Connecting Standards

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Terri L. Sebring

Florida Department of Education
fldoe.org

Expressions

Numeric Expressions

MAFS.5.OA.1.1
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

MAFS.5.OA.1.2
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

MAFS.6.EE.1.1
Write and evaluate numerical expressions involving whole-number exponents.

MAFS.8.EE.1.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Expressions

Connects numeric expressions with algebraic expressions.

MAFS.6.EE.1.2

Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers.
b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Algebraic Expressions

MAFS.6.EE.1.3

Apply the properties of operations to generate equivalent expressions.

MAFS.6.EE.2.6

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Algebraic Expressions

MAFS.7.EE.1.1
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

MAFS.7.EE.1.2
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Group Discussion

Mathematical expressions express calculations with numbers. An expression can consist of just a single number. As students progress through MAFS, expressions grow from numeric to algebraic.

The standards also build a student's understanding of equations. ***How can you help teachers understand the importance of teaching their students the distinct difference between expressions and equations as well as how an equation is a relationship between two expressions?***

In Grades K–5 students have been writing numerical equations and simple equations involving one operation with a variable. In Grade 6 they start the systematic study of equations and inequalities and methods of solving them. Solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number is a solution.

Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off, such as in $4x + 3x = 3x + 20$, where they can see that $4x$ must be 20 to make the two sides equal.

Progressions for the Common Core State Standards in Mathematics

MAFS.6.EE.2.5

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Equations

Equations

Writing numeric equations begins as early as Kindergarten.

MAFS.K.OA.1.1

Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MAFS.5.NF.2.6

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Progression of Solving Equations

MAFS.6.EE.2.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all non-negative rational numbers.

MAFS.7.EE.2.4a

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach

Progression of Solving Equations

MAFS.8.EE.3.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).
b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Do Algebra 1 and Algebra 2 teachers know how solving equations have progressed through the standards?

MAFS.8.EE.3.7b
Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

What should the rigor be for Algebra 1 and Algebra 2?

REI.1.1 **(shared)** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Mathematical Context

REI.2.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Mathematical Context

Group Discussion

Kindergarten through 7th grade lays the foundation for a student's understanding the concept of a function. Before a student learns the term "function" he/she gains experience with functions. The notion of a function is introduced in Grade 8. Linear functions are a major focus, but students are also expected to give examples of functions that are not linear.

Elementary: How are the "pattern" standards (4.OA.3.5 and 5.OA.2.3) laying the foundation for the concept of a function?

Middle: How is 7.RP.1.2 laying the foundation for the concept of a function?

High: How could spiraling back to "patterns" and proportional relationships as well as 8.F.1.1 and 8.F.1.3 help students distinguish when an equation could also be written using function notation?

MAFS.4.OA.3.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

MAFS.5.OA.2.3

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

MAFS.7.RP.1.2

Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

MAFS.8.F.1.1

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

MAFS.8.F.1.3

Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

The Algebra category in high school is very closely alligned with the Functions category.

The separation of algebra and functions is not intended to indicate a preference between an equation-based or a function-based approach. It is, however, intended to specify the difference as mathematical concepts between expressions and equations on the one hand and functions on the other. Students often enter college-level mathematics courses apparently conflating all three of these.

Progressions Documents for the Common Core Math Standards

Expressions, Equations and Functions

A-SSE.2.3 **(shared)** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Real-World Context

A-REI.2.4a **(shared)** Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Mathematical Context

F-IF.3.8a **(shared)** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Mathematical or Real-World Context

Create Equations and Functions

Create Equations

MAFS.5.NF.2.6

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *Several 5th grade standards require students to write numeric equations.*

MAFS.6.EE.2.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all non-negative rational numbers.

MAFS.7.EE.2.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

MAFS.8.F.2.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Things to note:

- **The standard does not mention a certain form of the line.**
- **The standard does not state that student will change from one form of the line to another.**
- **Equivalence is a theme throughout the standards.**
- **Rigor should increase as students move from 8th grade to Algebra 1 as well as to Algebra 2.**

CED.1.1 **(shared)** Create equations and inequalities in one variable and use them to solve problems. **Real-World Context**

CED.1.2 **(shared)** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **Real-World Context**

LE.1.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). **Real-World Context**

BF.1.1a **(shared)** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **Real-World Context**

ID.2.6a Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* **Real-World Context**

Solving equations
Algebra 1 and Algebra 2

REI.2.4 (shared) Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Mathematical Context

MAFS.8.EE.3.8

Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

c. Solve real world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

REI.3.6 (shared) Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Mathematical or Real-World Context

REI.4.11 (shared) Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

This standard is not about solving.

Mathematical or Real-World Context

Equivalent forms

Equivalence

In K-5, students produce equivalent numeric expressions using properties of operations as they work with whole numbers, decimals and fractions. Equivalence continues as students work with algebraic expressions in middle school. The student is also introduced to understanding equivalence within a real-world context.

Equivalent Expressions

MAFS.6.EE.1.3
Apply the properties of operations to generate equivalent expressions

MAFS.6.EE.1.4
Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

MAFS.7.EE.1.1
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

MAFS.7.EE.1.2
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

SSE.1.2 **(shared)** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Mathematical or Real-World Context

SSE.2.3 **(shared)** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Real-World Context

IF.3.8 **(shared)** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Real-World Context

SSE.2.3 **(shared)** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15t can be rewritten as* $(1.15^{1/12})^{12t} \approx (1.012)^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Real-World Context

IF.3.8 **(shared)** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$ and classify them as representing exponential growth or decay.

Mathematical or Real-World Context

Domain and Constraints

CED.1.3 **(shared)** Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. **Real-World Context**

IF.2.5 **(shared)** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the* positive integers would be an appropriate domain for the function. **Real-World Context**

Key Features

MAFS.8.F.1.2
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

IF.2.4 **(shared)** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Real-World Context

IF.3.9 **(shared)** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Mathematical or Real-World Context

Interpreting

MAFS.8.F.2.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

SSE.1.1 **(shared)** Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

Real-World Context

IF.2.6 **(shared)** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Real-World Context

ID.3.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Real-World Context

LE.2.5 **(shared)** Interpret the parameters in a linear or exponential function in terms of a context.

Real-World Context

Common Standards

A-APR.1.1	F-BF.1.1
A-APR.2.3	F-BF.2.3
A-CED.1.1	F-IF.2.4
A-CED.1.2	F-IF.2.5
A-CED.1.3	F-IF.2.6
A-CED.1.4	F-IF.3.7 a, b, c, d, and e
A-REI.1.1	F-IF.3.8
A-REI.2.4	F-IF.3.9
A-REI.3.6	F-LE.2.5
A-REI.4.11	N-RN.1.1
A-SSE.1.1	N-RN.1.2
A-SSE.1.2	Real World Context
A-SSE.2.3	Real World or Mathematical Context

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