Girls' Angle Bulletin

To Foster and Nurture Girls' Interest in Mathematics

From the Founder

Much of this issue is devoted to the concept of the center of mass, a topic that provides a fascinating mixture of math and physics. This issue serves mainly as an introduction. After reading this issue, you'll be able to design mobiles, apply the technique of "mass points," understand the difference between mass and weight, and have become acquainted with vectors. As always, the Bulletin is met to instigate the study of more math, and if anything confounds you, please email us or ask the mentors at the club!

- Ken Fan, President and Founder

On the cover: A fantasy mobile by Julia Zimmerman. Mobiles, center of mass, gravity, weight, and the law of the lever are all themes in this issue.

An Interview with Karen Smith

Dr. Karen Smith is the Keeler Professor of Mathematics at the University of Michigan, Ann Arbor. She received her doctoral degree at the same university under the supervision of Melvin Hochester. She was recently a Clay scholar at the Mathematical Sciences Research Institute.

Ken: Will you please relate the childhood experiences that led you to want to become a mathematician?

Karen: I always liked math and was good at it. Although in sixth grade, I was placed in a 7th grade class (with about 10 other students) and was intimidated, sat in the back without asking questions, and did very poorly! So I actually repeated seventh grade math the next year!

Ken: You are quoted as saying, "I was lucky my high school teacher, Mr. Drinfeld, was willing to run an extra class for a few top seniors. One of the books we read was Underwood Dudley's 'Elementary Number theory.' I loved it! It definitely made me a mathematician." What did you love about number theory? Can you describe something specific that you found lovable in number theory?

I think the main qualities a mathematician needs is a willingness to work hard and not give up, an interest in and curiosity about mathematics…, a willingness to dream about what might be possible, and to accept being wrong most of the time... Also, you need to be willing to ask a lot of questions.

Karen: Number theory is very simple and elegant. That appealed to me. I thought it was neat, for example, how you can check a number is divisible by 9 just by adding its digits. For example, 1,232,406 is divisible by 9 because 1 + 2 + 3 + 2 + 4 + 0 + 6 is divisible by 9. We got to prove these kinds of facts and many others. It was also rewarding to work hard on a problem for days, and finally crack it!

Ken: I found it fascinating that after receiving your bachelor's degree from Princeton, instead of going directly to graduate school in mathematics, you first taught in a high school as a math teacher. Will you please describe your decision process that led you back to academia and into math graduate school?

Karen: I was not from a very privileged background. I had never heard of graduate school, and my parents were concerned about my being a math major, because they thought (incorrectly) that I wouldn't be able to find a job with a math degree. I had a lot of student loans and never even considered continuing my education after college. No one at Princeton offered any kind of career advice (at least at that time), so I responded to my parents' worries about my employability by getting a teaching certificate through a program Princeton offered. So at graduation, the natural step was to get a teaching job, and I did, as a high school math teacher. However, some of the other math majors told me in May that they were going off to study more math and that they *would get paid* to do so! I had had no idea that such an option was available, or at least, that I was in any way capable or eligible for such an opportunity. Not one of my professors or friends had ever suggested it. It sounded great! Someone was paying them to

study math? I was incredulous! Looking back, my cluelessness was partly my own naivety, but perhaps mainly the huge class difference I felt at Princeton which kept me isolated from many peers and events there even as everyone else assumed I would have heard of graduate school or viewed myself as eligible. In more recent years, I have also wondered whether there may have been a substantial amount of gender bias: were my professors encouraging the other students (nearly all male) to apply to grad school? Probably if someone had suggested it to me, I would have gone directly.

As it was, I accepted a teaching job at a high school in a working class area of industrial New Jersey with a lot of urban problems. I still love teaching. But that job was incredibly

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Smith, and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

Content Removed from Electronic Version

Content Removed from Electronic Version

Ken: Relatively speaking, there aren't very many women in mathematics today. Do you think there is gender bias against women in the field of mathematics?

Karen: Yes, I do. Unfortunately, I think that there is gender bias against women in nearly every profession and human activity (excepting perhaps the most important: motherhood). This does not mean that I think a woman can't be successful in math, or that I think most male mathematicians are sexist. (Most are not! And some female mathematicians are!) In fact, I think that academics is one of the better places to find refuge from the gender bias and sexism of our society: an academic career is flexible, and deans tend to be progressive; there are excellent programs to help with the thorny issues of combining parenting and career; there is little emphasis on appearance and other annoying issues women face.

The most insidious kind of sexism that women face, in my opinion, is self-imposed by the brain-washing we get from society. Many girls and young women waste an enormous amount of time and energy and money and emotional health worrying about their appearance, or beating themselves up for not being more like the stereotypes depicted in advertisements, music videos, movies, or even in popular shows like "Big Bang theory." A strong woman who wants to do math will have no problem seeking out good mentors, and collaborators; when she encounters sexism, she will find plenty of supportive people to help, provided she has the confidence to seek and accept that help. I think this especially true in academics.

My main advice for avoiding gender bias in all fields is to stay away from TV, most mainstream movies, and pop culture. Instead, read classics, study math and science, stay physically active, and learn about feminism!

Ken: What do you like to do when you're not doing mathematics?

Karen: Spend time with my kids, outdoors, mostly, or watching them play sports.

Ken: Do you have any advice for the girls at Girls' Angle?

Karen: Work hard in school, stay physically fit, find people you can help (like maybe tutoring others!). Also, find people who support *you*. Try to avoid spending a lot of time in front of the computer, unless you are doing something productive like learning to program! Try to avoid pop-culture and TV and social networking websites, or at least learn how biased and brainwashing these are so you don't fall into that trap yourself.

Ken: Thank you for this interview!

Investigate Further

In her interview, Prof. Smith pointed out that "5 points determine a conic." Recall that a conic is a hyperbola, parabola, or ellipse. One implication of this statement is that given any 4 points, no 3 of which are collinear, there should be a family of conics that pass through these 4 points. To that end, consider the 4 points (1, 0), (0, 1), (-1, 0), and (0, -1) in the Cartesian plane. Find all the conics that pass through these 4 points.

Given 5 points, no 3 of which are collinear, in the plane, can you devise a criterion that will tell you if the unique conic that passes through the 5 points is an ellipse, parabola, or hyperbola?

Can you show that every conic section in the plane can be realized as the solutions to an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

Learn by Doing

**Vectors**[1]

by Robert Donley

A **vector** can be thought of as an arrow. It has a direction and a magnitude (which can be thought of as the length of the arrow). By contrast, a **scalar** has a magnitude but no direction. Numbers, temperatures, and masses are example of scalars. Velocities, accelerations, and forces are examples of vectors. Here are some vectors all contained inside the plane of the paper:

(The foot of this one is 50 light years over to the right.)
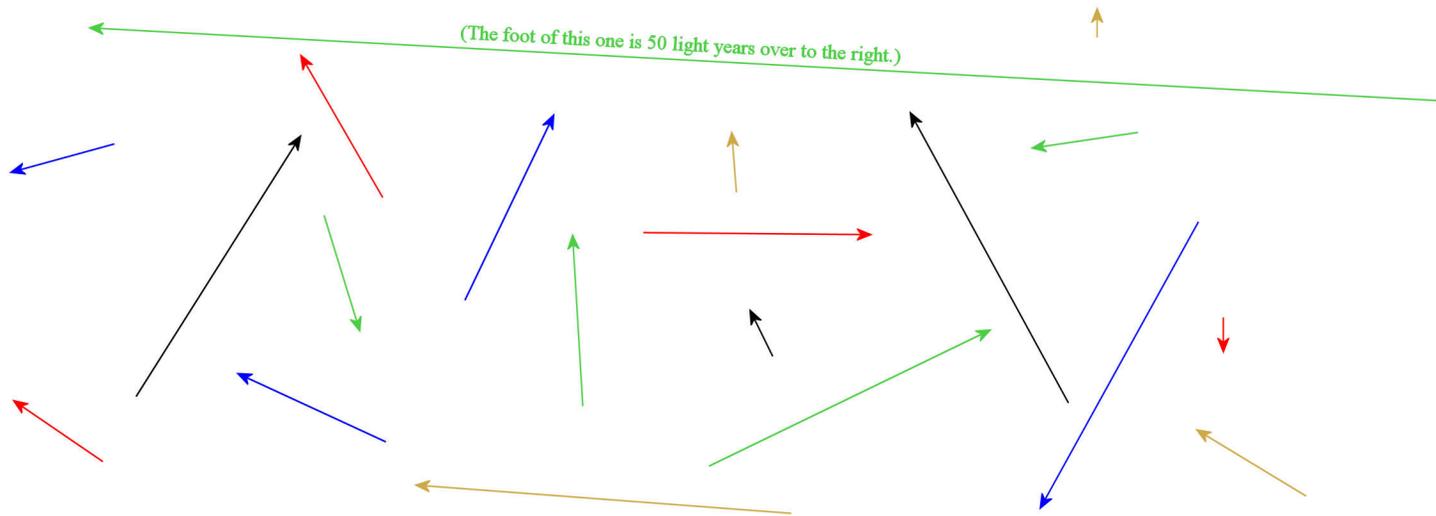
Can you imagine vectors in 3 dimensions? How about in 1 dimension? In 4 or more dimensions?

Two vectors are considered to be equal if and only if they point in the same direction and have the same magnitude. Note that vectors do not have to be drawn in the same place to be considered equal; they just have to point in the same direction and be of the same length.

The tip of the arrow is called the **head** of the vector and its other end is called its **foot**. The magnitude of the vector v is denoted by $|v|$.
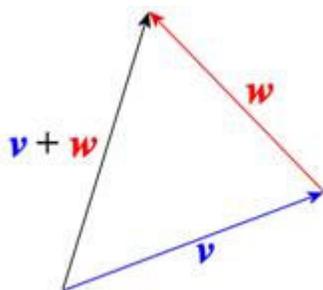
Vector Addition

Let v and w be two vectors. The sum of the two vectors, which we denote by $v + w$, is obtained as follows: place the foot of w on the head of v. Now draw an arrow from the foot of v to the head of w. This arrow represents the vector sum $v + w$, and is illustrated in the diagram at left.

Problem 1. Draw two vectors a and b. Sketch $a + b$. Convince yourself that $a + b$ and $b + a$ are equal. Consider a third vector c. Convince yourself that you get the same sum regardless of the order that you add together the 3 vectors a, b, and c.

The vector with no length at all is called the **zero vector**. Since the zero vector doesn't have a length, it can't have a direction either. It is the only vector that doesn't have a direction. We'll denote the zero vector by **0**.

---

[1] This content was supported in part by a grant from MathWorks.

Problem 2. For any vector v, show that $v + 0 = 0 + v = v$.

Problem 3. Given a vector v, describe the vector w that has the property that $v + w = 0$.

Problem 4. Make a sketch that illustrates the associative property of addition, namely

$$a + (b + c) = (a + b) + c$$

for any vectors a, b, and c.

Scalar Multiplication

You can lengthen and shorten a vector to get other vectors that point in the same direction but have a different magnitude. Stretching and shrinking a vector without changing its direction is called **scalar multiplication**. That is, let r be a positive real number and let v be a vector. The scalar multiple of v by r, denoted by rv, is the vector that points in the same direction as v, but has magnitude r times that of v.

It's convenient to extend scalar multiplication to all real numbers by defining $(-r)v$ to be the vector with the same length as rv, but pointing in the opposite direction, and defining $0v$ to be the zero vector 0.

Problem 5. Sketch a vector v. For each value of $r = -2, -1, 0, 1,$ and 2, sketch rv.

Problem 6. In the setup of Problem 3, show that $w = (-1)v$. (Note: $(-1)v$ is usually written simply as $-v$.)

When we write $v - w$, we mean $v + (-w)$.

Problem 7. Verify that scalar multiplication distributes over vector addition. That is, convince yourself that $r(v + w) = rv + rw$.

Vectors in R^2 and R^3

Let's fix a plane and temporarily restrict vectors to this plane. Sometimes, it is important to be able to specify vectors precisely. One way to do this is to introduce coordinates. So let's imagine that our plane is a Cartesian plane with a horizontal x-axis and a vertical y-axis.

Let v be a vector drawn in this Cartesian plane. We can specify v precisely with 2 numbers a and b. To get a, we subtract the x-coordinate of the foot of v from the x-coordinate of the head of v. To get b, we subtract the y-coordinate of the foot of v from the y-coordinate of the head of v. We write these two numbers as an ordered pair (a, b).
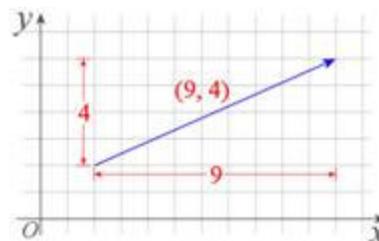
The notation "(a, b)" is exactly the same as the notation used to specify points in the Cartesian plane by their coordinates, but don't get confused! When used to specify a vector, the two numbers tell how to get from the foot to the head of the vector, irrespective of where the vector may or may not be drawn. But when "(a, b)" is used to specify a point with Cartesian coordinates $x = a$ and $y = b$, the numbers give the location of the point with respect to the origin of coordinates.
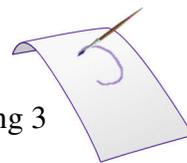
For vectors in 3 dimensions, we introduce a third axis and specify vectors by providing 3 numbers in an analogous way. For each new dimension, we add another axis. It takes n numbers to specify an n-dimensional vector.

Problem 8. What other ways can you think of to specify vectors in the plane precisely?

Problem 9. Draw vectors that represent (0, 3), (5, 1), (-3, -2), and (-1, 1) in a Cartesian plane.

Let $v = (x_1, y_1)$ and $w = (x_2, y_2)$. Vector addition is

$$v + w = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2).$$

Scalar multiplication is
$$rv = r(x_1, y_1) = (rx_1, ry_1),$$

where r is a real number. Also, $|v| = \sqrt{x_1^2 + y_1^2}$.

Problem 10. Show that $v = w$ if and only if $x_1 = x_2$ and $y_1 = y_2$. Because of this fact, an equation involving vectors implies as many scalar equations as the dimension of the vector.

In Problems 11-13, let $u = (3, 4)$, $v = (-2, 1)$, and $w = (-3, -1)$.

Problem 11. Sketch all three vectors. Find $u + v$ and sketch it. Find $u + v + w$ and sketch it.

Problem 12. For each of the values $r = -2, -1, 0, 1$, and 2, compute ru and sketch it.

Problem 13. Compute the magnitude of u. For a real number r, show that $|ru| = |r||u|$, where $|r|$ is the absolute value of r.

The Dot Product

Let $v = (x_1, y_1, z_1)$ and $w = (x_2, y_2, z_2)$ be 3-dimensional vectors. The **dot product** of v and w is denoted by $v \cdot w$ and is equal to $x_1 x_2 + y_1 y_2 + z_1 z_2$. Note that the dot product of 2 vectors is a *scalar*, not a vector.

Problem 14. Show that $v \cdot v = |v|^2$.

Problem 15. Use the law of cosines to show that $v \cdot w = |v||w| \cos \theta$, where θ is equal to the angle between v and w. (If you place v and w so that their feet coincide, θ is the angle measured from v to w. Note that it does not matter if you measure clockwise or counterclockwise because $\cos \theta = \cos -\theta$.) Show that nonzero vectors v and w are perpendicular if and only if $v \cdot w = 0$.

Problem 16. Generalize the dot product to higher dimensions in such a way that the equation in Problem 15 remains valid. If $v = (a_1, a_2, a_3, \ldots, a_n)$ and $w = (b_1, b_2, b_3, \ldots, b_n)$, what is $v \cdot w$ in terms of the a_k and b_k? (Suggestion: Start with 1-dimensional vectors, then do 2-dimensional vectors, and so on.)

Problem 17. Show that $v \cdot w = w \cdot v$ and $v \cdot (w + u) = v \cdot w + v \cdot u$.

10

The Cross Product

In contrast to the dot product, the cross product takes two 3D vectors and returns a vector and *only applies* to 3D vectors. (If we think of the plane as a slice in 3D space, then we can also apply the cross product to the vectors in the plane by thinking of them as belonging to the 3D space, but there is no cross product for vectors in 4 or higher dimensions.)

First, a word about why the cross product is limited to 3 dimensional vectors. Suppose you want to create an operation other than vector addition that takes two vectors and returns a vector. Not only would we have to specify the magnitude of the result, we would also have to specify its direction. If you take two generic vectors in 3D, they will determine a unique direction (up to sign): the direction perpendicular to both vectors. In 4 dimensions and higher, this is no longer true. For example, in 4D, if you fix 2 generic vectors, there will be a whole circle of directions that are perpendicular to both of them. Without additional structure, there's no natural way to pick out 1 direction from among a whole circle of directions.

So the cross product of two vectors is going to point, in general, in the direction perpendicular to both of them. But, which way should it point? The choice is a matter of convention. To explain the convention that has been adopted, we have to be careful about which way things point, starting with the coordinate axes.

By convention, the axes should be set up according to the **right-hand rule**: with your right hand, point your index finger in the direction of the positive *x*-axis, your middle finger in the direction of the positive *y*-axis, and extend your thumb; your thumb should point in the direction of the positive *z*-axis.

Now we're ready to define the cross product $v \times w$ of two 3-dimensional vectors v and w. The magnitude of the cross product is equal to $|v||w| \sin \theta$, where $0 \le \theta \le 180$ is the angle between v and w. Notice that the magnitude is 0 if either vector has magnitude 0. It is also 0 if θ is 0 or 180 degrees (which means that v and w point in the same or in opposite directions to each other). Therefore, we only have to specify the direction of the cross product in the generic case when neither v nor w is **0** and they do not point in the same or in opposite directions. In this case, v and w, as discussed above, define a unique direction up to sign, namely the direction that is perpendicular to both vectors. To figure out which of the 2 perpendicular directions to use, we use the right-hand rule. First, position v and w so that their feet are in the same place. If you point your index finger in the direction of v (the first factor) and your middle finger in the direction of w (the second factor), then your thumb will tell you which way to point $v \times w$.

Problem 18. Show that $(1, 0, 0) \times (0, 1, 0) = (0, 0, 1)$. Show that $(1, 0, 0) \times (0, 0, 1) = (0, -1, 0)$.

Problem 19. Let $v = (x_1, y_1, z_1)$ and $w = (x_2, y_2, z_2)$. Let $p = (y_1z_2 - y_2z_1, x_2z_1 - x_1z_2, x_1y_2 - x_2y_1)$. Show that $|p| = |v \times w|$. Show that p is perpendicular to both v and w. (Hint: Use the dot product.) Convince yourself that p points in the direction that is consistent with the right-hand rule. Conclude that $v \times w = p$.

Problem 20. Let u, v, and w be 3-dimensional vectors. Show that $u \times v = -v \times u$. For any real number r, show that $(ru) \times v = r(u \times v)$. Show that $u \times (v + w) = u \times v + u \times w$.

Problem 21. Show that $|v \times w|$ is equal to the area of the parallelogram spanned by v and w.

For more details on vectors, consult any book on vector analysis or linear algebra, such as the free Google ebook *Vector Analysis* by Gibbs and Wilson.

Mobile Magic: The Law of the Lever

by Addie Summer
edited by Jennifer Silva

Have you ever played on a seesaw with a partner who was much heavier than you? If you have, you may have noticed that it's not much fun if you both sit the same distance away from the pivot. Your heavier friend will sink to the ground and the seesaw will basically be stuck there. But there's an easy remedy. All your friend has to do is scoot in toward the pivot. At some point, the seesaw will balance.

The same principle applies when you're trying to keep a door shut from the inside while someone is trying to enter from the outside. The farther away from the hinge that you push, the easier it will be for you to keep the person out.
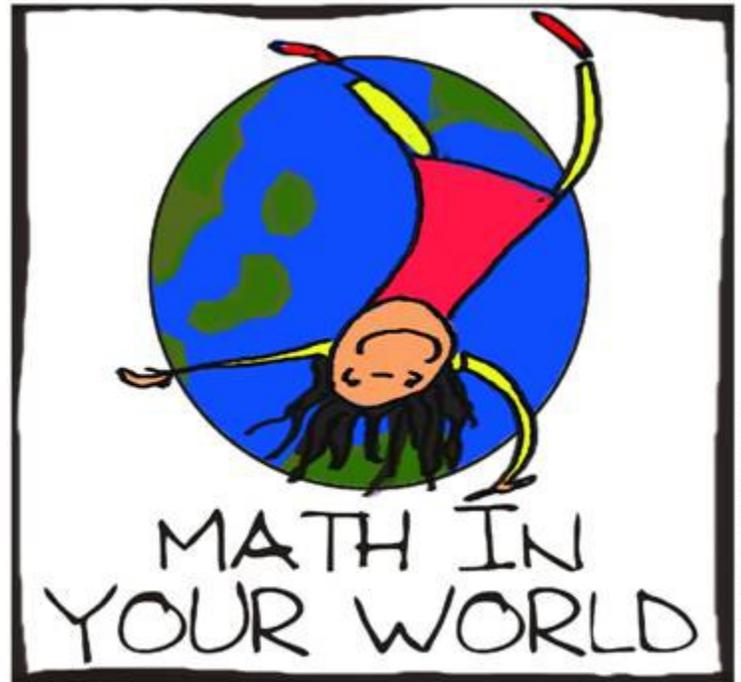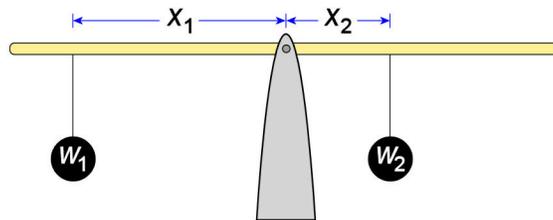
These are all examples of the **law of the lever**. Look at the figure above. Each weight dangles from the horizontal beam at a different distance from the pivot. Let's think of the horizontal beam as a number line, with the pivot located at 0 (so x_1 is negative). If we multiply a weight by its coordinate on the number line, we get a quantity called the **torque**. The law of the lever says that the horizontal beam will balance exactly when the sum of all torques is equal to 0. For the example above, this means that the beam balances when $w_1x_1 + w_2x_2 = 0$.
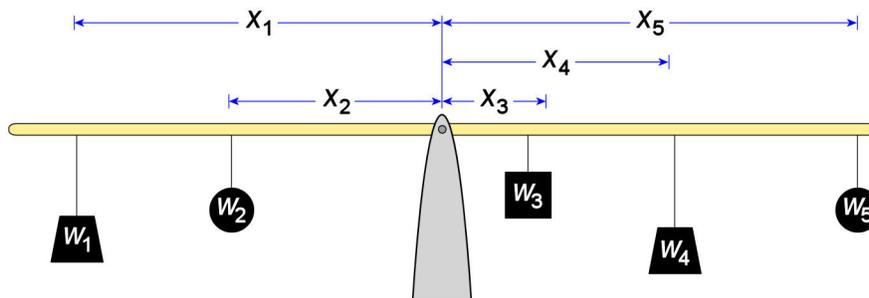
If there are several weights hanging from our horizontal beam, we simply add up the torques for all of the weights. When the total torque is negative, the left side of the beam will drop, and the right side will drop when the total torque is positive. If the total torque is 0, then the beam will balance. The beam above balances when $w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 = 0$.

The law of the lever is not a mathematical fact. It is a physical law discovered through experiment. The law can also be derived from Newton's laws of motion; if you are curious about this, we recommend that you consult a physics book such as *The Feynman Lectures on Physics*, particularly Volume 1, Chapter 4. We will assume the law of the lever and explore an application.

Design a Mobile Let's use the law of the lever to figure out how to design the mobile shown at right. The mobile consists of 2 sticks (the horizontals), 5 pieces of thread (the verticals), and 3 colored weights. Although the sticks themselves have some weight and will contribute to the total torque, we will assume that their contribution to the torque is negligible and can be ignored. The numbers indicate distances with respect to some unspecified unit of length.
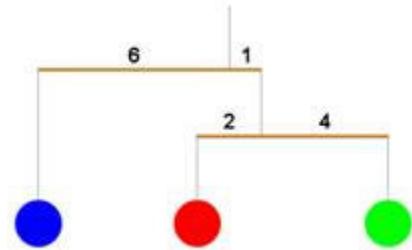
Our problem: how much should each circle weigh so that the mobile will balance?

To figure this out, let x, y, and z be the weights of the blue, red, and green weights, respectively. If everything is balanced as indicated, the thread hanging from the right tip of the top stick will feel the same as if it were holding a single weight equal to the sum of the weights of the red and green circles. By applying the law of the lever to the top stick, we require that $6x = 1(y + z)$; by applying the law of the lever to the bottom stick, we know that $2y = 4z$, or $y = 2z$. If we substitute $2z$ for y in $6x = y + z$, we find that $6x = 3z$, or $2x = z$. Hence, $y = 2z = 4x$. We can now express all of the weights in terms of a single number x: the blue weight weighs x pounds, the red one weighs $4x$ pounds, and the green one weighs $2x$ pounds. We are free to choose x to be anything we please (as long as it is significantly heavier than the sticks). The math tells us that even though the blue weight is but one-sixth of the red and green weights, it will counterbalance them.

Seeing Is Believing The photograph above shows an actual mobile, based on our specified design, made out of two straws, thread, 7 quarters, and tape. Note that I hung each straw from its midpoint. By doing so, there is excess straw, but I don't have to worry about torque from the weight of the straw (even if it isn't negligible) because the straws themselves are balanced.

Another mobile design is shown at right. The circular weights weigh A, B, C, D, E, and F pounds, as indicated. Assuming that the weights of the sticks and thread are negligible, express weights B, C, D, E, and F in terms of A.

13
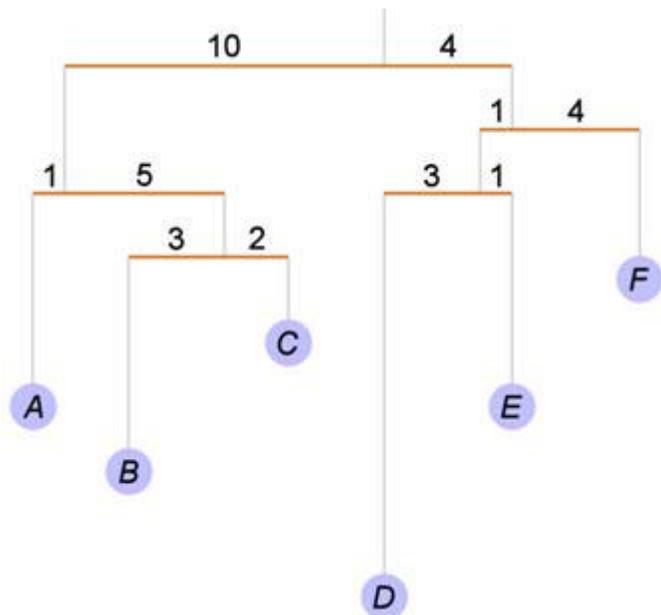
More on Torque We have been computing torque under very special circumstances. In every case, the force of the weight pulled straight down from a horizontal stick. Also, our problems involved levers and mobiles that fit inside of a flat plane. If you create more complex mobiles, you will need to understand how to compute the torques around each pivot in more general settings.
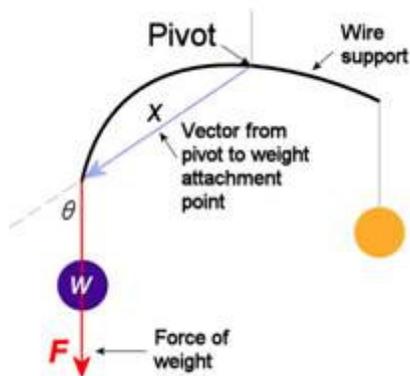
For example, perhaps you wish to design a mobile where objects hang from curved wires instead of straight sticks, like the situation depicted at left. In this more general setting, it is useful to draw in two vectors (see *Learn by Doing* on page 8). One of these vectors extends from the pivot point to the place where the weight is attached; we'll call this the "position" vector x. The other vector will represent the force of the weight, which in our case points straight down; we'll call this the "force" vector F. The length of the force vector equals the weight of the hanging object.

Notice that the position vector and the force vector are no longer perpendicular, unlike in all of our previous examples. Torque is a measure of rotary force. Some of the weight of the object does tend to rotate the object about the pivot, but some of the weight pulls directly away from the pivot. We can see just how much of the weight rotates and how much pulls away from the pivot by breaking the force vector into a sum of two pieces, one perpendicular to the position vector and one pointing in the same direction as the position vector (see the diagram below right). Since torque measures rotary force, we might imagine that the component of the force that points in the same direction as the position vector does not contribute anything to the torque; in fact, this is the case. To compute the torque of the weight about the pivot, we multiply the length of the position vector by the magnitude of the component of the force perpendicular to the position vector. We also have to establish a convention for the sign of the torque. For example, one might use negative torques for torques that would cause counterclockwise rotation about the pivot, and positive torques for torques that would cause clockwise rotation. With this convention, the torque caused by the purple weight about the pivot is equal to $-|x||F|\sin\theta$.
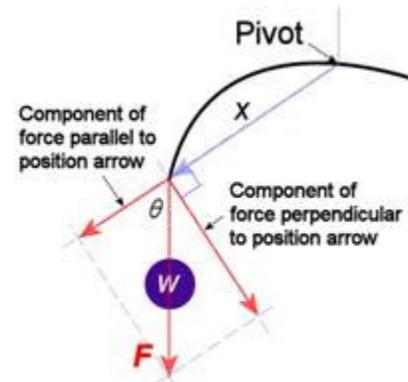
Torques in 3D The above example was planar in the sense that the mobile could still fit inside a plane. What do we do if our mobile is fully 3-dimensional? Perhaps there is a tripod at the end of a thread, instead of a stick. In this case, we really have to use torque in its full generality.

The general formula for torque is best expressed using vectors. In fact, torque itself is really a vector. If r is the position vector pointing from the pivot point to the point where a force represented by the vector F is applied, then the torque produced by F about the pivot point is given by $r \times F$, where $\times$ denotes the vector cross product (see *Learn by Doing* on page 8). In fully planar situations, the vectors representing torque will always point in a direction perpendicular to the plane. That's why for planar mobiles we could model torque with positive and negative numbers; we used negative torques to represent vectors coming straight out of the page toward you, and positive torques for vectors pointing into the page directly away from you.

Take It To Your World

Design and build a fantastic mobile. Send us a picture!

Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna takes a break from products of consecutive integers and finds all solutions to $x^x = (0.5)^{0.5}$.

> Ken mentioned to me that 1/2 to the 1/2 can be expressed as x to the x for another value of x other than 1/2. I decided to take a break from products of consecutive numbers and investigate.

> Since the base of the exponent is 1/2, I'll express x as a power of 1/2 too. I can do that without loss of generality.

> The upshot is that 1/2 to the 1/2 is also equal to 1/4 to the 1/4...that's amusing!

> I wonder what other examples of this there are. What numbers of the form x to the x can be written as y to the y, with y different from x. I'll just worry about nonnegative, real x.

> After calculating the derivative, I realized that this part of the graph should start out tangent to the vertical axis.

$$x^x = \left(\tfrac{1}{2}\right)^{1/2} \quad \Rightarrow \quad x \log_{1/2} x = \tfrac{1}{2}$$

$$x = \left(\tfrac{1}{2}\right)^n \qquad \left(\left(\tfrac{1}{2}\right)^n\right)^{\left(\tfrac{1}{2}\right)^n} = \left(\tfrac{1}{2}\right)^{n\left(\tfrac{1}{2}\right)^n} = \left(\tfrac{1}{2}\right)^{1/2}$$

$$n\left(\tfrac{1}{2}\right)^n = \tfrac{1}{2}$$

$$2n = 2^n$$

$$n = 1 \text{ or } 2$$

$$\boxed{\left(\tfrac{1}{4}\right)^{1/4} = \left(\tfrac{1}{2}\right)^{1/2}}$$

$$x^x = y^y \ ?$$

> Taking the log of both sides doesn't seem to help.

> I get down to this equation. Hmm. Well, I know 1 is a solution, since $x = 1/2$ is a solution. And, by inspection, 2 is a solution. I'll draw a graph of the left and right sides...

> ...from the graph, I'm pretty certain n equals 1 or 2 are the only solutions...I think if I had to prove this rigorously, I could, so I'll move on.

> I'll start by graphing x to the x.

> Actually, what is 0 to the 0? It's not really defined. I guess I'll see if the limit of x to the x, as x goes to 0 from above exists.

> I can use L'hopital's rule.

> So the graph starts at (0, 1), dips down and must rise up to (1, 1), and then blasts off. I wonder where the minimum is located.

$$\lim_{x \to 0} x^x = \lim_{x \to 0} e^{x \log x}$$

$$\lim_{x \to 0} x \log x = \lim_{x \to 0} \frac{\log x}{\tfrac{1}{x}}$$

$$= \lim_{x \to 0} \frac{\tfrac{1}{x}}{-\tfrac{1}{x^2}}$$

$$= \lim_{x \to 0} -x = 0$$

$$\Rightarrow \lim_{x \to 0} x^x = 1.$$

$$0 = \frac{d}{dx} x^x = \frac{d}{dx} e^{x \log x} = e^{x \log x}\left(\frac{d}{dx} x \log x\right)$$

$$= e^{x \log x}(\log x + 1)$$

$$\log x = -1$$

$$x = 1/e$$

> Solving for where the derivative of x to the x with respect to x is 0 works out nicely...how about that... $x = 1/e$ is where the unique minimum is. That means that for all values of x between 0 and 1, inclusive, other than $1/e$, there will be a y, different from x, with x to the x equal to y to the y.

Key:

(Anna's thoughts)

[Anna's afterthoughts]

[*Editor's comments*]

I'll summarize what I've learned so far.

If $\left(\frac{1}{e}\right)^{1/e} < n \leq 1$ then there are 2 solutions to $x^x = n$.

I'll try the same approach that worked for $x = 1/2$. I'll write y as a power of x.

$$y^y = x^x$$

Say $y = x^a$.

$$(x^a)^{x^a} = x^{ax^a} = x^x$$

$$ax^a = x$$

Since I'm restricting myself to positive real values, and since I know both x and y are less than or equal to 1, I can assume that a is nonnegative. I'll make note of this explicitly.

$a \geq 0$

I can see that $a = 1$ is a solution, as it should be, but I can't see what the other solution is.

$$a = x^{1-a}$$

Hm. I think I'll try the same idea, except I'll express x as a power of a.

Say $x = a^b$

$$a = a^{b(1-a)}$$
$$1 = b(1-a)$$
$$b = \frac{1}{1-a}$$

That does enable me to go further. It lets me parametrize solutions to x to the x equals y to the y.

$$x = a^{\frac{1}{1-a}} \qquad y = \left(a^{\frac{1}{1-a}}\right)^a = a^{\frac{a}{1-a}}$$

I get this funky identity.

$$\boxed{\left(a^{\frac{1}{1-a}}\right)^{\left(a^{\frac{1}{1-a}}\right)} = \left(a^{\frac{a}{1-a}}\right)^{\left(a^{\frac{a}{1-a}}\right)}}$$

Just for kicks, I'm going to work out a few examples.

Ex. $a = \frac{1}{2}$ $\qquad a^{\frac{1}{1-a}} = \frac{1}{2}^{\frac{1}{1-\frac{1}{2}}} = \frac{1}{4} \qquad \Rightarrow \left(\frac{1}{2}\right)^{1/2} = \left(\frac{1}{4}\right)^{1/4}$

$a^{\frac{a}{1-a}} = \left(\frac{1}{4}\right)^{1/2} = \frac{1}{2}$

When $a = 1/2$, I get the case I just did: 1/2 to the 1/2 equals 1/4 to the 1/4.

$a = \frac{1}{3}$ $\qquad a^{\frac{1}{1-a}} = \left(\frac{1}{3}\right)^{\frac{1}{2/3}} = \sqrt{\frac{1}{27}} \qquad \Rightarrow \left(\sqrt{\frac{1}{3}}\right)^{\sqrt{\frac{1}{3}}} = \left(\sqrt{\frac{1}{27}}\right)^{\sqrt{\frac{1}{27}}}$

$a^{\frac{a}{1-a}} = \left(\sqrt{\frac{1}{27}}\right)^{1/3} = \sqrt{\frac{1}{3}}$

When $a = 1/3$, I get this strange identity.

$a = 0$ $\qquad a^{\frac{1}{1-a}} = 0^1 = 0$

$a^{\frac{a}{1-a}} = 1$

When $a = 0$, I get the case 0 to the 0 equals 1 to 1.

$$a^{\frac{1}{1-a}} \neq a^{\frac{a}{1-a}} \quad \text{unless } a = 1.$$

$$\Rightarrow \quad \lim_{a \to 1} a^{\frac{1}{1-a}} = \lim_{a \to 1} a^{\frac{a}{1-a}} = \frac{1}{e}.$$

Hey! Wait a sec... When you raise something to the a power, it always changes unless $a = 1$...that must mean that the limit, as a tends to 1, of both expressions must be $1/e$... is that easy to check directly?

$$a^{\frac{1}{1-a}} = e^{\frac{\log a}{1-a}}$$

$$\lim_{a \to 1} \frac{\log a}{1-a} = \lim_{a \to 1} \frac{1/a}{-1} = -1 \qquad \checkmark$$

I guess it is...L'hospital's rule works nicely again. Still, it was neat to deduce a limit without having to do a direct computation!

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

ABB 2.27.14

Newton's Law of Gravitation

by Lightning Factorial | edited by Jennifer Silva

Content Removed from Electronic Version

Content Removed from Electronic Version

Center of Mass

by Ken Fan | edited by Jennifer Silva

In *Math In Your World* (page 12), several weights were placed on a number line and we answered the following question: How much must the weights weigh, relative to each other, in order for the number line to balance about the origin? Let's turn the question around and instead ask this: From what point should a stick bearing weights be hung so that it balances? That is, suppose we have a number line and hang weights on it. We hang a weight w_1 from the point x_1, a weight w_2 from the point x_2, and so on, for n weights, so that weight w_k is hanging from point x_k, where $1 \leq k \leq n$. We think of the number line as a rigid stick with no weight at all. Where on the number line should we place a pivot so that the number line balances?

Since we don't know where the pivot should be yet, we'll just give it a name. Let p denote the point where the number line will balance. From the law of the lever, if the number line balances, then

$$(x_1 - p)w_1 + (x_2 - p)w_2 + (x_3 - p)w_3 + \ldots + (x_n - p)w_n = 0.$$

Solve this equation for p to find

$$p = \frac{x_1 w_1 + x_2 w_2 + x_3 w_3 + \ldots + x_n w_n}{w_1 + w_2 + w_3 + \ldots + w_n}. \tag{1}$$

This expression is an example of a **weighted average**. (That's an unintentional pun!)

Center of *Mass* or Center of *Weight*?

You might have noticed that we obtained an expression involving weights, and yet the title of this article is "Center of *Mass*." Mass is a measure of an object's resistance to force. Weight is the force on an object due to gravity. Since mass and weight are different concepts, we need to be explicit about the difference between the center of mass and the center of weight (which is more commonly called the **center of gravity**).

If our weights are part of an actual object that isn't too big, then the gravitational forces on the weights, to a high degree of accuracy, will equal the mass of each weight multiplied by the gravitational constant g; this is explained at the end of *Newton's Law of Gravitation* on page 17. That is, if m_k is the mass of the kth weight, located at x_k on the number line, then $w_k = gm_k$. (Recall that $g = GM_e/R_e^2$, where G is the universal gravitational constant, M_e is the mass of the Earth, and R_e is the Earth's radius.) Furthermore, to a high degree of accuracy, the forces due to gravity will all point in the same direction because the center of the Earth is so distant. This means that we can substitute gm_k for w_k in our formula for p to obtain

$$p = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \ldots + x_n m_n}{m_1 + m_2 + m_3 + \ldots + m_n}. \tag{2}$$

This expression is the correct formula for the center of mass. If the varying distances of the weights from the center of the Earth becomes significant or the gravitational forces begin to noticeably point in different directions, then the center of mass and the center of gravity will generally not be the same.

I'll focus on the center of *mass* for the rest of this article.

Center of Mass in Higher Dimensions

If the masses are not collinear, we can compute the center of mass by treating each coordinate separately. A shorthand way of expressing this is to use vectors. Designate a special point in space as the origin O. The vector pointing from O to the kth mass will be denoted r_k and is called its **position vector**. The formula for the center of mass p becomes

$$p = \frac{r_1 m_1 + r_2 m_2 + r_3 m_3 + \ldots + r_n m_n}{m_1 + m_2 + m_3 + \ldots + m_n}.$$

In general, p will be located within the convex hull of the points $r_1, r_2, r_3, \ldots, r_n$.

We could define the center of mass as the point where a system of objects would balance in a uniform gravitational field; however, I urge you to consult Chapter 18 of Volume 1 of *The Feynman Lectures on Physics* for a more widely applicable physical definition.

Computing the Center of Mass

A very nice property of the center of mass is that it can be computed piece by piece.

Suppose I split the objects into two sets. One set consists of the first j masses, where $j < n$, and the other set consists of the remaining $n - j$ masses. To compute the center of mass of all n masses, one can begin by computing the center of mass of the two sets separately. Then compute the center of mass of two point masses, one located at the center of mass of the first set and with mass equal to the total of all masses in that set, and the other located at the center of mass of the second set and with mass equal to the total of all masses in the second set.

Here's the proof: let p_1 be the center of mass of the masses in the first set and p_2 be the center of mass of the masses in the second set. By applying formula 2 to each set, we find that

$$p_1 = \frac{r_1 m_1 + r_2 m_2 + r_3 m_3 + \ldots + r_j m_j}{m_1 + m_2 + m_3 + \ldots + m_j} \quad \text{and} \quad p_2 = \frac{r_{j+1} m_{j+1} + r_{j+2} m_{j+2} + r_{j+3} m_{j+3} + \ldots + r_n m_n}{m_{j+1} + m_{j+2} + m_{j+3} + \ldots + m_n}.$$

We now find the center of mass of two point masses, one of mass $M_1 = m_1 + m_2 + m_3 + \ldots + m_j$ and located at p_1, and the other of mass $M_2 = m_{j+1} + m_{j+2} + m_{j+3} + \ldots + m_n$ and located at p_2:

$$\frac{p_1 M_1 + p_2 M_2}{M_1 + M_2} = \frac{\dfrac{r_1 m_1 + r_2 m_2 + r_3 m_3 + \ldots + r_j m_j}{m_1 + m_2 + m_3 + \ldots + m_j} M_1 + \dfrac{r_{j+1} m_{j+1} + r_{j+2} m_{j+2} + r_{j+3} m_{j+3} + \ldots + r_n m_n}{m_{j+1} + m_{j+2} + m_{j+3} + \ldots + m_n} M_2}{M_1 + M_2}.$$

After simplifying the expression on the right, it is seen to be the same as the formula 2.
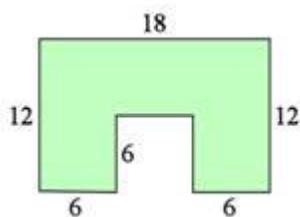
This tells us that if we want to find the center of mass of a complicated object with lots of different parts, we can do it in stages. First, we can compute the center of mass for each part. Then, we can pretend that we have point masses at the center of mass of each part with each mass equal to the total mass of each part. We can then compute the center of mass of these point masses to find the center of mass of the original object.

For example, let's compute the location of the center of mass of the 2-dimensional region shown above. It has a uniform mass density of 1 unit of mass for every unit of area. Also, its

sides meet in right angles. To find the center of mass, we can split the object into pieces whose centers of mass are easier to find, such as rectangles. By symmetry, the center of mass of a rectangle with uniform mass density must be located at its center, where the diagonals intersect.

The figure at right shows the piece split up into 2 rectangles and a square. The center of mass of each of these 3 rectangles is indicated with a red dot, and the associated number is the mass of that rectangle (which is equal to its area since the mass density is 1).
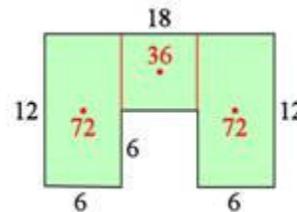
We have reduced the problem to finding the center of mass of the 3 red point masses. The two masses that are 72 units will have a center of mass located halfway between them, which is the midpoint of the lower edge of the sandwiched 6 by 6 square.

Thus, we simply need to find the center of mass of the 36-unit point mass and a 144-unit point mass located at the midpoint of the bottom edge of the sandwiched square. Using formula 2, we find that the center of mass of these 2 point masses (and, hence, of the entire object), is located
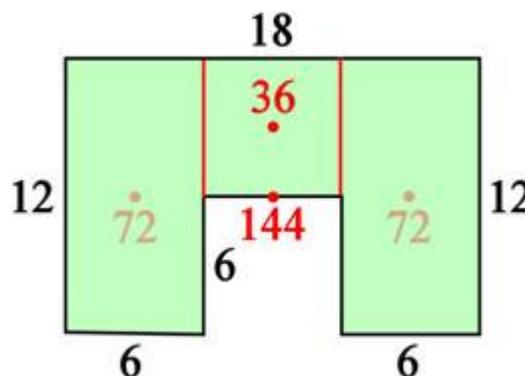
$$\frac{144(0)+36(3)}{144+36}$$

units directly above the 144-unit point mass, which simplifies to 3/5 of a unit.

The notion of the center of mass comes from physics. However, this fact about computing the center of mass piece by piece is a mathematical consequence of formula 2. That is, we could take formula 2 as the *definition* of the center of mass, irrespective of any physical motivation, and view our piece-by-piece computational fact as a mathematical property or theorem about the center of mass. By abstracting out the physics, we can see that we are discovering properties that apply to any situation that involves weighted averages.

Because we can take this mathematical point of view, we can use the center of mass concept to prove mathematical theorems. Let's do that next!
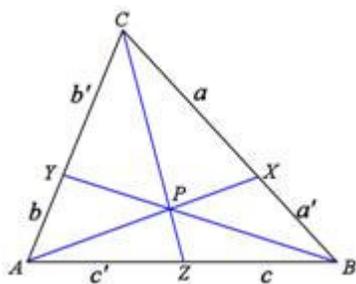
Ceva's Theorem

A **cevian** is a line connecting a vertex of a triangle to a point on the opposite side. The figure at right shows 3 cevians that intersect in a point P. Ceva's theorem says that when 3 cevians intersect in a point, then $\dfrac{a'}{a}\dfrac{b'}{b}\dfrac{c'}{c}=1$,

where a, b, c, a', b', and c' are the lengths as indicated in the figure.

We're going to prove this theorem using the center of mass concept. The idea is to place point masses at the vertices of the triangle in such a way that their center of mass is P. For reasons that will become clear in a moment, let's start by placing a point mass of mass $b'c$ at vertex A, as indicated in the figure at right.
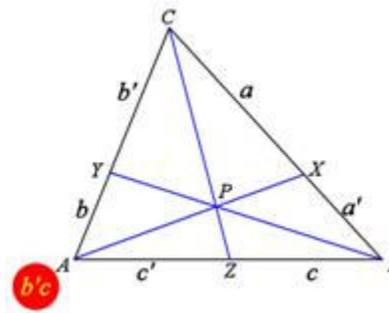
If we put a point mass of mass *b'c'* at *B*, then we know by the law of the lever that the center of mass of the point masses at vertices *A* and *B* will be located at point *Z*. Similarly, if we place a point mass of mass *bc* at vertex *C*, then the center of mass of the point masses at vertices *A* and *C* will be located at point *Y*. Let's place these masses on our triangle. (Now we can explain that the reason why we started with a mass of *b'c* at vertex *A* is merely to ensure that the expressions for the desired masses at vertices *B* and *C* have no denominators.)
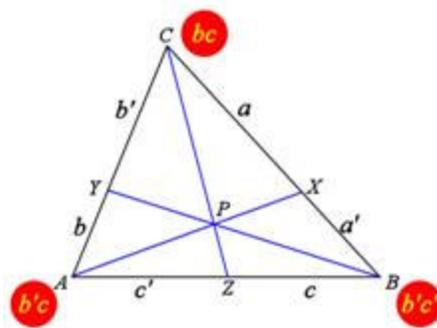
I claim that the center of mass of all 3 point masses is located at point *P*. To see this, we repeatedly use the "piecemeal" property of the center of mass. We can compute the center of mass of all 3 point masses by first computing the center of mass of the point masses at vertices *A* and *B* and pretending that their total mass is located there, then computing the center of mass of this pretend point mass and the point mass at vertex *C*. By construction, the center of mass of the point masses at vertices *A* and *B* is located at point *Z*. Since the center of mass of point masses located at *C* and *Z* will be on cevian $\overline{CZ}$, we learn that the center of mass of all 3 point masses must lie along cevian $\overline{CZ}$. If, on the other hand, we compute the center of mass by first computing the center of mass of the point masses at vertices *A* and *C*, we learn by the same reasoning that the center of mass of all 3 masses must lie along cevian $\overline{BY}$. Therefore, the center of mass of all 3 point masses must be on the intersection of $\overline{CZ}$ and $\overline{BY}$, which is *P*!

But we can also compute the center of mass of all 3 point masses by first computing the center of mass of the point masses at vertices *B* and *C*. Suppose this center of mass (of the point masses at vertices *B* and *C*) is located at the point *W*. We already know that the center of mass of all 3 point masses is located at *P*. Therefore, *P* must lie along the line segment from *A* to *W*, *which means that W must coincide with X!* By the law of the lever, in order for *W* = *X*, it must be that *b'c'a'* = *bca*; that is Ceva's theorem.

Can you prove the converse of Ceva's theorem by balancing point masses?

Theorem of Menelaus

In triangle *ABC*, draw a cevian from vertex *C*. The cevian will split side *AB* into two pieces. In fact, we can specify which cevian from vertex *C* by specifying the ratio *c/c'*. Now draw another cevian from vertex *A*. We can specify this cevian by specifying the ratio *a/a'*. What is the ratio of lengths *x/y*?

As *a/a'* tends to infinity, *x/y* approaches *c/c'*. As *a/a'* tends to 0, *x/y* also tends to 0. How can we figure out exactly what *x/y* is in terms of *a*, *a'*, *c*, and *c'*? We place point masses at *A*, *B*, and *C* so that their center of mass is located at the intersection of the two cevians. We'll leave the details to you. The upshot is that
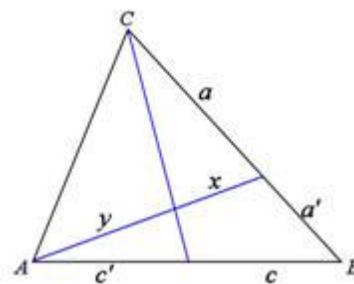
$$\frac{x}{y} = \left(\frac{a}{a+a'}\right)\frac{c}{c'}.$$

In other words, *x/y* is equal to *c/c'*, adjusted by the fraction that tells how far along side *CB* the cevian from vertex *A* intersects it. This result is essentially equivalent to Menelaus' theorem.

COACH BARB'S CORNER

by Barbara Remmers | edited by Jennifer Silva
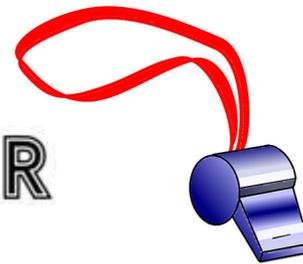
Owning it: Fraction Satisfaction, Part 14

You: Hi 3/7. Might you happen to know the least common multiple of 3827, 3847, and 3867?

$\dfrac{3}{7}$: Off the top of my head, I do not. Whyever are you asking?

You: Look at my homework. It's only two questions. Usually, I like short homework assignments because they contain problems that are interesting to think about, as opposed to calculation practice. This time, however, massive calculations are unavoidable. For each problem, I'm going to have to give all of the fractions the same bottom number so I can compare them.

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on the complete interview with Prof. Smith, and some other content.   We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

Content Removed from Electronic Version

Content Removed from Electronic Version

Girls!

Learn Mathematics!

Make new Friends!

Meet Professonal Women who use math in their work!

Improve how you Think and Dream!

Girls' Angle
A math club for ALL girls, grades 5-12.

girlsangle@gmail.com
girlsangle.org

Girls' Angle

Hexagon/Equilateral Triangle
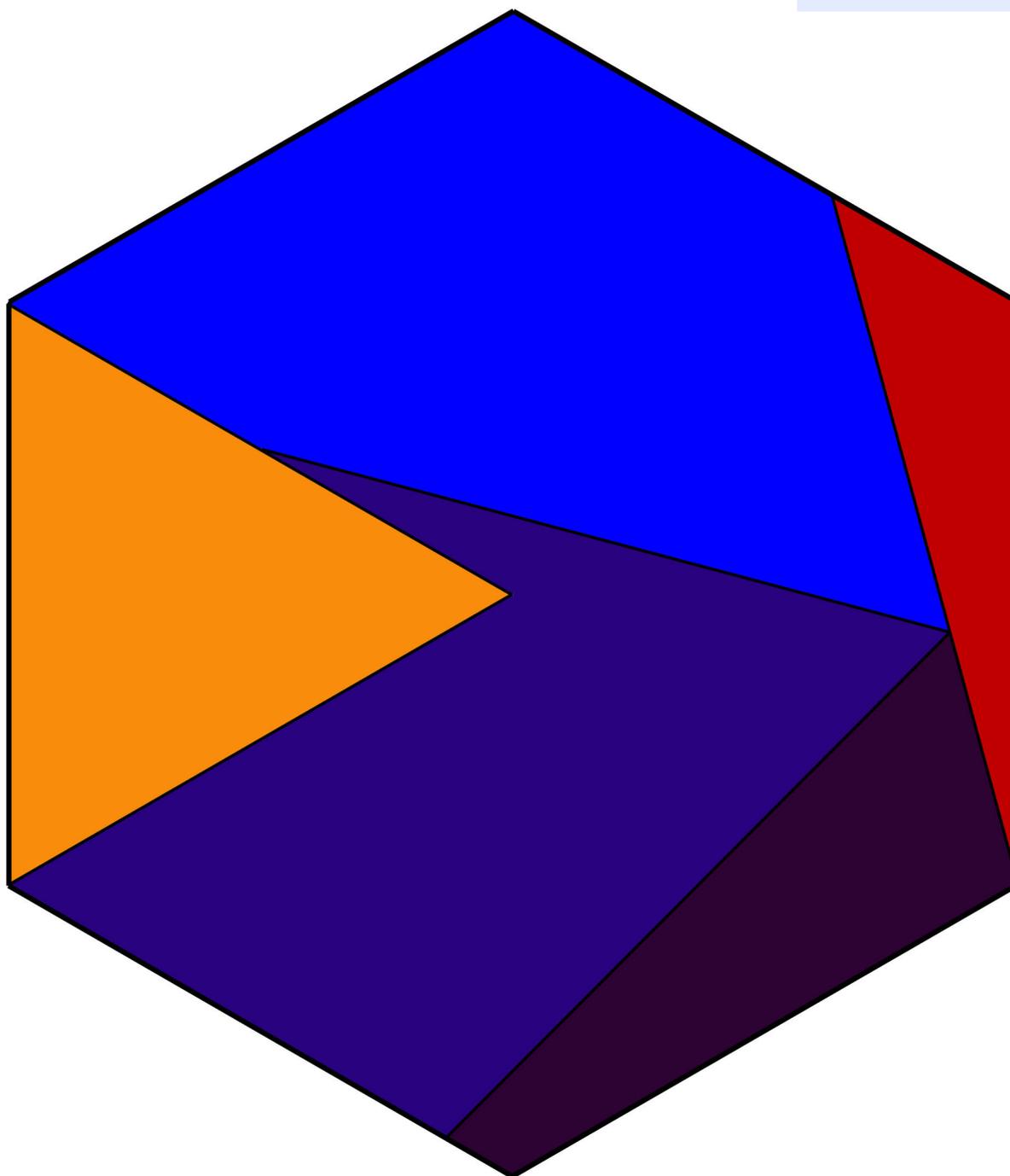
When challenged to devise a tiling of a regular hexagon with pieces that can be rearranged to form an equilateral triangle, **Henri** dreamed up the tiling shown above. He also specified the shapes of all 5 tiles by giving their exact side lengths and angles measures. We're deliberately withholding this information to provide you with a fun puzzle:

> Figure out how these 5 tiles form an equilateral triangle and determine their exact measurements up to a scale factor.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 14 - Meet 1   Mentors: Hannah Larson, Jennifer Matthews, Liz Simon, Isabel Vogt
January 30, 2014

We played a variation of One Minute! Please see *Notes from the Club* of Volume 7, Number 1 of this Bulletin for more information on One Minute! For one of the tasks, members had to successfully get a blindfolded mentor to feed water to another blindfolded mentor.

Session 14 - Meet 2   Mentors: Elenna Capote, Hannah Larson, Wangui Mbuguiro,
February 6, 2014   Liz Simon, Isabel Vogt

This semester, each member will embark on their own "personal math project." Mathematical journeys begin with a question and launch as soon as one seeks an answer. So to begin these journeys, all members were asked to come up with a math question. The only conditions were that the member had to come up with the question on her own and it had to be a question that she found she could not answer.

Unrelated to the personal projects, here's a question that arose at this meet: Imagine two lines rotating clockwise around 2 different pivot points in the plane. The lines rotate at the same rate and are not parallel to each other. What curve is traced out by their intersection? See figure at left.

Session 14 - Meet 3   This meet was cancelled due to lots of snow.
February 13, 2014

Session 14 - Meet 4   Mentors: Hannah Larson, Jennifer Matthews, Liz Simon
February 27, 2014

Members who have been studying parametric equations were presented with the following problem: In a coordinate plane, 3 points were selected in a random way. A member would start at the first point. Her goal was to make it to the third point and visit the second point along the way. She had to produce parametric equations each defined by single expression that achieved these objectives.

Calendar

Session 13: (all dates in 2013)

September	12	Start of the thirteenth session!
	19	No meet
	26	
October	3	
	10	
	17	Anna Frebel, Department of Physics, MIT
	24	
	31	
November	7	
	14	Kate Jenkins, Akamai Technologies
	21	
	28	Thanksgiving - No meet
December	5	
	12	

Session 14: (all dates in 2014)

January	30	Start of the fourteenth session!
February	6	
	13	
	20	No meet
	27	
March	6	
	13	Sarah Spence Adams, Olin College
	20	
	27	No meet
April	3	
	10	Anna Frebel, Department of Physics, MIT
	17	
	24	No meet
May	1	Karen Willcox, Dept. of Aeronautics and Astronautics, MIT
	8	

Solution to mobile weights on page 13: $B = 2A$, $C = 3A$, $D = A$, $E = 3A$, and $F = A$.

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

All Girls' Angle Members and Subscribers are invited to email math questions, solutions, comments, and suggestions. We will respond!

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) _____ (first) _____

Parents/Guardians: _____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: _____   Cell Phone: _____

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

_____

The $36 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

☐   Enclosed is a check for $36 for a 1-year Girls' Angle Membership.

☐   I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is located about 12 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

31

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

> Connie Chow, executive director of Science Club for Girls
> Yaim Cooper, graduate student in mathematics, Princeton
> Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
> Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
> Grace Lyo, Moore Instructor, MIT
> Lauren McGough, MIT '12
> Mia Minnes, SEW assistant professor of mathematics, UC San Diego
> Beth O'Sullivan, co-founder of Science Club for Girls.
> Elissa Ozanne, assistant professor, UCSF Medical School
> Kathy Paur, Kiva Systems
> Bjorn Poonen, professor of mathematics, MIT
> Gigliola Staffilani, professor of mathematics, MIT
> Bianca Viray, Tamarkin assistant professor, Brown University
> Lauren Williams, assistant professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) _____ (first) _____

Parents/Guardians: _____

Address: _____ Zip Code: _____

Home Phone: _____ Cell Phone: _____ Email: _____

Please fill out the information in this box.

Emergency contact name and number: _____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

_____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

_____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes   No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

_____   Date: _____
(Parent/Guardian Signature)

Participant Signature: _____

Members: Please choose one.

- ☐ Enclosed is $216 for one session
     (12 meets)

- ☐ I will pay on a per meet basis at $20/meet.

Nonmembers: Please choose one.

- ☐ I will pay on a per meet basis at $30/meet.

- ☐ I'm including $36 to become a member,
     and I have selected an item from the left.

☐   I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

_____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: _____Date: _____

Print name of applicant/parent: _____

Print name(s) of child(ren) in program: _____