Physics 1A

Lecture 2B

"Your education is ultimately the flavor left over after the facts, formulas, and diagrams have been forgotten."

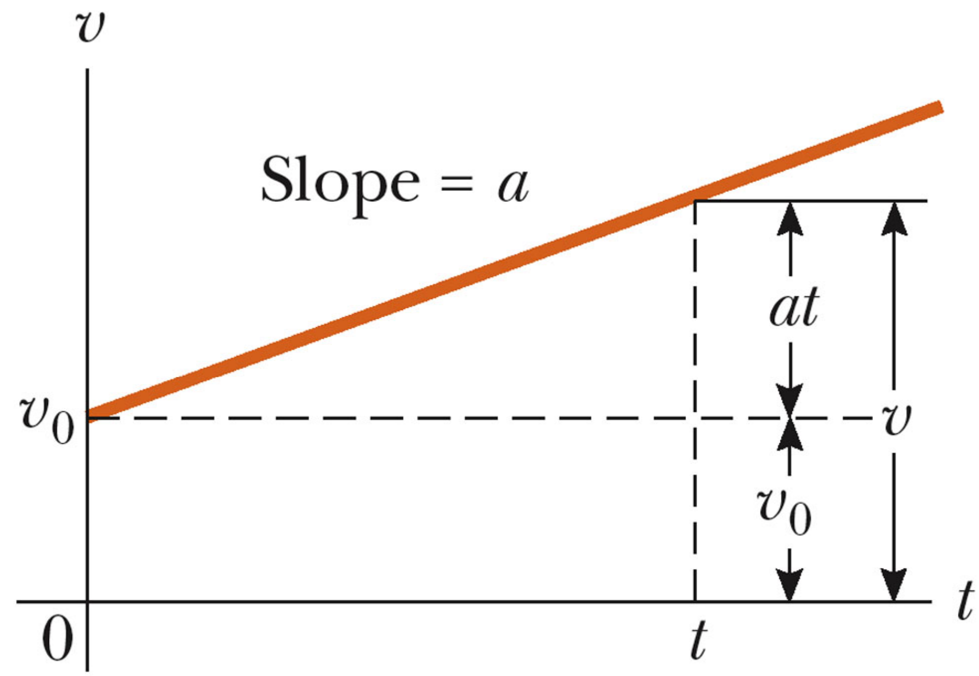
--Paul G. Hewitt

Kinematics

- With the basic definitions now in place we can turn to kinematics.
- Kinematics is the study of motion.
- A set of equations to describe a body in motion has been derived known as the kinematic equations.
- These equations assume that the acceleration of the body in motion is constant in time.

Kinematics

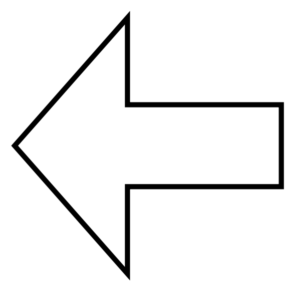
An object starts with an initial velocity, v_o, at t = 0 and accelerates with a constant acceleration, a. What is its final velocity, v, at time, t?



$$a = \text{constant} = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a\Delta t$$

$$(v - v_o) = a(t - 0) \qquad v = v_o + at$$

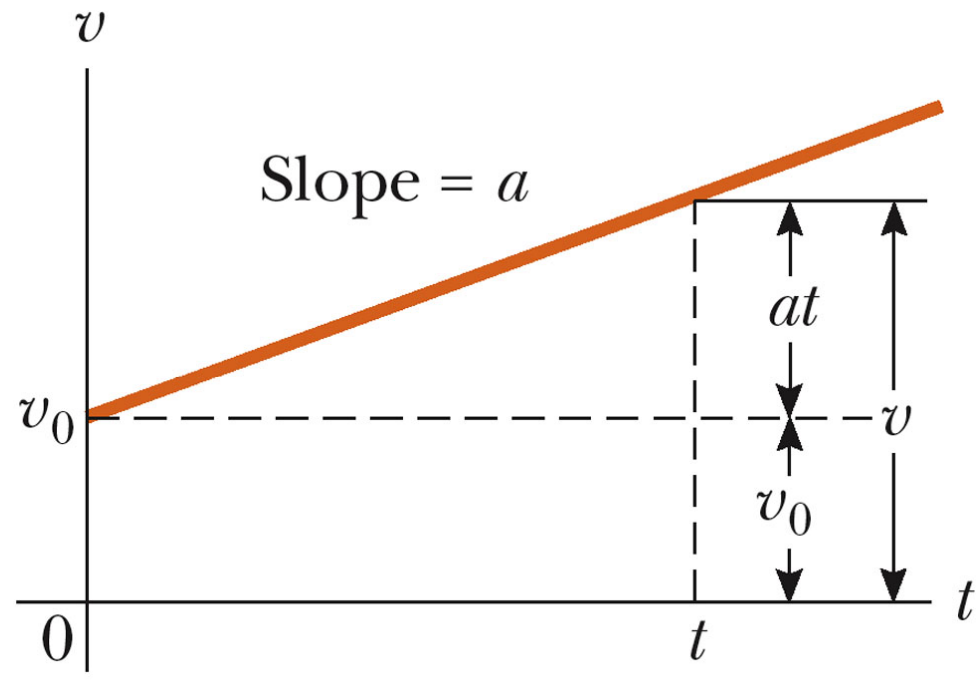


This is the first kinematics equation!

Kinematics

- Now let's look at the definition of v_{avg} again assuming a is constant and we start at t = 0:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{t-0} = \frac{\Delta x}{t}$$



Also by looking at the graph, we find:

$$v_{avg} = \frac{v_o + v}{2}$$

$$\tfrac{1}{2}(v_o + v) = \frac{\Delta x}{t} \qquad \Delta x = \tfrac{1}{2}(v_o + v)t$$

This is the second kinematics equation!

Kinematics

- Recall our first kinematics equation:

$$v = v_o + at$$

Re-substitute this velocity, v, back into our second kinematics equation:

$$\Delta x = \tfrac{1}{2}(v_o + v)t$$

$$\Delta x = \tfrac{1}{2}(v_o + (v_o + at))t$$

$$\Delta x = \tfrac{1}{2}v_o t + \tfrac{1}{2}v_o t + \tfrac{1}{2}at^2$$

$$\Delta x = v_o t + \tfrac{1}{2}at^2$$

This is the third kinematics equation!

Kinematics

- Recall our first kinematics equation:

$$v = v_o + at \qquad v - v_o = at$$

$$t = \frac{v - v_o}{a}$$

Re-substitute this time, t, back into our second kinematics equation:

$$\Delta x = \tfrac{1}{2}(v_o + v)t \qquad \Delta x = \tfrac{1}{2}(v_o + v)(v - v_o)\tfrac{1}{a}$$

$$\Delta x = \tfrac{1}{2a}\left(v_o v + v^2 - v_o^2 - v v_o\right) \qquad \Delta x = \tfrac{1}{2a}\left(v^2 - v_o^2\right)$$

$$2a\Delta x = \left(v^2 - v_o^2\right) \qquad v^2 = v_o^2 + 2a\Delta x$$

This is the fourth kinematics equation!

Kinematics

- Recall that all 4 equations are based on the fact that acceleration is constant.
- Each kinematic equation has a missing quantity:

Equation	**Missing Quantity**
$v = v_o + at$	Δx
$\Delta x = \frac{1}{2}(v + v_o)t$	a
$\Delta x = v_o t + \frac{1}{2}at^2$	v
$v^2 = v_o^2 + 2a\Delta x$	t

Know how to use all of the equations.

Also, a warning for the third equation, if you are solving for t you may have to use the quadratic formula.

Kinematics

- Example
- A sprinter accelerates at $2.5 m/s^2$ until reaching his top speed of 15 m/s, he then continues to run at top speed. How long does it take him to run the 100m dash?

- Answer
- First, you must define a coordinate system.
- Let's choose the direction of motion as positive, and $x = 0$ where the sprinter starts at $t = 0$.

Kinematics

- Answer
- Here we have two different time periods with two different accelerations.
- In the first part, the sprinter accelerates, then in the second part he has a constant velocity.
- You must use the kinematics equations separately for both parts.
- Let's list the quantities we know for the first part:
- $v = +15 m/s$
- $a = +2.5 m/s^2$
- $v_o = 0$ <-- it starts from rest
- t <-- finding
- Δx <-- finding

Kinematics

- Answer
- Looks like it is the first equation for us:

$$v = v_o + at$$

But we know that $v_o = 0$, so it becomes:

$$v = at \qquad t = \frac{v}{a} = \frac{15\,\text{m/s}}{2.5\,\text{m/s}^2} = 6.0\text{s}$$

How far did he run in that time period?

Use the second equation, again with $v_o = 0$:

$$\Delta x = \tfrac{1}{2}(v_o + v)t$$

$$\Delta x = \tfrac{1}{2}(v)t \qquad \Delta x = \tfrac{1}{2}(15\,\text{m/s})6.0\text{s} = 45\text{m}$$

Kinematics

- Answer
- This means that for the second part of the motion (a = 0), the sprinter only needs to travel:
- 100m – 45m = 55m
- Let's list the quantities we know for the second part:
- $v = +15 m/s = v_o$
- $a = 0$
- $\Delta x = 55m$
- t <-- finding
- Use the second equation: $\Delta x = \frac{1}{2}(v_o + v)t$ $\Delta x = \frac{1}{2}(2v)t = (v)t$

$$t = \frac{\Delta x}{v} = \frac{55\text{m}}{15\,\text{m/s}} = 3.7\text{s}$$

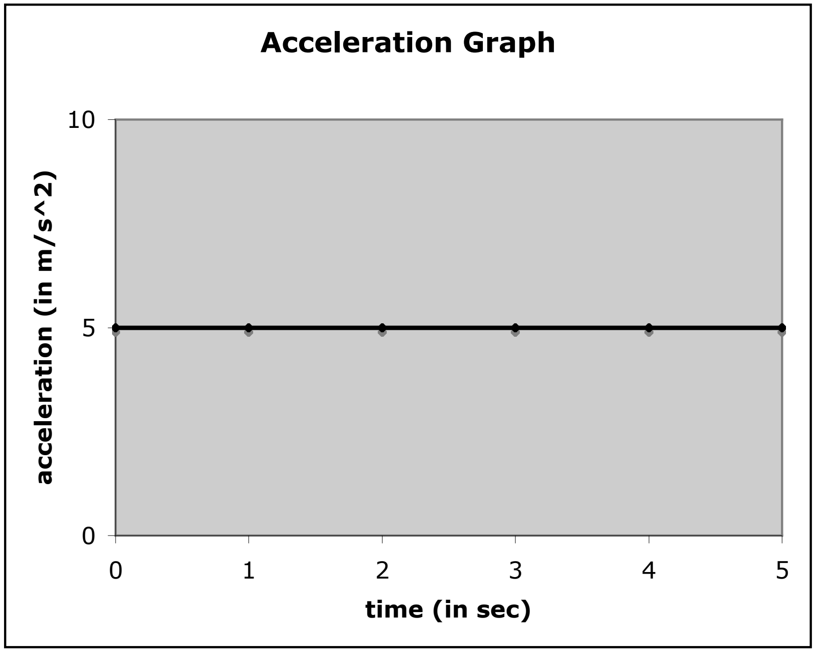
- The total time will be: 6.0s + 3.7s = 9.7s

Graphical Kinematics

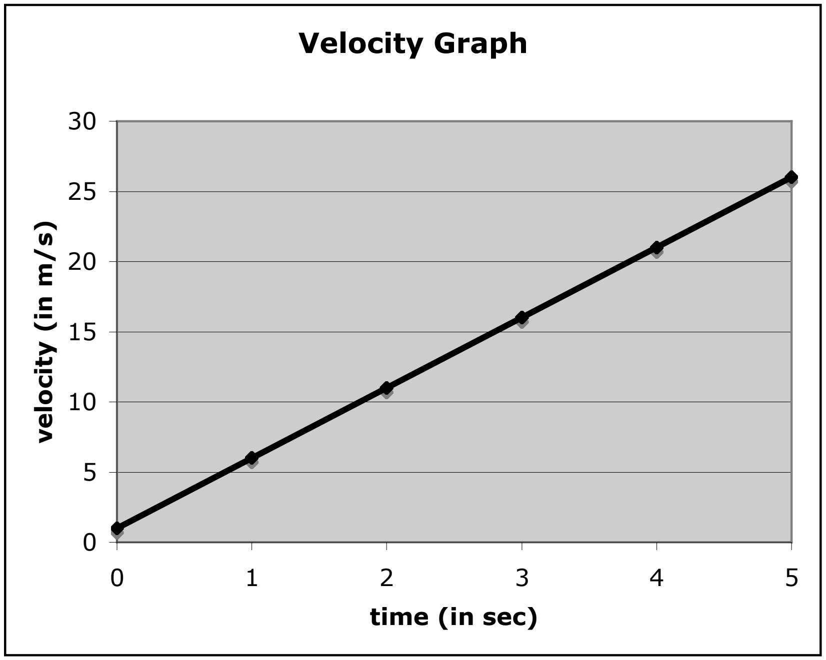
- Another way to examine constant acceleration problems is do it graphically.
- This relies on the fact that:
- 1) The slope of position is velocity.
- 2) The slope of velocity is acceleration.
- Essentially, you can get one graph (say v vs t) by knowing another graph (say x vs t).

Graphical Kinematics

For example, if you are given the following a vs t graph and the fact that v_o = 1m/s. Plot the corresponding v vs t graph and find the velocity at t = 3s.



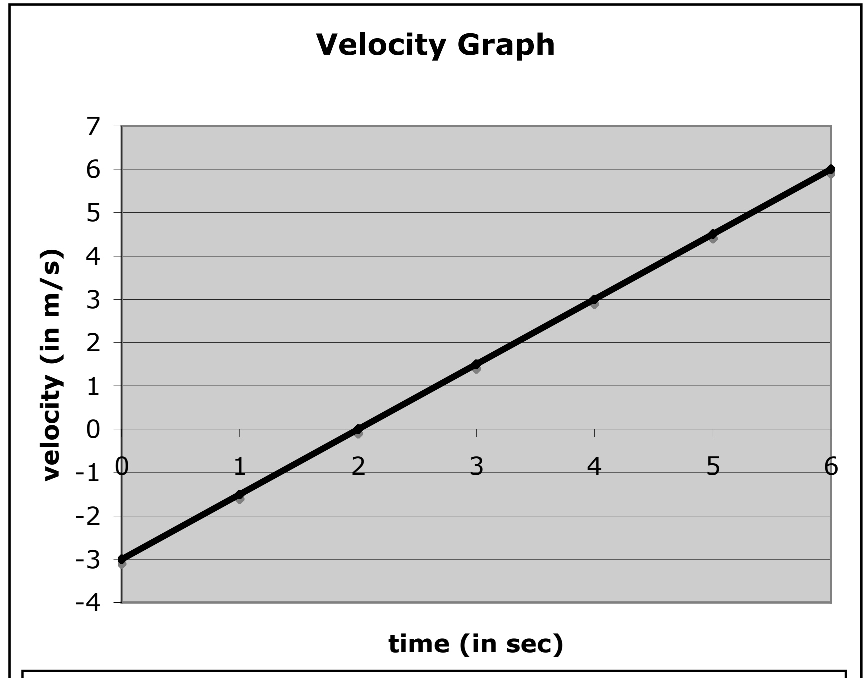
Since a is the slope for v, then the v vs t graph is:



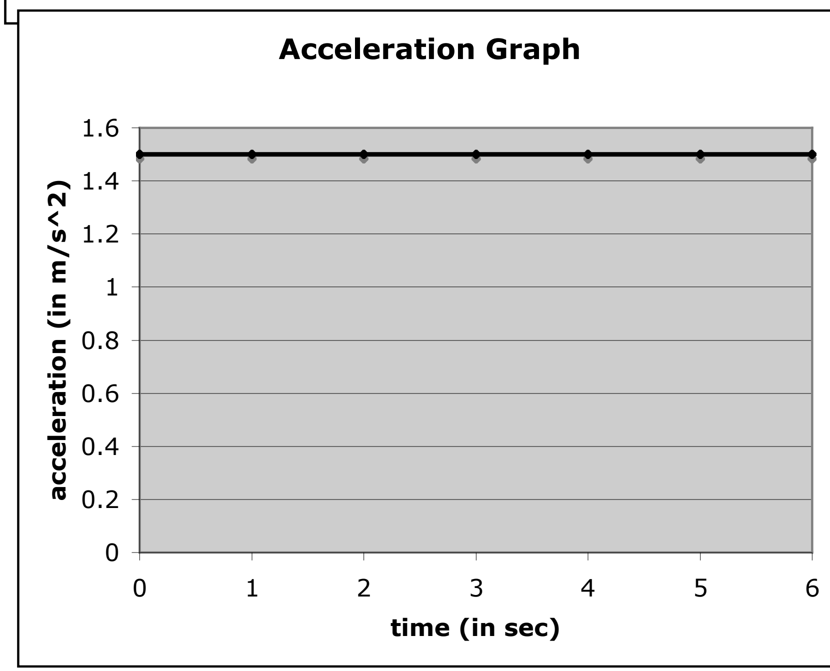
We can then read off of the graph that v = 16 m/s at t = 3s.

Graphical Kinematics

- Or, you could be given the following v vs t graph and asked to plot the corresponding a vs t graph.



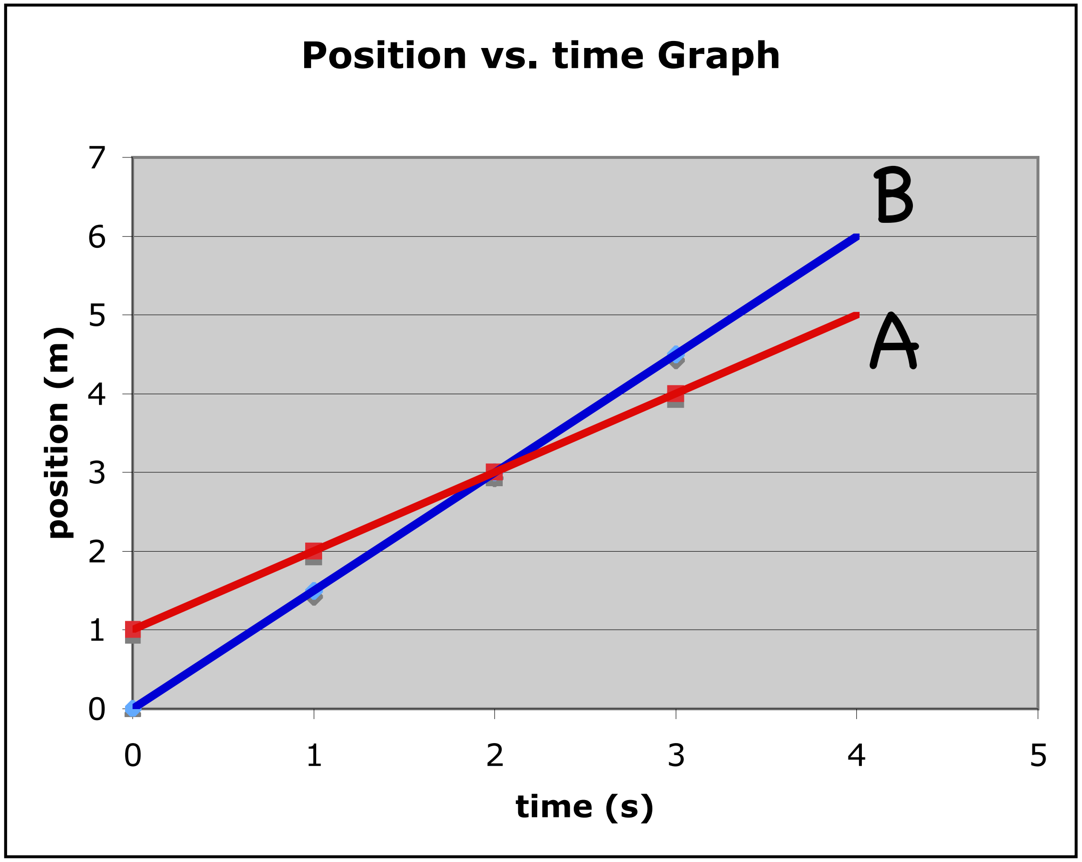
Since a is the slope for v, then the a vs t graph is:



Note: Even though the velocity was zero on the original graph as t = 2s, acceleration was not zero at that time.

Conceptual Question

- In the following position vs. time graph, when will object A and object B have the same speed?
- A) At t = 1s.
- B) At t = 2s.
- C) At t = 3s.
- D) They have the same speed during the graph's entire time period.
- E) They never have the same speed during the graph's entire time period.



Falling Bodies

- Objects moving under the influence of only gravity are said to be in free fall.
- Before 1600, it was widely believed that heavier bodies fell faster than lighter bodies.
- People at the time believed that the speed of the fall of an object is proportional to how heavy it is.
- Galileo was the first to predict that falling bodies on Earth would fall with a distance proportional to time squared ($D \propto t^2$).

Falling Bodies

- Galileo surmised through repeated experiment that:
- "at a given location on the Earth and in the absence of air resistance, all objects fall with the same uniform acceleration."
- Galileo felt that earlier work neglected the effect that air friction (i.e. resistance) had on different objects.
- The uniform acceleration Galileo talked about was the acceleration due to gravity, g.
- Seen here in this Ball Drop Demo.

Falling Bodies

- In SI units, g is generally regarded as being $9.80 m/s^2$.
- In English units, $g = 32 ft/s^2$.
- g varies slightly depending on where you are on Earth, but it always points downward towards the Earth.

City	value of g (in m/s^2)	Altitude (in m)	Latitude (in degrees)
New York	9.803	38	41
San Francisco	9.800	114	38
Denver	9.796	1638	40
Equator	9.780	0	0

Falling Bodies

- Example
- In the original Superman comics, Superman was unable to fly but could simply "leap tall buildings in a single bound." Superman's range was about one eighth of a mile (201m). What initial velocity would Superman need to reach his maximum height?

- Answer
- First, you must define a coordinate system.
- Let's choose the upward direction as positive, and $x = 0$ where Superman starts at $t = 0$.

Falling Bodies

- Answer
- Let's list the quantities we know:
- $\Delta x = +201\text{m}$
- $a = -9.80\text{m/s}^2 = \text{constant}$
- $v = 0$ <-- velocity at max height
- v_o <-- finding
- t <-- don't know

- Looks like it is the fourth equation for us:

$$v^2 = v_o^2 + 2a\Delta x$$

Falling Bodies

Answer

$$v^2 = v_o^2 + 2a\Delta x$$

But we know that v = 0 at max height, so it becomes:

$$v_o^2 = -2a\Delta x$$

$$v_o = \sqrt{-2a\Delta x}$$

$$v_o = \sqrt{-2\left(-9.80\,{}^{\text{m}}\!/_{\text{s}^2}\right)\left(201\text{m}\right)} = 62.8\,{}^{\text{m}}\!/_{\text{s}}$$

This is the initial velocity that Superman needs to attain maximum height.

Conceptual Question

- A tennis player on serve tosses a ball straight up. When the ball is at its maximum height, the ball's:
  - A) velocity is non-zero and acceleration is zero.
  - B) velocity is zero and acceleration is non-zero.
  - C) velocity is zero and acceleration is zero.
  - D) velocity is non-zero and acceleration is non-zero.

For Next Time (FNT)

- Finish up the homework for Chapter 2.
- Start Reading Chapter 3.