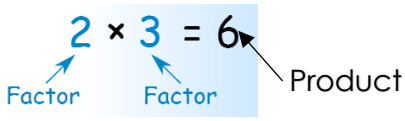
TIPS & REMINDERS: TABLE OF CONTENTS

Factors



- **T-Chart**: Helps organize factors. Write in the factor pairs, starting with "1".

48

1	48
2	24
3	16
4	12
6	8

Factors of 48:
1, 2, 3, 4, 6, 8, 12, 16, 24, 48

1 x 48, 2 x 24, 3 x 16,
4 x 12, 6 x 8

Greatest Common Factor (GCF)

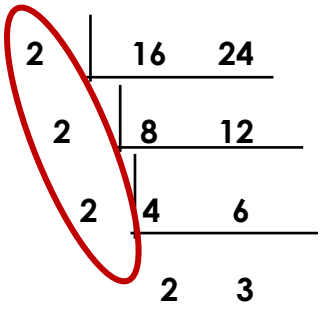
GCF: The highest number that divides evenly into two or more numbers.
To find the GCF, first list all the factors of each number:

16: 1, 2, 4, 8, 16
24: 1, 2, 3, 4, 6, 8, 12, 24

- The common factors of 16 and 24 are: 1, 2, 4, and 8.
- The **greatest (largest) common factor** of 16 and 24 is 8.

Finding the GCF using the LADDER:

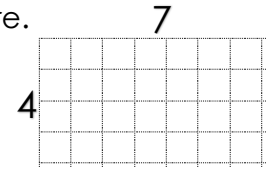
- Divide by prime numbers until no number can be divided evenly into both 16 & 24.



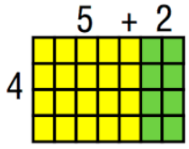
- Multiply the numbers on the side: 2 x 2 x 2 = 8
- The GCF is 8.
- 2 x 8 = 16 and 3 x 8 = 24

Using Area Models to Represent Products & factors

The **area** of a figure is the number of square units inside the figure.



4 x 7 = 28



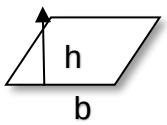
$4 \times 7 = (4 \times 5) + (4 \times 2) = 28$

Distributive Property- states that for any numbers a, b, and c: a(b + c) = ab + ac.
Used to find the GCF of two numbers.

- 6 (4 + 5) = 6 x 4 + 6 x 5 = 54
- 6 (4 + 5) = 24 + 30 = 54
- 6 = the **GCF** of 24 & 30.

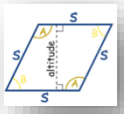
Quadrilaterals

Parallelogram: Opposite sides are parallel & congruent. Opposite angles are congruent.

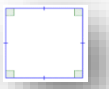


Rectangle: Parallelogram with 4 right angles

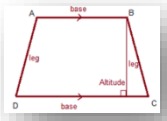
Rhombus: Parallelogram with 4 congruent sides (s)



Square: Parallelogram with 4 congruent sides and 4 right angles

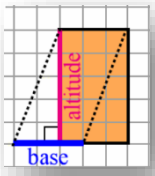


Trapezoid: Quadrilateral with exactly 2 parallel sides; may have 2 right angles



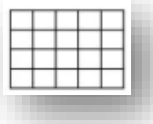
Area of Parallelograms
A = bh

Altitude (h): perpendicular distance from the base of the parallelogram or triangle to the opposite side, represented by a line segment.



Area of Rectangles
A = lw

The length is 4 units & the width is 5 units. Since A=lw, Area = 4 x 5 = 20 u^2.



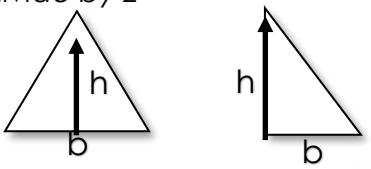
Area of Squares
A = s^2

If one side = 3 units, then 3^2 = 9 units^2.

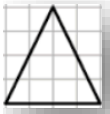
Area of Triangles:

A= ½ bh or A= $\frac{bh}{2}$

"Area of triangles are easy to do…base times height and divide by 2"

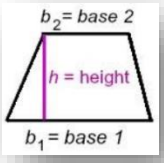


If b = 4 units, h = 4 units
(4 • 4) ÷ 2 = 8 units²
OR
½ (4 • 4) = 8 units²



Area of Trapezoids

A= ½ h (b_1+ b_2)



If b_1= 6 units & b_2 = 4 units & h = 5 units:
Then ½ h(b_1+ b_2)=
½ (5) (6 + 4) =
½ (5)(10)=
25 units²

Perimeter

Perimeter is the distance (outside area) around a figure:

"Add up all lengths as you go around…perimeter is what you've found!"

***Perimeter of a rectangle, square or parallelogram*: P=2ℓ+ 2w**

3 + 3 + 3 + 3 = 12 units

Prime & Composite

Composite number:
A number that has three or more factors:
10 (1, 2, 5, 10)

Prime number: Has only two factors, 1 and itself:
1 • 3 = 3

Relatively Prime: When the GCF of two numbers is 1 and the LCM equals the product of both numbers.

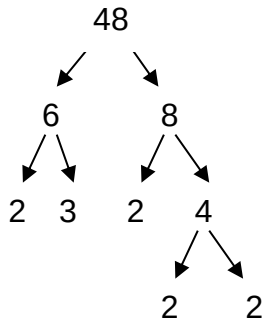
- 1 (a + b), GCF = 1

First 25 Prime Numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Prime Factorization

Factor Tree



- Write the number you are factoring at the top of the "tree"

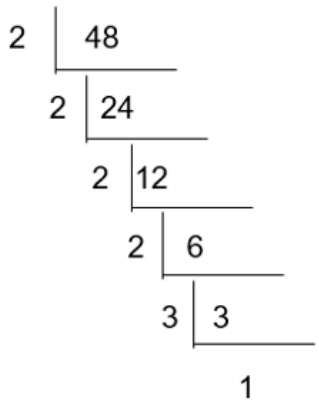
- Choose any pair of factors as branches. If either of these factors are not prime, you need to factor again

- Continue until all the branches end in a prime number

- Write only the prime factors in **prime factorization form**.

$2^4 \cdot 3$

Factor Ladder/ Hockey Sticks



- Only divide by **prime** numbers. Start with 2, 3, 5, 7, 11

-Write the number you are factoring at the top of the "ladder"

- See if you can divide it by "2". If so, write a two on the outside of the ladder, then write what the number divided by 2 equals (48 ÷ 2 =24).

- Continue this until the number cannot be divided by 2 again. Go to three, and divide by three as many times as you can.

- Continue with 4, 5, etc., if necessary, until you end up with a prime number.

- Write the prime factors (excluding 1) in prime factorization form.

Least Common Multiple (LCM)

Multiples of a number are the products of the number and other factors:
5 • 1 = 5; 5 • 2 = 10; 5 • 3 = 15
Multiples of 5: 5, 10, 15, 20,

To find the least common multiple **(LCM)** of two numbers, first list out their multiples:

3: 3, 6, 9, **12**, 15, 18, 21, **24**…

4: 4, 8, **12** 16, 20, **24**, 28….
First two common multiples:
12, 24

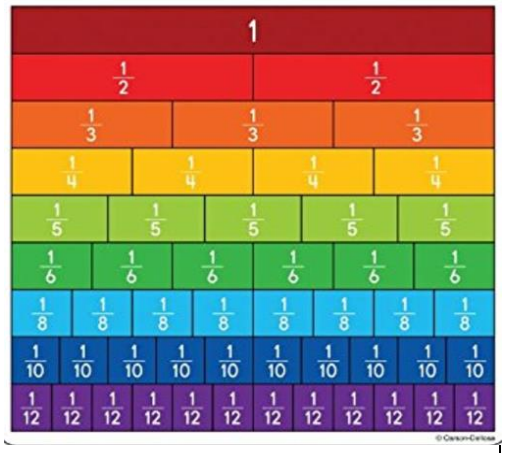
Least (or lowest) **C**ommon **M**ultiple: 12

Fraction, Decimal & Percent Benchmarks

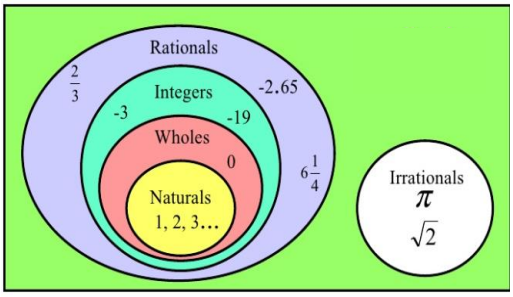
Percent	Fraction	Decimal
10%	$\frac{1}{10}$	.10 or 0.1
12.5%	$\frac{1}{8}$	.125
11%	$\frac{1}{9}$	$.\overline{11}$
17%	$\frac{1}{6}$	$.1\overline{66}$ = .17
20%	$\frac{1}{5}$	.20 or 0.2
22%	$\frac{2}{9}$	$.\overline{22}$
25%	$\frac{1}{4}$	.25
30%	$\frac{3}{10}$	.30 or 0.3
33%	$\frac{1}{3}$	$.\overline{33}$
37.5%	$\frac{3}{8}$	.375
40%	$\frac{2}{5}$	.40 or 0.4
50%	$\frac{1}{2}$	.50 or 0.5
60%	$\frac{3}{5}$	.60 or 0.6
62.5%	$\frac{5}{8}$	.625
66.6%	$\frac{2}{3}$	$.\overline{66}$ or .67
75%	$\frac{3}{4}$	.75
80%	$\frac{4}{5}$	.80 or 0.8
83%	$\frac{5}{6}$	$.8\overline{3}$ = .83
87.5%	$\frac{7}{8}$	.875
100%	$\frac{1}{1}$	1.0 or 1

- To convert $\frac{1}{9}$ to an equivalent decimal or percent, multiply the decimal .11 x the numerator and round if needed.
- To make a percent, multiply the decimal x 100.
- EX: $\frac{5}{9}$ = (.11 x 5) = .55 or .56 rounded
- .56 x 100 = 56%

Fraction Equivalencies



Real Numbers



- **Natural Numbers:** 1, 2, 3…
- **Whole Numbers**: 0, 1, 2, 3…
- **Integers**: Positive whole numbers & their opposites.
  - Negative & positive numbers

Rational Numbers: a number that can be expressed as an integer or the quotient of an integer divided by a nonzero integer. (Fraction)
If written as a decimal:

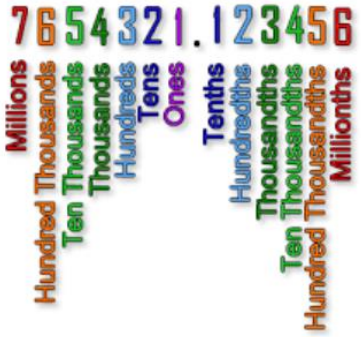
- ***Repeating Decimal:*** A decimal in which one or more digits repeat infinitely.
  - Ex: 0.7575…
  - Ex: 0.3333333…
  - Write with a repeating bar over the numbers that repeat.
    $0.\overline{75}$ $0.\overline{3}$
- ***Terminating Decimal:*** A decimal the ends on its own. Ex: 0.75

Irrational Numbers: A continuing decimal.

- ***Continuing Decimal***: A decimal that neither terminates, nor repeats. Ex: 0.548759314…

  - ***Pi***: 3.14159265359

Decimal Place Value



Converting Fractions to Decimals

Divide the numerator by the denominator.
Annex zeros if needed. Write the fraction in lowest terms.

$\frac{1}{4} \Longrightarrow$

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ -\ 8 \\ \hline 20 \\ -\ 20 \\ \hline 0 \end{array}$$

(Terminating Decimal)

Converting Decimals to Fractions

Decimals are fractions with a special set of denominators (tenths, hundredths, thousandths, etc.) and a special written form.

- To write a decimal as a fraction, say it aloud.
- You'll notice it sounds like a fraction:

Decimal: 0.9
Word name: nine tenths
Fraction: $\frac{9}{10}$

Decimal: 0.47
Word name: forty-seven hundredths
Fraction: $\frac{47}{100}$

Decimal: 3.2
Word name: three and two tenths
Fraction: $3\frac{2}{10}$

Decimal: 5.25
Word name: five and twenty five hundredths
Fraction: $5\frac{25}{100} = 5\frac{1}{4}$

Converting Decimals & Percent using 100

MR. DL

Decimal to percent ⟶

Move the decimal two places to the right because you are **multiplying** by 100.
"BUTTCHEEKS"
Add the percent symbol.

$0.46 = 46\%$

$0.305 = 30.5\%$

Percent to Decimal ⟵

Move the decimal two places to the left because you are **dividing** by 100. If one is not present, add it to the end, then move it.
Drop the percent sign.

$54\% = 54. = 0.54$

$25.8\% = 25.8 = 0.258$

Simplifying Fractions

Whenever the numerator and denominator of a fraction can be divided by the same non-zero whole number **(GCF)** it can be "reduced" or written in lower terms.

When the numerator and denominator can no longer be divided by the same non-zero whole number, it is in lowest terms or simplest form.

EX: $\frac{10}{15}$

Both can be divided evenly by "5" which is the GCF of 10 & 15.

$\frac{10 \div 5 = 2}{15 \div 5 = 3}$

There is not a whole number that can be evenly divided into 2 and 3, so $\frac{2}{3}$ is in lowest terms.

Complex Fraction: A fraction is complex if it has a fraction in the numerator, denominator, or both.

Improper Fractions & Mixed Numbers

Improper Fraction:
a fraction where the numerator is larger than the denominator $\frac{9}{4}$

Mixed Number:
a fraction with a whole number. 2 ¼

Convert Improper Fraction to Mixed Number

$$\frac{11}{2} \longrightarrow 5\frac{1}{2}$$

Divide the numerator by the denominator. If it divides evenly, then the answer is a whole number.

If it does not divide evenly, keep the whole number and then the remainder becomes the new numerator and the denominator stays the same.

$\frac{11}{2} = 11 \div 2 = 5$ remainder 1

$= 5\frac{1}{2}$

Converting a Mixed Number into an Improper Fraction

Using the order of operations, multiply the denominator by the whole number, then add the numerator. This becomes the new numerator. The denominator stays the same.

$5\frac{1}{2} = 2 \bullet 5 + 1 = 11 = \frac{11}{2}$

Adding/Subtracting Fractions with LIKE Denominators

Add or subtract the numerators. Write the new numerator, the denominator stays the same. Simplify when necessary.

$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$

Common Denominators

Common denominators may be found by different methods:

- Multiply each fraction by denominator of the opposite fraction:

EX: $\frac{2}{3}$; $\frac{1}{2}$

$\frac{2}{3} \bullet \frac{2}{2} = \frac{4}{6}$

$\frac{1}{2} \bullet \frac{3}{3} = \frac{3}{6}$

- Find the LCM (least common denominator) of the 2 denominators:

  3: 3, (6,) 9

  2: 2, 4, (6)

- Then, multiply the numerator by the number that you would need to multiply both the numerator and the denominator by to get the LCM as the new denominator.

$\frac{2}{3} \bullet \frac{2}{2} = \frac{4}{6}$

$\frac{1}{2} \bullet \frac{3}{3} = \frac{3}{6}$

Equivalent Fractions

Multiply the numerator and denominator by the same number to find equivalent fractions:

$\frac{3}{4} = \frac{3 \bullet 2}{4 \bullet 2} = \frac{6}{8}$

$\frac{3}{4} = \frac{3 \bullet 3}{4 \bullet 3} = \frac{9}{12}$

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$

Compare Fractions with LIKE Denominators

If fractions have the same denominator, compare the numerators.

$\frac{4}{7} > \frac{2}{7}$ because 4 is greater than 2.

$\frac{1}{9} < \frac{5}{9}$ because 1 is less than 5.

Ordering Fractions with UNLIKE Denominators

Either find common denominators or convert to decimals, then put in order.

Comparing Fractions with UNLIKE Denominators

One method is to get a common denominator by multiplying each fraction by the denominator of the opposite fraction, and then comparing numerators:

$\frac{2}{3}$ and $\frac{3}{4}$

$\frac{2 \bullet 4}{3 \bullet 4} = \frac{8}{12}$ $\quad$ $\frac{3 \bullet 3}{4 \bullet 3} = \frac{9}{12}$

$\frac{8}{12} < \frac{9}{12}$, so $\frac{2}{3} < \frac{3}{4}$

Another method is to convert each fraction to a decimal, by dividing the numerator by the denominator, and then compare the decimals:

$\frac{2}{3}$ and $\frac{3}{4}$

$\frac{2}{3} = 0.67 < \frac{3}{4} = 0.75$

$\frac{2}{3} < \frac{3}{4}$

Adding or Subtracting Fractions with UNLIKE Denominators

Since you need to have the same denominator to add or subtract fractions, multiply each fraction by denominator of the opposite fraction to find a common denominator.

$\frac{2}{3} + \frac{1}{2}$

$\frac{2}{3} \bullet \frac{2}{2} = \frac{4}{6}$

$+ \frac{1}{2} \bullet \frac{3}{3} = + \frac{3}{6}$

$\frac{7}{6} = 1\frac{1}{6}$

Adding or Subtracting Mixed Numbers

Addition:

Add the whole numbers, then add the fractions. If you do not have common denominators, you need to get common denominators before you may add. Write answer in lowest terms (reduce if necessary).

$5\frac{3}{8} + 4\frac{1}{8} = 9\frac{4}{8} = 9\frac{1}{2}$

Subtraction:

Do subtraction problems the same way. If you do not have common denominators, you need to get common

denominators before you may subtract. Write answer in lowest terms (reduce if necessary).

$$5\frac{2}{3} + 9\frac{1}{2}$$

$$5\frac{2}{3} \cdot \frac{2}{2} = 5\frac{4}{6}$$

$$+\ 9\frac{1}{2} \cdot \frac{3}{3} = +\ 9\frac{3}{6}$$

$$14\frac{7}{6}$$

$$14 + \frac{7}{6} = 14 + 1\frac{1}{6} = 15\frac{1}{6}$$

Subtraction with Renaming (Borrowing)

$$6 - 3\frac{2}{5}$$

1. If the fraction is a terminating fraction, then change to a decimal & subtract from a whole number. 6.0 – 3.4= 2.6 & change back into a fraction.

$$6 = \not{6}^{5}\frac{5}{5}$$

$$-3\frac{2}{5} = -3\frac{2}{5}$$

$$2\frac{3}{5}$$

2. Otherwise, look at the denominator of the fraction. You will need to borrow 1 from the whole number. But instead of borrowing 1, you will borrow 1 in the form of whatever the denominator is. EX: $1 = \frac{5}{5}$
3. Subtract 1 from the whole number and rewrite as $5\frac{5}{5}$ because $5 + \frac{5}{5} = 6$.
4. Subtract & simplify if needed.

Multiplying Fractions

- Multiply the numerators.
- Multiply the denominators.
- Write the product in lowest terms (reduce) if necessary.

$$\frac{2}{3} \cdot \frac{3}{7} = \frac{6}{21} = \frac{2}{7}$$

If multiplying a whole number by a fraction, make the whole number a fraction by placing it over 1.

$$9 \cdot \frac{3}{7} = \frac{9}{1} \cdot \frac{3}{7} =$$

$$\frac{27}{7} = 3\frac{6}{7}$$

Dividing Fractions

1. Figure out what you have… that's the whole. DON'T EVER TOUCH THE WHOLE!!!!
2. If there is a whole number, put a 1 under it to make it a fraction.
3. If there are mixed numbers, make them improper fractions.
4. Instead of dividing… multiply by the inverse or <u>multiplicative inverse</u>:
   - $\frac{a}{b}$ is the <u>reciprocal</u> of $\frac{b}{a}$ where a and b are nonzero numbers.
   - Invert (flip) the numerator & denominator of the second fraction (the right reciprocal).
5. Multiply the numerators
6. Multiply the denominators
7. Write the product in lowest terms (reduce) if necessary.

$$\frac{2}{3} \div \frac{3}{7} = \frac{2}{3} \cdot \frac{7}{3} =$$

$$\frac{14}{9} = 1\frac{5}{9}$$

Fraction Jingle
(By: Mrs. Mackey)

I don't know what you've been told, fractions are the way to go.
Fractions, fractions, don't you know? Each operation has a different flow.

(CHORUS)
Sound off, 7/8
Knock it on down, 3/4
All the way down, 5/8, 1/2, 3/8, 1/4! *(clap, clap)*

Adding and subtracting are so cool. It's quite easy, here's the rule. Change the denominators so they match. Then add the numerators, that's the catch.
(CHORUS)

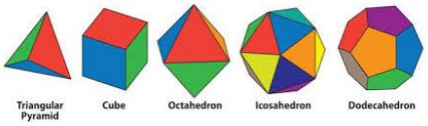
Multiplication rules, we can name. The denominators aren't the same. Multiply the tops and then the bottoms. Simplify and then you got 'em!
(CHORUS)

Dividing fractions, that's the test. It's more confusing than the rest. We don't divide we multiply. By the right reciprocal, and that's no lie!

(CHORUS)
OOH RAH!!!

Three Dimensional Shapes

<u>Polyhedron</u>: Three-dimensional object, or solid figure, with flat surfaces called polygons.

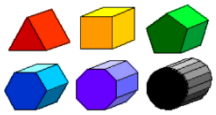


Regular Polyhedron: a three-dimensional solid that has

CONGRUENT regular polygons as faces and has CONGRUENT angles between all faces.

Polygon: A CLOSED two-dimensional shape with at least three sides. The faces of polyhedrons.

- **Triangle:** 3 sides
- **Quadrilateral:** 4 sides
- **Pentagon:** 5 sides
- **Hexagon:** 6 sides
- **Heptagon:** 7 sides
- **Octagon:** 8 sides
- **Nonagon:** 9 sides
- **Decagon:** 10 sides

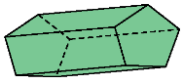


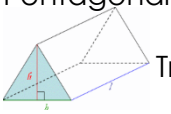
Prism: A polyhedron with 2 congruent, parallel bases and other faces that are all parallelograms.

- Prisms are named for the shape of its 2 **bases**. EX:

Cube 

Rectangular Prism

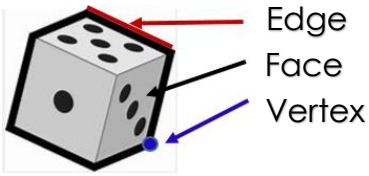
Pentagonal Prism 



Triangular Prism

Every Prism has:

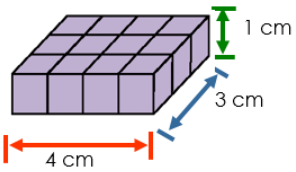
- **Faces: Flat surfaces** that make up polyhedrons.
- **Edges:** The **side** that 2 faces share.
- **Vertex (Vertices):** A point at which 3 or more edges meet. (**Corner**)



Volume of Prisms
V=Bh

Volume: The amount of three dimensional space an object can hold: capacity.

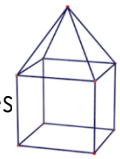
- Labeled in units³
- B= area of the base
- h = height



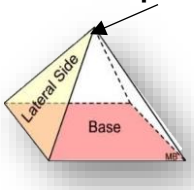
V = Bh
V = Rectangle Base ($\ell \cdot w$)
V = (4 · 3) · 1
V = 12 · 1
V = 12 cm^3

Composite Solid: Made up of more than one geometric solid.

- Volume = adding/ subtracting volumes common solids.



Pyramid: Has 1 polygon shaped base, and the other faces are triangles that come to a point called an **apex**.

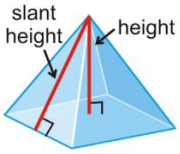


Apex: The point of a pyramid where all faces come together. Also known as a vertex.

Lateral Face: The remaining faces of a prism, not including the bases.

Pyramid Slant Height: altitude of the lateral faces.

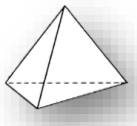
- Height of the triangles on the **OUTSIDE.**



Pyramid Height: the perpendicular distance from the **apex** of the pyramid to the **base** of the pyramid.

- Height of pyramid on the **INSIDE**.

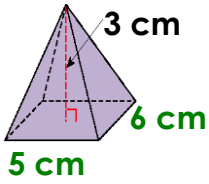
Triangular Pyramid: a solid having four plane triangular faces also known as a **tetrahedron.**



Volume of Pyramid

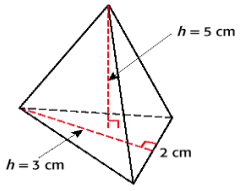
$V = \frac{1}{3}Bh$ OR $V = \frac{Bh}{3}$

- *B= area of the base*
  h= INSIDE height



V = $\frac{1}{3}Bh$
V = B(area of the base= $\ell \cdot w$)
B = 5 x 6 = 30
V = 30 x 3 = 30
V = 30 ÷3 = 10 cm^3

Volume of a Triangular Pyramid



$V = \frac{1}{3}Bh$

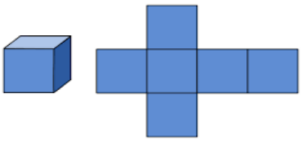
V = B (area of the base= ½ bh)

B = ½ (3 x 2) = 3

V = 3 x 5 = 15

V =15 ÷ 3 = 5 cm^3

Nets & Surface Area

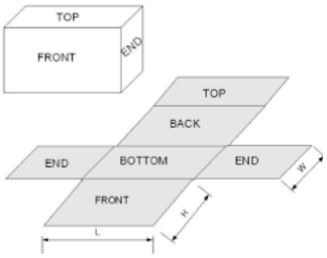


Nets: Two dimensional pattern of a three dimensional solid.

- It's cut out from a single piece of paper, folded, and taped to create a model of a geometric solid.

Surface Area: The total area of all the two dimensional surfaces added together required to cover a three dimensional shape.

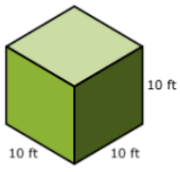
- Area of the OUTSIDE of a shape.
- Like wrapping a present.



Surface Area Formulas

Cube

SA = 6s²



s = side

SA = 6 (10)²

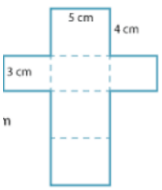
SA = 6(100) =

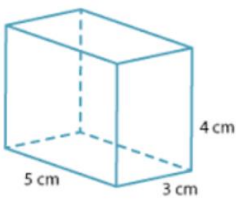
600 ft²

Rectangular Prism

SA = 2(ℓw) + 2(wh) + 2 (hℓ)

ℓ = length, w = width, h =height





SA = 2(3 · 4) + 2(4 · 5) + 2(5 · 3)

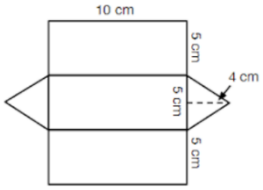
= 2(12) + 2(20) + 2(15)

= 24 + 40 + 30

= 94 cm²

Triangular Prism

SA = 2(½bh) + bh + bh + bh



b = base, h = height

SA = 2(½ (5 · 4)) + (10 · 5) + (5 · 10) + (5 · 10)

= 2(10) + 50 + 50 + 50

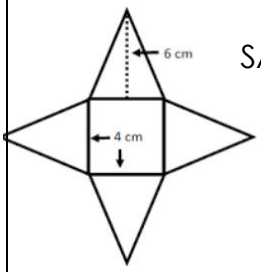
= 20 + 50 + 50 + 50

= 170 cm²

Square Pyramid

SA = 2bs + b²

b = base, s = slant height



SA = 2(4 ·6) + 4²

= 2(24) + 16

= 48 + 16

= 64 cm²

Division Vocabulary

12 ÷ 3 = 4

dividend ÷ divisor = quotient

Dividend: 12- the whole that is being divided.

Divisor: the amount of parts that is being divided.

Quotient: the answer to a division problem.

Whole Number Division

Division without Remainders

2214 ÷ 9 =

```
      0  2  4  6
   ┌──────────
 9 │ 2  2  1  4
     - 1  8
     ------
        4  1
      - 3  6
      ------
           5  4
         - 5  4
         ------
              0
```

2214 ÷ 9 = 246

Check your work by multiplying 246 by 9:

```
        4   5
        2   4   6
  x             9
-------------------
  2   2   1   4
```

Rounding Whole Numbers or Decimals (Right Round)

Look to the right of the number to be rounded.

- If that number is 5 or greater, round up
- If that number is 4 or less, round down
- Drop all digits to the right of the rounded number and fill in with zeros

Ex: 46,375

- The digit to the right of the underlined number is 7, so round up.
- Drop the remaining digits and fill with zeros

46,375 = 46,400

Ex: 963.154

- The digit to the right of the underlined number is 1, so round down.

- Drop the remaining digits and fill with zeros

963.154 = 963.0 or 963

Ex: 0.4<u>8</u>52

- The digit to the right of the underlined number is 5, so round up.
- Drop the remaining digits.

0.4<u>8</u>52 = 0.49

Decimals

Adding/Subtracting

- Line up the decimal points.
- Annex (add) zeros if necessary
- Add or subtract as you would with whole numbers
- Remember to bring down the decimal point in the exact same spot into the answer.

Ex: 24.7 + 48.92 =

```
  1 1
  24.70  (annexed zero)
+ 48.92
  73.62
```

Ex: 59.45 - 17.3 =

```
  59.45
- 17.30  (annexed zero)
  42.15
```

Multiplying Decimals

- Multiply as you would with whole numbers. Ignore the decimals.
- **DO NOT LINE UP THE DECIMALS.**

Count the total number of digits **behind** the decimals in the problem. That is how many places will be after the decimal in the product (answer).

Ex: 5.4 x 13

```
    1
    5.4
x   1 3
  1 6 2
+ 5 4 0
  7 0.2
```

Dividing Decimals by Whole Numbers:

43.26 ÷ 6

```
   07.21
6 | 43.26
  - 42
    12
  - 12
      06
    - 06
      0
```

Bring up the decimal point into the quotient. Divide as you would with whole numbers. Annex zeros if necessary.

Dividing Decimals by Decimals

```
4.2 | 28.56
```

Change the divisor to a whole number by moving the decimal point to the right. Move the decimal point in the dividend the same number of spaces to the right. Annex zeros if necessary.

```
4 2. | 2 8 5.6
```

Divide as you would with whole numbers. Remember to bring up the decimal point into the quotient.

```
     6.8
4 2 | 285.6
  - 252
    336
  - 336
      0
```

Division with Decimal Remainders

(Repeating Decimal)

You won't know if there is a remainder until you do the problem.

Use divisibility rules to help you figure out if there will be a remainder.

Set it up just like any other division problem.

Since you won't leave the remainder as "R", you need to annex up to 3 zeros.

DO NOT USE the BIG 7 with decimals!!!

```
   0345.666
6 | 2074.000
  - 18
    27
  - 24
     34
   - 30
      40
    - 36
      40
    - 36
      40
    - 36
       4
```

$= 345.\overline{6}$

Rates, Ratios & Proportions

Ratio: is a comparison of two quantities using division. Reduce when possible. Ratios can be compared as **part to part** or **part to whole**.

★ ★ ★ ★ ★ ❤ ❤

Part to Part

$\frac{5}{2}$ 5:2 5 to 2

Part to Whole

$\frac{5}{7}$ 5:7 5 to 7

Rate: Compares 2 quantities that have different units of measure. (Must be labeled)

- 2-liter bottle of soda costs $1.98.

rate = $\frac{Price}{\#\ of\ Liters} = \frac{\$1.98}{2\ liters}$

Distance Formula

- Distance = Rate x Time
- d=rt
- Rate = Speed = $\frac{distance}{time}$

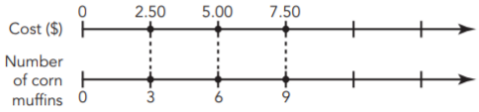
Ex: Speed = $\frac{60\ miles}{1\ hour}$ = 60 mph

Unit Rate: The comparison to one unit. Unit rates make it easier to compare quantities.

- Best Buys

$\frac{\$1.98}{2\ liters} = \frac{\div 2}{\div 2} = \frac{\$0.49}{1\ liter}$

- **Scale Up**: MULTIPLY the numerator & denominator by the same factor.
- **Scale Down**: DIVIDE the numerator & denominator by the same factor.
- **Double Number Line:** Model made up of two number lines used to represent the equivalence of 2 related numbers.
  - Intervals on each number line maintain the same ratio.



Proportion: An equation that shows two equivalent ratios.

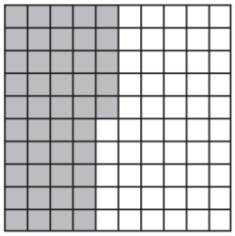
$\frac{3}{8} = \frac{6}{?}$ $\frac{3 \cdot 2}{8 \cdot 2} = \frac{6}{16}$

- The numbers representing the same quantity must be placed in both numerators or in both denominators.
- The unit of measurement must be consistent among the ratios.

Percents

Percent: a ratio whose denominator is 100.

- Out of 100
- Percent is another name for hundredths.
- 1 whole = 100%

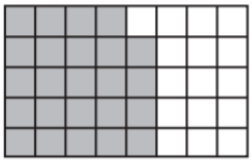


45%

$\frac{45}{100}$

0.45

What percent is shaded?



1. Count the whole = 5 x 8 = 40
2. Count the shaded part = 24
3. Write as a part to whole ratio & simplify. $\frac{24}{40} = \frac{3}{5}$
4. Turn into percent by dividing the numerator by the denominator= 60%

$\frac{part}{whole} = \frac{\%}{100}$

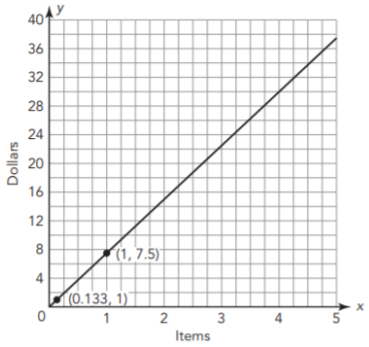
10/5/1: Benchmark Percents

100%	240
50%	120
10%	24
1%	2.4
5%	12
25%	60
75%	180

Equivalent Ratios

Equivalent ratios represented on the coordinate plane form a *straight line* that *passes through the origin.*

- $k = \frac{y}{x}$

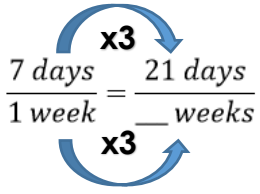


- A ratio $\frac{y}{x}$ is plotted as the ordered pair (x, y).

Scaling & Conversions

(Convert Using Equivalent Ratios)

1. Write down the unit conversion as a fraction.
2. Set up a proportion with labels matching side by side.
3. Scale up or down.
4. LABEL



- **Converting Units:**
  - **Small to Large** = Large has fewer units of measure.

- **Large to Small** = Small has more units of measure

Metric Conversions

Metric Prefixes

Kilo = 1000	**Deci** = 0.1
Hecto = 100	**Centi** = 0.01
Deka = 10	**Milli** = 0.001

King Henry Doesn't Usually Drink Chocolate Mini-Milks

K H D U D C MM

Metric Weight/ Mass - GRAMS

1 kg	1000 g
1 kg	1,000,000 mg
1 g	1000 mg

Metric Capacity- Liters

1 L	1,000 mL
1 kL	1,000 L

Metric Length- Meters

1 km	1,000,000 mm
1 km	1,000 g
1 km	100,000 cm
1 m	100 cm
1 m	1,000 mm
1 cm	10 mm

Customary Conversions

Customary Length

1 ft	12 in
1 yd	36 in
1 yd	3 ft
1 mi	5280 ft
1 mi	1760 yd

Customary Time

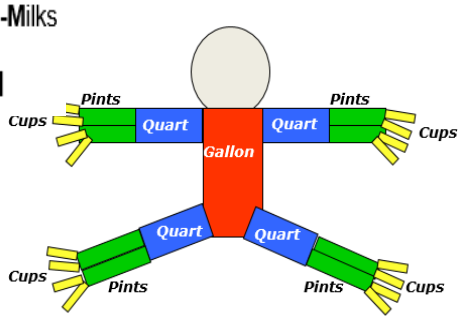
1 min	60 sec
1 hr	60 min
1 day	24 hr
1 wk	7 days
1 yr	12 mo
1 leap year	366 days
1 yr	365 days
1 yr	52 wks

Customary Weight

1 lb	16 oz
1 ton	2000 lbs

Customary Volume

1 cup	8 fl. oz
1 gal	4 qts
1 gal	8 pts
1 gal	16 cups
1 qt	4 cups
1 pt	2 cups



Customary & Metric Cross-Conversions

MASS/WEIGHT

1 oz	28.35 g
1 g	0.0035 oz
1 lb	0.45 kg
1 kg	2.2 lb

CAPACITY

1 pt	0.47 L
1 L	2.11 pt
1 qt	0.95 L
1 L	1.06 qt
1 gal	3.79 L
1 L	0.26 gal

LENGTH

1 in	2.54 cm
1 cm	0.39 in
1 ft	30.48 cm
1 m	3.28 ft
1 mi	1.61 km
1 km	0.62 mi
1 m	39.37 in
1 in	0.0254 m
1 m	1.09 yd

Exponents (Powers)

An exponent tells the number of times a base is multiplied by itself (power).

Anything raised to the **zero power** equals 1.

- **$7^0 = 1$**

Anything raised to the **1st power** equals itself.

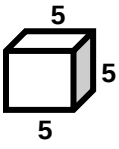
- **Identity Property of One**
- **$8^1 = 8$**

Anything raised to the **2nd power** is called **"squared".**
4^2
4 = base, 2 = exponent
$4 \bullet 4 = 16 = 4^2$

Anything raised to the **3rd power** is called **"cubed".**
5^3
5 = base, 3 = exponent
$5 \bullet 5 \bullet 5 = 125 = 5^3$

16, Base 2
The base is 2 and 16 is the answer. Figure out, to what power the base should be raised.
2 x 2 x 2 x 2 = 16
So, **16 Base 2** = 2^4

Powers & Roots

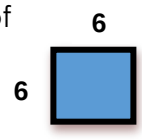


Cubed Root: One of the three equal factors of a number.

Cubes & Cubed Roots

1^3	1		$\sqrt[3]{1}$	1
2^3	8		$\sqrt[3]{8}$	2
3^3	27		$\sqrt[3]{27}$	3
4^3	64		$\sqrt[3]{64}$	4
5^3	125		$\sqrt[3]{125}$	5
10^3	1000		$\sqrt[3]{1000}$	10

Square Root: One of the two equal factors of a number.



Squares & Square Roots

1^2	1		$\sqrt{1}$	1
2^2	4		$\sqrt{4}$	2
3^2	9		$\sqrt{9}$	3
4^2	16		$\sqrt{16}$	4
5^2	25		$\sqrt{25}$	5
6^2	36		$\sqrt{36}$	6
7^2	49		$\sqrt{49}$	7
8^2	64		$\sqrt{64}$	8
9^2	81		$\sqrt{81}$	9
10^2	100		$\sqrt{100}$	10
11^2	121		$\sqrt{121}$	11
12^2	144		$\sqrt{144}$	12

Order of Operations

PEMDAS

Left → **Right**

1st: **P**arenthesis
2nd: **E**xponents
3rd: **M**ultiplication OR **D**ivision
4th: **A**ddition OR **S**ubtraction

1. Do all computations within brackets [ ], then parenthesis ( ) , if any.
2. Compute all the exponents 3^2
3. Multiply or divide, in order that they are given, from left to right.
4. Add or subtract, in order that they are given, from left to right.

" ***P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally*"

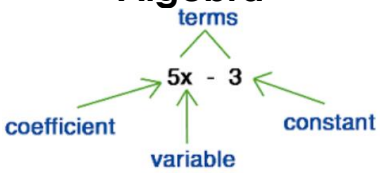
P $5 \cdot 7 + \mathbf{(10-4)} + 3^2 - 2$
6

E $5 \cdot 7 + 6 + \mathbf{3^2} - 2$
9

M OR D $\mathbf{5 \cdot 7} + 6 + 9 - 2$
35

A OR S $35 + 6 + 9 - 2$
$41 + 9 - 2$
$50 - 2$
48

Algebra



Variable: A letter or symbol used to measure a quantity that can change. LOWERCASE

Numeric Coefficient: the number next to the variable.

- A number, or quantity, that is multiplied by a variable in an algebraic expression.
- If a variable does not have a coefficient, then it is understood to be 1

Constant: A number, or quantity, that does not change its value.

Term: Either a single number or variable, or numbers and variables multiplied together.

- Separated by + or − signs.

Evaluate: Substitute the given values for the variables and then apply the order of operation rules to the numerical expression.

Numeric Expression: a mathematical phrase containing numbers, symbols, sometimes variables and operations with no equal sign.

- Multiplication can be represented as:
  5 x 7, 5 • 7, 5 * 7, (5)(7)

Equation: A mathematical sentence that shows that two expressions are equivalent. Contains an equal (=) sign.

Inverse Operation: The operation that reverses the effect of another operation (UNDO)

Properties of Equality: If the same operation is performed on both sides of the equation, then equality is maintained.

Isolate the Variable: Getting the variable by itself on one side of the equal sign.

7 Steps of Algebra

1. Write down the problem
2. Isolate the variable by doing the inverse operation on both sides
3. Cross out- on variable side
4. Draw Line
5. Drop down variable
6. Solve
7. Check ☆

Algebraic Expressions in Words

Addition Words: x + 4

- Added to
- Plus
- Sum
- More than

Subtraction Words: x - 4

- Subtracted
- Minus
- Difference
- Less than
- Take away

Multiplication Words: 4x

- Times
- Multiplied By
- Product
- Groups of

Division Words: $\frac{x}{4}$

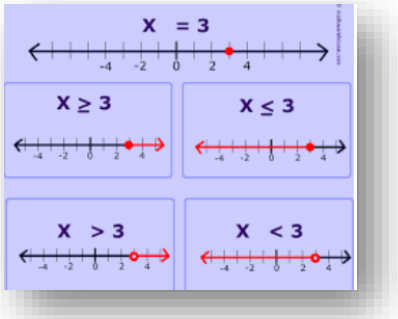
- Divided by
- Quotient
- Into

Inequalities

Symbol	Graph	Meaning	Word Phrases
<	○	is less than	Fewer than, below
>	○	is greater than	More than, above
≤	●	is less than or equal to	At most, no more than
≥	●	is greater than or equal to	At least, no less than

Inequality: States that two quantities either are NOT EQUAL or MAY NOT BE EQUAL.

- Use a number line to graph inequalities.



- **Solution Set of the Inequality**: In one variable, is the set of ALL points on a number line that make the inequality true.

Properties

Commutative Property of Addition: Add numbers in any order.

- a + b = b + a
- 6 + 1 = 1 + 6

Commutative Property of Multiplication: Multiply numbers in any order.

- ab = ba
- 6 x 1 = 1 x 6

Associative Property of Addition: When adding, group numbers together with parentheses.

- (a + b) + c = a + (b + c)
- (9+3) + 2 = 9+ (3 + 2)

Associative Property of Multiplication: When multiplying, group numbers together with parentheses.

- $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
- (9 x 3) x 2 = 9 x (3 x 2)

Identity Property of Zero: The sum of any number and zero is equal to the number.

- a + 0 = a
- 9 + 0 = 9

Identity Property of One: The product of any number and one is equal to the number.

- $b \cdot 1 = b$
- 6 x 1 = 6

Property of Zero: The product of any number and zero is zero.

- $c \cdot 0 = 0$
- 4 x 0 = 0

Distributive Property: Use to find the GCF of two numbers. Also known as the **product of two factors.**

- **Over Addition**
  - a(b + c) = ab + ac
  - 2(3 + 4) = 2 x 3 + 2 x 4 = 6 + 8 = 14
- **Over Subtraction**
  - a(b - c) = ab - ac
  - 3(4 - 2) = 3 x 4 - 3 x 2 = 12 – 6 = 6
- **Over Division**
  - $\frac{(a+b)}{c} = \frac{a}{c} + \frac{b}{c}$
  - $\frac{(6+4)}{2} = \frac{6}{2} + \frac{4}{2} = 3 + 2 = 5$
  - $\frac{(a-b)}{c} = \frac{a}{c} - \frac{b}{c}$
  - $\frac{(6-4)}{2} = \frac{6}{2} - \frac{4}{2} = 3 - 2 = 1$

Graphing Quantitative Relationships

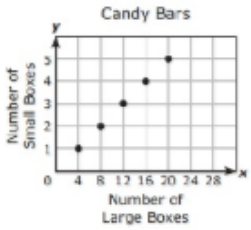
Dependent Quantity: When one quantity depends on another in a situation.

- **Dependent Variable**
- **y-axis**

Independent Quantity: The quantity on which the dependent quantity depends.

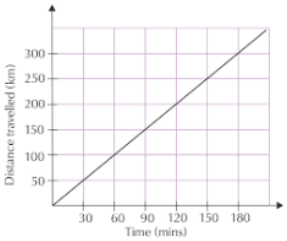
- **Independent Variable**
- **x-axis**

Discrete Graph: Graph of isolated points.



Continuous Graph: Graph with no breaks in it.

- Each point on it represents a solution to the graphed scenario.
- Represents fractional parts in between points.



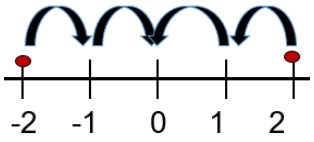
Integers

Integers: Whole numbers and their opposites. Cannot be fractions or decimals.

Opposites: Two numbers whose sum is 0.

- -4 is opposite to 4
- Same distance from zero on number line, but on opposite sides of zero.
- Zero (0) is its own opposite.

Absolute Value: The distance of a number from zero on the number line. **(Magnitude)**



- Numbers that are the same distance from 0 have the same absolute value.

|2| = 2
|-2| = 2

- Distance can **NEVER** be negative**.**

Coordinate Plane

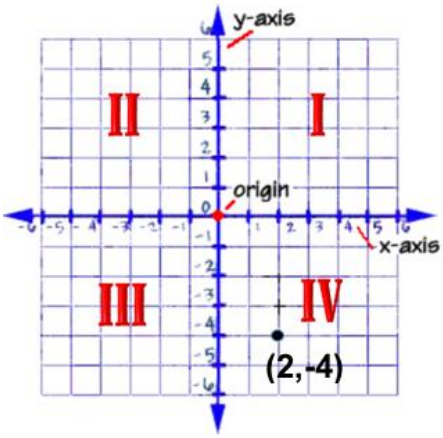
- Formed by a horizontal number line **(x-axis)** and a vertical number line **(y-axis)**.

- The x-axis & y-axis intersect at (0, 0) called the **origin**.

- The coordinate plane is divided into **4 quadrants** - written in Roman numerals.

- **Ordered Pair:** An x and y coordinate that represents an exact point on the coordinate plane.

- (2,-4) represents an ordered pair. 2 is the x coordinate & -4 is the y coordinate.



- **Distance** between two points are found by using **absolute value equations.**
  - The two points share either an x coordinate or y coordinate.
  - EX: The distance between (2, -4) and (2, 5). Since both points have the same x coordinate, you must find the distance between -4 and 5.
  - To find distance, find the absolute value: $|-4 + 5| = 9$ units

Data Collection

Data: The facts or numbers that describe the results of an experiment, observational studies, or survey.
- **Categorical** (words)
- **Quantitative** (numbers)

Statistical Question: A question about a population or sample.
- Gathers data.
- Answers to questions that have variability.

Population: An entire set of items from which data can be collected.
  - **Parameter:** The characteristic of a population.

Sample: A selection from a population.
  - **Statistic:** Characteristic of a sample.

Measures of Center

Variability: Describes how spread out the data is.

Mean: (Average)
Make everything into equal parts or fair share.
- Affected by extreme highs and lows in a data set.

Add the numbers and divide by the total number in the set:

{4, 4, 2, 3, 5, 5} =
4 + 4 + 2 + 3 + 5 + 5 = 23

23 ÷ 6 = 3.8 Mean = 3.8

- Round to the hundredths place if necessary.

Mean Absolute Deviation (MAD): The average distance of all of the data values in a data set from the mean of the distribution.

Median: (Middle Number)
- NOT affected by extreme highs and lows of data.

Place the numbers in ORDER. Find the middle number.
"Whack, Whack"
10, 4, 2, 3, 5, 5, 1=
1, 2, 3, 4, 5, 5, 10

Median = 4
- If there isn't one middle number, find the median by adding the two middle numbers (5 & 6) and divide by 2.

6, 9, 2, 7, 5, 1 = 1, 2, 5, 6, 7, 9
5 + 6 = 11 ÷ 2 = 5.5
Median = 5.5

Frequency: The number of times an item or number occurs in a data set.

Mode: (*Most Frequent*) Find the number that occurs most frequently. There can be more than one mode if two or more numbers occur "most often"
{4, 4, 2, 3, 4, 5, 5}
Mode= 4

{4, 4, 2, 3, 5, 5}
Modes= 4 & 5

{4, 1, 2, 3, 5, 6}
No Mode

Range: The difference between the greatest number and the least number:
{4, 4, 2, 3, 4, 5, 5} =
5 – 2 = 3
Range = 3

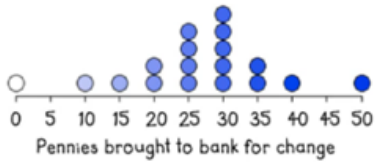
Outlier: The number(s) that do not fit with the rest of the data.

Data Displays & Distribution

Line Plot/ Dot Plot: A way to organize data along a number line where the x's (or dots) above a number represent how often each value is mentioned.

- Shows shape of data.
- Represents quantitative data.

Data Distribution: The overall shape of the graph.



Symmetric (Bell Curve): Has a high point or **peak** in the middle of the data set.

- Data points to left and right are mirror images of each other.
- Mean & median are about equal.
- **MEAN** = best measure of center.

Gap: Represents no data. Usually next to clusters.

- May include an **outlier**.

Cluster: Sudden spirt often surrounded by gaps.

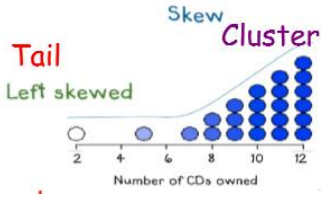
- Seem to stand separate from the remainder of the answers.

Uniform Distribution: Graphs with no shape at all. Leveled data with very few clusters, gaps, peaks or curves.

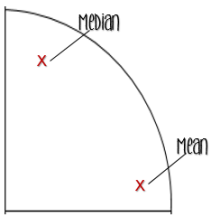
- Most answers have the same amount of responses.

Skew: Named after its tail.

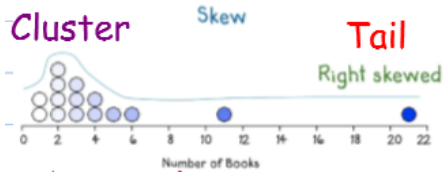
- **Median** is not affected by the skew.



Left Skewed: Has much of its data clustered to the right with only a few data values being plotted to the left (TAIL).

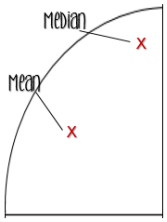


- Mean is less than the median.
- Median is the BEST measure of center



Right Skewed: Has much of its data clustered to the left with only a few data values being plotted to the right (TAIL).

- Mean is greater than the median.
- Median is the BEST measure of center.



Stem-and-Leaf Plot: A type of graph used to represent & organize data values for a large number of quantitative data.

- **Leaf:** represents the one's place.

Stem	Leaf
5	4 4 4 4 6 6 8 9
6	1
7	7

5|6 = 56 years

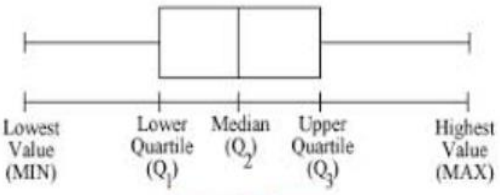
Box-and-Whisker Plot: Shows the distribution of values (**median**) in a data set separated into 4 equal size groups.

5 Number Summary

- Minimum Value or Lower Extreme
- Lower Quartile
- Median
- Upper Quartile
- Maximum Value or Upper Extreme

Interquartile Range (IQR): The difference between the values of the upper quartile (Q3) and the lower quartile (Q1).

Box-and-Whisker Plot

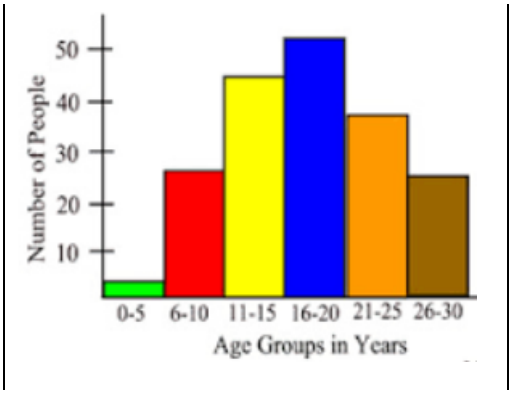


Histogram: Shows the distribution of numerical data represented continuously in intervals or ranges displayed on the horizontal axis.

- Ranges must be the same size.

- The vertical axis shows frequency in numbers or in percentages.

- The height of the bar over each interval indicates the count or percent of data values in that interval.
- The bars are fat & touching & are usually different colors.
- No key is needed.



Divisibility Rules

2	3	4	5	6	8	9	10
Ends in an Even Number 0, 2, 4, 6, 8	Sum of Digits	Last 2 Digits	Ends in 0 or 5	Divisible by **BOTH** 2 & 3	Last 3 Digits	Sum of Digits	Ends in 0
1,560	1,560 = 1+5+6+0 = 12 12 ÷ 3 = 4	1,560= 60 ÷ 4= 15	1,560	Even Number & Sum of Digits	1,560 560 ÷ 8 =70	Sum of Digits 1,560 = 1+5+6+0 = 12 12 ÷ 9 = NO	1,560
				Even Number 1,560 &		279= 2+7+9= 18 ÷ 9= 2 YES	
				Sum of Digits 1,560 = 1+5+6+0 = 12 12 ÷ 3 = 4			

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	**1**	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	**4**	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	**9**	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	**16**	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	**25**	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	**36**	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	**49**	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	**64**	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	**81**	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	**100**	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	**121**	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	**144**	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	**169**	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	**196**	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	**225**	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	**256**	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	**289**	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	**324**	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	**361**	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	**400**