FE Review

Kinematics, Dynamics, & Vibration

Tactics

- Answer easy questions you know first
- Come back to hard questions
- Once answered don't look back
- Use books and tables to save time

Topics

Basics
- Center of gravity
- Mass & weight
- Inertia
- Mass moment of inertia
- Parallel axis theorem
- Radius of gyration
- Principle axes

Kinematics:
- Particles & rigid Bodies
- Coordinate systems
- Linear particle motion
- Distance & speed
- Uniform motion
- Uniform acceleration
- Linear acceleration
- Projectile motion
- Rotational particle motion
- Relation between linear and rotational variables
- Normal acceleration
- Coriolis acceleration
- Particle motion in polar coordinates
- Relative motion
- Dependant motion
- General planar motion
- Rotation about a fixed axis
- Instananious center of acceleration
- Sliders
- Crank Sliders
- 4 bar linkages

Kinetics:
- Rigid Body motion
- Equilibrium & Stability
- Constant forces
- Linear forces
- Ballistic pendulum
- Angular momentum
- Newton's laws of motion
- Centripital force
- Dynamic equilibrium
- Friction
- Roadway/conveyor banking
- Rigid body motion
- Constrained motion
- Cables & tension from suspended accelerating masses
- Impluse
- Impulse – momentum
- Impacts
- Coefficient of restitution
- Gravitation

Definitions

- Kinematics : Geometry of motion independent of forces
  - Linkages
- Kinetics : Acceleration and motion
- Vector : magnitude & direction
- I = Mass moment of inertia

General Problem Solving

- Understand the problem
  - Identify desired result
  - Identify unknowns
  - Identify knowns
  - Draw Diagrams
- Establish a coordinate system
- Identify theory and equations that relate results to knowns
- Does it make sense?

Vector Properties

- Magnitude & direction
  - Commonly break into orthogonal components:
    $$\vec{A} = \langle A_x\hat{x}, A_y\hat{y}, A_z\hat{z}\rangle$$
  - Magnitude:
    $$|\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$
  - Direction:
    $$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$
- Add
  - Sum components:
    $$\vec{A} + \vec{B} = \langle A_x + B_x, A_y + B_y, A_z + B_z\rangle$$
- Subtract
  - Add negative

Vector Manipulation

- Dot Product: projection of one vector onto another
  - Scalar : $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
  - Commutative: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
  - Distributive : $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
- Cross Product: Perpendicular vector of magnitude of the orthogonal space between 2 vectors
  - Vector :$\vec{A}\ x\ \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y -$

Basics: Properties of Solid Bodies

- Center of Gravity
- Mass and Weight
- Inertia
- Mass Moment of Inertia
- Parallel Axis Theorem
- Radius of Gyration
- Principle Axes

Center of Gravity = Center of Mass

- Volumetric centroid of a homogenous body
- On axis of symmetry
  - $x_c = \frac{\int x\,dm}{m}$
  - $y_c = \frac{\int y\,dm}{m}$
  - $z_c = \frac{\int z\,dm}{m}$

Mass & Weight

- Mass:
  - Independent of gravitational field
  - Measure of inertia
- Weight:
  - Depends on gravitational field acting on mass

Inertia

- Resistance to changes in linear speed
- Inertial force = $m\vec{a}$

Mass Moment of Inertia

- Resistance to changes in rotational speed
- References about axis of rotation
- About centroid:
  - $I_x = \int (y^2 + z^2)dm$
  - $I_y = \int (x^2 + z^2)dm$
  - $I_z = \int (x^2 + y^2)dm$
- Look it up

Parallel Axis Theorem

- Mass moment of inertia about other axes
- $I_{parallel} = I_{centroid} + md^2$

Radius of Gyration : k

- Equivalent distance form rotational axis at which entire mass could be located

- $k = \sqrt{\frac{I}{m}}$
- $I = mk^2$

Kinematics

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Kinematics

- Study of a body's motion independent of forces involved
- Position, velocity, & acceleration

Particles vs. Rigid Bodies

- Particles aren't concerned with rotation
  - Rotation is absent or insignificant
  - Kinetic energy of rotation is small
  - All parts have the same instantaneous displacement, velocity, and acceleration
- Rigid bodies
  - Body does not deform (significantly)
  - Can be idealized as a combination of 2 or more particles that remain a fixed finite distance from each other
  - Particles can have differing displacement, velocity and acceleration

Coordinate Systems

- Need a reference!
  - Rectangular : Cartesian
  - Polar
  - Cylindrical
  - Spherical
- Coordinates in 3D
  - 3 location
  - 3 orientation

Position, Velocity, & Acceleration

- Position
  - $\vec{s} = \int \overrightarrow{v(t)}dt = \int(\int \overrightarrow{a(t)}dt)dt$
  - Relative to a reference
- Velocity
  - $\vec{v} = \frac{d\vec{s}}{dt} = \int \overrightarrow{a(t)}dt$
- Acceleration
  - $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{s}}{dt^2}$

Distance vs. Dispalcement

- Displacement : Net change in particle position
  - Path independent
  - Final position – initial position
- Distance : Path dependant change in particle location
  - Accumulated length of path traveled including reversals

$$Distance \geq Displacement$$

Speed vs. Velocity

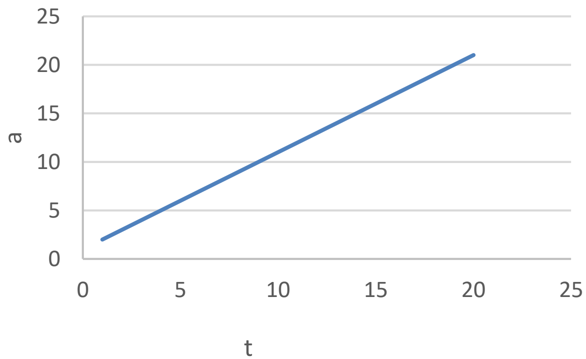
- Speed is a scalar
  - Magnitude of velocity
  - Direction not considered
- Velocity is a vector
  - Magnitude and direction.

Uniform Motion & Acceleration

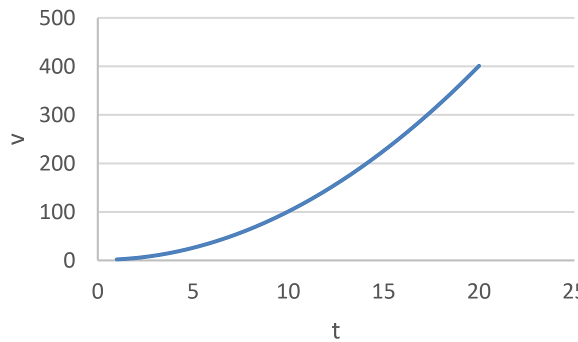
- Uniform Motion
  - Constant velocity : zero acceleration
    - Linear change in position with time

- Uniform Acceleration
  - Constant acceleration
    - Linear change in velocity with time
    - Second order, parabolic change in position with time

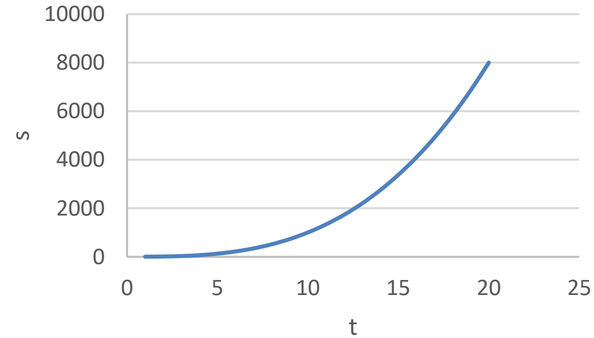
Linear Acceleration



Linear



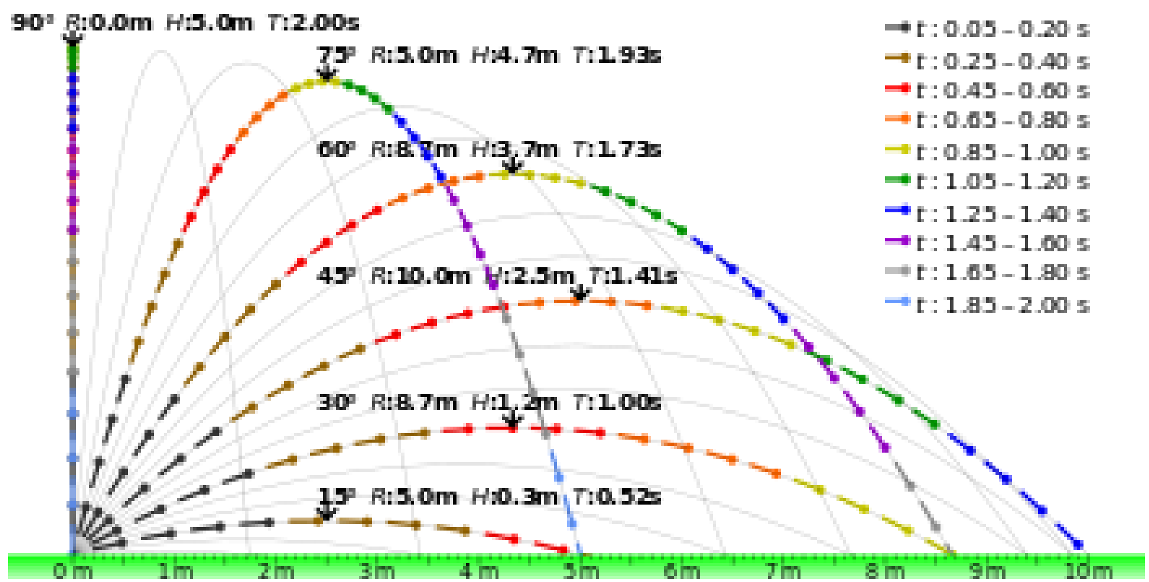
2^{nd} order = parabolic



3^{rd} order = cubic

Projectile Motion (Without Drag)

- Great example of decoupling orthogonal equations
- Horizontal: no forces = constant velocity
- Vertical: constant gravitational acceleration



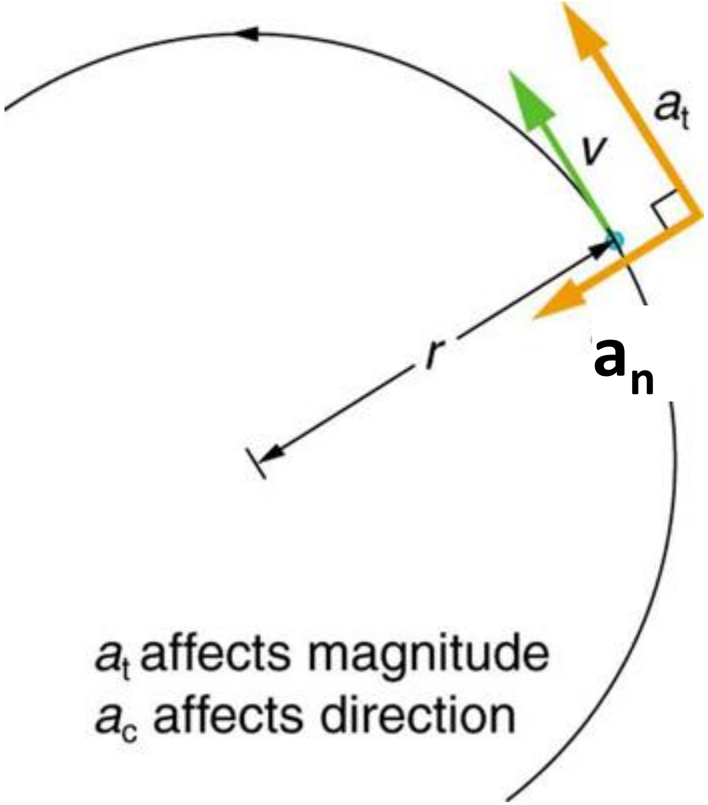
Rotational Particle Motion

- Position : angle
  - $\vec{\theta} = \int \overrightarrow{w(t)}dt = \int(\int \overrightarrow{\alpha(t)}dt)dt$
  - Relative to a reference
- Angular velocity
  - $\vec{w} = \frac{d\vec{\theta}}{dt} = \int \overrightarrow{\alpha(t)}dt$
- Angular acceleration
  - $\vec{\alpha} = \frac{d\vec{w}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$

Linear & Angular Relations

- $v_t = wr$
- $a_t = \alpha r$
- $a_n = \frac{v_t^2}{r} = rw^2 = v_t w$

- Coriolis
  $a_c = 2v_r w$



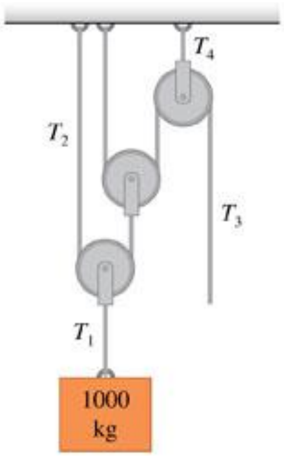
Relative Motion

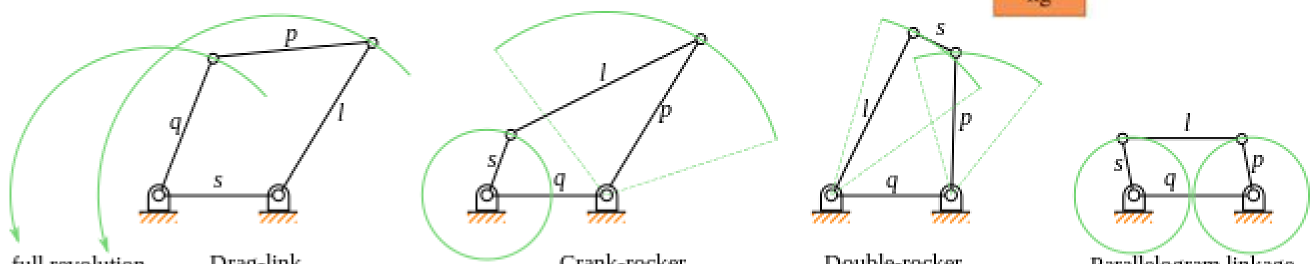
- Motion of a particle compared to another particle that is in motion

- $\overrightarrow{s_{B/A}} = \overrightarrow{s_B} - \overrightarrow{s_A}$
- $\overrightarrow{v_{B/A}} = \overrightarrow{v_B} - \overrightarrow{v_A}$
- $\overrightarrow{a\ _{B/A}} = \overrightarrow{a_B} - \overrightarrow{a_A}$

Dependent Motion

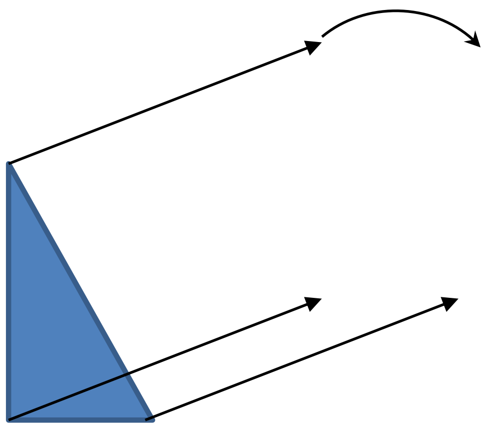
- Connected masses and systems
- Motion of one part is coupled to motion of other parts
  - Pulleys
  - Linkages





General Planar Motion

- Translation & rotation



General Planar Motion

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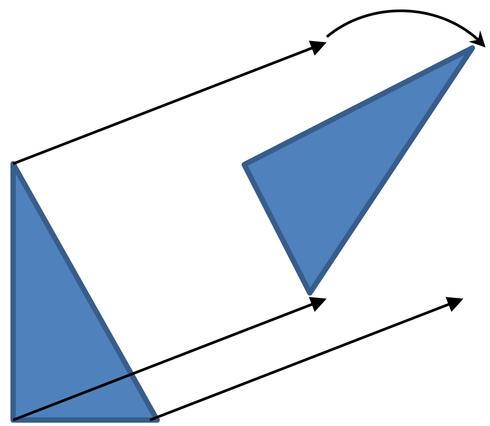
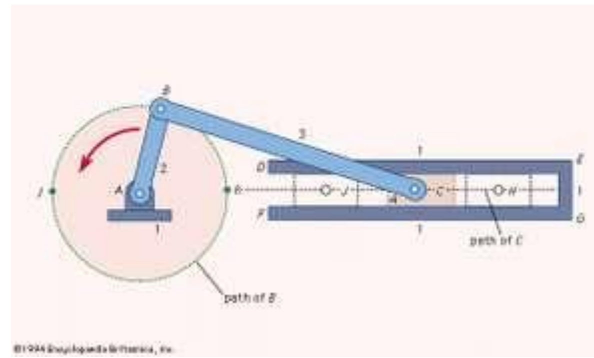


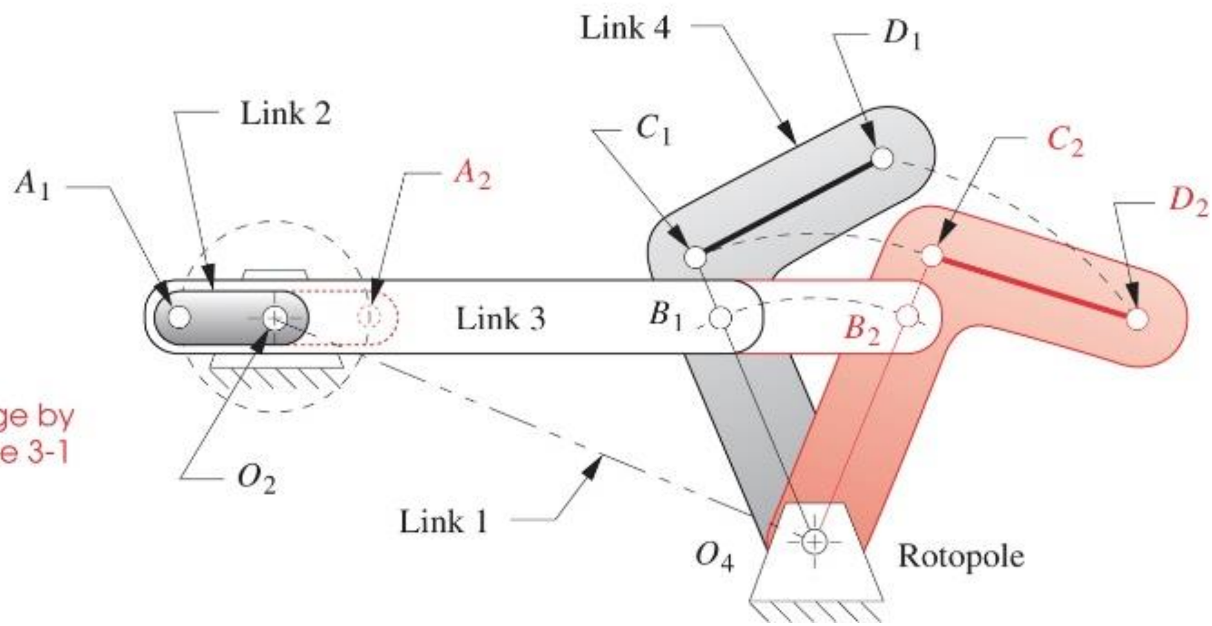
Instantaneous Center

- 2 points on rigid body with known velocity
  - Draw lines perpendicular to velocities
  - Intersection is instantaneous center
- If velocities are parallel but different magnitude
  - Draw a line perpendicular to velocities
  - Draw line from vector tips
  - Intersection is instantaneous center

Linkages Crank Slider

- Review 360
  - Positions
  - Velocities
  - Accelerations



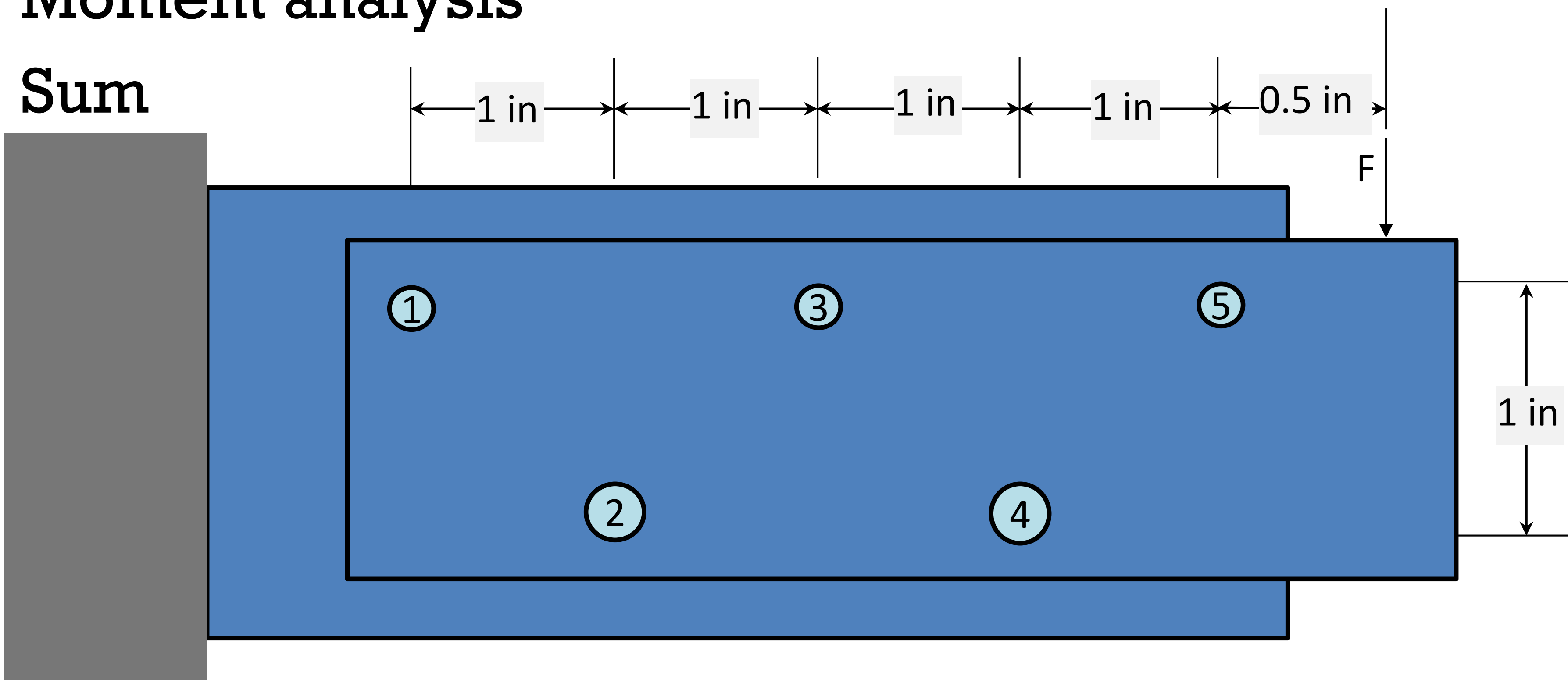


(b) Constructing the linkage by the method in Example 3-1

Fastener Patterns (366)

- Force analysis
- Find centroid
- Moment analysis
- Sum

$d_1 = d_3 = d_5 = 0.25$ inch
$d_2 = d_4 = 0.5$ inch
$F = 1000$ lbf



Kinetics

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Main Topics

- Momentum
- Forces & acceleration
- Energy

- Linear
- Rotational

Newton's Laws of Motion

1) A particle at remain in a state of rest or in a state of constant velocity unless an unbalanced force acts upon it.

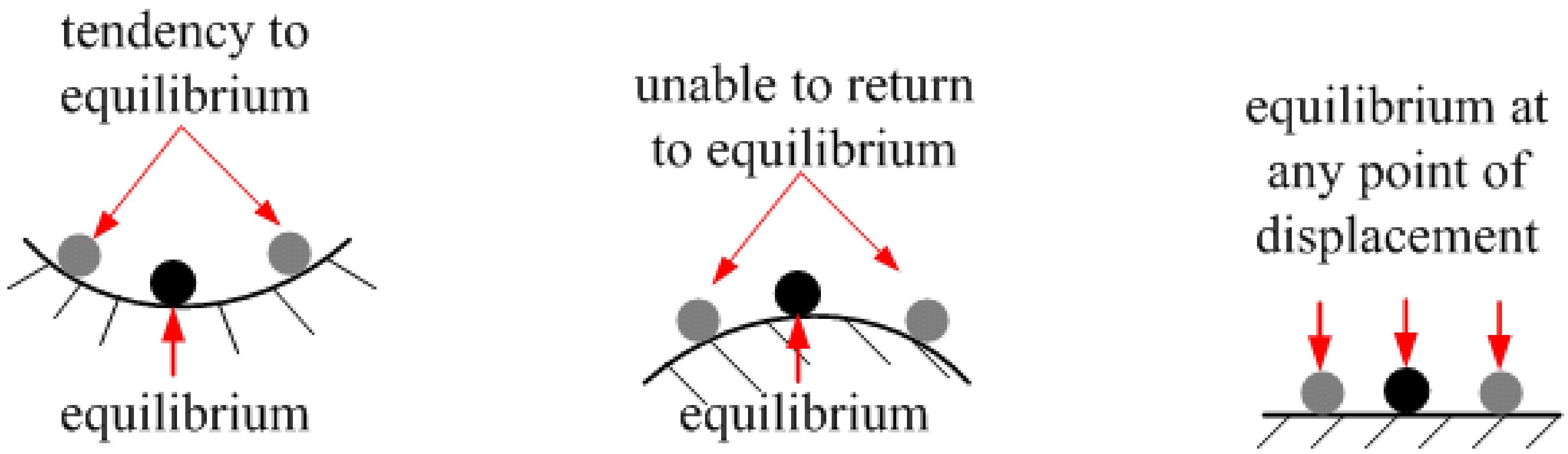
2) The acceleration of a particle is directly proportional to the force acting on it and inversely proportional to its mass. The direction of acceleration is aligned with the force.

3) For every acting force between two bodies, there is an equal but opposite reacting force on the same line of action.

Rigid Body Motion

- Pure translation
- Rotation about a fixed axis
- General planar motion (2D translation and rotation)
- Motion about a fixed point (3D rotation)
- General motion (motion not falling into other categories)

Stability & Equilibrium



(a) stable equilibrium

(b) unstable equilibrium

(c) neutral equilibrium

Momentum

- Momentum is conserved when no external forces act on a particle or system

- Linear:

$$\vec{p} = m\vec{v}$$

- Angular:

$$\vec{h} = \vec{r} \; x \; m\vec{v}$$

Energy

- Kinetic Energy
  - Liner : $K = \frac{1}{2}mv^2$
  - Angular: $K = \frac{1}{2}Iw^2$

- Potential
  - Gravitational : $E = mgh$
  - Spring: $E = \frac{1}{2}kx^2$

Equations of Motion

- Linear
  - $\vec{F} = m\vec{a}$
  - $\vec{F} = \frac{dm\vec{v}}{dt}$
- Angular
  - $\vec{M} = I\vec{\alpha}$

Impulse

- *Linear*
  - $\vec{F} = \frac{dm\vec{v}}{dt}$
  - $m(\overrightarrow{v_2} - \overrightarrow{v_1}) = \int_{t_1}^{t_2} \vec{F}\, dt$
  - $Imp = \int_{t_1}^{t_2} \vec{F}\, dt$
- Angular
  - $Imp = \int_{t_1}^{t_2} T\, dt$

Example 1

Impacts

- Momentum of system is conserved
- Kinetic energy may be transformed to heat

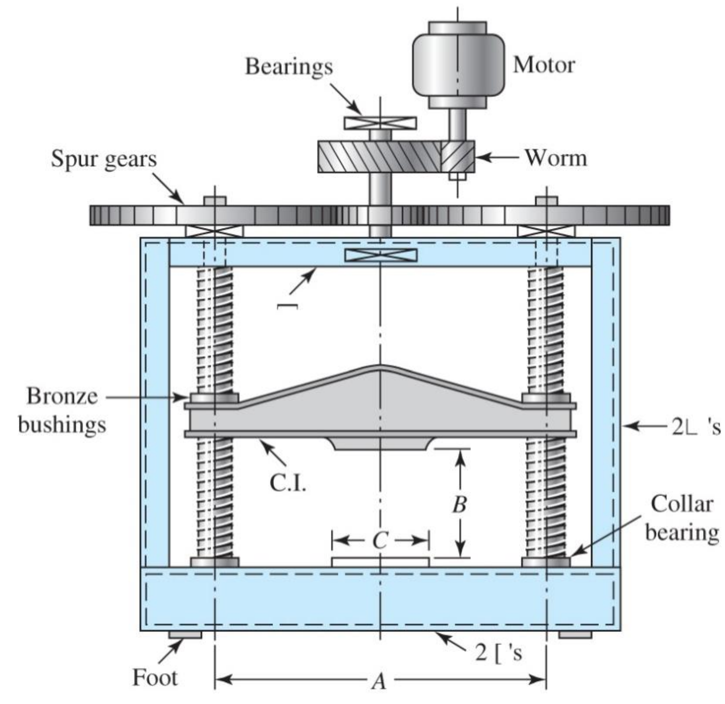
- Coefficient of restitution : e
  - Perfectly elastic e = 1
  - Perfectly inelastic e = 0
  - $e = \frac{relative\ separation\ velocity}{relative\ approach\ velocity} = \frac{v'_1 - v'_2}{v_2 - v_1}$

Mechanisms & Power Transmission

- Power Screws
- Energy Storage in Flywheels
- Rotating Solid Disks
- Rotating Rings
- Rotating Hubs
- Rotating Fluid Masses
- Rim Flywheels
- Gear Trains
- Cams
- More

Review 366

- A press with twin screws has been rated to 5000 lbf. Screws have ACME threads with outer diameter of 2 inches and a pitch of ¼ inch. Coefficients of friction are 0.05 for the threads and 0.08 for the collars. Collars have a diameter of 3.5 inches. Gears have a ratio of 60:1 and an efficiency of 95%. The full load motor speed is 1720 RPM
- How fast will the press move & how much power is required?



Example 42

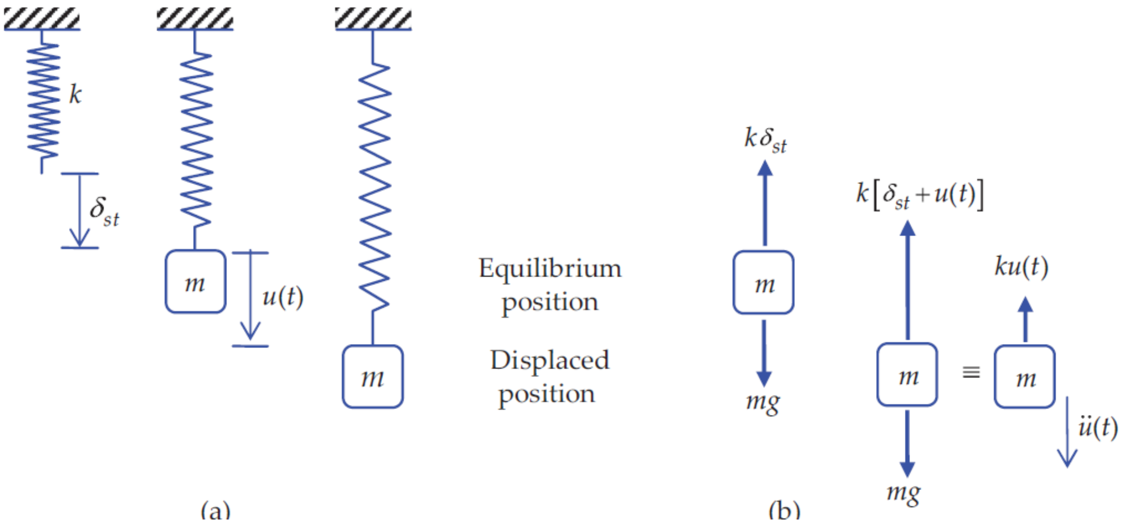
Vibration

- Free / Forced
- Undamped / Damped

- Boundary conditions

Fundamental Equation(s)

- Spring Mass
- Relate forces and accelerations
- $-ks = m\ddot{s}$
- $0 = m\ddot{s} + ks$
- $s(t) = e^{rt}$
- $0 = mr^2 + k$
- $\mathrm{r} = \pm i\sqrt{\frac{k}{m}}$
- natural frequency of undamped free vibration
- $w_n = \sqrt{\frac{k}{m}}$



Flexural Vibration of a Beam

- From Euler Bernoulli Beam Theory

$$M = EI\frac{d^2 s(x,t)}{dx^2} = IE\ s''$$

$$V = IE\ s'''$$

$$F = IE\ s''''$$

- Equation of motion

$$IE\ s'''' + m\ddot{s} = 0$$

- Pinned Pinned Natural Frequencies
  - $w_j = \left(j\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}}$

Torsional

- *Equation of motion*
  - $-k_r\theta = I\ddot{\theta}$
- $k_r = \frac{GJ}{L}$ for a round section $J = \frac{\pi d^4}{32}$
- $w = \sqrt{\frac{\pi d^4 G}{32LI}}$

Other

- Forced Vibration : Add a forcing term to the equations of motion
- Damped vibration : Add a velocity dependent damping force to the equations of motion

Recommended Resources

- Hibbeler, Engineering Mechanics series : Statics, Dynamics, Mechanics of Materials
- Beer & Johnson, Vector Mechanics for Engineers series: Statics, Dynamics, Mechanics of Materials
- Norton, Design of Machinery (360)
- Shigley's Design of Machine Elements (366)
- Mechanical Engineering Reference Manual for the PE Exam