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FDEAHP Analysis to Reduce Defects in an Aluminium Extrusion Industry

Arun Mathews Varikatt, Dr. Haris Naduthodi

Abstract— Aluminium extrusions are popularly used in transportation, construction, energy, electrical, manufacturing and defence industries. Technical and economic viability of the process depends on reduction of defects that lead to product rejection or rework. Product rejection can be due to material defects, tooling defects and processing anomalies. Some of the defects found in aluminium extrusions are kink, blister, holes, die lines, length, bend, twist, speed crack, inclusion, damages, off shape, die mark, die scores, dents etc. This paper deals with the collection of defect data from an extrusion facility, analyzing the data so as to find the major defects and find the critical causes for each of these major defects.

The research began by collecting the rejection data. Statistical data from the defect records were utilized to find critical defects. The causes of these defects were analyzed and ranked by Fuzzy Data Envelopement Analytical Hierarchy Process (FDEAHP). Two causes each with the highest Final Weight (FW) of each problem were selected for improvement and appropriate suggestions were made.

Index Terms— AHP, DEA, Fuzzy logic, Hot aluminium extrusions, Product defects, FMEA, Analysis of defects.

1 Introduction

Hot aluminum extrusion is a process with high productivity. Rejected product is undesirable because it leads to material and manpower/processing loss, in addition to the extra cost required to make up for the rejected items. This rejection may be due to defective billets, faulty or unsuitable tooling and processing anomalies. Recovery is a very important factor in any production facility; it is the amount of output produced per unit input. The plant's planned recovery is set as 75% by the factory management and the plant is unable to reach its target recovery rate.

There are three main losses that take place during extrusion process.

- During extrusion process the billet is never completely extruded, the end part with impurities and oxides are sheared away called shearing off butt end.
- After extrusion when the extruded rods are cut into the length required by the customers, some parts such as the front and rear ends of the extrusion and the part near the billet to billet weld joint are removed known as cutting losses.
- Finally various defects occur during extrusion process which are rejected during inspection called loss due to extrusion defects.

The loss due to shearing of butt end, cutting and a certain amount of rejection losses are considered while setting the planned recovery for the plant. So it can be inferred that the plant is not able to attain its planned recovery rate due to the increase in number of defects formed during extrusion.

In this paper FDEAHP is used to find the major defects and the critical causes that cause them with an objective of reducing the rejection rates in the aluminium extrusion plant.

2 LITERATURE REVIEW

Product defects in an aluminium extrusion facility were previously analysed by S Z Qmar *et al*(2004). In this paper the rejection and acceptance percentages have been worked out relative to individual cost center production and in relation to total plant production. It also briefly explains certain causes for some extrusion defects. Arif *et* al(2002) categories the reason for extrusion defects into four broad categories i.e defective billets, faulty or unsuitable tooling ,defect arising during extrusion and post extrusion defects. Post extrusion defects are not taken into consideration in this paper.

The analytical hierarchy process (AHP) was first proposed by Saaty (1980). It is a widely used decision-making analysis tool to deal with complicated, unstructured decision problems, especially in situations where there are important qualitative aspects that must be considered, in conjunction with various measurable quantitative factors based on hierarchical structures and the judgment of decision maker(s). It has unique advantages when important elements of the decision are difficult to quantify or compare, or where communication among team members is impeded by their different specializations, terminologies or perspectives. It has successfully been applied to many decision situations in areas such as selection, evaluation, planning and development, decision making and forecasting.

- *Arun Mathews Varikatt is currently pursuing masters degree program in Manufacturing Systems Management in Production Engineering Department in Government Engineering College,Thrissur , India E-mail: arunmathewv@gmail.com*
- *Dr.Haris Naduthodi is Associate Professor in Production Engineering Department in Government Engineering College,Thrissur , India*

In general, the AHP concept for decision making requires four steps.

- In the first step, the hierarchy structure of decision must be constructed. The first layer of the hierarchy structure is the main objective of the problem. The second is decision criteria. Sometimes, when the problem is complex, criteria can be divided further into sub-criteria and sub-sub-criteria and so on. The last layer is the alternative, which must be chosen.
- In the second step, the decision-maker(s) must build the judgment matrix by having pair-wise comparison criteria and alternatives in each criterion, based on discrete scales 1-9 Each scale a_{ij} of scale of the judgment matrix are the three rules: $a_{ij} > 0$, $a_{ij} = 1/a_{ji}$, and $a_{ii} = 1$ for all i.
- In the third step, the local weights (LW) of each judgment matrix are calculated. Based on Saaty, the eigen-vector method (EVM) is used to yield priorities for criteria and for alternative criteria. There are also other methods for calculating weights, including the logarithmic least-square technique (LLST) and goal programming (GP).
- The last step is to synthesize the priorities of the alternative criteria into composite measures to arrive at a set of ratings for the alternatives or final weights (FW), based on the hierarchical arithmetic aggregation.

The concept of data envelopment analysis, which was first proposed by Charnes *et al.*(1978)for generating LW from the judgment matrices and aggregating them to be FW in AHP. The first DEA model is a CCR (Charnes, Cooper and Rhodes) model. It used for evaluate relative efficiencies of decision making units (DMUs) in a case of constant returns to scale (CRS) of efficiency production frontiers in input or output oriented models in a form of linear programming (LP), and is extended to other models. In DEA the DMU with the maximum efficiency is given a value 1 and all the other inefficient DMUs are ranked with respect to the most efficient DMU. One of the advantages of the DEA model is that it does not require either a priori weights or explicit specification of functional relations between the multiple outputs and inputs.

The data envelopment analytical hierarchy process (DEAHP) was first proposed by Ramanathan (2006). In DEAHP the concept of DEA is applied while finding the LWs. The LWs are got from the LP model with the most efficient DMU having a value of 1 as relative efficiency and the other DMUs having values between 0 and 1. Further the LWs are aggregated to get the FWs. DEAHP has a unique advantage over traditional AHP which is, the independence of irrelevant alternatives i.e. if an alternative is eliminated from consideration, then the new ordering for the remaining alternatives is equivalent to the original ordering for the same alternatives.

Since the judgment matrices of AHP are obtained using a suitable semantic scale, it is unrealistic to expect that the decision-maker(s) have either complete information or a full understanding of all aspects of the problem, which are represented as exact (or crisp, according to the fuzzy set terminology) numbers. So, the fuzzy set theory and possibility theory, which were proposed by Zadeh (1978), are used to confront the fuzzy uncertainty. References to possibility theory can be found in Dubois and Prade (1980) and Zimmermann (1996). It is called the fuzzy AHP (FAHP). Since the triangular fuzzy number has one discrete value at = 1 and linear spread, then it is easier to model. In this paper, the fuzzy scales of fuzzy DEAHP (FDEAHP) are assumed to be the triangular fuzzy number. Let minimum scale 1 be the crisp value, fuzzy judgments scale (2-8) be symmetry triangular fuzzy numbers with the lower and upper spreads = 1, thus scale 2-8 can be respectively rewritten in terms of α-level set as follow; $2 = [\alpha + 1, 3 - \alpha]$, $3 = [\alpha + 2, 4 - \alpha]$, …, $8 = [\alpha + 7, 9 - \alpha]$, and maximum scale 9 be a triangular fuzzy number with the lower spreads = 1, thus scale 9 can be rewritten in terms of α-level set as $9 = [\alpha + 8, 9]$. Since 2-9 are positive fuzzy numbers, therefore 1/2-1/9 can be calculated by extended division operator of fuzzy arithmetic proposed by Zimmerman (1998) e.q.

$$1(/)\tilde{\Lambda} = [\min\{1/(\tilde{\Lambda})^{L}_{\alpha},\ 1/(\tilde{\Lambda})^{U}_{\alpha}\},\ \max\{1/(\tilde{\Lambda})^{L}_{\alpha},\ 1/(\tilde{\Lambda})^{U}_{\alpha}\}] = [1/(\tilde{\Lambda})^{L}_{\alpha}, 1/(\tilde{\Lambda})^{U}_{\alpha}] \qquad (1)$$

where $\tilde{\Lambda}$ is a positive fuzzy number. Therefore, $1/2 = [1/(3 - \alpha), 1/(\alpha + 1)]$, …, $1/8 = [1/(9 - \alpha), 1/(\alpha+7)]$, and $1/9 = [1/9, 1/(\alpha + 8)]$.

Focusing on FDEAHP, let A be a fuzzy judgment matrix of size n x n (compare n elements) and triangular fuzzy number a_{ij} be entities of A. Thus, there are 1 dummy input, n outputs and n DMUs of the FDEAHP model. The FDEAHP model in a case of input oriented and constant return to scales (CRS) or FDEAHP-CCR-I and its dual problem or FDEAHP-DCCR-I is the following linear programming (LP) problem.

$$\text{(FDEAHP-CCR-I)}\ \text{Max}\ \theta = \textstyle\sum_{r=1}^{n} u_i\, \tilde{a}_{io} \qquad (2)$$
$$\text{Subject to } v = 1 \qquad (3)$$
$$\textstyle\sum_{r=1}^{n} u_i - v < 0 \text{ for } j=1,\ldots, n \qquad (4)$$
$$u_i, v > 0 \qquad (5)$$
$$\text{(FDEAHP-DCCR-I) Min } \theta \qquad (6)$$
$$\text{Subject to } \theta - \textstyle\sum_{j=1}^{n} \lambda_j > 0 \qquad (7)$$
$$\tilde{a}_{i0} - \textstyle\sum_{j=1}^{n} \lambda_j\, \tilde{a}_{ij} < 0;\ i= 1, \ldots, n \qquad (8)$$
$$\theta \text{ Unrestricted}, \lambda_j > 0 \qquad (9)$$

where u_i for i = 1, …,n and v are decision variables of the primal problem, θ and λ_j for j = 1, …, n are dual variables. Since the traditional DEA and DEAHP are formulated in the form of LP, then it basically requires exact crisp inputs and outputs of all DMUs. The concept of possibility theory and lemma is used to transform the FDEAHP to be the equivalent crisp DEAHP (E-CDEAHP). The derivation of E-CDEAHP model is shown in S Ketsarapong and V Punyangarm (2010).The E-CDEAHP-CCR-I and E-CDEAHP-DCCR-I will be transformed to be LP, as follows:

(E-CDEAHP-CCR-I) Max $\theta = \Psi$ (10)
Subject to $\sum_{r=1}^{n} u_i (\tilde{a}_{io})_\alpha^U \geq \Psi$ (11)
$\sum_{i=1}^{n} u_i (\tilde{a}_{io})_\alpha^L \leq 1$ for j= 1, ……, n (12)
$u_i > 0$ (13)

(E-CDEAHP-DCCR-I) Min θ (14)
Subject to $\theta - \sum_{j=1}^{n} \lambda_j > 0$ (15)
$(\tilde{a}_{io})_\alpha^L - \sum_{j=1}^{n} \lambda_j \left(\tilde{a}_{ij}\right)_\alpha^U \leq 0$ for i= 1, ……, n (16)
θ Unrestricted, $\lambda_j > 0$ (17)

Where $(\,.\,)_\alpha^L$ and $(\,.\,)_\alpha^U$ are the lower and upper bounds of the α-level set of comparison entities. The fuzzy relative efficiencies (θ^*) from the E-CDEAHP CCR- I or E-CDEAHP-DCCR-I must be converted to be the fuzzy LWs, and will be aggregated to be the FW by the concept of the fuzzy hierarchical arithmetic aggregation, based on the extension principle.

3 METHODOLOGY

3.1 Using Statistical Data to Choose the Critical Defects

Statistical methods are used to find the critical defects formed during extrusion. The inspection data recorded by the company was used for this purpose. The defects where found to be Kink, Blister, Bend, Twist and Speed Crack. Kink defect is eliminated from further analysis as this defect usually occurs at the beginning of extrusion of a new section and is caused due to process limitations.

3.2 Finding the Causes of the Problems by using Cause and Effect Diagrams

From literature review, it was discovered that each defect has five controllable root causes. The various reasons for the occurrence of the four problems are found and shown in Table 3.1, Table 3.2, Table 3.3 and Table 3.4 respectively

BLISTER

Symbol	Description
A_1	Improper shearing of butt end
A_2	Due to use of excess grafting lubricant
A_3	Due to pores and cracks on billets
A_4	Due to keeping the butt end too low
A_5	Oxides and impurities on container wall

Table 3.1 : The causes of blister

BEND

Symbol	Description
B_1	Improper flow of metal through die
B_2	Improper alignment of die, stem and container
B_3	Stretching of hot sections
B_4	Differential cooling
B_5	Improper handling of sections by employees

Table 3.2 : The causes of bend

TWIST

Symbol	Description
C_1	Improper alignment of die, stem and container
C_2	Improper handling of sections by employees
C_3	Improper stretching
C_4	Improper quenching
C_5	Improper flow of metal through die

Table 3.3 : The causes of twist

SPEED CRACK

Symbol	Description
D_1	High billet temperature
D_2	High container temperature
D_3	High speed of extrusion
D_4	Improper die, stem and container alignment
D_5	Due defect in dies

Table 3.4 : The causes of speed crack

3.3 Ranking The Root Causes Of The Problems

In this paper, the controllable causes of each problem were ranked, based on the FMEA framework. An AHP structure is formed with criteria as severity, occurrence and detection as in an FMEA framework i.e. Severity (S), Occurrence (O), and Detection (D). The alternatives are the various causes for these defects. . The AHP judgement matrices are then formed by pair-wise comparison of the criteria and alternatives. Pair-wise comparison of the elements of the criteria in the AHP structure is shown in matrix (1) and matrices (2) to (4) are the comparison between alternatives under each criteria for blister defect. Matrices (5) to (13) are the comparison between alternatives under each criteria for the other three defects.

$$\tilde{\Theta}_{\text{Criteria/FMEA}} = \begin{bmatrix} 1 & \tilde{3} & \tilde{5} \\ \frac{1}{\tilde{3}} & 1 & \tilde{3} \\ \frac{1}{\tilde{5}} & \frac{1}{\tilde{3}} & 1 \end{bmatrix} \tag{1}$$

$$\tilde{\Theta}_{1/S} = \begin{bmatrix} 1 & \frac{1}{\tilde{3}} & \tilde{6} & \tilde{4} & \tilde{9} \\ \tilde{3} & 1 & \tilde{7} & \tilde{5} & \tilde{9} \\ \frac{1}{\tilde{6}} & \frac{1}{\tilde{7}} & 1 & \frac{1}{\tilde{3}} & \tilde{5} \\ \frac{1}{\tilde{4}} & \frac{1}{\tilde{5}} & \tilde{3} & 1 & \tilde{2} \\ \frac{1}{\tilde{9}} & \frac{1}{\tilde{9}} & \tilde{2} & \frac{1}{\tilde{3}} & 1 \end{bmatrix} \tag{2}$$

$$\tilde{\Theta}_{1/O} = \begin{bmatrix} 1 & \frac{1}{\tilde{5}} & \tilde{4} & \tilde{3} & \tilde{6} \\ \tilde{5} & 1 & \tilde{6} & \tilde{3} & \tilde{9} \\ \frac{1}{\tilde{4}} & \frac{1}{\tilde{6}} & 1 & \frac{1}{\tilde{4}} & \tilde{3} \\ \frac{1}{\tilde{3}} & \frac{1}{\tilde{3}} & \tilde{4} & 1 & \tilde{5} \\ \frac{1}{\tilde{6}} & \frac{1}{\tilde{9}} & \frac{1}{\tilde{3}} & \frac{1}{\tilde{5}} & 1 \end{bmatrix} \tag{3}$$

$$\tilde{\Theta}_{1/D} = \begin{bmatrix} 1 & \tilde{3} & \frac{1}{\tilde{5}} & 1 & \frac{1}{\tilde{5}} \\ \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{5}} & 1 & \frac{1}{\tilde{4}} \\ \tilde{5} & \tilde{5} & 1 & \tilde{5} & \tilde{3} \\ 1 & 1 & \frac{1}{\tilde{5}} & 1 & \frac{1}{\tilde{5}} \\ \tilde{5} & \tilde{4} & \frac{1}{\tilde{3}} & \tilde{5} & 1 \end{bmatrix} \tag{4}$$

$$\tilde{\Theta}_{2/S} = \begin{bmatrix} 1 & \frac{1}{\tilde{6}} & \frac{1}{\tilde{5}} & \tilde{3} & \frac{1}{\tilde{3}} \\ \tilde{6} & 1 & \tilde{3} & \tilde{6} & \tilde{4} \\ \tilde{5} & \frac{1}{\tilde{3}} & 1 & \tilde{5} & 1 \\ \frac{1}{\tilde{3}} & \frac{1}{\tilde{6}} & \frac{1}{\tilde{5}} & 1 & \frac{1}{\tilde{4}} \\ \tilde{3} & \frac{1}{\tilde{4}} & 1 & \tilde{4} & 1 \end{bmatrix} \tag{5}$$

$$\tilde{\Theta}_{2/O} = \begin{bmatrix} 1 & \frac{1}{\tilde{7}} & \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{4}} \\ \tilde{7} & 1 & 1 & \tilde{5} & \tilde{4} \\ \tilde{3} & 1 & 1 & \tilde{4} & \tilde{3} \\ 1 & \frac{1}{\tilde{5}} & \frac{1}{\tilde{4}} & 1 & 1 \\ \tilde{4} & \frac{1}{\tilde{4}} & \frac{1}{\tilde{3}} & 1 & 1 \end{bmatrix} \tag{6}$$

$$\tilde{\Theta}_{2/D} = \begin{bmatrix} 1 & \frac{1}{\tilde{4}} & \frac{1}{\tilde{4}} & \frac{1}{\tilde{7}} & \frac{1}{\tilde{4}} \\ \tilde{4} & 1 & \tilde{1} & \frac{1}{\tilde{4}} & 1 \\ \tilde{4} & 1 & 1 & \frac{1}{\tilde{3}} & 1 \\ \tilde{7} & \tilde{4} & \tilde{3} & 1 & \tilde{5} \\ \tilde{4} & 1 & 1 & \frac{1}{\tilde{5}} & 1 \end{bmatrix} \tag{7}$$

$$\tilde{\Theta}_{3/S} = \begin{bmatrix} 1 & \frac{1}{\tilde{5}} & \frac{1}{\tilde{4}} & \frac{1}{\tilde{8}} & \frac{1}{\tilde{6}} \\ \tilde{5} & 1 & \frac{1}{\tilde{2}} & \frac{1}{\tilde{5}} & \frac{1}{\tilde{3}} \\ \tilde{4} & \tilde{2} & 1 & \frac{1}{\tilde{4}} & \tilde{2} \\ \tilde{8} & \tilde{5} & \tilde{4} & 1 & \tilde{6} \\ \tilde{6} & \tilde{3} & 1 & \frac{1}{\tilde{6}} & 1 \end{bmatrix} \tag{8}$$

$$\tilde{\Theta}_{3/O} = \begin{bmatrix} 1 & \frac{1}{\tilde{7}} & \frac{1}{\tilde{4}} & \frac{1}{\tilde{5}} & \frac{1}{\tilde{4}} \\ \tilde{7} & 1 & \tilde{3} & \tilde{2} & \tilde{3} \\ \tilde{4} & \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{2}} & 1 \\ \tilde{5} & \frac{1}{\tilde{2}} & \tilde{2} & 1 & \tilde{2} \\ \tilde{4} & \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{2}} & 1 \end{bmatrix} \tag{9}$$

$$\tilde{\Theta}_{3/D} = \begin{bmatrix} 1 & \frac{1}{\tilde{7}} & \frac{1}{\tilde{3}} & \tilde{3} & \frac{1}{\tilde{4}} \\ \tilde{4} & 1 & \tilde{2} & \tilde{5} & 1 \\ \tilde{3} & \frac{1}{\tilde{2}} & 1 & \tilde{3} & \tilde{2} \\ \frac{1}{\tilde{3}} & \frac{1}{\tilde{5}} & \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{5}} \\ \tilde{4} & 1 & \frac{1}{\tilde{2}} & \tilde{5} & 1 \end{bmatrix} \tag{10}$$

$$\tilde{\Theta}_{4/S} = \begin{bmatrix} 1 & \frac{1}{\tilde{3}} & \frac{1}{\tilde{6}} & \tilde{4} & \tilde{4} \\ \tilde{3} & 1 & \frac{1}{\tilde{3}} & \tilde{5} & \tilde{3} \\ \tilde{6} & \tilde{3} & 1 & \tilde{9} & \tilde{9} \\ \frac{1}{\tilde{4}} & \frac{1}{\tilde{5}} & \frac{1}{\tilde{9}} & 1 & 1 \\ \frac{1}{\tilde{4}} & \frac{1}{\tilde{3}} & \frac{1}{\tilde{9}} & 1 & 1 \end{bmatrix} \tag{11}$$

$$\tilde{\Theta}_{4/O} = \begin{bmatrix} 1 & \tilde{3} & \frac{1}{\tilde{5}} & \tilde{7} & \tilde{7} \\ \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{6}} & \tilde{3} & \tilde{3} \\ \tilde{5} & \tilde{6} & 1 & \tilde{9} & \tilde{9} \\ \frac{1}{\tilde{7}} & \frac{1}{\tilde{3}} & \frac{1}{\tilde{9}} & 1 & \tilde{1} \\ \frac{1}{\tilde{7}} & \frac{1}{\tilde{3}} & \frac{1}{\tilde{9}} & 1 & 1 \end{bmatrix} \tag{12}$$

$$\tilde{\Theta}_{4/D} = \begin{bmatrix} 1 & \tilde{3} & \frac{1}{\tilde{5}} & \frac{1}{\tilde{3}} & \frac{1}{\tilde{5}} \\ \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{8}} & \frac{1}{\tilde{5}} & \frac{1}{\tilde{8}} \\ \tilde{5} & \tilde{8} & 1 & \tilde{3} & 1 \\ \tilde{3} & \tilde{5} & \frac{1}{\tilde{3}} & 1 & \frac{1}{\tilde{3}} \\ \tilde{5} & \tilde{8} & 1 & \frac{1}{\tilde{3}} & 1 \end{bmatrix} \tag{13}$$

From the judgment matrix of criteria in equation (1) and the E-CDEAHP-DCCR-I model, the fuzzy relative efficiencies of S, O, and D can be calculated by the following LP model,

(E-CDEAHP-DCCR-I) $\text{Min}\theta_{\text{Criteria/FMEA}}$ (14)

Subject to $\theta_{\text{Criteria/FMEA}} - \lambda_1 - \lambda_2 - \lambda_3 < 0$ (15)

$(\tilde{a}_{1o})^L_\alpha$ –(1) λ_1-(1/(α+2)) λ_2-(1/(α+4)) λ_3< 0 (16)
$(\tilde{a}_{2o})^L_\alpha$ –(4-α) λ_1-(1) λ_2-(1/(α+2)) λ_3< 0 (17)
$(\tilde{a}_{3o})^L_\alpha$ –(6-α) λ_1-(4-α) λ_2-(1) λ_3< 0 (18)
Θ Unrestricted, λ_j > 0 (19)

where $(\tilde{a}_{1o})^L_\alpha \in$ { 1, 1/(4-α), 1/(6-α) }, $(\tilde{a}_{2o})^L_\alpha \in$ { α+2, 1,(1/(4-α) }, and $(\tilde{a}_{3o})^L_\alpha \in$ { α+4, α+2, 1 }.

To obtain LW of each criterion, the E-CDEAHP-DCCR-I model must be solved at the specified α-level set. In this paper, eleven levels of α-level set, which were 0, 0.1, 0.2, …, 1, was specified. The relative efficiency of criteria S, O, and D at each α-level set are shown in Table 3.5

α- level	Relative Efficiency ($\theta_{Criteria/FMEA}$)		
	DMU 1 (S)	DMU 2 (O)	DMU 3 (D)
0	1	0.3333	0.1667
0.1	1	0.3559	0.1695
0.2	1	0.3793	0.1724
0.3	1	0.4035	0.1754
0.4	1	0.4286	0.1786
0.5	1	0.4545	0.1818
0.6	1	0.4815	0.1852
0.7	1	0.5094	0.1887
0.8	1	0.5385	0.1923
0.9	1	0.5686	0.1961
1	1	0.6000	0.2000

Table 3.5: .Relative efficiency of criteria

From Table 3.5, the relative efficiency of DMU 1 is crisp, and others are fuzzy. The membership functions of relative efficiency of DMU 2 and 3 can be approximated to be the one side triangular membership functions by regression analysis. The regression equations are given below:

$\theta_{S/FMEA}$ = 1

$\theta_{O/FMEA}$= [0.326 + 0.266α, 0.6] R^2= 99.7% $\theta_{D/FMEA}$= [0.166 + 0.033α, 0.2] R^2= 99.7

By the extension principle, summation of fuzzy relative efficiency from DEA model is [1.492 + 0.299α, 1.8]. Therefore, the fuzzy LW in a form of traditional AHP can be rewritten as follows:

rom the judgment matrix of alternatives in equation (2) and the E-CDEAHP-DCCR-I model, the fuzzy relative efficiencies of A1, A2, A3, A4 and A5 based on severity can be calculated by the following LP model,

(E-CDEAHP-DCCR-I) Min $\theta_{A/S}$ (23)
Subject to $\theta_{A/S}$- λ_1- λ_2- λ_3-λ_4-λ_5> 0 (24)
$(\tilde{a}_{1o})^L_\alpha$–(1)$\lambda_1$-(1/(α+2))$\lambda_2$-(1/(α+4))$\lambda_3$-(1/(α+3))$\lambda_4$-(1/(α+8))$\lambda_5$< 0 (25)
$(\tilde{a}_{2o})^L_\alpha$–(1/(α+2))$\lambda_1$-(1)$\lambda_2$-(1/(α+6))$\lambda_3$-(1/(α+4))$\lambda_4$-(1/(α+8))$\lambda_5$< 0 (26)
$(\tilde{a}_{3o})^L_\alpha$ –(7-α) λ_1-(8-α) λ_2-(1) λ_3-(4-α)λ_4-(3-α)λ_5< 0 (27)
$(\tilde{a}_{4o})^L_\alpha$–(5-α)$\lambda_1$-(6-α)$\lambda_2$-(1/(α+2))$\lambda_3$-(1)$\lambda_4$-(1/(α+2))$\lambda_5$<0 (28)
$(\tilde{a}_{5o})^L_\alpha$–(9)$\lambda_1$-(9)$\lambda_2$-(6-α)$\lambda_3$-(3-α)$\lambda_4$-(1)$\lambda_5$<0 (29)

where $(\tilde{a}_{1o})^L_\alpha \in$ {1, 1/(4-α), 1/(7-α), 1/(5-α), 1/9}, $(\tilde{a}_{2o})^L_\alpha \in${1/(4-α), 1, 1/(8-α), 1/(6-α), 1/9}, $(\tilde{a}_{3o})^L_\alpha \in${α+5, α+6, 1, α+2, α+1}, $(\tilde{a}_{4o})^L_\alpha \in${α+3, α+4, 1/(4-α), 1, 1/(4-α)}, $(\tilde{a}_{5o})^L_\alpha \in${α+8, α+8, α+4, α+1, 1}

α-level	Relative Efficiency ($\theta_{Alternative/S}$)				
	A1	A2	A3	A4	A5
0	0.8889	1	0.4444	0.2500	0.1250
0.1	0.9000	1	0.4556	0.2658	0.1392
0.2	0.9111	1	0.4667	0.2821	0.1538
0.3	0.9222	1	0.4778	0.2987	0.1688
0.4	0.9333	1	0.4889	0.3158	0.1842
0.5	0.9444	1	0.5000	0.3333	0.2000
0.6	0.9556	1	0.5111	0.3514	0.2162
0.7	0.9667	1	0.5222	0.3699	0.2329
0.8	0.9778	1	0.5333	0.3889	0.2500
0.9	0.9889	1	0.5444	0.4085	0.2676
1	1	1	0.5556	0.4286	0.2857

Table 3.6:.Relative efficiency of causes of blister based on severity

Table 3.2 shows the relative efficiencies for the alternatives based on the criteria S for blister defect. The regression equations formed from the relative efficiencies are shown below.

$\theta_{A1/S}$= [0.889 + 0.111α, 1] R^2= 100%

$\theta_{A2/S}$= 1

$\theta_{A3/S}$= [0.444 + 0.111α, 0.5556] R^2= 100%

$\theta_{A4/S}$= [0.247 + 0.178α, 0.4286] R^2= 99.9%

$\theta_{A5/S}$= [0.122 + 0.161α, 0.2857] R^2= 99.9%

Summation of fuzzy relative efficiencies from DEA model = [2.702 + 0.561α, 3.2699].

Similarly based on the criteria O in blister defect the re-

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gression equations were found to be

$\theta_{A1/O}$= [0.506 + 0.461α, 1] R^2= 97.4%
$\theta_{A2/O}$= 1
$\theta_{A3/O}$= [0.222 + 0.111α, 0.3333] R^2= 100%
$\theta_{A4/O}$= [0.430 + 0.228α, 0.6666] R^2= 99.2%
$\theta_{A5/O}$= 0.1111

Summation of fuzzy relative efficiencies from DEA model = [2.269 + 0.8α, 3.1111]

Based on the criteria D in problem 1 the regression equations were found to be

$\theta_{A1/D}$= [0.326 + 0.266α, 0.6] R^2= 99.7%
$\theta_{A2/D}$= $\theta_{A4/D}$= [0.166 + 0.033α, 0.2] R^2= 99.7%
$\theta_{A3/D}$= 1
$\theta_{A5/D}$= [0.658 + 0.333α, 1] R^2= 99.7%

Summation of fuzzy relative efficiencies from DEA model = [2.316 + 0.665α, 3]

The fuzzy relative efficiency of each controllable causes is converted to be traditional fuzzy LWs by the concept extension principle. The results are shown as follows:

$$\mathrm{LW}_{\mathrm{A1/S}} = \left[\frac{0.889+0.111\alpha}{3.2699}, \frac{1}{2.702+0.561\alpha}\right] \quad (30)$$

$$\mathrm{LW}_{\mathrm{A2/S}} = \left[\frac{1}{3.2699}, \frac{1}{2.702+0.561\alpha}\right] \quad (31)$$

$$\mathrm{LW}_{\mathrm{A3/S}} = \left[\frac{0.444+0.111\alpha}{3.2699}, \frac{0.5556}{2.702+0.561\alpha}\right] \quad (32)$$

$$\mathrm{LW}_{\mathrm{A4/S}} = \left[\frac{0.247+0.178\alpha}{3.2699}, \frac{0.4286}{2.702+0.561\alpha}\right] \quad (33)$$

$$\mathrm{LW}_{\mathrm{A5/S}} = \left[\frac{0.122+0.161\alpha}{3.2699}, \frac{0.2857}{2.702+0.561\alpha}\right] \quad (34)$$

$$\mathrm{LW}_{\mathrm{A1/O}} = \left[\frac{0.506+0.461\alpha}{3.1111}, \frac{1}{2.269+0.8\alpha}\right] \quad (35)$$

$$\mathrm{LW}_{\mathrm{A2/O}} = \left[\frac{1}{3.1111}, \frac{1}{2.269+0.8\alpha}\right] \quad (36)$$

$$\mathrm{LW}_{\mathrm{A3/O}} = \left[\frac{0.222+0.111\alpha}{3.1111}, \frac{0.3333}{2.269+0.8\alpha}\right] \quad (37)$$

$$\mathrm{LW}_{\mathrm{A4/O}} = \left[\frac{0.430+0.228\alpha}{3.1111}, \frac{0.6667}{2.269+0.8\alpha}\right] \quad (38)$$

$$\mathrm{LW}_{\mathrm{A5/O}} = \left[\frac{0.1111}{3.1111}, \frac{0.1111}{2.269+0.8\alpha}\right] \quad (39)$$

$$\mathrm{LW}_{\mathrm{A1/D}} = \left[\frac{0.326+0.266\alpha}{3}, \frac{0.6}{2.316+0.665\alpha}\right] \quad (40)$$

$$\mathrm{LW}_{\mathrm{A2/D}} = \mathrm{LW}_{\mathrm{A4/D}} = \left[\frac{0.166+0.033\alpha}{3}, \frac{0.2}{2.316+0.665\alpha}\right] \quad (41)$$

$$\mathrm{LW}_{\mathrm{A3/D}} = \left[\frac{1}{3}, \frac{1}{2.316+0.665\alpha}\right] \quad (42)$$

$$\mathrm{LW}_{\mathrm{A5/D}} = \left[\frac{0.658+0.333\alpha}{3}, \frac{1}{2.316+0.665\alpha}\right] \quad (43)$$

The FW of A1-A5 can be calculated by the fuzzy hierarchical arithmetic aggregation based on the fuzzy LWs in equations (20) to (22) and (30) to (43). The upper and lower of fuzzy FW of the controllable cause A1-A5 for all α-level set can be respectively calculated by:

$$(\mathrm{FW}_\vartheta)_\alpha^{\mathrm{L}} = \sum_{\varphi\epsilon\{S,O,D\}} (\mathrm{LW}_{\varphi/\mathrm{FMEA}})_\alpha^{\mathrm{L}} (\mathrm{LW}_{\vartheta/\varphi})_\alpha^{\mathrm{L}}$$
$$(\mathrm{FW}_\vartheta)_\alpha^{\mathrm{U}} = \sum_{\varphi\epsilon\{S,O,D\}} (\mathrm{LW}_{\varphi/\mathrm{FMEA}})_\alpha^{\mathrm{U}} (\mathrm{LW}_{\vartheta/\varphi})_\alpha^{\mathrm{U}}$$

where $\vartheta \in$ {A1, …, A5}. For example, the lower and upper of fuzzy FW of A1 can be calculated by:

$$\begin{aligned}(\mathrm{FW}_{\mathrm{A1}})_\alpha^{\mathrm{L}} = {} & (\mathrm{LW}_{\mathrm{S/FMEA}})_\alpha^{\mathrm{L}} (\mathrm{LW}_{\mathrm{A1/S}})_\alpha^{\mathrm{L}} + \mathrm{LW}_{\mathrm{O/FMEA}})_\alpha^{\mathrm{L}} (\mathrm{LW}_{\mathrm{A1/O}})_\alpha^{\mathrm{L}} + (\mathrm{LW}_{\mathrm{D/FMEA}})_\alpha^{\mathrm{L}} (\mathrm{LW}_{\mathrm{A1/D}})_\alpha^{\mathrm{L}} \\ & \left(\frac{1}{1.8}\right)\left(\frac{0.889+.111\alpha}{3.2699}\right) + \left(\frac{0.326+0.266\alpha}{1.8}\right)\left(\frac{0.506+0.461\alpha}{3.111}\right) + \left(\frac{0.1666+0.033\alpha}{1.8}\right)\left(\frac{0.326+0.266\alpha}{3}\right)\end{aligned}$$

$$\begin{aligned}(\mathrm{FW}_{\mathrm{A1}})_\alpha^{\mathrm{U}} = {} & (\mathrm{LW}_{\mathrm{S/FMEA}})_\alpha^{\mathrm{U}} (\mathrm{LW}_{\mathrm{A1/S}})_\alpha^{\mathrm{U}} + \mathrm{LW}_{\mathrm{O/FMEA}})_\alpha^{\mathrm{U}} (\mathrm{LW}_{\mathrm{A1/O}})_\alpha^{\mathrm{U}} + (\mathrm{LW}_{\mathrm{D/FMEA}})_\alpha^{\mathrm{U}} (\mathrm{LW}_{\mathrm{A1/D}})_\alpha^{\mathrm{U}} \\ & \left(\frac{1}{1.492+0.299\alpha}\right)\left(\frac{1}{2.702+0.561\alpha}\right) + \left(\frac{0.6}{1.492+0.299\alpha}\right)\left(\frac{1}{2.269+0.8\alpha}\right) + \left(\frac{0.2}{1.492+0.299\alpha}\right)\left(\frac{0.6}{2.316+0.665\alpha}\right)\end{aligned}$$

Similarly all the other FWs for blister was calculated. Substituting the value of α from 0, 0.1......,1 in the equations of the FWs, the resultant upper and lower final weights are shown in Table 4.1

4 RESULTS AND DISCUSSIONS

The same procedure was applied for bend, twist and speed crack. The FWs for bend, twist and speed crack are shown in table 4.2, table 4.3, table 4.4.

The top two causes for each defect was selected for improvement. The selected reasons for the occurrence each defect is shown :

a. Blister
   - Improper shearing of butt end.
   - Due to excess use of grafting lubricant.
b. Bend
   - Improper alignment of die, stem and container
   - Stretching of hot sections
c. Twist
   - Improper flow of metal through the die
   - Improper quenching
d. Speed Crack
   - High extrusion speed
   - High billet temperature

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Improper Shearing of Butt End.

It is often seen that during extrusion process the butt end is not always properly sheared by the shearing mechanism this can cause blisters in the end product. This defect can be avoided or reduced by providing burp cycle between extrusions. During burp cycle the next billet is slightly pressed against the rear end of the sheared billet causing deformation of both billets. Then the press retracts slightly so as to allow the entrapped air to escape. It is mainly recommended that burp cycle should be automatically during the use of spreader and port hole dies for all billets.

Due To Excess Use of Grafting Lubricant.

Use of excess dag or lubricant causes blisters. This can be avoided by cleaning the surface of the dummy block and container of excess lubricant.

α- level	A_1		A_2		A_3		A_4		A_5	
	L	U	L	U	L	U	L	U	L	U
0	0.191	0.460	0.233	0.437	0.119	0.255	0.072	0.236	0.047	0.148
0.1	0.199	0.439	0.238	0.417	0.123	0.243	0.079	0.225	0.052	0.142
0.2	0.207	0.420	0.243	0.399	0.128	0.233	0.087	0.215	0.057	0.136
0.3	0.217	0.402	0.248	0.382	0.132	0.223	0.094	0.205	0.062	0.130
0.4	0.226	0.385	0.253	0.365	0.137	0.214	0.102	0.196	0.067	0.125
0.5	0.236	0.369	0.258	0.350	0.141	0.205	0.110	0.188	0.071	0.120
0.6	0.247	0.354	0.263	0.336	0.146	0.197	0.119	0.180	0.076	0.115
0.7	0.258	0.340	0.268	0.323	0.151	0.189	0.128	0.173	0.081	0.110
0.8	0.269	0.327	0.273	0.311	0.156	0.182	0.137	0.166	0.086	0.106
0.9	0.281	0.314	0.278	0.299	0.161	0.175	0.146	0.160	0.091	0.102
1	0.294	0.303	0.283	0.288	0.166	0.169	0.155	0.154	0.096	0.098
RANK	1		2		3		4		5	

Table 4.1 : Final weights of reasons for blister

α- level	B_1		B_2		B_3		B_4		B_5	
	L	U	L	U	L	U	L	U	L	U
0	0.068	0.195	0.245	0.455	0.169	0.409	0.069	0.142	0.107	0.301
0.1	0.073	0.186	0.250	0.435	0.179	0.391	0.071	0.136	0.115	0.288
0.2	0.079	0.178	0.256	0.417	0.189	0.375	0.073	0.130	0.124	0.276
0.3	0.084	0.171	0.262	0.400	0.199	0.360	0.076	0.125	0.132	0.264
0.4	0.090	0.164	0.267	0.384	0.209	0.346	0.078	0.120	0.141	0.254
0.5	0.096	0.157	0.273	0.369	0.220	0.333	0.080	0.115	0.150	0.244
0.6	0.102	0.151	0.279	0.355	0.230	0.320	0.083	0.111	0.159	0.234
0.7	0.108	0.145	0.284	0.342	0.240	0.308	0.086	0.107	0.168	0.225
0.8	0.114	0.140	0.290	0.329	0.251	0.297	0.088	0.103	0.177	0.217
0.9	0.120	0.135	0.296	0.317	0.261	0.287	0.091	0.099	0.187	0.209
1	0.127	0.130	0.302	0.306	0.271	0.277	0.094	0.095	0.197	0.202
RANK	4		1		2		5		3	

Table 4.2 : Final weights of reasons for bend

α- level	C_1		C_2		C_3		C_4		C_5	
	L	U	L	U	L	U	L	U	L	U
0	0.036	0.086	0.166	0.387	0.095	0.280	0.219	0.406	0.149	0.349
0.1	0.038	0.082	0.175	0.371	0.103	0.268	0.223	0.389	0.156	0.334

0.2	0.040	0.079	0.184	0.355	0.111	0.257	0.227	0.372	0.163	0.320
0.3	0.042	0.075	0.193	0.340	0.119	0.246	0.232	0.357	0.171	0.307
0.4	0.043	0.072	0.201	0.326	0.127	0.236	0.236	0.343	0.179	0.294
0.5	0.045	0.069	0.210	0.313	0.136	0.227	0.241	0.330	0.187	0.283
0.6	0.047	0.067	0.219	0.301	0.145	0.218	0.246	0.317	0.195	0.272
0.7	0.050	0.064	0.227	0.289	0.154	0.209	0.252	0.305	0.203	0.261
0.8	0.052	0.062	0.236	0.279	0.163	0.202	0.257	0.294	0.212	0.252
0.9	0.054	0.059	0.245	0.268	0.173	0.194	0.263	0.283	0.220	0.243
1	0.056	0.057	0.254	0.259	0.183	0.187	0.269	0.273	0.229	0.234
RANK	5		2		4		1		3	

Table 4.3 : Final weights of reasons for twist

α- level	D_1		D_2		D_3		D_4		D_5	
	L	U	L	U	L	U	L	U	L	U
0	0.134	0.289	0.158	0.315	0.345	0.541	0.048	0.085	0.065	0.103
0.1	0.141	0.281	0.165	0.305	0.351	0.525	0.049	0.082	0.066	0.100
0.2	0.149	0.273	0.172	0.297	0.357	0.509	0.051	0.080	0.067	0.097
0.3	0.157	0.265	0.180	0.288	0.362	0.495	0.052	0.077	0.068	0.094
0.4	0.165	0.258	0.187	0.280	0.368	0.481	0.054	0.075	0.069	0.091
0.5	0.173	0.250	0.195	0.272	0.374	0.467	0.055	0.073	0.070	0.089
0.6	0.181	0.244	0.203	0.265	0.380	0.454	0.057	0.071	0.072	0.086
0.7	0.190	0.237	0.211	0.258	0.386	0.442	0.058	0.069	0.073	0.084
0.8	0.198	0.231	0.219	0.251	0.392	0.430	0.060	0.067	0.074	0.081
0.9	0.207	0.225	0.227	0.244	0.398	0.419	0.061	0.065	0.075	0.079
1	0.216	0.219	0.235	0.238	0.404	0.408	0.063	0.064	0.076	0.077
RANK	3		2		1		4		5	

Table 4.4 : Final weights of reasons for speed crack

Improper alignment of die, stem and container

Bend defect can be caused due to improper alignment. Press alignment should always be the first item on any list of extrusion practices. The approximate alignment of the press can be easily found by observing the butt end of the extruded billet. If a perfect concentric round impression is formed at its end due to the pressing of the plunger it can be said to be perfectly aligned. Regular inspection (at least every shift) is essential, with emphasis always on preventing rather than correcting misalignment. Operating parameters like extrusion pressure, speed, operating temperature etc and regular maintenance plays an important role in maintaining proper alignment of press.

Stretching Of Hot Sections

Bend defect may be formed due the above reason. The proper remedy is to avoid such practices and allowing the sections to properly cool before stretching. The air cooling fans also have to be regularly monitored so that all parts of the section are equally cooled. Infrared cameras can also be used to check whether the sections are properly cooled

Improper Flow of Metal Through The Die

Twist defects are usually formed due to improper flow of metal through the die. This can be due to overheating or irregular heating of the die. The remedy to this cause is the use of single-cell die oven. This will bring the die quickly and uniformly to operating temperature. To avoid the initial capital expense of a complete battery of single-cell ovens, dies may be held at a moderate temperature for some time in a traditional chest oven, then the necessary heating quickly completed in a single-cell oven when the die is needed.

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Improper Quenching

Twist defect may also be formed due to improper quenching. This defect can be rectified by using modular coolers whose nozzles can be adjusted according to the shape of the section.

High Extrusion Speed

Speed cracks are formed due to sticking of metal on the surface of the die during extrusion. The sticking of aluminium on the die occurs due to the temperature of the die bearing, billet and die bearing condition. Speed cracks can be prevented by operating the press at beginning of a new lot at slower speed than the normal speed and then gradually increasing the speed. This defect is more common in certain aluminium alloys compared to others. Therefore operator must be provided with die history, alloy speciality and various speed for operating the press for that alloy and die which are not available to him at present.

High Billet Temperature

Speed cracks are also formed due to high billet temperature. Temperature measuring device has to be regularly checked. Moreover the billet temperature is not measured at the point of extrusion which also has to be made available. Infrared cameras can be used to check if billets are properly taper heated.

5 Conclusion

If The main objective of the paper was to identify the critical defects and rank their causes using Fuzzy Data Envelopment Analytical Hierarchy Process(FDEAHP). Blister, bend, twist and speed crack were found to be the critical defects. The reasons for these defects were found and FDEAHP analysis was done. After FDEAHP analysis the critical causes for each defect were found and suggestions were made so as to reduce the probability of occurrence of the above defects.

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