

IENG441 FACILITIES PLANNING AND DESIGN
LECTURE NOTES

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CHAPTER 1

FACILITIES

Facilities can be broadly defined as buildings where people, material, and machines come together for a stated purpose – typically to make a tangible product or provide a service.

The facility must be properly managed to achieve its stated purpose while satisfying several objectives.

Such objectives include producing a product or producing a service

- at lower cost,
- at higher quality,
- or using the least amount of resources.

1.1. Definition of Facilities Planning

1.1.1. Importance of Facilities Planning & Design

Manufacturing and Service companies spend a significant amount of time and money to design or redesign their facilities. This is an extremely important issue and must be addressed before products are produced or services are rendered.

A poor facility design can be costly and may result in:

- poor quality products,
- low employee morale,
- customer dissatisfaction.

1.1.2. Disciplines involved in Facilities Planning (FP):

Facilities Planning (FP) has been very popular. It is a complex and a broad subject.

Within the engineering profession:

- civil engineers,
- electrical engineers,
- industrial engineers,
- mechanical engineers are involved in FP.

Additionally,

- architects,
- consultants,

- general contractors,
- managers,
- real estate brokers, and
- urban planners are involved in FP.

1.1.3. Variety of Facility Planning (FP) Tools:

Facility Planning (FP) tools vary from checklists, cookbook type approaches to highly sophisticated mathematical modeling approaches.

In this course, a practical approach to facilities planning will be employed taking advantage of empirical and analytical approaches using both traditional and contemporary concepts.

1.1.4. Applications of Facilities Planning (FP):

Facilities Planning (FP) can be applied to planning of:

- a new hospital,
- an assembly department,
- an existing warehouse,
- the baggage department in an airport,
- department building of IE in EMU,
- a production plant,
- a retail store,
- a dormitory,
- a bank,
- an office,
- a cinema,
- a parking lot,
- or any portion of these activities etc...

Facilities Planning (FP) determines how an activities tangible fixed assets best support achieving the activity's objectives.

i.e. what is the objective of the facility? How the facility achieves that objective?

- **In the case of a manufacturing firm:**

Facilities Planning (FP) involves the determination of how the manufacturing facility best supports production.

- **In the case of an airport:**

Facilities Planning (FP) involves determining how the airport facility is to support the passenger-airplane interface.

- **In the case of a hospital:**

Facilities Planning (FP) for a hospital determines how the hospital facility supports providing medical care to patients.

Facilities Planner considers the facility as a dynamic entity. Therefore continuous improvement is an integral element of FP cycle.

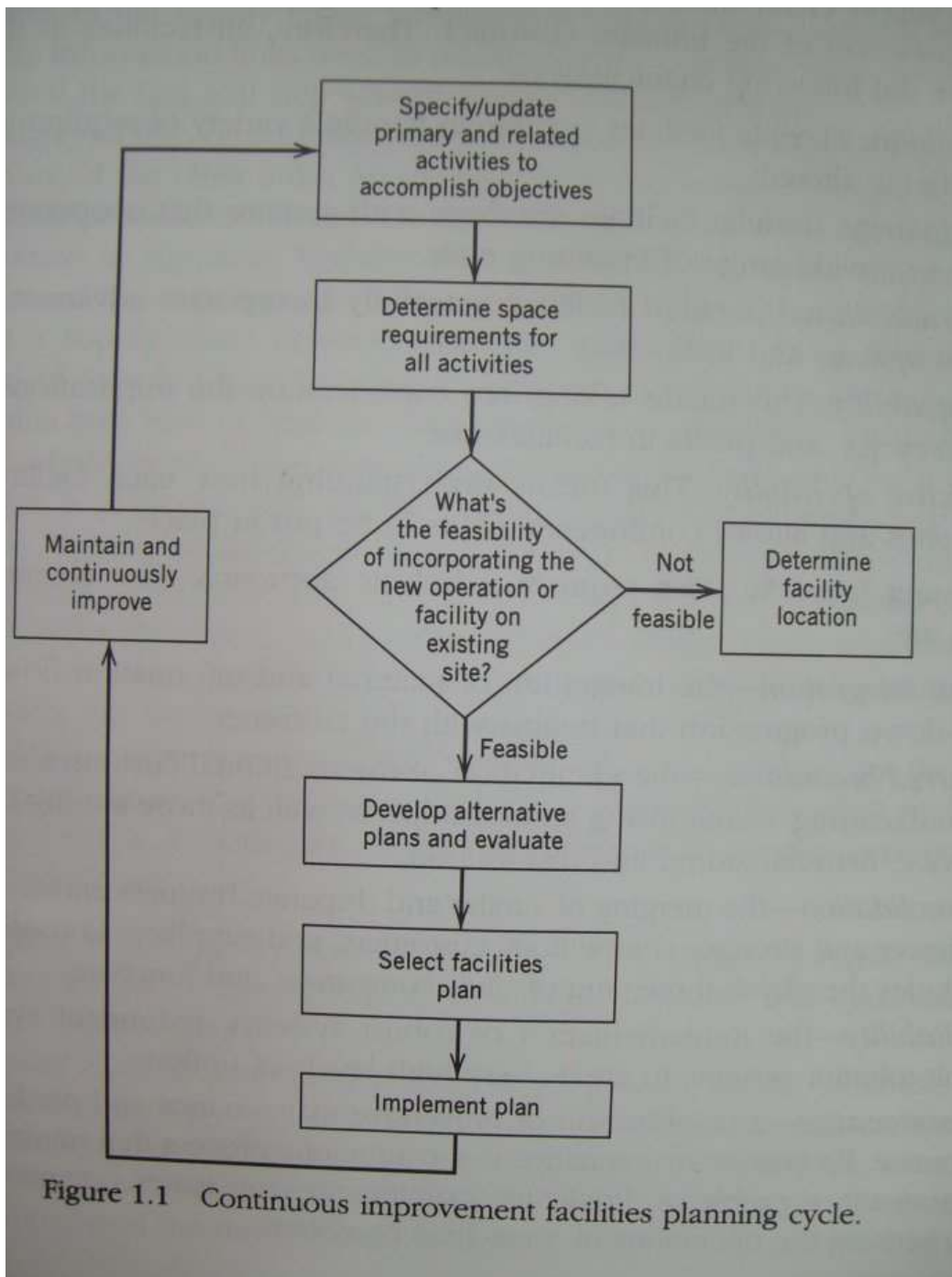
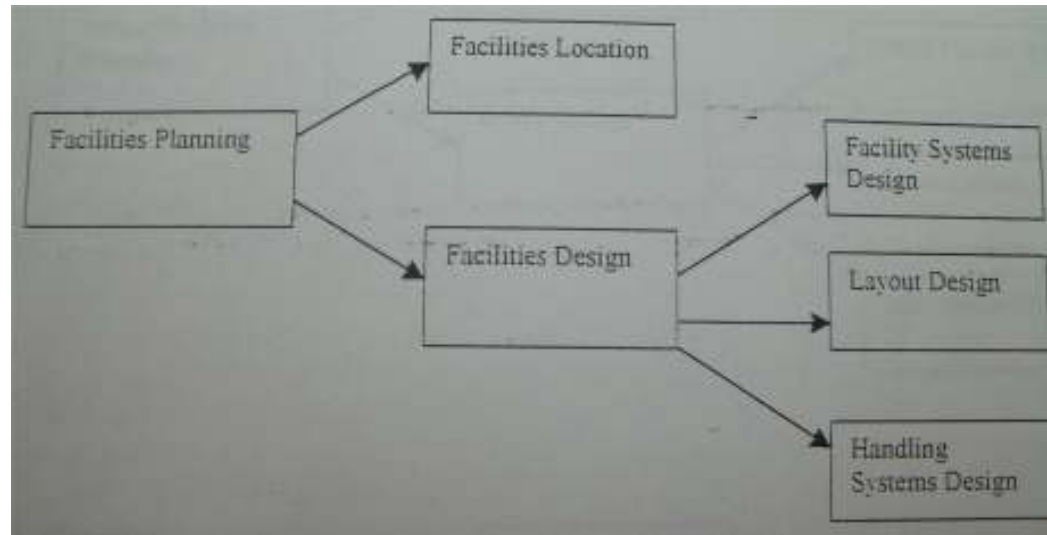


Figure 1.1. Continuous improvement facilities planning cycle

It is important to recognize that we do not use the term facilities planning as a synonym for such related terms as facilities location, facilities design, facilities layout, or plant layout. It is convenient to divide a facility into its location and design components.

Facilities Planning \neq Facility Location
 Facilities Design
 Facilities Layout
 Plant Layout

Facilities Planning Hierarchy:



1.1.5. Facilities Location (Macro Aspect of FP):

Location of the facility refers to its placement with respect to customers, suppliers, and other facilities with which it interfaces.

1.1.6. Facilities Design (Micro Aspect of FP):

Design components of a facility consists of the facility systems, the layout and the handling systems.

Facilities Systems:

Consists of the structural systems, the atmospheric systems, the lighting/electricity/communication systems, the life safety systems and the sanitation systems.

Layout:

Consists of all equipment, machinery and furnishings within the building.

Handling Systems:

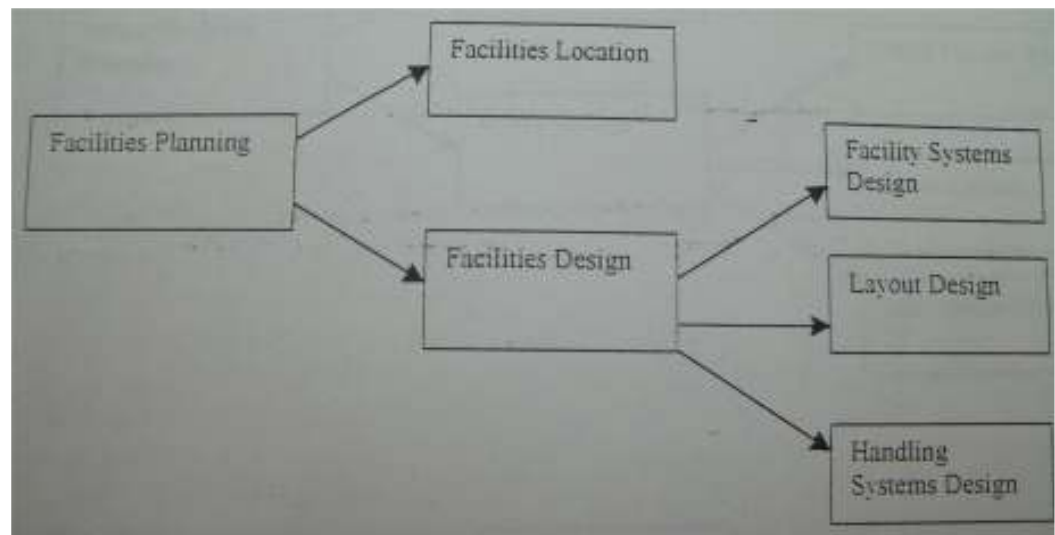
Consists of the mechanism need to satisfy the required facility interactions.

e.g. for a manufacturing system:

- **Facility Systems** – the structure (of building), power, light, gas, heat, ventilation, air-conditioning, water and sewage needs.
- **Layout** – the production areas, related support areas, personnel areas.
- **Handling Systems** – the materials- personnel, information, and equipment to support manufacturing.

1.1.7. Application of FP Hierarchy to a Number of Different Types of Facilities:

FP Hierarchy:



Facilities Planning for specific types of facilities:

- a) Manufacturing plant
- b) Office
- c) Hospital
- d) Emergency room

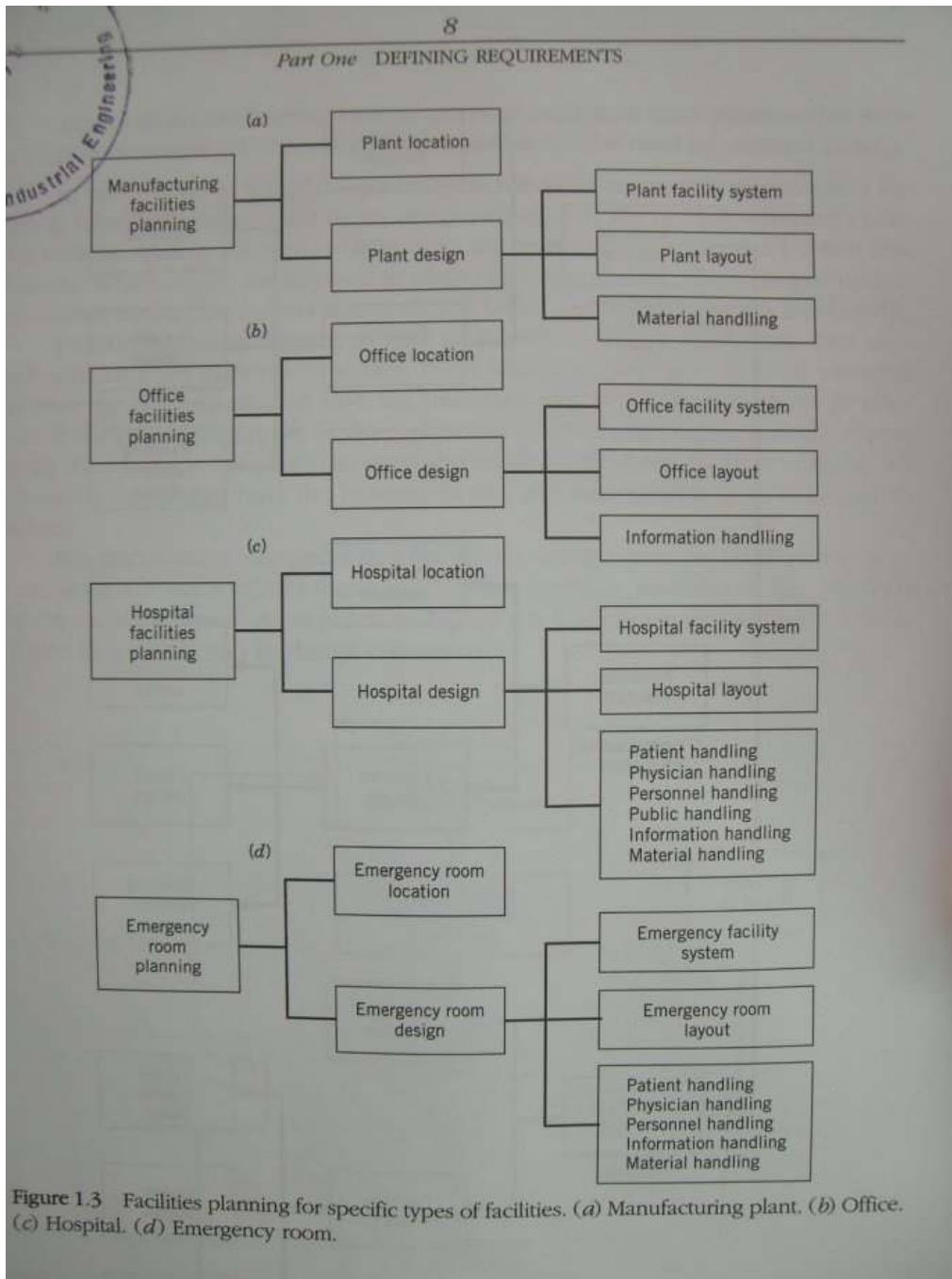


Figure 1.3. Facilities planning for specific types of facilities

1.2. Significance of Facilities Planning

To understand the significance of Facilities Planning (FP) consider the following questions:

- What impact does facilities planning have on handling and maintenance cost?
- What impact does facilities planning have on employee morale, and how does employee morale impact operating costs?

- In what do organizations invest the majority of their capital, and how convertible is their capital once invested?
- What impact does facilities planning have on the management of a facility?
- What impact does facilities planning have on facility's capability to adapt to change and satisfy future requirements?

1.3. Objectives of Facilities Planning

Objectives of FP is to plan a facility that achieves both facilities location and design objectives.

1.3.1. Objectives of Industrial Facility Location:

Objective of Industrial Facility Location is to determine the location which, in consideration of all factors affecting deliver-to-customers cost of the products to be manufactured, will be minimized.

1.3.2. Some Typical Facilities Design Objectives are to:

1. Support the organization's vision through improved material handling, material control, and good housekeeping.
2. Effectively utilize people, equipment, space and energy.
3. Minimize capital investment.
4. Be adaptable and promote ease of maintenance.
5. Provide for employee safety and job satisfaction.

1.4. Facilities Planning Process

Although facility is planned only once, it is frequently replanned to synchronize the facility and its constantly changing objectives. Planning and Replanning are linked by the continuous improvement FP cycle (Figure 1).

FP is not an exact science, but it can be approached using an organized and systematic approach.

Traditionally, the ENGINEERING DESIGN PROCESS (EDP) can be applied (similar to problem solving approach).

It consists of following 6 steps:

- Define the problem,
- Analyze the problem,
- Generate alternative designs,
- Evaluate the alternatives,
- Select the preferred design,
- Implement the design.

Applying the engineering design process to facilities planning results in the following process:

1. Define (or redefine) the objective of the facility,
2. Specify the primary and support activities to be performed in accomplishing the objective.
Requirements in terms of:
 - Operations,
 - Equipment,
 - Personnel,
 - Material flows should be satisfied.
3. Determine the interrelationships among all activities,
4. Determine the space requirements for all activities,
5. Generate alternative facilities plans,
6. Evaluate alternative facilities plans (alternative locations and alternative designs),
7. Select a facilities plan,
8. Implement the facilities plan,
9. Maintain and adapt the facilities plan,
10. Redefine the objective of the facility.

An Organization's Model of Success:

Experience has shown that in order for the facilities plan to be successful, not only a clear understanding of the vision is needed, but also the mission, the requirement of success, the guiding principles, and the evidence of success.

Five elements that form an organization's model of success:

- **Vision:** a description of where you are headed.
- **Mission:** how to accomplish the vision.
- **Requirements of success:** the science of your business.
- **Guiding principles:** the values to be used, while pursuing the vision.
- **Evidence of success:** measurable results that will demonstrate when an organization is moving towards their vision.

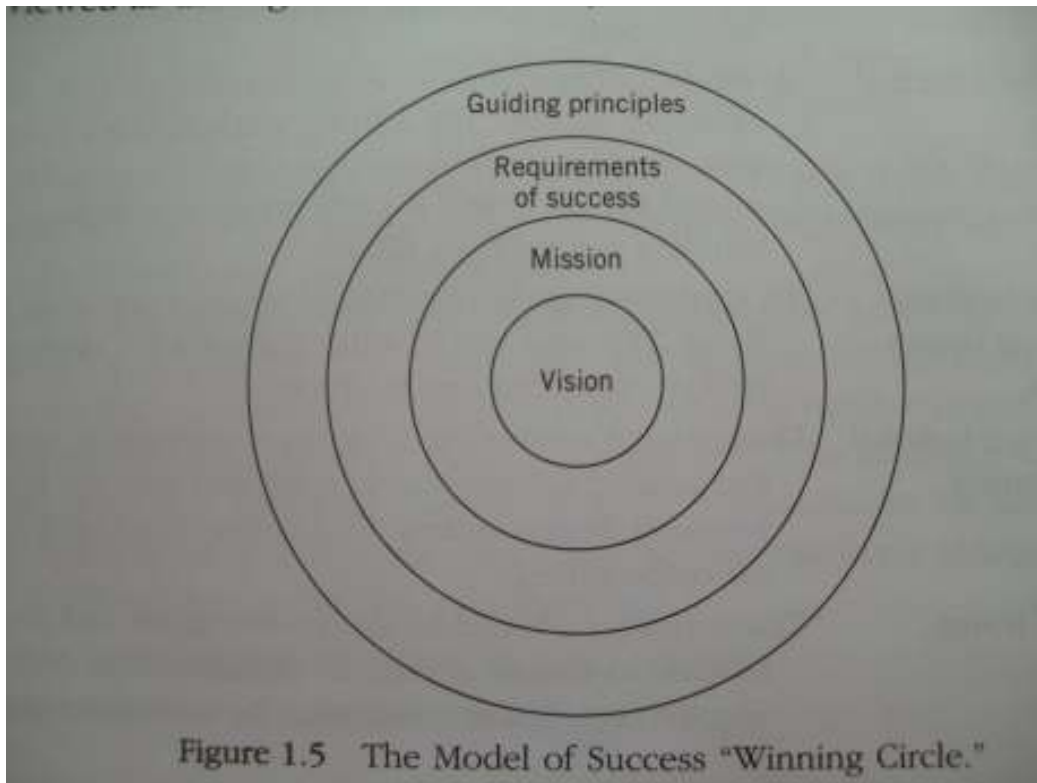


Figure 1.4. The model of success “winning circle”

SUMMARY

- FP determines how an activity’s tangible fixed assets should contribute to meeting the activity’s objectives.
- FP consists of facilities location and facilities design.
- Partly art, partly science.
- Can be approached using the engineering design process.
- Represents one of the most significant opportunity for cost reduction and productivity improvement.

CHAPTER 2

FACILITIES IN THE MANUFACTURING CONTEXT

In the manufacturing context, a facility is a place where raw materials, processing equipment, and people come together to make a finished product.

2.1. Logistics Management

Logistics management can be defined as the management of the transportation and distribution of goods.

Goods → Raw materials
Subassemblies obtained from suppliers
Finished goods shipped from plants to warehouses or customers

Logistics management includes all distribution and transportation activities from suppliers through to customers.

Logistics management is the management of a series of macro-level transportation and distribution activities with the main objective of delivering the right amount of material at the right place at the right time at the right cost using the right methods.

The decisions typically encountered in logistics management concern facility location, transportation and goods handling and storage.

Logistics management problems can be classified into three categories:

1. Location Problems:

Location Problems involve determining the location of one or more new facilities in one or more of several potential sites. The number of sites must at least equal the number of new facilities being located.

The cost of locating each new facility at each of the potential sites is assumed to be unknown.

It is the fixed cost of locating a new facility at a particular site plus the operating and transportation cost of serving customers from this facility-site combination.

2. Allocation Problems:

Allocation Problems assume that the number and location of facilities are known and attempt to determine how each customer is to be served. That is, given the demand for goods at each customer center, the production or supply capacities at each facility, and the cost of serving each customer from each facility, the allocation problem determined how much each facility is to supply to each customer center.

3. Location – Allocation Problems:

Location – Allocation Problems involve determining not only how much each customer is to receive from each facility but also the number of facilities along with their locations and capacities.

2.2. Classification of Facility Location Problems

Facility Location problems can be classified as:

- Single-Facility Location Problems
Single-Facility location problems deal with the optimal determination of the location of a single facility.
- Multifacility Location Problems
Multifacility location problems deal with the simultaneous location determination for more than one facility.

Generally, single-facility location problems are location problems, but multifacility location problems can be location as well as location-allocation problems.

Another classification of location problems is based on whether the set of possible locations for a facility is finite or infinite

- Continuous Space Location Problem
If a facility can be located anywhere within the confines of a geographic area, then the number of possible locations is infinite, and such a problem is called a *Continuous Space Location Problem*.
- Discrete Space Location Problem
Discrete Space Location Problems have a finite feasible set of sites in which to locate a facility.

Because facilities can be located anywhere in a two-dimensional space, sometimes the optimal location provided by the continuous space model may be infeasible. For example, a continuous space model may locate a manufacturing facility on a lake!

2.3. Facility Location Problem

The facility location problem consists of selecting a site for new facilities that will minimize the production and distribution cost of products and/or services to potential customers.

2.3.1. Reasons for considering Location Problems

- Significant changes in the level of demand,
- Significant changes in the geographical distribution of demand,
- Changes in the cost or quality requirements of critical production inputs (labor, raw materials, energy or others),
- Significant increases in the real-estate value of existing or adjacent sites or in their taxation,
- Need to change as a result of fire or flood for reasons of prestige or improved public relations.

2.3.2. Alternatives to New Location

- The increase of existing capacity by additional shifts or overtime, especially for capital-intensive systems.
- The use of seasonal inventories to reduce the need for maintaining capacity for peak demand.
- The use of subcontractors.
- The purchase of new equipment for the present location.

2.3.3. Important Factors in Location Decisions

- Production inputs (raw materials, human resources, etc...),
- Process techniques,
- Environmental factors
 - The availability and reliability of supporting systems
 - Social and cultural conditions
 - Legal and political considerations.

Example:

Consider the NIKE distribution center in Laakdal, Belgium.

- This warehouse employs 800 people,
- It has an annual turnover of 10.5 million of units of footwear and apparel,
- It covers 25 acres,
- It cost \$139 million to build.

A location and design study was done in 1992 and the building was completed in two phases – the last in 1995.

WHY Nike selected Laakdal from several available locations in Europe?

1. Nike's main business objective was to service 75% of its customers in less than 24 hours. Because of its proximity to major customer markets. Laakdal was a natural choice.
2. Proximity to ports of entry for footwear and apparel manufactured overseas, the road network in and around Laakdal, and access to major highways were superb.
3. Because its citizens are required to go to school until at least age of 18, Belgium has an educated workforce.
4. Other factors also favored Laakdal.

In practice, many factors have an important impact on location decisions. The relative importance of these factors depends on whether the scope of a particular location problems is international, national, statewide, or communitywide.

Example:

If we are trying to determine the location of a manufacturing facility in a foreign country, factors such as;

- Political stability,
- Foreign exchange rates,
- Business climate,
- Duties, and
- Taxes

play a role.

If the scope of the location problem is restricted to few communities, the factors like;

- Community services,
- Property tax incentives,
- Local business climate, and
- Local government regulations

are important.

2.3.4. Factors that affect Location Decisions

- Proximity to source of raw materials,
- Cost and availability of energy and utilities,
- Cost, availability, skill and productivity of labor,
- Government regulations at the federal, state, county and local levels,
- Taxes at the federal, state, county and local levels,
- Insurance,

- Construction costs and land price,
- Government and political stability,
- Exchange rate fluctuation,
- Export and import regulations, duties and tariffs,
- Transportation system,
- Technical expertise,
- Environmental regulations at the federal, state, county and local levels,
- Support services,
- Community services – schools, hospitals- recreation and so on,
- Weather,
- Proximity to customers,
- Business climate,
- Competition-related factors.

Example:

Suppose that the Waterstill Manufacturing Company has narrowed its choice down to two locations, city A and city B. all cost calculations have been made and there is no clear-cut distinction. In fact, for simplicity, assume that all costs are equal at the two locations. How can the decision be made?

Step 1: make a list of all important factors. Noncost factors in plant location:

- | | |
|--------------------------------|---|
| (1) Nearness to market | (12) Churches and religious facilities |
| (2) Nearness to unworked goods | (13) Recreational opportunities |
| (3) Availability of power | (14) Housing |
| (4) Climate | (15) Vulnerability to air attacks |
| (5) Availability of water | (16) Community attitude |
| (6) Capital availability | (17) Local ordinances |
| (7) Momentum of early start | (18) Labor laws |
| (8) Fire protection | (19) Future growth of community |
| (9) Police protection | (20) Medical facilities |
| (10) Schools and colleges | (21) Employee transportation facilities |
| (11) Union activity | |

Step 2: assign relative point values for each of the factor for specific company and plant to be located. Therefore, maximum point values for each factor:

<i>Factor-Value</i>	<i>Factor-Value</i>	<i>Factor-Value</i>
1 - 280	8 - 10	15 - 10
2 - 220	9 - 20	16 - 60
3 - 30	10 - 20	17 - 50
4 - 40	11 - 60	18 - 30
5 - 10	12 - 10	19 - 30
6 - 60	13 -20	20 - 10
7 - 10	14 - 10	21 - 20

Step 3: assign degrees and points within each factor. Usually, from 4 to 6 degrees are used with linear assignment of points between degrees.

Degrees and points for factor 16 (community attitude):

<u>Degrees</u>		<u>Point Assignment</u>
0	Hostile, bitter, noncooperative	0
1	Parasitic in nature	15
2	Noncooperative	30
3	Cooperative	45
Maximum	Friendly and more than cooperative	60

At this point Waterstill has its evaluation scheme completely defined, so it now must assign each of the two locations (A and B) degrees and corresponding points for each factor. The hypothetical results are;

<i>Factor</i>	CITY A		CITY B	
	<i>Degree</i>	<i>Points</i>	<i>Degree</i>	<i>Points</i>
1	Maximum	280	3	168
2	4	176	4	176
3	2	12	4	24
4	0	0	4	24
5	4	8	2	4
6	3	36	4	48
7	2	4	1	2
8	Maximum	10	2	4
9	4	16	2	8
10	2	8	3	12
11	3	36	3	36
12	2	6	2	6
13	3	15	Maximum	20
14	4	8	0	0
15	1	2	2	5
16	3	45	2	30
17	2	20	4	40
18	3	23	1	8
19	0	0	3	18
20	1	2	1	2
21	4	12	2	8
Total		719		643

Waterstill now can compare these results with the cost calculations and make a decision. City A has a total point value of 719 compared to 643 for City B. City A would probably be preferred since all cost calculations were assumed equal.

It is often extremely difficult to find a single location that meets all these objectives at the desired level. For example, a location may offer a highly skilled labor pool, but construction and land costs may be too high.

Similarly, another location may offer low tax rates and minimal government regulations but may be too far from the raw materials source or customer base.

Thus, facility location problem is to select a site (among several available alternatives) that optimizes a weighted set of objectives.

If we examine the inputs required to produce a product or provide a service, two things stand out:

- People, and
- Raw materials.

For a location to be effective, it must be in close proximity to relatively less expensive, skilled labor pools and raw materials sources.

Example:

- One of the reasons for electronics and software companies locating in Silicon Valley is availability of highly skilled computer professionals.
- Similarly, many U.S. companies are opening manufacturing facilities in Mexico and Far East to take advantage of lower labor wage rates. Many companies look for labor pools with higher productivity, a strong work ethic, and absence of unionization.

With respect to raw materials, some industries find it more important to be close to raw materials sources than others. These tend to be industries for which raw materials are bulky or otherwise expensive to transport. Companies that have implemented just-in-time (JIT) strategies are likely to be located near inventories and thereby reduce costs. Other inputs that have an impact on location decisions are cost and availability of energy and utilities, land prices and construction costs.

In addition to the input-related factors, one output-related factor plays an important role in the evaluation of location – proximity to customers. This factor is important because the product's shelf life may be short, the finished product may be bulky or may require special care during transportation, and duties and tariffs may be high, necessitating that the facility location be close to the market area.

2.4. Techniques for Discrete Space Location Problems

Our focus is on the single-facility location problem.

The single facility for which we seek a location may be;

- The only one that will serve all the customers,
- An addition to a network of existing facilities that are already serving customers.

1. Qualitative Analysis
2. Quantitative Analysis
3. Hybrid Analysis

2.4.1. Qualitative Analysis

Qualitative Analysis => Location Scoring Method

This is a very popular, subjective decision-making tool that is relatively easy to use.

Qualitative Analysis consists of these steps:

- Step 1:** List all the factors that are important – that have an impact on the location problem.
- Step 2:** Assign an appropriate weight (typically between 0 and 1) to each factor based on the relative importance of each.
- Step 3:** Assign a score (typically between 0 and 100) to each location with respect to each factor identified in step 1.
- Step 4:** Compute the weighted score for each factor for each location by multiplying its weight by the corresponding score.
- Step 5:** Compute the sum of the weighted scores for each location and choose a location based on these scores.

Example:

A payroll processing company has recently won several major contracts in the Midwest region of the United States and Central Canada, and wants to open a new, large facility to serve these areas. Because customer service is so important, the company wants to be as near its “customers” as possible. A preliminary investigation has shown that Minneapolis, Winnipeg, and Springfield are the three most desirable locations, and the payroll company has to select one of these. Using the location scoring method (Qualitative Analysis), determine the best location for the new payroll processing facility.

Solution:

A through investigation of each location with respect to eight important factors generated the raw scores and weights listed in the table below.

Table 1: Factors and weights for three locations:

Weight	Factor	Score		
		Minneapolis	Winnipeg	Springfield
0.25	Proximity to customer	95	90	65
0.15	Land and construction prices	60	60	90
0.15	Wage rates	70	45	60
0.10	Property taxes	70	90	70
0.10	Business taxes	80	90	85
0.10	Commercial travel	80	65	75
0.08	Insurance costs	70	95	60
0.07	Office services	90	90	80

Steps 1, 2, and 3 have been completed. That is, all the factors that are important (which have an impact on the location decision) are listed. Appropriate weights (typically between 0 and 1) are assigned to each factor based on the relative importance of each. A score (typically between 0 and 100) is assigned to each location with respect to each factor identified above.

We now need to compute the weighted score for each location-factor pair, add these weighted scores and determine the location based on the scores.

Table 2: Weighted scores for the three locations:

Factor	Weighted Score		
	Minneapolis	Winnipeg	Springfield
Proximity to customer	23.75	22.50	16.25
Land and construction prices	9.00	9.00	13.50
Wage rates	10.50	6.75	9.00
Property taxes	7.00	9.00	7.00
Business taxes	8.00	9.00	8.50
Commercial travel	8.00	6.50	7.50
Insurance costs	5.60	7.60	4.80
Office services	6.30	6.30	5.60
Sum of Weighted Scores	78.15	76.65	72.15

From the analysis in the table above, it is clear that Minneapolis is the best location on the subjective information.

Although step 5 calls for the location decision to be made solely on the basis of the weighted scores, those scores were arrived at in a subjective manner, and hence a final location decision must also take into account objective measures such as transportation costs, loads and operation costs.

2.4.2. Quantitative Analysis

Several quantitative techniques are available to solve the discrete space, single-facility location problem. Each is appropriate for a specific set of objectives and constraints.

e.g. the so-called minimax location model is appropriate for determining the location of an emergency service facility (such as a fire station, police station, hospital), where the objective is to minimize the maximum distance travelled between the facility and any customer.

If the objective is to minimize the total distance travelled, the transportation model is appropriate.

That is, we have m plants in a distribution network that serves n customers. Due to an increase in demand at one or more of these n customers, it has become necessary to open an addition plant. The new plant could be located at p possible sites. To evaluate which of the p sites will minimize distribution (transportation) costs, we can set up p

transportation models, each with n customers and $m+1$ plants, where $(m+1)^{th}$ plant corresponds to the new location being evaluated.

Solving the model will tell us not only the distribution of goods from the $m+1$ plants (including the new one from the location being evaluated) but also the cost of distribution.

The location that yields the least overall distribution cost is the one where the new facility should be located.

Example:

Seers Inc. has two manufacturing plants at Albany and Little Rock that supply Canmore brand refrigerators to four distribution centers in Boston, Philadelphia, Galveston and Raleigh. Due to an increase in the demand for this brand of refrigerators that is exported to last for several years, Sears Inc. has decided to build another plant in Atlanta or Pittsburgh.

The unit transportation costs, expected demand at the four distribution centers and the maximum capacity at the Albany and Little Rock plants are given in the following table. Determine which of the two locations, Atlanta or Pittsburgh, is suitable for the new plant Sears Inc. wishes to utilize all of the capacity available at its Albany and Little Rock locations. Costs, demand and supply capacity information:

<i>Factors and weights for the three locations</i>	Demand					
	Location	<i>Boston</i>	Philadelphia	Galveston	Raleigh	Supply Capacity
	Albany	10	15	22	20	250
	Little Rock	19	15	10	9	300
	Atlanta	21	11	13	6	No Limit
	Pittsburgh	17	8	18	12	No Limit
	Total	200	100	300	280	

Solution:

Manufacturing Plants

Albany
 Little Rock
 +
 New Plant
 in Atlanta?
 or
 in Pittsburgh?

Distribution Centers

Boston
 Philadelphia
 Galveston
 Raleigh

Maximum capacity of the new plant required at either location is 330 because the capacity at Albany and Little Rock is to be fully utilized.

$$\begin{aligned}
 \text{Total demand} &= 200 + 100 + 300 + 280 = 880 \\
 \text{Total supply} &= 250 + 300 + \chi = 550 + \chi \\
 550 + \chi &= 880 \\
 \chi &= 330
 \end{aligned}$$

(I) Transportation model with plant in Atlanta

Transportation model with plant in Atlanta	Demand					Supply Capacity
	Location	Boston	Philadelphia	Galveston	Raleigh	
Albany	10	15	22	20	250	
Little Rock	19	15	10	9	300	
Atlanta	21	11	13	6	330	
Total	200	100	300	280	880	

Distribution pattern is as follows:

Location	Demand				Supply Capacity
	Boston	Philadelphia	Galveston	Raleigh	
Albany	10	15	22	20	250 50
Little Rock	19	15	10	9	300 300
Atlanta	21	11	13	6	330 50
Total	200	100	300	280	880

$$\text{Total Cost} = (200 \times 10) + (50 \times 15) + (50 \times 11) + (300 \times 10) + (280 \times 6) = \$7980$$

(II) Transportation model with plant in Pittsburgh

Transportation model with plant in Pittsburgh	Demand					Supply Capacity
	Location	Boston	Philadelphia	Galveston	Raleigh	
Albany	10	15	22	20	250	
Little Rock	19	15	10	9	300	
Pittsburgh	17	8	18	12	330	
Total	200	100	300	280	880	

Distribution pattern is as follows:

Location	Demand				Supply Capacity
	Boston	Philadelphia	Galveston	Raleigh	
Albany	10	15	22	20	250 50
Little Rock	19	15	10	9	300 300
Pittsburgh	17	8	18	12	330 50
Total	200	100	300	280	880

$$\text{Total Cost} = (200 \times 10) + (50 \times 15) + (50 \times 8) + (300 \times 10) + (280 \times 12) = \$9510$$

Because the Atlanta location minimizes the cost, the decision is to construct the new plant in Atlanta.

2.4.3. Hybrid Analysis

A disadvantage of the Qualitative method discussed earlier is that location decision is made based entirely on a subjective evaluation. Although Quantitative method overcomes this disadvantage, it does not allow us to incorporate unquantifiable factors that have a major impact on the location decision.

Example:

The Quantitative techniques can easily consider:

- transportation cost, and
- operational costs,

but intangible factors such as;

- the attitude of a community toward businesses,
- potential labor unrest,
- reliability of auxiliary service providers

are difficult to capture though these are important in choosing a location decision.

Therefore, we need a method that incorporates subjective as well as quantifiable cost and other factors.

Hybrid Analysis

A multiattribute, single-facility location model based on the ones presented by Brown and Gibson (1972) and Buffa and Sarin (1987).

This model classifies the objective and subjective factors important to the specific location problem being addressed as:

- critical,
- objective, and
- subjective.

The meaning of objective and subjective factors is obvious. The meaning of critical factors needs some discussion.

Critical Factors:

In every location decision, usually at least one factor determines whether or not a location will be considered for further evaluation.

For instance, if water is used extensively in a manufacturing process (e.g. a brewery), then a site that does not have an adequate water supply now or in the

future is automatically removed from consideration. This is an example of a critical factor.

After the factors are classified, they are assigned numeric values:

$$CF_{ij} \begin{cases} 1 & \text{if location } i \text{ satisfies critical factor } j \\ 0 & \text{otherwise} \end{cases}$$

OF_{ij} : cost of objective factor j at location i

SF_{ij} : numeric value assigned (on a scale of 0–1) to subjective factor j for location i

w_j : weight assigned to subjective factor j ($0 \leq w_j \leq 1$)

Assume that we have m candidate locations and p critical, q objective and r subjective factors. We can determine overall critical factor measure (CFM_i), objective factor measure (OFM_i), and Subjective Factor Measure (SFM_i) for each location i with these equations.

CFM_i : overall critical factor measure for location i ,

OFM_i : objective factor measure for location i ,

SFM_i : subjective factor measure for location i ,

LM_i : location measure for location i .

$$CFM_i = CF_{i1}, CF_{i2}, \dots, CF_{ip} = \prod_{j=1}^p CF_{ij} \quad i = 1, 2, \dots, m$$

$$OFM_i = \frac{\max_i \left[\sum_{j=1}^q OF_{ij} \right] - \sum_{i=1}^q OF_{ij}}{\max_i \left[\sum_{j=1}^q OF_{ij} \right] - \min_i \left[\sum_{j=1}^q OF_{ij} \right]} \quad i = 1, 2, \dots, m$$

$$SFM_i = \sum_{j=1}^r w_j SF_{ij}$$

The location measure, LM_i for each location is then calculated as:

$$LM_i = CFM_i \left[\alpha(1 - OFM_i) + (1 - \alpha)SFM_i \right]$$

where α is the weight assigned to the objective factor measure.

After LM_i is determined for each candidate location, the next step is to select the one with the greatest LM_i value.

Example:

Mole-Sun Brewing Company is evaluating six candidate location; Montreal, Plattsburgh, Ottawa, Albany, Rochester, and Kingston for a new brewery. The two critical, three objective and four subjective factors that management wishes to incorporate in its decision making are summarized in the table below. The weights of the subjective factors are also provided in the table.

Determine the best location if the subjective factors are to be weighted 50% more than the objective factors.

Location	CRITICAL FACTORS		OBJECTIVE FACTORS			SUBJECTIVE FACTORS			
	Water Supply	Tax Incentives	Revenue	Labor Cost	Energy Cost	Community Attitude (0.3)	Ease of Transportation (0.4)	Labor Unionization (0.25)	Support Services (0.05)
Albany	0	1	185	80	10	0.5	0.9	0.6	0.7
Kingston	1	1	150	100	15	0.6	0.7	0.7	0.75
Montreal	1	1	170	90	13	0.4	0.8	0.2	0.8
Ottawa	1	0	200	100	15	0.5	0.4	0.4	0.8
Plattsburgh	1	1	140	75	8	0.9	0.9	0.9	0.55
Rochester	1	1	150	75	11	0.7	0.65	0.4	0.8

Solution:

$\alpha = 0.4$ so that the weight of the subjective factors ($1-\alpha = 0.6$) is 50% more than that of the objective factors.

Calculate:

Sum of objective factors = Revenue – Costs

Location	CRITICAL FACTORS		OBJECTIVE FACTORS			Sum of Objective Factors	SUBJECTIVE FACTORS			
	Water Supply	Tax Incentives	Revenue	Labor Cost	Energy Cost		Community Attitude (0.3)	Ease of Transportation (0.4)	Labor Unionization (0.25)	Support Services (0.05)
Albany	0	1	185	-80	-10	95	0.5	0.9	0.6	0.7
Kingston	1	1	150	-100	-15	35	0.6	0.7	0.7	0.75
Montreal	1	1	170	-90	-13	67	0.4	0.8	0.2	0.8
Ottawa	1	0	200	-100	-15	85	0.5	0.4	0.4	0.8
Plattsburgh	1	1	140	-75	-8	57	0.9	0.9	0.9	0.55
Rochester	1	1	150	-75	-11	64	0.7	0.65	0.4	0.8

Max= 95

Min= 35

$$CFM_i = CF_{i1}, CF_{i2}, \dots, CF_{ip} = \prod_{j=1}^p CF_{ij} \quad i = 1, 2, \dots, m$$

$$CFM_{Albany} = 0 \times 1 = 0$$

$$CFM_{Kingston} = 1 \times 1 = 1$$

$$CFM_{Montreal} = 1 \times 1 = 1$$

$$CFM_{Ottawa} = 1 \times 0 = 0$$

$$CFM_{Plattsburgh} = 1 \times 1 = 1$$

$$CFM_{Rochester} = 1 \times 1 = 1$$

$$OFM_i = \frac{\max_i \left[\sum_{j=1}^q OF_{ij} \right] - \sum_{i=1}^q OF_{ij}}{\max_i \left[\sum_{j=1}^q OF_{ij} \right] - \min_i \left[\sum_{j=1}^q OF_{ij} \right]} \quad i = 1, 2, \dots, m$$

$$OFM_{Albany} = \frac{95 - 95}{95 - 35} = 0$$

$$OFM_{Kingstony} = \frac{95 - 35}{95 - 35} = 1$$

$$OFM_{Moptreal} = \frac{95 - 67}{95 - 35} = 0.467$$

$$OFM_{Ottawa} = \frac{95 - 85}{95 - 35} = 0.167$$

$$OFM_{Plattsburgh} = \frac{95 - 57}{95 - 35} = 0.633$$

$$OFM_{Rochester} = \frac{95 - 64}{95 - 35} = 0.517$$

$$SFM_i = \sum_{j=1}^r w_j SF_{ij}$$

$$SFM_{Albany} = (0.3 \times 0.5) + (0.4 \times 0.9) + (0.25 \times 0.6) + (0.05 \times 0.7) = 0.695$$

$$SFM_{Kingstony} = (0.3 \times 0.6) + (0.4 \times 0.7) + (0.25 \times 0.7) + (0.05 \times 0.75) = 0.6725$$

$$SFM_{Moptreal} = (0.3 \times 0.4) + (0.4 \times 0.8) + (0.25 \times 0.2) + (0.05 \times 0.8) = 0.53$$

$$SFM_{Ottawa} = (0.3 \times 0.5) + (0.4 \times 0.4) + (0.25 \times 0.4) + (0.05 \times 0.8) = 0.45$$

$$SFM_{Plattsburgh} = (0.3 \times 0.9) + (0.4 \times 0.9) + (0.25 \times 0.9) + (0.05 \times 0.55) = 0.8825$$

$$SFM_{Rochester} = (0.3 \times 0.7) + (0.4 \times 0.65) + (0.25 \times 0.4) + (0.05 \times 0.8) = 0.61$$

$$LM_i = CFM_i [\alpha(1 - OFM_j) + (1 - \alpha)SFM_i]$$

$$\begin{aligned} LM_{Albany} &= CFM_{Alb} [\alpha(1 - OFM_{Alb}) + (1 - \alpha)SFM_{Alb}] \\ &= 0[0.4(1 - 0) + (1 - 0.4)0.695] = 0 \end{aligned}$$

$$\begin{aligned} LM_{Kingston} &= CFM_{King} [\alpha(1 - OFM_{King}) + (1 - \alpha)SFM_{King}] \\ &= 1[0.4(1 - 1) + (1 - 0.4)0.6725] = 0.4035 \end{aligned}$$

$$\begin{aligned} LM_{Montreal} &= CFM_{Mont} [\alpha(1 - OFM_{Mont}) + (1 - \alpha)SFM_{Mont}] \\ &= 1[0.4(1 - 0.467) + (1 - 0.4)0.53] = 0.5312 \end{aligned}$$

$$\begin{aligned} LM_{Ottawa} &= CFM_{Ottw} [\alpha(1 - OFM_{Ottw}) + (1 - \alpha)SFM_{Ottw}] \\ &= 0[0.4(1 - 0.167) + (1 - 0.4)0.45] = 0 \end{aligned}$$

$$\begin{aligned} LM_{Plattsburgh} &= CFM_{pla} [\alpha(1 - OFM_{pla}) + (1 - \alpha)SFM_{pla}] \\ &= 1[0.4(1 - 0.633) + (1 - 0.4)0.8825] = 0.6763 \end{aligned}$$

$$\begin{aligned} LM_{Rochester} &= CFM_{Roc} [\alpha(1 - OFM_{Roc}) + (1 - \alpha)SFM_{Roc}] \\ &= 1[0.4(1 - 0.517) + (1 - 0.4)0.61] = 0.5592 \end{aligned}$$

$$LM_{Albany} = 0$$

$$LM_{Kingston} = 0.4035$$

$$LM_{Montrela} = 0.5312$$

$$LM_{Ottawa} = 0$$

$$LM_{Plattsburgh} = 0.6763 \quad \leftarrow \text{Highest!!!}$$

$$LM_{Rochester} = 0.5592$$

Therefore; based on an α value of 0.4, the Plattsburgh location seems favorable. However, as the weight of the objective factors, α , increases more than 0.6, the Montreal location becomes attractive.

Assignment: Show how Montreal location will be attractive with ($\alpha = 0.7$).

2.5. Techniques for Continuous Space Location Problems

Continuous space location models determine the optimal location of one or more facilities on a two-dimensional plane. The obvious disadvantage is that the optimal location suggested by the model may not be a feasible one—for example, it may be in the middle of a water body, a river, lake, or sea. Or the optimal location may be in a community that prohibits such a facility. Despite this drawback, these models are very useful because they lend themselves to easy solution. Furthermore, if the optimal location is infeasible, techniques that find the nearest feasible and optimal locations are available.

The most important and widely used distance metrics:

- **Euclidean distance** is the "ordinary" distance between two points that one would measure with a ruler, and is given by the Pythagorean formula.

The *Euclidean distance* between points \mathbf{p} and \mathbf{q} is the length of the line segment $\overline{\mathbf{PQ}}$. In Cartesian coordinates, if $\mathbf{p} = (p_1, p_2, p_n)$ and $\mathbf{q} = (q_1, q_2, \dots, q_n)$ are two points in Euclidean n -space, then the distance from \mathbf{p} to \mathbf{q} is given by:

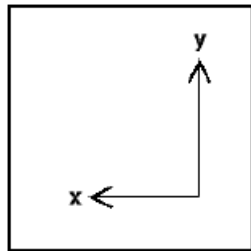
$$d(\mathbf{p}, \mathbf{q}) = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + \dots + (p_n - q_n)^2} = \sqrt{\sum_{i=1}^n (p_i - q_i)^2}.$$

- **Squared Euclidean distance** uses the same equation as the Euclidean distance metric, but does not take the square root. As a result, clustering with the Euclidean Squared distance metric is faster than clustering with the regular Euclidean distance.
- **Rectilinear distance** is known as city block distance or Manhattan distance as well. The distance, d_1 , between two vectors \mathbf{P}, \mathbf{Q} in an n -dimensional real vector space with fixed Cartesian coordinate system, is the sum of the lengths of the projections of the line segment between the points onto the coordinate axes. More formally,

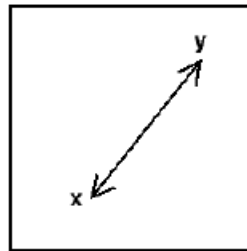
$$d_1(\mathbf{p}, \mathbf{q}) = \|\mathbf{p} - \mathbf{q}\|_1 = \sum_{i=1}^n |p_i - q_i|,$$

where

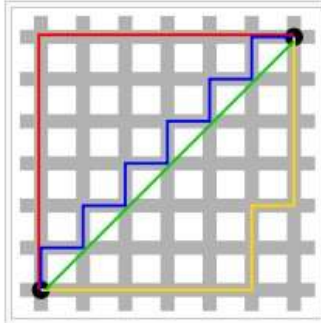
$$\mathbf{p} = (p_1, p_2, \dots, p_n) \text{ and } \mathbf{q} = (q_1, q_2, \dots, q_n) \text{ are vectors.}$$



Manhattan



Euclidean



Taxicab geometry versus Euclidean distance: The red, blue, and yellow lines have the same length (12) in Manhattan geometry for the same route. In Euclidean geometry, the green line has length $6 \times \sqrt{2} \approx 8.48$, and is the unique shortest path.

Single-facility location models, each incorporating a different distance metric, along with the solution methods or algorithms for these models will be introduced in this section. Because the optimal solution for a continuous space model may be infeasible, where available, we also discuss techniques that enable us to find feasible and optimal locations.

Techniques for Continuous Space Location Problem:

4. Median Method
5. Contour Line Method
6. Gravity Method
7. Weiszfeld Method

2.5.1. Median Method

As the name implies, the median method finds the median location and assigns the new facility to it. This method is used for single-facility location problems with rectilinear distance. Consider m facilities in a distribution network. Due to marketplace reasons (e.g., increased customer demand), it is desired to add another facility to this network. The interaction between the new facility and existing ones is known. The problem is to locate the new facility to minimize the total interaction cost between each existing facility and the new one.

At the macro level, this problem arises, for example, when deciding where to locate a warehouse that is to receive goods from several plants with known locations. At the micro level, this problem arises when we have to add a new machine to an existing network of machines on the factory floor. Because the routing and volume of parts processed on the shop floor are known, the interaction (in number of trips) between the new machine and existing ones can be easily calculated. Other non-manufacturing applications of this model are given in Francis, McGinnis, and White (1992).

Consider this notation:

- c_i cost of transportation between existing facility i and new facility, per unit
- f_i traffic flow between existing facility i and new facility
- x_i, y_i coordinates of existing facility i

The median location model is then to:

$$\text{Minimize } TC = \sum_{i=1}^m c_i f_i \left[|x_i - \bar{x}| + |y_i - \bar{y}| \right] \quad (1)$$

where TC is the total cost of distribution and \bar{x}, \bar{y} are the optimal coordinates of the new facility.

Because the c_{if_i} product is known for each facility, it can be thought of as a weight w_i corresponding to facility i . Using the notation w_i instead of c_{if_i} , let's rewrite the above expression (1) as follows:

$$\text{Minimize } TC = \sum_{i=1}^m w_i |x_i - \bar{x}| + \sum_{i=1}^m w_i |y_i - \bar{y}| \quad (2)$$

Because the x and y terms can be separated, we can solve for the optimal x and y coordinates independently. Here is the median method:

Median Method

Step 1: List the existing facilities in nondecreasing order of the x coordinates.

Step 2: Find the j^{th} x coordinate in the list (created in step 1) at which the cumulative weight equals or exceed half of the total weight for the first time;

$$\sum_{i=1}^{j-1} w_i < \sum_{i=1}^m \frac{w_i}{2} \text{ and } \sum_{i=1}^j w_i \geq \sum_{i=1}^m \frac{w_i}{2} \quad (3)$$

Step 3: List the existing facilities in nondecreasing order of the y coordinates.

Step 4: Find the k^{th} y coordinate in the list (created in step 3) at which the cumulative weight equals or exceeds half of the total weight for the first time:

$$\sum_{i=1}^{k-1} w_i < \sum_{i=1}^m \frac{w_i}{2} \text{ and } \sum_{i=1}^k w_i \geq \sum_{i=1}^m \frac{w_i}{2} \quad (4)$$

The optimal location of the new facility is given by the j^{th} x coordinate and the k^{th} y coordinate identified in steps 2 and 4, respectively.

Four points about the model and algorithm are worth mentioning. First, the total movement cost—that is, the OFV of Equation (2)—is the sum of the movement costs in the x and y directions. These two cost functions are independent in the sense that the solution of one does not influence the solution of the other. Moreover, both cost functions have the same form. This means that we can solve the two functions separately using the same basic procedure, as we do in the median method.

Second, in step 2 the algorithm determines a point on the two-dimensional plane such that no more than half of the total traffic flow cost is to the left or right of the point. In step 4 the same is done so that no more than half of the total traffic flow cost is above or below the point. Thus the optimal location of the new facility is a median point.

Third, it can be shown that any other x or y coordinate will not be the same as the optimal location's coordinates; in other words, the median method is optimal. We offer an intuitive explanation. Because the problem can be decomposed into x axis and y axis problems and solved separately, let us examine the x axis problem—the following x axis movement cost function:

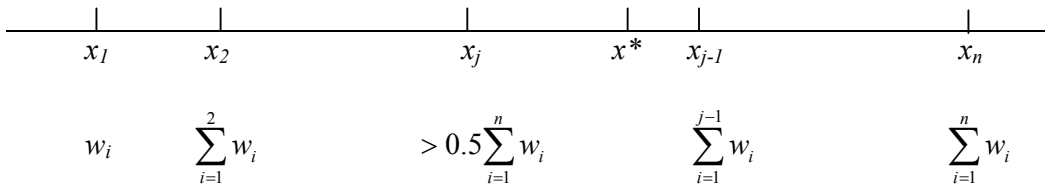
$$\sum_{i=1}^m w_i |x_i - \bar{x}|$$

Suppose the facilities are arranged in nondecreasing order of their x coordinates as shown in figure below. Let us assume that the x coordinate at which the cumulative weight exceeds half the total weight (for the first time) is the point shown as x_j in the figure.

(The cumulative weights are shown below the respective coordinates in figure below. For coordinate x , we indicate that the cumulative weight exceeds half the total weight.)

Let us also assume that the optimal x coordinate of the new facility falls at the coordinate indicated as x in the figure. For every unit distance we move to the left of x , the x axis movement cost decreases by more than half the total weight and increases by less than half the total weight. This is because the facilities to the left of x have a combined weight exceeding half the total weight and therefore those to the right of x (including x^*) must have a combined weight of less than half the total weight.

Since every unit distance movement to the left improves the cost function, it is beneficial to keep moving to the left until we reach the x . coordinate. Any more movement to the left increases the total cost. Thus x_j must be the optimal coordinate for the new facility. In a similar manner we can establish the result for the optimal y coordinate.



Fourth, these coordinates could coincide with the x and y coordinates of two different existing facilities or possibly one existing facility. In the latter case, the new facility must be moved to another location because it cannot be located on top of an existing one!

Example 2.1:

Two high-speed copiers are to be located on the fifth floor of an office complex that houses four departments of the Social Security Administration. The coordinates of the centroid of each department as well as the average number of trips made per day between each department and the copiers' yet-to-be-determined location are known and given in the following table.

Assume that travel originates and ends at the centroid of each department. Determine the optimal location—the x , y coordinates—for the copiers.

Department Number	x Coordinate	y Coordinate	Average Number of Daily Trips to Copiers
1	10	2	6
2	10	10	10
3	8	6	8
4	12	5	4

Solution:

We use the median method to get the solution.

Step 1:

Department Number	x Coordinates in Nondecreasing order	Weights	Cumulative Weights
3	8	8	8
1	10	6	14
2	10	10	24
4	12	4	28

Step 2: Because the second x coordinate—namely, 10—in the list is where the cumulative weight equals half the total weight of $28/2 = 14$, the optimal x coordinate is 10.

Step 3:

Department Number	y Coordinates in Nondecreasing order	Weights	Cumulative Weights
1	2	6	6
4	5	4	10
3	6	8	18
2	10	10	28

Step 4: Because the third y coordinate in the above list is where the cumulative weight exceeds half the total weight of $28/2 = 14$, the optimal coordinate is 6. Thus the optimal coordinates of the new facility are (10, 6).

Although the median method is the most efficient algorithm for the rectilinear distance, single facility location problem, we present another method for solving it that is used in the following chapters for the location of multiple facilities. It involves transforming the nonlinear, unconstrained model given by Equation (2) into an equivalent linear, constrained. Consider the following notation:

$$x_i^+ = \begin{cases} (x_i - \bar{x}) & \text{if } (x_i - \bar{x}) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$x_i^- = \begin{cases} (\bar{x} - x_i) & \text{if } (x_i - \bar{x}) \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

We can observe that;

$$|x_i - \bar{x}| = x_i^+ + x_i^- \quad (7)$$

$$x_i - \bar{x} = x_i^+ - x_i^- \quad (8)$$

A similar definition of y_i^+, y_i^- yields

$$|y_i - \bar{y}| = y_i^+ + y_i^- \quad (9)$$

$$y_i - \bar{y} = y_i^+ - y_i^- \quad (10)$$

Thus; the transformed linear model is:

$$\text{Minimize} \quad \sum_{i=1}^n w_i (x_i^+ + x_i^- + y_i^+ + y_i^-) \quad (11)$$

$$\text{Subject to} \quad x_i - \bar{x} = x_i^+ + x_i^- \quad i = 1, 2, \dots, n \quad (8)$$

$$y_i - \bar{y} = y_i^+ + y_i^- \quad i = 1, 2, \dots, n \quad (10)$$

$$x_i^+, x_i^-, y_i^+, y_i^- \geq 0 \quad i = 1, 2, \dots, n \quad (12)$$

$$\bar{x}, \bar{y} \text{ unrestricted in sign} \quad (13)$$

For this model to be equivalent to (2), the solution must be that either x_i^+ or x_i^- , but not both, is greater than zero [if both are, then the values x_i^+ and x_i^- do not satisfy their definitions in (5) and (6)]. Similarly, only one of y_i^+, y_i^- must be greater than zero.

Assume that in the solution to the transformed model, x_i^+ and x_i^- take on values p and q , where $p, q > 0$. We can immediately observe that such a solution cannot be optimal because one can choose another set of values for x_i^+, x_i^- as follows:

$$x_i^+ = p - \min[p, q] \text{ and } x_i^- = q - \min[p, q] \quad (14)$$

and obtain a feasible solution to the model that yields a lower objective value than before because the new x_i^+, x_i^- , values are less than their previously assumed values. Moreover, at least one of the new values of x_i^+ or x_i^- is zero according to the Expression (14). This means that the original set of values for x_i^+, x_i^- could not have been optimal. Using a similar argument, we can show that either y_i^+ or y_i^- , will take on a value of zero in the optimal solution.

The model described by Expressions (7), (9), and (11)-(13), can be simplified by noting that x_i can be substituted as $\bar{x} - x_i^- + x_i^+$ from equality (8) and the fact that x is unrestricted in sign. Also y_i may be substituted similarly, resulting in a model with $2n$ fewer constraints and variables. Next we set up a constrained linear programming model for Example 2.1 and solve it using LINDO.

The solution obtained, which has a total cost of 92, is the same as the one from the Median method. Notice that XBAR, XPi, and XNi in the model stand for \bar{x}, x_i^-, x_i^+ respectively. Also, only one of XPi, XNi and YPi, YNi take on positive values. If XPi is positive in the optimal solution, it means that the new facility is to the left of existing facility i according to (5) and (6). Similarly, if YPi is positive, then the new facility is below existing facility i . Obviously, XBAR and YBAR give us the coordinates of the new facility's optimal location. As expected, we get the same solution obtained in Example 2.1.

MIN 6 XP1 + 6 XN1 + 6 YP1 + 6 YN1 + 10 XP2 + 10 XN2 + 10 YP2 + 10 YN2 + 8 XP3 + 8 XN3 + 8 YP3 + 8 YN3 + 4 XP4 + 4 XN4 + 4 YP4 + 4 YN4

SUBJECT TO

- 2) XP1 - XN1 + XBAR = 10
- 3) XP2 - XN2 + XBAR = 10
- 4) XP3 - XN3 + XBAR = 8
- 5) XP4 - XN4 + XBAR = 12
- 6) YP1 - YN1 + YBAR = 2
- 7) YP2 - YN2 + YBAR = 10
- 8) YP3 - YN3 + YBAR = 6
- 9) YP4 - YN4 + YBAR = 5

END

LP OPTIMUM FOUND AT STEP 11

OBJECTIVE FUNCTION VALUE

1) 92.00000

VARIABLE	VALUE	REDUCED COST
XP1	.000000	.000000
XN1	.000000	12.000000
YP1	.000000	12.000000
YM1	4.000000	.000000
XP2	.000000	12.000000
XN2	.000000	8.000000
YP2	4.000000	.000000
YN2	.000000	20.000000
XP3	.000000	16.000000
XN3	2.000000	.000000
YP3	.000000	8.000000
YN3	.000000	8.000000
XP4	2.000000	.000000
XN4	.000000	8.000000
YP4	.000000	8.000000
YN4	1.000000	.000000
XBAR	10.000000	.000000
YBAR	6.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	6.000000
3)	.000000	-2.000000

4)	.000000	-8.000000
5)	.000000	4.000000
6)	.000000	-6.000000
7)	.000000	10.000000
8)	.000000	.000000
9)	.000000	-4.000000

2.5.2. Contour Line Method

Suppose that the weight of facility 2 in Example 2.1 is increased to 20. Using the median method, we can verify that the optimal location's x, y coordinates are 10, 10. This location may not be feasible because it is department 2's centroid, and locating the two photocopiers in the middle of one department may not be acceptable to the others. We therefore wish to determine adjacent feasible locations that minimize the total cost function. To do so, we use the contour line method, which graphically constructs regions bounded by contour lines. Locating the new facility on any point along the contour line incurs the same total cost. Contour lines are important because if the optimal location determined is infeasible, we can move along the contour line and choose a feasible point that will have a similar cost. Also, if subjective factors need to be incorporated, we can use contour lines to move away from the optimal location determined by the median method to another point that better satisfies the subjective criteria.

We now provide an algorithm to construct contour lines, describe the steps, and illustrate with a numeric example.

Algorithm for Drawing Contour Lines

Step 1: Draw a vertical line through the x coordinate and a horizontal line through the y coordinate of each facility.

Step 2: Label each vertical line $V_i = 1, 2, \dots, p$, and horizontal line $H_j = 1, 2, \dots, q$, where;

$$V_i = \text{sum of weights of facilities whose } x \text{ coordinates fall on vertical line } i.$$

$$H_j = \text{sum of weights of facilities whose } y \text{ coordinates fall on vertical line } j.$$

Step 3: Set $i = j = 1$ and $N_0 = D_0 = -\sum_{i=1}^m w_i$

Step 4: Set $N_i = N_{i-1} + 2V_i$ and $D_j = D_{j-1} + 2H_j$. Increment $i = i + 1$ and $j = j + 1$. If $i \leq p$ or $j \leq q$, repeat 4. Otherwise, set $i = j = 0$.

Step 5: Determine S_{ij} , the slope of the contour lines through the region bounded by vertical lines i and $i+1$ and horizontal line j and $j+1$ using the equation

$$S_{ij} = -N_i / D_j \quad \text{Increment } i = i + 1 \text{ and } j = j + 1$$

Step 6: If $i \leq p$ or $j \leq q$, go to step 5. Otherwise, select any point (x, y) and draw a contour line with slope S_{ij} , in the region $[i, j]$ in which (x, y) appears so that

the line touches the boundary of this region. From one of the endpoints of this line, draw another contour line through the adjacent region with the corresponding slope. Repeat this until you get a contour line ending at point (x, y) . You now have a region bounded by contour lines with (x, y) on the boundary region.

We discuss four points about this algorithm. First, the numbers of vertical and horizontal lines need not be equal. Two facilities may have the same x coordinate but not the same y coordinate, thereby requiring one horizontal line and two vertical lines. In fact, this is why the index i of V_i ranges from one to p and that of H_i ranges from one to q .

Second, the N_i and D_j computed in steps 3 and 4 correspond to the numerator and denominator, respectively, of the slope equation of any contour line through the region bounded by the vertical lines i and $i + 1$ and the horizontal lines j and $j + 1$. To verify this, consider the objective function (11) when the new facility is located at some point (x, y) —that is, $x = x, y = y$:

$$TC = \sum_{i=1}^m w_i |x_i - x| + \sum_{i=1}^m w_i |y_i - y| \quad (15)$$

By noting that the V_i 's and H_j 's calculated in step 2 of the algorithm correspond to the sum of the weights of facilities whose x, y coordinates are equal to the x, y coordinates, respectively, of the $i^{\text{th}}, j^{\text{th}}$ distinct lines and that we have p, q such coordinates or lines ($p < m, q < m$), we can rewrite (15) as follows:

$$TC = \sum_{i=1}^p V_i |x_i - x| + \sum_{i=1}^q H_j |y_i - y| \quad (16)$$

Suppose that x is between the s^{th} and $(s + 1)^{\text{th}}$ (distinct) x coordinates or vertical lines (since we have drawn vertical lines through these coordinates in step 1). Similarly, let y be between the t^{th} and $(t + 1)^{\text{th}}$ vertical lines. Then;

$$TC = \sum_{i=1}^s V_i |x_i - x| + \sum_{i=s+1}^{ps} V_i |x_i - x| + \sum_{i=1}^t H_j |y_i - y| + \sum_{i=t+1}^q H_j |y_i - y| \quad (17)$$

Rearranging the variable and constant terms in Equation (17) we get:

$$TC = \left[\sum_{i=1}^s V_i - \sum_{i=s+1}^p V_i \right] x + \left[\sum_{i=1}^t H_j - \sum_{i=t+1}^q H_j \right] y - \sum_{i=1}^s V_i x_i + \sum_{i=s+1}^p V_i x_i - \sum_{i=1}^t H_j y_i + \sum_{i=s+1}^q H_j y_i \quad (18)$$

The last four terms in Equation (18) are constants. To make our discussion simpler, we substitute another constant term c . Also, the coefficients of x can be rewritten as follows:

$$\sum_{i=1}^s V_i + \sum_{i=s+1}^p V_i - \sum_{i=1}^t H_j + \sum_{i=s+1}^q H_j \quad (19)$$

Notice that all we have done in (19) is added and subtracted sum of V_i to the original coefficient. Because it is clear from step 2 that

$$\sum_{i=1}^p V_i = \sum_{j=1}^m w_j$$

The coefficient of x from (19) can be rewritten as:

$$2 \sum_{i=1}^s V_i - \left[\sum_{i=1}^s V_i + \sum_{i=s+1}^p V_i \right] = 2 \sum_{i=1}^s V_i - \sum_{i=1}^p V_i = 2 \sum_{i=1}^s V_i - \sum_{i=1}^m w_i \quad (20)$$

Similarly, the coefficient of y is:

$$2 \sum_{i=1}^t H_j - \sum_{i=1}^m w_i \quad (21)$$

Thus, equation (18) can be rewritten as:

$$TC = \left[2 \sum_{i=1}^s V_i - \sum_{i=1}^m w_i \right] x + \left[2 \sum_{i=1}^t H_j - \sum_{i=1}^m w_i \right] y + c \quad (22)$$

The N_i computation in step 4 is in fact this calculation of the coefficient if x . To verify this, note that $N_i = N_{i-1} + 2V_i$. Making the substitution for N_{i-1} , we get $N_i = N_{i-2} + 2V_{i-1} + 2V_i$. Repeating this procedure of making substitutions for N_{i-2}, N_{i-1}, \dots we get

$$N_i = N_0 + 2V_1 + 2V_2 + \dots + 2V_{i-1} + 2V_i = \sum_{i=1}^m w_i + 2 \sum_{k=1}^t V_k \quad (23)$$

Similarly, we can verify that

$$D_t = - \sum_{i=1}^m w_i + 2 \sum_{k=1}^t H_k \quad (24)$$

Hence the equation (22) is:

$$TC = N_i x + D_t y + c$$

Which can be written as:

$$y = - \frac{N_i}{D_t} x + (TC - c) \quad (25)$$

This expression for the total cost function at x, y or, in fact, any other point in the region $[s, t]$ has the form $y = mx + c$, where the slope $m = -N/D_t$. This is exactly how the slopes are computed in step 5 of the algorithm.

We have shown that the slope of any point x, y within a region $[s, t]$ bounded by vertical lines s and $s + 1$ and horizontal lines t and $t + 1$ can be easily computed. Thus the contour line (or isocost line) through x, y in region $[s, t]$ may be readily drawn. Proceeding from one line in one region to the next line in the adjacent region until we come back to the starting point (x, y) then gives us a region of points in which any point has a total cost less than or equal to that of (x, y) .

Third, the lines V_0, V_{p+1} and H_0, H_{q+1} are required for defining the "exterior" regions. Although they are not included in the algorithm steps, the reader must take care to draw these lines.

Fourth, once we have determined the slopes of all the regions, the user may choose any point (x, y) other than a point that minimizes the objective function and draw a series of contour lines in order to get a region that contains points (i.e., facility locations) yielding as good or better objective function values than (x, y) . Thus step 6 could be repeated for several points to yield several such regions. Beginning with the innermost region, if any point in it is feasible, we use it as the optimal location. If not, we can go to the next innermost region to identify a feasible point. We repeat this procedure until we get a feasible point.

We now illustrate the contour line method with a numeric example.

Example 2.2:

Consider Example 2.1. Suppose that the weight of facility 2 is not 10, but 20. Applying the median method, we can verify that the optimal location is (10, 10)—the centroid of department 2, where immovable structures exist. It is now desired to find a feasible and "near-optimal" location using the contour line method.

Solution:

The contour line method is illustrated in the figure at the end of the solution.

Step 1 The vertical and horizontal lines V_1, V_2, V_3 and H_1, H_2, H_3, H_4 are drawn as shown. In addition to these lines, we draw lines V_0, V_4 and H_0, H_5 to identify the "exterior" regions.

Step 2 The weights $V_1, V_2, V_3, H_1, H_2, H_3,$ and H_4 are calculated by adding the weights of the points that fall on the respective lines. Note that for this example, $p = 3$ and $q = 4$.

Step 3 Because

$$\sum_{i=1}^4 w_i = 38$$

Step 4 Set

$$\begin{aligned} N_1 &= -38 + 2(8) = -22 \\ N_2 &= -32 + 2(26) = 30 \\ N_3 &= 30 + 2(4) = 38 \end{aligned}$$

$$\begin{aligned} D_1 &= -38 + 2(6) = -26 \\ D_2 &= -26 + 2(4) = -18 \\ D_3 &= -18 + 2(8) = -2 \\ D_4 &= -2 + 2(20) = 3 \end{aligned}$$

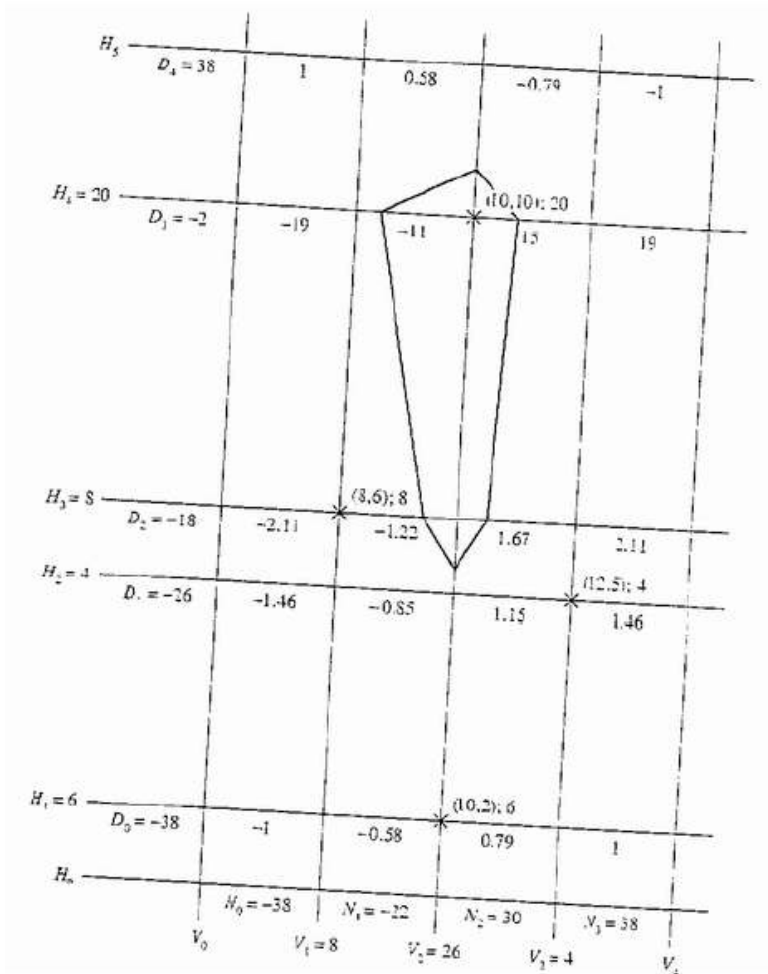
(These values are entered at the bottom of each column and to the left of each row in the following figure)

Step 5 Compute the slope of each region:

$$\begin{array}{lll} S_{00} = -(-38/-38) = -1 & S_{01} = -(-38/-26) = -1.46 & S_{02} = -(-38/-18) = -2.11 \\ S_{03} = -(-38/-2) = -19 & S_{04} = -(-38/38) = 1 & S_{10} = -(-22/-38) = -0.58 \\ S_{11} = -(-22/-26) = -0.85 & S_{12} = -(-22/-18) = -1.22 & S_{13} = -(-22/-2) = -11 \\ S_{14} = -(-22/38) = 0.58 & S_{20} = -(30/-38) = 0.79 & S_{21} = -(30/-26) = 1.15 \\ S_{22} = -(30/-18) = 1.67 & S_{23} = -(30/-2) = 15 & S_{24} = -(30/38) = -0.79 \\ S_{30} = -(38/-38) = 1 & S_{31} = -(38/-26) = 1.46 & S_{32} = -(38/18) = 2.11 \\ S_{33} = -(38/-2) = 19 & S_{34} = -(38/38) = -1 & \end{array}$$

(These slope values are shown inside each region.)

Step 6 When we draw contour lines through (9, 10), we get the region as shown in the following figure.



Because the copiers cannot be placed at the (10, 10) location, we drew contour lines through another nearby point, (9, 10). Locating the copiers anywhere possible within this region will give us a feasible, near-optimal solution.

2.5.3. Gravity Method (Center-of-Gravity or Centroid)

In some location problems the distance function may not be linear, but nonlinear. If it is quadratic, then determining the optimal location of the new facility is rather simple. To understand the method of solving such problems, consider the following objective function for single-facility location problems with a squared Euclidean distance metric:

$$\text{Minimize } TC = \sum_{i=1}^m c_i f_i \left[(x_i - \bar{x})^2 + (y_i - \bar{y})^2 \right] \quad (3.1)$$

As before, we substitute $w_i = c_i f_i$ where, $i = 1, 2, \dots, m$ and rewrite the objective function as:

$$\text{Minimize } TC = \sum_{i=1}^m w_i \left[(x_i - \bar{x})^2 + (y_i - \bar{y})^2 \right] \quad (3.2)$$

Because this objective function can be shown to be convex, partially differentiating TC with respect to \bar{x} and \bar{y} , setting the two resulting equations to zero and solving for \bar{x}, \bar{y} provide the optimal location of the new facility.

$$\frac{\partial TC}{\partial \bar{x}} = 2 \sum_{i=1}^m w_i \bar{x} - 2 \sum_{i=1}^m w_i x_i = 0 \quad (3.3)$$

$$\therefore \bar{x} = \frac{\sum_{i=1}^m w_i x_i}{\sum_{i=1}^m w_i} \quad (3.4)$$

$$\frac{\partial TC}{\partial \bar{y}} = 2 \sum_{i=1}^m w_i \bar{y} - 2 \sum_{i=1}^m w_i y_i = 0 \quad (3.5)$$

$$\therefore \bar{y} = \frac{\sum_{i=1}^m w_i y_i}{\sum_{i=1}^m w_i} \quad (3.6)$$

It is easy to see that the optimal locations x and y are simply the weighted averages of the x and y coordinates of the existing facilities. This method of determining the optimal location is popularly known as the center-of-gravity or gravity or centroid method.

If the optimal location determined by the gravity method is infeasible, we can again draw contour lines from neighboring points to find a feasible, near-optimal location. The contour lines will not be lines, however, but a circle through the point under consideration that has the optimal location as its center! Thus, if the gravity method yields an optimal location (x,y) that is infeasible for the new facility, all we need to do is find any feasible point (x,y) that has the shortest Euclidean distance to (x,y) and locate the new facility at (x,y) .

Example:

Consider Example 2.1, suppose the distance metric to be used is squared Euclidean. Determine the optimal location of the new facility using the gravity method.

Solution:

Department i	x_i	y_i	w_i	$w_i x_i$	$w_i y_i$
1	10	2	6	60	12
2	10	10	10	100	100
3	8	6	8	64	48
4	12	5	4	48	20
Total			28	272	180

$$\bar{x} = \frac{272}{28} = 9.7 \qquad \bar{y} = \frac{180}{28} = 6.4$$

If this location is not feasible, we find another feasible point that has the nearest Euclidean distance to (9.7, 6.4) and that is a feasible location for the new facility.

2.5.4. Weiszfeld Method

The objective function for the single facility location problem with Euclidean distance can be written as:

$$\text{Minimize } TC = \sum_{i=1}^m c_i f_i \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2} \quad (4.1)$$

As before, substituting $w_i = c_i f_i$, taking the derivate of TC with respect to \bar{x}, \bar{y} , setting the derivatives to zero, and solving for \bar{x}, \bar{y} yield:

$$\frac{\partial TC}{\partial \bar{x}} = \frac{1}{2} \sum_{i=1}^m \frac{w_i [2(x_i - \bar{x})]}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} \quad (4.2)$$

$$\frac{\partial TC}{\partial \bar{x}} = \sum_{i=1}^m \frac{w_i x_i}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} - \sum_{i=1}^m \frac{w_i \bar{x}}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} = 0 \quad (4.3)$$

$$\therefore \bar{x} = \frac{\left[\sum_{i=1}^m \frac{w_i x_i}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} \right]}{\left[\sum_{i=1}^m \frac{w_i}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} \right]} \quad (4.4)$$

$$\frac{\partial TC}{\partial \bar{y}} = \frac{1}{2} \sum_{i=1}^m \frac{w_i [2(y_i - \bar{y})]}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} \quad (4.5)$$

$$\frac{\partial TC}{\partial \bar{y}} = \sum_{i=1}^m \frac{w_i y_i}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} - \sum_{i=1}^m \frac{w_i \bar{y}}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} = 0 \quad (4.6)$$

$$\therefore \bar{y} = \frac{\left[\sum_{i=1}^m \frac{w_i y_i}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} \right]}{\left[\sum_{i=1}^m \frac{w_i}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} \right]} \quad (4.7)$$

Because $\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}$ appears twice in the denominators in Equations (4.4) and (4.7), the solution of \bar{x}, \bar{y} is not defined when $\bar{x} = x_i$, and $\bar{y} = y_i$ for some i . This means that if the new facility's optimal coordinates coincide with those of an existing facility, Equations (4.4) and (4.7) are not defined and we therefore cannot use them in computing the optimal coordinates \bar{x}, \bar{y} . The possibility of the optimal location of the new facility coinciding with that of an existing facility is very rare in practice, but cannot be ruled out, so we need to devise another method for solving the single facility Euclidean distance problem. Although (theoretically) optimal algorithms do not exist for this problem, a method from Weiszfeld (1936) is guaranteed to converge to the optimal location. This iterative algorithm is relatively straightforward.

Weiszfeld Method:

Step 0: Set iteration counter $k=1$

$$\bar{x}^{-k} = \frac{\sum_{i=1}^m w_i x_i}{\sum_{i=1}^m w_i}, \quad \bar{y}^{-k} = \frac{\sum_{i=1}^m w_i y_i}{\sum_{i=1}^m w_i}$$

Step 1:

$$\bar{x}^{-k+1} = \frac{\left[\sum_{i=1}^m \frac{w_i x_i}{\sqrt{(x_i - \bar{x}^{-k})^2 + (y_i - \bar{y}^{-k})^2}} \right]}{\left[\sum_{i=1}^m \frac{w_i}{\sqrt{(x_i - \bar{x}^{-k})^2 + (y_i - \bar{y}^{-k})^2}} \right]}$$

$$\bar{y}^{-k+1} = \frac{\left[\sum_{i=1}^m \frac{w_i y_i}{\sqrt{(x_i - \bar{x}^{-k})^2 + (y_i - \bar{y}^{-k})^2}} \right]}{\left[\sum_{i=1}^m \frac{w_i}{\sqrt{(x_i - \bar{x}^{-k})^2 + (y_i - \bar{y}^{-k})^2}} \right]}$$

Step 2: If $\bar{x}^{-k+1} \cong \bar{x}^{-k}$ and $\bar{y}^{-k+1} \cong \bar{y}^{-k}$, stop. Otherwise set $k = k + 1$ and go to step 1.

Notice that the initial seeds for \bar{x} and \bar{y} were obtained from Equations (3.4) and (3.6), which were used in the gravity method. Although the Weiszfeld method is theoretically suboptimal, it provides \bar{x} and \bar{y} values that are very close to optimal. For practical purposes the algorithm works very well and can be readily implemented on a spreadsheet.

If the optimal location provided by the Weiszfeld method is not feasible, we can once again use the contour line method to draw contour lines and then choose a suitable, feasible, near optimal location for the new facility. However, the methods for drawing the contour lines for the Euclidean distance metric, single-facility location problem are not exact. These approximate methods basically compute TC for a given point (x,y) , choose a neighboring x (or y) coordinate, and search for the y (or x) coordinate that yields the same TC value previously computed. This procedure is repeated until we come back to the starting point.

Example:

Consider Example 2.2. Assume the distance metric is Euclidean and determine the optimal location of the new facility using the Weiszfeld method. Data for this problem are given table below:

Department Number	x_i	y_i	w_i
1	10	2	6
2	10	10	20
3	8	6	8
4	12	5	4

Solution:

The gravity method finds the initial seed (9.8, 7.4). With this as the starting solution, we apply step 1 of the Weiszfeld method repeatedly until two consecutive \bar{x} and \bar{y} values are equal. As shown in the following table, this occurs in the 25th iteration. For convenience, the total costs at the first 12th, 20th, and 25th iterations are also shown in the table. The optimal location for this problem, (10,10), is the same as that of an existing facility-department 2. This is no accident, it occurs because department 2's weight is more than half of the cumulative weights. In fact, when facility i 's weight is greater than or equal to half of the sum of the weights for all the remaining facilities, the new facility's optimal location will be the same as that of facility i . this is true under the rectilinear as well as Euclidean distance metrics. We therefore must use an approximate contour line method to identify alternative, feasible solution.

Iteration Number	x	y	TC
1	9.7	7.8	113.4
2	9.7	8.2	111.9
3	9.8	8.4	110.8
4	9.8	8.7	109.9
5	9.8	8.9	109.1
6	9.9	9.0	108.5
7	9.9	9.2	108.0
8	9.9	9.3	107.6
9	9.9	9.4	107.2
10	9.9	9.5	106.9
11	9.9	9.6	106.7
12	10	9.6	106.5
:	:	:	:
:	:	:	:
20	10	9.9	105.6
:	:	:	:
:	:	:	:
25	10	10	105.5

2.6. Advanced Location Models

- Location problems
 - Multiple Facility problems
 - Rectilinear distance
 - Euclidean distance
- Allocation problems
 - Two-Stage Transportation Model (Transshipment)
- Location-Allocation problems

Main issues in the general location-allocation problem;

- How many new facilities are to be located in the distribution network that consists of previously establish facilities and customers?
- Where should the new facilities be located?
- How large should each new facility be? In other words, what is the capacity of the new facility?
- How should customers be assigned to the new and existing facilities? More specifically which facilities should be serving each customer?
- Can more than one facility serve a customer?

A model that can answer all or most of these questions would be desirable, but we know by now that the more features we add to a model, the more difficult it is to solve. For the multifacility location problem, however, we do have a model that captures a variety of issues and considerations and yet is relatively easy to solve.

2.6.1. n-Facility Location Problems

$$\text{Minimum } f(x) = \sum_{j=1}^n \sum_{k=1}^n v_{jk} d(x_j, x_k) + \sum_{j=1}^n \sum_{i=1}^n w_{ji} d(x_j, p_i)$$

$X = (x_j, y_j)$ Location of new facility $j, j=1, 2, \dots, n$

$P = (a_i, b_i)$ Location of existing facility $i, i=1, 2, \dots, n$

v_{jk} = cost per unit distance travel between new facilities j and k

w_{ji} = cost per unit distance travel between j and i

$d(x_j, x_k)$ = distance between new facility j and new facility k

$d(x_j, p_i)$ = distance between new facility j and existing facility i

Rectilinear Distance

$$d(x_j, x_k) = |x_j - x_k| + |y_j - y_k| \quad j=1, 2, \dots, n \ \& \ k=1, 2, \dots, n$$

$$d(x_j, p_i) = |x_j - a_i| + |y_j - b_i| \quad j=1, 2, \dots, n \ \& \ i=1, 2, \dots, n$$

Nonlinear Unconstrained Model

- Distribution network with m facilities
- Desired to add n new facilities
- Coordinates of the i^{th} existing facility are (a_i, b_i)
- Coordinates of the i^{th} new facility are (x_i, y_i)
- Flow from new to an existing facility, g_{ij}
- Flow between new facilities, f_{ij}

2.6.2. Multiple-Facility Problems with Rectilinear Distances

$$\text{Minimize } \sum_{i=1}^n \sum_{j=1}^n c_{ij} f_{ij} [|x_i - x_j| + |y_i - y_j|] + \sum_{i=1}^n \sum_{j=1}^m d_{ij} g_{ij} [|x_i - a_j| + |y_i - b_j|] \quad (1)$$

$$x_{ij}^+ = \begin{cases} (x_i - x_j) & \text{if } (x_i - x_j) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij}^- = \begin{cases} (x_j - x_i) & \text{if } (x_i - x_j) \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

We can observe that;

$$\begin{aligned} |x_i - x_j| &= x_{ij}^+ + x_{ij}^- \\ x_i - x_j &= x_{ij}^+ - x_{ij}^- \end{aligned}$$

A similar definition of y_i^+, y_i^- yields

$$\begin{aligned} |y_i - y_j| &= y_{ij}^+ + y_{ij}^- \\ y_i - y_j &= y_{ij}^+ - y_{ij}^- \end{aligned}$$

Thus; the transformed linear model is:

$$\text{Minimize } \sum_{i=1}^n \sum_{j=1}^n c_{ij} f_{ij} (x_{ij}^+ + x_{ij}^- + y_{ij}^+ + y_{ij}^-) + \sum_{i=1}^n \sum_{j=1}^m d_{ij} g_{ij} (x a_{ij}^+ + x a_{ij}^- + y b_{ij}^+ + y b_{ij}^-) \quad (2)$$

$$\begin{aligned} \text{Subject to } \quad x_i - x_j &= x_{ij}^+ - x_{ij}^- & i, j &= 1, 2, \dots, n \\ y_i - y_j &= y_{ij}^+ - y_{ij}^- & i, j &= 1, 2, \dots, n \\ x_i - a_j &= x a_{ij}^+ + x a_{ij}^- & i &= 1, 2, \dots, n \quad j = 1, 2, \dots, m \\ y_i - b_j &= y b_{ij}^+ + y b_{ij}^- & i &= 1, 2, \dots, n \quad j = 1, 2, \dots, m \\ x_{ij}^+, x_{ij}^-, y_{ij}^+, y_{ij}^- &\geq 0 & i, j &= 1, 2, \dots, n \\ x a_{ij}^+, x a_{ij}^-, y b_{ij}^+, y b_{ij}^- &\geq 0 & i &= 1, 2, \dots, n \quad j = 1, 2, \dots, m \\ x_i, y_j &\text{unrestricted in sign} & i &= 1, 2, \dots, n \end{aligned}$$

For this model to be equivalent to expression (1), the solution must be such that either of the two new variables introduced, x_{ij}^+ or x_{ij}^- , but not both, is greater than zero. [If both are, then the values of x_{ij}^+ and x_{ij}^- do not satisfy their definitions in Equations (2) and (3).] Similarly, only one of the pairs y_{ij}^+, y_{ij}^- and xa_{ij}^+, xa_{ij}^- and yb_{ij}^+, yb_{ij}^- must be greater than zero. Recall that this condition had to be satisfied for the LMIP models as well as the median location model.

Fortunately, they are automatically satisfied in the linear model presented here, just as they were in the median location model. It turns out that the optimal x coordinate of each new facility is the same as that of an existing facility or customer. The same is true for the y coordinates. If it turns out that the x and y coordinates of a new facility coincide with the x and y coordinates of a *single* existing facility, we must find alternative feasible locations heuristically using rules of thumb for example, locate a new facility in a feasible location that is within 5 miles of the optimal one. It is rather difficult to use the contour line methods that worked so well for the single-facility case, (1) can be simplified by noting that x_i can be substituted as $a_j + xa_{ij}^+ - xa_{ij}^-$ and the fact that x_i is unrestricted in sign. Similarly y_i may also be substituted, resulting in a model with $2n$ fewer constraints and variables than (1).

Assignment:

Tires and Brakes, Inc., is an automobile service company that specializes in tire and brake replacement. It has four service centers in a metropolitan area. It also has a warehouse that supplies tires, brakes, and other components to the service centers. The company manager has determined that he needs to add two more warehouses to improve component delivery service.

At the same time he wants the location of the two new warehouses to minimize the cost of delivering components from the new warehouses to the existing facilities (four service centers and the existing warehouse) as well as between the new warehouses. The four service centers and warehouse have these coordinate locations: (8, 20), (8, 10), (10, 20), (16,30), and (35, 20).

It is anticipated that there will be one trip per day between the new warehouses. The numbers of trips between the new warehouses (W_1, W_2) and the four service centers ($SC_1 - SC_4$) as well as the existing warehouse (SC_5) are provided in the matrix.

		SC_1	SC_2	SC_3	SC_4	SC_5
W_1	$\begin{bmatrix}$	7	7	5	4	2
W_2		3	2	4	5	2

Develop a model similar to the transformed (1) to minimize the distribution cost and solve it using LINDO.

2.6.3. Multiple-Facility Problems with Euclidean Distances

Consider the following objective function for the Euclidean distance problem (Recall that the notation was introduced earlier for the rectilinear distance problem).

Minimize

$$\sum_{i=1}^n \sum_{j=1}^n c_{ij} f_{ij} \left[\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \right] + \sum_{i=1}^n \sum_{j=1}^m d_{ij} g_{ij} \left[\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \right] \quad (3)$$

As in the single-facility model, we can take the partial derivative of expression (3) with respect to the variables x_i and y_i , set the equations to zero, and solve for the variables because (3) can be shown to be a convex function. Taking the partial derivatives, we get:

$$\sum_{j=1}^n \frac{c_{ij} f_{ij} (x_i - x_j)}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} + \sum_{j=1}^m \frac{d_{ij} g_{ij} (x_i - a_j)}{\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}} = 0 \quad i=1, 2, \dots, n \quad (4)$$

$$\sum_{j=1}^n \frac{c_{ij} f_{ij} (y_i - y_j)}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} + \sum_{j=1}^m \frac{d_{ij} g_{ij} (y_i - b_j)}{\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}} = 0 \quad i=1, 2, \dots, n \quad (5)$$

Because we have $2n$ variables and an equal number of constraints, we can solve Equations (4) and (5) to get the optimal (x, y) coordinates for all the n new facilities.

As noted in the single facility Euclidean distance model, however, we must be able to guarantee that the optimal location of any new facility does not coincide with that of any existing facility. Because the latter is not possible, we can develop an iterative heuristic procedure similar to what was done in the single-facility case.

We add a small quantity to the denominator in each term on the lefthand side of Equations (4) and (5). Because Equations (4) and (5) are now defined even when the optimal location of a new facility coincides with that of an existing one, we can begin with an initial value for x_i, y_i for each new facility i and substitute these values into the following Equations (6) and (7) to get the new values of x_i, y_i , (denoted as x'_i, y'_i respectively).

Notice that Equations (6) and (7) have been obtained by adding to the denominator of each term on the left-hand sides of Equations (4) and (5) and rewriting the equations:

$$x'_i = \frac{\sum_{j=1}^n \frac{c_{ij} f_{ij} x_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + \varepsilon}} + \sum_{j=1}^m \frac{d_{ij} g_{ij} a_j}{\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2 + \varepsilon}}}{\sum_{j=1}^n \frac{c_{ij} f_{ij}}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + \varepsilon}} + \sum_{j=1}^m \frac{d_{ij} g_{ij}}{\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2 + \varepsilon}}} \quad i=1, 2, \dots, n \quad (6)$$

$$y'_i = \frac{\sum_{j=1}^n \frac{c_{ij} f_{ij} y_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + \varepsilon}} + \sum_{j=1}^m \frac{d_{ij} g_{ij} b_j}{\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2 + \varepsilon}}}{\sum_{j=1}^n \frac{c_{ij} f_{ij}}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + \varepsilon}} + \sum_{j=1}^m \frac{d_{ij} g_{ij}}{\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2 + \varepsilon}}} \quad i=1, 2, \dots, n \quad (7)$$

The new values of x_i, y_i are substituted into the right-hand sides of Equations of (6) and (7) to get the next set of values. This procedure is continued until two successive x_i, y_i values or the objective function values [obtained by substituting x_i, y_i values in expression (3)] are nearly equal.

Although it cannot be proved, we assume convergence has occurred at this point and stop. Upper and lower bounds on the optimal objective function value for the Euclidean distance problem can be found by looking at the rectilinear distance solution [see Francis and White (1974) and Pritsker and Ghare (1970) for more details]. Based on these bounds, we can tell how far off a given Euclidean solution is for a particular problem.

For many practical problems, it has been found that the x_i, y_i values for the new facilities determined via the iterative procedure are very close to optimal. The iterative procedure is rather easy to set up in a spreadsheet. Note that large values of ε will ensure a faster convergence, but the quality of the final solution is inferior compared with that obtained with a smaller ε value. Thus the user has to trade off quick convergence and solution quality and choose an appropriate value.

Example:

Consider the above assignment. Assume the Euclidean distance metric is more appropriate and that Tire and Brakes, Inc., does not currently have a warehouse. Determine where the two new warehouses are to be located.

Solution:

Because there is no existing warehouse, we disregard that information in assignment. A spreadsheet set up to iteratively calculate the x_i and y_i values is shown in table below. Also shown in the spreadsheet are the flow and values as well as the coordinate locations of the existing service centers. The columns labeled C₁ through C₄ give the values of the following part of Equation (6) calculated for each service center's coordinate location (a_j, b_j):

$$\frac{d_{ij} g_{ij}}{\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2 + \varepsilon}}$$

Because this factor does not change for Equation (7), we do not show the values again in the y_i rows. The column labeled C₅ in the following table shows the values for the following part of Equations (6) and (7):

$$\frac{c_{ij} f_{ij}}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} + \varepsilon}$$

Once again, because it is the same in both expressions, it is not shown in the y_i rows. Notice that in each iteration this value is the same for each x_i row because we have only two new warehouses to be located. The column labeled C_6 gives the sums of the values in columns C_1 through C_5 and is the denominator of Equations (6) and (7).

Using an initial seed of (8,10) and (9,10) for the two facilities, we begin the iterative procedure. To determine the coordinates of the two new warehouses for the k th iteration, we use the ε , flow, (a_j, b_j) values, values in columns C_1 through C_6 for the previous $(k - 1)$ th iteration, and Equations (6) and (7).

This procedure is repeated until two successive x_i, y_i values are equal. This occurs in the 13th iteration, and we therefore stop the procedure (If we had used the total cost, shown in the last column as TC , to determine whether convergence had occurred, we would have stopped at the 12th iteration because solutions in this and the 11th yield the same total cost of 218). If we had used large values of ε , convergence would have occurred much earlier, but then we may have obtained a solution inferior to the current one.

			SC ₁	SC ₂	SC ₃	SC ₄			
	W ₁		7	7	5	4			Flow
	W ₂		3	2	4	5			values
			A ₁	a ₂	a ₃	a ₄			
	x coordinate		8	8	10	16			
			B ₁	b ₂	b ₃	b ₄			
	y coordinate		20	10	20	30			
	ε		0.02						
Iteration	Coordinates		C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	TC
1	x ₁	8	0.6999	49.5	0.4902	0.183	0.99	52	387
	y ₁	10							
	x ₂	9	0.2985	1.98	0.398	0.236	0.99	3.9	
	y ₂	10							
...									
11	x ₁	8.41	7.70711	0.734	3.8643	0.197	0.78	12.7	
	y ₁	19.497							218
	x ₂	9.967	1.5198	0.194	23.146	0.431	0.78	26.1	
	y ₂	20.094							
12	x ₁	8.794	7.5662	0.731	3.8568	0.197	0.77	13.1	
	y ₁	19.546							218
	x ₂	9.967	1.5198	0.194	23.149	0.431	0.77	26.1	
	y ₂	20.094							

$$\text{Minimize} \quad \sum_{i=1}^p \sum_{j=1}^q c_{ij} x_{ij} + \sum_{j=1}^q \sum_{k=1}^r d_{jk} y_{jk} \quad (1)$$

$$\text{Subject to} \quad \sum_{j=1}^q x_{ij} \leq S_i \quad i=1, 2, \dots, p \quad (2)$$

$$\sum_{i=1}^p x_{ij} \leq P_j \quad i=1, 2, \dots, q \quad (3)$$

$$\sum_{i=1}^q y_{jk} \geq D_k \quad k=1, 2, \dots, r \quad (4)$$

$$\sum_{i=1}^p x_{ij} = \sum_{k=1}^r y_{jk} \quad j=1, 2, \dots, q \quad (5)$$

$$x_{ij}, y_{jk} \geq 0 \quad i=1, 2, \dots, p, j=1, 2, \dots, q, \\ k=1, 2, \dots, r \quad (6)$$

The objective function (1) minimizes the cost of inbound as well as outbound shipments. Constraint (2) ensures that the raw material shipped out from each supply source does not exceed its capacity limits. Constraint (3) ensures that the raw material shipment received from all the supply sources at each plant does not exceed its capacity limits. Constraint (4) requires that the total amount of finished products shipped from the plants to each customer be sufficient to cover the demand. Constraint (5) is a material balance equation ensuring that all the raw material that comes into each plant is shipped out as finished product to customers.

Notice that we are implicitly assuming that a unit of finished product requires one unit of raw material. If this is not the case, we can adjust the model easily, as discussed in Das and Heragu (1988).

For the above model to be transformed into an equivalent transportation model, either the plants or the raw material supply sources (but not both) must have limited capacity. (Otherwise, the problem cannot be set up as a transportation model and hence we cannot use the well-known transportation algorithm.

The problem may be formulated in the above model, however, and solved via the simplex algorithm.) Depending on whether supply sources or plants have limited capacities and whether supply exceeds demand, these four cases arise:

1. Supply source capacity is unlimited, plant capacity is limited, and total plant capacity is greater than total demand.
2. Supply source capacity is unlimited, plant capacity is limited, and total demand exceeds total plant capacity.
3. Plant capacity is unlimited, supply source capacity is limited, and total supply source capacity exceeds total demand.
4. Plant capacity is unlimited, supply source capacity is limited, and total demand exceeds total supply source capacity.

Example:

Two-stage distribution problem: RIFIN Company has recently developed a new method of manufacturing a type of chemical. The method involves refining a certain raw material that can be obtained from four overseas suppliers, A, B, C, and D, who have access to the four ports at Vancouver, Boston, Miami, and San Francisco, respectively. RIFIN wants to determine the location for plants that will refine the material. Once refined, the chemical will be transported via trucks to five outlets located in Dallas, Phoenix, Portland, Montreal, and Orlando.

After an initial study, the choice of location for RIFIN's refineries has been narrowed down to Denver, Atlanta, and Pittsburgh. Assume that one unit of the raw material is required to make one unit of the chemical. The amount of raw material that can be obtained from suppliers A, B, C, and D and the amount of chemical required at the five outlets are given in the following table (a). The cost of transporting the raw material from each port to each potential refinery and the cost of trucking the chemical to outlets are provided in tables (b) and (c), respectively. Determine the locations of RIFIN's refining plants, the capacities at these plants, and the distribution pattern for the raw material and processed chemical.

(a) Supply and demand for four sources and five outlets

Raw Material			
Source	Supply	Outlet	Demand
A	1000	Dallas	900
B	800	Phoenix	800
C	800	Portland	600
D	700	Montreal	500
		Orlando	500

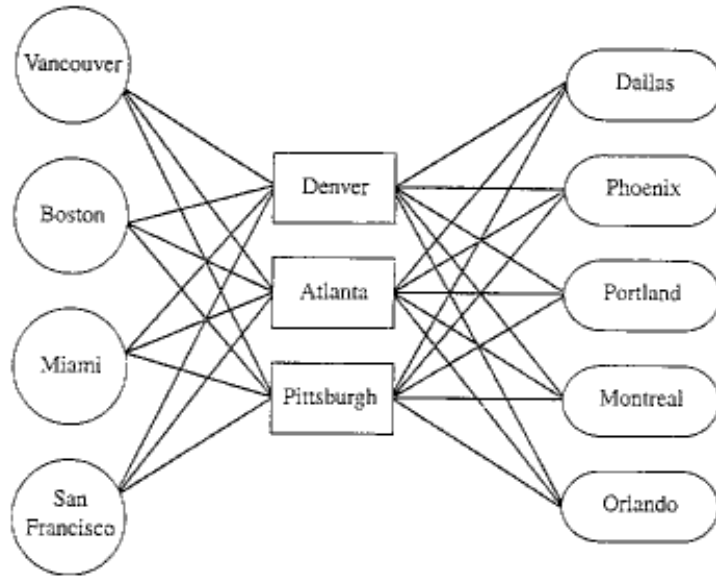
(b) Inland raw material transportation cost

	To	Denver	Atlanta	Pittsburgh
From				
Vancouver		4	13	9
Boston		8	8	5
Miami		12	2	9
San Francisco		11	11	12

(c) Chemical trucking cost

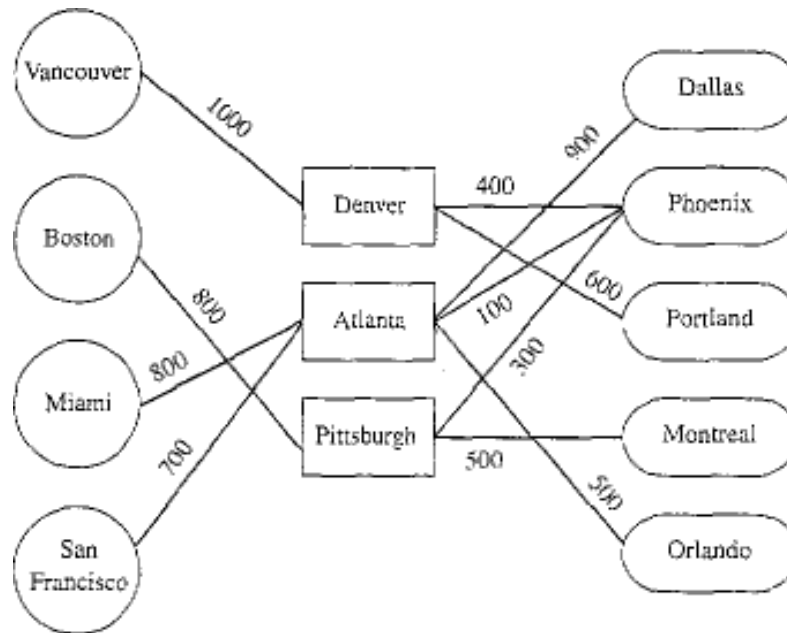
	To	Dallas	Phoenix	Portland	Montreal	Orlando
From						
Denver		28	26	12	30	30
Atlanta		10	22	23	29	8
Pittsburgh		18	21	23	18	21

Solution:



Above figure is a pictorial representation of the RIFIN problem. We can reasonably assume that there is no practical limit on the capacity of the refineries at any of the three locations, Atlanta, Denver, and Pittsburgh, because the refineries have not been built yet. This assumption allows us to use the two-stage transportation method.

The transportation problem may be solved to yield the solution (with a total cost of \$65.400) in the following figure, which indicates that refineries should be built at all three locations.



2.8. Location-Allocation Models

Generalized assignment problem, can be used to formulate location-allocation problems in which the objective is to determine the location of facilities to minimize the cost of assigning facilities to customers subject to the constraint that each facility be assigned to a prespecified number of customers. Similarly, the quadratic assignment model discussed in the context of a layout problem can be used at a macro level to determine the location of facilities given that these facilities have flow (interaction) among themselves.

In this section we consider three other location-allocation models, each with specific applications:

1. Set covering model
2. Uncapacitated location-allocation model
3. Comprehensive location-allocation model

The models are discussed in order of the difficulty in solving them. For all the models, we present good heuristic or optimal solution procedures. The models determine the number of facilities to be located, where they are to be located, and the interaction between the facilities and customers. The first two are rather simple. The first considers only the cost of covering each customer with a facility. The second model considers a single product, one stage of distribution, facilities with unlimited capacity, and a customer to be served from several facilities. The third model relaxes several of these assumptions and therefore better represents the real-world location allocation problem. To facilitate understanding of the third model, to provide a sound introduction, and to illustrate the use of efficient branch and bound algorithms, we begin our discussion of location-allocation problems with the first two simple models.

2.8.1. Set Covering Model

The set covering problem arises when it is necessary to ensure that each customer is covered by at least one service facility. For example, fire stations and other emergency facilities, libraries, community colleges, and state university campuses have to be located so that each population area or "customer" is within a certain range of distance from at least one facility. If a customer is within the desired range, we say the customer is covered. These are the parameters of the model:

c_j cost of locating facility at site j

$$a_{ij} \begin{cases} 1 & \text{if facility located at site } j \text{ can cover customer } i \\ 0 & \text{otherwise} \end{cases}$$

$$x_j \begin{cases} 1 & \text{if facility is located at site } j \\ 0 & \text{otherwise} \end{cases}$$

The set covering problem is given here:

$$\text{Minimize } \sum_{j=1}^n c_j x_j \quad (1)$$

$$\text{Subject to } \sum_{j=1}^n a_{ij} x_j \geq 1 \quad i=1, 2, \dots, m \quad (2)$$

$$x_j = 0 \text{ or } 1 \quad j=1, 2, \dots, n \quad (3)$$

In this 0-1 integer programming model, there are m customers and n facilities. Constraint (2) ensures that each customer is covered by at least one facility. The objective function (1) minimizes the cost of locating the required number of facilities.

The model may be solved optimally using a general-purpose branch and bound technique, but that may be too time consuming for large problems. Hence the following greedy algorithm is used to obtain suboptimal solutions efficiently. It assumes that $c_j \geq 0, j = 1, 2, \dots, n$.

Example:

A rural county administration wants to locate several medical emergency response units so that they can respond to any call in the county within 8 minutes. The county is divided into seven population zones. The distances between the centers of the zones are known and are given in the matrix in the following figure. The response units can be located in the centers of population zones 1-7 at a cost (in \$10,000s) of 100, 80, 120, 110, 90, 90 and 110, respectively.

Assuming the average travel speed during an emergency is 60 miles per hour; formulate an appropriate set covering model to determine where the units are to be located and how the population zones are to be covered. Solve the model using the greedy heuristic and calculate the solution cost.

Distance between seven zones:

$$[d_{ij}] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 4 & 12 & 6 & 15 & 10 & 8 \\ 8 & 0 & 15 & 60 & 7 & 2 & 3 \\ 50 & 13 & 0 & 8 & 6 & 5 & 9 \\ 9 & 11 & 8 & 0 & 9 & 10 & 3 \\ 50 & 8 & 4 & 10 & 0 & 2 & 50 \\ 30 & 5 & 7 & 9 & 3 & 0 & 27 \\ 8 & 5 & 9 & 7 & 25 & 27 & 0 \end{bmatrix} \end{matrix}$$

Solution:

We define;

$$a_{ij} = \begin{cases} 1 & \text{if zone } i\text{'s center can be reached from the center of zone } j \text{ within 8 minutes} \\ 0 & \text{otherwise} \end{cases}$$

and note that $d_{ij} > 8$, $d_{ij} \leq 8$ yield a_{ij} values of 0, 1 respectively. We can then set up the $[a_{ij}]$ matrix in figure below.

Revised binary distance matrix:

$$[a_{ij}] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

The corresponding set covering model is:

Minimize $100x_1 + 80x_2 + 120x_3 + 110x_4 + 90x_5 + 90x_6 + 110x_7$

Subject to

$$\begin{matrix} x_1 + & x_2 + & & & x_4 + & & & x_7 \geq 1 \\ x_1 + & x_2 + & & & & x_5 + & x_6 + & x_7 \geq 1 \\ & & x_3 + & x_4 + & x_5 + & x_6 & & \geq 1 \\ & & x_3 + & x_4 + & x_5 + & & & x_7 \geq 1 \\ & & & x_2 + & x_3 + & & x_5 + & x_6 + & \geq 1 \\ & & & x_2 + & x_3 + & & x_5 + & x_6 + & \geq 1 \\ x_1 + & x_2 + & & & x_4 + & & & & x_7 \geq 1 \\ x_1, & x_2, & x_3, & x_4, & x_5, & x_6, & & & x_7 = 0 \text{ or } 1 \end{matrix}$$

2.8.2. Uncapacitated Location-Allocation Model

Consider this notation:

- m Number of potential facilities
- n Number of customers
- c_{ij} Cost of transporting one unit of product from facility i to customer j
- F_i Fixed cost of opening and operating facility i
- D_j Number of units demanded at customer j
- x_{ij} Number of units shipped from facility i to customer j
- y_i $\begin{cases} 1 & \text{if facility is opened} \\ 0 & \text{otherwise} \end{cases}$

The basic location-allocation model is given here:

$$\text{Minimize} \quad \sum_{i=1}^m F_i y_i + \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (4)$$

$$\text{Subject to} \quad \sum_{j=1}^n x_{ij} = D_j \quad j=1, 2, \dots, n \quad (5)$$

$$\sum_{j=1}^n x_{ij} \leq y_i \sum_{j=1}^n D_j \quad i=1, 2, \dots, m \quad (6)$$

$$x_{ij} \geq 0 \quad i=1, 2, \dots, m, j=1, 2, \dots, n \quad (7)$$

$$y_i = 0 \text{ or } 1 \quad i=1, 2, \dots, m \quad (8)$$

The objective function (4) minimizes the variable transportation cost as well as the fixed cost of opening and operating the facilities needed to support the distribution activities. Constraint (5) ensures that each of the n customers demand is met fully by one or more of the m facilities. The objective function (4) and constraints (6) and (8) ensure that if a facility i ships goods to one or more customers, a corresponding fixed cost is incurred, and that the total number of units shipped does not exceed the total demand at all the customers. On the other hand, if a facility does not ship goods to any customer, then no fixed cost is incurred. Constraint (7) is a nonnegativity constraint.

This model may be solved using the general-purpose branch and bound technique found in most introductory operations research textbooks (e.g., Winston 1994; Hillier and Lieberman 1995). This entails setting up a root node, solving this subproblem using the simplex algorithm, selecting a y variable say, y_i with a fractional value, branching on this variable, setting up two subproblems (nodes), one with a subproblem at the root node plus the constraint $y_i = 0$ and another with $y_i = 1$, solving the two subproblems (again using simplex), and deciding whether or not to prune a node based on these two tests:

1. The bound at the node is greater than or equal to the objective function value (OFV) of the best known feasible solution. (If no feasible solution has been identified yet, we proceed to test 2.)
2. The solution to the subproblem at the node is an all-integer (binary) solution. If a node passes either of the two tests, it is pruned and we update the best known OFV if necessary. Otherwise, we determine (arbitrarily or using specialized branching rules) the fractional y_i variable on which to branch, set up two additional subproblems (nodes), solve, and make pruning decisions as before. This procedure is repeated until all the nodes are pruned. At this point we have the optimal solution to the problem.

The central idea of the branch and bound algorithm is based on the following result: Suppose, at some stage of the branch and bound solution process, we are at a node where some facilities are closed (corresponding $y_i = 0$), some are open ($y_i = 1$), and the remaining are free; that is, a decision whether to open or close has not yet been made ($0 < y_i < 1$). We then define these parameters:

- S_0 the set of facilities whose y_i value is equal to zero; $\{i: y_i = 0\}$
- S_1 the set of facilities whose y_i value is equal to one; $\{i: y_i = 1\}$
- S_2 the set of facilities whose y_i value is greater than zero but less than one $\{i: 0 < y_i < 1\}$

Revising the basic location-allocation model, we find:

$$\begin{aligned} \text{Min} \quad & \sum_{i \in S_1} F_i + \left\{ \sum_{i \in S_1} \sum_{j=1}^n c_{ij} x_{ij} + \left[\sum_{i \in S_2} F_i \sum_{j=1}^n \frac{x_{ij}}{n} + \sum_{i \in S_2} \sum_{j=1}^n c_{ij} x_{ij} \right] \right\} \\ & = \sum_{i \in S_1} F_i + \text{Min} \left\{ \sum_{i \in S_1} \sum_{j=1}^n c_{ij} x_{ij} + \sum_{i \in S_2} \sum_{j=1}^n \left(c_{ij} + \frac{F_i}{n} \right) [x_{ij}] \right\} \end{aligned} \quad (9)$$

In order to solve the above model, we only need to find for a specific y , the smallest coefficient of x_{ij} in Equation (9), $i = 1, 2, \dots, m$, and set the corresponding x_{ij} equal to one and all other x_{ij} s to zero. This is to be done for each j as shown next. We list the coefficients for each j as follows:

$$\begin{aligned} & c_{ij} \text{ if } i \in S_1 \\ & c_{ij} + \frac{F_i}{n} \text{ if } i \in S_2 \end{aligned} \quad (10)$$

Select the smallest coefficient, and set the corresponding x_{ij} to one and all other x_{ij} 's to zero. This method of determining the x_{ij} 's is called as the minimum coefficient rule. Notice that (10) does not include facility $i \in S_0$ because these are closed. Since the x_{ij} 's are known, the y_i values for $i \in S_2$ can be determined by:

$$y_i = \frac{1}{n} \sum_{j=1}^n x_{ij} \quad (11)$$

Moreover, a lower bound on the partial solution of the node under consideration can be obtained via Equation (9) or simply by adding $\sum_{i \in S_0} F_i$ to the sum of the coefficients

of the x_{ij} variables that have taken on a value of one (since all the other x_{ij} 's are equal to zero per the minimum coefficient rule). If it turns out that all the y_i values ($i \in S_2$) obtained from Equation (11) are binary, then we have a feasible solution and the lower bound obtained for the node from Equation (10) is also an upper bound for the original location-allocation problem. The node can therefore be pruned. If, on the other hand, one or more y_i variables take on fractional values, then we need to branch on *one* of these variables, first by setting it equal to zero (and then to one), creating two corresponding nodes, updating S_0 or S_1 as appropriate, and lower bound via the minimum coefficient rule discussed earlier, Equations (9) and (10). If the solution at a node has a lower bound greater than or equal to the best upper bound determined so far for the overall location-allocation problem, then it can be pruned because branching further on this node can only lead to worse solutions. We repeat the procedure of branching on nodes, solving the problem at each newly created node, determining the lower bound, and making pruning decisions until all the nodes are pruned. At that time, we have an optimal solution to the location-allocation model given by the node that has a *feasible* solution with the least cost among all the nodes.

2.8.3. Comprehensive Location-Allocation Model

In all the models we have studied so far in this chapter and the preceding one, we did not explicitly consider multiple commodities. Now we present a comprehensive model that considers real-world factors and constraints. Consider this problem: Different types of products are produced at several plants that have known production capacities. The demand for each product type at each of several customer areas is also known. The products are shipped from plants to customer areas via intermediate warehouses with the restriction that each customer area be serviced by only one warehouse. This is done to improve customer service. Upper and lower bounds on the capacity of each warehouse, potential locations for the warehouses, inbound and outbound transportation costs at each of the warehouses (i.e., from each plant and to each customer area), and the fixed cost of opening and operating a warehouse at each potential location are known.

The problem is to find the locations for the warehouses, the corresponding capacities, the customers served by each warehouse, and how products are to be shipped from each plant to minimize the fixed and variable costs of opening and operating warehouses as well as the distribution costs. We use this notation:

S_{ij}	Production capacity of product i at plant j
D_{il}	Demand for product i at customer zone l
F_k	Fixed cost of operating warehouse k
V_{ik}	unit variable cost of handling product i at warehouse k
c_{ijkl}	Average unit cost of producing and transporting product i from plant j via warehouse k to customer area l
UC_k	Upper bound on capacity of warehouse k
LC_k	Lower bound on capacity of warehouse k
x_{ijkl}	Number of units of product i transported from plant j via warehouse k to customer area l
y_{kl}	1 if warehouse k serves customer area l 0 otherwise
z_k	1 if warehouse is opened at location k 0 otherwise

Here is the model for location-allocation:

$$\text{Minimize} \quad \sum_{i=1}^p \sum_{j=1}^q \sum_{k=1}^r \sum_{l=1}^s c_{ijkl} x_{ijkl} + \sum_{i=1}^p \sum_{l=1}^s D_{il} \sum_{k=1}^r V_{ik} y_{kl} + \sum_{k=1}^r F_k z_k \quad (12)$$

$$\text{Subject to} \quad \sum_{k=1}^r \sum_{l=1}^s x_{ijkl} \leq S_{ij} \quad i=1, 2, \dots, p, j=1, 2, \dots, q \quad (13)$$

$$\sum_{j=1}^q x_{ijkl} \geq D_{il} y_{kl} \quad i=1, 2, \dots, p, k=1, 2, \dots, r$$

$$l=1, 2, \dots, s \quad (14)$$

$$\sum_{k=1}^r y_{kl} = 1 \quad l=1, 2, \dots, s \quad (15)$$

$$\sum_{i=1}^p \sum_{l=1}^s D_{il} y_{kl} \geq LC_k z_k \quad k=1, 2, \dots, r \quad (16)$$

$$\sum_{i=1}^p \sum_{l=1}^s D_{il} y_{kl} \leq UC_k z_k \quad k=1, 2, \dots, r \quad (17)$$

$$x_{ijkl} \geq 0 \quad i=1, 2, \dots, p, j=1, 2, \dots, q \quad (18)$$

$$y_{kl} z_k = 0 \text{ or } 1 \quad k=1, 2, \dots, r, l=1, 2, \dots, s \quad (19)$$

The objective function (12) of model 8 minimizes the inbound and outbound transportation costs as well as the production costs for each product at each warehouse. It also minimizes the fixed and variable costs of opening and operating the required number of warehouses.

Constraint (13) ensures for each product that the capacity constraints at each plant are not violated. Constraint (14) ensures that the demand for each product at each customer zone is met. Constraints (15) and (19) require that each customer area be serviced by a single warehouse. Constraints (16) and (17) have a dual purpose. Not only do they enforce the upper and lower bounds on the warehouse capacity, but they also “connect” the y_{kl} and z_k variables. Because a warehouse can serve a customer area only if it is open, we must have $y_{kl}=1$ when $z_k=1$ and $y_{kl}=0$ when $z_k=0$ for each warehouse-customer area $\{k, l\}$ pair. These two conditions are satisfied by constraints (16) and (17), respectively.

We can easily add more linear constraints (not involving x_{ijkl} variables) to model 8 to:

- Impose upper and lower limits on the number of warehouses that can be opened;
- Enforce precedence relationships among warehouses (e.g., open warehouse at location 1 only if another is opened at location 3); and
- Enforce service constraints (e.g., if it is decided to open a certain warehouse, then a specific customer area must be served by it).

Other constraints that can be added are discussed further in Geoffrion and Graves (1974). Such constraints reduce the solution space, so they allow quicker solution of the model while giving the modeler much flexibility.

Model above can be solved using available mixed integer programming software, but due to the presence of binary integer variables y_{kl} and z_k , only small problems can be solved. Real world problems such as Hunt-Wesson Foods, Inc., location allocation problem considered in Geoffrion and Graves (1974), which had more than 11,000 constraints, 23,000 x_{ijkl} variables, and 700 y_{kl} and z_k binary variables, cannot be solved via general mixed integer programming algorithms. Such large problems have been rather easily solved using modified Bender’s decomposition algorithm.

CHAPTER 3

STRATEGIC FACILITIES PLANNING

While the concerns of facilities planning are the location and the design of the facility, there exists another primary responsibility – planning!

“The plan is nothing, but planning is everything.” – Dwight Eisenhower

As an indicator of its importance in facilities planning, consider the process of planning and designing a manufacturing facility, building it, and installing and using the equipment. As shown in Figure 3.1, the costs of design changes increase exponentially as a project moves beyond the planning and designing phases.

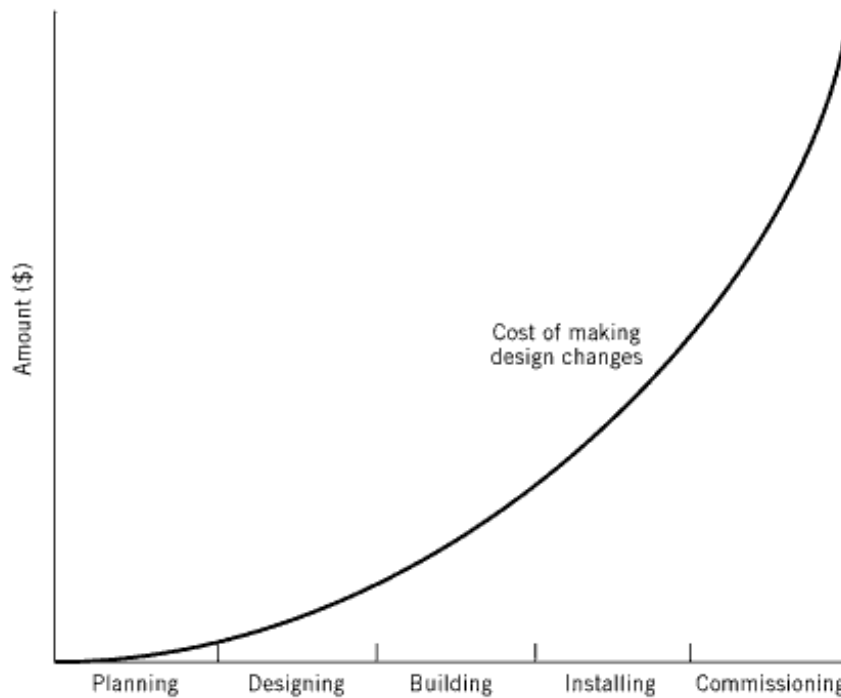


Figure 3.1. Cost of design changes during a process

Strategic Planning is a special type of planning, which is frequently used in politics, sports, investments, and business. Our concern is with the latter usage. Business strategies can be defined as the art and science of employing the resources of a firm to achieve its business objectives.

Among the resources that are available are marketing resources, manufacturing resources, and distribution resources. Hence, marketing strategies, manufacturing strategies, and distribution strategies can be developed to support the achievement of the business objectives.

Recall, facilities planning was defined as determining how a firm's resources (tangible fixed assets) best support achieving the business objectives. In a real sense, facilities planning is itself a strategic process and must be an integral part of overall corporate strategy.

3.1. Developing Facilities Planning Strategies

Strategies are needed for such functions as marketing, manufacturing, distribution, purchasing, facilities, material handling, and data processing/information systems among others. It is important to recognize that each strategy is multidimensional. Namely, each must support or contribute to the strategic plan for the entire organization. Furthermore, each must have its own set of objectives, strategies and tactics.

A number of internal functional areas tend to have a significant impact on facilities planning, including marketing, product development, manufacturing, production and inventory control, human resources, and finance.

Marketing decisions affect the location of facilities and the handling systems design. For example, material handling will be affected by decisions related to unit volume, product mix, packaging, service levels for spares, and delivery times.

Product development and design decisions affect processing and materials requirements, which in turn affect layout and material handling. Changes in materials, component shapes, product complexity, number of new part numbers and package sizes introduced, stability of product design, and the number of products introduced will affect the handling, storage, and control of materials.

Manufacturing decisions will have an impact on both facilities location and facilities design. Decisions concerning the degree of vertical integration, types and levels of automation, types and levels of control over tooling and work-in-process, plant sizes, and general-purpose versus special-purpose equipment can affect the location and design of manufacturing and support facilities.

Production planning and inventory control decisions affect the layout and handling system. Lot size decisions, production scheduling, in-process inventory requirements, inventory turnover goals, and approaches used to deal with seasonal demand affect facilities plan.

Human resources and finance decisions related to capital availability, labor skills and stability, staffing levels, inventory investment levels, organization design, and employee services and benefits will impact the size and design of facilities, as well as their number and location. Space and flow will be affected by financial and human resources decisions; in turn, they have an impact on the storage, movement, protection, and control of material.

3.2. Examples of Inadequate Planning

- A major manufacturer made a significant investment in storage equipment for a parts distribution center. The selection decision was based on the need for a “quick fix” to a pressing requirement for increased space utilization. The company soon learned that the “solution” would not provide the required throughput and was not compatible with long-term needs.
- An electronics manufacturer was faced with rapid growth. Management received proposals that required approximately equivalent funding for large warehouses at two sites having essentially the same storage and throughput requirements. Management questioned the rationale for one “solution” being a high-rise AS/RS and the other being a low-rise warehouse with computer-controlled industrial trucks.
- An electronic manufacturer was planning to develop a new site. The facilities planner and architects were designing the first building for the site. No projections of space and throughput had been developed since decisions had not been made concerning the occupant of the building.
- A manufacturer of automotive equipment acquired the land for a new manufacturing plant. The manufacturing team designed the layout and the architect began designing the facility before the movement, projection, storage, and control system was designed.

CHAPTER 4

PRODUCT, PROCESS, AND SCHEDULE DESIGN

Recall the facilities planning process for manufacturing can be listed as:

1. Define the products to be manufactured.
2. Specify the manufacturing processes and related activities required to produce the products.
3. Determine the interrelationships among all activities.
4. Determine the space requirements for all activities.
5. Generate alternative facilities plans.
6. Evaluate the alternative facilities plans.
7. Select the preferred facilities plan.
8. Implement the facilities plan.
9. Maintain and adapt the facilities plan.
10. Update the products to be manufactured and redefine the objective of the facility.

Among the questions to be answered before alternative facility plans can be generated are the following:

1. What is to be produced?
2. How are the products to be produced?
3. When are the products to be produced?
4. How much of each product will be produced?
5. For how long will the products be produced?
6. Where are the products to be produced?

The answers to the first five questions are obtained from product design, process design, and schedule design, respectively. The sixth question might be answered by facilities location determination or it might be answered by schedule design when production is to be allocated among several existing facilities.

Product designers specify what the end product is to be in terms of dimensions, material composition, and perhaps, packaging. The process planner determines how the product will be produced. The production planner specifies the production equipment the facilities planner is dependent on timely and accurate input from product, process, and schedule designers to carry out his task effectively.

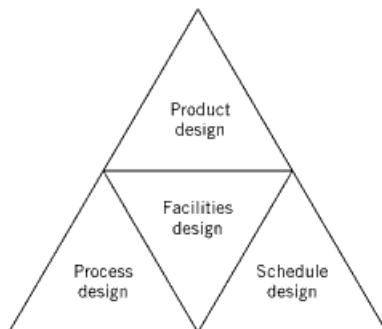


Figure 4.1. Relationship between product, process, and schedule (PP&S) design and facilities planning

4.1. Product Design

Product design involves both the determination of which products are to be produced and the detailed design of individual products. Decisions regarding the products to be produced are generally made by top management based on input from marketing, manufacturing, and finance concerning projected economic performance.

Detailed operational specifications, pictorial representations, and prototypes of the product are important inputs for the facilities planner. Exploded assembly drawings, such as given in figure 4.2, are quite useful in designing the layout and handling system. These drawings generally omit specifications and dimensions, although they are drawn to scale.

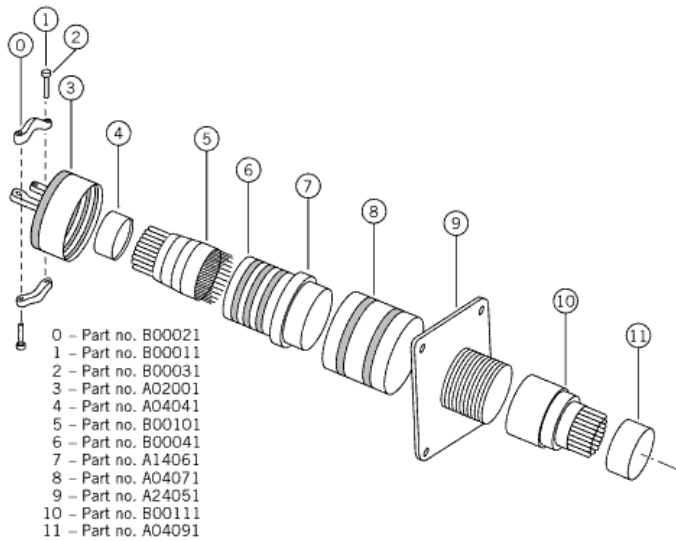


Figure 4.2. Exploded assembly drawing

As an alternative to the exploded assembly drawing, a photograph can be used to show the parts properly oriented. Such a photograph is given in figure 4.3. photographs and drawings allow the planner to visualize how the product is assembled, provide a reference for part numbers, and promote clearer communications during oral presentations.

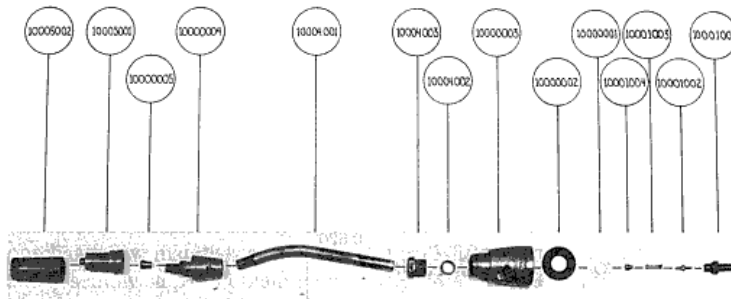


Figure 4.3. Exploded part photograph

Detailed component part drawings are needed for each component part. The drawings should provide part specifications and dimensions in sufficient detail to allow part fabrication. The combination of exploded assembly drawings and component part drawings fully documents the design of the products.

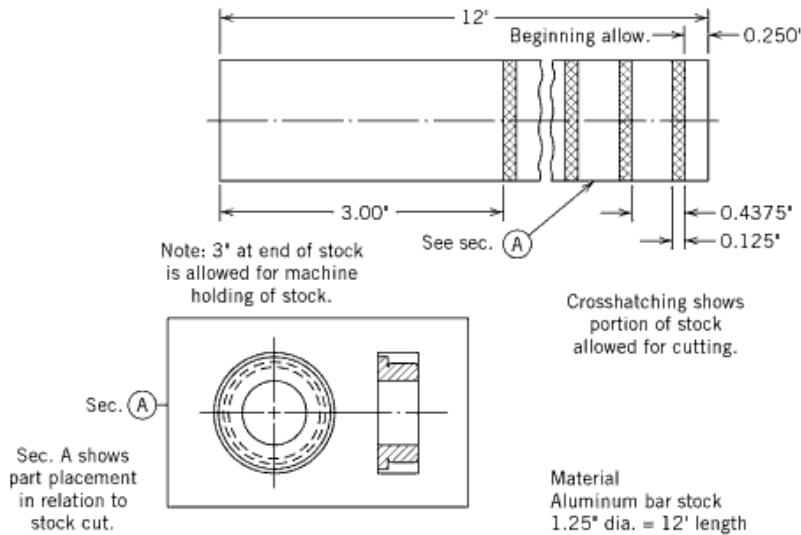


Figure 4.4. Component part drawing of a plunger

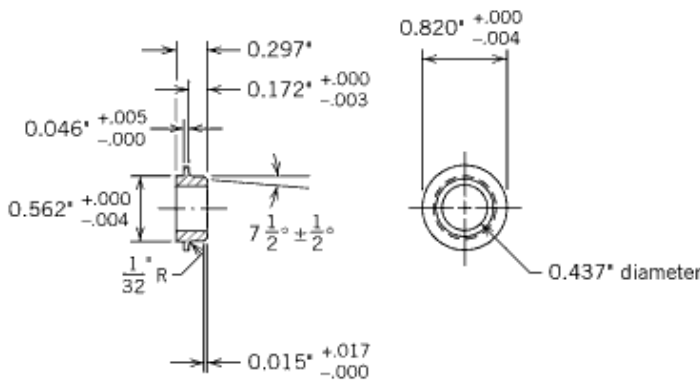


Figure 4.5. Component part drawing of a seat

The drawings can be prepared and analyzed with *computer aided design* (CAD) systems. CAD is the creation and manipulation of design prototypes on a computer to assist the design process of the product.

In addition to CAD, concurrent engineering (CE) can be used to improve the relationship between the function of a component or product and its cost. CE provides a simultaneous consideration in the design phase of life cycle factors such as product, function, design, materials, manufacturing processes, testability, serviceability, quality, and reliability. As a result of this analysis, a less expensive but functionally equivalent product design might be identified. CE is important because it is at the design stage that many of the costs of a product are specified. It has been estimated that more than 70% of a product's manufacturing cost is dictated by design decisions.

4.2. Process Design

The process designer or process planner is responsible for determining how the products to be produced. As a part of that determination, the process planner addresses who should do the processing; namely, should a particular product, subassembly, or part be produced in-house or subcontracted to an outside supplier or contractor? The “make-or-buy” decision is part of the process planning function.

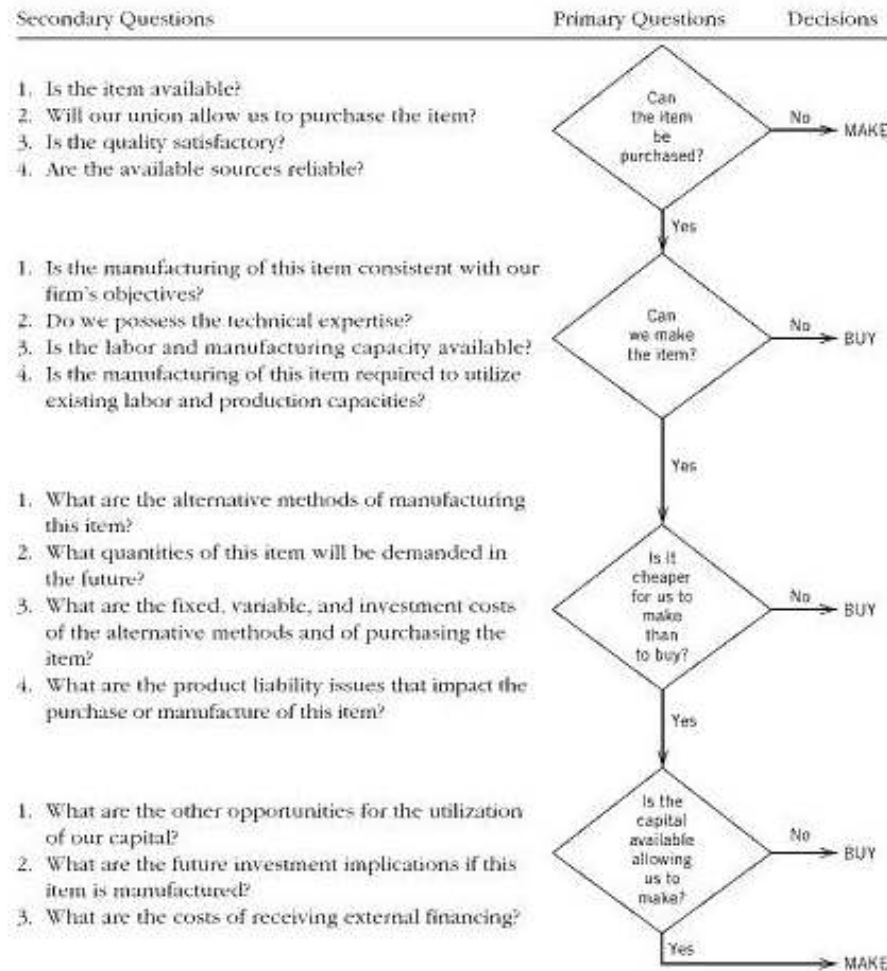


Figure 4.6. The make-or-buy decision process

4.2.1. Identifying Required Processes

Determining the scope of a facility is a basic decision and must be made early in the facilities planning process. For a hospital whose objective is to serve the health needs of a community, it may be necessary to limit the scope of the facility by not including in the facility a burn-care clinic, specific types of x-ray equipment, and/or a psychiatric ward. The excluded services, although needed by the community, may not be feasible for a particular hospital. Patients requiring care provided elsewhere would be referred to other hospitals.

Similarly, the scope of a manufacturing facility must be established by determining the processes that are to be included within the facility. The extremes for a manufacturing facility may range from a vertically integrated firm that purchases raw materials and proceeds through a multitude of refining, processing, and assembly steps to obtain a finished product, to another firm that purchases components and assembles finished products. Therefore, it is obvious that the scope and magnitude of activities within a manufacturing facility are dependent on the decisions concerning the level of vertical integration. Such decisions are often referred to as “make-or-buy” decisions.

Make-or-buy decisions are typically managerial decisions requiring input from finance, industrial engineering, marketing, process engineering, purchasing, and perhaps human resources, among others. The input to the facilities planner is a listing often takes the form of a parts list or a bill of material.

The **part list** provides a listing of the component parts of a product. In addition to make-or-buy decisions, a part list includes at least the following:

1. Part numbers.
2. Part name.
3. Number of parts per product.
4. Drawing references

PARTS LIST							
Company		T.W., Inc.			Prepared by		J.A.
Product		Air Flow Regulator			Date		
Part No.	Part Name	Drwg. No.	Quant./ Unit	Material	Size	Make or Buy	
1050	Pipe plug	4006	1	Steel	.50" × 1.00"	Buy	
2200	Body	1003	1	Aluminum	2.75" × 2.50" × 1.50"	Make	
3250	Seat ring	1005	1	Stainless steel	2.97" × .87"	Make	
3251	O-ring	—	1	Rubber	.75" dia.	Buy	
3252	Plunger	1007	1	Brass	.812" × .715"	Make	
3253	Spring	—	1	Steel	1.40" × .225"	Buy	
3254	Plunger housing	1009	1	Aluminum	1.60" × .225"	Make	
3255	O-ring	—	1	Rubber	.925" dia.	Buy	
4150	Plunger retainer	1011	1	Aluminum	.42" × 1.20"	Make	
4250	Lock nut	4007	1	Aluminum	.21" × 1.00"	Buy	

Figure 4.7. Part list for an air flow regulator

A **bill of materials** is often referred to as a structured parts list, as it contains the same information as a parts list plus information on the structure of the product. Typically, the product structure is a hierarchy referring to the level of product assembly. Level 0 usually indicates the final product; level 1 applies to subassemblies and components that feed directly into the final product; level 2 refers to the subassemblies and components that feed directly into the first level, and so on.

BILL OF MATERIALS

Company T.W., Inc. Prepared by J.A.
 Product Air Flow Regulator Date _____

Level	Part No.	Part Name	Drwg. No.	Quant./ Unit	Make or Buy	Comments
0	0021	Air flow regulator	0999	1	Make	
1	1050	Pipe plug	4006	1	Buy	
1	6023	Main assembly	—	1	Make	
2	4250	Lock nut	4007	1	Buy	
2	6022	Body assembly	—	1	Make	
3	2200	Body	1003	1	Make	
3	6021	Plunger assembly	—	1	Make	
4	3250	Seat ring	1005	1	Make	
4	3251	O-ring	—	1	Buy	
4	3252	Plunger	1007	1	Make	
4	3253	Spring	—	1	Buy	
4	3254	Plunger housing	1009	1	Make	
4	3255	O-ring	—	1	Buy	
4	4150	Plunger retainer	1011	1	Make	

Figure 4.8. Bill of materials for an air flow regulator

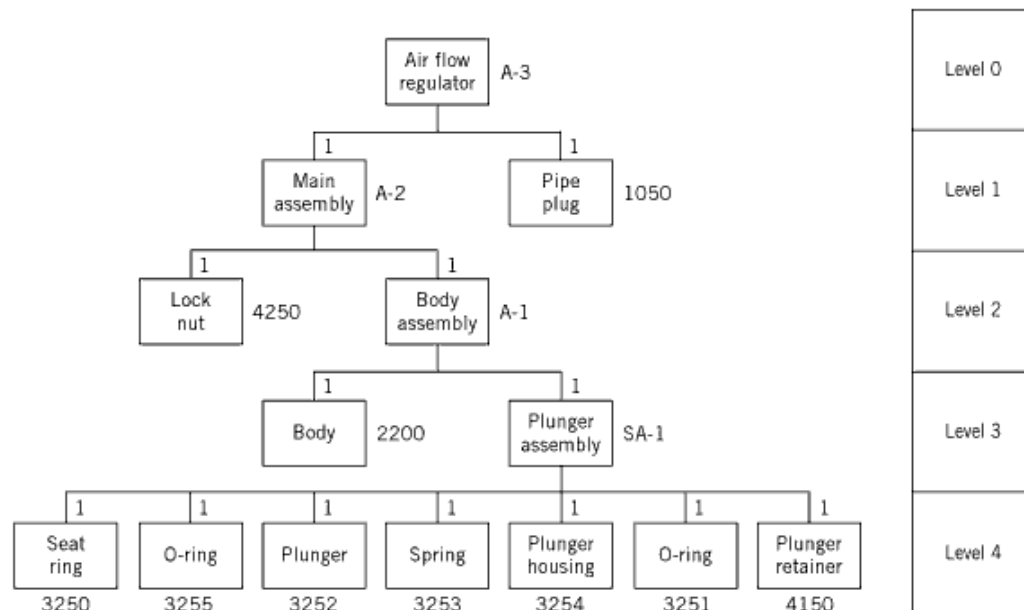


Figure 4.9. Bill of materials for an air flow regulator

4.2.2. Selecting Required Processes

Once a determination has been made concerning the products to be made “in-house”, decisions are needed as to how the products will be made. Such decisions are based on previous experiences, related requirements, available equipment, production rates, and future expectations. Therefore, it is not uncommon for different processes to be selected in different facilities to perform identical operations. However, the selection procedure used should be the same.

Process Identification	
Define elemental operations	Step 1
Identify alternative process for each operation	Step 2
Analyze alternative processes	Step 3
Standardize processes	Step 4
Evaluate alternative processes	Step 5
Select processes	Step 6

Figure 4.10. Process selection procedure

The outputs from the process selection procedure are the processes, equipment, and raw materials required for the in-house production of products. Output is generally given in the form of a route sheet. A **route sheet** should contain at least the data given in table 4.1. Figure 4.11 is a route sheet for the production given in part in table 1.

Data	Production Example
Component name and number	Plunger housing—3254
Operation description and number	Shape, drill, and cut off—0104
Equipment requirements	Automatic screw machine and appropriate tooling
Unit times	Setup time—5 hr operation time—.0057 hr per component
Raw material requirements	1 in. dia. × 12 ft. aluminum bar per 80 components

Table 4.1. Route sheet data requirements

4.2.3. Sequencing Required Processes

The only process selection information not yet documented is the method of assembling the product. An **assembly chart** (figure 12) provides such documentation. The easiest method of constructing an assembly chart is to begin with the completed product and to trace the product disassembly back to its basic components.

ROUTE SHEET

Company A.R.C., Inc. Part Name Plunger Housing Prepared by J.A.
 Produce Air Flow Regulator Part No. 3254 Date _____

Oper No.	Operation Description	Machine Type	Tooling	Dept.	Set-up Time (hr.)	Operation Time (hr.)	Materials or Parts Description
0104	Shape, drill, cut off	Automatic screw machine	.50 in. dia. collar, feed fingers, cir. form tool, .45 in. dia. center drill, .129 in. twist drill, finish spiral drill, cut off blade		5	.0057	Aluminum 1.0 in. dia. × 12 ft.
0204	Machine slot and thread	Chucker	.045 in. slot saw, turret slot attach. 3/8-32 thread chaser		2.25	.0067	
0304	Drill 8 holes	Auto. dr. unit (chucker)	.078 in. dia. twist drill		1.25	.0038	
0404	Deburr and blow out	Drill press	Deburring tool with pilot		.5	.0031	
SA1	Enclose subassembly	Dennison hyd. press	None		.25	.0100	

Figure 4.11. Route sheet for one component of the air flow regulator

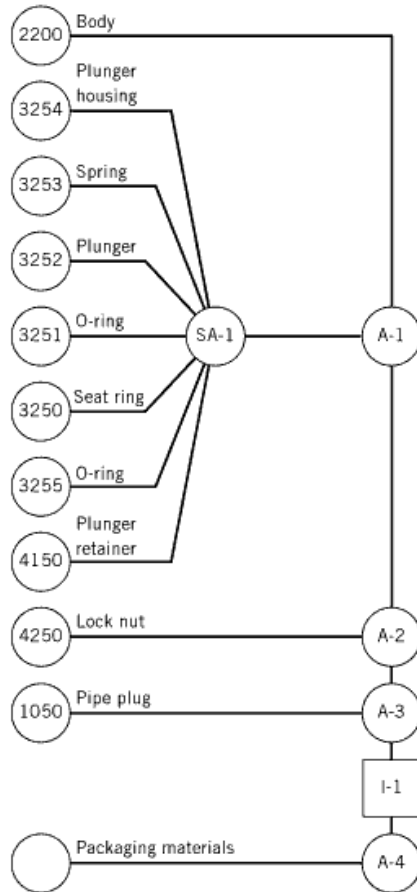


Figure 4.12. Assembly chart for an air flow regulator

For example, the assembly chart given in figure 4.12 would be constructed by beginning in the lower right-hand corner of the chart with a finished air flow regulator. The first disassembly operation would be to unpackage the air flow regulator (operation A-4). The operation that precedes packaging is the inspection of the air flow regulator. Circles denote assembly operation; inspections are indicated on assembly charts as squares. Therefore, in figure 4.12, a square labeled I-1 immediately precedes operation A-4. The first component to be disassembled from the air flow regulator is part number 1050, the pipe plug, indicated by operation A-3. The lock nut is then disassembled, followed by the disassembly of the body assembly (the subassembly made during subassembly operation SA-1) and the body. The only remaining steps required to complete the assembly chart are the labeling of the circles and lines of the seven components following into SA-1.

Although route sheets provide information on production methods and assembly charts indicate how components are combined, neither provides an overall understanding of the flow within the facility. However, by superimposing the route sheets on the assembly chart, a chart results that does give an overview of the flow within the facility. This chart is an **operation process chart**.

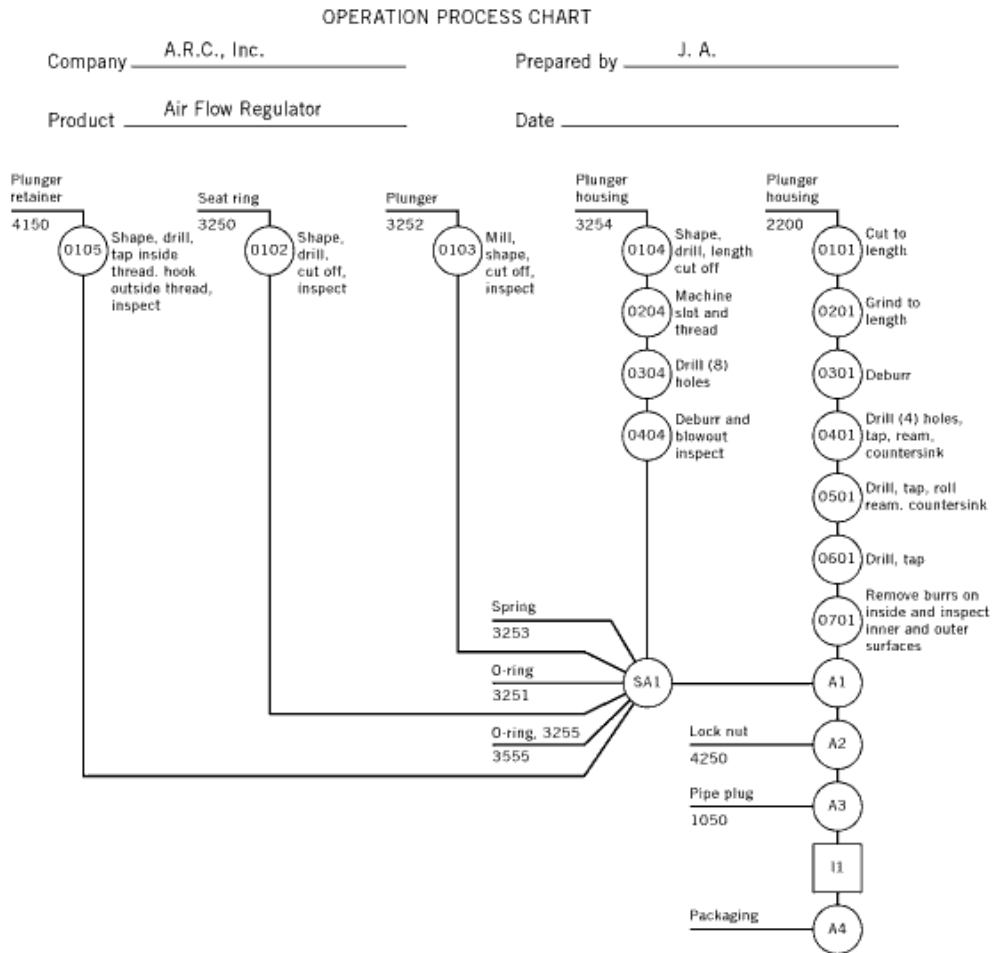


Figure 4.13. Operation process chart for the air flow regulator

To construct an operation process chart, begin at the upper right side of the chart with the components included in the first assembly operation. If the components are purchased, they should be shown as feeding horizontally into the appropriate assembly operation. If the components are manufactured, the production methods should be extracted from the route sheets and shown as feeding vertically into the appropriate assembly operation. The operation process chart may be completed by continuing in this manner through all required steps until the product is ready for release to the warehouse.

The operation process chart can be complemented with transportations, storages, and delays (including distances and times) when the information is available.

A second viewpoint is to interpret the charts as network representations, or more accurately, tree representations of production processes. A variation of the network viewpoint is to treat the assembly chart and the operation process chart as special cases of a more general graphical model, the **precedence diagram**. The precedence diagram is a directed network and is used in project planning; critical path diagrams and PERT charts are examples of precedence diagrams.

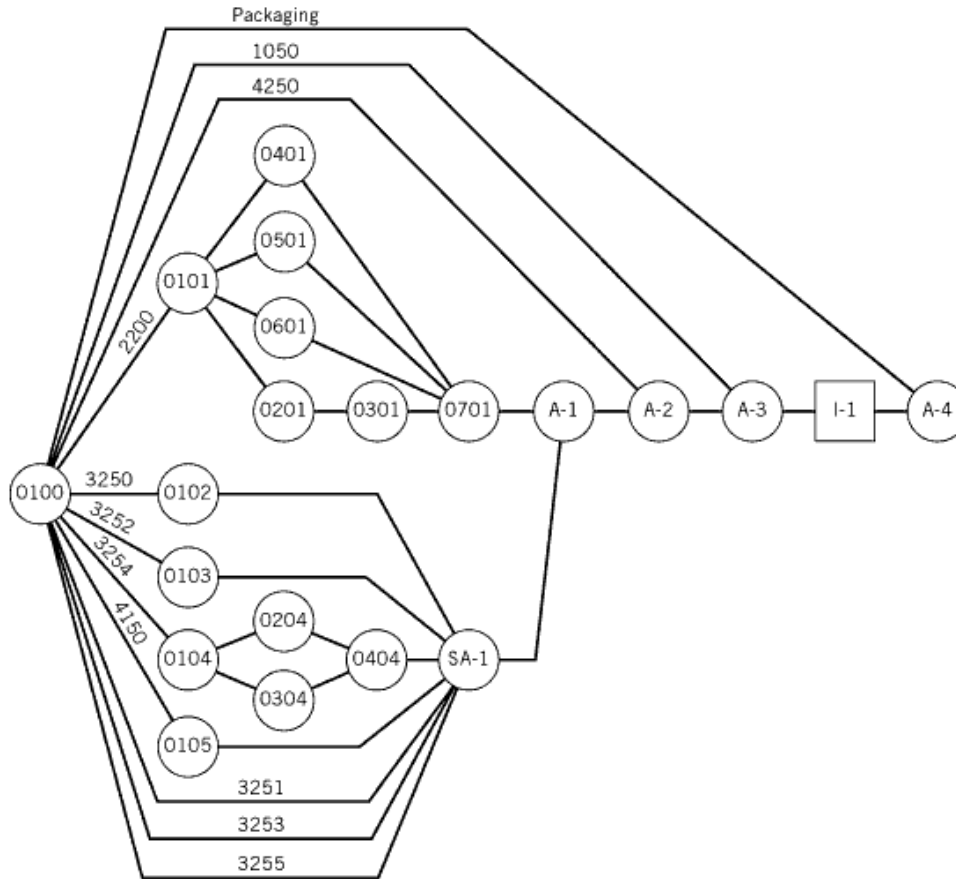


Figure 4.14. Precedence diagram for the air flow regulator

The precedence diagram shows part numbers on the arcs and denotes operations and inspection by circles and squares, respectively. A procurement operation, 0100, is used in figure 4.14 to initiate the process.

Because of the limitations of the assembly chart and the operation process chart, it is recommended a precedence diagram to be constructed first. Based on the precedence diagram, alternative assembly charts and operation process charts should then be constructed.

Other techniques to generate and evaluate assembly sequences have been explored. These techniques consider the assembly according to the relationship among parts instead of the order in which parts will be assembled.

Group technology is having an impact on product and process design. Group technology refers to the grouping of parts into families and then making decisions based on family characteristics. Groupings are typically based on part shapes, part sizes, material types, and processing requirements. In those cases where there are thousands of individual parts, the number of families might be less than 100. Group technology is an aggregation process that has been found to be quite useful in achieving standardized part numbers, standard specifications of purchased parts, for example, fasteners and standardized process selection.

4.3. Schedule Design

Schedule design provide answers to questions involving how much to produce and when to produce. Production quantity decisions are referred to as lot size decisions; determining when to produce is referred to as production scheduling. In addition to how much and when, it is important to know how long production will continue; such a determination is obtained from market forecasts.

Schedule design decisions impact machine selection, number of machines, number of shifts, number of employees, space requirements, storage equipment, material handling equipment, personnel requirements, storage policies, unit load design, building size, and so on.

4.3.1. Market Information

A minimum market information given in table 4.2 is needed. Preferably, information regarding the dynamic value of demands to be place on the facility is desired.

Product or Service	First Year Volume	Second Year Volume	Fifth Year Volume	Tenth Year Volume
A	5000	5000	8000	10,000
B	8000	7500	3000	0
C	3500	3500	3500	4000
D	0	2000	3000	8000

Table 4.2. Minimum market information required for facilities planning

Ideally, information of the type shown in table 4.3 would be provided. If such information is available, a facilities plan can be developed for each demand state and a facility designed with sufficient flexibility to meet the yearly fluctuations in product mix.

Product or Service	Demand State	First Year		Second Year		Fifth Year		Tenth Year	
		Probability	Volume	Probability	Volume	Probability	Volume	Probability	Volume
A	Pessimistic	.3	3000	.2	3500	.1	5500	.1	7000
	Most likely	.5	5000	.6	5500	.8	8000	.9	10,000
	Optimistic	.2	6000	.2	6500	.1	9500		
B	Pessimistic	.1	7000						
	Most likely	.6	8000	.7	7000	.9	3000	1.0	0
	Optimistic	.3	8500	.3	8000	.1	3500		
C	Pessimistic	.2	2000	.2	2000	.2	2000	.2	2000
	Most likely	.7	4000	.7	4000	.7	4000	.6	4000
	Optimistic	.1	4500	.1	4500	.1	4500	.2	5000
D	Pessimistic			.1	1500	.1	2500	.2	7000
	Most likely	1.0	0	.8	2000	.8	5000	.6	8000
	Optimistic					.1	3500	.2	9000
Confidence level or degree of certainty		90%		85%		70%		59%	

Table 4.3. Market analysis indicating the stochastic nature of future requirements for Facilities Planning

Unfortunately, information of the type given in table 4.3 is generally unavailable. Therefore, facilities typically are planned using deterministic data. The assumptions of deterministic data and known demands must be dealt with when evaluating alternative facilities plans.

In addition to the volume, trend, and predictability of future demands for various products, the qualitative information listed in table 3 should be obtained.

Information to Be Obtained from Marketing	Facilities Planning Issues Impacted by the Information
Who are the consumers of the product?	<ol style="list-style-type: none"> 1. Packaging 2. Susceptibility to product changes 3. Susceptibility to changes in marketing strategies
Where are the consumers located?	<ol style="list-style-type: none"> 1. Facilities location 2. Method of shipping 3. Warehousing systems design
Why will the consumer purchase the product?	<ol style="list-style-type: none"> 1. Seasonability 2. Variability in sales 3. Packaging
Where will the consumer purchase the product?	<ol style="list-style-type: none"> 1. Unit load sizes 2. Order processing 3. Packaging
What percentage of the market does the product attract and who is the competition	<ol style="list-style-type: none"> 1. Future trends 2. Growth potential 3. Need for flexibility
What is the trend in product changes?	<ol style="list-style-type: none"> 1. Space allocations 2. Materials handling methods 3. Need for flexibility

Table 4.4. Valuable information that should be obtained from marketing and used by a facilities planner

4.3.2. Process Requirements

Process design determines the specific equipment types required to produce the product. Schedule design determines the number of each equipment type required to meet the production schedule.

Specification of process requirements typically occurs in three phases.

1. Determining the quantity of components that must be produced, including scrap allowance, in order to meet the market estimate.
2. Determining equipment requirements for each operation.
3. Combination of the operation requirements to obtain overall equipment requirements.

4.3.2.1. Scrap Estimates

The market estimate specifies the annual volume to be produced for each product. To produce the required amount of product, the number of units scheduled through production must equal the market estimate plus a scrap estimate. Hence, production capacity must be planned for the production of scrap. Otherwise, when scrap is produced the market estimate will not be met.

Scrap is the material waste generated in the manufacturing process due to geometric or quality considerations. For example, scrap due to geometry is generated when a rectangular steel plate is used to create circular components or when rolls of fabric are used to make shirt.

Let;

P_k = percentage of scrap produced on the k^{th} operation,

O_k = the desired output of nondefective product from operation k ,

I_k = the production input to operation k .

$$O_k = I_k - P_k I_k \quad \text{or} \quad O_k = I_k(1 - P_k)$$

Hence;

$$I_k = \frac{O_k}{1 - P_k}$$

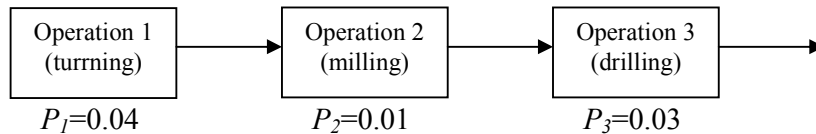
Thus, the expected number of units to start into production for a part having n operations is

$$I_k = \frac{O_n}{(1 - P_1)(1 - P_2) \dots (1 - P_n)}$$

where in this case O_n is the market estimate.

Example:

A product has a market estimate of 97,000 components and requires three processing steps (turning, milling, and drilling) having scrap estimates of $P_1=0.04$, $P_2=0.01$, $P_3=0.03$. Calculate the production input to operation 1.



$$I_3 = \frac{97,000}{1 - 0.03} = 100,000 \quad \left(I_k = \frac{O_k}{1 - P_k} \right)$$

$$I_2 = \frac{100,000}{1 - 0.01} = 101,010$$

$$I_1 = \frac{101,010}{1 - 0.04} = 105,219 \quad \left(\text{or; } I_1 = \frac{97,000}{(1 - 0.04)(1 - 0.01)(1 - 0.03)} = 105,219 \right)$$

Summary of Production Requirements		
Operation	Production Quantity Scheduled (units)	Expected number of good units produced
Turning	105,219	101,010
Milling	101,010	100,000
Drilling	100,000	97,000

Table 4.5. Summary of production requirements

4.3.2.2. Equipment Fractions

The quantity of equipment required for an operation is referred to as the equipment fraction. The equipment fraction may be determined for an operation by dividing the total time required to perform the operation by the time available to complete the operation. The total time required to perform an operation is the product of the standard time for the operation and the number of times the operation is to be performed.

$$F = \frac{SQ}{EHR}$$

where; F = number of machines required per shift
 S = standard time (minutes) per unit produced
 Q = number of units to be produced per shift
 E = actual performance, expressed as a percentage of standard time
 H = amount of time (minutes) available per machine
 R = reliability of machine, expressed as percent “up time”

Equipment requirements are a function of the following factors:

- Number of shifts (the same machine can work in more than one shift).
- Setup times (if machines are not dedicated, the longer the setup, the more machines needed).
- Degree of flexibility (customers may require small lot sizes of different products delivered frequently – extra machine capacity will be required to handle these requests).
- Layout type (dedicating manufacturing cells or focused factories to the production of product families may require more machines).
- Total productive maintenance (will increase machine up time and improve quality, thus fewer machines will be needed).

Example:

A machine part has a machinery time of 2.8 min per part on a milling machine. During an 8-hr shift 200 units are to be produced. Of the 480 min available for production, the milling machine will be operational 80% of the time. During the time the machine is operational, parts are produced at a rate equal to 95% of the standard rate. How many milling machines are required?

$S = 2.8$ min per part,
 $Q = 200$ units per shift,
 $E = 0.95$,
 $H = 480$ min per shift,
 $R = 0.80$.

$$F = \frac{SQ}{EHR} = \frac{2.8 \times 200}{0.95 \times 480 \times 0.80} = 1.535 \text{ machines per shift}$$

4.4. Facilities Design

Once the product, process and schedule design decisions have been made, the facilities planner needs to organize the information and generate and evaluate layout, handling, storage, and unit load design alternatives.

The seven management and planning tools are the **affinity diagram**, the **interrelationship digraph**, the **tree diagram**, the **matrix diagram**, the **contingency diagram**, the **activity network diagram**, and the **prioritization matrix**.

4.4.1. Affinity Diagram

The affinity diagram is used to gather verbal data, such as ideas and issues, and organize it into grouping.

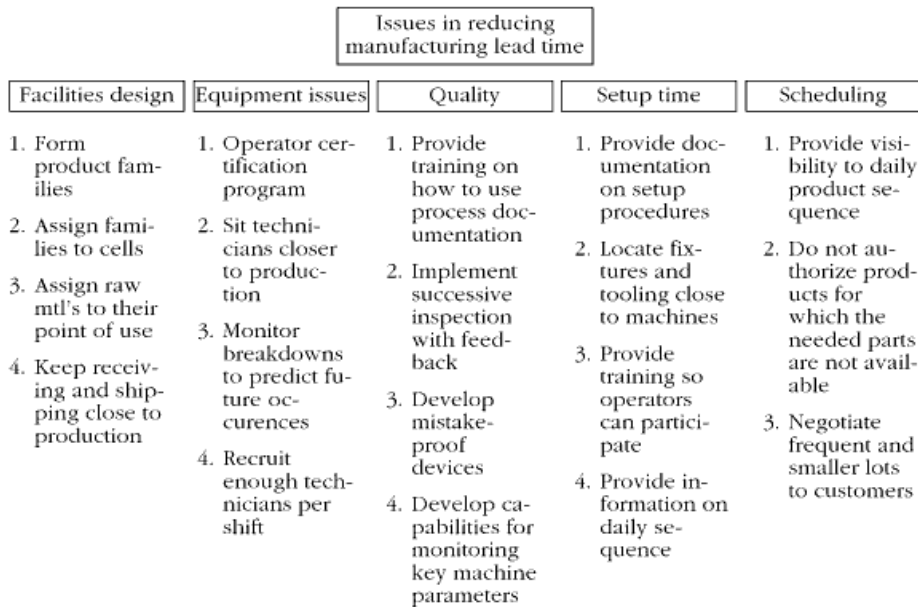


Figure 4.15. Affinity diagram example for reducing manufacturing lead time

4.4.2. Interrelationship Diagram

The interrelationship diagram is used to map the logical links among related items, trying to identify which items impact others the most. This graph helps us understand the logical sequence of steps for the facilities design.

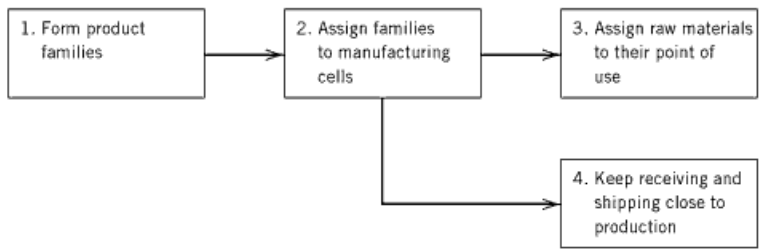


Figure 4.16. Interrelationship diagram for facilities design

4.4.3. Tree Diagram

The tree diagram is used to map in increasing detail the actions that need to be accomplished in order to achieve a general objective.

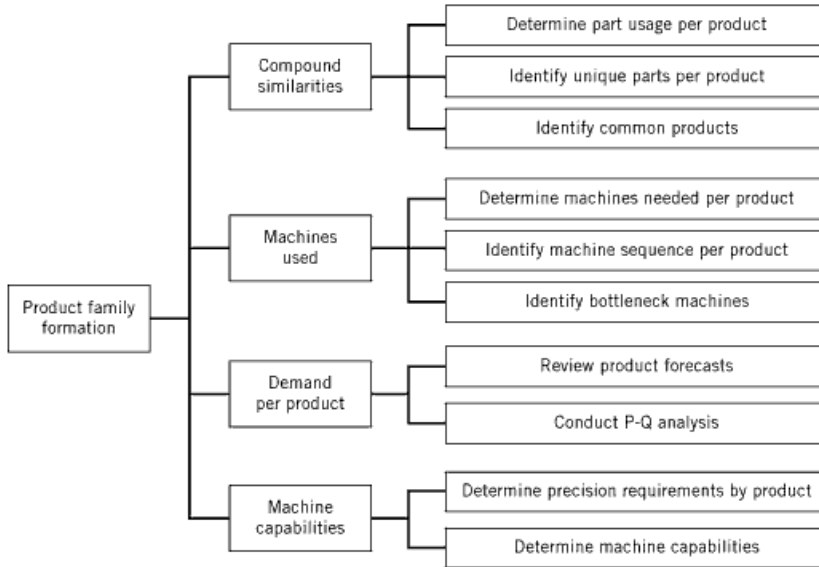


Figure 4.17. Tree diagram for the formation of product families

4.4.4. Matrix Diagram

The matrix diagram organizes information such as characteristics, functions, and tasks into sets of items to be compared. This tool provides visibility to key contact on specific issues and helps identify individuals who are assigned to too many teams.

Team\Participants	Joe	Mary	Jerry	Lou	Linda	Daisy	Jack
Part usage team	P	C	P	L			P
Machine use & cap team	L		C				P
Demand forecast team				P	C	L	

Note: L: Team Leader
 C: Team Coordinator
 P: Team Participant

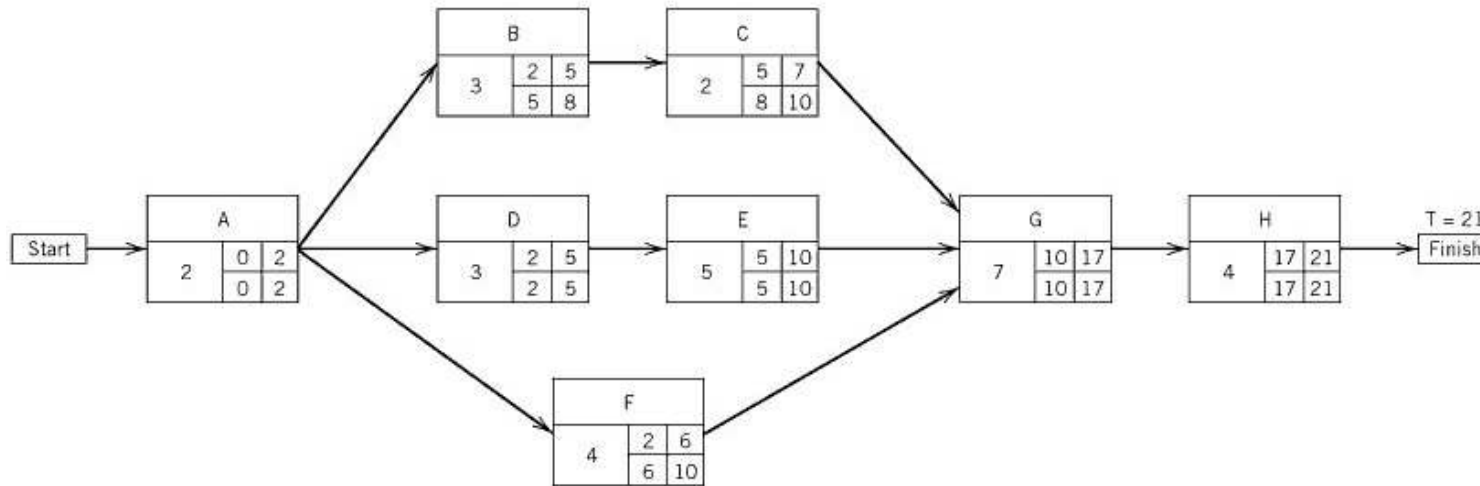
Table 4.6. Matrix diagram for team participation

4.4.5. Contingency Diagram

The contingency diagram formally known as process decision program chart, maps conceivable events and contingencies that might occur during implementation. It particularly is useful when the project being planned consists of unfamiliar tasks.

4.4.6. Activity Network Diagram

The activity network diagram is used to develop a work schedule for the facilities design effort. This diagram is synonymous to the critical path method (CPM) graph. It can also be replaced by a Gantt chart.



Legend

- A: Schedule shutdown periods for equipment movement and installation
- B: Interview, evaluate, select, and hire new employees
- C: Train new employees
- D: Interview, evaluate, and select equipment vendors
- E: Order equipment
- F: Interview, evaluate, select, and hire construction contractors
- G: Meet to review installation plan (facilities design team, contractors, new employees, vendor representatives, and management)
- H: Executive installation plan and test system

Activity			
Di	ES	EF	
	LS	LF	

Di: Activity duration
 ES: Early start
 EF: Early finish
 LS: Late start
 LF: Late finish

Figure 4.18. Activity network diagram example for a production line expansion facilities design project

4.4.7. Prioritization Matrix

In developing facilities design alternatives it is important to consider:

- (a) Layout characteristics
 - total distance travelled
 - manufacturing floor visibility
 - overall aesthetics of the layout
 - ease of adding future business
- (b) Material handling equipment
 - use of current material handling equipment
 - investment requirements on new equipment
 - space and people requirements
- (c) Unit load implied
 - impact on WIP levels
 - space requirements
 - impact on material handling equipment
- (d) Storage strategies
 - space and people requirements
 - impact on material handling equipment
 - human factor risk
- (e) Overall building impact
 - estimated cost of the alternative
 - opportunities for new business.

The prioritization matrix can be used to judge the relative importance of each criterion as compared to each other.

- A. Total distance travelled
- G. Space requirements
- B. Manufacturing floor visibility
- H. People requirements
- C. Overall aesthetics of the layout
- I. Impact on WIP levels

	Criteria											Row totals (%)
	A	B	C	D	E	F	G	H	I	J	K	
A	1	5	10	5	1	1	1	1	1	5	1	32 (9.9)
B	1/5	1	5	1/5	1/5	1/10	1/5	1/5	1/10	1/5	1/5	7.6 (2.4)
C	1/10	1/5	1	1/10	1/10	1/10	1/5	1/5	1/10	1/10	1/10	2.3 (0.7)
D	1/5	5	10	1	1/5	1/5	1/5	1/5	1/10	1/5	1/10	17.4 (5.4)
E	1	5	10	5	1	1	5	5	1/5	1	1/5	34.4 (10.7)
F	1	10	10	5	1	1	5	5	1	1	1	41 (12.7)
G	1	5	5	5	1/5	1/5	1	5	1/5	1/5	1/58	23 (7.1)
H	1	5	5	5	1/5	1/5	5	1	1/10	1/5	1/5	22.9 (7.1)
I	1	10	10	10	5	1	5	10	1	1	5	59 (18.3)
J	1/5	5	10	5	1	1	5	5	1	1	5	39.2 (12.2)
K	1	5	10	10	5	1	5	5	1/5	1/5	1	43.4 (13.5)
Column total	7.7	56.2	86	51.3	14.9	6.8	32.6	37.6	5	10.1	14	322.2
												Grand total

Table 7. Prioritization matrix for the evaluation of facilities design alternatives

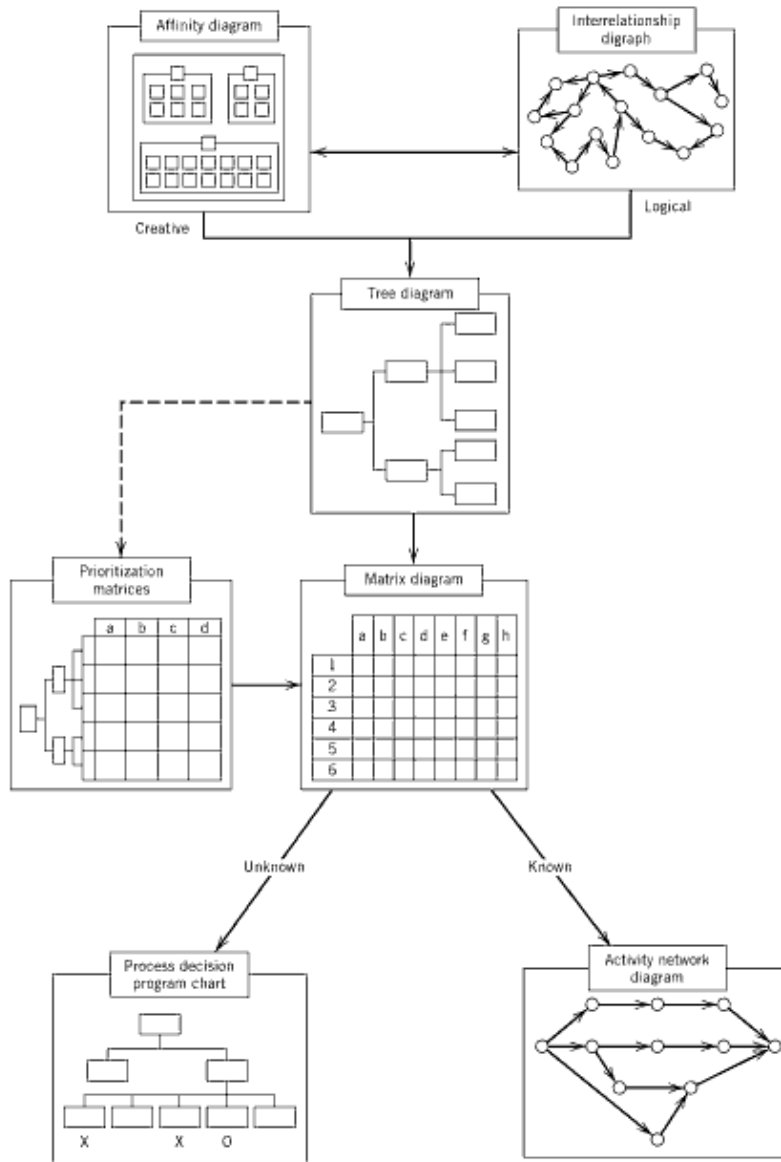


Figure 4.19. Logical application sequence of the seven management and planning tool

CHAPTER 5

FLOW, SPACE, AND ACTIVITY RELATIONSHIPS

5.1.Introduction

In determining the requirements of a facility, three important considerations are flow, space, and activity relationships.

- **Flow** depends on lot sizes, unit load sizes, material handling equipment and strategies, layout arrangement, and building configuration.
- **Space** is a function of lot sizes, storage system, production equipment type and size, layout arrangement, building configuration, housekeeping and organization policies, material handling equipment, and office, cafeteria, and restroom design.
- **Activity relationships** are defined by material or personnel flow, environmental considerations, organizational structure, continuous improvement methodology (teamwork activities), control issues, and process requirements.

5.2. Departmental Planning

To facilitate the consideration of flow, space, and activity relationships, it is helpful to introduce the subject of departmental planning. **Planning departments** can involve production, support, administrative, and service areas (called production, support, administrative, and service planning departments).

Production planning departments are collections of workstations to be grouped together during the facilities layout process. The formulation of organizational units should parallel the formation of planning departments.

As a general rule, planning departments may be determined by combining workstations that perform “like” functions. “Like” could refer to workstations performing operations on similar products or components or to workstations performing similar processes.

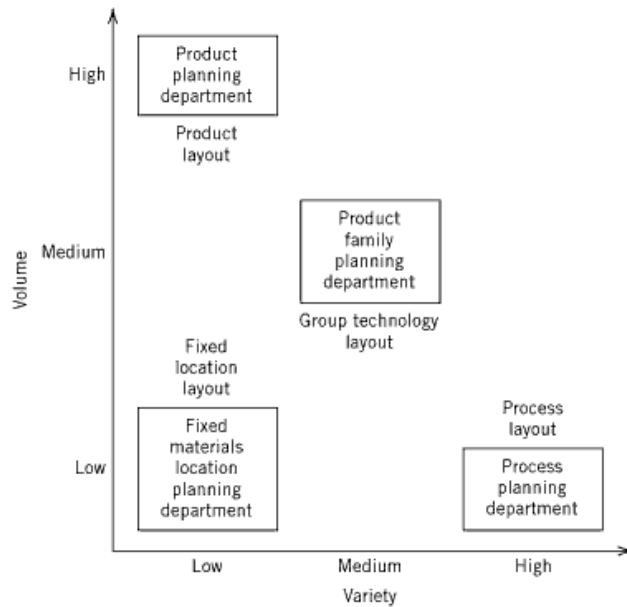


Figure 5.1. Volume-variety layout classification

Depending on the product volume-variety, production planning departments can also be classified as product, fixed materials location, product family (or group technology), and process planning departments (figure 5.1).

Examples of **product planning departments** that consist of a combination of workstations performing operations on similar products or components are; engine block production line departments, aircraft fuselage assembly departments, and uniform flat sheet metal departments. Product planning departments may be further subdivided by the characteristics of the product being produced.

Suppose a large, stable demand for a standardized product, like an engine block, is to be met by production. In such situation, the workstations should be combined into a planning department so that all workstations required to produce the product are combined. The resulting product planning department may be referred to as a **production line department**.

Suppose a low, sporadic demand exists for a product that is very large and awkward to move, for example, an aircraft fuselage. The workstations should be combined into a planning department that includes all workstation required to produce the product and the staging area. This type of product planning department may be referred to as a **fixed materials location department**.

A third type of product planning department may be identified when there exists a medium demand for a medium number of similar components. Similar components form, a family of components that may be produced via a “group” of workstations. The combination of the group of workstations referred to as a product family department may be referred to as a **product family department**.

Examples of planning departments based on the combination of the workstations containing “similar” processes are metal cutting departments, gear cutting

departments, and hobbing departments. Such planning departments are referred to as **process departments** because they are formed by combining workstations that perform “similar” processes.

Most facilities consist of a mixture of product and process planning departments. For example, in a facility consisting of mainly process planning departments producing a large variety of rather unrelated products, the detailed placement of individual workstations within a process department might be based on a product planning department philosophy.

As an example, all painting activities might be grouped together in a painting process department. However, the layout of the painting department can consist of a painting line designed on the basis of a product planning department philosophy.

Many companies using modern manufacturing approaches are converting their facilities to combination of product and product family (group technology) planning departments. Group technology layouts are combined with *Just-In-Time* (JIT) concepts in cellular manufacturing arrangements.

If the Product Is	The Type of Planning Department Should Be	And the Method of Combining Workstations into Planning Departments Should Be
Standardized and has a large stable demand	Production line, product department	Combine all workstations required to produce the product
Physically large, awkward to move, and has a low sporadic demand	Fixed materials location, product department	Combine all workstations required to produce the product with the area required for staging the product
Capable of being grouped into families of similar parts that may be produced by a group of workstations	Product family, product department	Combine all workstations required to produce the family of products
None of the above	Process department	Combine identical workstations into initial planning departments and attempt to combine similar initial planning departments without obscuring important interrelationships within departments

Table5. 1. Procedural guide for combining workstations in planning departments

5.2.1. Manufacturing Cells

Product family or group technology departments aggregate medium volume-variety parts into families based on similar manufacturing operations or design attributes. The machines that are required to manufacture the part family are grouped together to form a “cell”.

Manufacturing cells group machines, employees, materials, tooling, and material handling and storage equipment to produce family of parts.

The most important benefits of cellular manufacturing are achieved when manufacturing cells are designed, controlled, and operated using *Just-In-Time* (JIT), Total Quality Management (TQM), and Total Employee Involvement (TEI) concepts. These benefits are:

- *Reduction* of inventories, space, machine breakdowns, rework, paperwork, warranty claims, storage and handling equipments, employee turnover and absenteeism, production leadtimes, costs, and stockout;
- *Simplification* of communication, handling, and production scheduling; and
- *Improvement* of productivity, flexibility, inventory turnover, quality, and customer and employee satisfaction.

Successful implementation of manufacturing cells requires addressing selection, design, operation, and control issues. Selection refers to the identification of machine and part types for a particular cell. Cell design refers to a layout and production and material handling requirements. Operation of a cell involves determining lot sizes, scheduling, number of operators, type of operators, and type of production control (push vs. pull). Finally, control of a cell refers to the methods used to measure the performance of the cell.

The most popular approach to selection issues of manufacturing cells are classification, coding, production flow analysis, clustering techniques, heuristic procedures, and mathematical models.

Classification is the grouping of parts into classes or part families based on design attributes and coding is the representation of these attributes by assigning numbers or symbols to them.

Production flow analysis is a procedure for forming part families by analyzing the operation sequences and the production routing of a part or component through the plant.

Clustering methodologies are used to group parts together so that they can be processed as a family. This methodology lists parts and machines in rows and columns, and interchanges them based on some criterion like similarity coefficients. For example, Direct Clustering Algorithm (DCA) forms clustered groups based on sequentially moving rows and columns to the top and left.

Example:

The below matrix is a representation of the machines visited by a set of parts during production. Our objective is to group parts and machines together to form cells.

		M/C #																											
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26		
Part #	1	1		1		1			1	1		1							1	1				1				9	
	2	1		1		1			1	1		1							1	1				1				9	
	3	1		1		1			1	1		1							1	1				1				9	
	4	1		1		1			1	1		1							1	1				1				9	
	5	1		1			1			1	1		1						1	1				1				9	
	6	1		1			1			1	1		1						1	1				1				9	
	7	1		1			1			1	1		1						1	1				1				9	
	8		1		1			1				1		1							1	1			1			8	
	9		1		1			1				1		1							1	1			1			8	
	10	1		1											1	1	1	1					1			1	1	1	10
	11	1		1											1	1	1	1					1			1	1	1	10
	12	1		1											1	1	1	1					1			1	1	1	10
	13	1		1											1	1	1	1					1			1	1	1	10
		11	2	11	2	4	3	2	7	7	2	7	2	4	4	4	4	4	7	7	2	2	4	7	2	4	4	4	

Figure 5.2. A part-machine matrix

The DCA methodology accomplishes this by ranking each row and column by its number of occurrences, that is “1”s. This ranking is represented by the number of far right of each row and the bottom of each column. Once the ranking is performed, the rows and columns are to be sorted in descending order as shown figure 5.3.

		Machine #																										# of 1s		
		23	20	19	12	10	7	4	2	6	26	25	24	21	16	15	14	13	5	22	18	21	17	11	9	3	1			
Part #	13										1	1	1	1	1	1	1	1									1	1	10	
	12										1	1	1	1	1	1	1	1										1	1	10
	11										1	1	1	1	1	1	1	1										1	1	10
	10										1	1	1	1	1	1	1	1										1	1	10
	7									1											1	1	1	1	1	1	1	1	1	9
	6									1											1	1	1	1	1	1	1	1	1	9
	5									1											1	1	1	1	1	1	1	1	1	9
	4																				1	1	1	1	1	1	1	1	1	9
	3																				1	1	1	1	1	1	1	1	1	9
	2																				1	1	1	1	1	1	1	1	1	9
	1																				1	1	1	1	1	1	1	1	1	9
	9	1	1	1	1	1	1	1	1	1																				8
	8	1	1	1	1	1	1	1	1	1																				8
# of 1s	2	2	2	2	2	2	2	2	2	3	4	4	4	4	4	4	4	4	4	4	7	7	7	7	7	7	7	11	11	

Figure 5.3. Ordered machine parts

The next step in the methodology is to; starting with the first column, transfer all rows with occurrences to the top of the matrix.

Part #	3	1	26	25	24	21	16	15	14	13	22	18	17	11	9	8	6	5	23	20	19	12	10	7	4	2	# of 1s
13	1	1	1	1	1	1	1	1	1	1																	10
12	1	1	1	1	1	1	1	1	1	1																	10
11	1	1	1	1	1	1	1	1	1	1																	10
10	1	1	1	1	1	1	1	1	1	1																	10
7	1	1									1	1	1	1	1	1	1										9
6	1	1									1	1	1	1	1	1	1										9
5	1	1									1	1	1	1	1	1	1										9
4	1	1									1	1	1	1	1	1		1									9
3	1	1									1	1	1	1	1	1			1								9
2	1	1									1	1	1	1	1	1				1							9
1	1	1									1	1	1	1	1	1					1						9
9																			1	1	1	1	1	1	1	1	8
8																			1	1	1	1	1	1	1	1	8
# of 1s	11	11	4	4	4	4	4	4	4	4	7	7	7	7	7	7	7	3	4	2	2	2	2	2	2	2	2

Figure 5.4. Column-sorted machine part matrix

Once the rows have been transferred, the columns are adjusted in a similar manner.

Part #	3	1	26	25	24	21	16	15	14	13	22	18	17	11	9	8	6	5	23	20	19	12	10	7	4	2	
13	1	1	1	1	1	1	1	1	1	1																	
12	1	1	1	1	1	1	1	1	1	1																	
11	1	1	1	1	1	1	1	1	1	1																	
10	1	1	1	1	1	1	1	1	1	1																	
7	1	1									1	1	1	1	1	1	1										
6	1	1									1	1	1	1	1	1	1										
5	1	1									1	1	1	1	1	1	1										
4	1	1									1	1	1	1	1	1		1									
3	1	1									1	1	1	1	1	1											
2	1	1									1	1	1	1	1	1											
1	1	1									1	1	1	1	1	1											
9																				1	1	1	1	1	1	1	1
8																				1	1	1	1	1	1	1	1

Figure 5.5. Clustered machine parts to form 2 cells: Cell A and Cell B

Part #	3	1	26	25	24	21	16	15	14	13	22	18	17	11	9	8	6	5	23	20	19	12	10	7	4	2	
13	1	1	1	1	1	1	1	1	1	1																	
12	1	1	1	1	1	1	1	1	1	1																	
11	1	1	1	1	1	1	1	1	1	1																	
10	1	1	1	1	1	1	1	1	1	1																	
7	1	1									1	1	1	1	1	1	1										
6	1	1									1	1	1	1	1	1	1										
5	1	1									1	1	1	1	1	1	1										
4	1	1									1	1	1	1	1	1		1									
3	1	1									1	1	1	1	1	1											
2	1	1									1	1	1	1	1	1											
1	1	1									1	1	1	1	1	1											
9																				1	1	1	1	1	1	1	1
8																				1	1	1	1	1	1	1	1

Figure 5.6. Clustered machine parts to form three cells: Cell A, Cell B, and Cell C

Note that both cell A and B require the use of machine 1 and machine 3. Ideally, the machines in a cell are to be dedicated to its associated part family. However, this is not the case since machine 1 and machine 3 operate on the part families of cell A and

cell B. These machines are commonly called “bottleneck” machines since they bind the two cells together.

A common practice is to either address the parts of a particular part family that go to these machines or the bottleneck machines themselves. The parts can be removed from the part family, redesigned to operate on other machines within its corresponding cell, or outsourced.

Part #	Machine #																																			
	26	25	24	21	16	15	14	13	3a	1a	22	18	17	11	9	8	3b	1b	6	5	23	20	19	12	10	7	4	2								
13	1	1	1	1	1	1	1	1	1	1																										
12	1	1	1	1	1	1	1	1	1	1																										
11	1	1	1	1	1	1	1	1	1	1																										
10	1	1	1	1	1	1	1	1	1	1																										
7	Cell A										1	1	1	1	1	1	1	1	1	1	1	1														
6											1	1	1	1	1	1	1	1	1	1	1	1														
5											1	1	1	1	1	1	1	1	1	1	1	1														
4											1	1	1	1	1	1	1	1	1	1	1	1														
3											1	1	1	1	1	1	1	1	1	1	1	1														
2											1	1	1	1	1	1	1	1	1	1	1	1														
1											1	1	1	1	1	1	1	1	1	1	1	1														
9											Cell B										Cell C															
8																					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 5.7. Final solution to the clustering problem

5.3. Activity Relationship

Activity relationships provide the basis for many decisions in the facilities planning process. The primary relationships are considered are

1. Organizational relationships, influenced by a span of control and reporting relationships.
2. Flow relationships, including the flow of materials, people equipment, information, and money.
3. Control relationships, including centralized versus decentralized materials control, real time versus batch inventory control, shop floor control, and levels of automation and integration.
4. Environmental relationships, including safety considerations and temperature, noise, fumes, humidity, and dust.
5. Process relationships other than those considered above, such as floor loadings, requirements for water treatment, chemical processing, and special services.

Several relationships can be expressed quantitatively; others must be expressed qualitatively. Flow relationships are typically expressed in terms of the number of moves per hour, the quantity of goods to be moved per shift, the turnover rate for inventory, the number of documents processed per month, and the monthly expenditures for labor and materials.

A flow process may be described in terms of the **subject** of flow, the **resources** that bring about the flow, and the **communications** that coordinate the resources.

The subject is the item to be processed. The resources that bring about flow are the processing and transportation facilities required to accomplish the required flow. The communications that coordinate the resources include the procedures that facilitate the management of the flow process.

If the flow process being considered is the flow of materials into a manufacturing facility, the flow process is typically referred to as a *materials management system*. The subjects of material management systems are the materials, parts, and supplies purchased by a firm and required for the production of its product. The resources of material management systems include:

1. The production control and purchasing functions
2. The vendors
3. The transportation and material handling equipment required to move the materials, parts, and supplies
4. The receiving, storage, and accounting functions

If the flow of materials, parts, and supplies within a manufacturing facility is to be the subject of the flow process, the process is called the *material flow system*. The type of material flow system is determined by the makeup of the activities or planning departments among which materials flow. There are four types of production planning departments (figure 1):

1. Production line departments
2. Fixed material location departments
3. Product family departments
4. Process departments

The material flow systems for each department type are shown in figures 5.8-5.11. The subjects of material flow systems are the materials, parts, and supplies used by a firm in manufacturing its products. The resources of material flow systems include:

1. The production control and quality control departments
2. The manufacturing, assembly, and storage departments
3. The material handling equipment required to move materials, parts, and supplies.
4. The warehouse.

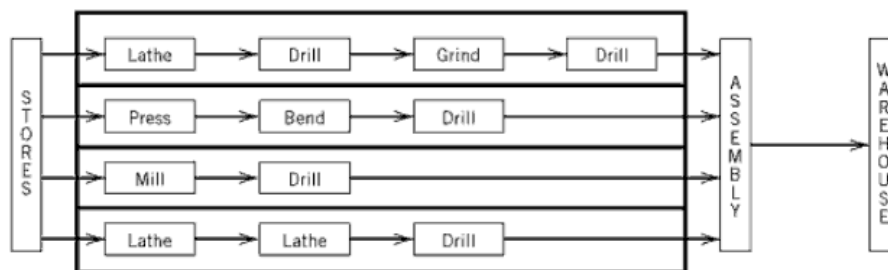


Figure 5.8. Material flow system for product planning departments

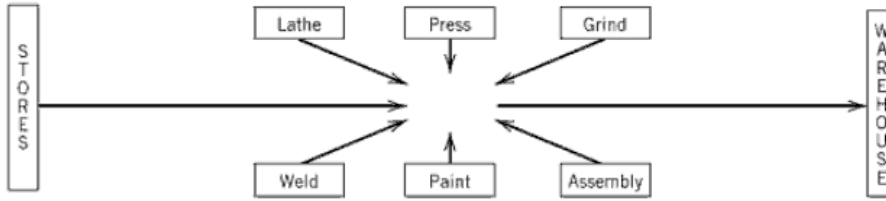


Figure 5.9. Material flow system for fixed materials location departments

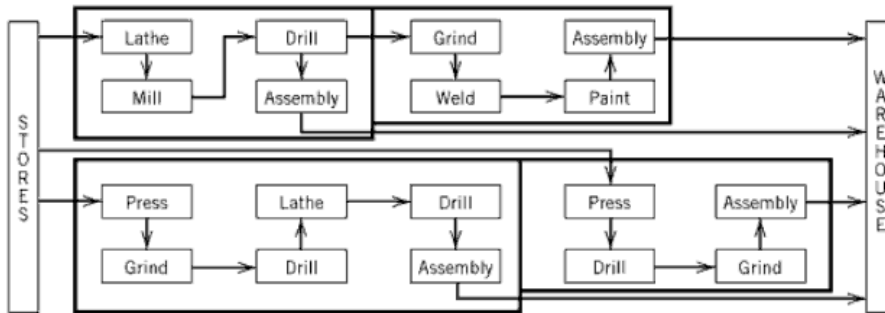


Figure 5.10. Material flow system for product family departments

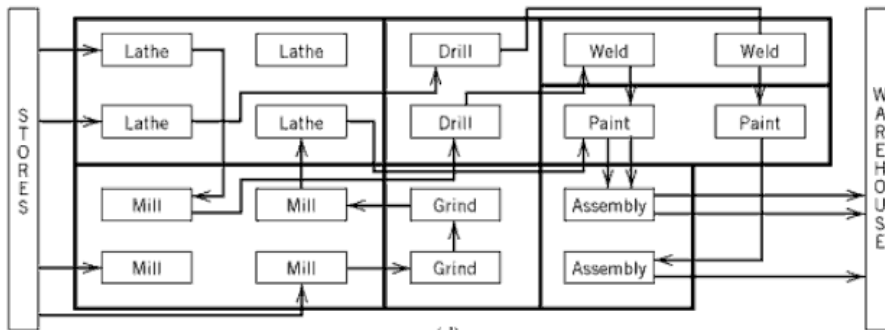


Figure 5.11. Material flow system for process planning departments

If the flow of products from a manufacturing facility is to be the subject of the flow, the flow process is referred to as the *physical distribution system*. The subject of physical distribution systems are the finished goods produced by a firm. The resources of physical distribution systems include:

1. The customer
2. The sales and accounting departments and warehouses
3. The material handling and transportation equipment required to move the finished product
4. The distributors of the finished products.

5.4. Flow Patterns

Patterns of flow may be viewed from the perspective of flow within workstations, within departments, and between departments.

5.4.1. Flow Within Workstations

Motion studies and ergonomics considerations are important in establishing the flow within workstations. For example, flow within a workstation should be simultaneous, symmetrical, natural, rhythmical, and habitual.

5.4.2. Flow Within Departments

The flow pattern within departments is dependent on the type of department. In a product and/or product family department, the flow of work follows the product flow.

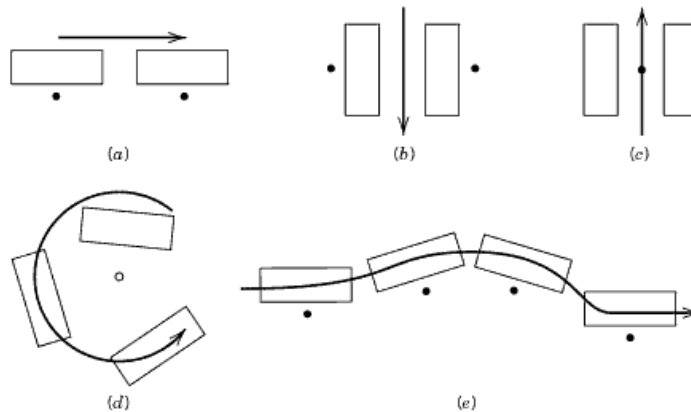


Figure 5.12. Flow within production departments. (a) End-to-end. (b) Back-to-back. (c) Front-to-front. (d) Circular. (e) Odd angle.

End-to-end, back-to-back, and odd-angle flow patterns are indicative of product departments where one operator works at each workstation. Front-to-front flow patterns are used when one operator works on two workstations and circular flow patterns are used when one operator works on more than two workstations.

In a process department, little flow should occur between workstations within departments. Flow typically occurs between workstations and aisles. Flow patterns are dictated by the orientation of the workstations to the aisles.

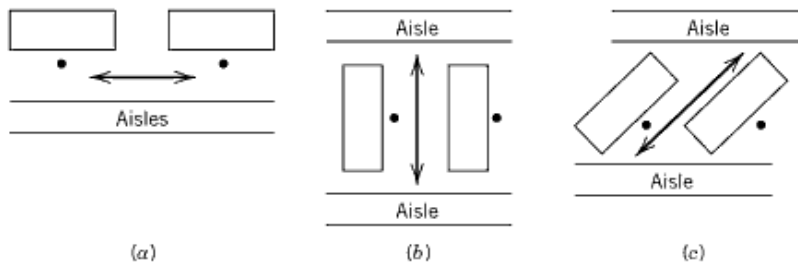


Figure 5.13. Flow within process departments. (a) Parallel. (b) Perpendicular. (c) Diagonal

Diagonal flow patterns are typically used in conjunction with one-way aisles. Aisles that support diagonal flow pattern often require less space than aisles with either parallel or perpendicular workstation-aisle arrangements. However, one-way aisles also result in less flexibility. Therefore, diagonal flow patterns are not utilized often.

5.4.3. Flow Between Departments

Flow typically consists of a combination of the four general flow patterns shown in figure 3.

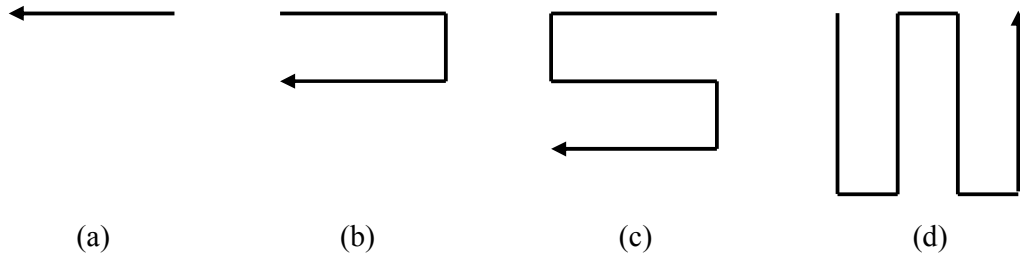


Figure 5.14. General flow patterns. (a) Straight line. (b) U-shaped. (c) S-shaped. (d) W-shaped

An important consideration in combining the flow patterns shown in figure 5.14 is the location of the entrance and exit. As a result of the plot plan or building construction, the location of the entrance (receiving department) and exit (shipping department) is often fixed at a given location and flow within the facility conform to these restrictions. A few examples of how flow within a facility may be planned to conform to entrance and exit restrictions are given in figure 5.15.

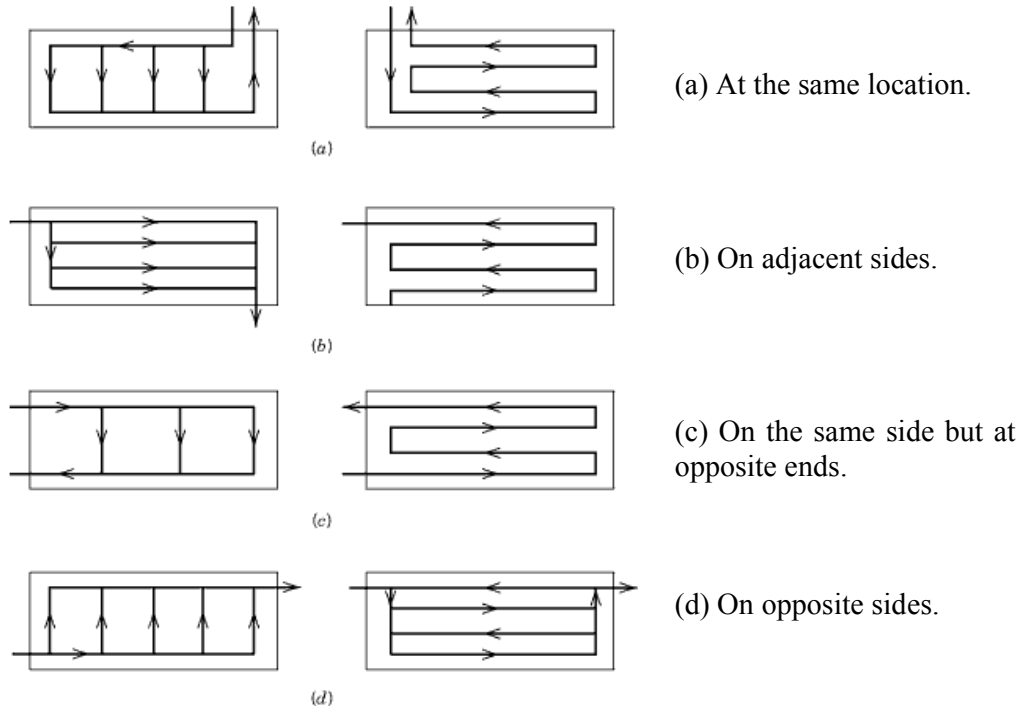


Figure 5.15. Flow within a facility considering the locations of the entrance and exit.

5.5. Flow Planning

Planning effective flow involves combining the flow patterns with adequate aisles to obtain a progressive movement from origination to destination.

Effective flow within a facility includes the progressive movement of materials, information, or people *between departments*.

Effective flow within a department involves the progressive movement of materials, information, or people *between workstations*.

Effective flow within a workstation addresses the progressive movement of materials, information, or people *through the workstation*.

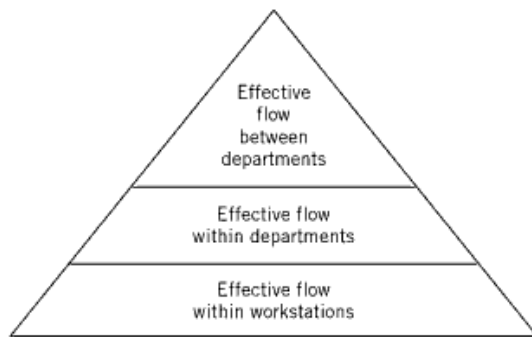


Figure 5.16. Flow planning hierarchy

A directed flow path is an uninterrupted flow path progressing directly from origination to destination. An uninterrupted flow path is a flow path that does not intersect with other paths. Figure 5.17 illustrates the congestion and undesirable intersections that may occur when flow paths are interrupted.

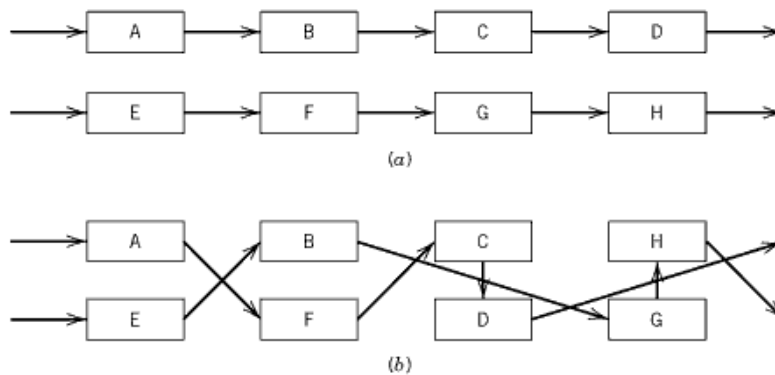


Figure 5.17.. The impact of interruptions on flow paths. (a) Uninterrupted flow paths. (b) Interrupted flow paths.

A directed flow path progressing from origination to destination is a flow path without backtracking. As can be seen in figure 5.18, backtracking increases the length of the flow path.

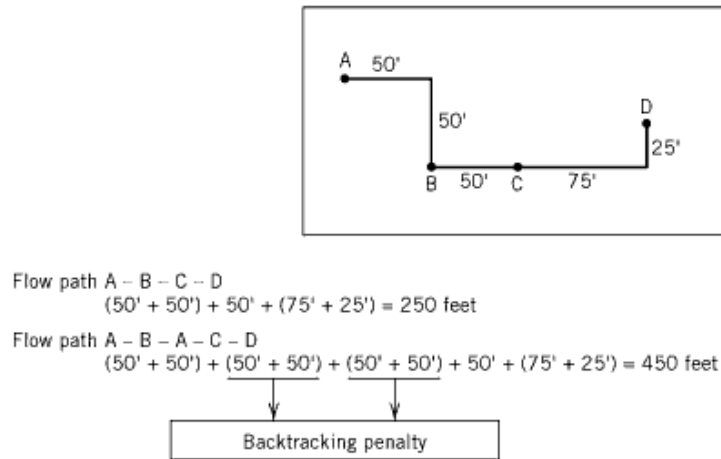


Figure 5.18. Illustration of how backtracking impacts the length of flow paths.

The principle of minimizing flow represents the work simplification approach to material flow. The work simplification approach to material flow includes:

1. Eliminating flow by planning for the delivery of materials, information, or people directly to the point of ultimate use and eliminate intermediate steps.
2. Minimizing multiple flows by planning for the flow between two consecutive points of use to take place in as few moments as possible, preferably one.
3. Combining flows and operations wherever possible by planning for the movement of materials, information, or people to be combined with a processing step.

The principle of minimizing the cost of flow may be viewed from either of the following two perspectives.

1. Minimize manual handling by minimizing walking, manual travel distances, and motions.
2. Eliminate manual handling by mechanizing or automating flow to allow workers to spend full time on their assigned tasks.

5.6. Measuring Flow

Flow among departments is one of the most important factors in the arrangement of departments within a facility. To evaluate alternative arrangements, a measure of flow must be established. Flows may be specified

- *Quantitatively* in terms of pieces per hour, moves per day, or pounds per week.
- *Qualitatively* in terms of an absolute necessity that two departments be close to each other or a preference that two departments not to be close to each other.

A chart that can be of use in flow measurement is the mileage chart shown in figure 5.19. Notice that the diagonal of the mileage chart is blank since it does not make any sense to ask “How far is it from New York to New York?”.

From \ To	Atlanta, GA	Boston, MA	Chicago, IL	Dallas, TX	New York, NY	Pittsburgh, PA	Raleigh, NC	San Francisco, CA
Atlanta, GA		1037	674	795	841	687	372	2496
Boston, MA	1037		963	1748	206	561	685	3095
Chicago, IL	674	963		917	802	452	784	2142
Dallas, TX	795	1748	917		1552	1204	1166	1753
New York, NY	841	206	802	1552		368	489	2934
Pittsburgh, PA	687	561	452	1204	368		445	2578
Raleigh, NC	372	685	784	1166	489	445		2843
San Francisco, CA	2496	3095	2142	1753	2934	2578	2843	

Figure 5.19. Mileage chart

Furthermore, the mileage chart is a symmetric matrix. When this occurs, the format of the mileage chart is often changed to a triangular matrix as shown in figure 5.20.

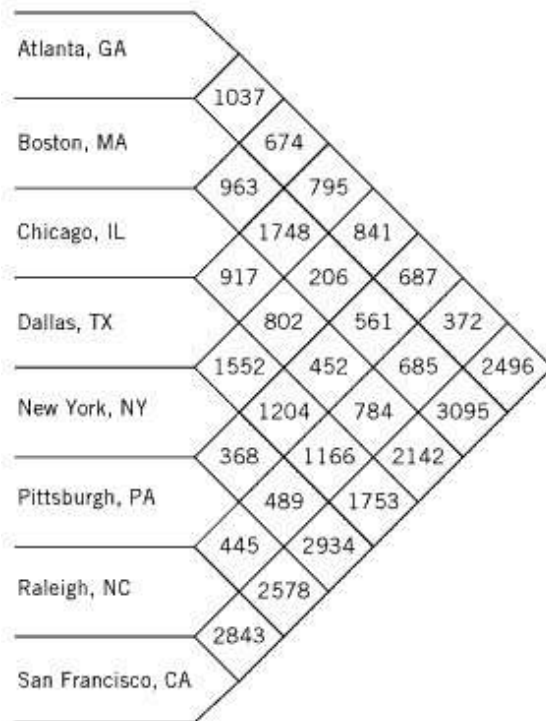


Figure 5.20. Triangular mileage chart

5.6.1. Quantitative Flow Measurement

Flows may be measured quantitatively in terms of the amount moved between departments. The chart most often used to record these flows is a from-to chart.

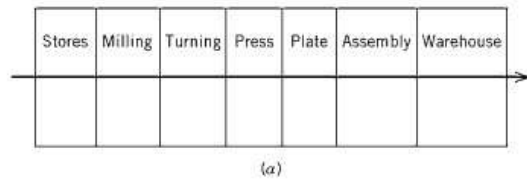
	To							
		Stores	Milling	Turning	Press	Plate	Assembly	Warehouse
From								
Stores			12	6	9	1	4	
Milling						7	2	
Turning			3			4		
Press						3	1	1
Plate			3	1			4	3
Assembly		1						7
Warehouse								

Figure 5.21. From-to chart

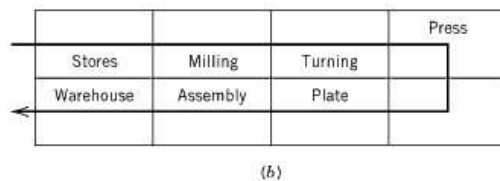
The from-to chart is a square matrix, but is seldom symmetric. The lack of symmetry is because there is no definite reason for the flows from stores to milling to be the same as the flows from milling to stores.

A from-to chart is constructed as follows:

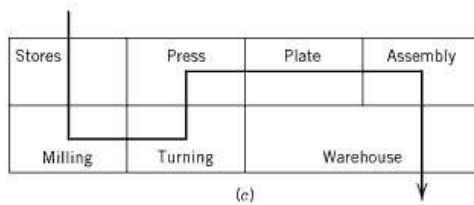
1. List all departments down the row and across the column following the overall flow pattern. For example figure 11 shows various flow patterns that result in the departments being listed as in figure 5.21.



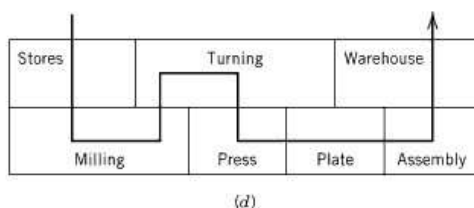
(a) Straight-line flow.



(b) U-shaped flow.



(c) S-shaped flow.



(d) W-shaped flow.

Figure 5.22. Flow patterns indicating the order of flow given

2. Establish a measure of flow for the facility that accurately indicates equivalent flow volumes.
3. Based on the flow paths for the items to be moved and the established measure of flow, record the flow volumes in the from-to chart.

Example:

A firm produces three components. Components 1 and 2 have the same size and weight and are equivalent with respect to movement. Component 3 is almost twice as large and moving two units of either component 1 or 2 is equivalent to moving 1 unit of component 3. The departments included in the facility are A, B, C, D, and E. The overall flow path is A-B-C-D-E. The quantities to be produced and the component routings are as follows:

Component	Production Quantities (per day)	Routing
1	30	A-C-B-D-E
2	12	A-B-D-E
3	7	A-C-D-B-E

Solution:

From \ To	A	C	B	D	E
A		① 30 ③ 2(7) = 14	② 12		
		44	12	0	0
C			① 30	③ 2(7) = 14	
	0		30	14	0
B				① 30 ② 12	③ 2(7) = 14
	0	0		42	14
D			③ 2(7) = 14		① 30 ② 12
	0	0	14		42
E	0	0	0	0	

In the above from-to chart, the circled numbers represent component numbers and the number of following the circled numbers indicate the volume of equivalent flows for the component.

Notice that flow volumes below the diagonal represent backtracking and the closer the flow volumes are to the main diagonal, the shorter will be the move in the facility.

5.6.2. Qualitative Flow Measurement

Flows may be measured qualitatively using the closeness relationships values developed by Muther and given in table 5.2.

Value	Closeness
A	Absolute necessary
E	Especially important
I	Important
O	Ordinary closeness okay
U	Unimportant
X	Undesirable

Table 5.2. Closeness relationship values

A relationship chart may be constructed as follows:

1. List all departments on the relationship chart.
2. Conduct interviews or surveys with persons from each department listed on the relationship chart and with the management responsible for all departments.
3. Define the criteria for assigning closeness relationships and itemize and record the criteria as the reasons for relationship values on the relationship chart.
4. Establish the relationship value and the reason for the value for all pairs of departments.
5. Allow everyone having input to the development of the relationship chart to have an opportunity to evaluate and discuss changes in the chart.

The values may be recorded in conjunction with the reasons for the closeness value using relationship chart given in figure 5.23.

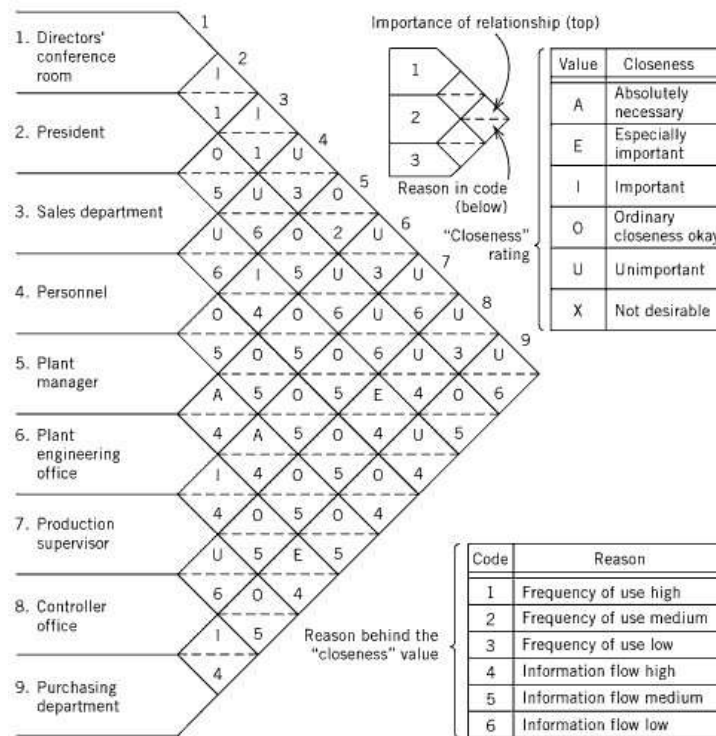


Figure 5.23. Relationship chart

5.7. Space Requirements

Perhaps the most difficult determination in facilities planning is the amount of space required in the facility. The design year for a facility is typically 5 to 10 years in the future.

Considerable uncertainty generally exists concerning the impact of technology, changing product mix, changing demand levels, and organizational designs for the future. The facilities planner then has the difficult task of projecting true space requirements for the uncertain future.

In determining space requirements for storage warehousing activities, inventory levels, storage units, storage methods and strategies, equipment requirements, building constraints, and personnel requirements must be considered.

In manufacturing and office environments, space requirements should be determined first for individual workstations; next, departmental requirements should be determined based on the collection of workstation in the department.

5.7.1. Workstation Specification

Workstations are places where specific operations are performed. Productivity of a firm is definitely related to the productivity of the workstation.

A workstation includes space for equipment, materials, and personnel. The equipment space for a workstation consists of space for:

1. The equipment
2. Machine travel
3. Machine maintenance
4. Plant services.

Floor area requirements for each machine, including machine travel, can be determined by multiplying total width (static width plus maximum travel to the left and right) by total depth (static depth plus maximum travel toward and away from the operator). To the floor area requirement of the machine add the maintenance and plant service area requirements. The resulting sum represents the total machinery area for a machine.

The materials areas for a workstation consist of space for:

1. Receiving and storing materials
2. In-process materials
3. Storing and shipping materials
4. Storing and shipping waste and scrap
5. Tools- fixtures, jigs, dies, and maintenance materials.

The personnel area for a workstation consists of space for:

1. The operator
2. Material handling
3. Operator ingress and egress (enter-exit).

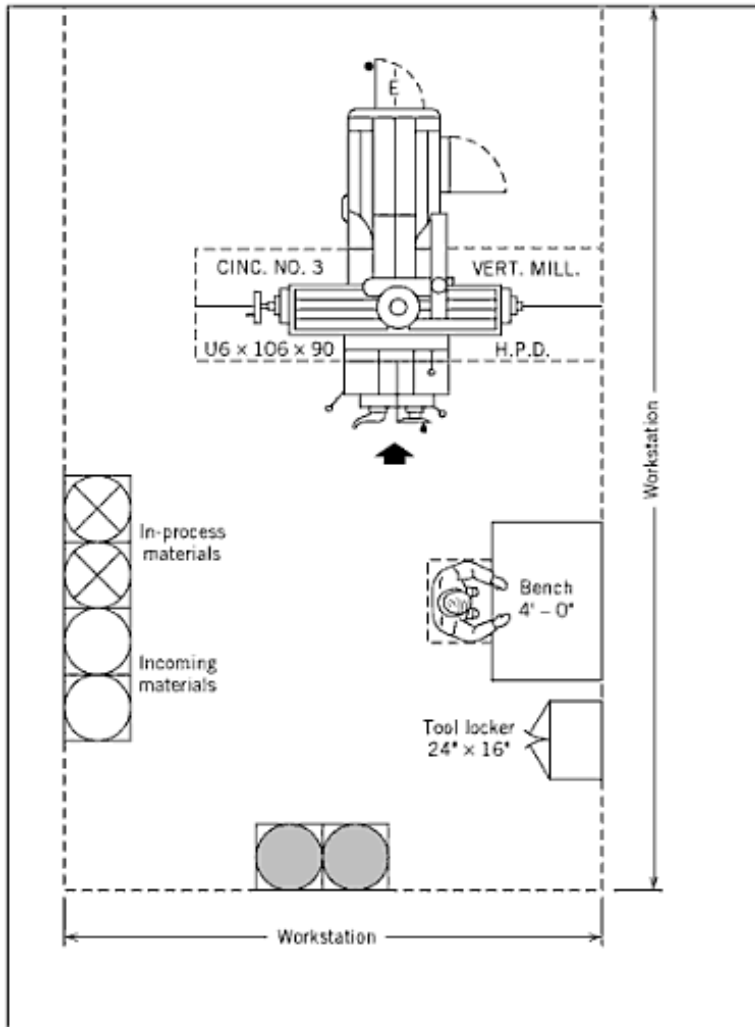


Figure 5.24. Workstation sketch required to determine total area requirements

5.7.2. Department Specification

Once the space requirements for individual workstations have been determined, the space requirements for each department can be established. To do this, we need to establish the departmental service requirements.

Departmental area requirements are not simply the sum of the areas of the individual workstations included with the department. It is quite possible tools, dies, equipment maintenance, plant services, housekeeping items, storage areas, operators, spare parts-kanban boards, information-communication-recognition boards, problem boards, and andons may be shared to save space and resources (figure 5.25).

Additional space is required within each department for material handling within the department. Aisle space requirements can be approximated, since relative sizes of the loads to be handled are known (table 5.3).

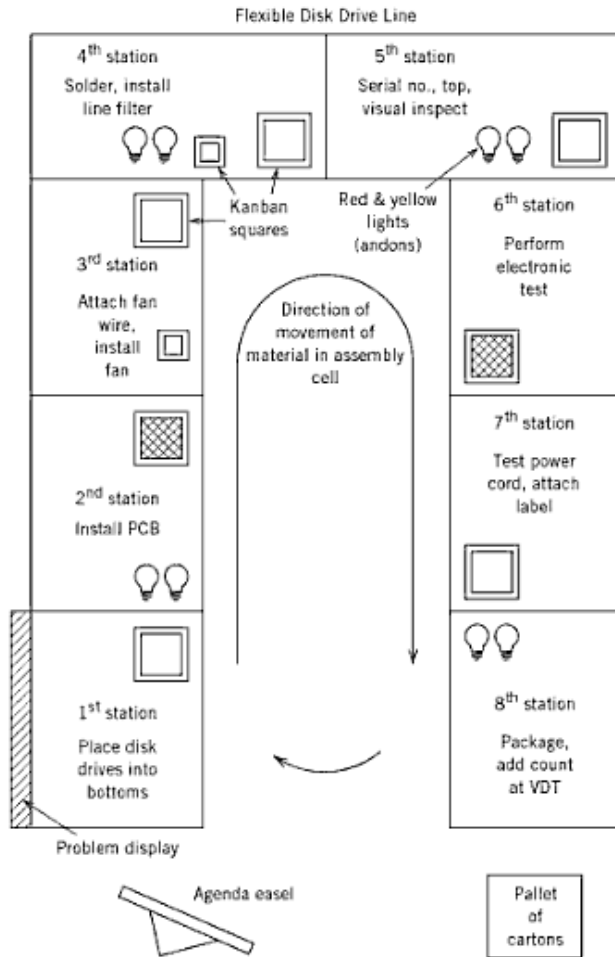


Figure 5.25. An assembly cell

If the largest load is	Aisle Allowance Percentage Is
Less than 6 ft ²	5-10
Between 6 and 12 ft ²	10-20
Between 12 and 18 ft ²	20-30
Greater than 18 ft ²	30-40

Table 5.3. Aisle allowance estimates

Example:

A planning department for the ABC Company consists of 13 machines that perform turning operations. Five turret lathes, six automatic screw machines, and two chuckers are included in the planning department.

Bar stock, in 8-ft bundles, is delivered to the machines. The “footprints” for the machines are 4 × 12 ft for turret lathes, 4 × 14 ft for screw machines, and 5 × 6 ft for chuckers. Personnel space footprints of 4 × 5 ft are used.

Materials storage requirements are estimated to be 20 ft² per turret lathe, 40 ft² per screw machine, and 50 ft² per chucker.

A aisle space allowance 13% is used. The space calculations are summarized in figure 15 a total of 1447 ft² of floor space is required for the planning department.

DEPARTMENTAL SERVICE AND AREA REQUIREMENTS SHEET											
Company		A.B.C., Inc.			Prepared by		J.A.				
Department		Turning			Date		Sheet 1 of 1				
Service Requirements											
Work Station	Quantity	Compressed			Floor Loading	Ceiling Height	Area (square feet)				
		Power	Air	Other			Equipment	Material	Personnel	Total	
Turret lathe	5	440 V AC	10 CFM @ 100 psi		150 PSF	4'	240	100	100	440	
Screw machine	6	440 V AC	10 CFM @ 100 psi		190 PSF	4'	280	240	120	640	
Chucker	2	440 V AC	10 CFM @ 100 psi		150 PSF	5'	60	100	40	200	
										Net area required	1280
										13% aisle allowance	167
										Total area required	1447

Figure 5.26. Department service and area requirement sheet

5.7.3. Aisle Arrangement

Aisles should be located in a facility to promote effective flow. Aisles may be classified as departmental aisles and main aisles.

Planning aisles that are too narrow may result in congested facilities having high levels of damage and safety problems. Conversely, planning aisles that are too wide may result in wasted space and poor housekeeping practices.

Aisles widths should be determined by considering the type and volume of flow to be handled by the aisle. The type of flow may be specified by considering the people and equipment types using the aisle.

Type of Flow	Aisle Width (feet)
Tractors	12
3-ton forklift	11
2-ton forklift	10
1-ton forklift	9
Narrow aisle truck	6
Manual platform truck	5
Personnel	3
Personnel with doors opening into the aisle from one side	6
Personnel with doors opening into the aisle from two sides	8

Table 5.4. Recommended aisle widths for various types of flow

CHAPTER 6

PERSONNEL REQUIREMENTS

The planning of personnel requirements includes planning for:

- Employee parking,
- Locker rooms,
- Restrooms,
- Food services,
- Drinking fountains, and
- Health services.

Personnel requirements became more important with the advent of the 1989 American with Disabilities Act (ADA) which addresses barrier-free designs.

Philosophies relating to personnel:

- “Our firm is responsible for our employees from the moment they leave their home until they return. We must provide adequate methods of getting to and from work”.
- “Employees spend one third of their life within our facility; we must help them enjoy working here”.

Assignment: Investigate the working conditions and personnel requirements at Googleplex, San Jose, CA.

6.1. The Employee-Facility Interface

An interface between an employee’s work and nonwork activities must be provided. The interface functions as a storage area for personal property of the employee during work hours. Personal property typically includes the automobile and the employee’s personal belongings, such as coats, clothes, purses, and lunches.

6.1.1. Employee Parking

The procedure of planning employee parking:

1. Determine the number of automobiles to be parked.
2. Determine the space required for each automobile.
3. Determine the available space for parking.
4. Determine alternative parking layouts for alternative parking patterns.
5. Select the layout that best utilizes space and maximizes employee convenience.

Care must be used when determining the number of automobiles to be parked. For remote sites not being serviced by public transportation, a parking space may be

required for every 1.25 employees. At the other extreme, a centralized location served by public transportation may require a parking space for every three employees.

The number of parking spaces to be provided must be specifically determined for each facility and must be in accordance with local zoning regulations. Attention should be paid to the requirement for handicapped parking. Although minimum requirements can be as low as two handicapped spaces per 100 parking spaces, five handicapped spaces per 100 parking spaces is not uncommon.

The size of parking space for an automobile can vary from 7×15 ft to 9.5×19 ft, depending on the type of automobile and the amount of clearance to be provided. The total area required for a parked automobile depends on the size of the parking space, the parking angle and the aisle width (figure 1).

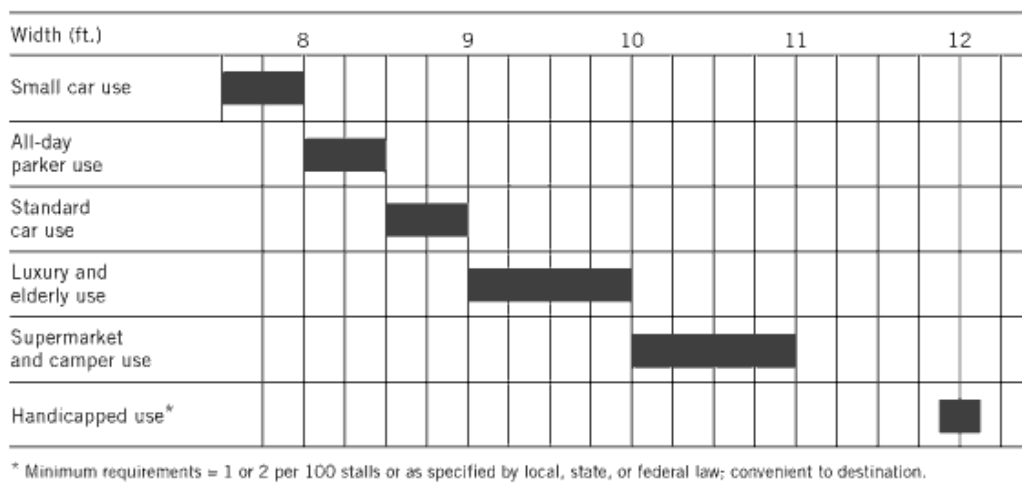


Figure 6.1. Recommended parking dimensions.

The factors to be considered in determining the specification for a specific parking lot are:

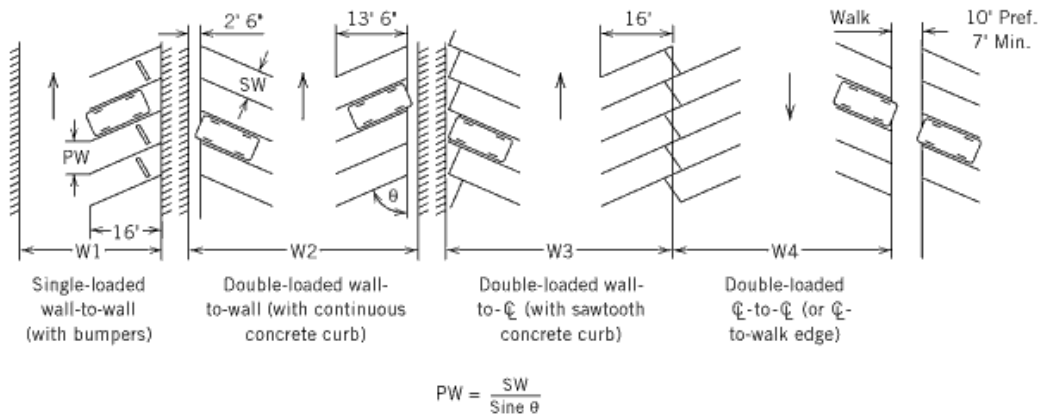
1. The percentage of automobiles to be parked that are compact automobiles. As a planning guideline, if more specific data are not available, 33% of all parking is often allocated to compact automobiles.
2. Increasing the area provided for parking decreases the amount of time required to park and de-park.
3. Angular configurations allow quicker turnover; perpendicular parking yields greater space utilization.
4. As the angle of parking space increases, so does the required space allocated to aisles.

In table 1, there are 3 car groups, (G1-small cars, G2-standard cars, G3-large cars). For a given groups, there are corresponding stall widths (SW) options. For each stall width option, there are 4 configurations; W1, W2, W3, and W4 (figure 2). Each configuration for a given SW has 10 corresponding angles of park, “ θ ” and a associated “W” dimension.

Using the information from figures 6.1, 6.2 and table 6.1, the facilities planner can generate several parking layout alternatives that will optimize the space allocated for parking and maximize employee convenience.

θ ANGLE OF PARK												
	SW	W	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
Group I: Small Cars	8'0"	1	25'9"	26'6"	27'2"	29'4"	31'9"	34'0"	36'2"	38'2"	40'0"	41'9"
		2	40'10"	42'0"	43'11"	45'8"	48'2"	50'6"	52'7"	54'4"	55'11"	57'2"
		3	38'9"	40'2"	41'5"	44'2"	47'0"	49'6"	51'10"	53'10"	55'8"	57'2"
		4	36'8"	38'3"	39'9"	42'9"	45'9"	48'6"	51'1"	53'4"	55'5"	57'2"
Group II: Standard Cars	8'6"	1	32'0"	32'11"	34'2"	36'2"	38'5"	41'0"	43'6"	45'6"	46'11"	48'0"
		2	49'10"	51'9"	53'10"	56'0"	58'4"	60'2"	62'0"	63'6"	64'9"	66'0"
		3	47'8"	49'4"	51'6"	54'0"	56'6"	59'0"	61'2"	63'0"	64'6"	66'0"
		4	45'3"	46'10"	49'0"	51'8"	54'6"	57'10"	60'0"	62'6"	64'3"	66'0"
	9'0"	1	32'0"	32'9"	34'0"	35'4"	37'6"	39'8"	42'0"	44'4"	46'2"	48'0"
		2	49'4"	51'0"	53'2"	55'6"	57'10"	60'0"	61'10"	63'4"	64'9"	66'0"
		3	46'4"	48'10"	51'4"	53'10"	56'0"	58'8"	61'0"	63'0"	64'6"	66'0"
		4	44'8"	46'6"	49'0"	51'6"	54'0"	57'0"	59'8"	62'0"	64'2"	66'0"
	9'6"	1	32'0"	32'8"	34'0"	35'0"	36'10"	38'10"	41'6"	43'8"	46'0"	48'0"
		2	49'2"	50'6"	51'10"	53'6"	55'4"	58'0"	60'6"	62'8"	64'6"	65'11"
		3	47'0"	48'2"	49'10"	51'6"	53'11"	57'0"	59'8"	62'0"	64'3"	65'11"
		4	44'8"	45'10"	47'6"	49'10"	52'6"	55'9"	58'9"	61'6"	63'10"	65'11"
Group III: Large Cars	9'0"	1	32'7"	33'0"	34'0"	35'11"	38'3"	40'11"	43'6"	45'5"	46'9"	48'0"
		2	50'2"	51'2"	53'3"	55'4"	58'0"	60'4"	62'9"	64'3"	65'5"	66'0"
		3	47'9"	49'1"	52'3"	53'8"	56'2"	59'2"	61'11"	63'9"	65'2"	66'0"
		4	45'5"	46'11"	49'0"	51'8"	54'9"	58'0"	61'0"	63'2"	64'10"	66'0"
	9'6"	1	32'4"	32'8"	33'10"	34'11"	37'2"	39'11"	42'5"	45'0"	46'6"	48'0"
		2	49'11"	50'11"	52'2"	54'0"	56'6"	59'3"	61'9"	63'4"	64'8"	66'0"
		3	47'7"	48'9"	50'2"	52'4"	55'1"	58'4"	60'11"	62'10"	64'6"	66'0"
		4	45'3"	46'8"	48'5"	50'8"	53'8"	57'0"	59'10"	62'2"	64'1"	66'0"
	10'0"	1	32'4"	32'8"	33'10"	34'11"	37'2"	39'11"	42'5"	45'0"	46'6"	48'0"
		2	49'11"	50'11"	52'2"	54'0"	56'6"	59'3"	61'9"	63'4"	64'8"	66'0"
		3	57'7"	48'9"	50'2"	52'5"	55'1"	58'4"	60'11"	62'11"	64'6"	66'0"
		4	45'3"	46'8"	48'5"	50'8"	53'8"	57'0"	59'10"	62'2"	64'1"	66'0"

Table 6.1. Parking dimensions for each car group as a function of single and double loaded model



θ is the parking angle, PW is parking width, and SW is the stall width. At an angle of 90° (sine $90^\circ = 1$), $PW = SW$. As the parking angle decreases, PW increases accordingly.

Figure 6.2. Single- and double-loaded module options

An important issue related to parking lot planning is the location of facility entrances and exists or ingress and egress conditions. Employees should not be required to walk more than 500 ft from their parking place to the entrance of the facility. The entrances should be convenient not only to their parking locations, but also to their place of work.

Example:

A new facility is to have 200 employees. A survey of similar facilities that one parking space must be provided for every two employees and that 40% of all automobiles driven to work are compact automobiles. Five percent of the spaces should be allocated for the handicapped. The available parking lot space is 180 ft and 200 ft deep. What is the best parking layout?

Solution:

If the facility were to have the same number of parking spaces as similar facilities, 100 spaces would be required. Of these 100 spaces, 40 could be for compact automobiles. However, not all drivers of compact cars will park in a compact space. Therefore, only 30 compact spaces will be provided.

Begin the layout of the lot using 90° double-loaded, two-way traffic because of its efficient use of space to determine if the available lot is adequate. From figure 2, W4 is the required module option. Using the W4 module and table 1, we can obtain the following:

Compact cars (8'0")	Module width
90° , W4	57'2"
Standard cars (8'6")	66'0"
90° , W4	

Check to see if the depth of the lot (200ft) can accommodate a parking layout consisting of 2 modules of standard cars and 1 compact module,

$$2(66 \text{ ft}) + 1(57'2") = 189'2"$$

$$189'2" < 200 \text{ ft, therefore depth requirement OK.}$$

Each compact module row will yield a car capacity based on the width of the lot (180 ft) divided by the width requirement per stall (8') times the rows per module (2).

$$\left(\frac{180}{8}\right) \times 2 = 44 \text{ potential number of compact cars}$$

Similarly, each standard module row will yield a car capacity based on the width of the lot (180 ft) divided by the width requirement per stall (8.5') times the number of rows per module (2) times the number of modules (2).

$$\left(\frac{180}{8.5}\right) \times 2 \times 2 = 84 \text{ potential number of standard cars}$$

Total possible = 44 + 84 = 128, which is greater than the required number. Therefore, module configuration (W4) is feasible. A possible alternative of (2 rows/modules × 2 standard rows) + (2 rows/module × 1compact row) for a total of six rows is a starting point for the layout.

Modifying the layout to account for handicap requirements and circulation reveals the following:

Row 1 will handle all five handicap spaces = 5(12') = 60

The remaining space will be occupied by standard cars

$$(180-60)/8.5 = 14 \text{ spaces}$$

Row adjusting for two circulation lanes of 15' each number 2 will handle

$$[180 - (15 \times 2)]/8.5 = 17 \text{ spaces}$$

Row 3 and 4 will yield the same number of spaces

Row 5 will have (180-30)/8 = 18 spaces

Row 6 will handle 180/8 = 22 spaces

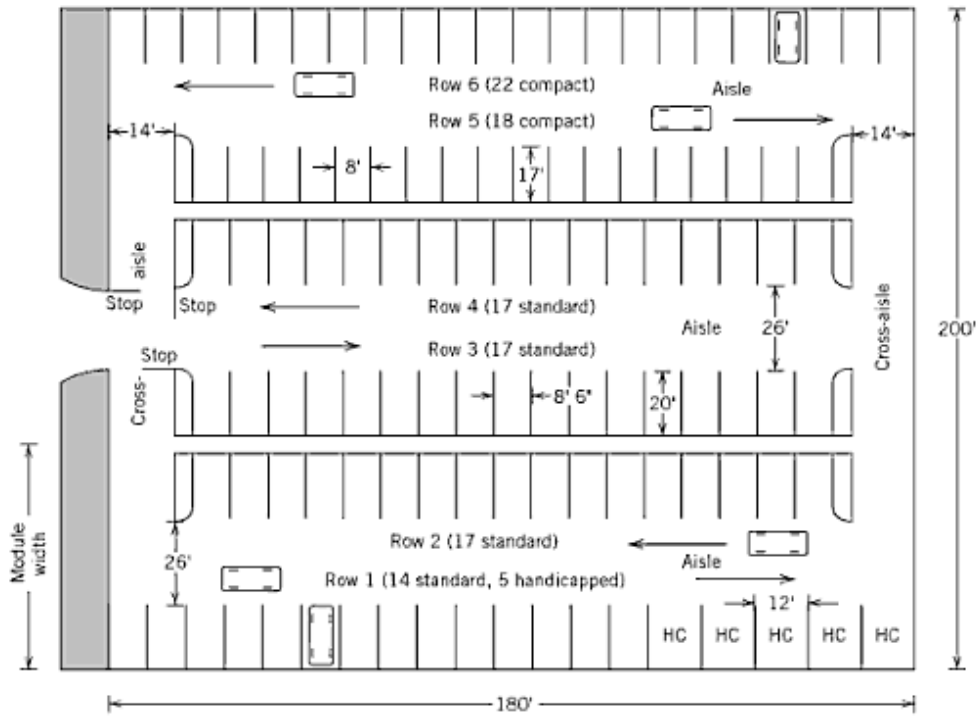


Figure 6.3. Parking lot for the example

The assignment of compact, standard, and handicap spaces is as follows:

Row	Compact	Standard	Handicap
#1		14	5
#2		17	
#3		17	
#4		17	
#5	18		
#6	<u>22</u>		
Total	40	<u>65</u>	<u>5</u>

6.1.2. Storage of Employees Personal Belongings

A location for storage of employee personal belongings should be provided between the employee entrance and work area. Employees typically store lunches, briefcases, and purses at their place of work.

Employees who are not required to change their clothes and who work in an environment where toxic substances do not exist need only to be provided with a coat rack. Employees who either change their clothes or work where toxic substances are present should be provided with lockers.

The lockers may be located in a corridor adjacent to the employee entrance if clothes changing does not take place. More commonly, locker rooms are provided for each sex even if clothes changing is not required. Each employee should be assigned a

locker. For planning purposes, 6 ft² should be allocated for each person using the locker room.

If shower facilities are to be provided, they should be located in the locker room. Sinks and mirrors are also typically included there. If toilet facilities are to be included, they must be physically separated from the locker room area (should lunches be stored in the locker).

Locker rooms are often located along an outside wall adjacent to the employee entrance. This provides excellent ventilation and employee convenience while not interfering with the flow of work within the facility.

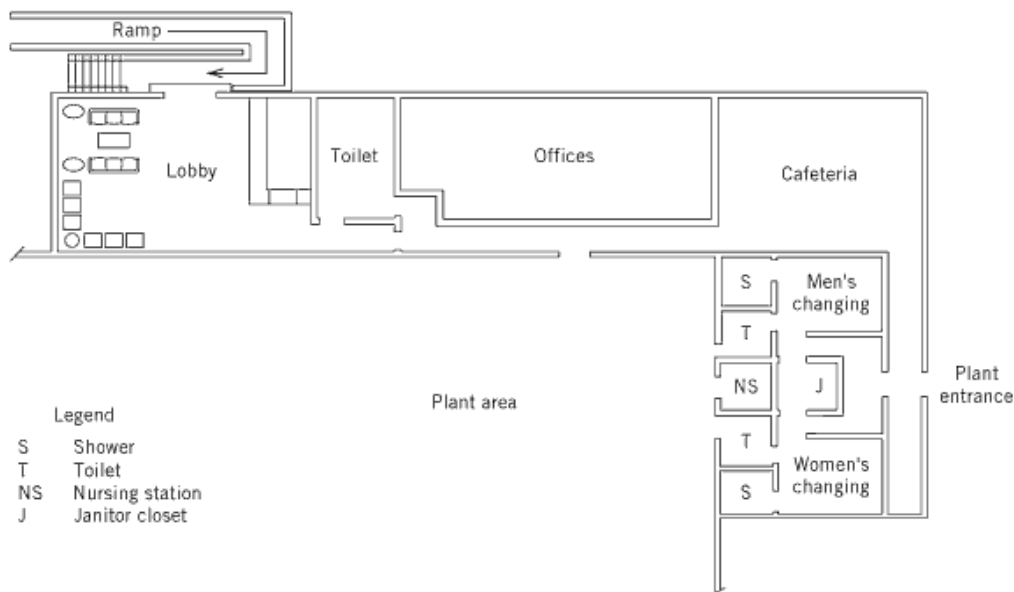


Figure 6.4. Plant entrance and changing room layout

6.2. Restrooms

A restroom should be located within 200 ft of every permanent workstation. Decentralized restrooms often provide greater employee convenience than large, centralized restrooms. Access to restrooms must be available to handicapped employees. Hence, some restrooms must be at ground level.

Unless restrooms are designed for single occupancy, separate restrooms should be provided for each sex. The recommended minimum number of toilets for the number of employees working within a facility is given in table 6.1.

Business, Mercantile, Industrial other than Foundry and Storage					
Water Closets	Employees	Lavatories	Employees		
1	1-15	1	1-20		
2	16-35	2	21-40		
3	36-55	3	41-60		
4	56-80	4	61-80		
5	81-110	5	81-100		
6	111-150	6	101-125		
7	151-190	7	126-150		
		8	151-175		
One additional water closet for each 40 employees in excess of 190.		One additional lavatory for each 30 employees in excess of 175.			
Industrial, Foundries, and Storage					
Water Closets	Employees	Lavatories	Employees		
1	1-10	1	1-8		
2	11-25	2	9-16		
3	26-50	3	17-30		
4	51-80	4	31-45		
5	81-125	5	46-65		
One additional water closet for each 45 employees in excess of 125.		One additional lavatory for each 25 employees in excess of 65.			
Assembly, Other Than Religious, and Schools					
Water Closets	Occupants	Urinal	Male Occupants	Lavatories	Occupants
1	1-100	1	1-100	1	1-100
2	101-200	2	101-200	2	101-200
3	201-400	3	201-400	3	201-400
4	401-700	4	401-700	4	401-700
5	701-1100	5	701-1100	5	701-1100
One additional water closet for each 600 occupants in excess of 1100.		One additional urinal for each 300 occupants in excess of 1100.		One additional lavatory for each 1500 occupants in excess of 1100. Such lavatories need not be supplied with hot water.	

Table 6.2. Plumbing fixture requirements for number of employees

In restrooms for males, a urinal may be substituted for a toilet, provided that the number of toilets is not reduced to less than two-thirds the minimum recommended in table 1.

For each space planning purposes, 15 ft² should be allowed for each toilet and 6 ft² for each urinal. Toilets and urinal must be designed to accommodate wheelchairs for handicapped employees as well.

In no restroom should less than one sink per three toilets be provided. When multiple users may use a sink at a time, 24 linear inches of sink or 20 inches or circular basin may be equated to one sink. For planning purposes, 6 ft² should be allowed for each sink.

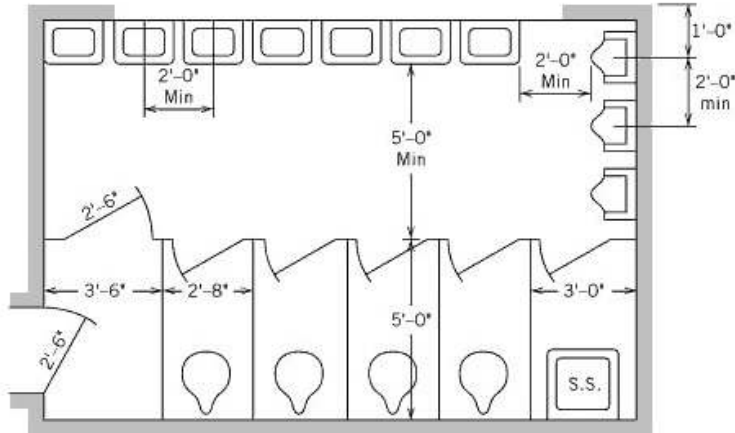


Figure 6.5. Restroom layout with typical clearances

Entrance doorways into restrooms should be designed such that the interior of the restroom is not visible from the outside when the door is open. A space allowance of 15 ft² should be used for the entrance. A sample layout of a commercial restroom is shown in figure 2.

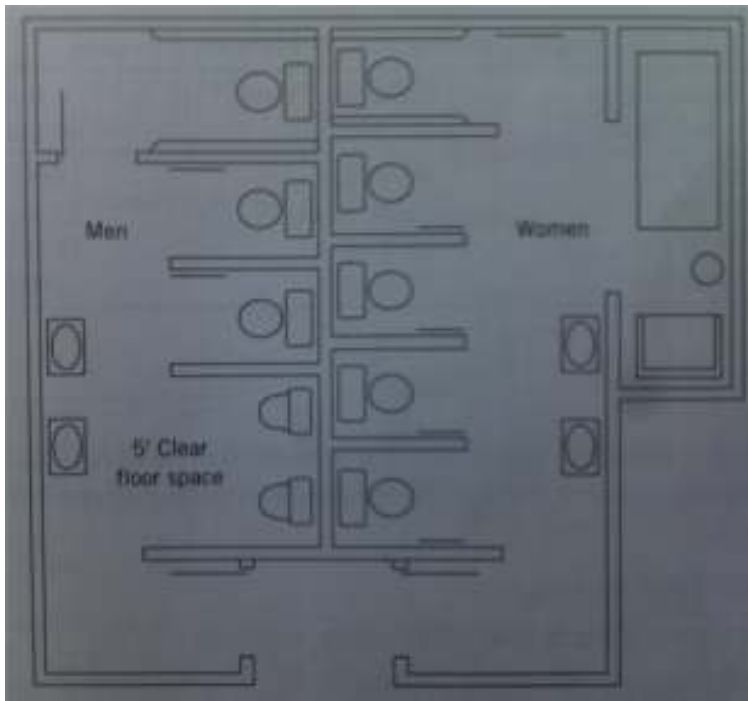


Figure 6.6. Typical commercial restroom layout

Beds or cots should be provided in restrooms for women. The area should be segregated from the restroom by a partition or curtain. If between 100 and 250 women are employed, two beds should be provided. One additional bed should be provided for each additional 250 female employees. A space allowance of 60 ft² should be used for each bed.

6.3. Food Services

Food service activities may be viewed by a firm as a necessity, a convenience, or a luxury. The viewpoint adopted, as well as a firm's policy on off-promises dining, subsidizing the costs of meals and the amount of time allowed for meals, has a significant impact on the planning of food service facilities.

Food service facilities should be planned by considering the number of employees who eat in the facilities during peak activity time. Kitchen facilities should be planned by considering the total number of meals to be served.

If employees eat in shifts, the first third of each shift will typically be used by the employee preparing to eat and obtaining the meal. The remainder of the time will be spent at a table eating. Therefore, if a 30-min meal break is planned, dining shifts, as shown in table 2, may begin every 20 min. In a like manner, if a 45-min. break is planned, shifts may begin every 30 min.

Beginning of Lunch Break	Time Sat Down in Chair	End of Lunch Break
11:30 A.M.	11:40 A.M.	12:00 Noon
11:50 A.M.	12:00 Noon	12:20 P.M.
12:10 P.M.	12:20 P.M.	12:40 P.M.
12:30 P.M.	12:40 P.M.	1:00 P.M.

Table 6.3. Shifting timing for 30-min lunch breaks

Food services requirements may be satisfied by any of the following alternatives:

1. Dining away from the facility
2. Vending machines and cafeteria
3. Serving line and cafeteria
4. Full kitchen and cafeteria

The first alternative certainly simplifies the task of the facilities planner. However, requiring employees to leave the facility for meals would result in the following disadvantages:

- Longer meal breaks,
- Lost supervision,
- A loss of worker interaction,
- A loss of worker concentration on the tasks to be performed.

For each of the three remaining food service alternatives, a cafeteria is required. Cafeterias should be designed so that employee can relax and dine conveniently.

An integral part of the cafeteria is the food preparation or serving facilities. The option of a serving line or full-service kitchen will be contingent on the number of employees to be served. If a facility employs over 200 people, a serving line is a feasible alternative.

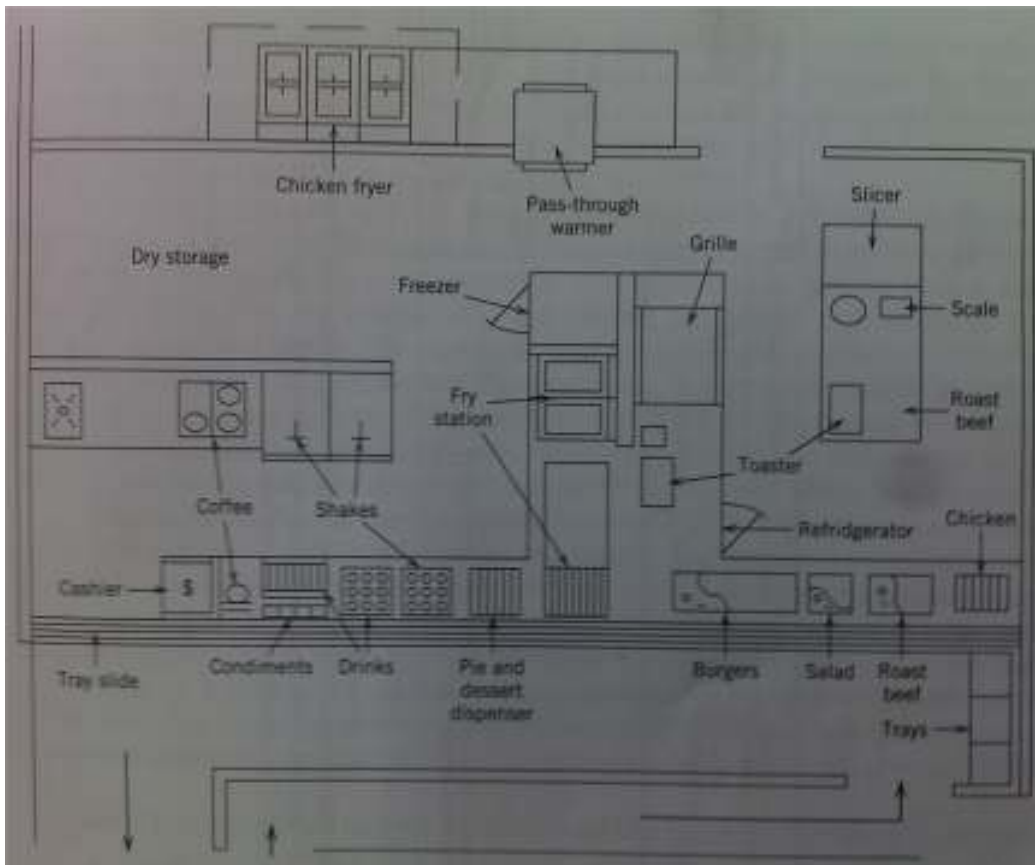


Figure 6.7. An efficient serving line

Space requirements for cafeterias should be based on the maximum number of employees to eat in the cafeteria at any one time.

Classification	Square Footage Allowance per Person
Commercial	16-18
Industrial	12-15
Banquet	10-11

Table 6.4. Space requirements for cafeterias

Popular table sizes are 36-, 42-, and 48- in. square tables and rectangular tables 30 in. wide and 6, 8, and 10 ft long. Square tables require more aisle space than rectangular tables, but results in more attractive cafeterias.

Table sizes depend on whether or not employees retain their trays during the meal. A 36-in. square table is adequate for four employees if they do not retain their trays. Standard trays are 14×18 in. therefore, a 48-in. square table is most suitable if employees retain their trays.

Rectangular tables 6, 8, and 10 ft long adequately seat three, four, and five employees, respectively, on each side of the table with no end seats.

The use of vending machines is the least troublesome way of providing food services for employees. It is also the most flexible on-site food service alternatives. Employees

not wishing to purchase their lunches typically feel more at ease if vending machines are utilized than if serving lines or full kitchens are provided. For space planning purposes 1 ft² per person should be allowed for the vending machine area, based on the maximum number of persons eating at one time.



Figure 6.8. Typically institutional vending area

If a facility employs over 200 people, a serving line is a feasible alternative. When a serving line is utilized, a caterer is frequently contracted to prepare all food off site and to serve the food to the employees. The cost of a meal is often quite competitive with the cost of running a full kitchen for facilities employing less than 400 people. A typical industrial service line requires 300 ft² and can service seven employees per minute.

When a full kitchen is used, a serving line (figure 3) and a kitchen must be included in the facility. A full kitchen can usually be justified economically if there are over 400 employees within a facility.

Space planning for kitchens to include space for food storage, food preparation- and dishwashing should be based on the total number of meals to be prepared.

Number of Meals Served	Area Requirements (ft ²)
100–200	500–1000
200–400	800–1600
400–800	1400–2800
800–1300	2400–3900
1300–2000	3250–5000
2000–3000	4000–6000
3000–5000	5500–9250

Table 6.5. Space required for full kitchens

Example:

If a facility employ 600 people and they are to eat in three equal 30-min shifts, how much space should be planned for a cafeteria with vending machines, serving lines, or a full kitchen?

Solution:

If 48-in. square tables are to be utilized, table 3 indicated 12 ft² are required for each of the 200 employees to eat per shift. Therefore, a 2400-ft² cafeteria should be planned.

If a vending machine area is to be used in conjunction with the cafeteria, an area of 200 ft² should be allocated for vending machines. Thus, a vending machine food service facility would require 2600 ft².

A serving line may serve 70 employees in the first third of each meal shift. Therefore, three serving lines of each 300 ft² each should be planned. A total of 3300 ft² should be required for a food service facility using serving lines.

A full kitchen will require 3300 ft² for serving lines plus (from table 4) 2400 ft² for the kitchen. Therefore, a total of 5700 ft² would be required for a full service food service facility.

6.4. Health Services

Some facilities have little more than a well-supplied first aid kit, while other facilities have small hospital. Therefore, local building codes should be checked in establishing a facility's requirements. The types of health services that may be provided within a facility include

1. Pre-employment examinations,
2. First aid treatment,
3. Major medical treatment,
4. Dental care, and
5. Treatment of illnesses.

The facilities planner should check the firm's operating procedure to determine what types of services are to be offered and what health services staff is to be housed within the facility.

At the very least, a small first aid room should be included. The minimal requirements for a first aid room are an approved first aid kit, a bed, and two chairs. A minimum of 100 ft² is required. If a nurse is to be employed, the first aid room should have two beds and should be expanded to 250 ft².

In addition, a 75 ft² waiting room should be included. For each additional nurse to be employed, 250 ft² should be added to the space requirements for the first aid room and 25 ft² should be added to the space requirements for the waiting room.

If a physician is to be employed on a part-time basis to perform pre-employment physicals, a 150 ft² examination room should be provided. If physicians are to be employed on a full-time basis the space requirement should be planned in conjunction with a physician, based on the types of services to be offered.

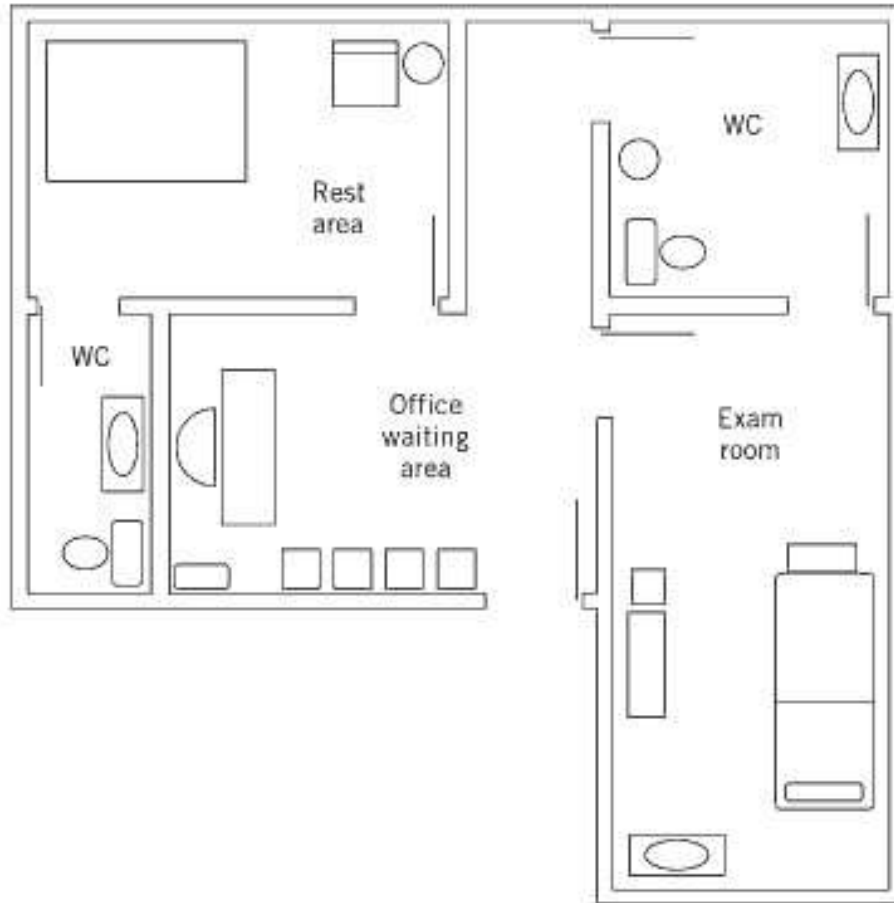


Figure 6.9. Nursing station layout

6.5. Barrier-Free Compliance

Facilities planner must incorporate the intent of the Americans with Disabilities Act (ADA). The intent is to ensure that disabled persons shall have the same rights as the able-bodied to the full and free use of all facilities that serve the public. To this extend, all barriers that would impede the use of the facility by the disabled person must be removed, thereby making the facility barrier free.

What are considered barriers? A barrier is a physical object that impedes a disabled person's access to the use of a facility, for example, a door that is not wide enough to accommodate a wheel chair or stairs without ramp access to the facility.

The facilities planner must recognize that this applies to all public facility use groups:

- Assembly
- Business
- Educational
- Factories and industrial
- Institutional.

The ADA will fundamentally impact the way industrial engineers approach the design of a facility – from the parking lot to entering the facility and exiting the facility, moving within the interior of the facility, workstations, offices, and restrooms.

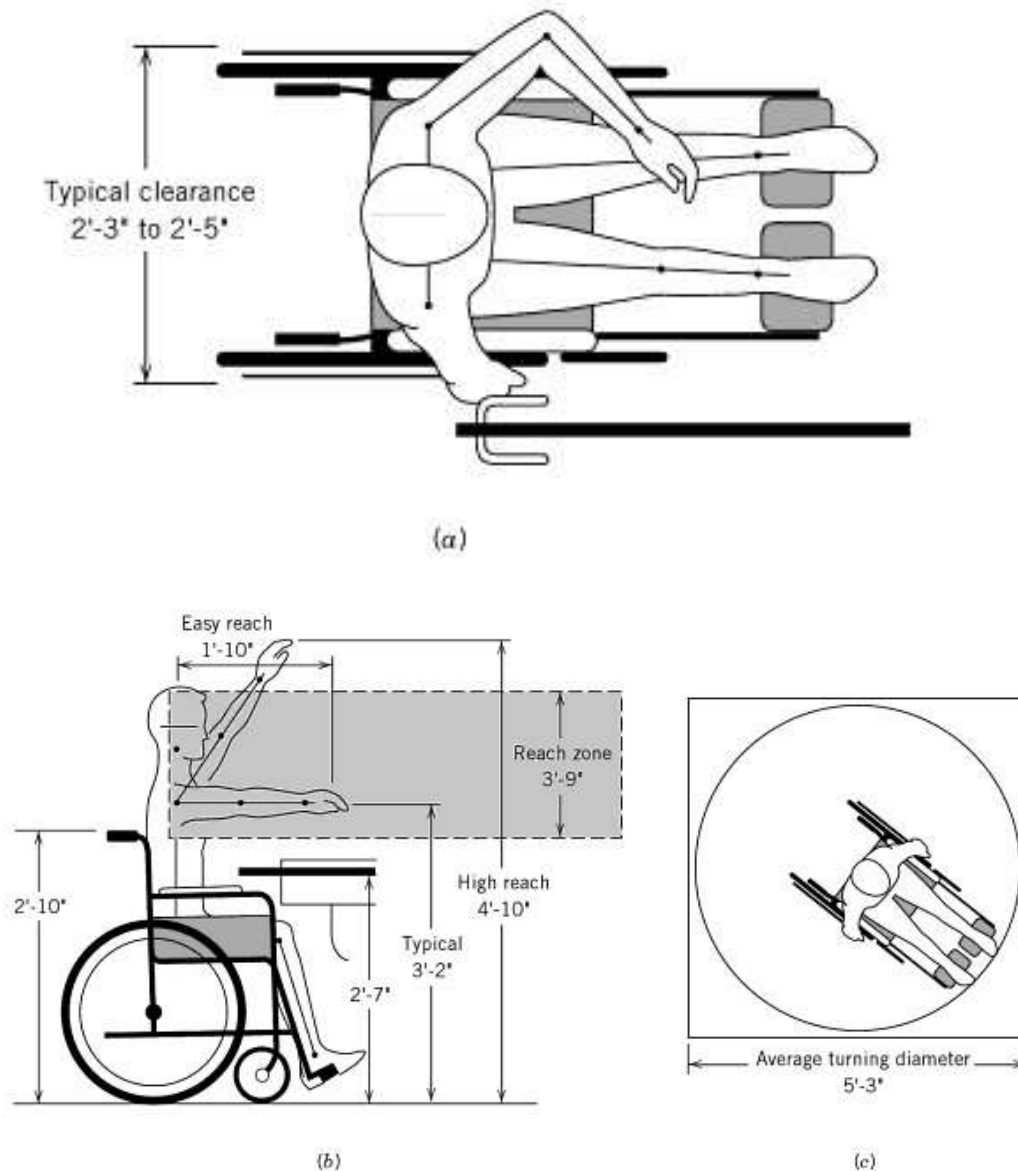


Figure 6.10. Wheel chair dimensions and turning radius

Compare these dimensions in figure 6.10 with the dimension of an able-bodied person's typical clearance and reach requirements as given in figure 6.11.

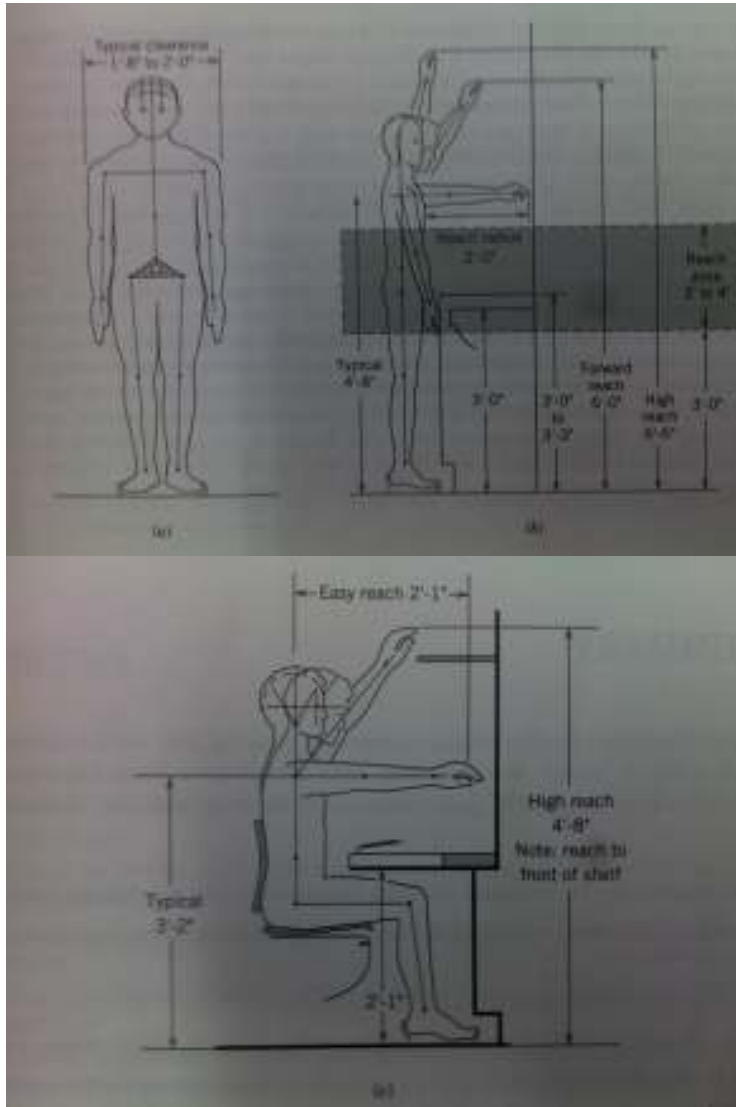


Figure 6.11. Able-bodied anthropometric clearance and reach requirements in standing and sitting positions

Although there are significant physical differences between able-bodied and physically disadvantaged individuals, there exists a reach zone where both groups can comfortably access objects placed in this zone (figure 6.10.b and 6.11.b). This zone, as shown, is typically 3 ft to 4 ft above floor level.

From analyzing the clearances for both groups, it is obvious that facility entrances, doors, hallways, and so on must be wide enough to accommodate the wheel chair, typically 3 ft minimum. Also, fixed facility elements in laboratories or other work study areas using workbenches requires a minimum clear width of 3 ft. additionally, by mapping the reach requirement of the average person against that of the handicapped, there exists a zone 3 ft to 4 ft where both groups can comfortably access items.

CHAPTER 7

MATERIAL HANDLING

The design of the material handling system is an important component of the facilities design problem. There exists a strong relationship between the layout design and the material handling design function.

7.1. Definition of Material Handling

In a typical factory handling accounts for 25% of all employees, 55% of all factory space, and 87% of production time. Material handling is estimated to represent between 15 and 70% of the total cost of a manufactured product. Certainly, material handling is one of the first places to look for cost reduction. Material handling is also one of the first places to look for quality improvements. It has been estimated that between 3 and 5% of all material handled becomes damaged.

Material handling is a means by which *total manufacturing costs are reduced* through reduced inventories, improved safety, reduced pilferage, and improved material control. Material handling is a means by which *manufacturing quality is improved* by reducing inventory and damage through improved handling. Finally, material handling is the means by which any production strategy is executed.

7.1.1. Understanding Material Handling

Two key definitions:

1. Material handling is the art and science of moving, storing, protecting, and controlling material.
2. Material handling means providing the right amount of the right material, in the right condition, at the right place, at the right time, in the right position, in the right sequence, and for the right cost, by using the right method(s).

7.2. Material Handling Principles

The material handling principles provide concise statements of the fundamentals of material handling practice. Condensed from decades of expert material handling experience, not unlike the material handling equation, they provide guidance and perspective to material handling system designers.

As with all design tools, the applicability of a material handling principle depends on the conditions that exist; due to many different conditions that might exist, it is unlikely that any principle will always be applicable.

Rather than being design axioms, the principles serve as rough guides or rules of thumb for material handling system design; hence, the principles should not be considered substitutes for sound judgment.

Material Handling Principles:

1. *Planning Principle.* A plan is prescribed course of action that is defined in advance of implementation. In its simplest form, a material handling plan defines the material (what) and the moves (when and where); together they define the method (how and who).
2. *Standardization Principle.* Standardization means less variety and customization in the methods and equipment employed.
3. *Work Principle.* The measure of work is material flow (volume, weight, or count per unit of time) multiplied by the distance moved.
4. *Ergonomic Principle.* Ergonomics is the science that seeks to adapt work or working conditions to suit the abilities of the worker.
5. *Unit Load Principle.* A unit load is one that can be stored or moved as a single entity at one time, such as a pallet, container, or tote, regardless of the number of individual items that make up the load.
6. *Space Utilization.* Space in material handling is three-dimensional and therefore is counted as cubic space.
7. *System Principle.* A system is a collection of interacting and/or interdependent entities that form a unified whole.
8. *Automation Principle.* Automation is a technology concerned with the application of electromechanical devices, electronics, and computer-based systems to operate and control production and service activities. It suggests the linking of multiple mechanical operations to create a system that can be controlled by programmed instructions.
9. *Environmental Principle.* Environmental consciousness stems from a desire not to waster natural resources and to predict and eliminate the possible negative effects of our daily actions on the environment.
10. *Life-Cycle Cost Principle.* Life-cycle cost include all cash flows that will occur from the time the first dollar is spent to plan or procure a new piece of equipment, or to put in place a new method, until that method and/or equipment is totally replaced.

7.3. Material Handling System Design

In designing new or improved material handling systems, the six-phased engineering design process should be used.

1. Define the objectives and scope for the material handling system.
2. Analyze the requirements for moving, storing, protecting, and controlling material.
3. Generate alternative designs for meeting material handling system requirements.
4. Evaluate alternative material handling system designs.
5. Select the preferred design for moving, storing, protecting, and controlling material.
6. Implement the preferred design, including the selection of suppliers, training of personnel, installation, debug and startup of equipment, and periodic audits of system performance.

Throughout the design process, a questioning attitude should prevail. The basic questions of why, what where, when, how, who and which should be asked constantly. In particular, the following questions should be addressed as a minimum:

1. Why
 - a. Is handling required?
 - b. Are operations to be performed as they are?
 - c. Are the operations to be performed in the given sequence?
 - d. Is material received as it is?
 - e. Is material shipped as it is?
 - f. Is material packaged as it is?
2. What
 - a. Is to be moved?
 - b. Data are available and required?
 - c. Alternatives are available?
 - d. Are the benefits and disbenefits (costs) for each alternative?
 - e. Is the planning horizon for the system?
 - f. Should be mechanized/automated?
 - g. Should be done manually?
 - h. Shouldn't be done at all?
 - i. Other firms have related problems?
 - j. Criteria will be used to evaluate alternative designs?
 - k. Exceptions can be anticipated?
3. Where
 - a. Is material handling required?
 - b. Do material handling problem exist?
 - c. Should material handling equipment be used?
 - d. Should material handling responsibility exist in the organization?
 - e. Will future change occur?
 - f. Can operations be eliminated, combined, simplified?
 - g. Can assistance be obtained?
 - h. Should material be stored?
4. When
 - a. Should material be moved?
 - b. Should I automate?
 - c. Should I eliminate?
 - d. Should I expand (contract)?
 - e. Should I consult vendors?
 - f. Should a postaudit of the system be performed?
5. How
 - a. Should material be moved?
 - b. Do I analyze the material handling problem?
 - c. Do I sell everyone involved?
 - d. Do I learn more about material handling?
 - e. Do I choose from among the alternatives available?
 - f. Do I measure material handling performance?
 - g. Should exceptions be accommodated?
6. Who
 - a. Should be handling material?
 - b. Should be involved in designing the system?
 - c. Should be involved in evaluating the system?
 - d. Should be involved in installing the system?
 - e. Should be involved in auditing the system?
 - f. Should be invited to submit equipment quotes?
 - g. Has faced a similar problem in the past?

7. Which
 - a. Operations are necessary?
 - b. Problems should be studied first?
 - c. Type equipment (if any) should be considered?
 - d. Materials should have real-time control?
 - e. Alternative is preferred?

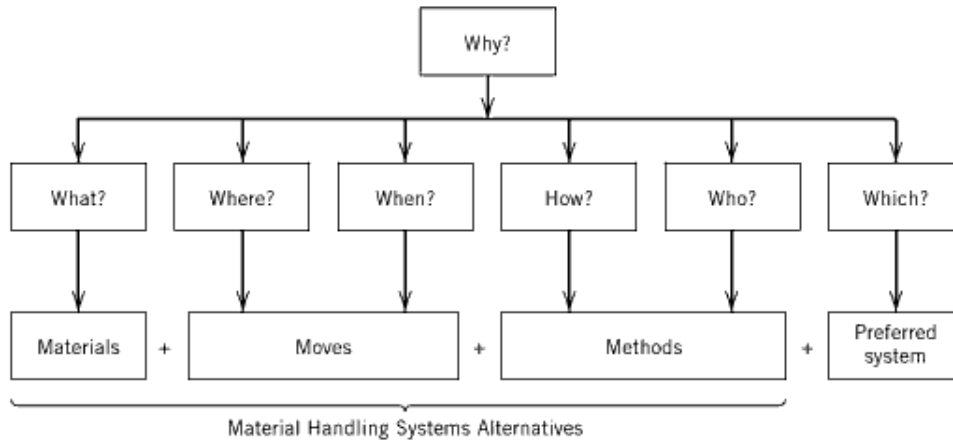


Figure 7.1. Material handling system equation

The combination of material characteristics and move or flow requirements is referred to as **material flow**. Hence, to develop material flow system requirements, one should focus on the material to be handled, stored, and controlled and the flow or throughput requirements for the system. Material flow is transformed to material handling by the method of handling, storing, and controlling the material.

Some frequently encountered reasons for considering changes in the way material is handled by an organization include:

1. reduce costs
2. reduce damage
3. increase space and equipment utilization
4. increase throughput
5. increase productivity
6. improve working conditions

While it is true that material handling improvements can result in the benefits listed above, it is also the case that a number of so-called “improvements” in the material handling system can result in the following disbenefits:

1. increased capital requirements
2. decreased flexibility
3. decreased reliability, maintainability, and operability

An alternative method of representing the importance of the questioning attitude in designing material handling systems is given by the following expression.

$$\sum_{moves} [Why(Where + What + When)]$$

The expression within brackets defines the method of performing each move within a facility. The moves are considered individually and without reference to other moves within a facility. The initial consideration for particular move is “Why should this move be performed?” The multiplication by “Why” represents the initial consideration and work simplification approach to material handling. Each move should be evaluated by asking the following questions:

1. Can the move be eliminated?
2. Can the move be combined with another or within an in-transit operation?
3. Can the move be simplified?
4. Can the sequence of moves be changed to advantages?

7.4. Unit Loads

The concept of a unit load is derived from the unit size principle; a unit load can be defined simply as the unit to be moved or handled at one time. In some cases the unit load is one item of production; in other situations the unit load is several cartons, each containing numerous items of production.

The unit load includes the container, carrier, or support that will be used to move materials. Unit loads consists of material in, on, or grouped together by something. The primary advantage of using unit loads is the capability of handling more items at a time and reducing the number of trips, handling costs, loading and unloading times, and product damage.

The size of the unit load can range from a single carton to an intermodal container (e.g. piggyback trailer). Additionally, the integrity of the unit load can be maintained in a number of ways. As examples, tote boxes, cartons, pallets, and pallet boxes can be used to “contain” the unit load.

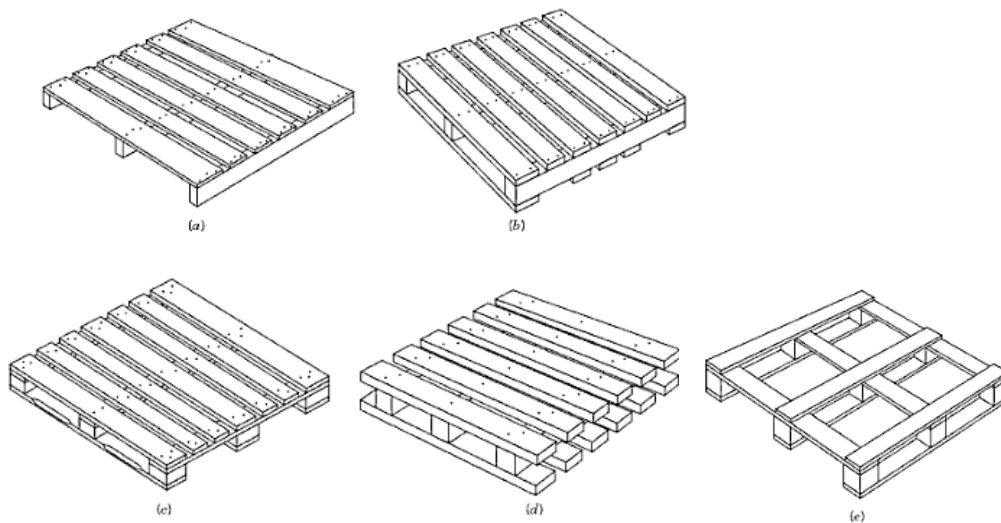


Figure 7.2. Shapes and sizes of pallets

Shapes and sizes of pallets:

- (a) Standard single-deck wooden pallet
- (b) Double-faced nonreversible pallet for pallet truck handling
- (c) Four-way block-leg pallet
- (d) Double-wing-type (stevedore) pallet
- (e) Three-board single-deck expandable shipping pallet

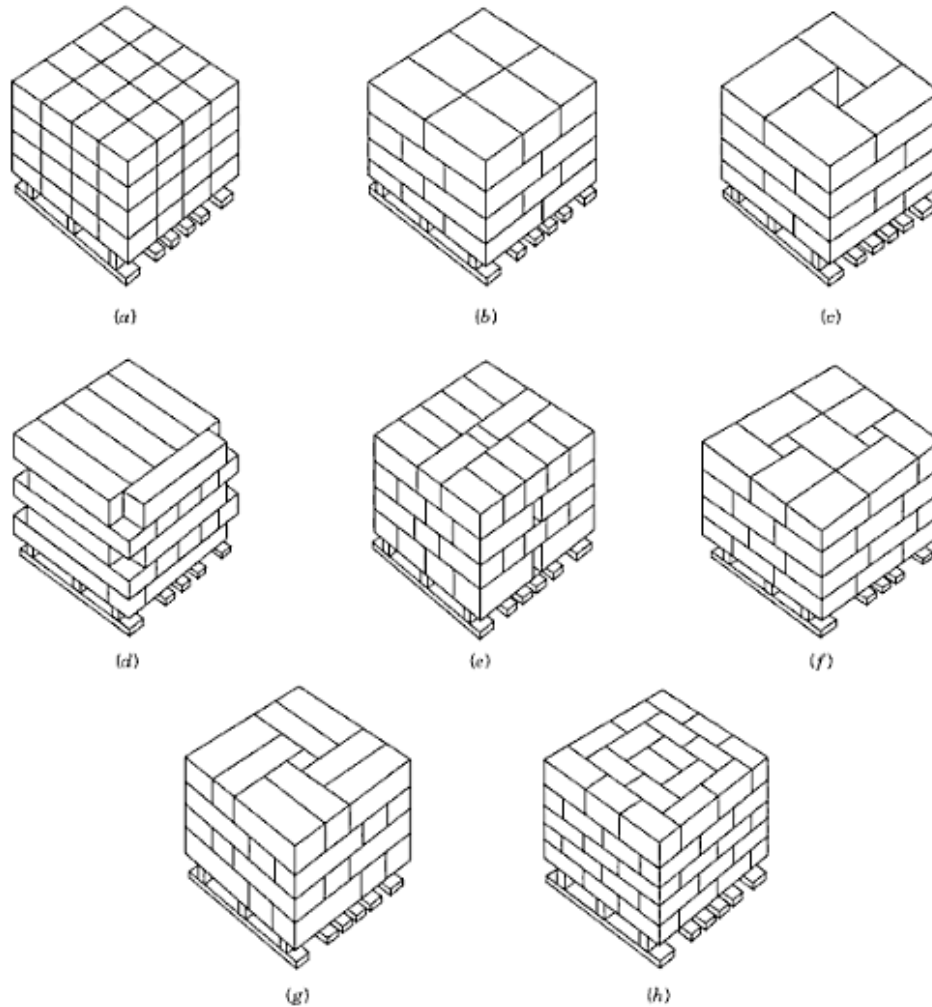


Figure 7.3. Stacking patterns for different pallet sizes

Stacking patterns for different pallet sizes:

- (a) Block pattern
- (b) Row pattern
- (c) Pinwheel pattern
- (d) Honeycomb pattern
- (e) Split-row pattern
- (f) Split-pinwheel pattern
- (g) Split-pinwheel pattern for narrow boxes
- (h) Brick pattern

Among the most popular pallet sizes are the following:

32 × 40 in.	42 × 42 in.
36 × 48 in.	48 × 40 in.
40 × 48 in.	48 × 48 in.

7.5. Material Handling Equipment

Classifications:

I. Containers and Unitizing Equipment

A. Containers

1. Pallets
2. Skids and Skid Boxes

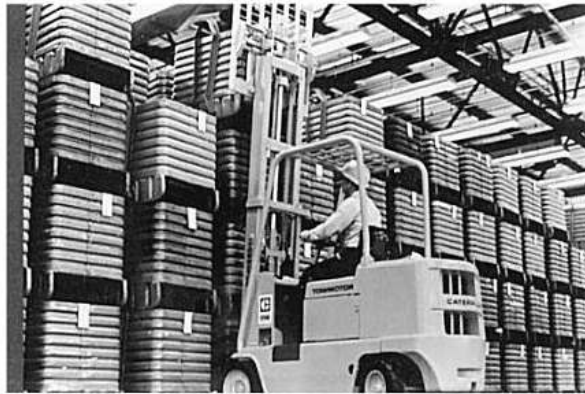


Figure 5.14 Skid boxes.

3. Tote Pans



Figure 5.15 Plastic reusable tote pans. (Courtesy of Buckhorn, Inc.)

B. Unitizers

1. Stretchwrap

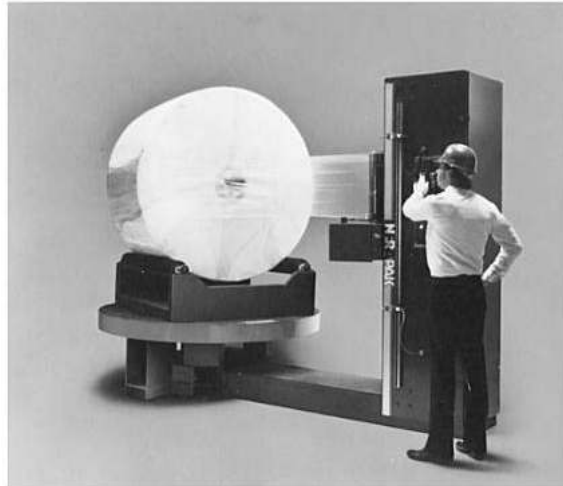


Figure 5.16 Stretchwrap equipment. (Courtesy of Infra Pak.)

2. Palletizers

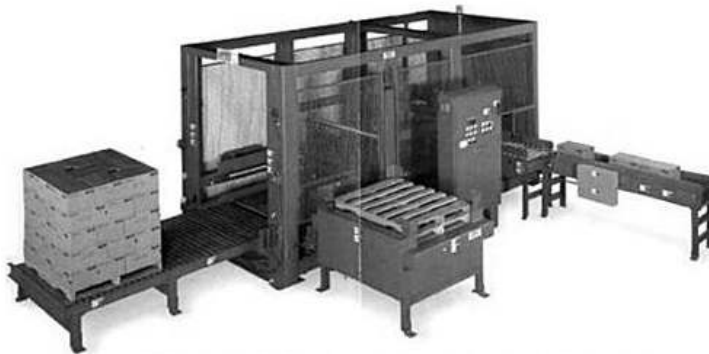


Figure 5.17 Carton palletizer. (Courtesy of Lambert Material Handling.)

II. Material Transport Equipment

A. Conveyors

1. Chute Conveyor

2. Belt Conveyor

a. Flat Belt Conveyor



Figure 5.19 Flat belt conveyor. (Courtesy of Siemens Dematic Corp.)

b. Telescoping Belt Conveyor



Figure 5.20 Telescoping conveyor. (Courtesy of Siemens Dematic Corp.)

c. Troughed Belt Conveyor

d. Magnetic Belt Conveyor



Figure 5.21 Magnetic belt conveyor. (Courtesy of Bunting Magnetics Co.)

3. Roller Conveyor



Figure 5.22 Roller conveyor. (Courtesy of Siemens Dematic Corp.)

4. Wheel Conveyor



Figure 5.23 Slats-wheel conveyor.

5. Slat Conveyor



Figure 5.24 Slat conveyor. (Courtesy of Fredenhagen, Inc. and Acco Babcock.)

6. Chain Conveyor



Figure 5.25 Chain conveyor. (Courtesy of Jervis B. Webb Co.)

7. Tow Line Conveyor



Figure 5.26. Tow line conveyor. (Courtesy of Jervis H. Webb Co.)

8. Trolley Conveyor



Figure 5.27. Trolley conveyor. (Courtesy of PWC/DFE Overhead Conveyors)

9. Power and Free Conveyor



Figure 5.28. Powered push and free conveyor. (Courtesy of Jervis H. Webb Co.)

10. Cart-on-Truck Conveyor



Figure 5.29 Cart-on-truck. (Courtesy of SE Systems, Inc.)

11. Sorting Conveyor

a. Deflector



Figure 5.30 Deflector arm sorter. (Courtesy of Siemens Dematic Corp.)

b. Push Diverter



Figure 5.31 Push diverter. (Courtesy of Embraco, Inc.)

- c. Rake Puller
- d. Moving Slat Conveyor



Figure 5.32 Moving slat sorter. (Courtesy of Siemens Dematic Corp.)

- e. Pop-up Skewed Wheels



Figure 5.33 Pop-up wheel sorter. (Courtesy of Siemens Dematic Corp.)

- f. Pop-up Belts and Chains



Figure 5.34 Pop-up belt. (Courtesy of Ermano, Inc.)

g. Pop-up Rollers



Figure 5.35 - Pop-up chain. (Courtesy of Eaton DHS.)

h. Tilting Slat Conveyor

i. Tilt Tray Sorter

j. Cross Belt Sorter



Figure 5.38 - Cross-belt sorter. (Courtesy of Systems Design, Corp.)

k. Bombardier Sorter



Figure 5.39 - Bombardier sorter. (Courtesy of G&B Data and Sorting Systems Inc.)

B. Industrial Vehicles

1. Walking

a. Hand Truck and Hand Cart

b. Pallet Jack



Figure 5.40 - Pallet jack. (Courtesy of Crown Equipment Corp.)

c. Walkie Stacker



Figure 5.42 Walkie stacker. (Courtesy of Crown Equipment Corp.)



Figure 5.43 Walkie reach straddle truck. (Courtesy of Raymond Corp.)

2. Riding

a. Pallet Truck



Figure 5.44 Stand-up rider pallet truck. (Courtesy of Crown Equipment Corp.)

b. Platform Truck



Figure 5.41 Walkie platform truck. (Courtesy of Yale Material Handling Corp.)

c. Tractor Trailer



Figure 5.45 Stand-up rider tow tractor. (Courtesy of Crown Equipment Corp.)

d. Counterbalanced Lift Truck



Figure 5.46 Sit-down rider counterbalanced lift truck. (Courtesy of Crown Equipment Corp.)

e. Straddle Carrier

f. Mobile Yard Crane

3. Automated

a. Automated Guided Vehicles

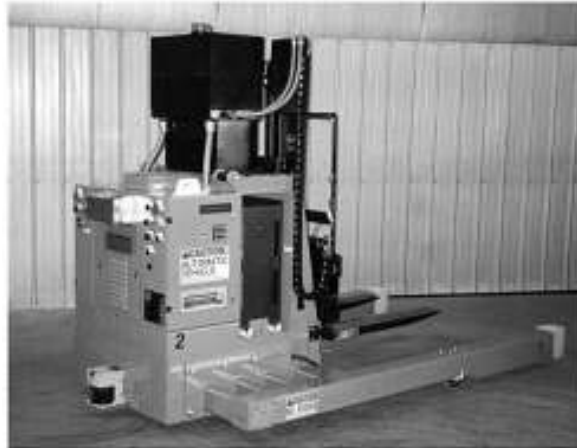


Figure 5.47 Track truck AGV. (Courtesy of Jervis B. Webb Co.)

i. Unit Load Carrier

ii. Small Load Carrier



Figure 5.48 Small load AGVs. (Courtesy of Jervis B. Webb Co.)



Figure 5.49 Medium load AGVs. (Courtesy of Jervis B. Webb Co.)



Figure 5.50 Heavy load AGV. (Courtesy of Jervis B. Webb Co.)

iii. Towing Vehicle



Figure 5.51 AGV towing vehicle. (Courtesy of Siemens Dematic Corp.)

iv. Assembly Vehicle



Figure 5.52 Work platform AUV. (Courtesy of Jervis B. Webb Co.)

v. Storage/Retrieval Vehicle



Figure 5.53 Work vehicle AUV. (Courtesy of Jervis B. Webb Co.)

b. Automated Electrified Monorail

c. Sorting Transfer Vehicle

C. Monorails, Hoists, and Cranes

1. Monorail

2. Hoist

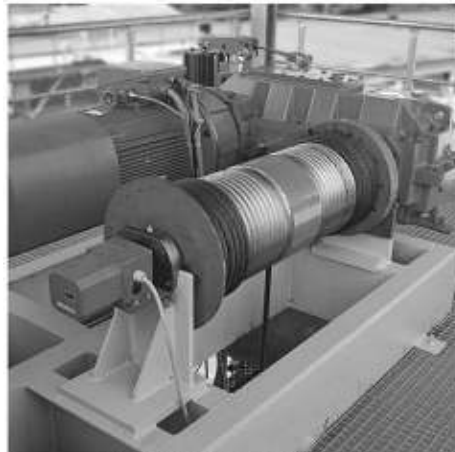


Figure 5.57 Hoist. (Courtesy of Demag Cranes AG.)

3. Cranes
a. Jib Crane

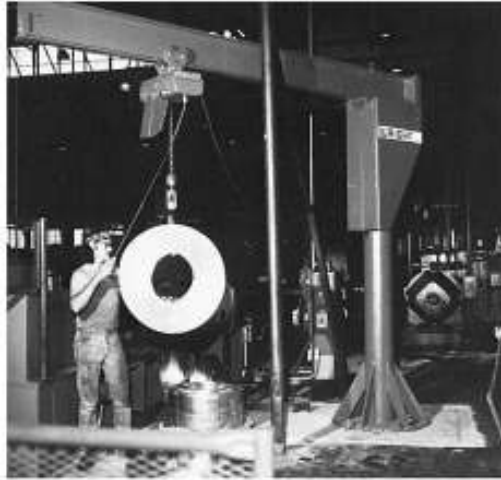


Figure 5.58 Jib cranes. (Courtesy of Accu Babcock.)

b. Bridge Crane



Figure 5.59 Bridge crane. (Courtesy of Mannesmann Demag.)

c. Gantry Crane

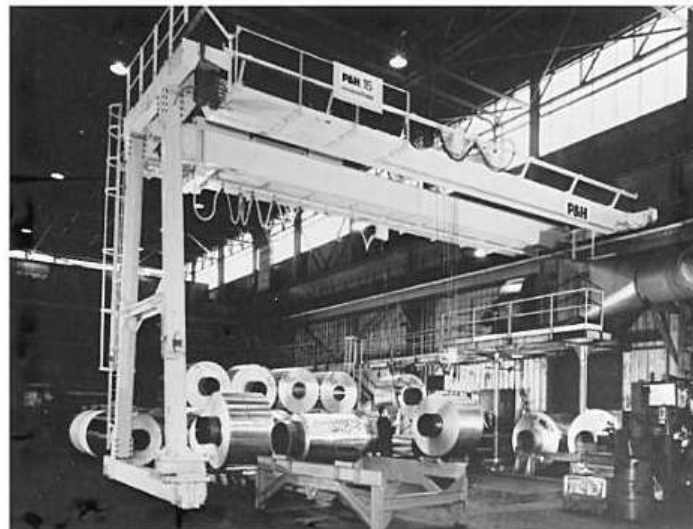


Figure 5.60 Gantry cranes. (Courtesy of Harnischfeger Corp.)

d. Tower Crane



Figure 5.61 Tower crane. (Courtesy of Potain/America, Inc.)

e. Stacker Crane

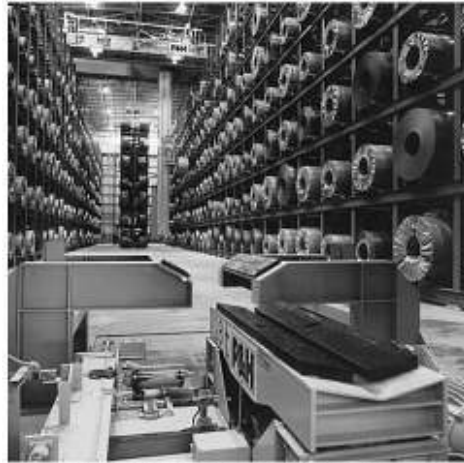


Figure 5.62 Stacker crane. (Courtesy of Harnischfeger Engineers, Inc.)

III. Storage and Retrieval Equipment

A. Unit Load Storage and Retrieval

1. Unit Load Storage Equipment

- a. Block Stacking
- b. Pallet Stacking Frame
- c. Single-Deep Selective Rack
- d. Double-Deep Rack



Figure 5.66 Double-deep selective pallet rack. (Courtesy of Ridg-U-Rak, Inc.)

e. Drive-In Rack



Figure 5.67 Drive-in rack. (Courtesy of UNARCO, Inc.)

f. Drive-Thru Rack



Figure 5.68 Drive-thru rack. (Courtesy of Boston Rack.)

g. Pallet Flow Rack



Figure 5.69 Pallet flow rack. (Courtesy of UNARCO, Inc.)

h. Push-Back Rack



Figure 5.70 Push-back rack without loads. (Courtesy of TITABCO, Inc.)

- i. Mobile Rack
 - j. Cantilever Rack
2. Unit Load Retrieval Equipment
- a. Walkie Stacker
 - b. Counterbalance Lift Truck
 - c. Narrow Aisle Vehicles
 - i. Straddle Truck
 - ii. Straddle Reach Truck



Figure 5.74 Straddle-reach truck with scissor mechanism. (Courtesy of Raymond Corp.)

iii. Sideloader Truck



Figure 5.75 Sideloader truck. (Courtesy of Raymond Corp.)

iv. Turret Truck



Figure 5.76 Swing-mast truck. (Courtesy of Drexel Industries LLC.)



Figure 5.77 Operator on-board turret truck. (Courtesy of Yale Materials Handling Corp.)

v. Hybrid truck



Figure 5.78 Hybrid truck. (Courtesy of Jervis B. Webb Co.)

d. Automated Storage/Retrieval Machines



Figure 5.79 Unit load AS/RS. (Courtesy of Sietests Derratic Corp.)

B. Small Load Storage and Retrieval Equipment

1. Operator-to-Stock – Storage Equipment

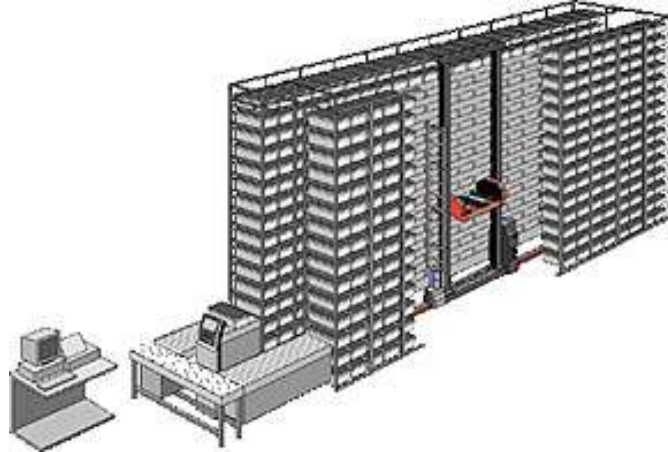
- a. Bin Shelving
- b. Modular Storage Drawers in Cabinets



Figure 5.81 Modular storage drawers in cabinets. (Courtesy of Equipto.)

- c. Mezzanine
 - d. Mobile Storage
2. Operator-to-Stock – Retrieval Equipment
- a. Picking Cart
 - b. Order Pick Truck
 - c. Person-aboard Automated Storage/Retrieval Machine
3. Stock-to-Operator Equipment
- a. Carousels
 - i. Horizontal Carousel
 - ii. Vertical Carousel
 - iii. Independent Rotating Carousel

b. Miniload Automated Storage and Retrieval Machine



- c. Vertical Lift Module
- d. Automatic Dispenser



Figure 5.92 Automatic dispenser. (Courtesy of SI Systems, Inc.)

IV. Automatic Identification and Communication Equipment

A. Automatic Identification and Recognition

1. Bar Coding

a. Bar Codes



Figure 5.93 Sample automatic identification product codes.

b. Bar Code Readers



Figure 5.94 Contact reader. (Courtesy of IXL.)



(a)

Figure 5.95(a) Handheld bar code scanner. (Motorola)



(b)

Figure 5.95(b) Stationary bar code scanner. (Accu-Sort, Inc.)

2. Optical Character Recognition
 3. Radio Frequency Tag
 4. Magnetic Stripe
 5. Machine Vision
- B. Automatic, Paperless Communication
1. Radio Frequency Data Terminal
 2. Voice Headset



Figure 5.98 Voice I/O system. (Courtesy of Vocollect, Inc.)

3. Light and Computer Aids



Figure 5.99 Light-aided order picking. (Courtesy of Lightning Pick Technologies.)

4. Smart Card



CHAPTER 8

LAYOUT

Long-range viewpoint and coordinating the facilities plan with the plans of the other organizational units is very important. A facilities layout strategy should emerge from the overall strategic plan. Product, manufacturing, marketing distribution, management, and human resource plans will be impacted by and will impact on the facilities layout.

It seems appropriate to ask the following question. Which comes first, the material handling system or the facility layout? Many appear to believe the layout should be designed first and then the material handling system should be developed. Yet, material handling decisions can have a significant impact on the effectiveness of a layout. For example, the following decisions will affect the layout:

1. Centralized versus decentralized storage of work-in-process (WIP), tooling, and, supplies.
2. Fixed path versus variable path handling.
3. The handling unit (unit load) planned for the systems.
4. The degree of automation used in handling.
5. The type of inventory control, physical control, and computer control of materials.

8.1. Basic Layout Types

1. Fixed Material Location Departments

In the case of fixed material location departments the workstations are brought to the material. It is used in aircraft assembly, shipbuilding, and most construction projects. The layout of the fixed material location department involves the sequencing and placement of workstations around the material or product.

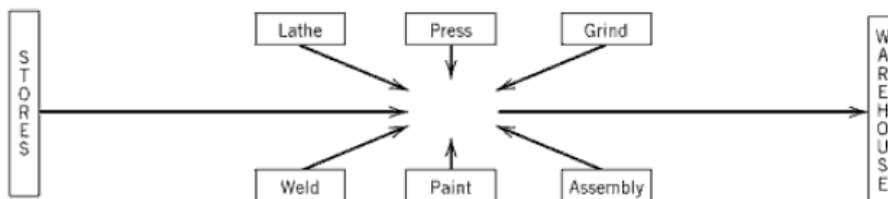


Figure 8.1. Fixed materials location product departments

Fixed Product Layout	
Advantages	Limitations
1. Material movement is reduced.	1. Personal and equipment movement is increased.
2. When a team approach is used, continuity of operations and responsibility results.	2. May result in duplicate equipment.
3. Provides job enrichment opportunities.	3. Requires greater skill for personnel.
4. Promotes pride and quality because an individual can complete the "whole job."	4. Requires general supervision.
5. Highly flexible; can accommodate changes in product design, product mix, and production volume.	5. May result in increased space and greater work-in-process.
	6. Requires close control and coordination in scheduling production.

2. Production Line Departments

The layout for a production line department is based on the processing sequence for the part(s) being produced on the line. Materials typically flow from one workstation directly to the next adjacent one. Nice, well-planned flow paths generally result in this high-volume environment (product layouts).

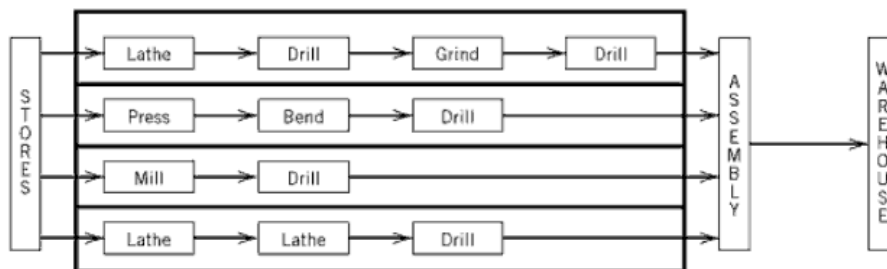


Figure 8.2. Production line product departments

Product Layout	
Advantages	Limitations
1. Smooth, simple, logical, and direct flow lines result.	1. Machine stoppage stops the line.
2. Small work-in-process inventories should result.	2. Product design changes cause the layout to become obsolete.
3. Total production time per unit is short.	3. Slowest station paces the line.
4. Material handling requirements are reduced.	4. General supervision is required.
5. Less skill is required for personnel.	5. Higher equipment investment usually results.
6. Simple production control is possible.	
7. Special-purpose equipment can be used.	

3. Product Family Departments

The layout for a product family department is based on the grouping of parts to form product families. Nonidentical parts may be grouped into families based on common

processing sequences, shapes, material composition, tooling requirements, handling/storage/control requirements, and so on.

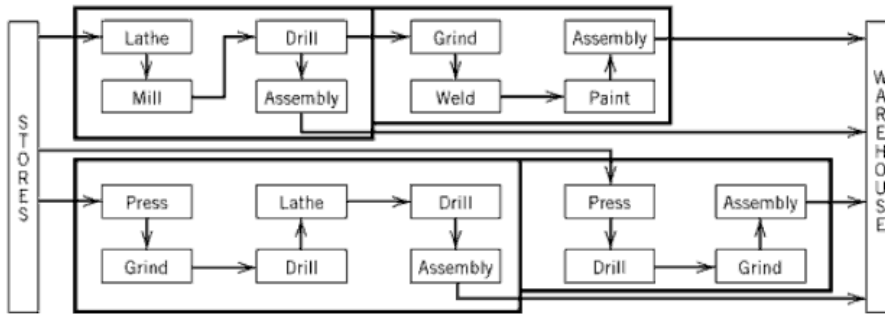


Figure 8.3. Product family product departments

Group Layout	
Advantages	Limitations
1. By grouping products, higher machine utilization can result.	1. General supervision required.
2. Smoother flow lines and shorter travel distances are expected than for process layouts.	2. Greater labor skills required for team members to be skilled on all operations.
3. Team atmosphere and job enlargement benefits often result.	3. Critically dependent on production control balancing the flows through the individual cells.
4. Has some of the benefits of product layouts and process layouts; it is a compromise between the two.	4. If flow is not balanced in each cell, buffers and work-in-process storage are required in the cell to eliminate the need for added material handling to and from the cell.
5. Encourages consideration of general-purpose equipment.	5. Has some of the disadvantages of product layouts and process layouts; it is a compromise between the two.
	6. Decreased the opportunity to use special-purpose equipment.

4. Process Departments

The layout for a process department is obtained by grouping like processes together and placing individual process departments relative to one another based on flow between the departments.

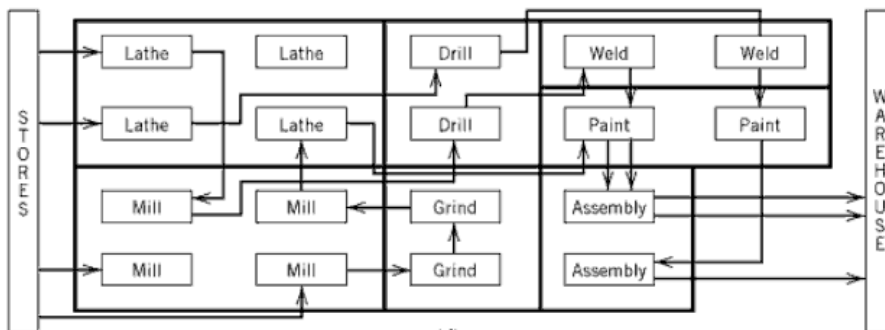


Figure 8.4. Process departments

Process Layout	
Advantages	Limitations
1. Increased machine utilization.	1. Increased material handling requirements.
2. General-purpose equipment can be used.	2. More complicated production control required.
3. Highly flexible on allocating personnel and equipment.	3. Increased work-in-process.
4. Diversity of tasks for personnel.	4. Longer production lines.
5. Specialized supervision is possible.	5. Higher skills required to accommodate diversity of tasks required.

8.2. Layout Procedures

In designing layouts, the procedures can be classified into two main categories:

1. *Construction type* layout methods basically involve developing a new layout “from scratch”, and
2. *Improvement* procedures generate layout alternatives based on an existing layout.

Apple’s Plant Layout Procedure

1. Procure the basic data
2. Analyze the basic data
3. Design the productive process
4. Plan the material flow pattern
5. Consider the general material handling plan
6. Calculate equipment requirements
7. Plan individual workstations
8. Select specific material handling equipment
9. Coordinate groups of related operations
10. Design activity interrelationships
11. Determine storage requirements
12. Plan service and auxiliary activities
13. Determine space requirements
14. Allocate activities to total space
15. Consider building types
16. Construct master layout
17. Evaluate, adjust, and check the layout with the appropriate persons
18. Obtain approvals
19. Install the layout
20. Follow up on implementation of the layout

Reed’s Plant Layout Procedure

1. Analyze the product or products to be produced
2. Determine the process required to manufacture the product
3. Prepare layout planning charts
4. Determine workstations

5. Analyze storage area requirements
6. Establish minimum aisle widths
7. Establish office requirements
8. Consider personnel facilities and services
9. Survey plant services
10. Provide for future expansion

Reed calls the layout planning chart (figure 8.5) “the most important single phase of the entire layout process”. It incorporates the following:

1. Flow process, including operations, transportation, storage, and inspection
2. Standard times for each operation
3. Machine selection and balance
4. Manpower selection and balance
5. Material handling requirement

LAYOUT PLANNING CHART																			
PART NO. <u>1</u>		PART NAME <u>PLASTIC CONTACT PAD</u>		PCS/ASSY <u>1</u>		PCS REQ/HR <u>70.4</u>		SHEET <u>1</u> OF <u>1</u>											
ASSY NO. <u>—</u>		ASSY NAME <u>—</u>		ASSY/PRODUCT <u>2</u>		PRODUCTION HRS/DAY <u>6.0</u>		PREPARED BY <u>J.G.D.</u> DATE <u>1-4-60</u>											
MATERIAL <u>PLASTIC</u>		SIZE <u>1 1/2" OD x 3/8" ID (from 4' x 8' x 1/4" SHEETS)</u>		PCS/DAY <u>422</u>		LOT SIZE <u>1</u>		APPROVED BY <u>—</u> DATE <u>—</u>											
ST NO.	F M S I	DESCRIPTION	OPER NO.	DEPT NO.	TIME PER PIECE	MACHINE OR EQUIPMENT	MACHINES REQD			OPER PER MACH	TOTAL MANPOWER				HANDLING REQUIREMENTS				REMARKS
							MACH FRAC	COMB WITH	MACH REQD		CREQ FRAC	MAN FRAC	COMB WITH	MEN REQD	HOW MOVED	CONT TYPE	LOAD SIZE	DIST MOVED	
1	⊙⊙△⊙	FROM MATERIALS STORAGE																	
2	⊙⊙△⊙	ON PALLET BY SAW		2															
3	⊙⊙△⊙	TO SAW TABLE																	
4	⊙⊙△⊙	SAW INTO STRIPS 2 1/2" x 8'	10	2	.02	TABLE SAW	.028	9-10 16-20	1	2	.028	.056	9-10 16-10	2					
5	⊙⊙△⊙	TO RACK BY SAW																	
6	⊙⊙△⊙	IN RACK		2															
7	⊙⊙△⊙	TO HEATER																	
8	⊙⊙△⊙	IN RACK BY HEATER		2															
9	⊙⊙△⊙	FEED INTO HEATER																	
10	⊙⊙△⊙	HEAT	10	2	.04	HEATER	.055	9-20 16-20	1	1	.055	.055	6-20 6-10	1					
	⊙⊙△⊙																		
	⊙⊙△⊙																		
	⊙⊙△⊙																		
	⊙⊙△⊙																		
11	⊙⊙△⊙	FEED TO PUNCH PRESS																	
12	⊙⊙△⊙	PUNCH TO SHAPE	10	2	.04	PUNCH PRESS	.055	4-10 4-20	1	1	.055	.055	4-10 4-20	1					
13	⊙⊙△⊙	TP BIN BY PUNCH PRESS						9-30 16-30					9-30 16-30						
14	⊙⊙△⊙	IN BIN		2				19A-10 19B-10					19A-10 19B-10						
15	⊙⊙△⊙	TO PARTS STORAGE											19A-20 5A-10		4 WHEEL HAND TRUCK	TOTE BOX	400 PER BOX	150'	
16	⊙⊙△⊙	IN PARTS STORAGE		5															
17	⊙⊙△⊙	TO ASSEMBLY													4 WHEEL HAND TRUCK	TOTE BOX	400 PER BOX	60'	
18	⊙⊙△⊙	IN BIN IN ASSEMBLY		6															
19	⊙⊙△⊙	TO TABLE																	
	⊙⊙△⊙																		
	⊙⊙△⊙																		

Figure 8.5. Layout planning chart

Muther's Systematic Layout Planning (SLP) Procedure

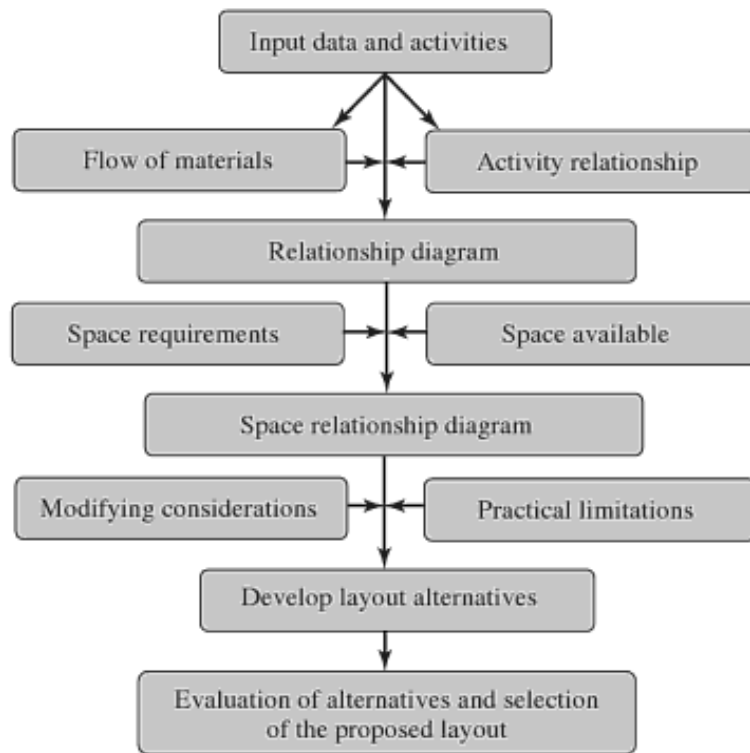


Figure 8.6. Systematic layout planning (SLP) procedure

The systematic layout planning (SLP) procedure uses as its foundation the activity relationship chart (figure 8.7).

Based on the input data and an understanding of the roles and relationships between activities, a material flow analysis (from-to-chart) and an activity relationship analysis (activity relationship chart) are performed. From the analyses performed, a relationship diagram is developed (figure 8.8).

The next two steps involve the determination of the amount of space to be assigned to each activity. Departmental service and area requirement sheets would be completed for each department. Once the space assignments have been made, space templates are developed for each planning department and the space is “hung on the relationship diagram” to obtain the space relationship diagram (figure 8.9).

Based on modifying considerations and practical limitations, a number of layout alternatives are developed (figure 8.10) and evaluated. The preferred alternative is then recommended.

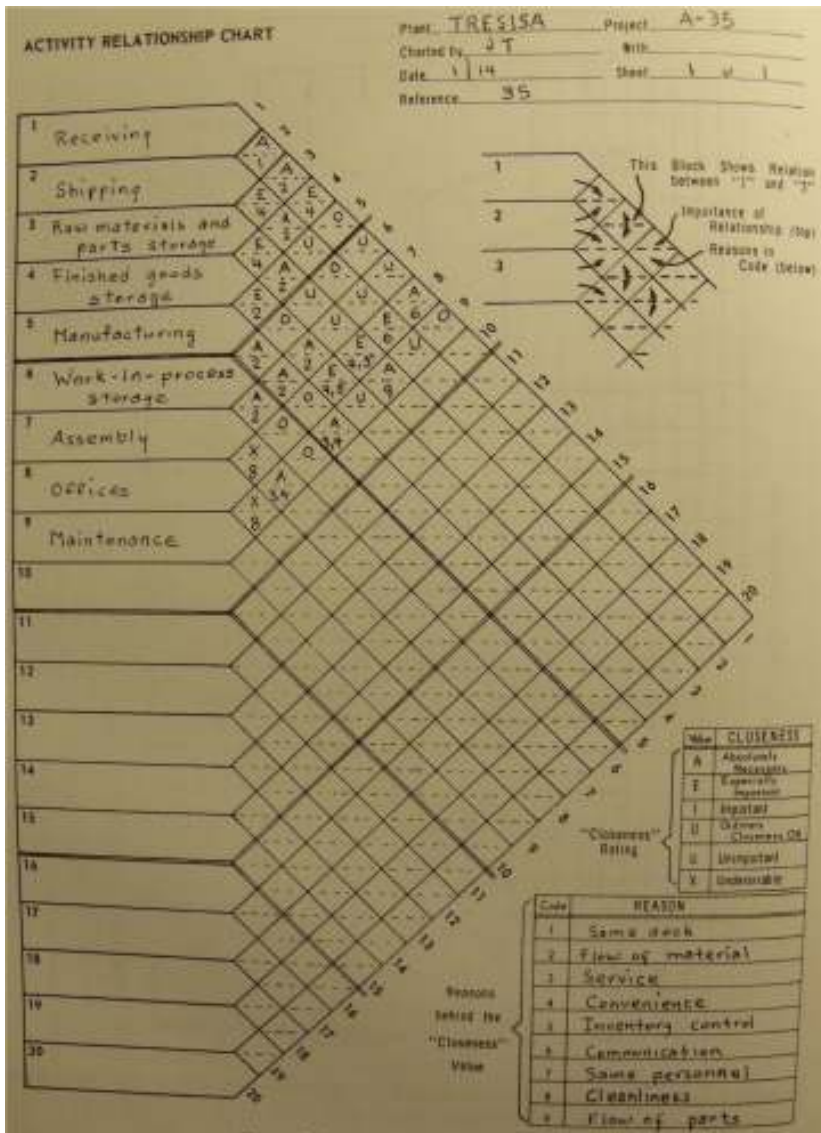


Figure 8.7. Activity relationship chart

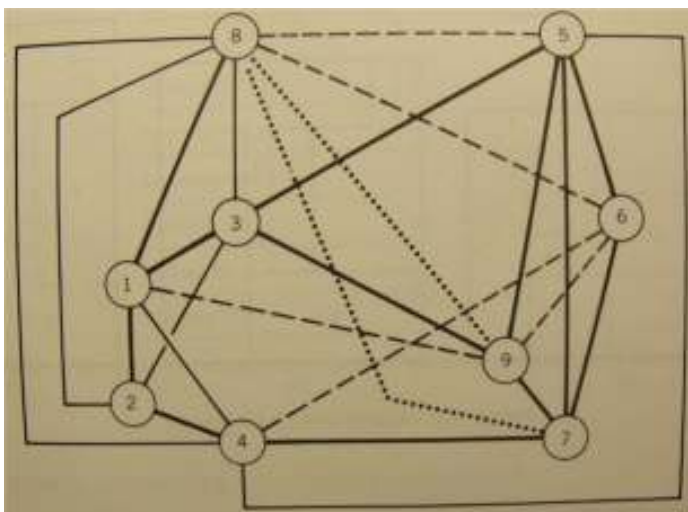


Figure 8.8. Relationship chart

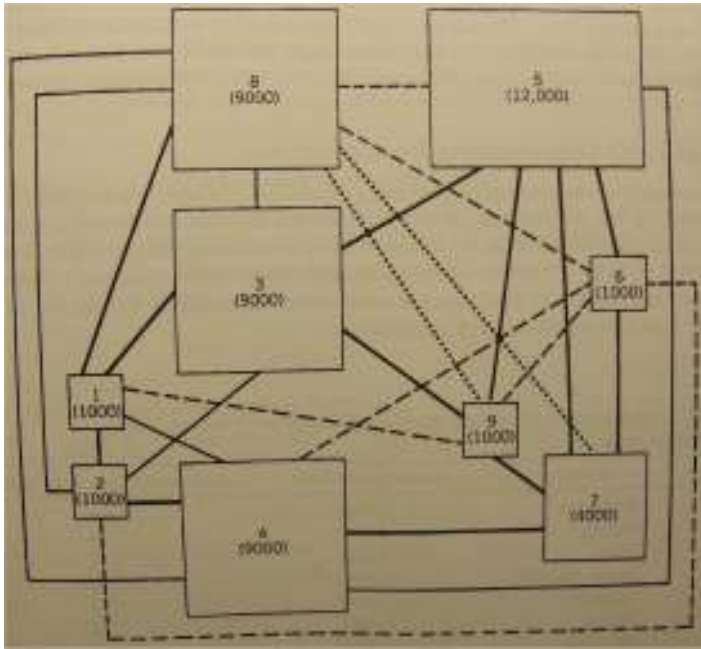


Figure 8.9. Space relationship diagram

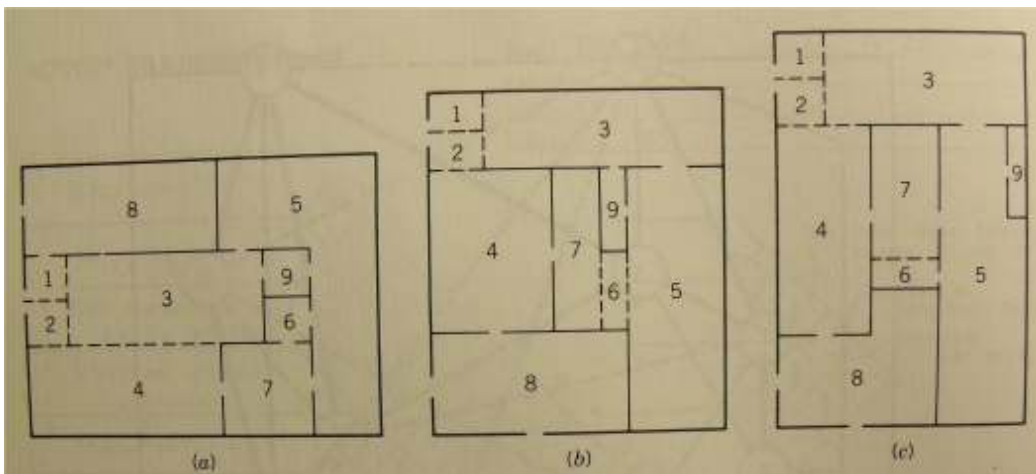


Figure 8.10. Alternative block layouts

8.3. Algorithmic Approaches

8.3.1. Relationship Diagramming for New Layouts

To illustrate the relationship diagramming procedure, consider the information given in table 1. The activity relationship chart for this illustration is shown in figure 8.11. the information in this table is converted into a relationship diagramming worksheet (table 8.2), which will be used as the basis for constructing a relationship diagram and layout.

Code	Function	Area (Square feet)	Number of Unit Area Templates
1	Receiving	12000	6
2	Milling	8000	4
3	Press	6000	3
4	Screw machine	12000	6
5	Assembly	8000	4
6	Plating	12000	6
7	Shipping	12000	6

Table 8.1. Department areas and number of unit area templates

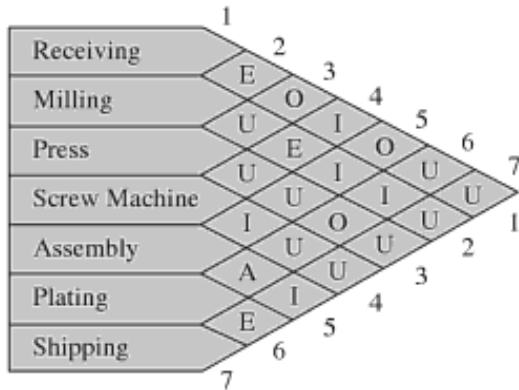


Figure 8.11. Activity relationship chart to illustrate a variation of the SLP

Rel.	Dept. 1	Dept. 2	Dept. 3	Dept. 4	Dept. 5	Dept. 6	Dept. 7
A					6	5	
E	2	1 4		2		7	6
I	4	5 6		1 5	2 4 7	2	5
O	3 5		1 6		1	3	
U	6 7	3 7	2 4 5 7	3 6 7	3	1 4	1 2 3 4
X							

Table 8.2. Relationship diagramming worksheet

The steps in constructing a relationship diagram are:

Step 1: Select the first department to enter the layout

The department with the greatest number of “A” relationship is selected. If a tie exists, the tie-breaking rule is based on the hierarchy of the relationships (greatest number of “E”, “I”, “O”, “U”, “X” relationships will enter respectively).

Step 2: Select the second department to enter the layout

The second department selected should have an “A” relationship with the first department selected. Additionally, it should have the greatest number of “A” relationships with the other departments not yet selected.

Step 3: Select the third department to enter the layout

The third department selected should have the highest combined relationships with the two departments already in the layout. The highest possible combine

relationships would be an “A” relationship with the both of the departments already selected. (Ranking: AA, AE, AI, A*, EE, EI, E*, II, and I*)

Step 4: Determine the fourth department to enter the layout

The fourth department selected is based on the same logic as in Step 3. The selection is based on the highest combined relationship with the three departments already in the layout. (Rankings: AAA, AAE, AAI, AA*, AEE, AEI, AE*, AII, AI*, A**, EEE, EEI, EE*, EII, EI*, E**, III, II*, and I**)

Step n: Department n is placed according to the rules described in Steps 3 and 4.

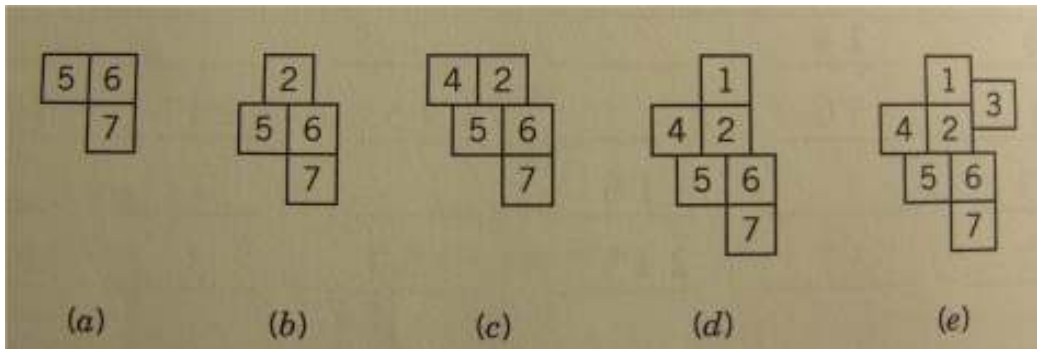


Figure 8.12. Relative location of block templates for the example

	1	1	1
4	4	1	1
4	4	3	1
4	2	3	3
4	2	2	2
5	6	6	6
5	6	6	6
5	7	7	7
5	7	7	7

Figure 8.13. Final layout by relationship diagramming technique

8.3.2. Pairwise Exchange Method

The majority of layout problems involves the redesign of an existing facility, which is typically triggered by the addition of new machines, changes in product mixes, decisions related to the contraction and expansion of storage areas, or a simple realization that the old layout is no longer adequate for its current needs.

We will discuss the layout improvement based on minimizing the total cost of transporting materials among all departments in a facility. We will assume that the distance between departments is rectilinear and is measured from the department centroids.

		To Department			
		1	2	3	4
From Department	1	---	10	15	20
	2		---	10	5
	3			---	5
	4				---

Table 8.3. Material flow matrix

Suppose a distance matrix is given as if in table 3. The total cost of existing layout computed as follows:

$$TC_{1234} = 10(1) + 15(2) + 20(3) + 10(1) + 5(2) + 5(1) = 125$$

The pairwise exchange method is simply states that for each iteration, all feasible exchanges in the locations of department pairs are evaluated and the pair that results in the largest reduction in total cost is selected. Since all departments areas are assumed to be of equal size, the feasible exchanges are 1-2, 1-3, 1-4, 2-3, 2-4, and 3-4.

- (a) Iteration 0: 1-2-3-4
- (b) Iteration 1: 3-2-1-4
- (c) Iteration 2: 2-3-1-4

$$TC_{2134}(1-2) = 10(1) + 15(1) + 20(2) + 10(2) + 5(3) + 5(1) = 105$$

$$TC_{3214}(1-3) = 10(1) + 15(2) + 20(1) + 10(1) + 5(2) + 5(3) = 95$$

$$TC_{4231}(1-4) = 10(2) + 15(1) + 20(3) + 10(1) + 5(1) + 5(2) = 120$$

$$TC_{1324}(2-3) = 10(2) + 15(1) + 20(3) + 10(1) + 5(1) + 5(2) = 120$$

$$TC_{1432}(2-4) = 10(3) + 15(2) + 20(1) + 10(1) + 5(2) + 5(1) = 105$$

$$TC_{1243}(3-4) = 10(1) + 15(3) + 20(2) + 10(2) + 5(1) + 5(1) = 125$$

Thus, we select pair 1-3 and perform the exchange in the layout [(b) iteration 1].

For the next iteration, we consider all feasible exchanges:

$$TC_{3124}(1-2) = 10(1) + 15(1) + 20(2) + 10(1) + 5(1) + 5(3) = 95$$

$$TC_{1234}(1-3) = 10(1) + 15(2) + 20(3) + 10(1) + 5(2) + 5(1) = 125$$

$$TC_{3241}(1-4) = 10(2) + 15(3) + 20(1) + 10(1) + 5(1) + 5(2) = 110$$

$$TC_{2314}(2-3) = 10(2) + 15(1) + 20(1) + 10(1) + 5(3) + 5(2) = 90$$

$$TC_{3412}(2-4) = 10(1) + 15(2) + 20(1) + 10(3) + 5(2) + 5(2) = 105$$

$$TC_{4213}(3-4) = 10(1) + 15(1) + 20(2) + 10(2) + 5(1) + 5(3) = 105$$

The pair 2-3 selected with a total cost value of 90 [(c) iteration 2].

Continuing on, the third iteration calculations are

$$TC_{3124}(1-2) = 10(1) + 15(2) + 20(1) + 10(1) + 5(2) + 5(3) = 95$$

$$TC_{1324}(1-3) = 10(2) + 15(1) + 20(3) + 10(1) + 5(1) + 5(2) = 120$$

$$\begin{aligned}
 TC_{3421}(1-4) &= 10(1) + 15(3) + 20(2) + 10(2) + 5(1) + 5(1) = 125 \\
 TC_{2134}(2-3) &= 10(1) + 15(1) + 20(2) + 10(2) + 5(3) + 5(1) = 105 \\
 TC_{3142}(2-4) &= 10(2) + 15(1) + 20(1) + 10(3) + 5(1) + 5(2) = 100 \\
 TC_{4123}(3-4) &= 10(1) + 15(2) + 20(1) + 10(1) + 5(2) + 5(3) = 95
 \end{aligned}$$

Since the lowest total cost for this iteration, 95, is worse than the total cost value of 90 in the second iteration, then the procedure is terminated. Thus, the final layout arrangement is 2-3-1-4.

8.3.3. Graph-Based Construction Method

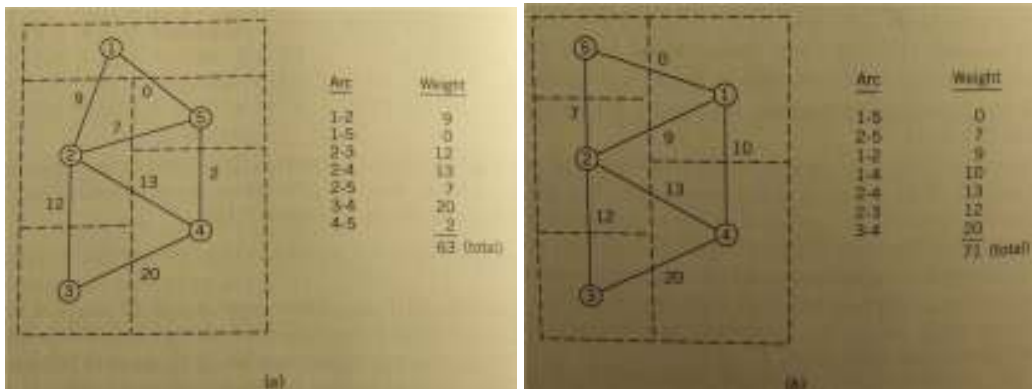


Figure 8.14. Adjacency graphs for alternative block layouts

Which block plan layout is better? We can score each block plan layout by summing the numerical weights assigned to each arc. On this basis, block plan layout (b) is better than block plan layout (a) with scores 71 and 63, respectively. Thus, finding a maximally weighted block plan layout is equivalent to obtaining an adjacency graph with the maximum sum of arc weights.

Observations:

- (a) The score does not account for distance, nor does it account for relationships other than those between adjacent departments.
- (b) Dimensional specifications of departments are not considered; the length of common boundaries between adjacent departments are also not considered.
- (c) The arcs do not intersect; this property of graphs is called planarity. We note that the relationship diagram is usually a nonplanar graph.
- (d) The score is very sensitive to the assignment of numerical weights in the relationship chart.

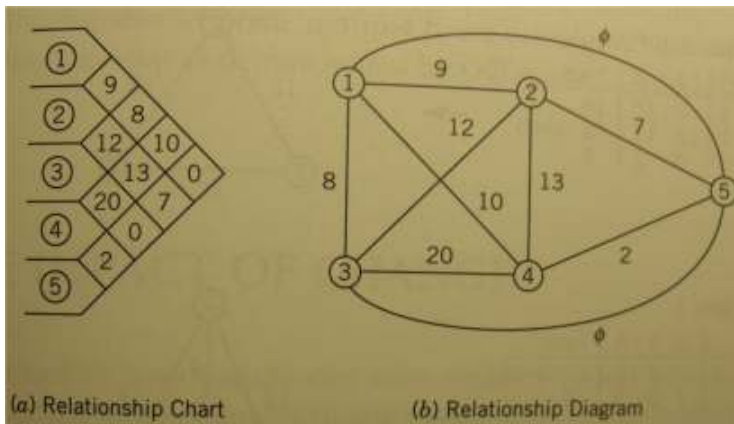


Figure 8.15. Relationship chart and relationship diagram

Procedure:

There are two strategies, we can follow in developing a maximally weighted planar adjacency graph. One way is to start with the relationship diagram and selectively prune connecting arcs while making sure that the final graph is planar.

A second approach is to iteratively construct an adjacency graph via a node insertion algorithm while retaining planarity at all times. A heuristic procedure is described below:

- Step 1:** From the relationship chart in figure 8.15(a), select the department pair with the largest weight. Thus departments 3 and 4 are selected to enter the graph.
- Step 2:** Select the third department to enter. The third department is selected based on the sum of the weights with respect to the departments 3 and 4. From figure 8.16(a) department 2 is chosen with a value of 25.
- Step 3:** The fourth department to enter by evaluating the value of adding one of the unassigned departments represented by a node on a face of the graph. A face of a graph is a bounded region of a graph. The value of adding departments 1 and 5 are 27 and 9, respectively. Department 1 is selected and placed inside the region 2-3-4, as shown in figure 8.16(b).
- Step 4:** Department 5 can be inserted on faces 1-2-3, 1-2-4, 1-3-4, and 2-3-4. Inserting 5 on faces 1-2-4 and 2-3-4 yields identical values of 9. We select arbitrarily 1-2-4. The final adjacency graph is given in figure 8.16(c). This solution is optimal with a total sum of arc weights equal to 81.
- Step 5:** A block layout based on the final adjacency graph is shown in figure 17. The manner by which we constructed the block layout is analogous to the SLP method.

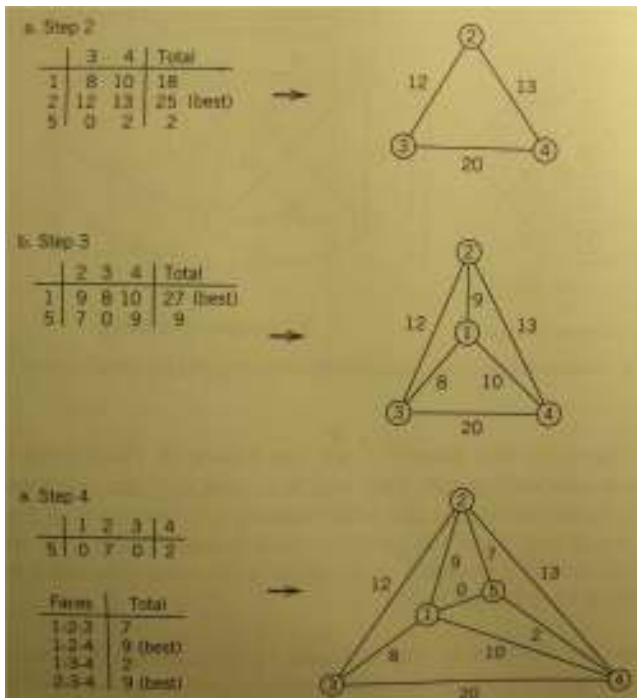


Figure 8.16. Steps in graph-based procedure

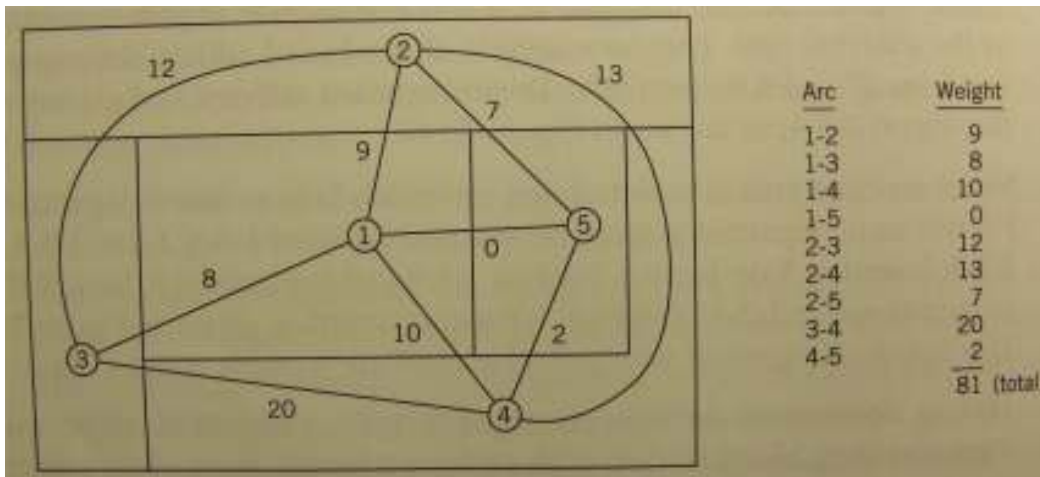


Figure 8.17. Block layout from the final adjacency graph