Detection and Analysis of Braess's Paradox in San Francisco Bay Area's Highway Network

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Abstract

This research aims to detect the occurrence of Braess's Paradox in the Bay Area's highway network. To do so, we set up linear equations to model the time each traveler takes to travel through each of the 12 highway segments between San Jose and San Francisco. Then we solved equations to find out the equilibrium traffic situation that all routes take travelers the same amount of time. We compared the time that travelers use in equilibrium situations with or without the closure of San Mateo Bridge and Dumbarton bridge, two bridges that could potentially cause the occurrence of the paradox. We found out that there are no occurrences of Braess's Paradox in this part of the Bay Area highway network under the tested traffic.

I. INTRODUCTION

When you're stuck in traffic jams, you might wonder, "Why can't the government build more roads to help relieve the car flow?" In fact, your government might have intervened too much by building extra roads that can sometimes be the cause of congestion.

Braess's Paradox describes the scenario that the addition of a new road slows down the overall traffic flow in a network(1). The basic situation of the paradox can be modeled as the following figures:

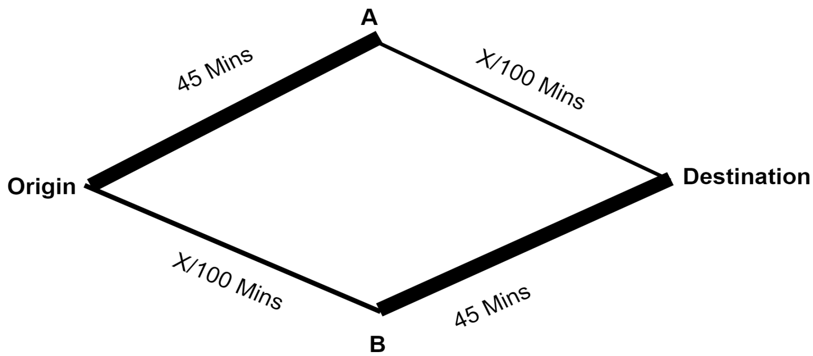

Figure 1: Basic Model of Braess's Paradox

Imagine there are 4000 drivers willing to travel from Origin to Destination. In the situation of figure 1, there are two symmetrical paths they can take: via junction A or junction B. As they're symmetrical, half of them (2000) will go via A, the other half (2000) via B. Each car will take 45 + (2000/100) = 65 minutes to travel from origin to destination. The system reaches an equilibrium.

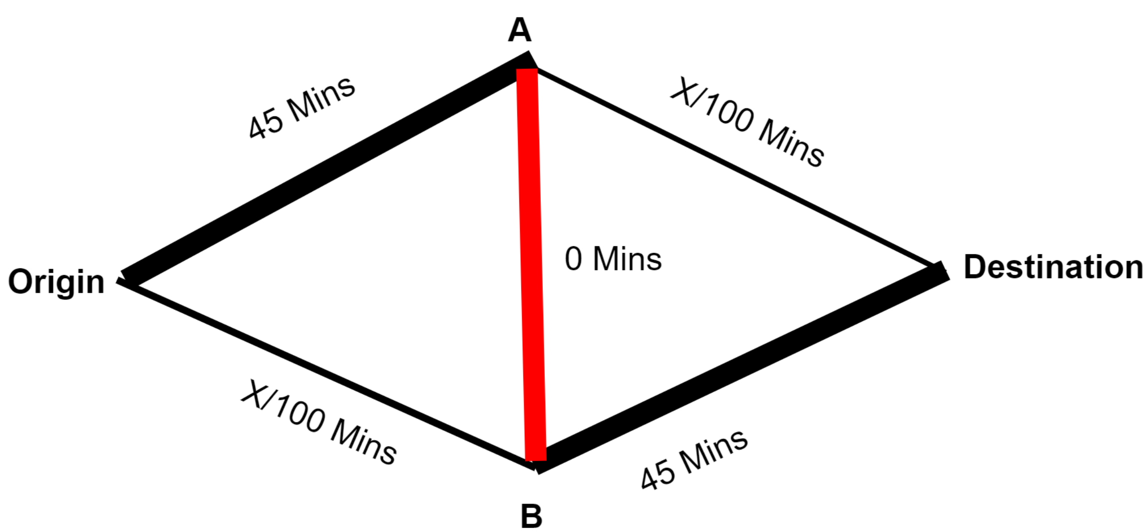

Figure 2: Basic Model of Braess's Paradox

Now when a new road that takes no time to travel and connects junction A and B is added (figure 2), the drivers see a new optimal path: Origin - B - A - Destination. As when X = 4000, X/100 < 45. Thus, all 4000 drivers will choose this Origin - B - A - Destination path, taking each of them: (4000/100) + 0 + (4000/100) = 80 minutes, 15 minutes more than the 65 minutes in the previous situation. The addition of a new road increased everyone's travel time. Furthermore, researchers have found that this paradox often ceases to exist when the traffic over the road network is too low or overwhelmingly high. (2)

The problem of Braess's Paradox occurs in some more complicated travel networks of big cities as well. Researchers have found that removing certain roads in the city of Boston, London, and New York City could improve overall traffic in the city. (3) Braess's Paradox can be effectively avoided through careful planning of road networks.

San Francisco Bay Area is among one of America's most congested regions, in which travelers spend on average over 100 hours a year in traffic jams in 2017. (4) In this research, the highway network of the southern San Francisco Bay Area will be analyzed and mathematically modeled to detect the occurrence of Braess's paradox and if it's the cause of congestion in this area.

II. METHODS

i. Model Design

The two largest cities in the San Francisco Bay area are San Francisco and San Jose, each home to around a million people. In this research, we will model the traffic flow between these two cities in the Bay Area highway network.

Figure 3: Highways between San Francisco and San Jose

As marked gray in the figure above, there are five routes via major highways that travelers can take to go from San Jose to San Francisco without taking detours. We observe that the shape of this network somehow resembles the basic Braes's Paradox model. Thus, we will determine if the two major bridges in the middle, San Mateo - Hayward Bridge (CA - 92) and Dumbarton Bridge (CA - 84), are the "shortcuts" that cause the Braess's Paradox.

To model the traffic flow, the time a car goes through a certain road segment must be modeled first. We divided the highway network into 12 segments. Even if there is no traffic, the drivers still need to follow speed limits and takes a certain amount of time to travel through. Thus, we model the time a car takes to go through a segment of the highway as the following. There are 12 such segments between San Francisco and San Jose that are all parts of interstate or state highways:

$T_x = A_x + B_x * F_x$

In which :

T_x: the amount of time a traveler takes to travel through highway segment x

A_x: the free-flow travel time on highway x

B_x: the delay parameter of highway x(increase in travel time per car increase on highway x)

F_x: the number of cars that travels on highway x

The parameter A_x is determined by:

$A_x = L_x$ *(miles)/ Speed Limit (mph)*

In which :

L_x: the length (miles) of the highway segment x

The parameter B_x (delay parameter) is determined by:

$B_x = (Actual\ T_x - A_x)/ Actual\ F_x$

For convenience and uniformity, we will use minutes as the unit of A_x.

ii. Constructing the Model

The length of highway segments is measured through *Google Maps* to determine A_x. To determine B_x, the actual traffic that travels through those highways is found in *Wikipedia*(5). The actual travel time is estimated with *Google Maps*. For highways that don't have actual traffic data, we estimated the delay parameter based on the number of lanes (not including carpool lanes) it has and delay parameter and lane count of other highway segments. The A_x and B_x are calculated as follows:

x	Ax	Bx
1	8.49	0.00463
2	14.27	0.00772
3	11.26	0.00772

4	7.46	0.0115
5	16.98	0.00579
6	7.72	0.00579
7	9.92	0.00579
8	7.21	0.00579
9	17.00	0.00579
10	5.94	0.00579
11	11.17	0.00579
12	4.02	0.00579

Table 4

After modeling each segment, we make systems of equations to determine the equilibrium traffic flow, that is the F_x of each segment. Under equilibrium traffic flow, the amount of time a driver would take traveling all 5 routes shall be the same, so that no drivers have the incentive to change route and all drivers spend the same amount of time. So we have:

$T_8 + T_7 + T_6 + T_5 = T_{12} + T_4 + T_7 + T_6 + T_5 = T_{12} + T_{11} + T_3 + T_6 + T_5 = T_{12} + T_{11} + T_{10} + T_2 + T_5 = T_{12} + T_{11} + T_{10} + T_9 + T_1$

We test two scenarios in which the total traffic leaving San Jose for San Francisco is 5000 and 10000:

$T_8 + T_{12} = 5000$ *or* $T_8 + T_{12} = 10000$

Solving it (via *Wolfram Mathematica*) gives us the traffic flow on F_x each highway segment. We can do a similar thing to model the situation when a certain bridge is closed. In this research, we modeled the situation when highway 2 (San Mateo Bridge) or Highway 3 (Dumbarton Bridge) is closed. In those situations, we set F_2 or F_3 to be 0.

We can then determine the time each driver spends from San Jose to San Francisco under each of those situations, and compare them to determine the occurrence of Braess's Paradox. If the travel time for drivers decreased with the closure of San Mateo Bridge or Dumbarton Bridge, the Braess's Paradox occurred in the Bay Area highway network. Otherwise, the model doesn't suggest the occurrence of the paradox.

III. RESULTS

Traffic \ Highway Closure	-	2 (San Mateo)	3(Dumbarton)
5000	107.1(mins)	107.3(mins)	107.1(mins)
10000	170(mins)	170.1(mins)	170(mins)

Table 5: The effect of closing certain bridges to travel time of San Jose to San Francisco drivers

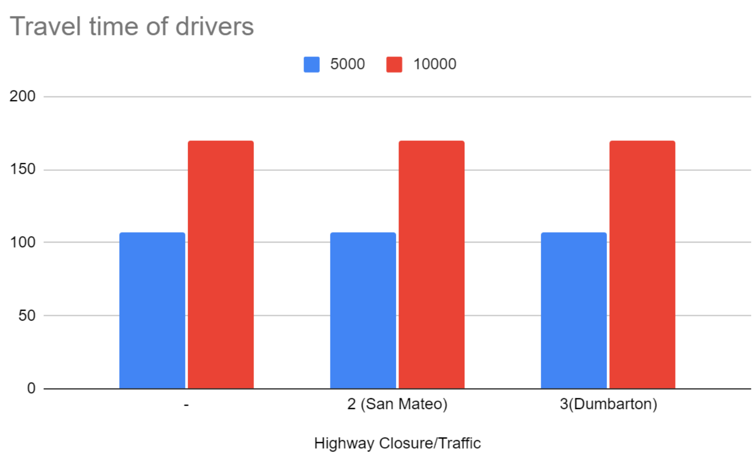

Figure 6: The effect of closing certain bridges to travel time of San Jose to San Francisco drivers

Highway Closure	Total Traffic	1	2	3	4	5	6
-	5000	2157	32	0	0	2843	2811
-	10000	4141	449	0	21	5859	5410
2 (San Mateo)	5000	2221	0(Closed)	0	2	2873	2873
2 (San Mateo)	10000	4371	0(Closed)	149	56	5629	5629
3(Dumbarton)	5000	2157	32	0 (Closed)	0	2843	2811
3(Dumbarton)	10000	4141	449	0(Closed)	21	5859	5410

Table 7.1: Traffic flow of highway segment 1- 6 under the modeled equilibrium

Highway Closure	Total Traffic	7	8	9	10	11	12
-	5000	2811	2811	2157	2189	2189	2189
-	10000	5410	5389	4141	4590	4590	4611
2 (San Mateo)	5000	2873	2871	2221	2221	2127	2129
2 (San Mateo)	10000	5480	5424	4371	4371	4520	4576
3(Dumbarton)	5000	2811	2811	2157	2189	2189	2189
3(Dumbarton)	10000	5410	5389	4141	4590	4590	4611

Table 7.2: Traffic flow of highway segment 7- 12 under the modeled equilibrium

IV. DISCUSSION

As the compared results in Table 5 and Figure 6 show, the closure of the San Mateo bridge (segment 2) or Dumbarton Bridge (segment 3) doesn't lead to many changes in the travel time of individual drivers from San Jose to San Francisco. This suggests that there are no occurrences of Braess's Paradox in this part of the Bay Area highway network under the tested traffic.

As figure 7.1 and 7.2 suggests, under this model, route 8 - 7 - 6 - 5 and route 12 - 11 - 10 - 9 - 1 via San Francisco - Oakland Bay Bridge each holds at least 40% of total traffic in all situations modeled. The San Mateo Bridge, Dumbarton Bridge and highway segment 4 (CA Route 237) in the middle had very limited traffic flowing through. Thus, the impact of these three connections on the network as a whole is very limited. In the best situation, it only holds 449 of 10,000 cars, which is less than 5%. This is completely different from the Braess's Paradox scenario in which the majority of travelers choose to travel on a "shortcut" in the middle, leaving it overwhelmingly congested.

In the future, many details of this model could be improved. First, instead of just 5000 and 10000, more situations of total traffic flow can be tested, as the occurrence of the paradox partially depends on the traffic amount. Next, the travel time model for each highway segment can be improved in many ways. For example, carpool or Express lanes could be added into consideration of the travel time model. Lastly, travelers with different smaller destinations can also be modeled. For example, travelers from Fremont to San Francisco taking routes 10 - 9 - 1 may make segments 10, 9, and 1 more congested, leaving significant changes to the overall model equilibrium.

V. REFERENCES

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