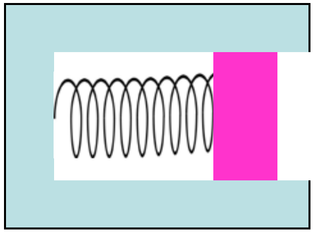
Last class we began discussion of pressure in fluids, with pressure defined as,

$$p = \frac{F}{A} \qquad ; \text{ units } \quad 1\,\text{Pa} = 1\frac{\text{N}}{\text{m}^2}$$

There are a number of other pressure units in common use having the following equivalence,

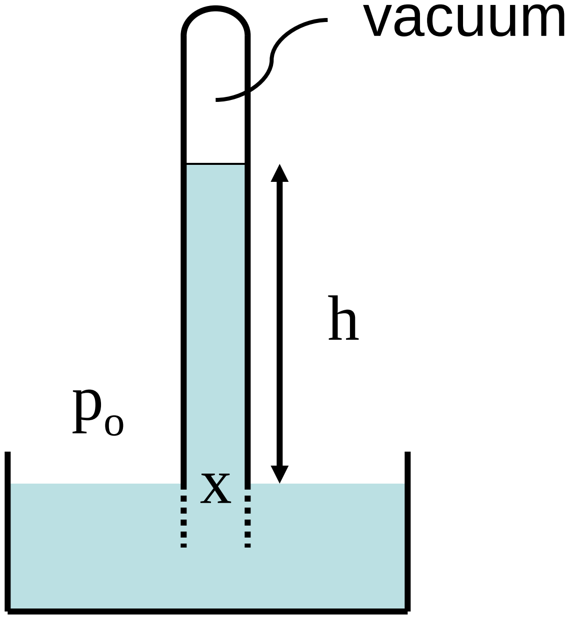
$$1\,\text{atm} = 1.01\text{x}10^5\ \text{Pa} = 760\ \text{Torr} = 14.7\ \text{lb}/\text{in}^2$$

We also discussed a pressure gauge based on an evacuated cylinder & spring arrangement.



A device for measuring atmospheric pressure is the barometer. This is a close bottomed tube, filled to overflowing with a fluid and then turned over in an open bath of the same fluid.

Done on the moon which has essentially no atmosphere, the fluid would just run out until $h = 0$.



On earth where the pressure on the exterior surface is around 1 atm the fluid at point x inside the tube has pressure equal to p_o, from the atmospheric pressure p_o outside the tube.

That means a force up given by solving

$$p_o = \frac{F}{A}$$

with A the cross-section of the tube.

Since the fluid is stationary, there must be an equal but opposite force down at point x having the same magnitude, but where we recognize the force as due to the weight of the fluid column,

$$p_o = \frac{F}{A} = \frac{mg}{A}$$

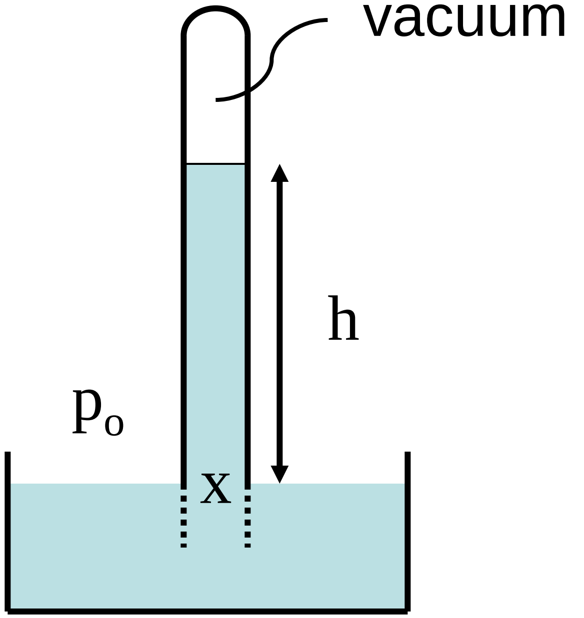
But since the density (rho) $\rho = \frac{m}{V}$ this can be rearranged to give,

$$m = \rho V = \rho A h,$$

which substituted above gives

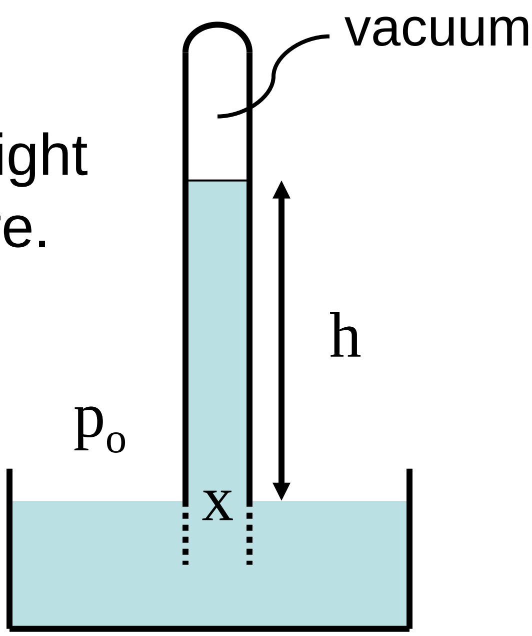
$$p_o = \frac{F}{A} = \frac{mg}{A} = \frac{\rho g A h}{A}$$

$$p_o = \rho g h$$



$$p_o = \rho g h$$

Taking $h = 0$ as zero pressure (absolute) the height of the fluid column is proportional to the pressure.



Mercury is often used. The height of a column of mercury at sea level is on average 760 mm, so you will often hear atmospheric pressures quoted in mm (or equivalently Torr) around this value.

Since the density of mercury depends on the temperature, and the pressure reading depends on density, for accurate readings of atmospheric pressure this must be corrected for the local temperature.

HITT

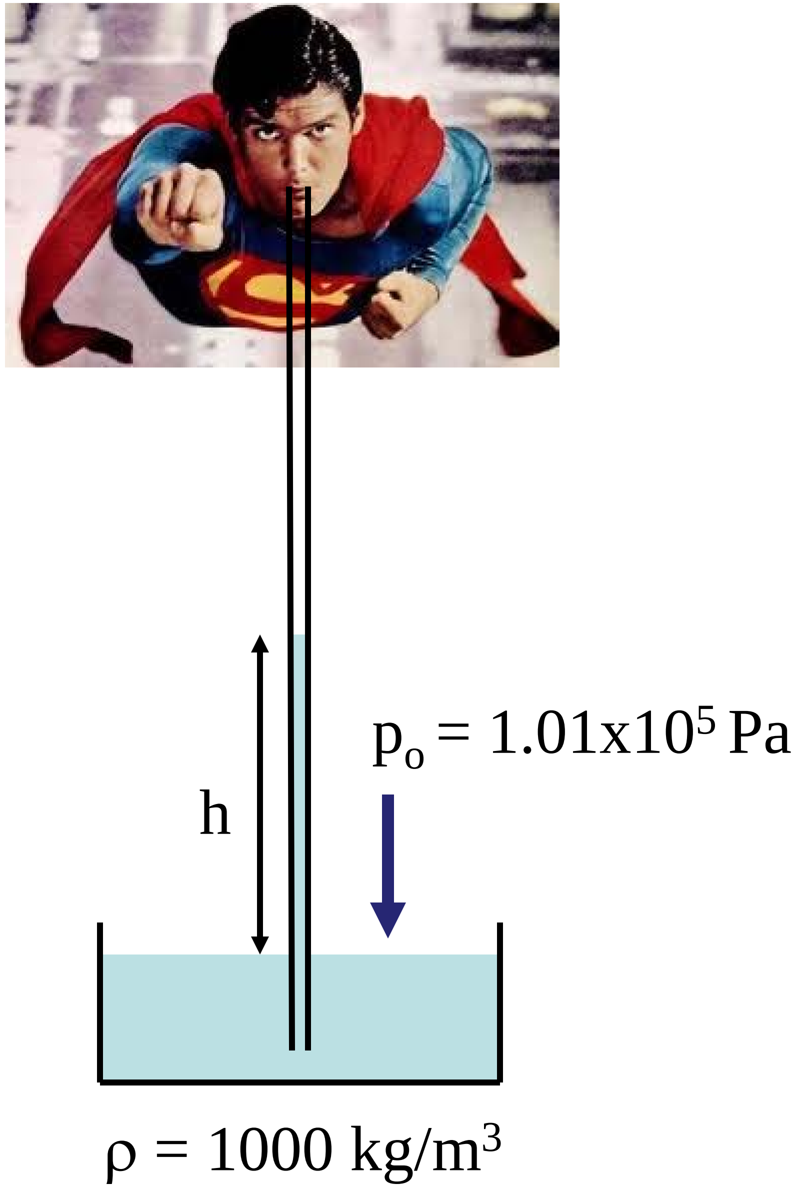
Superman (of course) has super suction, capable of pulling an infinite vacuum.

Given an appropriate straw how high could he suck the fresh water from a lake?

A) as high as he wants

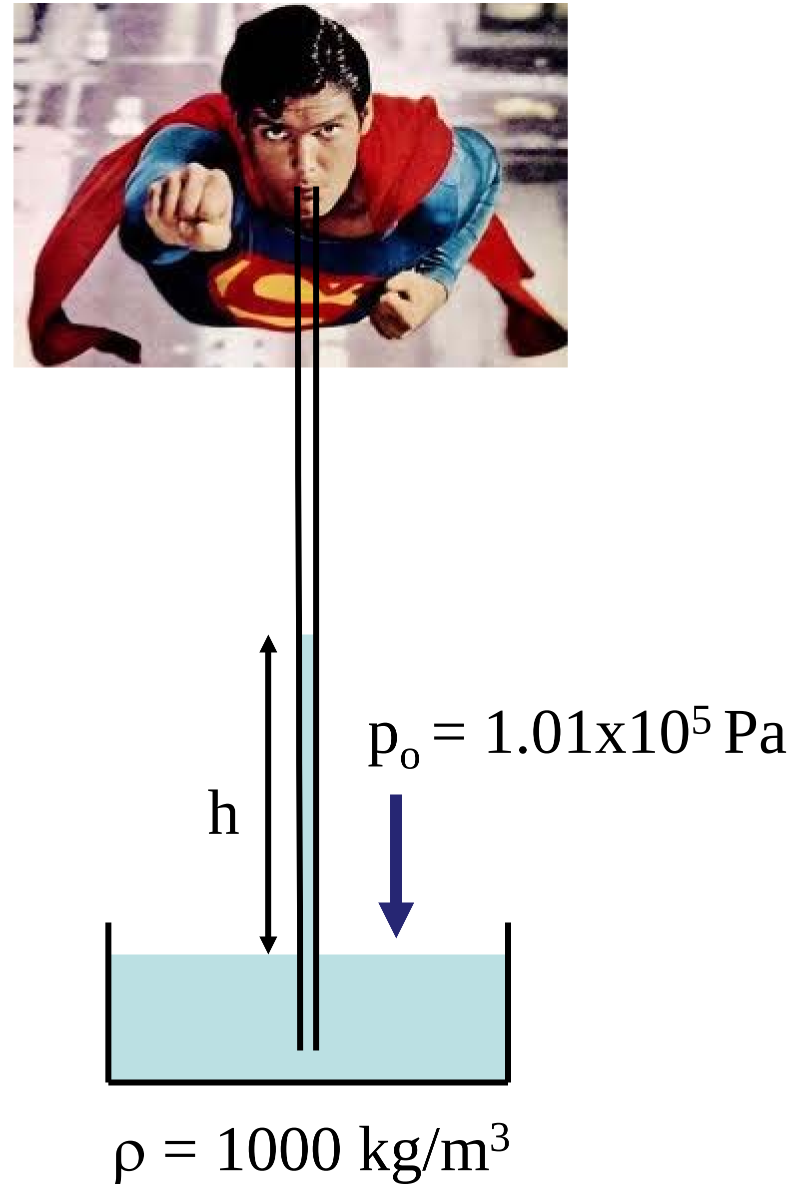
B) 32.0 m

C) 10.3 m



HITT

The barometer also has a vacuum in the upper part of the tube. Making that vacuum more perfect changes the height of the water negligibly.



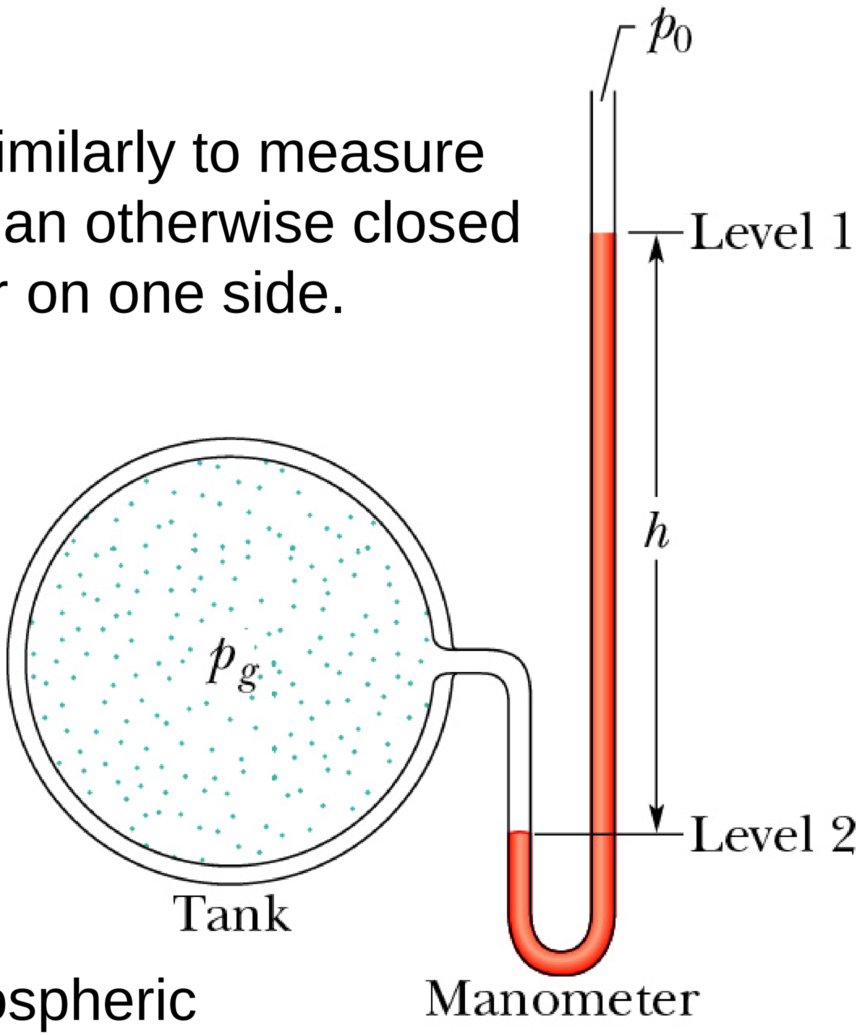
There we found $p_o = \rho g h$

so,

$$h = \frac{p_o}{\rho g} = \frac{1.01\times10^5\ \text{Pa}}{\left(1000\,\frac{\text{kg}}{\text{m}^2}\right)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)} = 10.3\,\text{m}$$

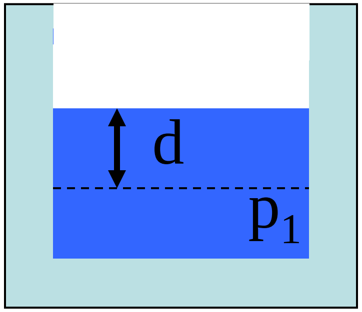
The open tube manometer works similarly to measure the *gauge* pressure of a gas inside an otherwise closed tank, that is open to the manometer on one side.
Here

$$p_g = \rho g h$$



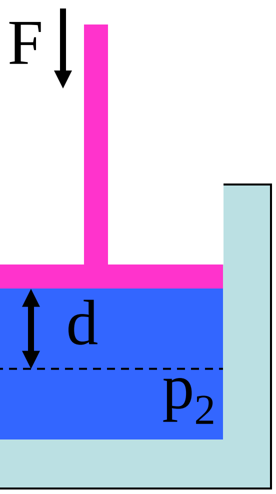
Where it measures gauge pressure (i.e. relative to the atmospheric pressure) because it is open to the atmosphere on the end of the tube not connected to the tank.

Pascal's Principle



Consider the circumstance to the right in which we have a cylinder of cross-section A filled with fluid.

The pressure at a depth d below the surface depends on the depth.



Let's now add a piston and apply an additional force to it. The pressure at depth d increases by

$$\Delta p = \frac{F}{A}$$

but that's true at every point of the fluid.

The pressure *change* is transmitted throughout the entire fluid.

Such a change in pressure would be transmitted independent of the source of the pressure change.

For example, if the temperature rises and the fluid expands, the resulting *change* in the pressure would occur throughout the volume of fluid.

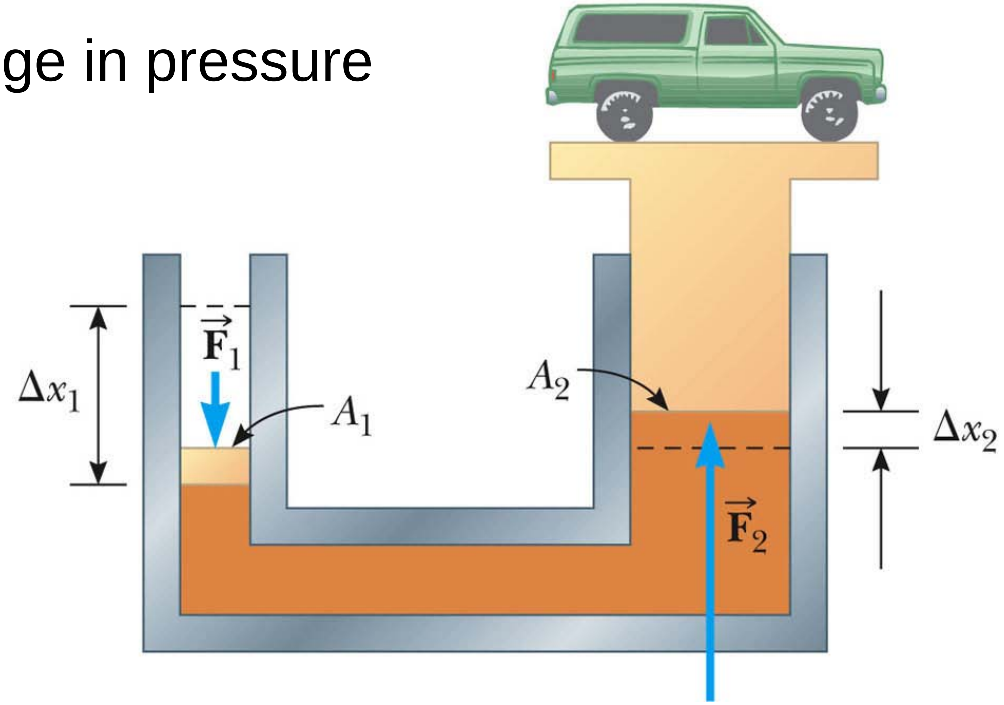
This is Pascal’s principle: the *change* in pressure occurring in an incompressible fluid, in a closed container, is transmitted undiminished to every portion of the fluid and to the walls of the container.

This allows for a hydraulic lever, which consists of different area pistons/cylinders connected together as shown here.

A force $\vec{F}_1$ is applied at the input piston and the force $\vec{F}_2$ occurs at the the output piston.

By Pascal's principle, the change in pressure is the same everywhere so

$$\Delta p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$



But then,

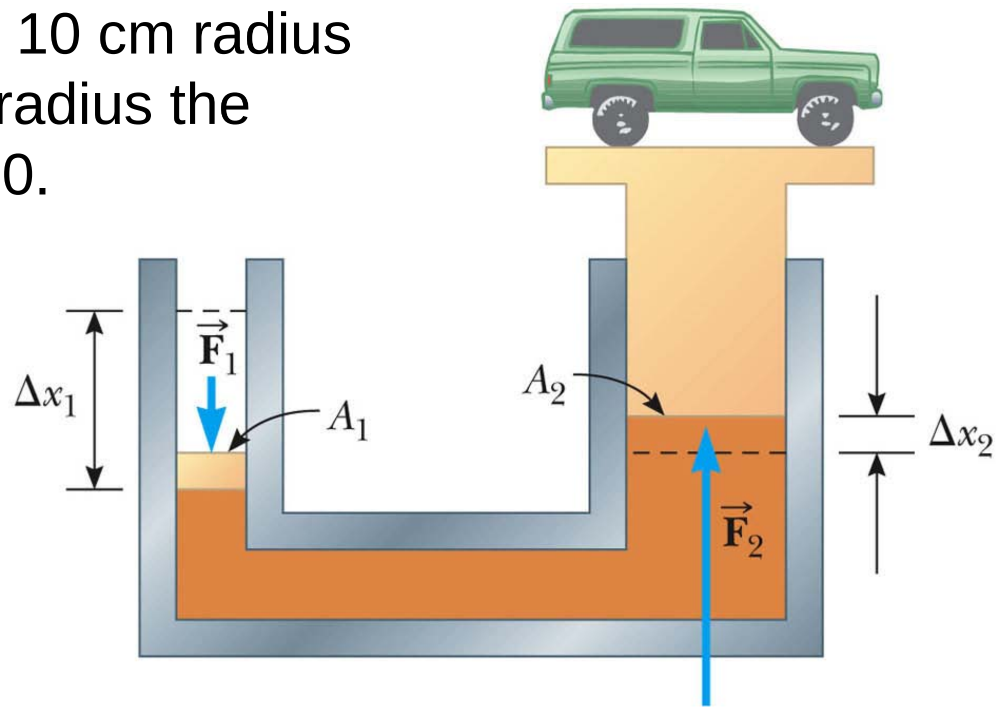
$$F_2 = \frac{A_2}{A_1} F_1$$

So the output force is the input force, times the ratio of the piston areas.

For round cylinder/pistons:

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi R_2^2}{\pi R_1^2} F_1 = \left(\frac{R_2}{R_1} \right)^2 F_1$$

So if the output cylinder has a 10 cm radius and the input cylinder a 1 cm radius the force multiplier is $(10/1)^2 = 100$.



This is how hydraulic lifts and the brakes in your car work.

Does this scheme defy conservation of energy?

(a wise thing to ask when we seem to be getting something seemingly extraordinary)

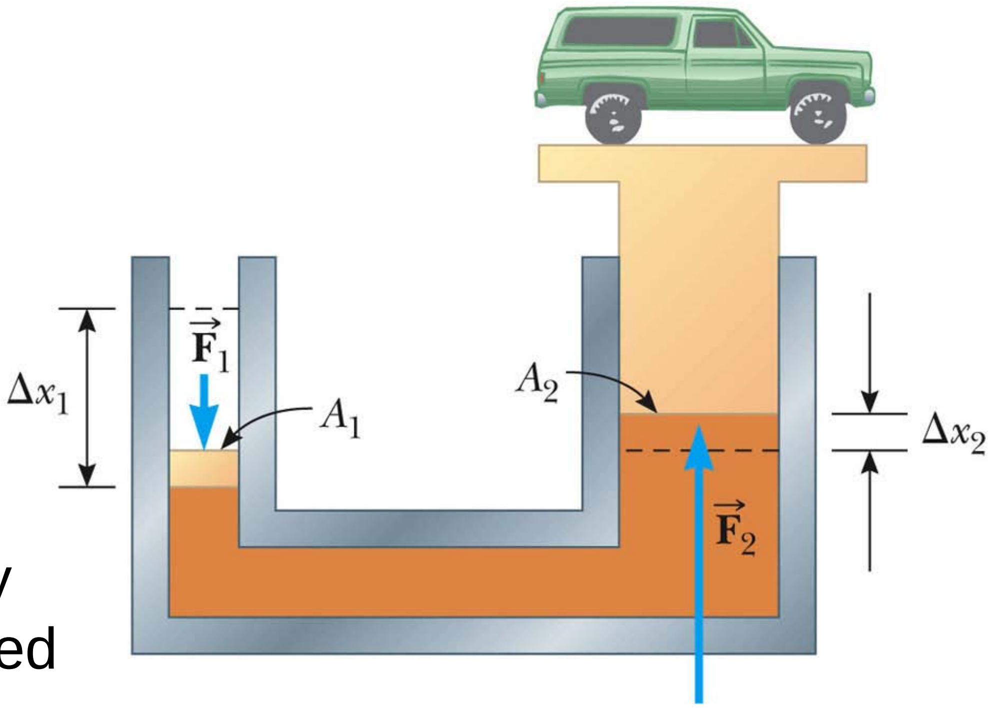
If the input piston moves down a distance Δx_1 the volume of fluid it displaces is $\Delta V = \Delta x_1 A_1$. Since the fluid is incompressible this must be the same volume displaced by the opposing piston so,

$$\Delta V = \Delta x_1 A_1 = \Delta x_2 A_2$$

so that,

$$\Delta x_2 = \frac{A_1}{A_2} \Delta x_1$$

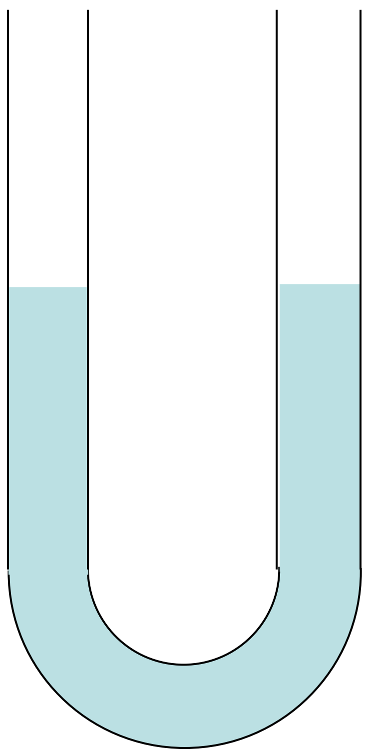
This is the inverse ratio of the force multiplier so the distance moved by the output piston is proportionately smaller than the distance moved by the input piston.



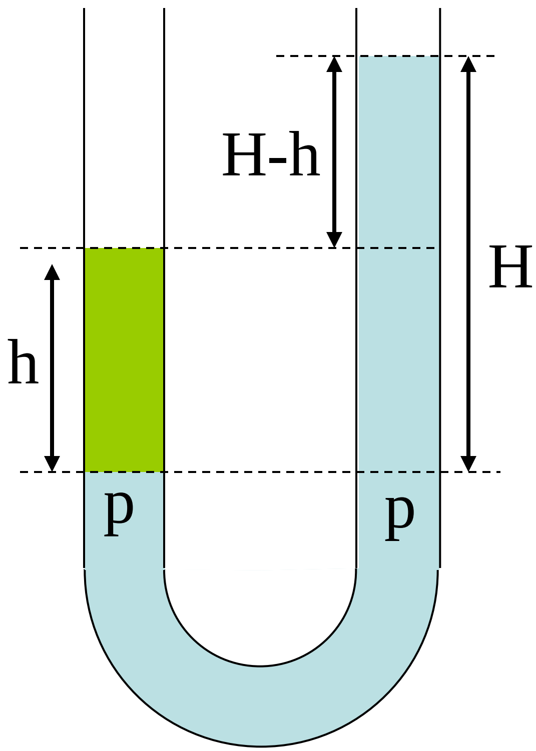
Since work is force times distance the same work is done on both sides & energy is conserved.

U-tubes (determining the density of an immiscible fluid)

Tube cross-sectional area A.



Initially – fluid of known density ρ_k



Add column h of fluid of unknown density ρ_u

($\rho_u > \rho_k$ case)

The pressure on the two sides at the lowest dashed line must be equal (or the fluid would move).

$$p_{left} = p_{right}$$

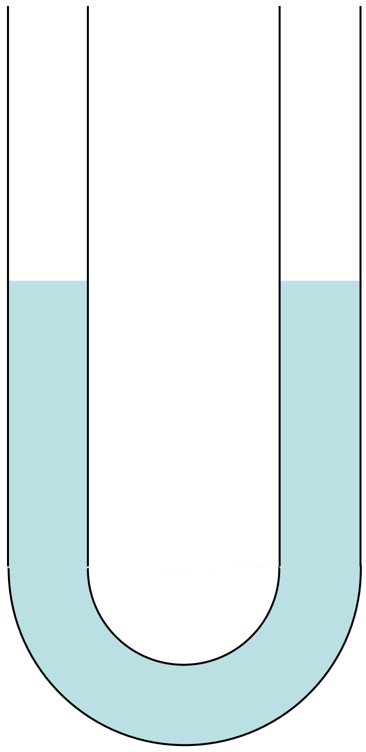
$$p_o + \frac{m_u g}{A} = p_o + \frac{m_k g}{A}$$

$$m_u = m_k$$

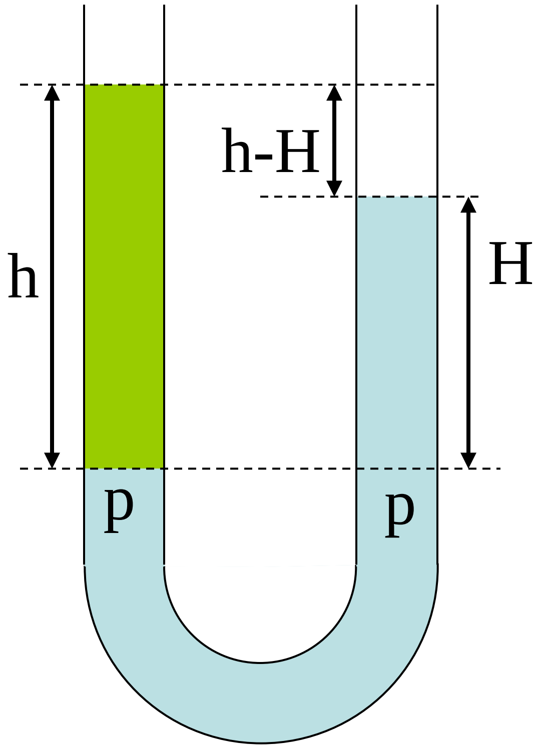
$$\rho_u A h = \rho_k A H$$

$$\rho_u = \frac{H}{h}\rho_k$$

If $\rho_u < \rho_k$



Initially – fluid of known density ρ_k



Add column h of fluid of unknown density ρ_u

The pressure on the two sides at the lowest dashed line must be equal (or the fluid would move).

$$p_{left} = p_{right}$$

$$p_o + \frac{m_u g}{A} = p_o + \frac{m_k g}{A}$$

$$m_u = m_k$$

$$\rho_u A h = \rho_k A H$$

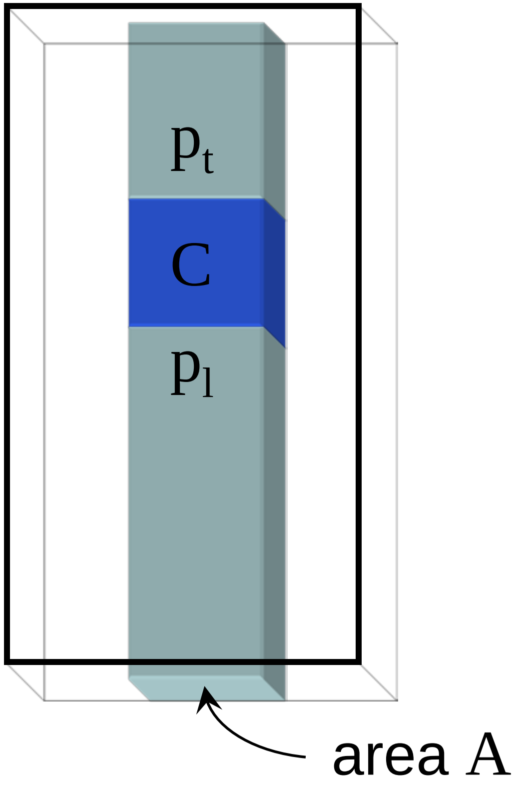
So again,

$$\rho_u = \frac{H}{h}\rho_k$$

But now $H < h$.

Archimedes' principle

Consider the pressure on the top and lower faces of the cube of water labeled C, somewhere in the tall stationary column of water.



The pressure at the upper face, p_t , is due to the mass of the column of water above C.

The pressure at the lower face, p_l , is due to the mass of the column above C plus the mass of C itself.

With m_f the mass of the fluid contained in C, we can write that,

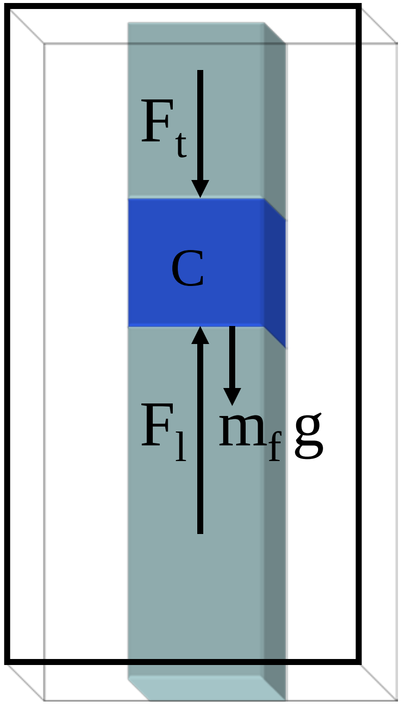
$$p_l = p_t + \frac{m_f g}{A}$$

Or since, $p = \frac{F}{A}$

$$\frac{F_l}{A} = \frac{F_t}{A} + \frac{m_f g}{A}$$

$$F_l = F_t + m_f g$$

F_l is the force downwards due to C and the column of water above it but since the water is stationary there must be an equal and opposite force, due to the surrounding water on the lower face of C upwards.



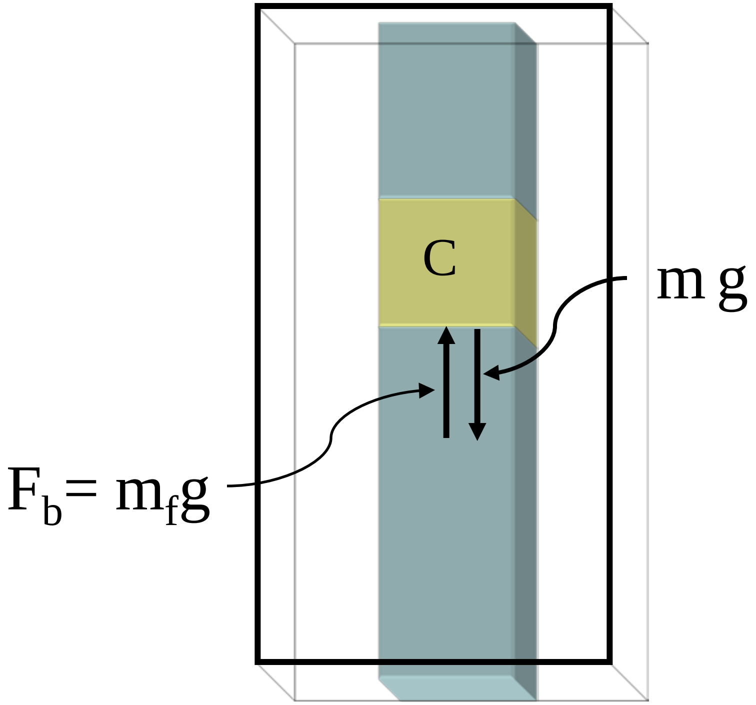
Hence the net forces acting on C are

$$F_l - F_t = F_t + m_f g - F_t = m_f g$$

Hence we've found that the water that surrounds C provides a buoyant force F_b upwards given by,

$$F_b = m_f g$$

This net force from the surrounding water it is actually independent of what material occupies the volume.



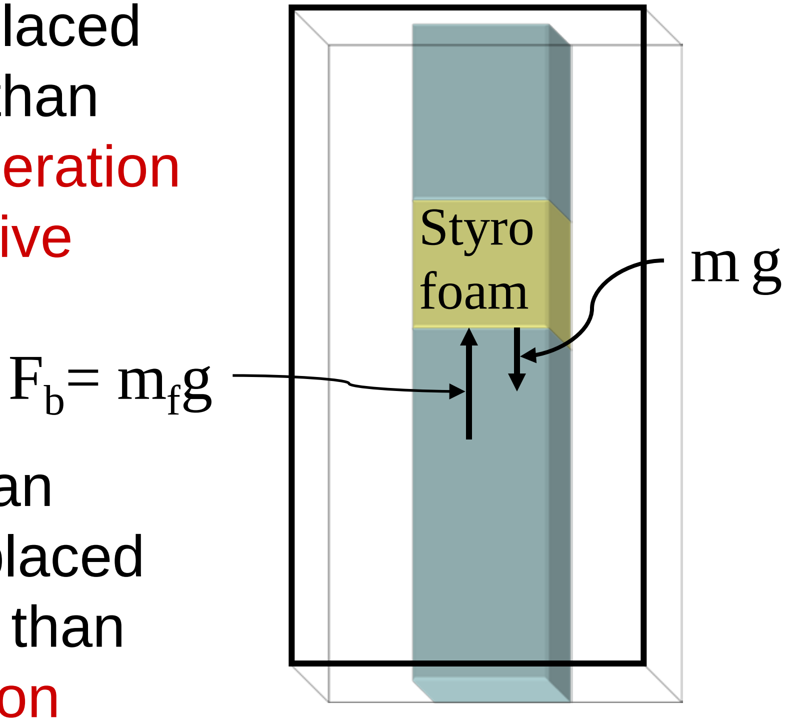
Hence if we replace the cube of water with a different material having a different mass, m, Newton's 2nd law gives,

$$F_b - mg = ma$$

$$m_f g - mg = ma$$

$$a = \frac{m_f - m}{m} g$$ (ignoring hydrodynamic drag)

If the material has a smaller mass than the equal volume of water that it displaced (meaning that it has a lower density than water, e.g. Styrofoam) then the acceleration due to the buoyant force will be positive (i.e. upwards).



If the material has a greater mass than the equal volume of water that it displaced (meaning that it has a higher density than water, e.g. metal) then the acceleration due to the buoyant force will be negative (i.e. downwards), but smaller than its free fall acceleration if only gravity were acting.

Another way to look at this is to to consider the *apparent weight* of the object. Suppose the object is placed on a spring scale (in the fluid). Then

$$F_S + F_b - mg = ma = 0$$

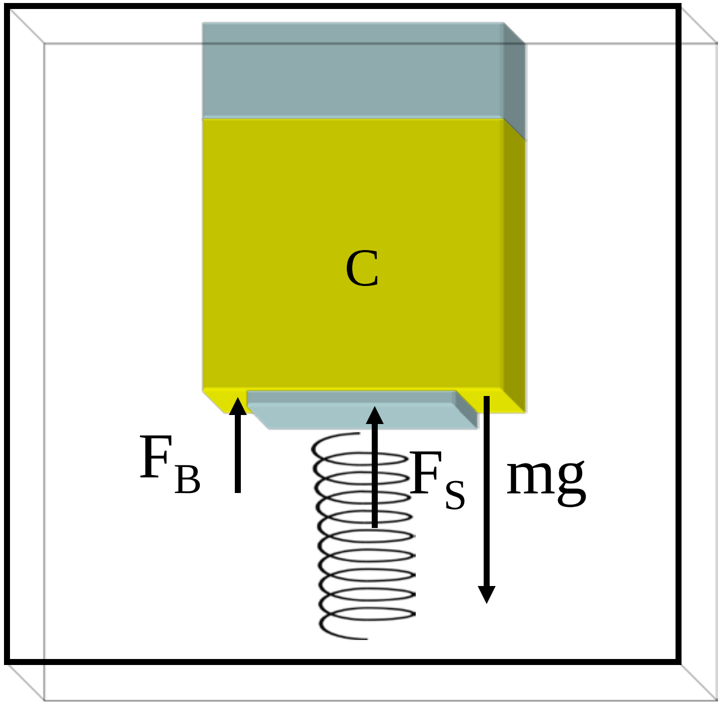
So the scale reads,

$$F_S = mg - F_b$$

Thus its *apparent weight* is the actual weight minus the buoyant force.

The buoyant force depends only on the weight of the fluid displaced, i.e.

$$F_b = m_f\, g$$

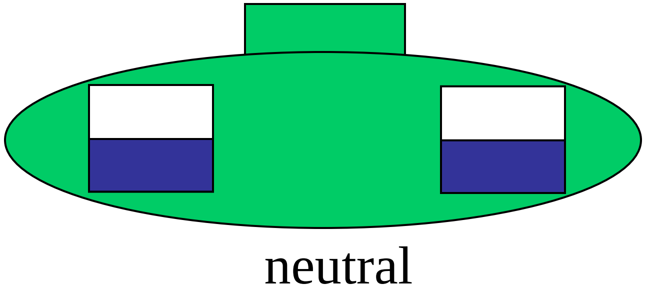


This is Archimedes' principle.

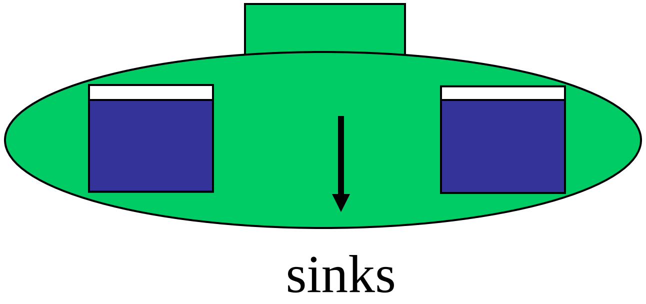
If the buoyant force exactly balances the weight of the object the apparent weight $F_S = 0$ and the object neither rises nor sinks. The object is then said to be neutrally buoyant.

$$F_S = mg - F_B = 0$$

This is what permits a submarine to hover at particular depth. A submarine has internal ballast tanks that are designed to be filled with seawater.



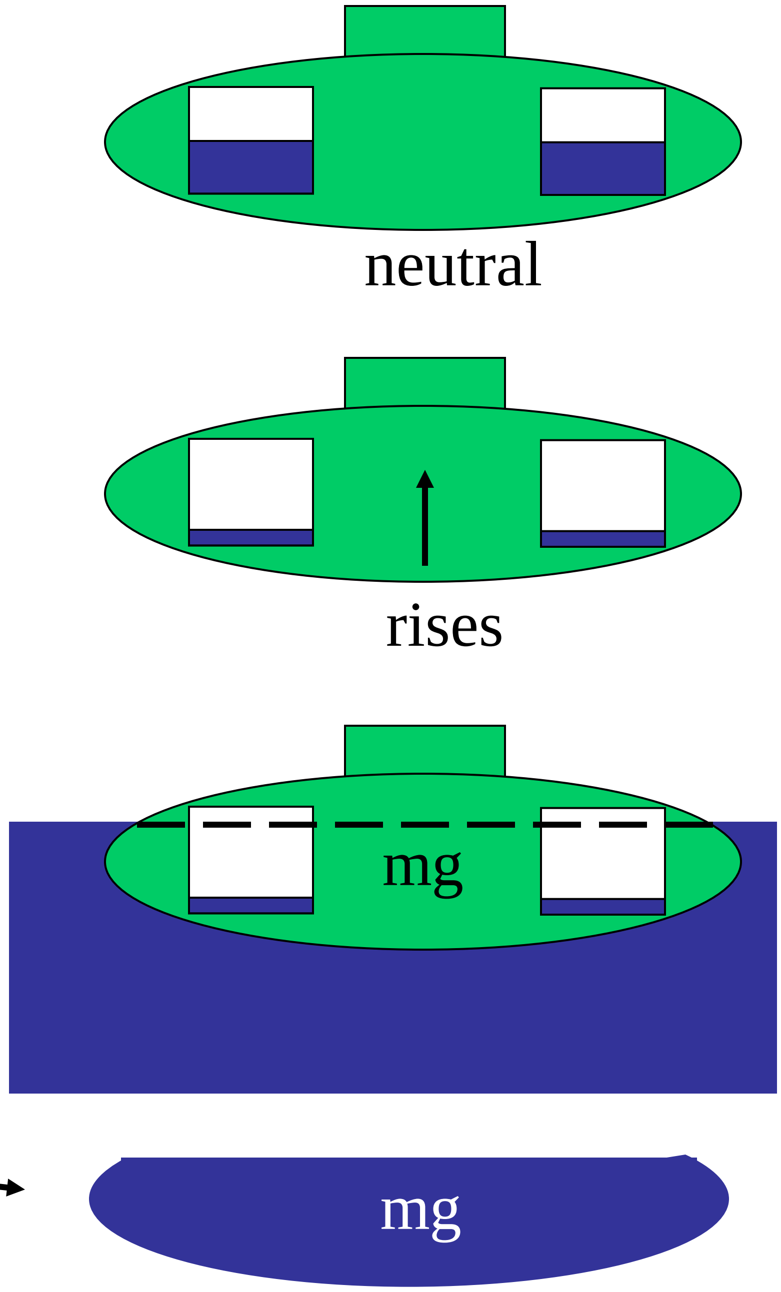
To dive a neutrally buoyant submarine pumps water into these chambers making the weight of the hull plus the water taken on greater than the weight of the water displaced by the hull (i.e. greater than the buoyant force).



As the desired depth is approached this water is blown out, making the ship neutrally buoyant again.

To rise again, still more water is blown out, making the submarine have an apparent weight that is negative (F_b > mg).

When the ship gets to the surface, it continues to rise until its total weight equals the weight of the water it displaces.



This sets the depth at which an object floats, i.e. an object will sink into a fluid to a level until the weight of the object equals the weight of fluid displaced.

Example

Floating in the very salt rich (dense) waters of the Dead Sea keeps about 1/3 of your body above the water line. What is the density of the water there? Assume your density to be 1 g/cm^3.

Let your mass be m,

$$F_B - mg = 0$$

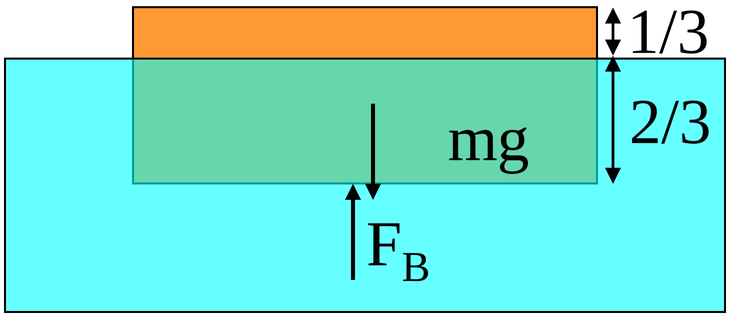
but

$F_B = m_f g$, m_f = displaced water

so

$$m_f g - mg = 0$$

$$m_f = m \longrightarrow \rho_f V_f = \rho V$$



$$m_f g - mg = 0$$

$$m_f = m \longrightarrow \rho_f V_f = \rho V$$

ρ_f = density of the seawater

ρ = 1 g/cm^3 (your density)

Now the volume of fluid displaced is 2/3 of your volume so,

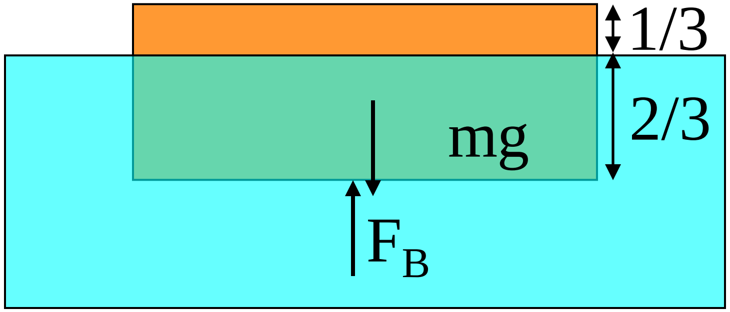
$V_f = 2/3V$ then,

$$\rho_f 2/3V = \rho V$$

$$\rho_f 2/3 = \rho$$

$$\rho_f = 3/2\rho$$

$$\rho_f = 1.5 \text{ g/cm}^3$$



HITT

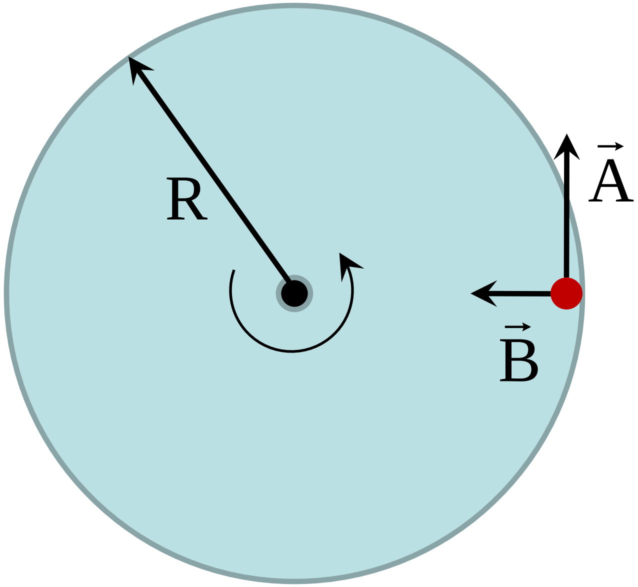
A merry-go-round of radius R has a constant angular acceleration α.

At the instant shown the *net* linear acceleration of the point in red on the rim is:

1) Along vector $\vec{A}$

2) Along vector $\vec{B}$

3) Between vectors $\vec{A}$ & $\vec{B}$

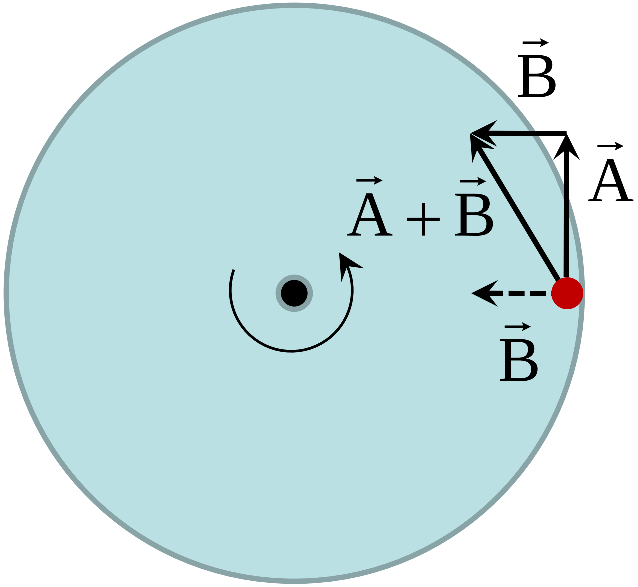


Since there is an angular acceleration α the red spot has a linear acceleration tangential to the rim along $\vec{A}$ that has magnitude,

$$a_t = R\alpha$$

Since the spot is moving on a circular path it has a centripetal acceleration toward the center of the circle along $\vec{B}$ of magnitude,

$$a_c = \frac{v^2}{R}$$



with v the linear speed of the spot at that instant.

The answer is the sum of these vectors so between $\vec{A}$ & $\vec{B}$.