. Transient stability is improved.

3. Due to increased damping provided, dynamic system stability is improved.
4. Steady-state and temporary voltages can be controlled.
5. Load power factor can be improved thereby increasing efficiency of transmission and lowering of line losses.
6. Damping is provided for SSR oscillations.

7. Overall improvement is obtained in power-transfer capability and in increased economy.
8. The fast dynamic response of SVC's have offered a replacement to synchronous condensers having fast excitation response.

In principle, when the voltage at a bus reduces from a reference value, capacitive Varshave to be provided, whereas inductive Vars are necessary to lower the bus voltage.Figure 12.14 shows schematically the control characteristics required and the connection scheme.The automatic voltage regulator AVR provides the gate pulses to the proper thyristors toswitch either capacitors or inductors. The range of voltage variation is not more than 5%between no load and full load.Many schemes are in operation and some of them are shown in Figure 12.15 and describedbelow.

## 5.9 SVC Schemes

### 5.9.1 The TCR-FC System (Figure 12.15 (a)).

This is the Thyristor Controlled Reactor-Fixed Capacitor system and provides leadingvars from the capacitors to lagging vars from the thyristor-switched reactors. Because of theswitching operation, harmonics are generated. Since there are 6 thyristors for the 3-phases, 6-pulse harmonics are generated in addition to the 3rd. These must be eliminated if they are notto affect normal system operation.

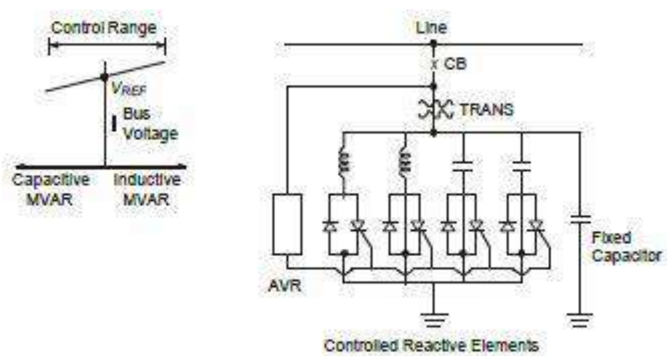


**Fig. 5.5** General circuit of Static Var Systems with controlled reactive elements and control range exercized.

This can be improved by using a large number of small reactors to reduce harmonics which normally depend on the reactor size. However, it requires correspondingly larger numberof thyristor switches. This is known as segmented TCR-FC.By using two transformers, one in *Y*/Δ and the other in *Y*/*Y*, and dividing the fixed capacitorsand controlled reactors into two groups, 12 thyristors can be used in a 12-pulse configuration.The lowest harmonic will be the 12th (600 Hz or 720 Hz in 50 or 60 Hz system). It introducescomplications in transformer design, and requires more thyristors. The gate-firing angle controlis made more difficult because of the 30° phase shift between the secondary voltages of the*Y*/Δ and *Y*/*Y* transformers.

(2) *The TCT Scheme* (Thyristor Controlled Transformer)

A transformer of special design with almost 100% leakage impedance is controlled on thesecondary side by thyristor switches. By connecting the 3 phases in Δ, 3rd harmonics are eliminated. This system has better over-load capacity and can withstand severe transientovervoltages. But it is more expensive than TCR scheme. See Figure 12.15(*b*).

(3) *TSC-TCR Scheme* (Figure 12.15)(*c*))

This is Thyristor Switched Capacitors and Thyristor-Controlled Reactors, It has TCR'sand capacitance changed in discrete steps. The capacitors serve as filters for harmonics whenonly the reactor is switched.

(4) *MSC-TCR Compensator Scheme*

This is the Mechanically Switched Capacitor-Thyristor Controlled Reactors scheme. Itutilizes conventional mechanical or $SF_6$ switches instead of thyristors to switch the capacitors.It proves more economical where there are a large number of capacitors to be switched thanusing TSC (thyristor-switched capacitors). The speed of switching is however longer and thismay impair transient stability of the system.

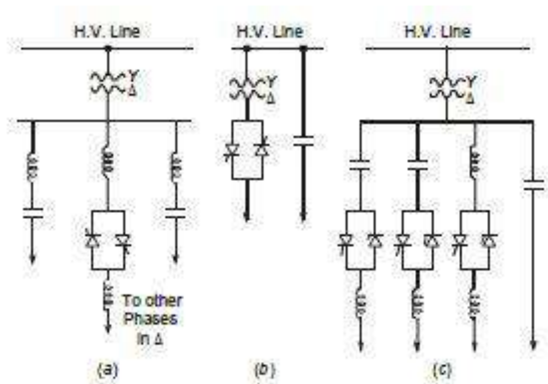


**Fig. 5.6** Different types of Static Var Compensators.
(*a*) TCR-FC System, (*b*) TCT scheme, (*c*) TSC-TCR scheme

(5) *SR Scheme* (Saturated Reactor)

In some schemes for compensation, saturated reactors are used. Fixed capacitors areprovided as usual. A slope-correction capacitor is usually connected in series with the saturatedreactor to alter the B-H characteristics and hence the reactance.

# Extra High Voltage A.C. Transmission Engineering

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