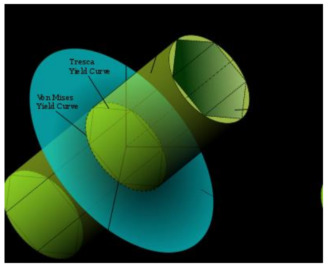
TKS 4108 - BAJA PLASTIS

Failure Criteria



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Stress-Based Criteria

The purpose of failure criteria is to predict or estimate the failure/yield of machine parts and structural members. A considerable number of theories have been proposed. However, only the most common and well-tested theories applicable to isotropic materials. These theories, dependent on the nature of the material in question (i.e. brittle or ductile), are listed in the following table:

Table 1 Failure theories

Material Type	Failure Theories
Ductile	Maximum shear stress criterion, von Mises criterion
Brittle	Maximum normal stress criterion, Mohr's theory

Stress-Based Criteria (cont'd)

All four criteria are presented in terms of principal stresses. Therefore, all stresses should be transformed to the principal stresses before applying these failure criteria.

Note:

1. Whether a material is *brittle* or *ductile* could be a subjective guess, and often depends on temperature, strain levels, and other environmental conditions. However, a 5% *elongation* criterion at break is a reasonable dividing line. Materials with a larger elongation can be considered ductile and those with a lower value brittle. Another distinction is a brittle material's compression strength is usually significantly larger than its tensile strength.

Stress-Based Criteria (cont'd)

2. All popular failure criteria rely on only a handful of basic tests (such as uniaxial tensile and/or compression strength), even though most machine parts and structural members are typically subjected to multi-axial loading. This disparity is usually driven by cost, since complete multi-axial failure testing requires extensive, complicated, and expensive tests.

Maximum Shear Stress Criterion

The maximum shear stress criterion, also known as **Tresca's** or **Guest's criterion**, is often used to predict the yielding of ductile materials.

Yield in ductile materials is usually caused by the *slippage* of crystal planes along the maximum shear stress surface. Therefore, a given point in the body is considered safe as long as the maximum shear stress at that point is under the yield shear stress σ_y obtained from a uniaxial tensile test.

Maximum Shear Stress Criterion

(cont'd)

With respect to 2D stress, the maximum shear stress is related to the difference in the two principal stresses (see Mohr's Circle). Therefore, the criterion requires the principal stress difference, along with the principal stresses themselves, to be less than the yield shear stress,

$$|\sigma_1| \leq \sigma_y,\ |\sigma_2| \leq \sigma_y,\ \text{and}\ |\sigma_1 - \sigma_2| \leq \sigma_y$$

Graphically, the maximum shear stress criterion requires that the two principal stresses be within the green zone indicated in **Fig. 1**.

Maximum Shear Stress Criterion
(cont'd)

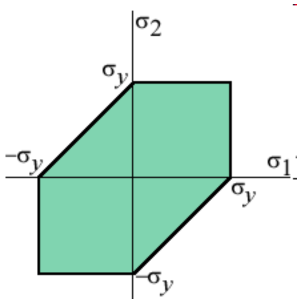

Fig. 1 Tresca's criterion

Von Mises Criterion

The von Mises Criterion (1913), also known as the maximum distortion energy criterion, octahedral shear stress theory, or Maxwell-Huber-Hencky-von Mises theory, is often used to estimate the yield of ductile materials.

The von Mises criterion states that failure occurs when the energy of distortion reaches the same energy for yield/failure in uniaxial tension. Mathematically, this is expressed as,

$$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]\leq\sigma_y{}^2$$

Von Mises Criterion (cont'd)

In the cases of plane stress, σ_3 = 0. The von Mises criterion reduces to,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \leq \sigma_y^2$$

This equation represents a principal stress ellipse as illustrated in **Fig. 2**. Also shown on this figure is the maximum shear stress criterion (dashed line). This theory is more conservative than the von Mises criterion since it lies inside the von Mises ellipse. In addition to bounding the principal stresses to prevent ductile failure, the von Mises criterion also gives a reasonable estimation of fatigue failure, especially in cases of repeated tensile and tensile-shear loading.

Von Mises Criterion (cont'd)

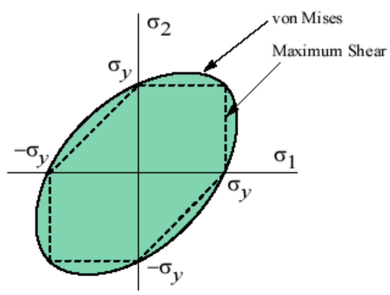

Fig. 2 Von Mises's criterion

Maximum Normal Stress Criterion

The maximum stress criterion, also known as the normal stress, Coulomb, or Rankine criterion, is often used to predict the failure of brittle materials.

The maximum stress criterion states that failure occurs when the maximum (normal) principal stress reaches either the uniaxial tension strength σ_t, or the uniaxial compression strength σ_c,

$$-\sigma_c < \{\sigma_1, \sigma_2\} < \sigma_t$$

where σ_1 and σ_2 are the principal stresses for 2D stress.

Maximum Normal Stress Criterion (cont'd)

Graphically, the maximum stress criterion requires that the two principal stresses lie within the green zone depicted below,

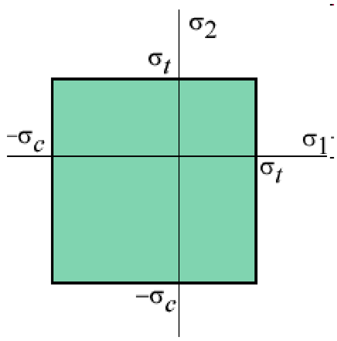

Fig. 3 The maximum stress criterion

Mohr's Theory

The Mohr Theory of Failure, also known as the Coulomb-Mohr criterion or internal-friction theory, is based on the famous Mohr's Circle. Mohr's theory is often used in predicting the failure of brittle materials, and is applied to cases of 2D stress.

Mohr's theory suggests that failure occurs when Mohr's Circle at a point in the body exceeds the envelope created by the two Mohr's circles for uniaxial tensile strength and uniaxial compression strength. This envelope is shown in **Fig. 4**,

Mohr's Theory (cont'd)

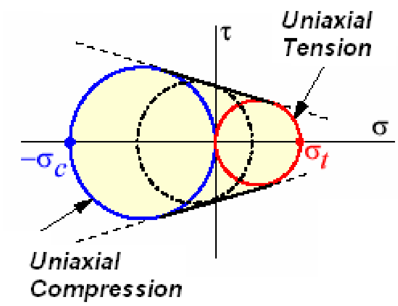

Fig. 4 The Mohr's circle

Mohr's Theory (cont'd)

The left circle is for uniaxial compression at the limiting compression stress σ_c of the material. Likewise, the right circle is for uniaxial tension at the limiting tension stress σ_t.

The middle Mohr's Circle on the figure (dash-dot-dash line) represents the maximum allowable stress for an intermediate stress state.

All intermediate stress states fall into one of the four categories as shown in **Table 2**. Each case defines the maximum allowable values for the two principal stresses to avoid failure.

Mohr's Theory (cont'd)

Table 2 The categories of intermediate stress states

Case	Principal Stresses		Criterion requirements
1	Both in tension	$s_1 > 0$, $s_2 > 0$	$s_1 < s_t$, $s_2 < s_t$
2	Both in compression	$s_1 < 0$, $s_2 < 0$	$s_1 > -s_c$, $s_2 > -s_c$
3	s_1 in tension, s_2 in compression	$s_1 > 0$, $s_2 < 0$	$\frac{\sigma_1}{\sigma_t} + \frac{\sigma_2}{-\sigma_c} < 1$
4	s_1 in compression, s_2 in tension	$s_1 < 0$, $s_2 > 0$	$\frac{\sigma_1}{-\sigma_c} + \frac{\sigma_2}{\sigma_t} < 1$

Mohr's Theory (cont'd)

Graphically, Mohr's theory requires that the two principal stresses lie within the green zone as shown in **Fig. 5**. Also shown on this figure is the maximum stress criterion (dashed line). This theory is less conservative than Mohr's theory since it lies outside Mohr's boundary.

Mohr's Theory (cont'd)

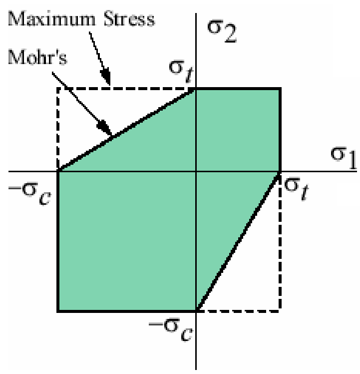

Fig. 5 The Mohr's theory

Principal Directions, Principal Stress

The normal stresses ($\sigma_{x'}$ and $\sigma_{y'}$) and the shear stress ($\tau_{x'y'}$) vary smoothly with respect to the rotation angle q, in accordance with the coordinate transformation equations. There exist a couple of particular angles where the stresses take on special values.

First, there exists an angle θ_p where the shear stress $\tau_{x'y'}$ becomes zero. That angle is found by setting $\tau_{x'y'}$ to zero in the above shear transformation equation and solving for θ (set equal to θ_p). The result is,

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Principal Directions, Principal Stress (cont'd)

The angle θ_p defines the *principal directions* where the only stresses are normal stresses. These stresses are called *principal stresses* and are found from the original stresses (expressed in the *x*,*y*,*z* directions) via,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

Principal Directions, Principal Stress (cont'd)

The transformation to the principal directions can be illustrated as shown in **Fig. 6**.

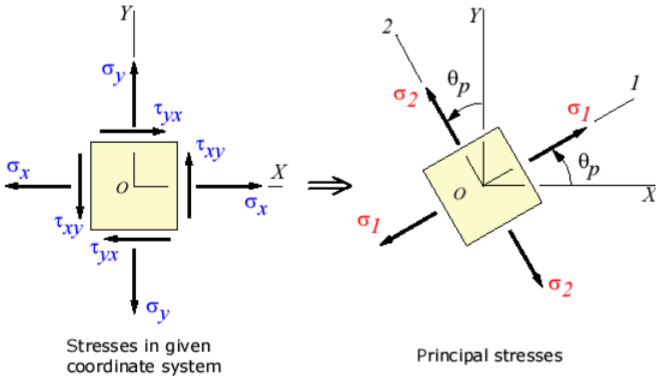

Fig. 6 The transformation of principal directions

Maximum Shear Stress Direction

Another important angle, θ_s, is where the maximum shear stress occurs. This is found by finding the maximum of the shear stress transformation equation, and solving for θ. The result is,

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\Rightarrow \theta_s = \theta_p \pm 45^\circ$$

Maximum Shear Stress Direction
(cont'd)

The maximum shear stress is equal to one-half the difference between the two principal stresses,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} = \frac{\sigma_1 - \sigma_2}{2}$$

Maximum Shear Stress Direction
(cont'd)

The transformation to the maximum shear stress direction can be illustrated as shown in **Fig. 7**.

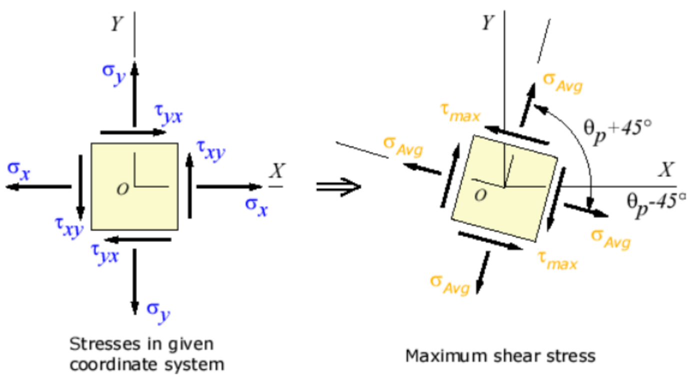

Fig. 7 The transformation of principal directions


Thanks for the Attention and
Success for Your Study!