Chapters 31 & 32

Faraday’s Law

Induction

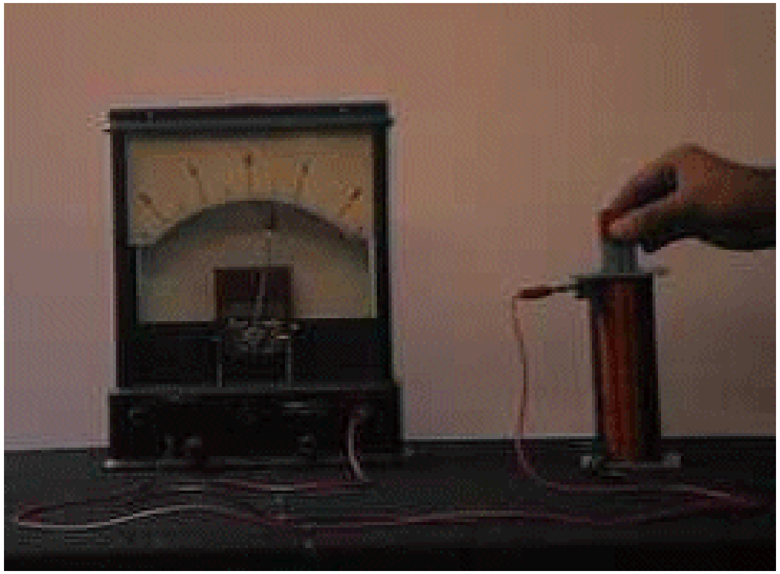
Michael Faraday

- Great experimental physicist
- 1791 – 1867
- Contributions to early electricity include:
  - Invention of motor, generator, and transformer
  - Electromagnetic induction
  - Laws of electrolysis

- A constant current produces a magnetic field.
- Does a constant magnetic field produce a current? Let’s try an experiment...

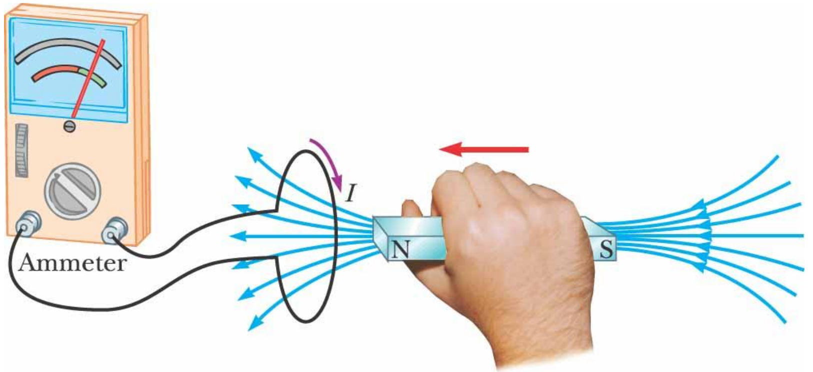
Ec6: Electromagnetic Induction

- Moving a permanent magnet inside a coil of wire induces an emf which then drives a current through the circuit.



EMF Produced by a Changing Magnetic Field, 1

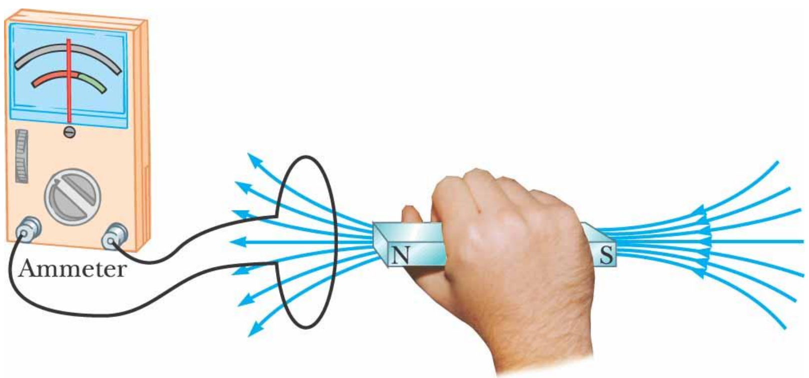
- A loop of wire is connected to a sensitive ammeter
- When a magnet is moved toward the loop, the ammeter deflects
  - The direction was chosen to be toward the right arbitrarily



(a)

EMF Produced by a Changing Magnetic Field, 2

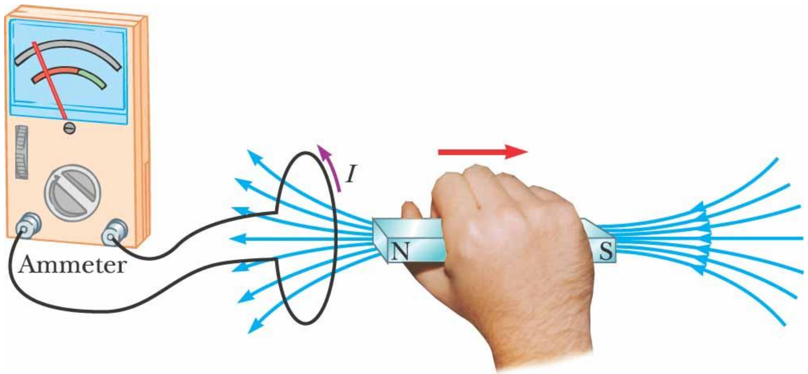
- When the magnet is held stationary, there is no deflection of the ammeter
- Therefore, there is no induced current
  - Even though the magnet is in the loop



(b)

EMF Produced by a Changing Magnetic Field, 3

- The magnet is moved away from the loop
- The ammeter deflects in the opposite direction



(c)

Induction

- An *induced current* is produced by a changing magnetic field
- There is an *induced emf* associated with the induced current
- A current can be produced without a battery present in the circuit
- Faraday's law of induction describes the induced emf

MFM06VD1: Faraday’s law

Faraday's Law of Induction

- The emf induced in a circuit is directly proportional to the rate of change of the magnetic flux through that circuit
- Mathematically,

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday's Law

- Remember Φ_B is the magnetic flux through the circuit and is found by

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

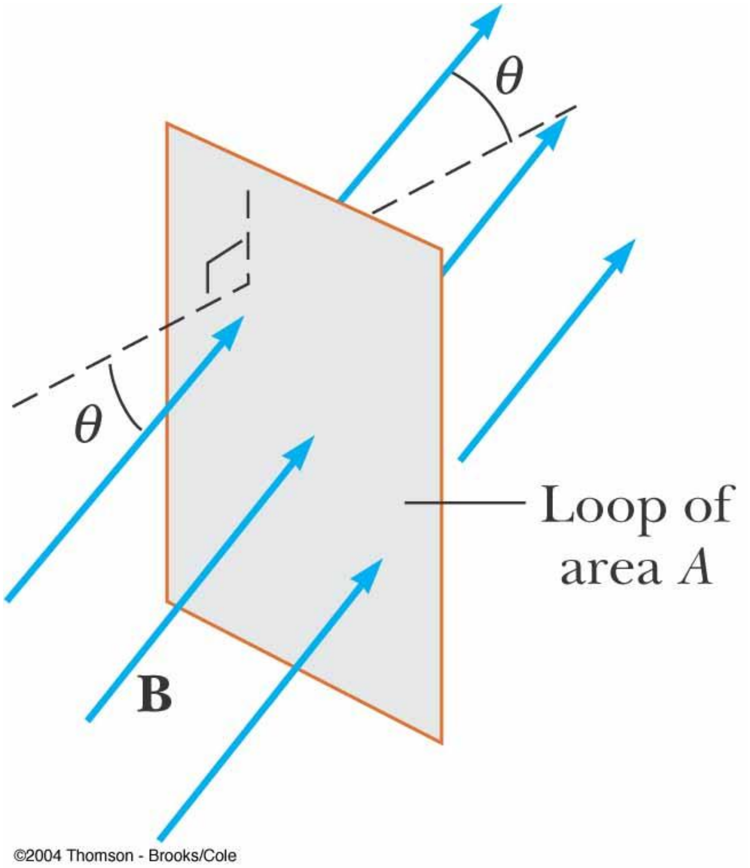
- For a circuit of *N* loops (all with Φ_B through them), an emf is induced in every loop. Faraday's law becomes

$$\varepsilon = -N \frac{d\Phi_B}{dt}$$

Faraday's Law – Example

- Assume a loop enclosing an area A lies in a uniform magnetic field **B**
- The magnetic flux through the loop is $\Phi_B = BA \cos \theta$
- The induced emf is

  $\varepsilon = - d/dt\,(BA \cos \theta)$



Ways of Inducing an emf

$$\varepsilon = - d/dt\ (BA \cos \theta)$$

- Magnitude of **B** can change with time
- Area enclosed, A, can change with time
- Angle θ can change with time
- Any combination of the above can occur

Quick Quiz 31.1

A circular loop of wire is held in a uniform magnetic field, with the plane of the loop perpendicular to the field lines. Which of the following will *not* cause a current to be induced in the loop?

(a) crushing the loop

(b) rotating the loop about an axis perpendicular to the field lines

(c) rotating the loop about an axis parallel to the field lines

(d) keeping the orientation of the loop fixed and moving it along the field lines

(e) pulling the loop out of the field

Quick Quiz 31.1

Answer: (c) and (d).

In all other cases there is a change in the magnetic flux through the loop.

Quick Quiz 31.2

The figure below shows the strength versus time for a magnetic field that passes through a fixed loop, oriented perpendicular to the plane of the loop. The magnitude of the magnetic field at any time is uniform over the area of the loop. Rank the magnitudes of the emf generated in the loop at the five instants indicated, from largest to smallest.

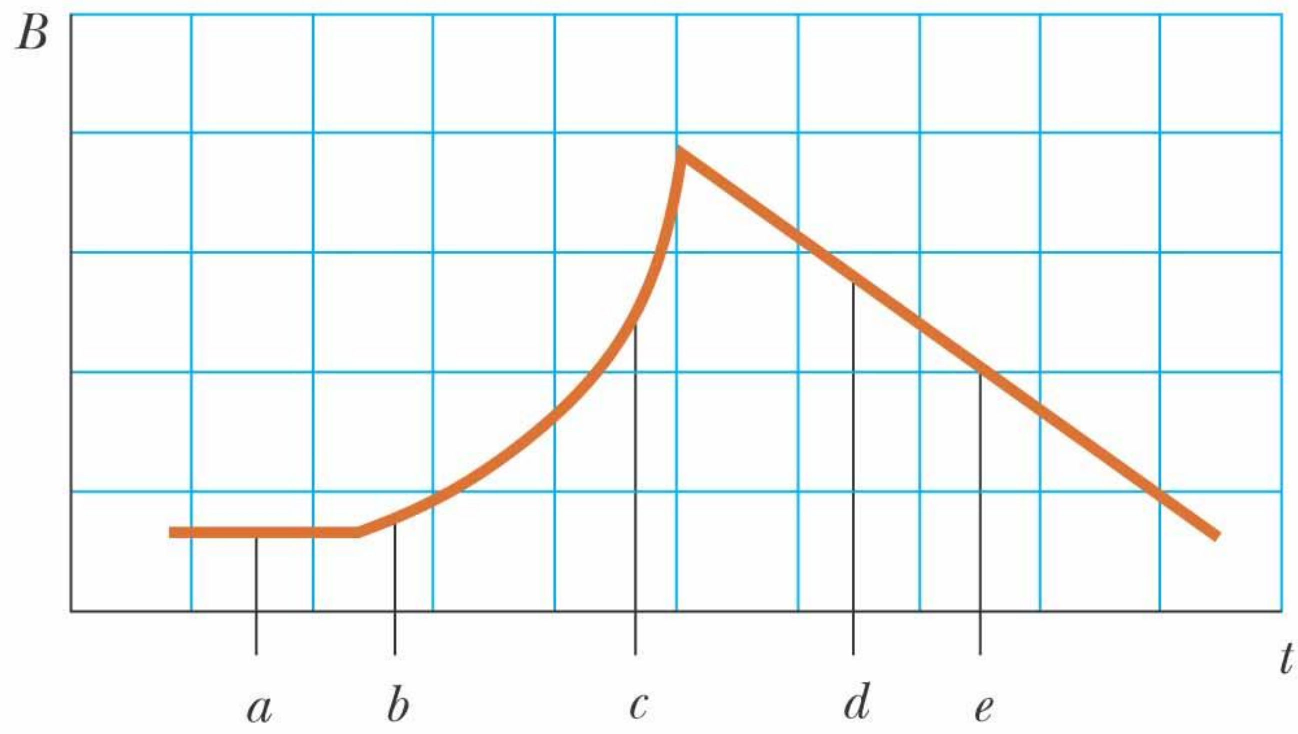
(a) a, b, c, d

(b) b, d, a, c

(c) c, d, b, a

(d) d, c, a, b

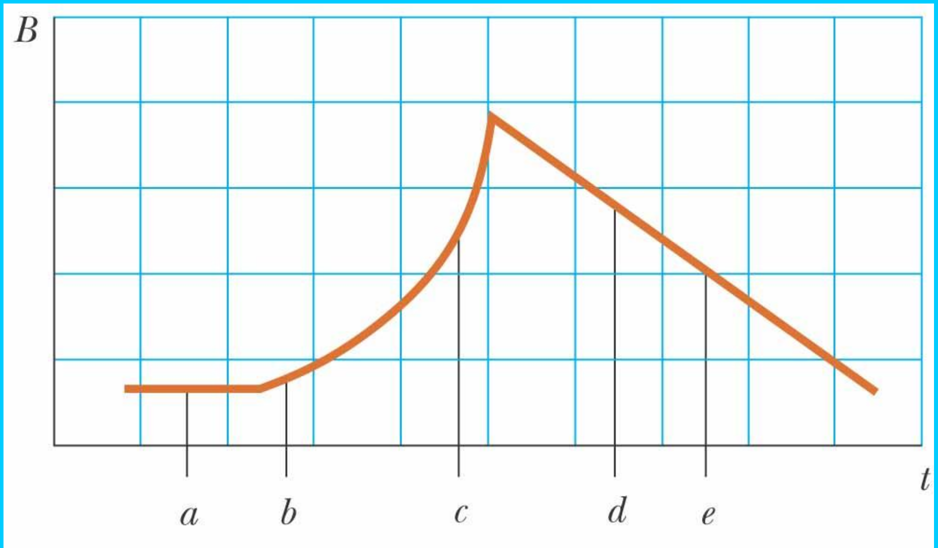
(e) e, a, d, c



Quick Quiz 31.2

Answer: (c). Specifically: *c*, *d* = *e*, *b*, *a*.

The magnitude of the emf is proportional to the rate of change of the magnetic flux. This is proportional to the rate of change of the magnetic field – i.e. the change in the slope of the graph. The magnitude of the slope is largest at *c*. Points *d* and *e* are on a straight line, so the slope is the same at each point. Point *d* represents a point of relatively small slope, while *a* is at a point of zero slope because the curve is horizontal at that point.



Quick Quiz 31.3

Suppose you would like to steal power for your home from the electric company by placing a loop of wire near a transmission cable, so as to induce an emf in the loop (*an illegal procedure!*). You would have to

(a) place your loop so that the transmission cable passes through your loop

(b) simply place your loop near the transmission cable

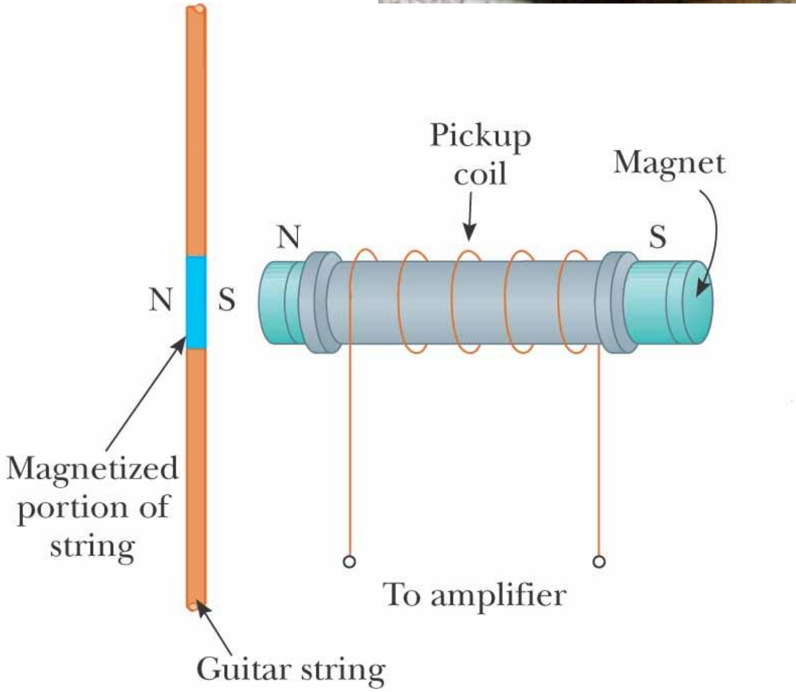
Quick Quiz 31.3

Answer: (b).

The magnetic field lines around the transmission cable will be circular, centred on the cable. If you place your loop around the cable, there are no field lines passing through the loop, so no emf is induced. The loop must be placed next to the cable, with the plane of the loop parallel to the cable to maximize the flux through its area.

Application Faraday's Law – Pickup Coil

- The pickup coil of an electric guitar uses Faraday's law
- The coil is placed near the vibrating string and causes a portion of the string to become magnetized
- When the string vibrates the magnetized segment produces a changing flux through the coil
- The induced emf is fed to an amplifier

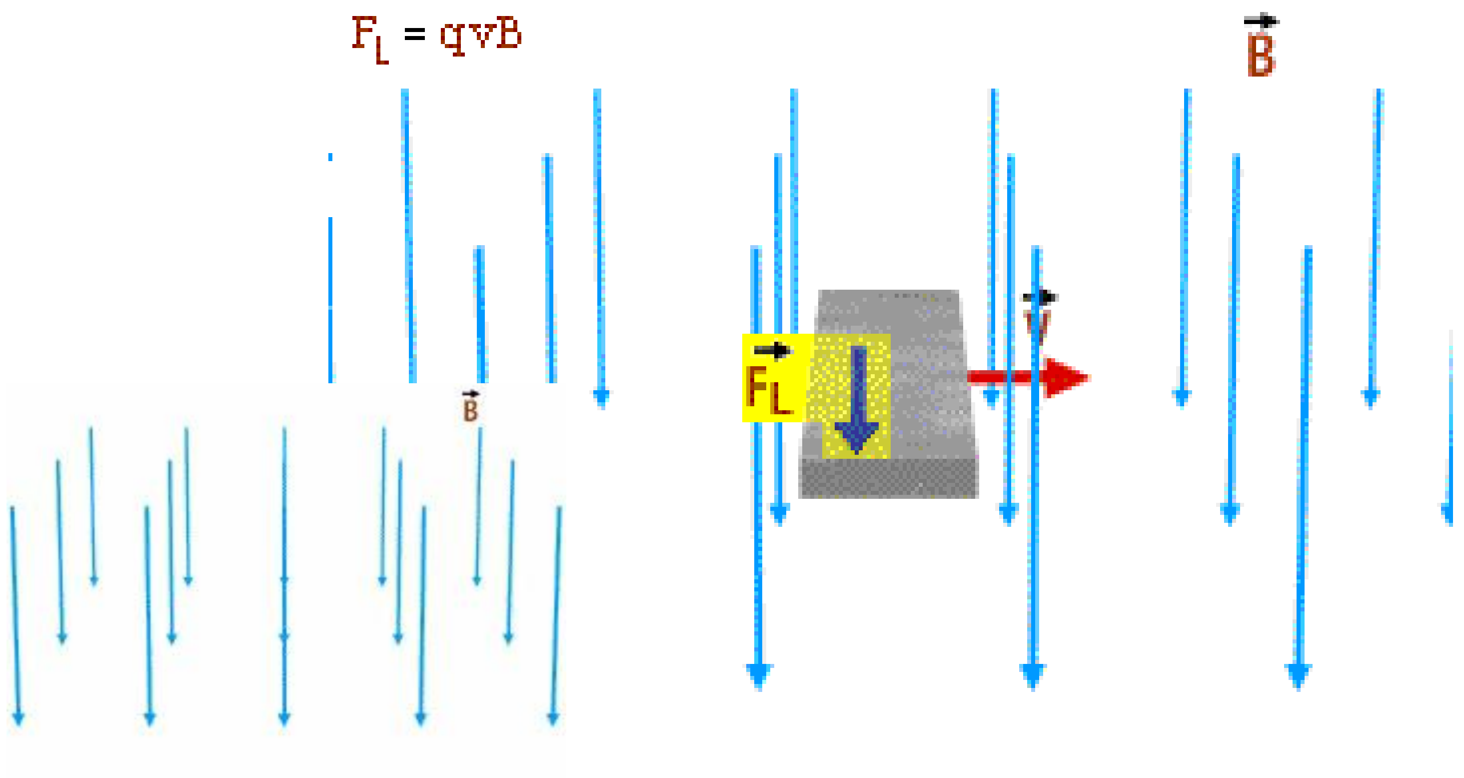


(a)

Ec17: Electromagnetic Induction – Jumping Rings

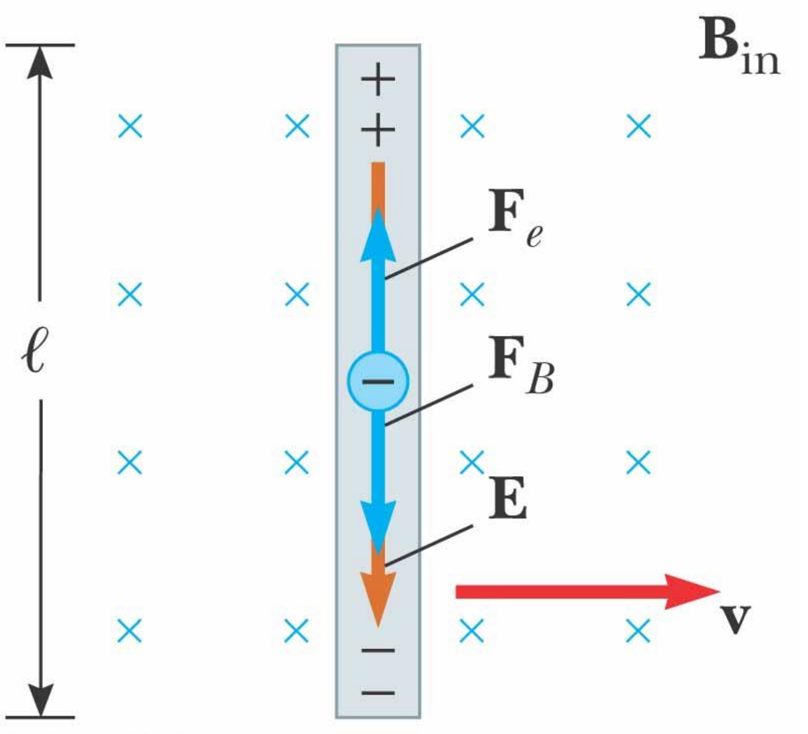
- Unbroken aluminium rings placed around the iron core of an inductor are repelled upwards when a conductor is connected to an AC source.
- The ring jumps much higher if first cooled in liquid nitrogen.
- Note: doesn't work if an iron ring is used because the ferromagnetic attraction is much larger than the repulsion due to the Eddy currents.

MFA06AN1: Lorentz force on a conductor moving through a magnetic field

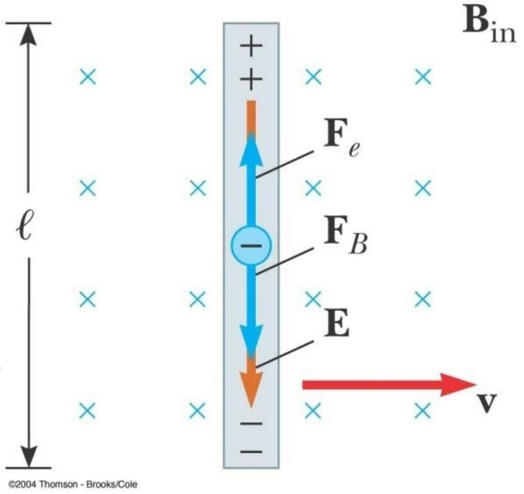


Motional emf

- A **motional emf** is one induced in a conductor moving through a constant magnetic field
- The electrons in the conductor experience a force, $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ that is directed along ℓ



Motional emf, cont.

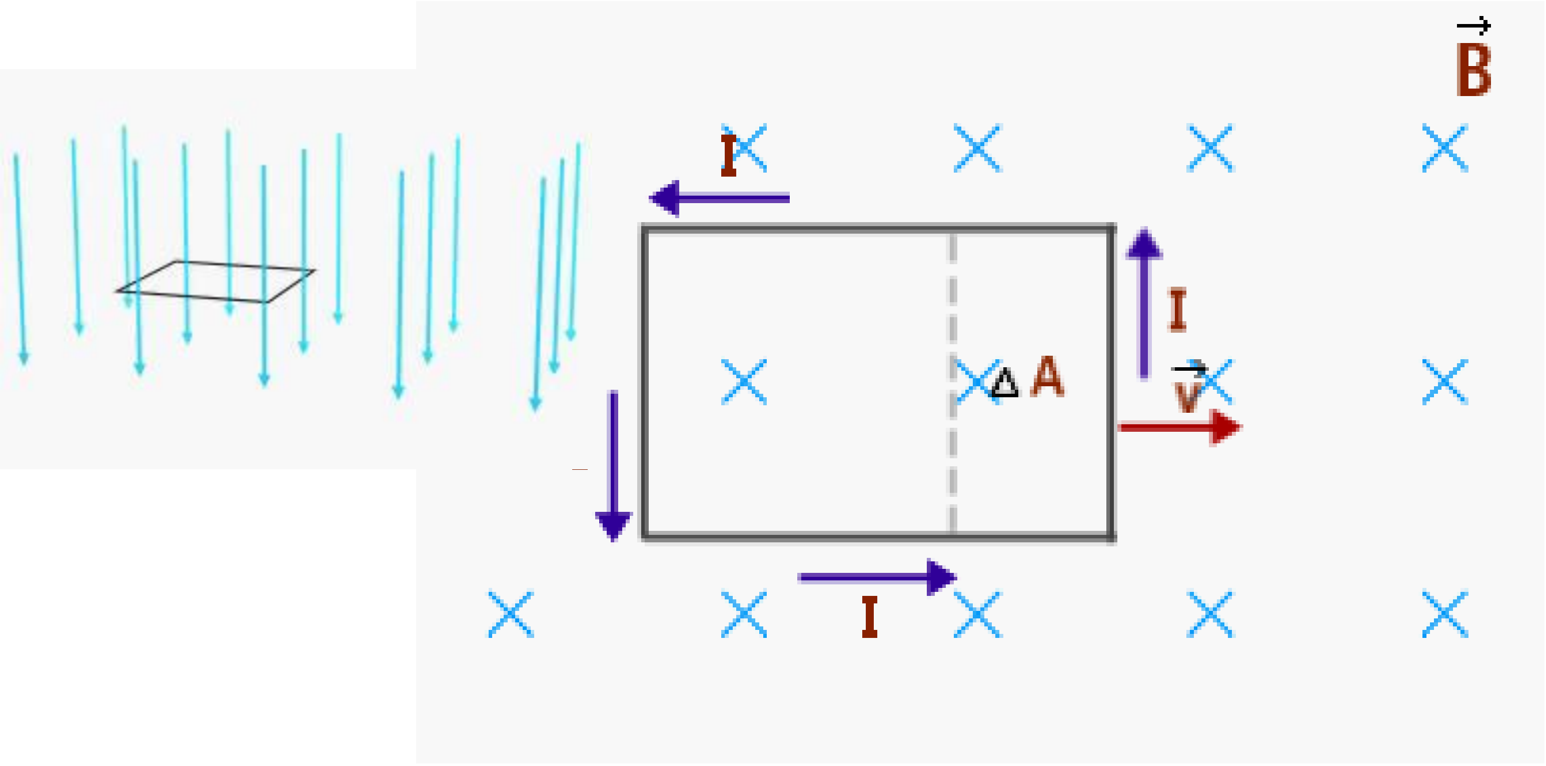


- Under the influence of the force, the electrons move to the lower end of the conductor and accumulate there
- As a result of the charge separation, an electric field **E** is produced inside the conductor
- The charges accumulate at both ends of the conductor until they are in equilibrium with regard to the electric and magnetic forces

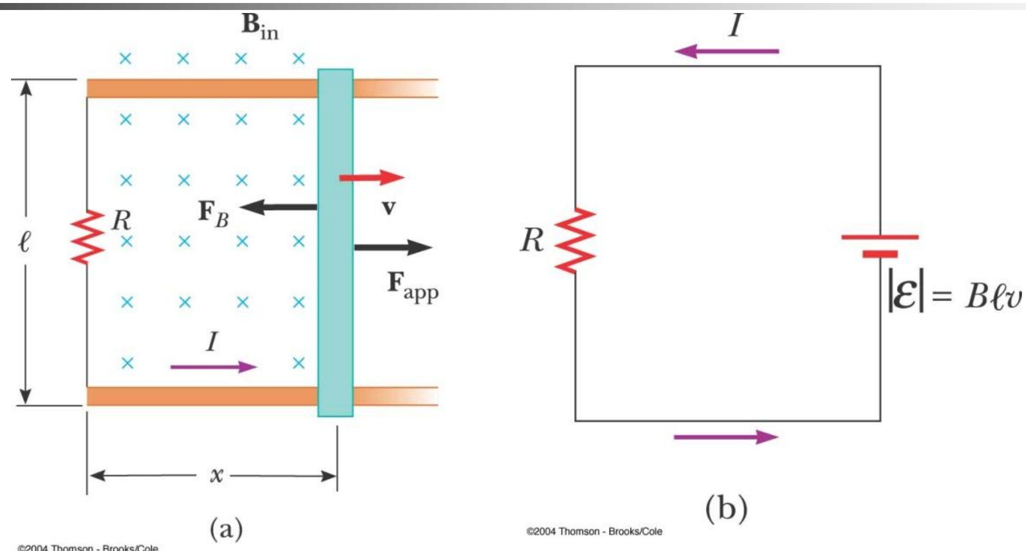
Motional emf, final

- In equilibrium, $qE = qvB$ or $E = vB$
- A potential difference is maintained between the ends of the conductor as long as it continues to move through the magnetic field
- If the direction of the motion is reversed, the sign of the potential difference is also reversed

MFM07AN1: emf generated by wire cutting magnetic field



Sliding Conducting Bar



- A bar moving through a uniform field and the equivalent circuit diagram
  - Assume the bar has zero resistance
- The work done by the applied force appears as internal energy in the resistor *R*

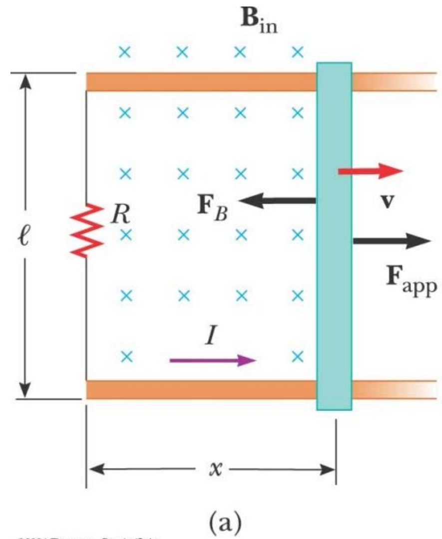
Sliding Conducting Bar, cont.

- Magnetic flux is $\Phi_B = Blx$
- The induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv$$

- Thus the current is

$$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$$



Sliding Conducting Bar: Forces

- The applied force F_{app} does work on the conducting bar
- This moves the charges through a magnetic field
- The magnetic force $F_B = BIl$ opposes the motion
- Its direction is opposite to the applied force (right-hand rule)
- Since the bar is moving at constant speed (i.e. no acceleration) we must have $F_{app} = F_B = BIl$

Sliding Conducting Bar, Energy Considerations

- The change in energy must be equal to the transfer of energy into the system by this work
- The power input is equal to the rate at which energy is delivered to the resistor
- Thus:

$$P = F_{app} v = (IlB)v = \frac{B^2 l^2 v^2}{R} = \frac{\varepsilon^2}{R}$$

Quick Quiz 31.4

As an airplane flies from Sydney to Melbourne, it passes through the Earth's magnetic field, which is directed upwards. As a result, a motional emf is developed between the wingtips. Which wingtip is positively charged?

(a) the left wing

(b) the right wing

Quick Quiz 31.4

Answer: (b).

The Earth’s magnetic field has an upward component in the southern hemisphere. As the plane flies south, the right-hand rule indicates that positive charge experiences a force directed toward the west. Thus, the right wingtip becomes positively charged and the left wingtip negatively charged.

Quick Quiz 31.5

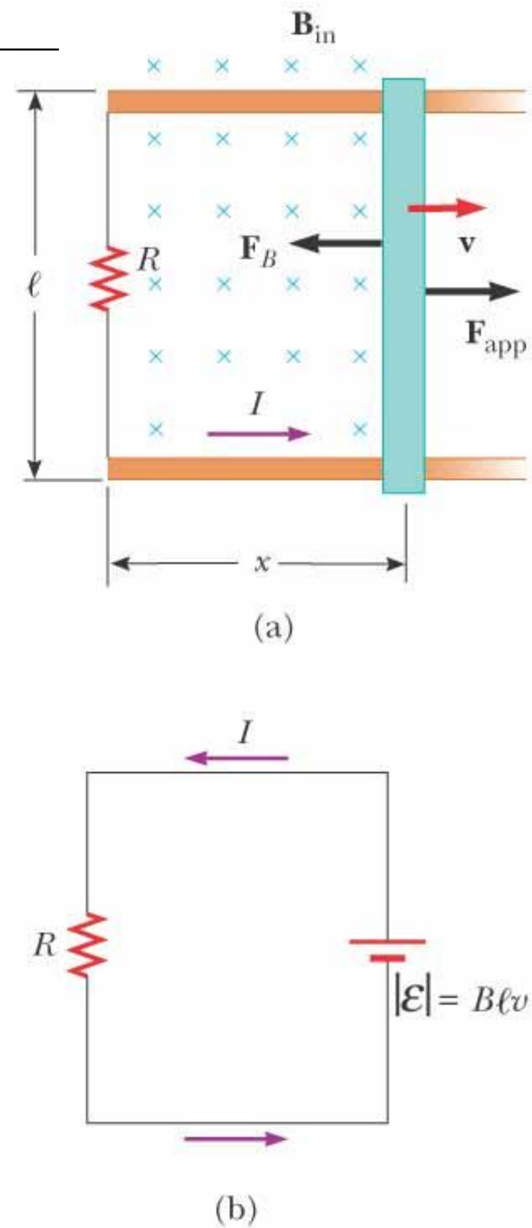
In the figure, a given applied force of magnitude F_{app} results in a constant speed v and a power input P. Imagine that the force is increased so that the constant speed of the bar is doubled to $2v$. Under these conditions, the new force and the new power input are

(a) $2F$ and $2P$

(b) $4F$ and $2P$

(c) $2F$ and $4P$

(d) $4F$ and $4P$



Quick Quiz 31.5

Answer: (c).

The force on the wire is of magnitude $F_{\text{app}} = F_B = I\ell B$, with $I = Blv/R$. Thus, the force is proportional to the speed and the force doubles. Because $P = F_{\text{app}}v$, the doubling of the force *and* the speed results in the power being four times as large.

Quick Quiz 31.6

You wish to move a rectangular loop of wire into a region of uniform magnetic field at a given speed so as to induce an emf in the loop. The plane of the loop remains perpendicular to the magnetic field lines. In which orientation should you hold the loop while you move it into the region of magnetic field in order to generate the largest emf?

(a) with the long dimension of the loop parallel to the velocity vector

(b) with the short dimension of the loop parallel to the velocity vector

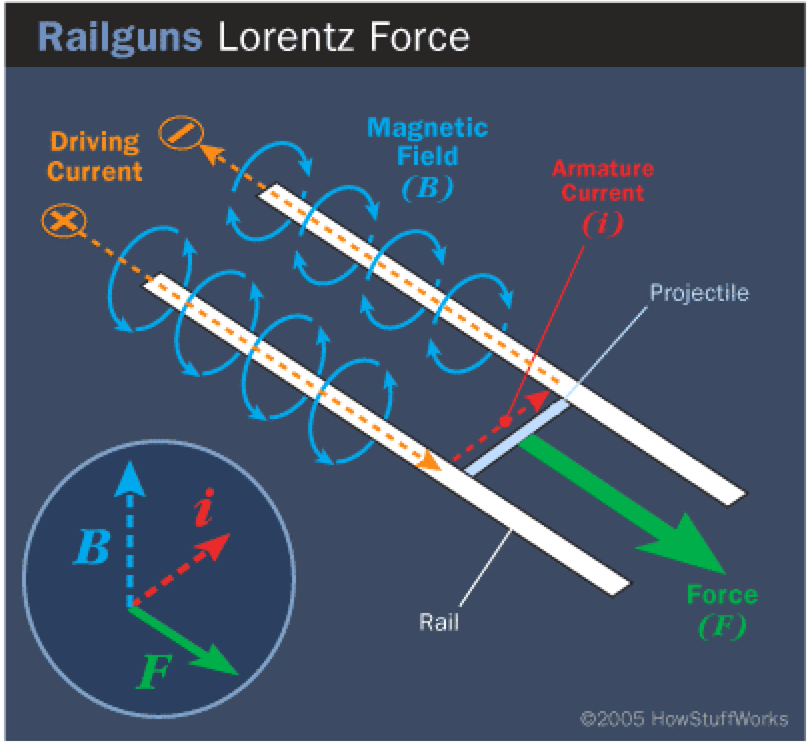
(c) either way—the emf is the same regardless of orientation.

Quick Quiz 31.6

Answer: (b).

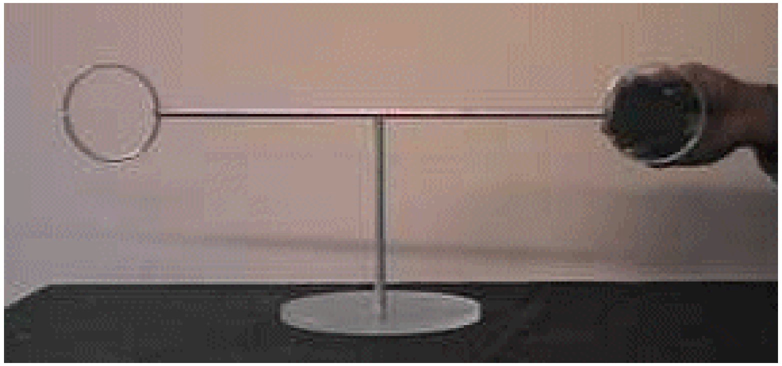
Since $\varepsilon = -Blv$, because B and v are fixed, the emf depends only on the length of the wire moving in the magnetic field. Thus, you want the long dimension moving through the magnetic field lines so that it is perpendicular to the velocity vector. In this case, the short dimension is parallel to the velocity vector.

Rail guns


Railguns Lorentz Force
Driving Current
Magnetic Field
(B)
Armature Current
(i)
Projectile
Rail
Force
(F)
B
i
F
©2005 HowStuffWorks

Ec15: Electromagnetic Induction – Lenz's rings

- An eddy current is induced in the unbroken ring but not in the broken one.
- The unbroken ring moves in a direction to oppose the force that caused the change in flux.



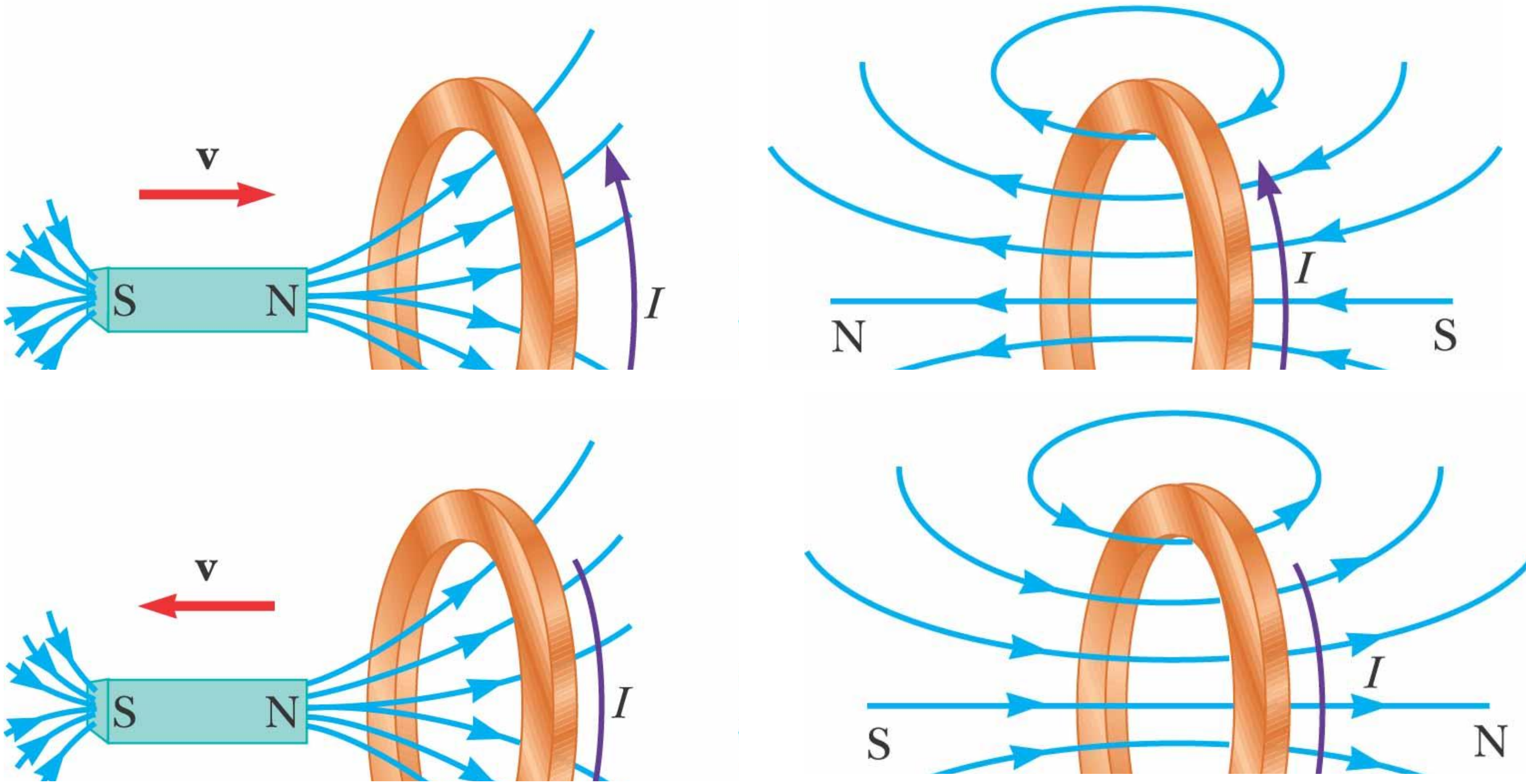
Lenz’s Law

- Faraday’s law indicates that the induced emf and the change in flux have opposite algebraic signs
- This has a physical interpretation that has come to be known as **Lenz’s law**
- Developed by German physicist Heinrich Lenz

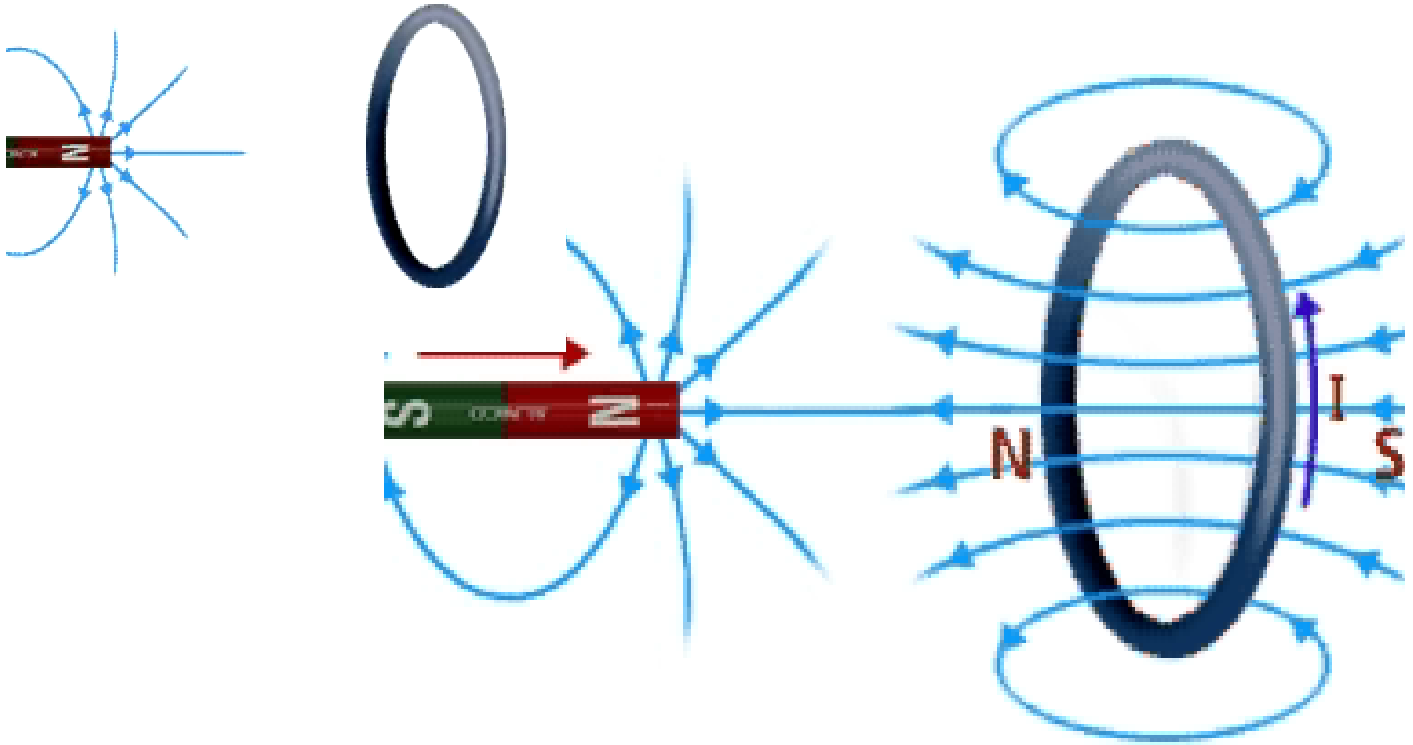
Lenz's Law, cont.

- **Lenz's law**: *the induced current in a loop is in the direction that creates a magnetic field that opposes the change that produced it*
- The induced current tends to keep the original magnetic flux through the circuit from changing

Lenz’s law in action



MFA07AN1: Lenz's law from magnet moving near wire



Quick Quiz 31.7

The figure below shows a magnet being moved in the vicinity of a solenoid connected to a sensitive ammeter. The south pole of the magnet is the pole nearest the solenoid, and the ammeter indicates a clockwise (viewed from above) current in the solenoid. The person is:

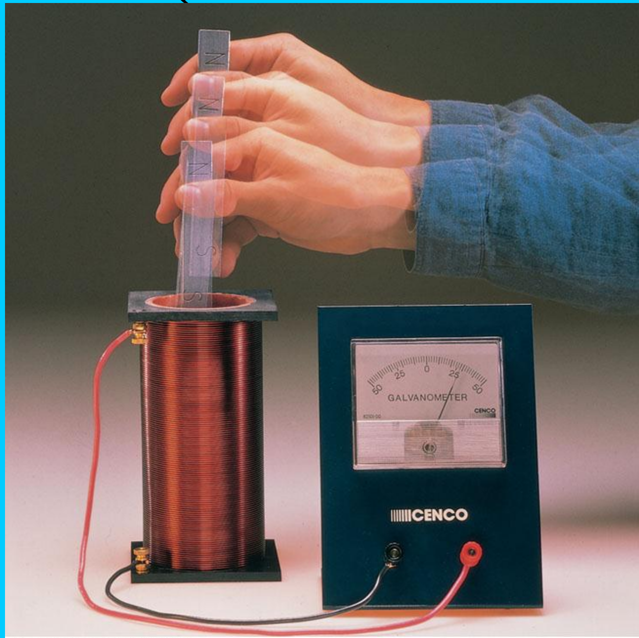
(a) inserting the magnet

(b) pulling it out

Quick Quiz 31.7

Answer: (a).

Because the current induced in the solenoid is clockwise when viewed from above, the magnetic field lines produced by this current point downward in the Figure. Thus, the upper end of the solenoid acts as a south pole. For this situation to be consistent with Lenz's law, the south pole of the bar magnet must be approaching the solenoid (as like poles repel).



Quick Quiz 31.8

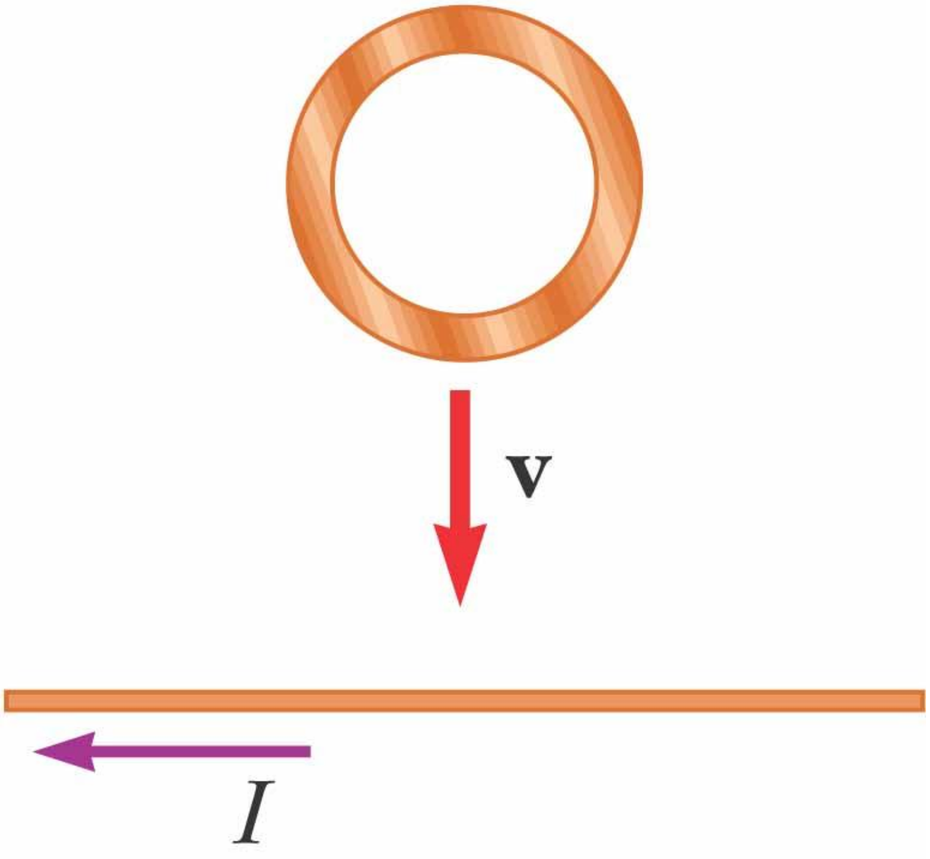
The figure below shows a circular loop of wire being dropped toward a wire carrying a current to the left. The direction of the induced current in the loop of wire is

(a) clockwise

(b) counterclockwise

(c) zero

(d) impossible to determine



Quick Quiz 31.8

Answer: (b).

At the position of the loop, the magnetic field lines point into the page. The loop is entering a region of stronger magnetic field as it drops toward the wire, so the flux is increasing. The induced current must set up a magnetic field that opposes this increase. To do this, it creates a magnetic field directed out of the page. By the right-hand rule for current loops, this requires a counterclockwise current in the loop.

Induced emf and Electric Field

- A changing magnetic flux induces an emf and a current in a conducting loop
- An electric field is created in a conductor by a changing magnetic flux
  - It is a *non-conservative* field.

Induced Electric Field in Conducting Loop

- Faraday's Law: $\varepsilon = -\frac{d\Phi_B}{dt}$
  - With $\Phi_B = B.A = \pi r^2 B$
- Work done moving once around loop is $q\varepsilon$
- This equals Force x distance moved

  i.e. $q\varepsilon = qE(2\pi r)$. So $E = \varepsilon/2\pi r$.
- Hence:

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{\pi r^2}{2\pi r}\frac{dB}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

Induced emf & Electric Fields

- The emf, ε, for any closed path can be expressed as the line integral of **E**·*d***s** over the path

$$\varepsilon = \oint \mathbf{E} \cdot d\mathbf{s}$$

- Thus, Faraday's law can be written in a general form:

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

- The field cannot be an electrostatic field because if it were the line integral of **E**·*d***s** would be zero. It isn't!

Quick Quiz 31.9

In a region of space, the magnetic field increases at a constant rate. This changing magnetic field induces an electric field that

(a) increases in time

(b) is conservative

(c) is in the direction of the magnetic field

(d) has a constant magnitude

Quick Quiz 31.9

Answer: (d).

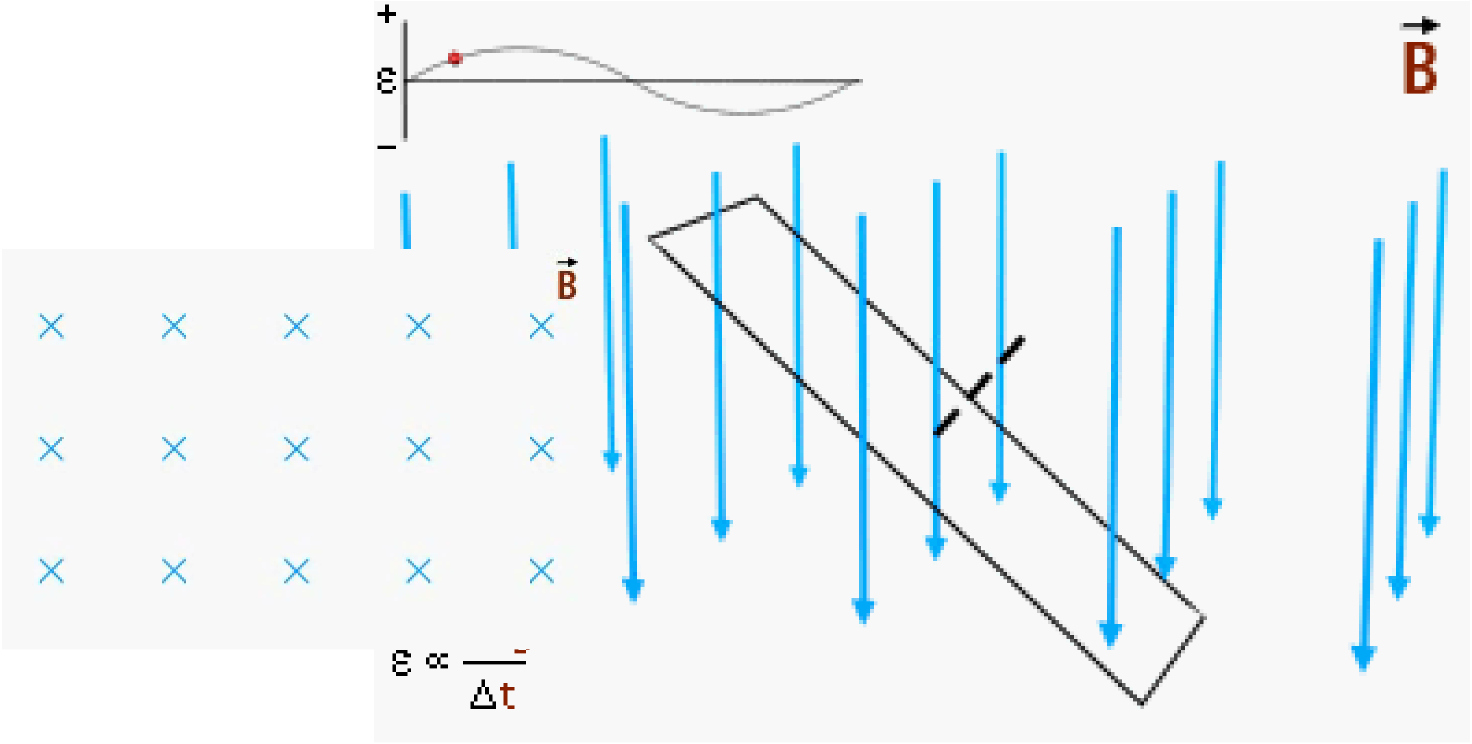
The constant rate of change of *B* will result in a constant rate of change of the magnetic flux. According to

$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$, if $d\Phi_B/dt$ is constant, **E** is constant in magnitude.

Or, consider the formula we derived: $E = -\frac{r}{2}\frac{dB}{dt}$

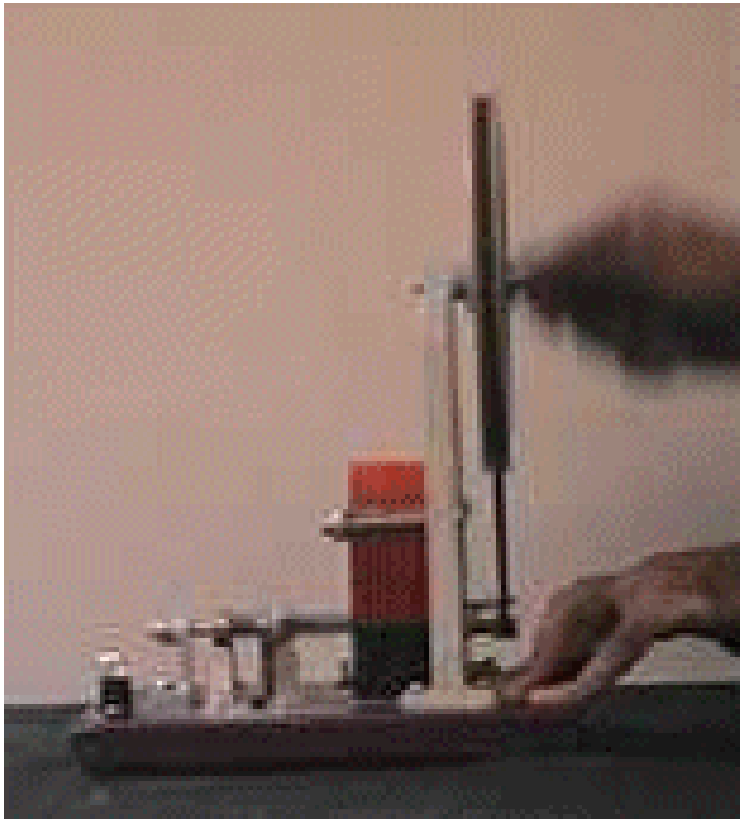
Then, since dB/dt is constant, E must be too.

MFM07AN3: emf generated by rotating current loop in a magnetic field



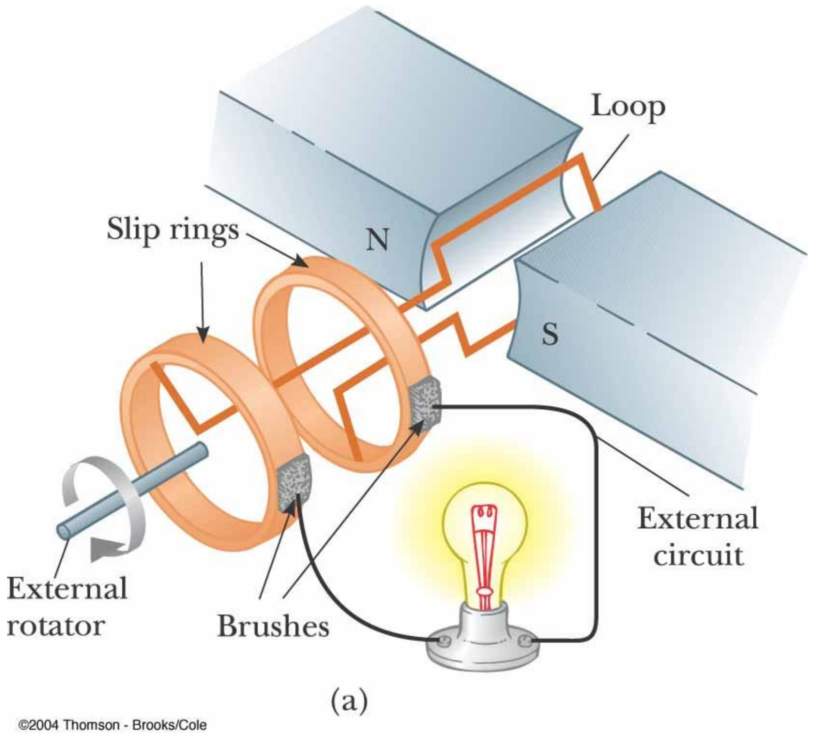
Ec8: The Electric Generator
AC or DC currents

- By vigorously turning a handle the globe can be made to glow.



Generators

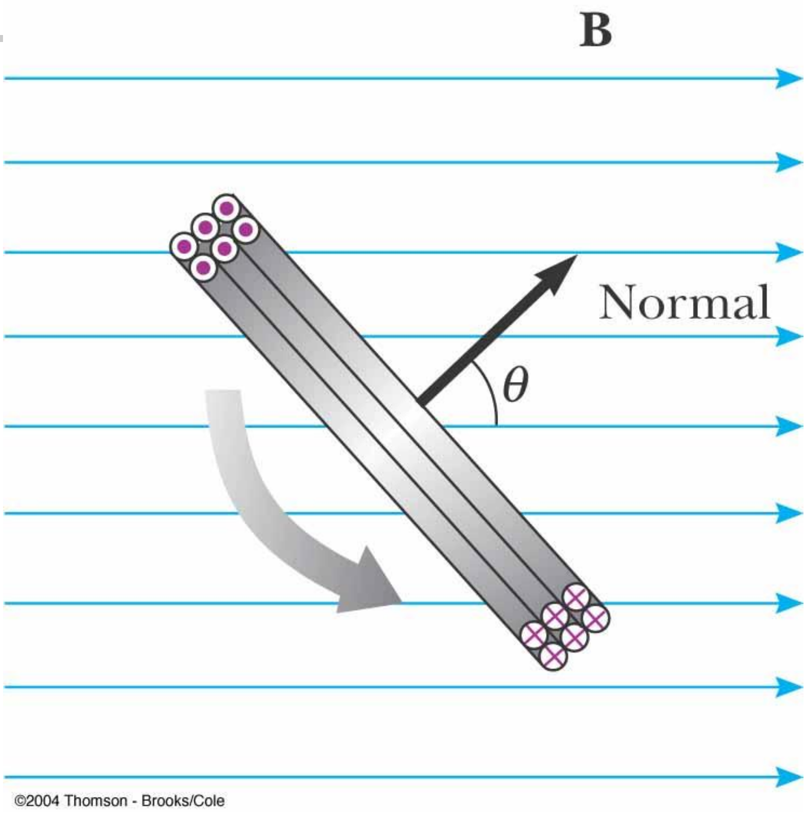
- Electric generators take in energy by work and transfer it out by electrical transmission
- The AC generator consists of a loop of wire rotated by some external means in a magnetic field



Rotating Loop

- Assume a loop with N turns, all of the same area, rotating in a magnetic field
- The flux through one loop at any time t is (with $\theta = \omega t$):

$\Phi_B = BA \cos \theta = BA \cos \omega t$



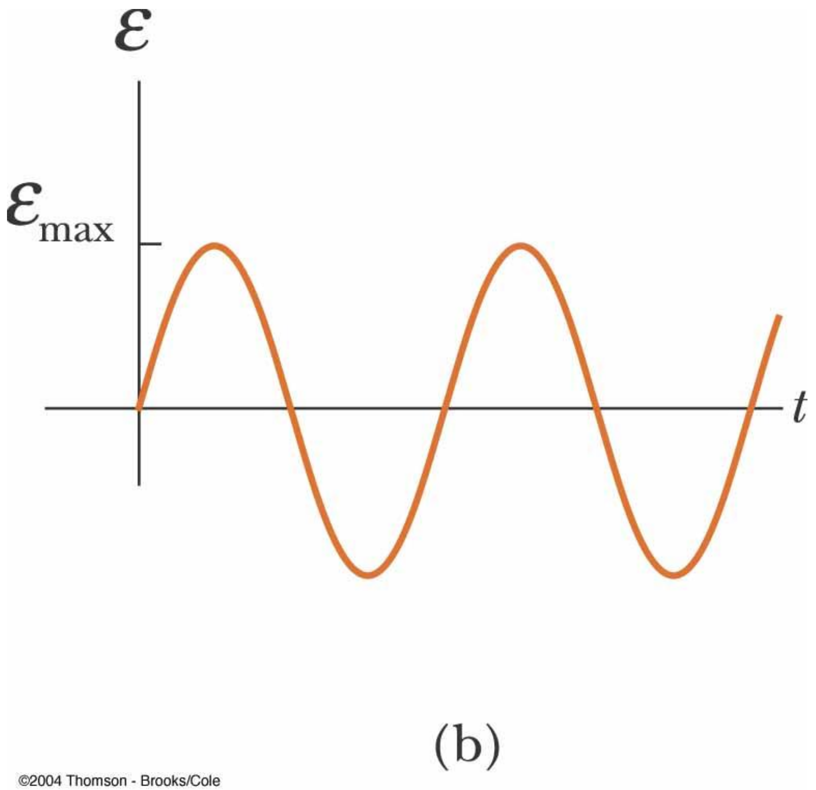
$$\therefore\ \varepsilon = -N \frac{d\Phi_B}{dt} = -NAB \frac{d}{dt}(\cos \omega t) = NAB\, \omega \sin \omega t$$

Induced emf in a Rotating Loop

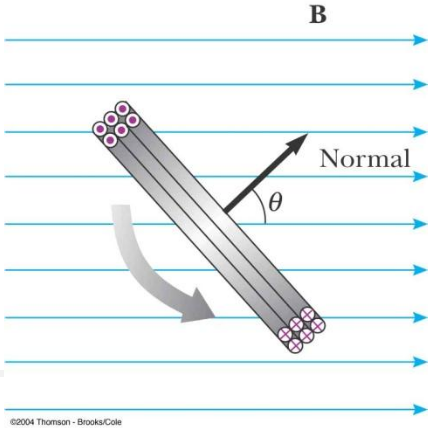
- The induced emf in the loop is

$$\varepsilon = NAB\ \ \omega \sin\ \omega t$$

- This is sinusoidal, with $\varepsilon_{max} = NAB\omega$

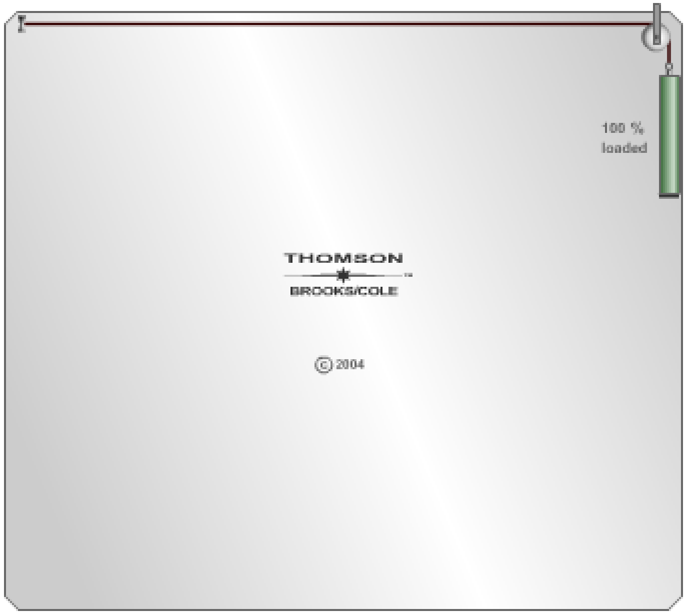


Induced emf in Rotating Loop



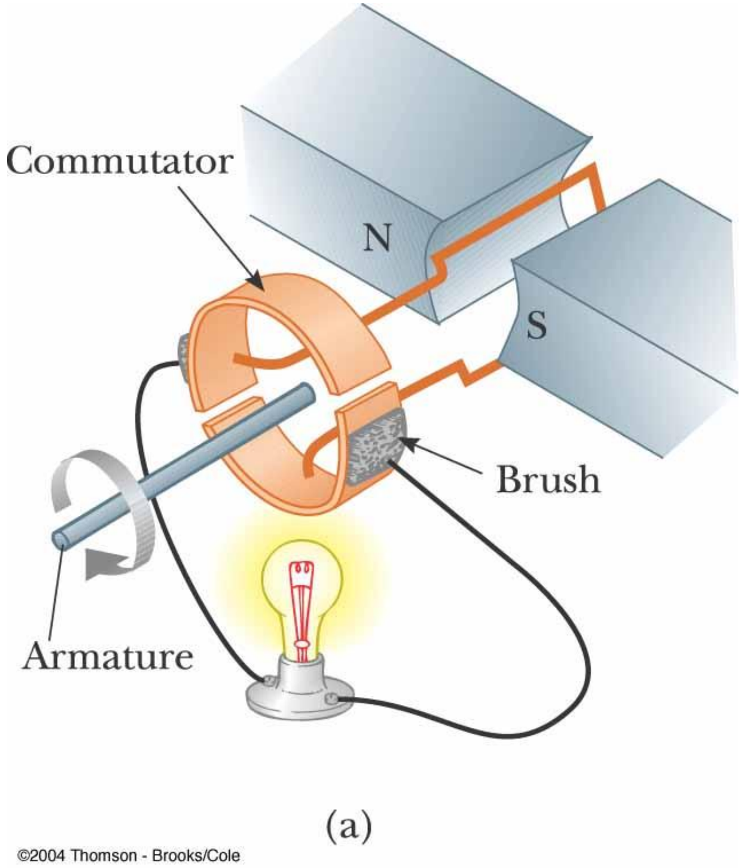
- ε_{max} occurs when ωt = 90º or 270º
  - Occurs when the magnetic field is in the plane of the coil and the rate of change of flux is a maximum
- ε = 0 when ωt = 0º or 180º
  - Occurs when **B** is perpendicular to the plane of the coil and the rate of change of flux is zero

AC motor animation



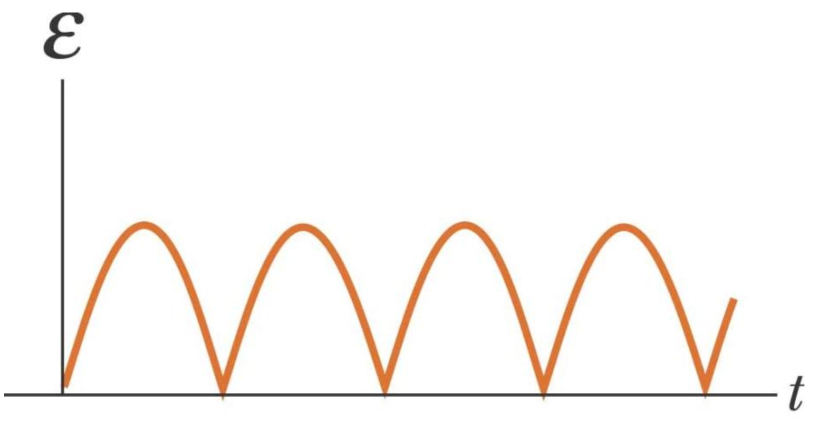
DC Generators

- The DC (direct current) generator has essentially the same components as the AC generator
- The main difference is that the contacts to the rotating loop are made using a split ring called a *commutator*



DC Generators, cont.

- In this configuration, the output voltage always has the same polarity
- It also pulsates with time
- To obtain a steady DC current, commercial generators use many coils and commutators distributed so the pulses are out of phase

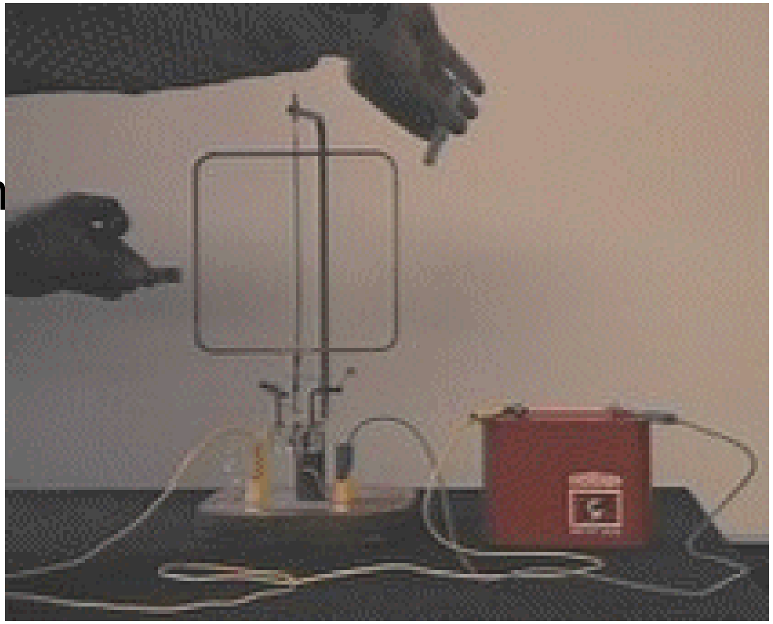


(b)

DC motor animation

Ec22: DC Motor

- An open working model of a DC motor with separate magnetic poles.
- On reversing these we change the direction of rotation.
- Force on vertical wire of height *a* given by $F=BaIN$
- Distance $b.\sin\theta$ from opposite wire
  - Force in opposite direction
- Torque = Force x Distance is thus: $BabIN \sin\theta = BAIN \sin\theta$
  - $A=ab$ is the area enclosed by the loop



Motors

- Motors are devices into which energy is transferred by electrical transmission while energy is transferred out by work
  - A motor is a generator operating in reverse
- A current is supplied to the coil by a battery and the torque acting on the current-carrying coil causes it to rotate
- As the coil rotates in a magnetic field, an emf is induced
  - This acts to reduce the current in the coil
  - This *back emf* increases in magnitude as the rotational speed of the coil increases

Quick Quiz 31.10

In an AC generator, a coil with *N* turns of wire spins in a magnetic field. Of the following choices, which will *not* cause an increase in the emf generated in the coil?

(a) replacing the coil wire with one of lower resistance

(b) spinning the coil faster

(c) increasing the magnetic field

(d) increasing the number of turns of wire on the coil

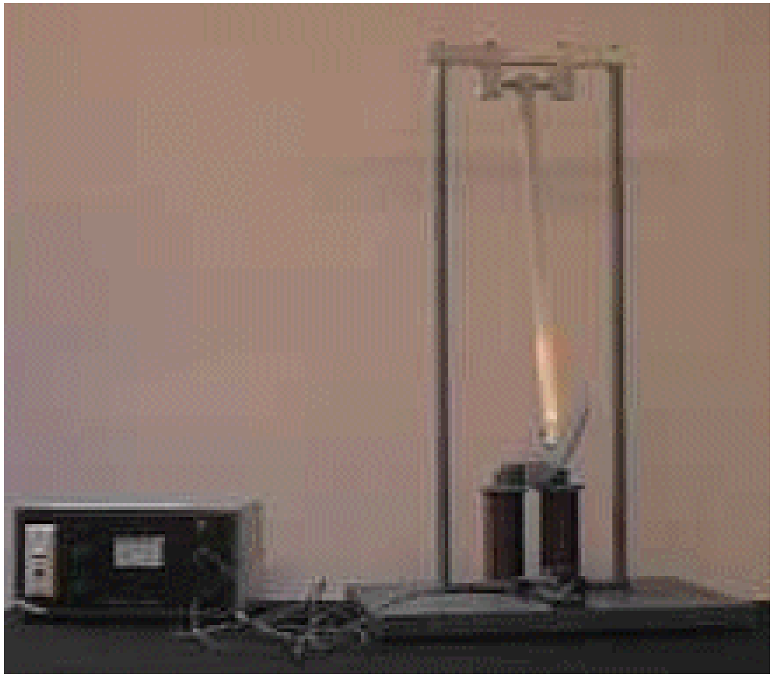
Quick Quiz 31.10

Answer: (a).

While reducing the resistance may increase the current that the generator provides to a load, it does not alter the emf. Since $\varepsilon = NAB\omega \sin \omega t$, the emf depends on ω, B and N, so all other choices increase the emf.

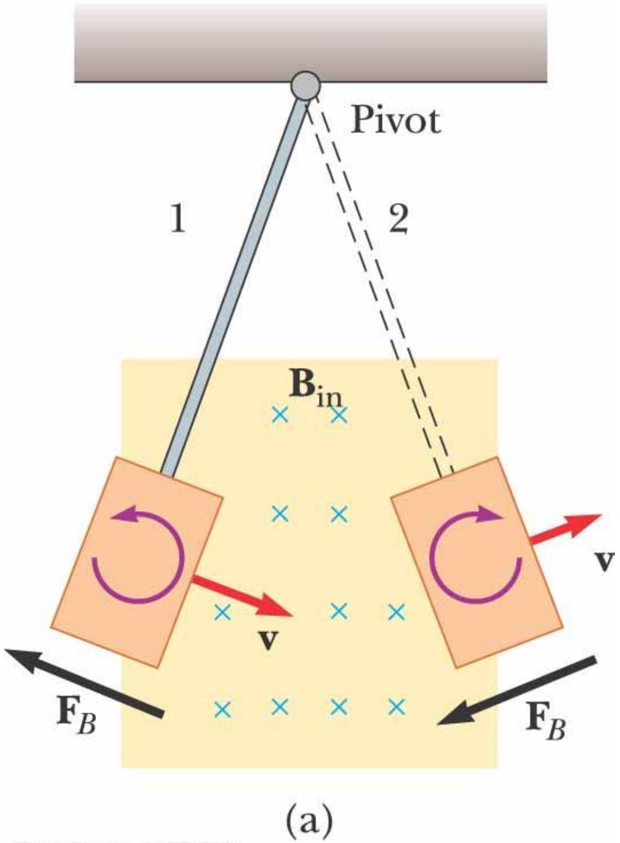
Ec16: Electromagnet Induction Eddy Current Pendulum

- When no magnetic field the pendulum swings freely.
- When electromagnet is turned on the pendulum is noticeably damped.
- If the disc of the pendulum is cooled to liquid nitrogen the pendulum is highly over-damped and will stop on the first swing!

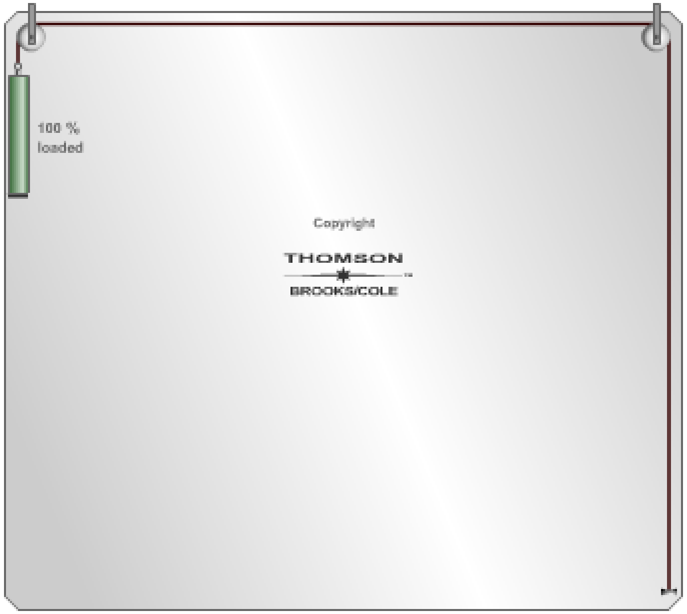


Eddy Currents

- Circulating currents called **eddy currents** are induced in bulk pieces of metal moving through a magnetic field
- From Lenz's law, their direction is to oppose the change that causes them.
  - The eddy currents are in opposite directions as the plate enters or leaves the field
- Eddy currents are often undesirable because they represent a transformation of mechanical energy into internal energy

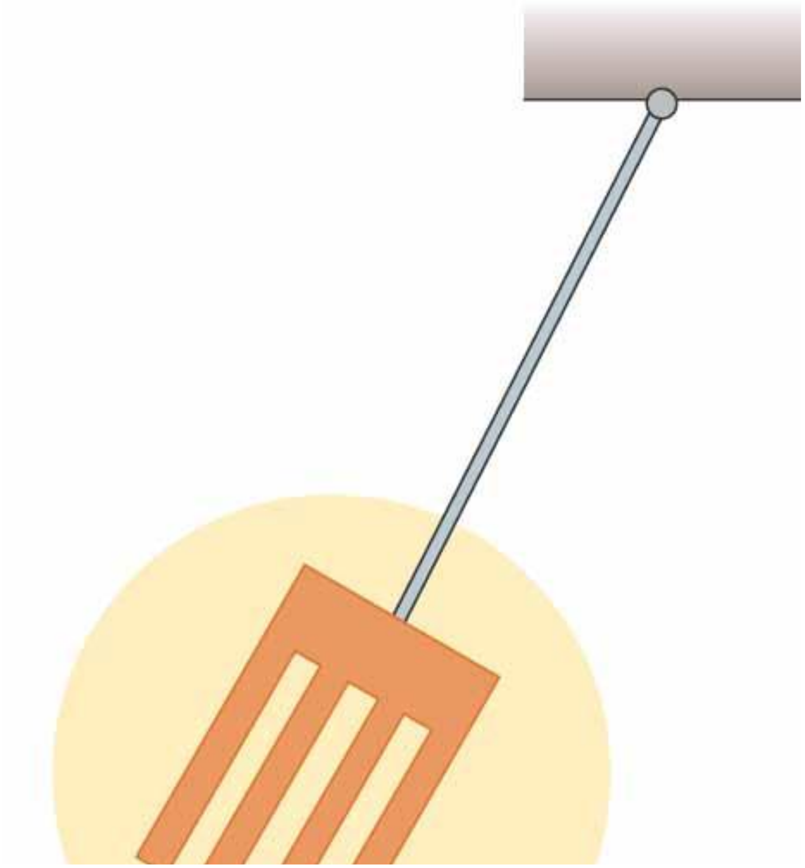


Eddy current animation



Mitigating Eddy Currents

- Cut slots into the plate, since these prevent the formation of large current loops.



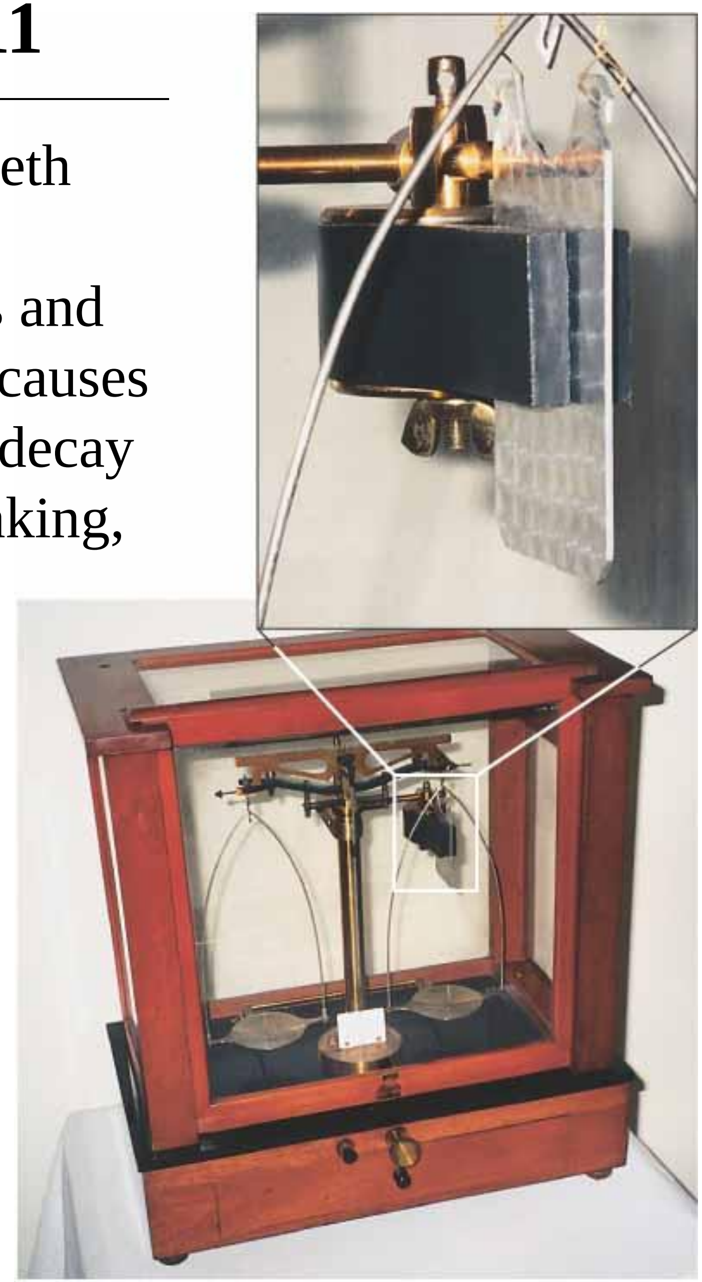
Quick Quiz 31.11

In equal-arm balances from the early twentieth century it is sometimes observed that an aluminum sheet hangs from one of the arms and passes between the poles of a magnet. This causes the oscillations of the equal-arm balance to decay rapidly. In the absence of such magnetic braking, the oscillation might continue for a very long time, so that the experimenter would have to wait to take a reading. The oscillations decay because

(a) the aluminum sheet is attracted to the magnet

(b) currents in the aluminum sheet set up a magnetic field that opposes the oscillations

(c) aluminum is paramagnetic



Quick Quiz 31.11

Answer: (b).

When the aluminum sheet moves between the poles of the magnet, eddy currents are established in the aluminum. According to Lenz’s law, these currents are in a direction so as to oppose the original change, which is the movement of the aluminum sheet in the magnetic field.

Maxwell's Equations *(not examinable!)*

$$\oint_s \mathbf{E}.d\mathbf{A} = \frac{q}{\varepsilon_0}$$ Gauss's law electric fields

$$\oint_s \mathbf{B}.d\mathbf{A} = 0$$ Gauss's law magnetism

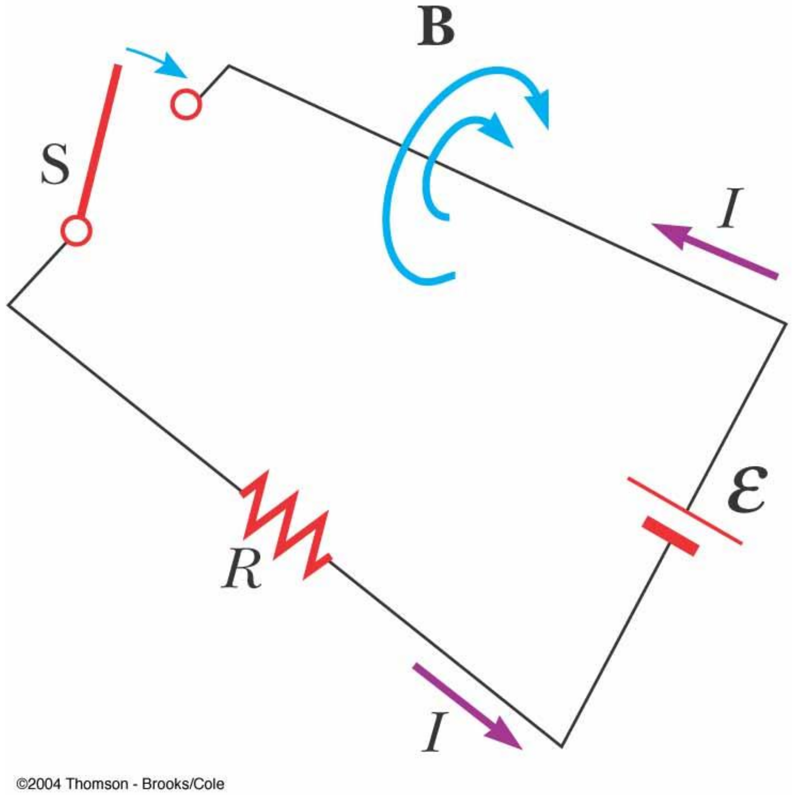
$$\oint \mathbf{E}.d\mathbf{s} = -\frac{d\Phi_B}{dt}$$ Faraday's law

$$\oint \mathbf{B}.d\mathbf{s} = \mu_0 I + \varepsilon_0\mu_0\frac{d\Phi_E}{dt}$$ Ampere - Maxwell

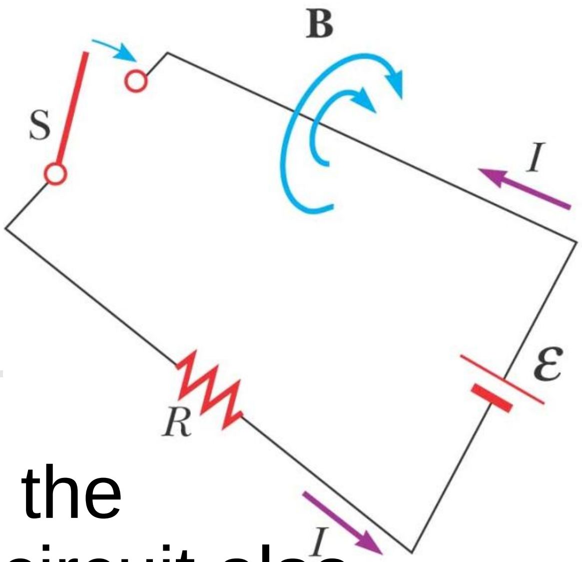
(displacement current)

Self-Inductance

- When the switch is closed, the current does not immediately reach its maximum value of ε/R due to **self-inductance**
- There is also a **self-induced emf,** ε_L
  - We use Faraday's law to describe the effect

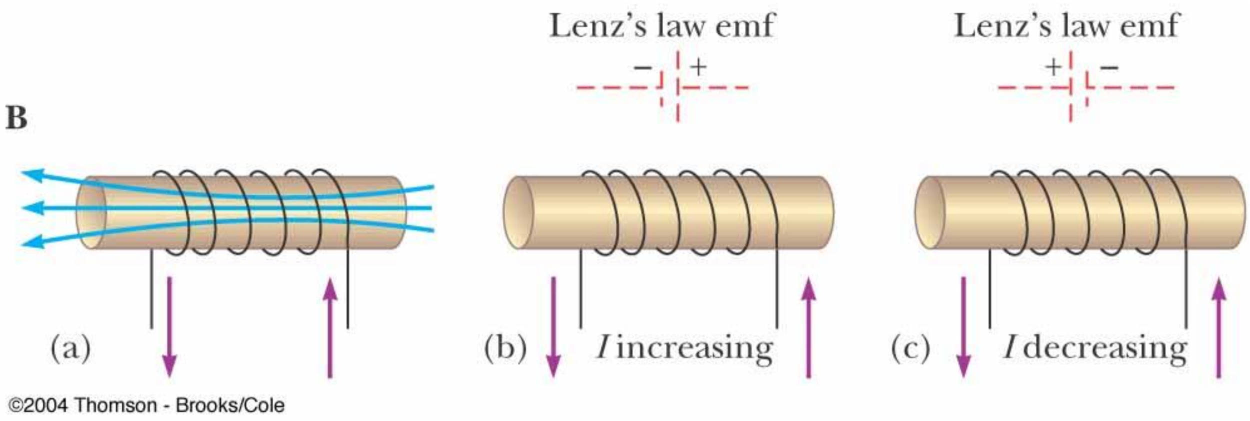


Self-Inductance, 2



- As the current increases with time, the magnetic flux induced through the circuit also increases with time
- This induces an emf in the circuit
- Direction of induced emf is to cause an induced current which establishes a magnetic field opposing the change in the field
  - i.e. opposite to direction of the emf of the battery
- The result is a gradual increase in the current to its final equilibrium value

Self-Inductance in a Coil



- (a) A current in the coil produces a magnetic field directed toward the left
- (b) If the current increases, the increasing flux creates an induced emf of the polarity shown
- (c) The polarity of the induced emf reverses if the current decreases

Equations for Self-Inductance

- The induced emf is proportional to the rate of change of the current

$$\varepsilon_L = -L\frac{dI}{dt}$$

- *L* is a constant of proportionality called the **inductance** of the coil. It depends on the geometry of the coil and its physical characteristics

Inductance of a Coil

- Closely spaced coil, *N* turns, current I:

$$\text{Since} \quad \varepsilon_{\mathrm{L}} = -N \frac{d\Phi_B}{dt} \text{ (Faraday)}$$

$$\text{We have} \quad L\frac{dI}{dt} = N\frac{d\Phi_B}{dt}$$

$$\therefore \; L\,dI = N\,d\Phi_B$$

$$\therefore \; L = \frac{N\Phi_B}{I}$$

- The Inductance is a measure of the opposition to a *change* in the current
- *c.f.* Resistance, which is a measure of the opposition to the current itself.

Inductance of a Solenoid

- Uniformly wound solenoid having *N* turns and length *ℓ*. Then we have:

$$B = \mu_0 nI = \mu_0 \frac{N}{l} I$$

$$\Phi_B = BA = \mu_0 \frac{NA}{l} I$$

$$\therefore L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{l}$$

- Note that inductance just depends on the *geometry*, just like capacitance!

Inductance Units

- The SI unit of inductance is the **henry** (H)

$$1\,\mathrm{H} = 1\,\frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}$$

- Named for Joseph Henry (pictured here)

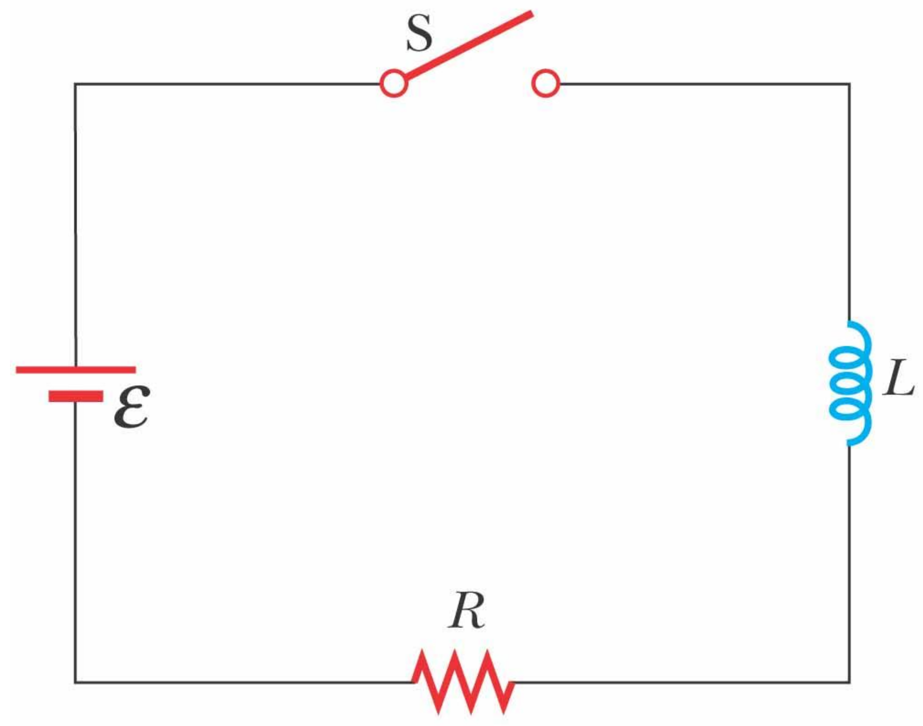
Quick Quiz 32.2a

The circuit in the figure below consists of a resistor, an inductor, and an ideal battery with no internal resistance. At the instant just after the switch is closed, across which circuit element is the voltage equal to the emf of the battery?

(a) the resistor

(b) the inductor

(c) both the inductor and resistor



Quick Quiz 32.2a

Answer: (b).

As the switch is closed, there is no current, so there is no voltage across the resistor.

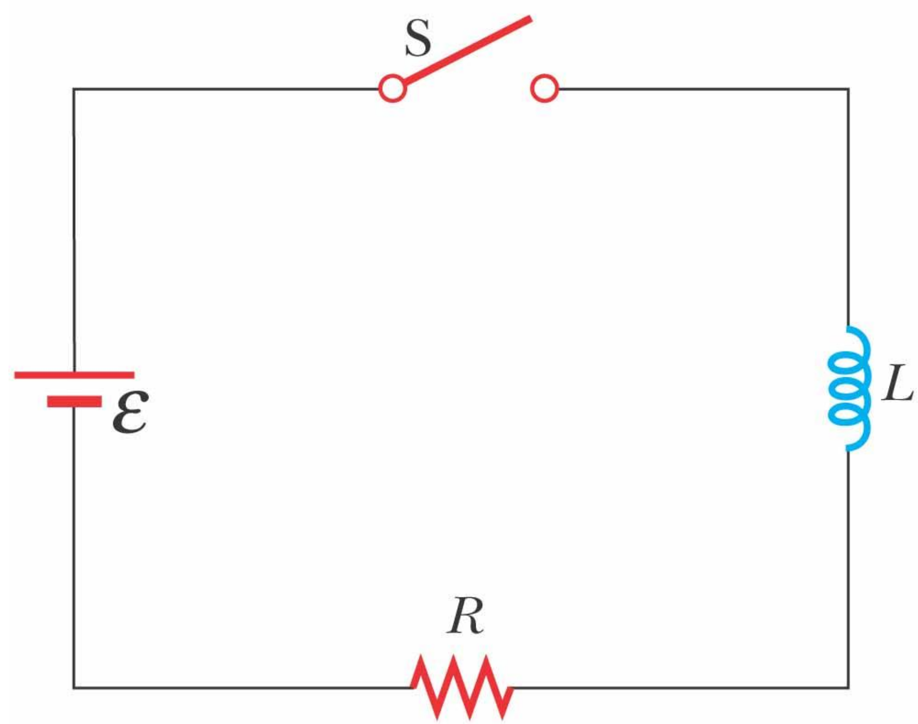
Quick Quiz 32.2b

After a very long time, across which circuit element is the voltage equal to the emf of the battery?

(a) the resistor

(b) the inductor

(c) both the inductor and resistor.



Quick Quiz 32.2b

Answer: (a). After a long time, the current has reached its final value, and the inductor has no further effect on the circuit.

Quick Quiz 32.1

A coil with zero resistance has its ends labeled *a* and *b*. The potential of the induced emf at *a* is higher than at *b*. Which of the following could be consistent with this situation?

(a) The current is constant and is directed from *a* to *b*

(b) The current is constant and is directed from *b* to *a*

(c) The current is increasing and is directed from *a* to *b*

(d) The current is decreasing and is directed from *a* to *b*

(e) The current is increasing and is directed from *b* to *a*

(f) The current is decreasing and is directed from *b* to *a*

Quick Quiz 32.1

Answer: (d), (e).

For the constant current in (a) and (b), there is no potential difference across the resistanceless inductor.

In (e), if the current increases, the emf induced in the inductor is in the opposite direction, and hence from *a* to *b*, making *a* higher in potential than *b*.

Similarly, in (d), the decreasing current induces an emf in the same direction as the current, from *a* to *b*, again making the potential higher at *a* than *b*.

Energy in a Magnetic Field

- In a circuit with an inductor, the battery must supply more energy than in a circuit without an inductor
- Part of the energy supplied by the battery appears as internal energy in the resistor
- The remaining energy is stored in the magnetic field of the inductor

Energy in a Magnetic Field

- Power supplied by battery =

  Dissipation across resistor, R +

  Rate of storing energy in inductor, L:

$$\varepsilon I = I^2 R + \varepsilon_L I$$

$$\text{But} \quad \varepsilon_L = L\frac{dI}{dt}. \text{ So } \quad \varepsilon I = I^2 R + L\frac{dI}{dt} I$$

  - εI is the rate at which energy is supplied by the battery
  - I^2R is the rate at which the energy is being delivered to the resistor
  - Therefore, $LI\, dI/dt$ must be the rate at which the energy is being stored in the magnetic field

Energy in a Magnetic Field

- Let *U* denote the energy stored in the inductor at any time
- The rate at which the energy is stored is

$$\frac{dU}{dt} = L\,I\,\frac{dI}{dt}$$

- To find the total energy, integrate:

$$U = L\int_0^I I\,dI = \tfrac{1}{2}LI^2$$

- c.f. Electric fields $U_E = 1/2\ Q^2/C$

Energy Density of a Magnetic Field in a Solenoid

- Given $U = 1/2\,LI^2$ with L $= \dfrac{\mu_0 N^2 A}{l}$, B $= \mu_0 nI$ and $n = \dfrac{N}{l}$

  Then $U = \dfrac{1}{2}\left(\mu_0 n^2 Al\right)\left(\dfrac{B}{\mu_0 n}\right)^2 = \dfrac{B^2}{2\mu_0}(Al)$

- Since $A\ell$ is the volume of the solenoid, the magnetic energy density, u_B, is

  $$u_B = \frac{U}{Al} = \frac{B^2}{2\mu_0}$$

- This applies to any region in which a magnetic field exists (not just the solenoid)
- c.f. $u_E = 1/2\,\varepsilon_0 E^2$ for electric fields

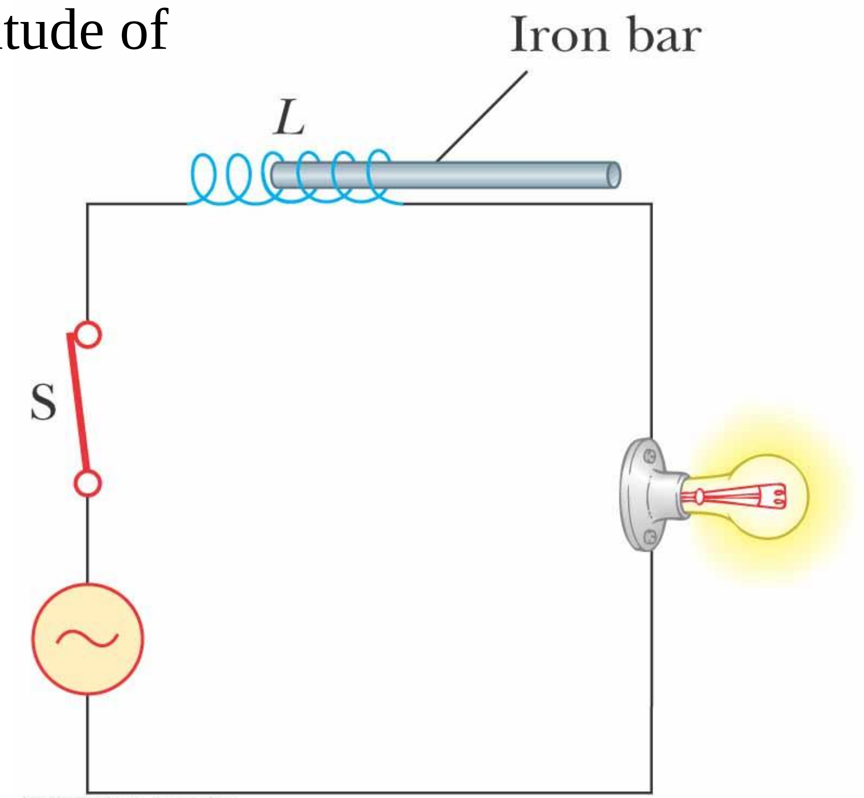
Quick Quiz 32.3

The circuit in the figure below includes a power source that provides a sinusoidal voltage. Thus, the magnetic field in the inductor is constantly changing. The inductor is a simple air-core solenoid. The switch in the circuit is closed and the lightbulb glows steadily. An iron rod is inserted into the interior of the solenoid, which increases the magnitude of the magnetic field in the solenoid. As this happens, the brightness of the lightbulb:

(a) increases

(b) decreases

(c) is unaffected



Quick Quiz 32.3

Answer: (b). When the iron rod is inserted into the solenoid, the inductance of the coil increases. As a result, more potential difference appears across the coil than before. Consequently, less potential difference appears across the bulb, so the bulb is dimmer.

Quick Quiz 32.5

You are performing an experiment that requires the highest possible energy density in the interior of a very long solenoid. Which of the following increases the energy density? (More than one choice may be correct.)

(a) increasing the number of turns per unit length on the solenoid

(b) increasing the cross-sectional area of the solenoid

(c) increasing only the length of the solenoid while keeping the number of turns per unit length fixed

(d) increasing the current in the solenoid

Quick Quiz 32.5

Answer: (a), (d).

Because the energy density depends on the magnitude of the magnetic field, to increase the energy density, we must increase the magnetic field. For a solenoid, $B = \mu_0 n I$, where n is the number of turns per unit length.

In (a), we increase n to increase the magnetic field.

In (b), the change in cross-sectional area has no effect on the magnetic field.

In (c), increasing the length but keeping n fixed has no effect on the magnetic field.

Increasing the current in (d) increases the magnetic field in the solenoid.

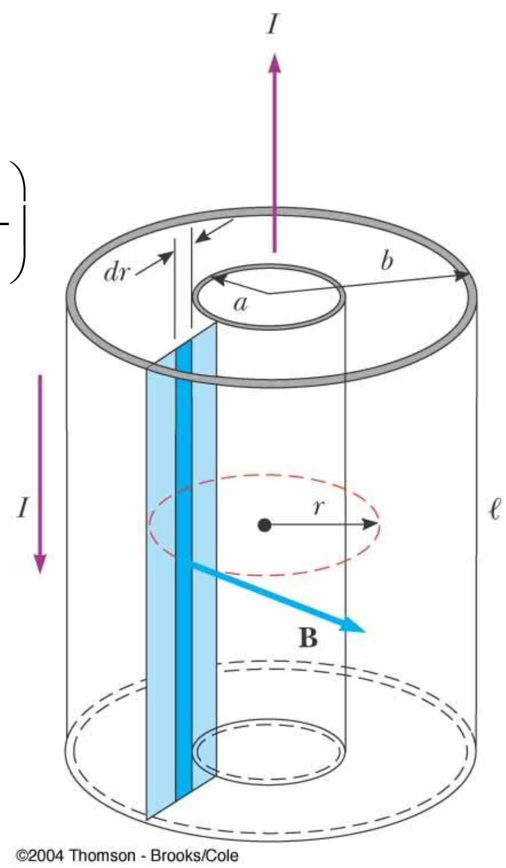
Example: The Coaxial Cable

From Ampere $B = \mu_0 I / 2\pi r$, then

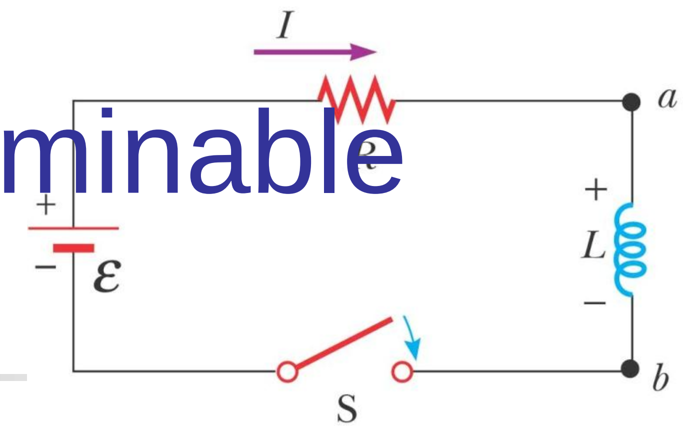
$$\Phi_B = \int \mathbf{B} . d\mathbf{A} = \int_a^b \frac{\mu_0 I}{2\pi r} l \, dr = \frac{\mu_0 Il}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 Il}{2\pi} \ln\left(\frac{b}{a}\right)$$

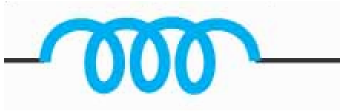
So, $L = \dfrac{\Phi_B}{I} = \dfrac{\mu_0 l}{2\pi} \ln\left(\dfrac{b}{a}\right).$

Thus energy $U = {}^1\!/_2 LI^2 = \dfrac{\mu_0 lI^2}{4\pi} \ln\left(\dfrac{b}{a}\right)$



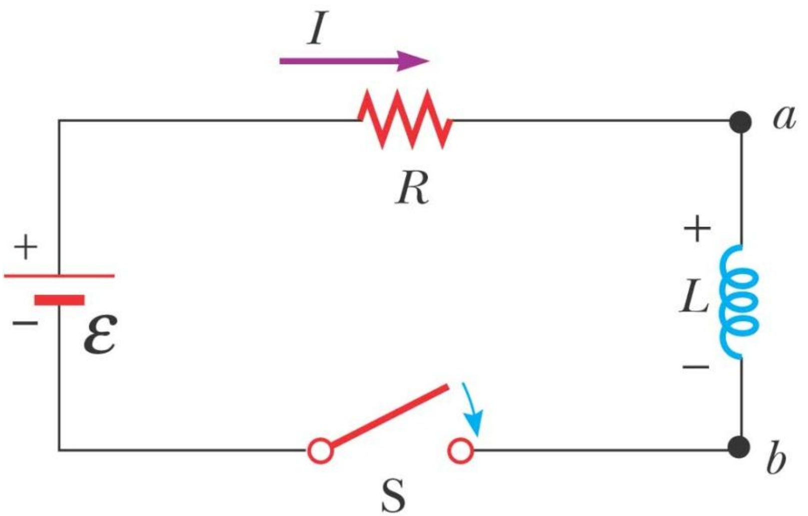
RL Circuit (not examinable from here on!)



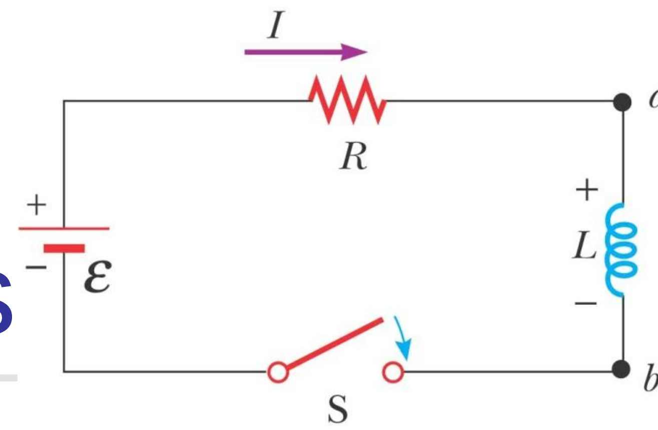
- A circuit element that has a large self-inductance is called an **inductor**
- The circuit symbol is 
- We assume the self-inductance of the rest of the circuit is negligible compared to the inductor
  - However, even without a coil, a circuit will have some self-inductance
- The inductance results in a back emf
- Therefore, the inductor in a circuit opposes changes in current in that circuit

RL Circuit, Analysis

- An *RL* circuit contains an inductor and a resistor
- When the switch is closed (at time $t = 0$), the current begins to increase
- At the same time, a back emf is induced in the inductor that opposes the original increasing current
  - Back emf given by $\varepsilon_L = -L\frac{dI}{dt}$



RL Circuit, Analysis



- Emf generated in circuit = Potential difference across resistor. i.e.

$$\varepsilon + \varepsilon_L = IR$$

$$\therefore\ \varepsilon - IR - L\frac{dI}{dt} = 0$$

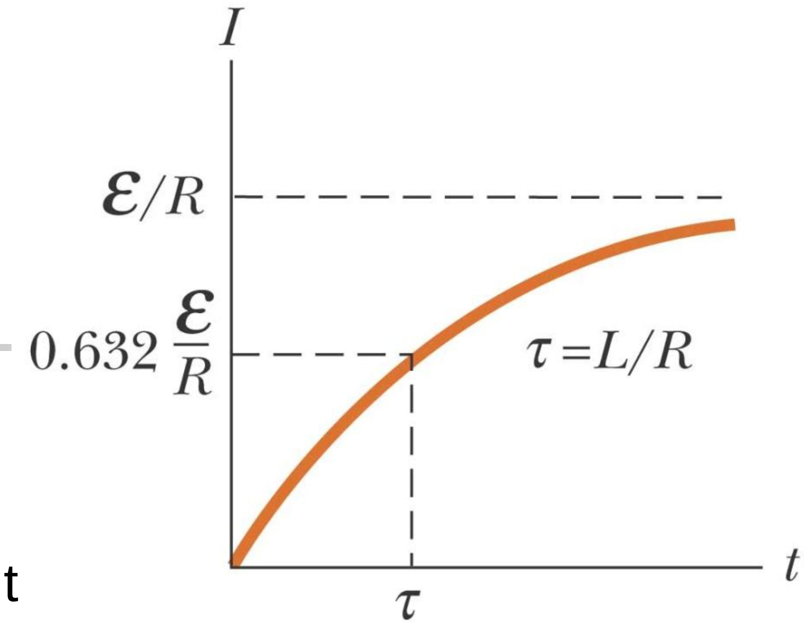
Let $x = \varepsilon/R - I$ so that $dx = -dI$

Then $x + \frac{L}{R}\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{x} = -\frac{R}{L}dt$

On integrating, with at $t = 0, I = 0, x = x_0$, so $x_0 = \varepsilon/R$

Then we get $\ln x/x_0 = -R/L\ t$ or $x = x_0 e^{-Rt/L}$

RL Circuit, Time Constant



- Thus, $I = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$
  - Time constant, $\tau = L / R$, for the circuit
  - τ is the time required for current to reach 63.2% of its max value
- The equilibrium value of the current is ε/R and is reached as t approaches infinity
- The current initially increases very rapidly
- Then gradually approaches the equilibrium value

RL Circuit, Rate of change of current

We have $$\frac{dI}{dt} = \frac{\varepsilon}{L} e^{-t/\tau}$$

- Rate of change of current is a maximum at $t = 0$
- Falls off exponentially as t approaches infinity

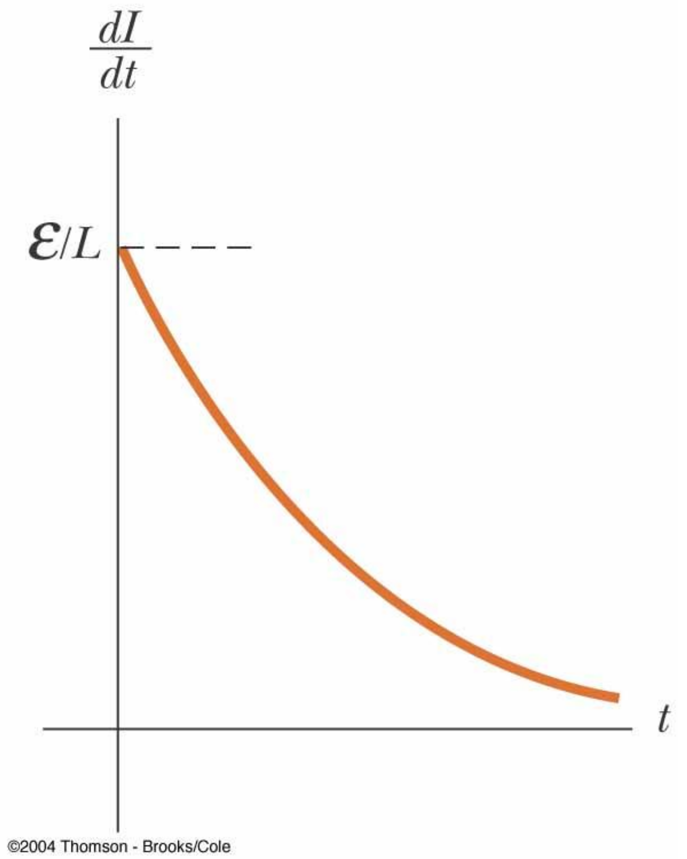


Table 32.1

Analogies Between Electrical and Mechanical Systems

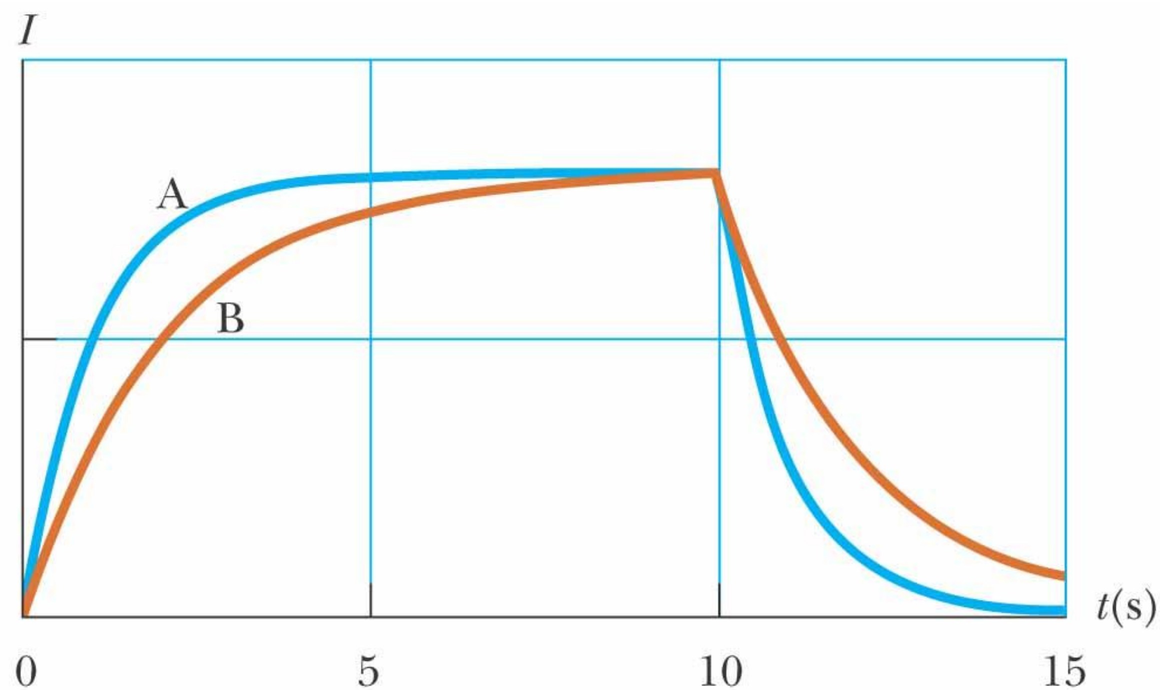
Electric Circuit		One-Dimensional Mechanical System
Charge	$Q \leftrightarrow x$	Position
Current	$I \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$I = \frac{dQ}{dt} \leftrightarrow v_x = \frac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \leftrightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_L = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving object
Energy in capacitor	$U_C = \frac{1}{2}\frac{Q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$	Damped object on a spring

End of Chapter

End of this part of the course!

Quick Quiz 32.4

Two circuits like the one shown in Figure 32.6 are identical except for the value of L. In circuit A the inductance of the inductor is L_A, and in circuit B it is L_B. Switch S is thrown to position a at $t = 0$. At $t = 10$ s, the switch is thrown to position b. The resulting time rates of change for the two currents are as graphed in the figure below. If we assume that the time constant of each circuit is much less than 10 s, which of the following is true?



(a) $L_A > L_B$

(b) $L_A < L_B$

(c) not enough information to tell

Quick Quiz 32.4

Answer: (b). Figure 32.10 shows that circuit B has the greater time constant because in this circuit it takes longer for the current to reach its maximum value and then longer for this current to decrease to zero after switch S_2 is closed. Equation 32.8 indicates that, for equal resistances R_A and R_B, the condition $\tau_B > \tau_A$ means that $L_A < L_B$.

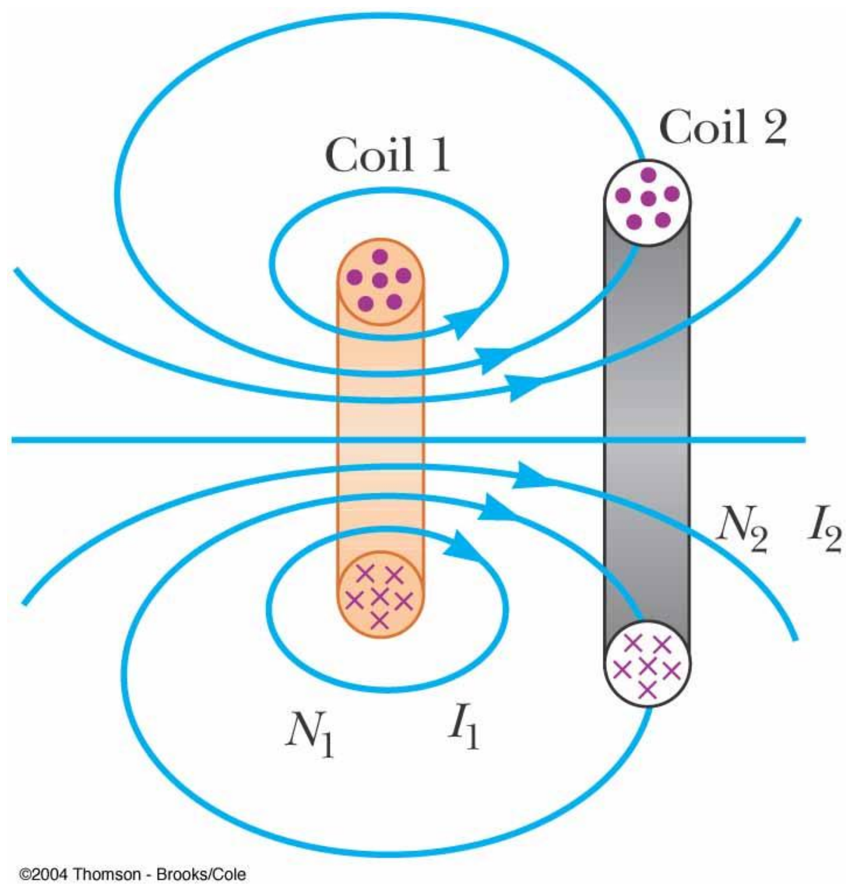
Quick Quiz 32.6

In the figure below, coil 1 is moved closer to coil 2, with the orientation of both coils remaining fixed. Because of this movement, the mutual induction of the two coils

(a) increases

(b) decreases

(c) is unaffected



Quick Quiz 32.6

Answer: (a). M_{12} increases because the magnetic flux through coil 2 increases.