Factoring Trinomials in One Step

THE INTRODUCTION

To this point you have been factoring trinomials using the product and sum numbers with factor by grouping. That process works great but requires a number of written steps that sometimes makes it slow and space consuming. Consider the steps involved when factoring $8x^2 + 10x - 3$:

The set up: $8x^2 + 10x - 3$ Product # = -24, Sum # = +10 The solution is +12 and -2.

Step 1: $= 8x^2 + 12x - 2x - 3$

Step 2: $= (8x^2 + 12x) + (-2x - 3)$

Step 3: $= 4x(2x + 3) + -1(2x + 3)$

Step 4: $= (4x - 1)(2x + 3)$

We'll consider this a four step process even though there's also

- the step of playing the Factor Game, and
- the step of verifying the resulting factors, where you multiply the two binomial factors using FOIL to make sure that their product is the original trinomial.

Let us verify the result and, at the same time, be reminded of the four FOIL products:

$$\left.\begin{aligned} &(4x - 1)(2x + 3) \\ = \;&\mathbf{F} + \mathbf{O} + \mathbf{I} + \mathbf{L} \\ = \;&8x^2 + 12x - 2x - 3 \end{aligned}\right\} \quad \left\{\begin{aligned} &\mathbf{F}\text{: }\mathbf{F}\text{irst product} = 8x^2 \\ &\mathbf{O}\text{: }\mathbf{O}\text{uter product} = 12x \\ &\mathbf{I}\text{: }\mathbf{I}\text{nner product} = -2x \\ &\mathbf{L}\text{: }\mathbf{L}\text{ast product} = -3 \end{aligned}\right.$$

Notice that these four FOIL products are the exact same four terms in Step 1 of the factor by grouping method, shown above. Take a close look at what was done when going from Step 2 to Step 3 in the factor by grouping method. We factored out the greatest common factor of the **F**irst and **O**uter products:

$$\mathbf{F} + \mathbf{O}$$

Step 2: $= 8x^2 + 12x$ The GCF of the **F** and **O** products is $\boldsymbol{4x}$, and this GCF is also the first term of the first binomial factor.

Step 3: $= \boldsymbol{4x}(2x + 3)$

Step 4: $= (\boldsymbol{4x} - 1)(2x + 3)$

The fact that the first term of the first binomial is the greatest common factor of the **F** and **O** products is a key element in being able to factor trinomials in one step. We must now learn how to find the other three terms of the binomial factors.

ONE-STEP FACTORING: THE OUTLINE

Here is an outline of how to factor a trinomial in one step using the Factor Game.

Factor $8x^2 + 10x - 3$ in just one step:

A. Identify the four FOIL products within the trinomial, and play the Factor Game.

The **F**irst (**F**) and **L**ast (**L**) products are already shown in the trinomial. **F** is the first term of the trinomial, $8x^2$, and **L** is the last term of the trinomial, -3.

First product ➔ $\boxed{8x^2} + 10x \boxed{-3}$ ← Last product

The Factor Game gives us two important pieces of information:

(1) If we are certain the Factor Game has no solution, then we can declare the trinomial *prime* and we are finished with that trinomial

(2) If the Factor Game does have a solution, the **O**uter (**O**) and **I**nner (**I**) products are the two factors in that solution.

$8x^2 + 10x - 3$

Product # = 8(-3) = -24
Sum # = +10

The solution is +12 and -2.

The factors +12 and -2 are the coefficients of the **O**uter and **I**nner products of FOIL, $+12x$ and $-2x$. We can choose either one to be the **O**uter product and the other to be the **I**nner product.

Note:	Which of the two factors you choose for the **O**uter product is up to you. Sometimes, one factor is a better choice than the other.

B. Create the framework.

Because the trinomial is factorable, it will factor into two binomials, and we can write two sets of parentheses in anticipation of those binomial factors:

$8x^2 + 10x - 3$

( )( )

first binomial second binomial

Instead of writing the **O**uter and **I**nner products as $8x^2 \ \underline{+\,12x\ -\ 2x}\ -\ 3$, we write the Outer product above the parentheses, as shown. (Surprisingly, the Inner product is not needed at this point.)

$$8x^2 + 10x - 3$$
$$+\,12x$$
$$(\qquad\quad)(\qquad\quad)$$

This next step is most critical to factoring a trinomial in one step, getting the first term of the first binomial.

C. Find the first term of the first binomial.

As was mentioned before, this first term is the greatest common factor of the **F**irst and **O**uter products, the GCF of $8x^2$ and $12x$, which is $4x$.

$$8x^2 + 10x - 3$$
$$+\,12x$$
GCF
$$(4x\qquad)(\qquad\quad)$$

D. Find the other three terms in the binomial factors.

Now that we have the "first of the first" we can use it and the FOIL products to find other terms within the two binomials. At this point we can use the first term, **4x**, and the **F**irst product, $\mathbf{8x^2}$, to help us find the first term of the *second* binomial.

F
$$8x^2 + 10x - 3$$
$$(4x\qquad)(2x\qquad)$$

Next, we find the **O**uter product. We already have the first term of the **O**uter product, $4x$. To create the **O**uter product, $+12x$, the second term of the second binomial must be +3.

O
$$8x^2 + 10x - 3$$
$$+\,12x$$
$$(4x\qquad)(2x\ +3)$$

To find the only remaining unknown term, we can use the **L**ast product, -3. This is the product of the two constant terms. Because one of the constant terms is already known, +3, the other must be -1, and this is the value that is placed in the first binomial.

L
$$8x^2 + 10x - 3$$
$$(4x\ -1)(2x\ +\ 3)$$

E. Verify that the factoring is correct.

To complete the factoring, we must verify that it is accurate by multiplying the two binomials together. This can be done mentally or on paper using FOIL.

Caution:	If a mistake has been made, it is common to repeat that same mistake when verifying the resulting factoring. You are encouraged to approach this important step with caution.

Notice that it appears as though we never used the **I**nner product in this one-step process. It's true that **I**nner product is not necessary to develop the terms of the binomial factors, but it is necessary in the Factor Game and in the verifying process.

What you have just seen is the explanation for the one-step method for factoring trinomials. This method has many parts to it, but when it's complete, it looks as though the trinomial has been factored in just one step. Also, the more you practice factoring this way, the more proficient you will become at it. Some may get so good at using this technique that they won't need to write the **O**uter product above the parentheses.

ONE-STEP FACTORING: PUTTING IT ALL TOGETHER

Here are the key components of the one-step factoring method:

1) **The Factor Game:** It indicates whether the trinomial is factorable, and if it is factorable, gives us the **O**uter and **I**nner products of FOIL.

2) **Find the first term:** With parentheses in place, the first term of the first binomial is the GCF of the **F**irst and **O**uter products.

3) **Place the remaining terms:** Use the First, Outer, and Last products, in that order, to place the remaining terms of the binomial factors.

4) **Verify the result:** Multiply the binomial factors together to verify whether the factors are correct.

For the next two example, follow the guidelines for one-step factoring method for trinomials. In the written explanation of each example, it will appear as though there are many steps, but in your actual work, they will be condensed to just one step.

Example 1: Factor $5x^2 + 14x + 8$ using the one-step method.

Procedure: Play the Factor Game to identify the **O**uter and **I**nner products. We may use either factor to be the **O**uter product. Verify that the result is accurate by using FOIL to multiply the results.

Answer:

$5x^2 + 14x + 8$

Product # = +40
Sum # = +14

The solution is +10 and +4.

Let's choose $+10x$ to be the **O**uter product.

(1) Set up the parentheses and show the **O**uter product

$$5x^2 + 14x + 8$$
$$+10x$$
$$(\qquad)(\qquad)$$

(2) The first term is the GCF of the **F**irst and **O**uter products.

$$5x^2 + 14x + 8$$
$$+10x$$
GCF
$$(5x \qquad)(\qquad)$$

(3) Use the **F**irst product to find the first term of the second binomial.

F

$$5x^2 + 14x + 8$$
$$(5x \qquad)(x \qquad)$$

(4) Use the **O**uter product to find the second term in the second binomial.

O

$$5x^2 + 14x + 8$$
$$+10x$$
$$(5x \qquad)(x + 2)$$

(5) Use the **L**ast product to find the second term in the first binomial.

L

$$5x^2 + 14x + 8$$
$$(5x + 4)(x + 2)$$

So, we conclude that

$$5x^2 + 14x + 8$$
$$= (5x + 4)(x + 2)$$

Verify this factoring by mentally multiplying the binomials.

Example 2: Factor $6x^2 - 13x - 5$ using the one-step technique.

Procedure: Play the Factor Game to identify the **O**uter and **I**nner products. We may use either factor to be the **O**uter product. Verify that the result is true by using FOIL to multiply the results.

Answer:

$6x^2 - 13x - 5$ Product # = -30

Sum # = -13

The solution is -15 and +2.

Let's choose $-15x$ to be the **O**uter product.

(1) Set up the parentheses and show the **O**uter product

$$6x^2 - 13x - 5$$

$$-15x$$

$$(\qquad)(\qquad)$$

(2) The first term is the GCF of the **F**irst and **O**uter products.

$$6x^2 - 13x - 5$$

$$-15x$$

GCF

$$(\mathbf{3x} \qquad)(\qquad)$$

(3) Use the **F**irst product to find the first term of the second binomial.

F $6x^2 - 13x - 5$

$$(3x \qquad)(2x \qquad)$$

(4) Use the **O**uter product to find the second term in the second binomial.

O $6x^2 - 13x - 5$

$$-15x$$

$$(3x \qquad)(2x - 5)$$

(5) Use the **L**ast product to find the second term in the first binomial.

L $6x^2 - 13x - 5$

$$(3x + 1)(2x - 5)$$

So, we conclude that

$$6x^2 - 13x - 5$$

$$= (3x + 1)(2x - 5)$$

Verify this factoring by mentally multiplying the binomials.

If you can't find a solution to the Factor Game, then either the trinomial is prime or you haven't found the right factor pair solution. If you are confident that there is no solution, then you should write "*prime*."

Also, if the two factors are exactly the same, then the factors can be written as a binomial squared. For example, $4x^2 + 12x + 9 = (2x + 3)(2x + 3) = (2x + 3)^2$.

Exercise 1: Factor each using the one-step method. If the trinomial factors into a perfect square, write it $(\qquad)^2$. Use Examples 1 and 2 as guides.

a) $10x^2 - 11x + 3$

b) $8m^2 - 2m - 3$

c) $10r^2 + 7r - 6$

d) $y^2 - 4y - 60$

e) $x^2 + 14x + 49$

f) $x^2 - 18x + 81$

g) $9y^2 - 6y + 1$

h) $9y^2 + 9y - 4$

Answers:

Exercise 1

Exercise 1	a)	$(5x - 3)(2x - 1)$	b)	$(4m - 3)(2m + 1)$
	c)	$(2r - 1)(5r + 6)$	d)	$(y + 6)(y - 10)$
	e)	$(x + 7)^2$	f)	$(x - 9)^2$
	g)	$(3y - 1)^2$	h)	$(3y - 1)(3y + 4)$