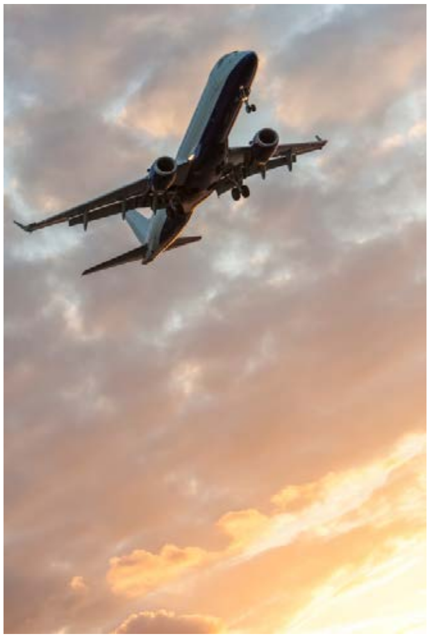
Autodesk® Helius PFA 2017

Fatigue Life Prediction in Composite Materials



Predicting fatigue life in composites is challenging because even simple load states lead to complex behavior in the individual fiber and matrix constituents. Furthermore, the effects of frequency and load history can only be dealt with adequately by using a physics-based approach. Autodesk Helius PFA addresses this challenges by combining multicontinuum technology with the kinetic theory of fracture to introduce an effective technology for progressive damage modeling of fatigue loading for both unidirectional and woven composites.

Contents

Table of Figures

Introduction

It is no news that the use of composite materials is on the rise. The extent of their use is increasing in many engineering applications from aerospace to infrastructure to alternative energy. Nearly 20 years ago, the Boeing 777 was introduced with composites comprising roughly 10% of the structural weight [1]. This included the cabin floor and much of the tail section. Recently, the state-of-the-art Boeing 787 Dreamliner aircraft was rolled out, which utilizes advanced composites in 50% of the aircraft. This includes 100% of the skin, entire sections of the fuselage, the tail, and the wing box and wing skins [2, 3]. General Motors recently indicated part of their strategy for future vehicle design, production, and globalization includes an expanded use of carbon fiber [4]. And, wind energy new capacity installation is expected to continue growth at an average rate of 13.8% per year over the next five years. The resultant need for new turbines is expected to increase the industry demand for composites at an annual growth rate of 18% to 1,243 million lbs by 2013 [5].

With the increasing use of composite materials comes an increasing need to understand their behavior and design life. Many of these applications include conditions that include repetitive loading cycles, thus demanding the ability to understand and evaluate fatigue in composite structures. Yet this understanding has remained a challenge for the design engineer.

The need for the ability to apply physics-based theories and principles to composite materials has been widely recognized. Efforts to develop this capability for fatigue analysis of large structures have been hindered by excess computational time and the inability to separate differences in physical behavior exhibited by each constituent of the composite.

In this paper, a new approach for fatigue life prediction in composites is introduced. The stresses and strains of the composite constituents are calculated using multicontinuum theory (MCT), which requires negligible additional computation time beyond that of standard finite element analysis of a homogenized material. The constituent stresses are used in combination with kinetic theory to predict fatigue life in large-scale composite structures under a variety of complex load states. This method dovetails readily with existing finite element analysis tools and requires minimal composite fatigue characterization. Thus, it provides the needed bridge between physics and large-scale structural analysis.

The Problem

Fiber reinforced polymer composites have become an attractive replacement for heavier metals due to their superior fatigue and corrosion properties. Although they may be less susceptible to fatigue failure than metals, fatigue can still occur. This is especially true when environmental factors such as temperature and humidity become significant.

In general, the lack of understanding about composite structural fatigue is a significant setback for many industries. Fatigue failures represent the greatest uncertainty with regard to the long-term service lifetime of the major structural components of a wind turbine, often leading the designer to either add excess weight to compensate or to inadequately size a blade resulting in fracture – both costly errors limiting the cost advantage of wind energy [6]. During the 2008 Society of Plastics Engineers' Automotive Composites Conference and Exhibition, the need for advanced predictive engineering techniques was discussed as a roadblock to a more widespread use of composites in automotive design [4]. A recent European study also concluded that for composites to take hold in automotive structural design, there will need to be continued development of failure theories, damage modeling, and fatigue life prediction [7].

For the design engineer or structural analyst, there are significant challenges to predicting fatigue behavior in composites:

- **Limited Fatigue Characterization Data**
  Coupon-level composite fatigue test data is an unreliable source for design because it doesn't capture the range of parameters that are often observed in real structures. Fatigue characterization is typically performed under simplified loading conditions and generally consists of uniaxial, constant amplitude tensile fatigue loading. However, many applications are subjected to mulitaxial, variable amplitude loads that involve different environments. In addition, fatigue failure depends on the specific layup and geometry of the structure – thus, characterization data is not likely to be applicable to structural design.

- **Physics-based Theories Cannot Be Easily Applied at Composite-Level**
  In a large structural analysis, typically only homogenized composite stresses are computed. The problem with utilizing these stresses to predict fatigue behavior is that they do not represent the stresses in the individual constituent materials. Fiber & matrix stresses drive composite fatigue behavior. As such, an accurate determination of composite fatigue requires access to the constituent properties. Historically, no commercial solution has existed which can reliably extract this information without exotic material data or substantial computational cost.

To meet the needs of composite structural evaluation, an effective solution to accurately model composite fatigue should:

- Account for material behavior at the atomic or defect level so that environmental effects (temperature, moisture, chemical exposure, etc.) can be incorporated
- Consider fatigue damage to a particular constituent material within the composite, not damage to the homogenized composite
- Require a minimal input data set for characterization of fatigue behavior
- Apply to multiple types of loading and load histories
- Apply to any composite laminate layup
- Be efficient enough to use for a routine structural analysis

Previous Options

Composite fatigue has been studied for decades and many of the proposed modeling approaches satisfy some of the above requirements. But currently a modeling approach does not exist that satisfies all of them. A range of approaches have considered several different types of fatigue loading: constant amplitude, block loading, and spectral loading. Yet no approach has been satisfactorily applied to the entire range of problems that must be addressed.

Early studies in predicting composite fatigue, such as those of Hashin and Rotem [8], relied on macroscopic composite strengths. This approach requires a minimal data set, but satisfies none of the other requirements. More sophisticated macroscopic approaches, such as calculating residual strength or residual stiffness, have also been used, but they require much more experimental data and still apply only to a particular laminate. Microstructural modeling of a composite can yield high-fidelity constituent stresses and strains but is too computationally intensive for large-scale structural analyses. Moreover, nearly all of these techniques require a large amount of experimental data to characterize the material.

Most predictive theories pertain to a specific load history at a specific temperature and are not easily generalized to capture multiaxial load states, variable amplitude or spectral loading, temperature changes, or environmental effects. In addition, any general solution must be able to be implemented into design codes, such as finite element analysis software, with computational efficiency.

This goal is unlikely to be achieved as long as composite fatigue predictions are based on purely empirical relationships. Physics-based concepts must be applied to the composite in order to use a minimal amount of characterization data to predict the fatigue life of a composite under a variety of loading and environmental conditions. But applying physics-based theories requires knowledge of constituent-level behavior. Because the majority of composite fatigue analyses use homogenized composite stresses/strains rather than constituent stresses/strains, direct application of relevant material physics is nearly impossible.

The Solution

The challenge in obtaining constituent material stresses in large-scale structural finite element analyses has been remedied by Autodesk® Helius PFA, which uses multicontinuum theory to efficiently extract average constituent stresses and strains from composite stress and strain. Using the Helius PFA's framework, we can now apply the appropriate physics - in this case polymer kinetics - to the constituent materials individually in order to predict composite fatigue response.

The fatigue solution implementation uses constituent stresses calculated using Helius PFA and then applies the well-established kinetic theory of fracture to produce a physics-based method for predicting fatigue life in composite materials. One crucial observation regarding the character of composite fatigue failure is that, with the exception of nearly perfect axial loading, fatigue failure is a matrix-dominated event [9, 10, 11]. This means that the physical behavior of polymers is the physics-based ingredient needed to predict fatigue. As such, the kinetic theory of fracture, a proven physics-based theory for polymer fatigue life, is applied to the matrix stress in the composite to predict the reduction in strength based on the number of cycles. In contrast, the fiber failure planes remain nearly unchanged as their strength is unaffected by cyclic loading. Figure 1 shows a representation of a static and fatigue failure envelope for a unidirectional graphite composite.

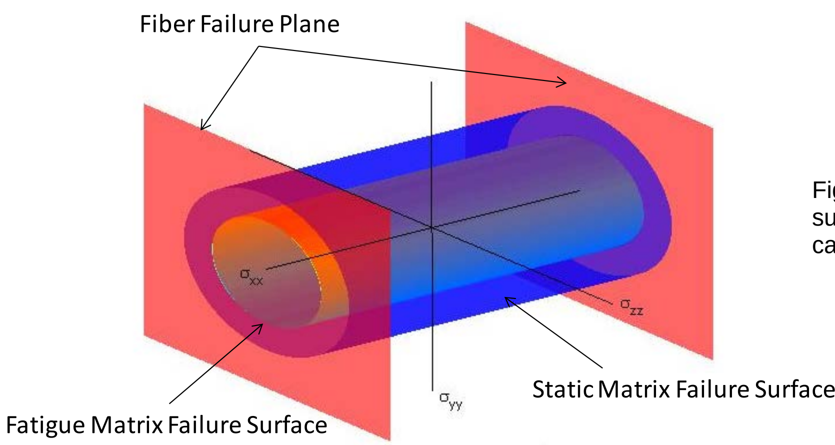


Figure 1. Typical three dimensional failure surfaces for static and fatigue failure in a carbon epoxy lamina

Theoretical Development: Fatigue Failure in Polymers

The relationship between polymer kinetics and mechanical behavior was developed more than five decades ago by Zhurkov [12, 13] and Coleman [14, 15] in parallel efforts. Zhurkov used experimental observations to show that the conceiving of strength in terms of molecular kinetics was well-founded. Most importantly, Zhurkov showed that the bond rupture rate determines the fracture strength of a polymer and the time to failure under a creep load, where the bond rupture rate K_b under a tensile load σ has the form

$$K_b = \nu_0 exp\left(-\frac{U-\gamma\sigma}{kT}\right). \tag{1}$$

In Equation (1), *U* is an activation energy that is closely related to bond energy, γ is an activation volume, and *k* is the Boltzmann constant. (Note: this is the Eyring equation, as discussed by Ward [17].) Approaching the problem from a statistical mechanics approach, Coleman developed a similar equation and noted that it could also be used to predict polymer fatigue life [15]. The value ν_0 is the oscillation frequency of the atom, which should be proportional to *kT/h*, where *h* is Planck's constant; at room temperature *kT/h* = $6.105\times10^{12}s^{-1}$. Zhurkov [12] reports a value of $10^{13}s^{-1}$ for this term, while Coleman [14] reports a value of $1.84\times10^{12}s^{-1}$. As a first-order approximation, we simply use *kT/h*, such that Equation (1) becomes

$$K_b = \frac{kT}{h} exp\left(-\frac{U_b-\gamma_b\sigma}{kT}\right). \tag{2}$$

Similarly, for bond healing we can write

$$K_h = \frac{kT}{h} exp\left(-\frac{U_h-\gamma_h\sigma}{kT}\right). \tag{3}$$

Hansen and Baker-Jarvis [16] combined these earlier works to develop a rate-dependent kinetic theory of fracture for polymers, which successfully predicted the strength of polymers subjected to a wide range of stress rates. In their formulation, they introduced a differential equation for the evolution of a damage variable *n* with time *t*, where the evolution of the damage variable is directly related to the bond rupture rate as:

$$\frac{dn}{dt} = (n_0 - n)^{\lambda}(K_b - K_h), \;\; n(0) = n_i \tag{4}$$

where n_0 is a parameter determined by enforcing the condition

$$\int_0^1 \frac{dn}{(n_0-n)^{\lambda}} = 1 \tag{5}$$

and n_i is the value of damage at the beginning of the fatigue loading. The distinction between this formulation and that of Hansen and Baker-Jarvis is that their formulation assumed *λ*=1. This value is the default value used in our current implementation[1]. For the case of *λ*=1,

$$n_0 = \frac{e}{e-1}.$$

Assuming a value of 1.0 for *λ*, and combining Equations (2), (3), and (4) gives the starting equation for determining the fatigue life of a polymer.

$$\frac{dn}{dt} = (n_0 - n)\frac{kT}{h}\left[exp\left(-\frac{U_b - \gamma_b \sigma_{eff}(t)}{kT}\right) - exp\left(-\frac{U_h}{kT}\right)\right],\ n(0) = n_i \qquad (6)$$

Note that the term containing the activation volume, γ, for healing in Equation (3) has been assumed zero. Healing generally only occurs at low stress values, and thus the contribution of the stress to healing at this level is negligible. Solving Equation (6) yields the evolution of the damage parameter with time, which can be written as[2]:

$$n(t) = n_0 - (n_0 - n_i)exp\left\{-\frac{kT}{h}\int_0^t \left[exp\left(-\frac{U_b - \gamma_b \sigma_{eff}(\tau)}{kT}\right) - exp\left(-\frac{U_h}{kT}\right)\right] d\tau\right\} \qquad (7)$$

The fatigue solution uses Equation (7) with calibrated values of U_b, γ_b, and U_h to determine the time at which the damage parameter becomes unity (*n*=1). Equation (7) must be numerically integrated over a load history to identify the number of cycles to ultimate failure.

Mean Stress Effects

Equation (7) can provide the fatigue life for a polymer under a defined load history. When applied to varying load amplitudes and frequencies, however, Equation (7) cannot capture the 'mean stress effect' appropriately. The mean stress effect is generally defined as the effect of varying frequency, load amplitude, or mean stress on the fatigue life of a material, as shown in Figure 2.

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[1] For a single linear elastic analysis, i.e. no progressive failure, the choice of *λ* is irrelevant. All solutions of *λ* will yield the same number of cycles to failure. However, this factor could be critical for block loading and progressive failure simulation.

[2] Note that we have assumed that the formulation in Equation (7) assumes that *U* and *T* are constant during the fatigue process.

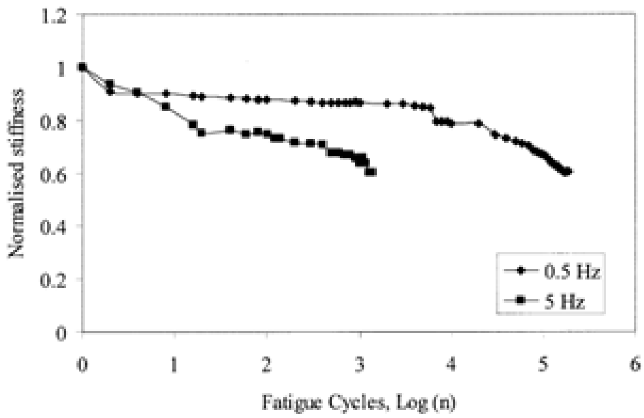

Figure 2. Stiffness reduction of woven fabric composites subjected to maximum fatigue stress of 80 MPa and a stress ratio R of 0.1 [21].

While there may be many fundamental physics which contribute to the mean stress effect in a polymer, one of the main contributors is through to be hysteresis heating, as shown in Figure 3.

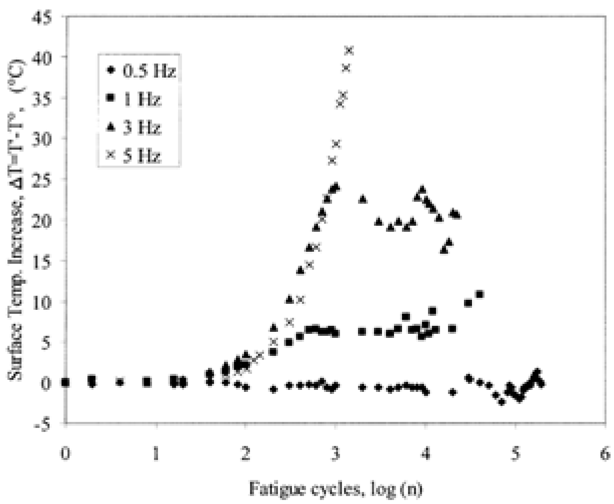

Figure 3. Hysteresis heating during fatigue tests in the bias direction. Woven fabric composite was subjected to maximum fatigue tress of 80 MPa and a stress ratio R of 0.1 [21].

We can then assume that the hysteresis heating contributes to a rise in temperature. This infers that the temperature in Equation (7) is a function of the frequency and stress amplitude, which would give

$$n(t) = n_0 - (n_0 - n_i)exp\left\{-\frac{kT^*}{h}\int_0^t \left[exp\left(-\frac{U_b - \gamma_b \sigma_{eff}(\tau)}{kT^*}\right) - exp\left(-\frac{U_h}{kT^*}\right)\right] d\tau\right\} \quad (8)$$

The new parameter, *T**, in Equation (8) is a modified temperature. We can write the change in temperature as an energy equation; the current form of which is

$$T_{i+1}^{*} = \left(T_i^{*} - T - \psi \frac{\Delta\sigma_{eff,i}{}^2}{\Delta t_i^2}\right) e^{-k\Delta t_i} + t + \psi \frac{\Delta\sigma_{eff,i}{}^2}{\Delta t_i^2 k} \qquad (8)$$

The above equation can be summed over the entire load history to identify the change in temperature as the number of cycles progresses. In the above, ψ is a constant of proportionality, $\Delta\sigma_{eff}$ is the magnitude of the effective stress change over a time period Δt, and *n* is the number of different stress ranges in a cycle.

Multicontinuum Theory for Unidirectional Composites

In order to apply kinetic theory to composite structures, the stress in the fiber and matrix constituents must be determined. Multicontinuum theory, as developed for two-constituent composite materials by Garnich and Hansen [19, 20,] and now commercially implemented in Helius PFA, provides an elegant and computationally efficient method to determine volume-averaged stresses of the matrix and fiber. In this report, we are interested in matrix-dominated fatigue failure, so the average matrix stress is the value of importance. The exact value of average stress in the matrix σ_m can be written as

$$\sigma_m = Q_m \sigma_c - \psi_m (\Delta T) \qquad (8)$$

where

$$Q_m = C_m \left\{ C_c \left(\phi_m I + \phi_f A \right) \right\}^{-1}$$
$$\psi_m = C_m \left\{ \phi_f \left[\left(C_c - C_f \right) \left(\phi_m I + \phi_f A \right) \right]^{-1} a + \eta_m - \left(\phi_m I + \phi_f A \right)^{-1} \eta_c \right\}$$
$$A = -\frac{\phi_m}{\phi_f} \left(C_c - C_f \right)^{-1} \left(C_c - C_m \right)$$
$$a = C_c \eta_c - \phi_f C_f \eta_f - \phi_m C_m \eta_m \qquad (9)$$

In Equations (8) and (9), σ_c is the six-component composite stress vector; ΔT is the temperature change from the stress-free state; C_i (*i* = *c*, *f*, *m*) are the reduced stiffness matrices for composite, fiber, and matrix, respectively; ϕ_f and ϕ_m are the fiber and matrix volume fractions, respectively; and η_i (*i* = *c*, *f*, *m*) are the thermal expansion coefficients of the composite, fiber, and matrix, written as six-component strain vectors where the shear components are zero.

Extracting Parameters for Use with Kinetic Theory

After determining the static *in situ* properties, the following material fatigue characterization process must be carried out for each identifiable fatigue failure mode within the composite lamina. The process is identical for each, so it is only described once. The material fatigue characterization process involves an initial step followed by an iterative step. The following processes define the initial step:

a) Read in S-N calibration curves and their corresponding parameters, *R*, *f*, and *T*, and material properties
b) Rotate the stress S into the principal coordinate system of the composite to get a composite stress tensor σ_c
c) Extract the matrix stress tensor σ_m from the composite stress tensor using MCT

d) Map the matrix stress tensor to an effective stress depending on the microstructure and whether the loading is on-axis or off-axis.

e) Use the value of R to calculate $\sigma_{eff,min} = R\sigma_{eff,max}$.

f) At the conclusion of this initial step, we have three vectors, $\sigma_{eff,max}, \sigma_{eff,min}$ and N_f, of length i, where i is the total number of data points in the S-N curve(s).

The iterative step of the material fatigue characterization involves adjusting U, γ, and Ψ to minimize the error between the log of the cycles to failure and the log of the predicted number of cycles to failure. Accomplishing this task requires an assumption about the load history. We assume that the load history has a saw-toothed shape, as shown by the red curve in Figure 4. As with the static *in situ* properties, the gradient method of steepest descent is used to optimize the fatigue parameters.

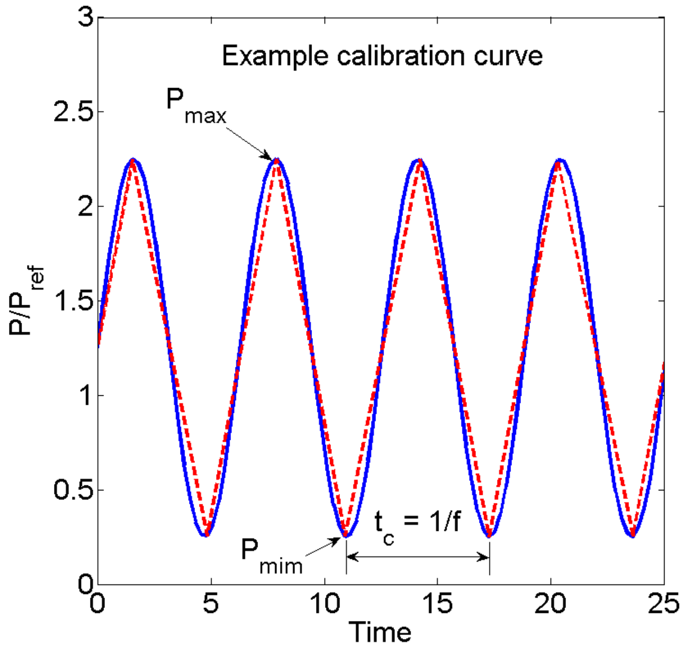

Figure 4. An example of simple cyclic loading and its saw-tooth approximation.

As a first-order approximation, we assume that the stiffness properties do not degrade with increasing n, as has been observed in some experimental work on composites [18]. Using these assumptions and solving Equation (7) gives the number of cycles to failure N_f.

Technology Demonstration

In the following subsection, we apply the fatigue model to off-axis fatigue of unidirectional T800H/2500EP carbon/epoxy composites. We compare our results with experimental data reported by Kawai *et al*. [9] at room temperature and show that our model can be used for prediction of fatigue failure.

Only two additional parameters beyond static failure data are needed: activation energy *U* and activation volume γ. Using off-axis room temperature fatigue data reported by Kawai *et al*. [9] for T800H/2500EP, we determined a best fit of *U* = 110.0 kJ/mol and γ = 1.312 kJ/MPa-mol. The results of these model predictions are shown in the graph below where the markers depict experimental data and the dashed lines indicate model predictions. Note that these parameters (*U* and γ) are determined from a typical S-N curve.

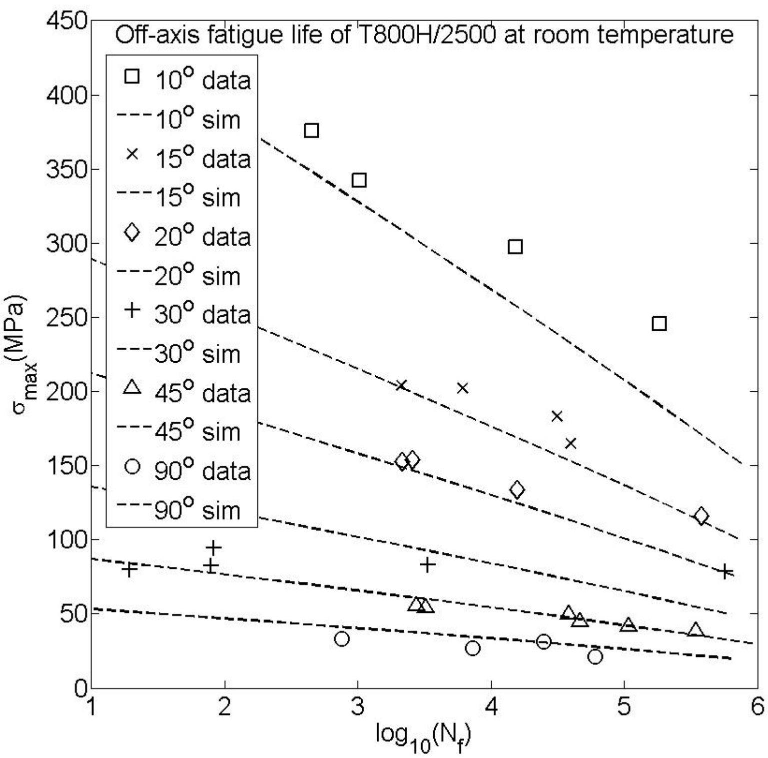

Figure 5. Off-axis composite tension-tension fatigue life predictions at room temperature (dashed lines) compared with experimental data (markers) from Kawai *et al*. [9].

Implementation

The use of multicontinuum theory to extract the matrix stresses allows the kinetic theory to be utilized in large-scale analyses. In a conventional finite element analysis, multicontinuum theory as used by Helius PFA, adds only about 3% to the computation time required for a large-scale structural analysis. Given the simple nature of the model proposed, we expect a similar computational burden implementing this fatigue life prediction model into existing finite element software. Together with Helius PFA, the fatigue feature permits rapid, robust fatigue life analysis of large-scale structures.

Summary

In the fatigue solution, kinetic theory is applied to *matrix* stresses and is used to predict *composite* fatigue life under *any* loading condition, including multiaxial load states and varying load histories. Results demonstrate that, using multicontinuum theory, kinetics can be applied to composite materials at the constituent level to predict fatigue life from fundamental physical properties of the polymer matrix. For further details regarding the fatigue feature of Helius PFA, refer to Section 11 of the Helius PFA Theory Manual.

References

1. *West, K.* "Boeing 777 Lights up with Plastics." *Seattle Post-Intelligencer*. 1 June 1993. 9 May 2009 < http://www.seattlepi.com/archives/1993/9306010037.asp>
2. *H. Baldwin. "787 Dreamliner™ Unmatched Economics, Performance and Passenger Appeal."* Aviation Week & Space Technology Market Supplement*. 14 March 2005.*
3. *J. Rakow and A. Pettinger. "Failure Analysis of Composite Structures in Aircraft Accidents."* Proceedings of the ISASI 2006 Annual Air Safety Seminar. *September 2006. p. 2.*
4. *M. LeGault. "2008 SPE Automotive Composites Conference and Exhibition." Composites Technology. 1 December 2008. 9 May 2009 < http://www.compositesworld.com/ articles/2008-spe-automotive-composites-conference-amp-exhibition.aspx>*
5. *"Future Material Needs for the Global Wind Energy Market, 2008-2013" 11 March 2009. 12 May 2009. <http://www.reuters.com/article/pressRelease/idUS121554+11-Mar-2009+BW20090311>*
6. *Commission on Engineering and Technical Systems. "Assessment of Research Needs for Wind Turbine Rotor Materials Technology." Washington, D.C.: National Academy Press, 1991.*
7. *J. Carruthers, E. Mangino and G. Pitarresi. "The Future Use of Structural Composite Materials in the Automotive Industry"* International Journal of Vehicle Design 2007 – Vol. 44, No. ¾ pp. 211-232
8. *Z. Hashin and A. Rotem, "A fatigue failure criterion for fiber reinforced materials," Journal of Composite Materials, vol. 7, 1973, p. 448.*
9. *M. Kawai, S. Yajima, A. Hachinohe, and Y. Takano, "Off-axis fatigue behavior of unidirectional carbon fiber-reinforced composites at room and high temperatures," Journal of Composite Materials, vol. 35, 2001, pp. 545-576.*
10. *J. Awerbuch and H. Hahn, "Off-axis fatigue of graphite/epoxy composite," Fatigue of Fibrous Composite Materials, San Francisco: ASTM, 1981, pp. 243-273*
11. *J. Petermann and A. Plumtree, "A unified fatigue failure criterion for unidirectional laminates," Composites: Part A, vol. 32, 2001, pp. 107-118*
12. *S. Zhurkov, "Kinetic Concept of the Strength of Solids," International Journal of Fracture, vol. 1, 1965, pp. 311-323*
13. *S. Zhurkov and V. Kuksenko, "The micromechanics of polymer fracture," International Journal of Fracture, vol. 11, 1975, pp. 629-639*
14. *B.D. Coleman, "Time Dependence of Mechanical Breakdown in Bundles of Fibers. I. Constant Total Load," Journal of applied physics, vol. 28, 1957, pp. 1058-1064*
15. B*. Coleman and D. Marquardt, "Time dependence of mechanical breakdown in bundles of fibers. II. The inifinite ideal bundle under linearly increasing loads," Journal of Applied Physics, vol. 28, 1957, pp. 1065-1067*
16. *A. Hansen and J. Baker-Jarvis, "A rate dependent kinetic theory of fracture for polymers," International Journal of Fracture, vol. 44, 1990, pp. 221-231*
17. *I. Ward, Mechanical Properties of Solid Polymers, New York: John Wiley & Sons, 1983.*
18. *H. El Kadi and F. Ellyin, "Effect of stress ratio on the fatigue of unidirectional glass fibre/epoxy composite laminae," Composites, vol. 25, 1994, pp. 917-924.*
19. *M.R. Garnich and A.C. Hansen, "A Multicontinuum Theory for Thermal-Elastic Finite Element Analysis of Composite Materials," Journal of Composite Materials, vol. 31, Jan. 1997, pp. 71-86.*
20. *M.R. Garnich and A.C. Hansen, "A Multicontinuum Approach to Structural Analysis of Linear Viscoelastic Composite Materials," J Appl Mech, vol. 64, Dec. 1997, pp. 795-803.*
21. K. Takemura and T. Fuji, "Fatigue Damage and Fracture of Carbon Fabric/Epoxy Composites under Tension-Tension Loading," *JSME International Journal,* pp. 472-480, 1994.