The following polynomial has four terms:

$$xy + 2y + 3x + 6$$

Notice that there is no common factor among the four terms (no GCF). However the first two terms do have a common factor of y and the last two terms have a common factor of 3. So while we can't factor the polynomial by taking out a GCF, we can **factor by grouping**. This means grouping the first two terms and factoring out a GCF, then grouping the last two terms and factoring out a GCF.

$$\underline{xy + 2y} + \underline{3x + 6}$$

$$\underline{y(x + 2)} + \underline{3(x + 2)}$$

We now have a sum of two terms, and both terms have a common factor of $(x + 2)$. If we take out the GCF of $(x + 2)$ we are left with the following:

$$y(x + 2) + 3(x + 2)$$

$$(x + 2)\left(\frac{y\cancel{(x + 2)}}{\cancel{(x + 2)}} + \frac{3\cancel{(x + 2)}}{\cancel{(x + 2)}}\right)$$

$$(x + 2)\left(\frac{y \cdot 1}{1} + \frac{3 \cdot 1}{1}\right)$$

$$\mathbf{(x + 2)(y + 3)}$$

This is an example of factoring a polynomial by grouping the terms.

Factor by grouping:

- grouping the terms of a polynomial and factoring out a GCF from each group
- you can group any terms that have a common factor
    - this means the order of the two middle terms can be reversed and the final factored answer will remain the same; this will be important on the next page when we use the ac-method to factor

$9x^3 + 36x^2 + 4x + 16$

$\underline{9x^3 + 36x^2} + \underline{4x + 16}$

$9x^2(x + 4) + 4(x + 4)$

$\mathbf{(x + 4)(9x^2 + 4)}$

$9x^3 + 4x + 36x^2 + 16$

$\underline{9x^3 + 4x} + \underline{36x^2 + 16}$

$x(9x^2 + 4) + 4(9x^2 + 4)$

$\mathbf{(9x^2 + 4)(x + 4)}$

Example 1: Factor the following polynomials by grouping.

a. $x^3 - 4x^2 + 6x - 24$

Always check to see if the terms in the polynomial have a GCF. In this problem, the four terms do not have a GCF, so I will simply factor by grouping.

$\underline{x^3 - 4x^2} + \underline{6x - 24}$

$x^2(x - 4) + 6(x - 4)$

$\mathbf{(x - 4)(x^2 + 6)}$

b. $24x^3 - 6x^2 + 8x - 2$

We have seen how to factor polynomials that contain a GCF, and how to factor polynomials where only certain groups of terms have a GCF. Next we will look at an algorithm for factoring quadratic trinomials (trinomials with a degree of 2, such as $12x^2 + 17x - 5$). In Lesson 7 we'll see examples of non-quadratic trinomials, such as $10x^6 - 13x^3 + 3$, and show how this algorithm can be used to factor those trinomials as well.

Using the ac-method to factor quadratic trinomials $\left(ax^2 + bx + c\right)$:

1. check for a GCF first
   a. this should be done regardless of how you are factoring or what type of polynomial you have (binomial, trinomial …)
2. find two numbers whose product is ac and whose sum is b
   a. a is the leading coefficient of the polynomial and c is the constant term, while b is the coefficient of x
3. replace the middle term of the original trinomial (bx) with an expression containing the two numbers from step 2
4. factor the resulting polynomial by grouping

Example 2: Factor the following polynomials completely.

a. $12x^2 + 17x - 5$

ac	b	**Think about the signs of the product and the sum.**

b. $6x^4 + 5x^3 + x^2$

ac	***b***	**Think about the signs of the product and the sum.**

c. $16x^2 + 24x + 9$

To factor this trinomial using the ac-method, I would start by trying to find two numbers with a of product ac $(16 \cdot 9 = 144)$ and a sum of b (24). In this case, those two numbers are 12 and 12, so I will replace $24x$ with $12x + 12x$ in order to then factor by grouping.

$$16x^2 + 12x + 12x + 9$$

$$4x(4x + 3) + 3(4x + 3)$$

$$(4x + 3)(4x + 3)$$

Since I end up with the same binomial twice, I can express it as a perfect square.

$$\mathbf{(4x + 3)^2}$$

When to factor out a negative factor rather than a positive factor:

There are two scenarios in which it is beneficial to factor out a negative factor rather than a positive factor:

1. When the leading coefficient is negative

$$18 + 15x - 3x^2$$

$$-3x^2 + 15x + 18$$

$$-3(x^2 - 5x - 6)$$

The trinomial $x^2 - 5x - 6$ should be easier to factor than $-x^2 + 5x + 6$, which we would have had if we'd factored out 3 instead of -3.

2. To make the binomials match when factoring by grouping

$$-3(x^2 - 5x - 6)$$

$$-3(x^2 + x \mathbf{- 6x - 6})$$

$$-3(x(\boldsymbol{x + 1}) - \mathbf{6}(\boldsymbol{x + 1}))$$

$$\mathbf{-3(x + 1)(x - 6)}$$

Had I factored out 6 from $-6x - 6$, I would have been left with the binomial $(-x - 1)$ which would not have matched $(x + 1)$. By factoring out a -6 instead, I had a common factor of $(x + 1)$, which I was then able to factor out.

Example 3: Factor the following polynomials completely.

a. $150 - 25x - x^2$

b. $-3x^3 + 17x^2 - 20x$

Since this trinomial has a negative leading coefficient, I will start by factoring out a negative GCF in order to make the leading coefficient positive.

$$-x(3x^2 - 17x + 20)$$

Next I will factor the trinomial that remains using the ac-method. To do so I will find two numbers with a of product ac $(3 \cdot 20 = 60)$ and a sum of b (-17). In this case, those two numbers are -5 and -12, so I would replace $-17x$ with $-5x - 12x$ in order to then factor by grouping.

$$-x(3x^2 - 5x - 12x + 20)$$

$$-x\big(x(3x - 5) - 4(3x - 5)\big)$$

$$\boldsymbol{-x(3x - 5)(x - 4)}$$

Answers to Examples:

1a. $(x-4)(x^2+6)$; *1b.* $2(4x-1)(3x^2+1)$;

2a. $(3x+5)(4x-1)$; *2b.* $x^2(3x+1)(2x+1)$; *2c.* $(4x+3)^2$;

3a. $-1(x-5)(x+30)$; *3b.* $-x(3x-5)(x-4)$