ASCE NATIONAL CAPITAL SECTION
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DR. Z's CORNER

Conquering the FE & PE exams
Examples & Applications

Topics covered in this month's column:

- FE CIVIL Exam Topics & Number of Questions
- Technology Usage (Casio fx-115 PLUS)
- Important Eight Modes in Your Calculator
- Statistics Mode, Decimal & Binary Conversions
- Mathematics, Equation of Spheres
- Ordinary Differential Equations
- Absolute Error & Relative Error
- Axial Stress & Strain / Hooke's Law
- Centroids & Moments of Inertia
- Deflection of Beams Using NCEES Formulas
- Determinate Beams & Frames, Free Body Diagrams / Support Reactions
- Shear Force & Bending Moment Diagrams

FUNDAMENTALS OF ENGINEERING

CIVIL EXAM TOPICS

Computer-Based Test (CBT)

Total Number of Questions: 110
Time: 6 hours

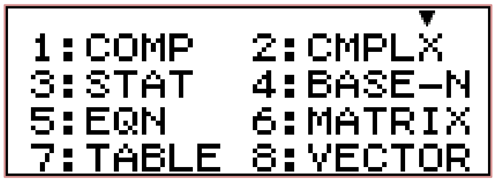
The new Civil FE Computer-Based Test (CBT) consists of 110 multiple-choice questions (Each problem only one question) the examinee will have 6 hours to complete the test.

- **Mathematics (Approx. 9 questions*)**
- **Probability and Statistics (5 questions)**
- **Computational Tools (5 questions)**
- **Ethics and Professional Practice (5 questions)**
- **Engineering Economics (5 questions)**
- **Statics (9 questions)**
- **Dynamics (5 questions)**
- **Mechanics of Materials (9 questions)**
- **Civil Engineering Materials (5 questions)**
- **Fluid Mechanics (5 questions)**
- **Hydraulics and Hydrologic Systems (10 questions)**
- **Structural Analysis (8 questions)**
- **Structural Design (8 questions)**
- **Geotechnical Engineering (12 questions)**
- **Transportation Engineering (10 questions)**
- **Environmental Engineering (8 questions)**

* Here the number of questions are the average values taken from the NCEES Reference Handbook (Version 9.3 / Computer-Based Test)

CALCULATOR MODES

The Casio fx-115 has eight "modes." These modes are accessed by pressing the **[MODE]** button.



Here are the 8 modes of Casio fx-115:

1:	**COMP**	computation mode (for most calculations)
2:	**CMPLX**	complex mode
3:	**STAT**	statistics mode
4:	**BASE-N**	base-N mode
5:	**EQN**	equation mode
6:	**MATRIX**	matrix mode
7:	**TABLE**	table mode
8:	**VECTOR**	vector mode

How to set your calculator into computation mode?

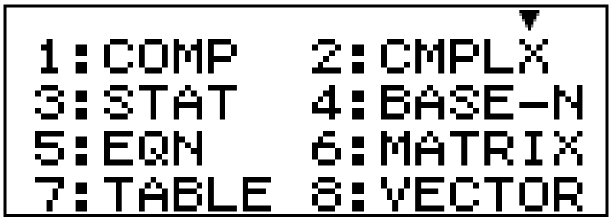
[MODE] [1]

How to set your calculator into statistics mode?

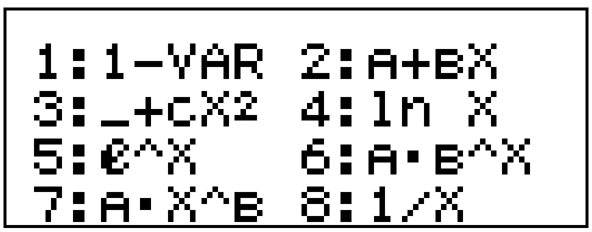
[MODE] [3]

STATISTICS MODE IN CASIO fx-115ES PLUS

First press the **[MODE]** button and you will get the following screen:



Then choose option 3 and you will see the following screen:



Let’s give an example using all the above steps.

FUNDAMENTALS OF ENGINEERING (FE)

PROBABILITY AND STATISTICS

FULL SOLUTION
NEXT PAGE

Problem:

A data set is given as listed below:

8, 25, 7, 5, 8, 3, 10, 12, 9

(1) The mean of this set is most nearly:

(A) 7.98
(B) 8.15
(C) 9.67
(D) 12.85

(2) The standard deviation is most nearly:

(A) 6.32
(B) 7.85
(C) 8.25
(D) 9.14

Problem:

8, 25, 7, 5, 8, 3, 10, 12, 9

Consider the data set given above:

(a) Calculate the mean ($\bar{y}$)
(b) Calculate the Standard Deviation (s_y)

Solution:

The **mean** is the sum of scores divided by n where n is the number of scores.

1. $\bar{y} = \frac{\sum y}{n}$ **= (8+25+7+5+8+3+10+12+9)/9**

= 9.67

2. the standard deviation may be calculated using the following formula:

$$s_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}} \qquad Deviation = (y_i - \bar{y})$$

In order to calculate the values in the standard deviation formula, the following table may be used:

Deviation = Score - Mean

Score	Mean	Deviation	$(\text{Deviation})^2$
8	9.67	- 1.67	2.79
25	9.67	+15.33	235.01
7	9.67	- 2.67	7.13
5	9.67	- 4.67	21.81
8	9.67	- 1.67	2.79
3	9.67	- 6.67	44.49
10	9.67	+ .33	.11
12	9.67	+ 2.33	5.43
9	9.67	- .67	.45

Σ **= 320.01**

Standard Deviation(S_y)

$$s_y = \sqrt{\frac{\sum (y_i - y)^2}{n-1}} = \sqrt{\frac{320.01}{9-1}} = 6.32$$

S_y = 6.32

Alternate method for calculating the Standard Deviation:
(The Raw Score Method)

Consider the raw scores 8, 25, 7, 5, 8, 3, 10, 12, 9.

1. First, square each of the scores.
2. Determine n, which is the number of scores.
3. Compute the sum of y_i and the sum of y_i^2
4. Then, calculate the standard deviation as illustrated below.

score(y_i)	y_i^2
8	64
25	625
7	49
5	25
8	64
3	9
10	100
12	144
9	81
---	----
87	1161

n = 9

$\Sigma y_i = 87$

$\Sigma y_i^2 = 1161$

Standard Deviation(S_y)

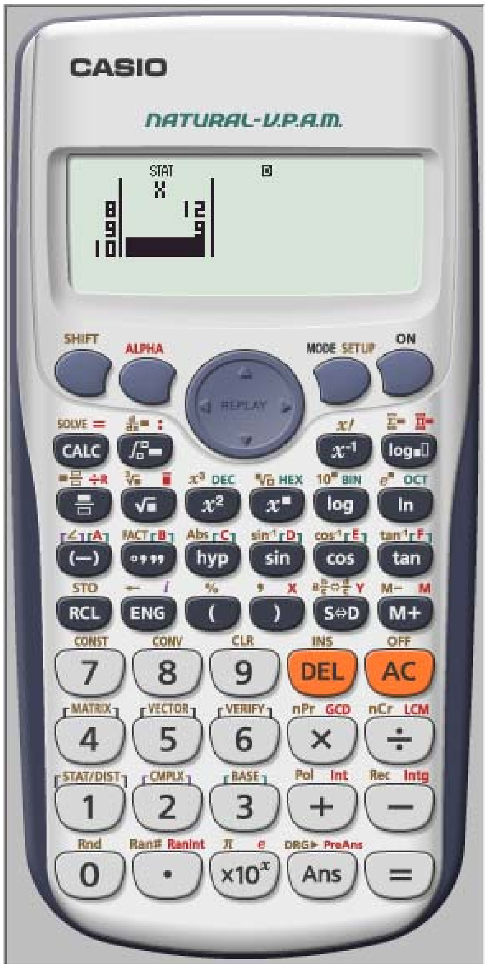
$$s_y = \sqrt{\frac{\sum y_i^2 - (\sum y_i)^2 / n}{n-1}}$$

= square root[(1161)-(87*87)/9)/(9-1)]

= square root[(1161-(7569/9)/8)] = 6.32

In *FE* exam CASIO fx-115 SE PLUS should be used

PROBABILITY & STATISTICS
WITH CASIO fx-115 ES PLUS




```
1:Type    2:Data
3:Sum     4:Var
5:Distr   6:MinMax
```

```
1:n       2:x̄
3:σx      4:sx
```

```
x̄
           9.666666667
```

```
SX
            6.32455532
```

MODE 3 1 8 = 2 5 = 7 = 5 = 8 = 3

= 1 0 = 1 2 = 9 =

AC SHIFT 1 4 2 =

AC SHIFT 1 4 4 =

CASIO / *fx*-115 ES PLUS

MODE	SHIFT SETUP
(1) COMP	(1) Mth-IO
(2) CMPLX	(2) Line-IO
(3) STAT	(3) Deg
(4) BASE-N	(4) Rad
(5) EQN	(5) Gra
(6) MATRIX	(6) Fix
(7) TABLE	(7) Sci
(8) VECTOR	(8) Norm

Frequently asked two Number Systems:

1- Decimal Number System (base 10)
2- Binary Number System (base 2)

In Decimal System 10 different digits are used to create any number, but in Binary System only 0s and 1s are used to create any number

DECIMAL	BINARY
2	10
3	11
5	101
6	110
8	1000
9	1001
10	1010
12	1100
14	1110
15	1111
19	10011
25	11001

ANGLE CONVERSIONS
FROM (DMS) TO DECIMAL DEGREES
MANUAL CALCULATIONS

Degrees, minutes and seconds: (d^{o} m' s'')

One degree is equal to 60 minutes and 3600 seconds:

$1^{o} = 60' = 3600''$

MANUAL CALCULATION

One minute is equal to 1/60 degrees:

$1' = (1/60)^{o} = 0.01666667^{o}$

One second is equal to 1/3600 degrees:

$1'' = (1/3600)^{o} = 0.000277778^{o}$

For an angle with d (integer) degrees, m minutes, and s seconds:

$d^{o}\ m'\ s''$

The formula to convert DMS to decimal degrees:

$\text{Angle} = d + m / 60 + s / 3600$

Example

Convert 5 degrees 25 minutes and 30 seconds angle to decimal degrees:

$\text{Angle} = 5^{o}\ 25'\ 30''$

The decimal degrees dd is equal to:

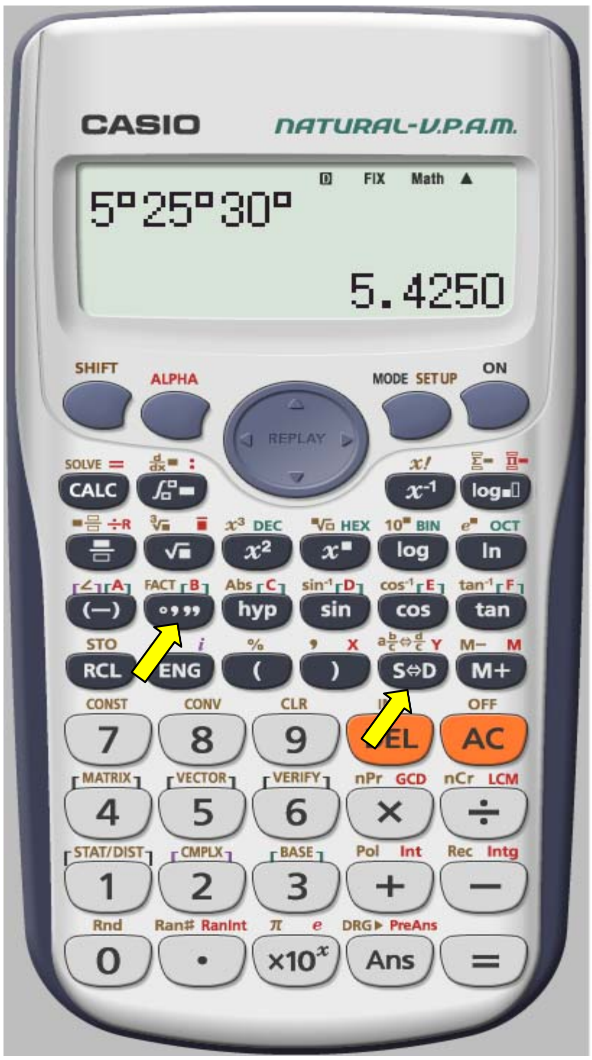
$$\text{Angle} = d + m/60 + s/3600 = 5^{o} + 25'/60 + 30''/3600$$
$$= 5.425^{o}$$

ANGLE CONVERSIONS
FROM (DMS) TO DECIMAL DEGREES
USING CALCULATOR

Problem:

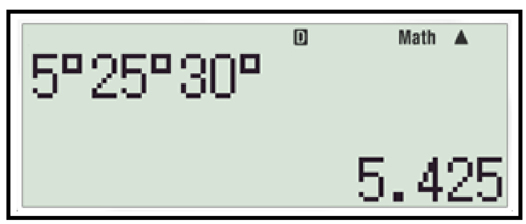
Convert the angle given as 5 degrees, 25 minutes, and 30 seconds to decimal degrees using your calculator.

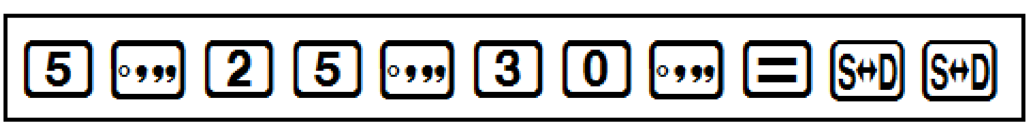
Solution:



TECHNOLOGY USAGE

Angle = 5º, 25', 30"





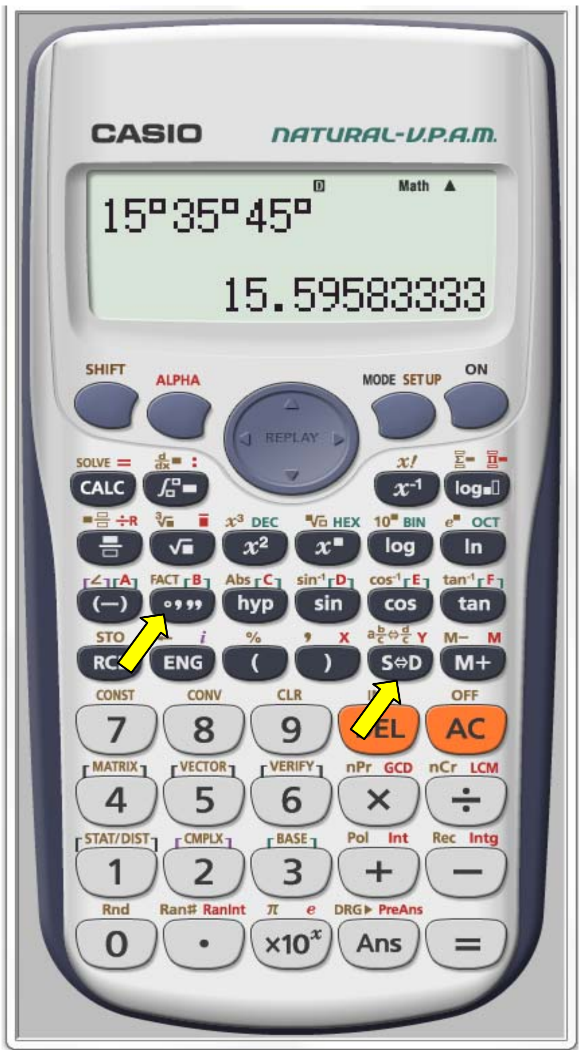
KEY SEQUENCE

ANGLE-315
ZEYTINCI
FALL-2015

ANGLE CONVERSIONS

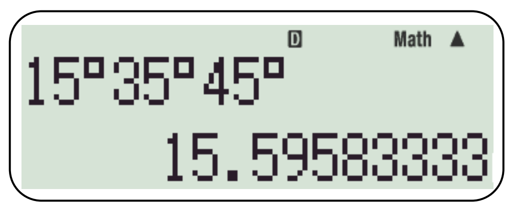
FROM (DMS) TO DECIMAL DEGREES

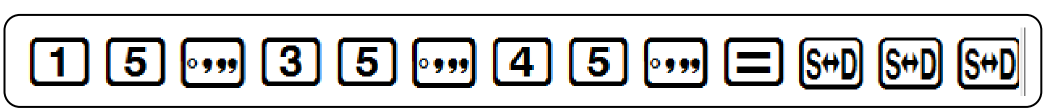
USING CALCULATOR



Important
Keys

Angle = 15º, 35', 45"





KEY SEQUENCE

ANGLE-315-1
ZEYTINCI
FALL-2015

ANGLE CONVERSIONS
FROM (DMS) TO DECIMAL DEGREES
SUPPLEMENTAL PROBLEMS

(1)

Degrees:	15	°
Minutes:	35	'
Seconds:	45	"

MANUAL CALCULATION

Decimal degrees:
15° 35' 45"
= 15° + 35'/60 + 45"/3600
= 15.59583°

(2)

Degrees:	12	°
Minutes:	20	'
Seconds:	36	"

Decimal degrees:
12° 20' 36"
= 12° + 20'/60 + 36"/3600
= 12.34333°

ANGLE CONVERSIONS
FROM DECIMAL TO DEGREES, MINUTES, SECONDS
SUPPLEMENTAL PROBLEMS

MANUAL CALCULATION

(1)

Decimal degrees: 5.425 °

Degrees,minutes,seconds:

$d = \text{int}(5.425°) = 5°$
$m = \text{int}((5.425° - 5°) \times 60)$
$= 25'$
$s = (5.425° - 5° - 25'/60)$
$\times 3600 = 30''$
$5.425°$
$= 5°\ 25'\ 30''$

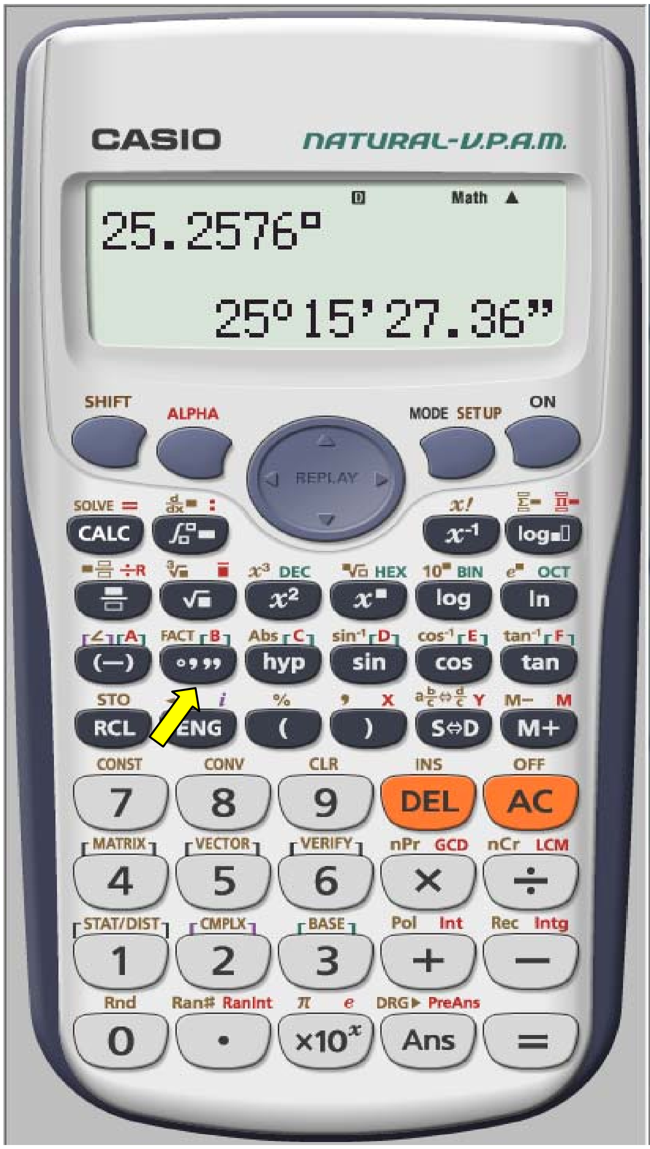
(2)

Decimal degrees: 25.2576 °

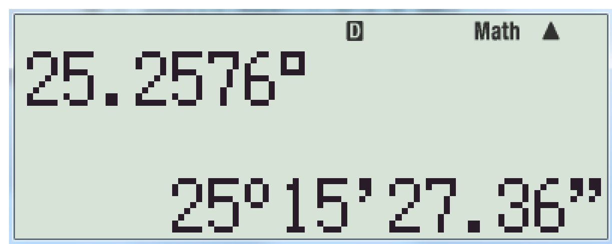
Degrees,minutes,seconds:

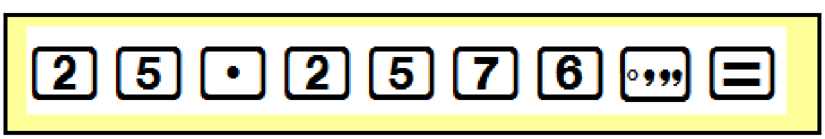
$d = \text{int}(25.2576°) = 25°$
$m = \text{int}((25.2576° - 25°) \times$
$60) = 15'$
$s = (25.2576° - 25° -$
$15'/60) \times 3600 = 27.36''$
$25.2576°$
$= 25°\ 15'\ 27.36''$

ANGLE CONVERSIONS
FROM DECIMAL TO DEGREES, MINUTES, SECONDS
USING CALCULATOR



TECHNOLOGY USAGE

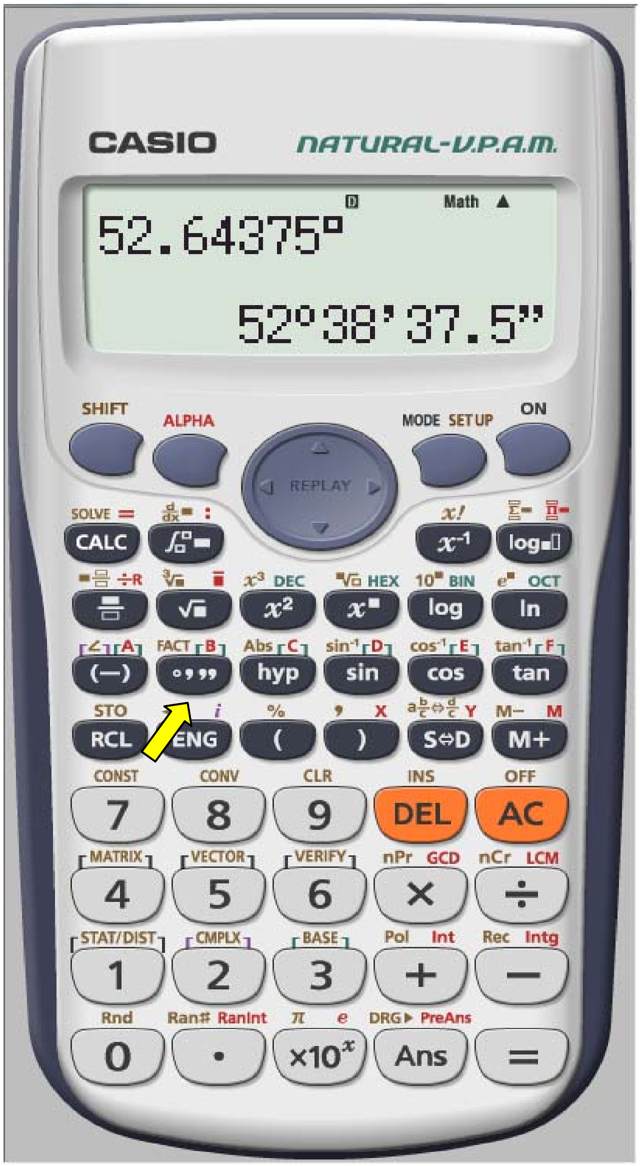




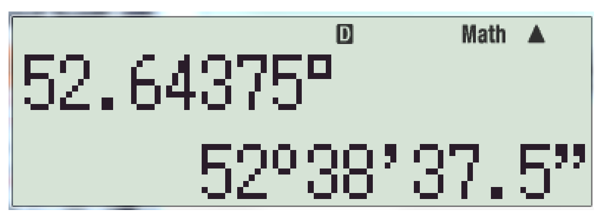
KEY SEQUENCE

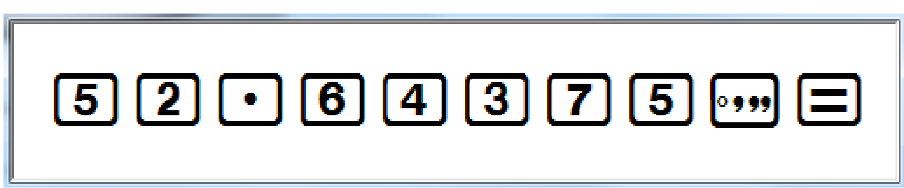
ANGLE-115-2
ZEYTINCI
FALL 2015

ANGLE CONVERSIONS
FROM DECIMAL TO DEGREES, MINUTES, SECONDS
USING CALCULATOR



TECHNOLOGY USAGE





ANGLE-115-3
ZEYTINCI
FALL 2015

FUNDAMENTALS OF ENGINEERING

DOMAIN: MATHEMATICS

CONIC SECTIONS

NCEES-Reference Handbook / Page-23

1- Parabola (eccentricity = 1)

$$(y - k)^2 = 2p(x - h)$$

Center: (h, k)

2- Ellipse (eccentricity < 1)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Center: (h, k)

3- Hyperbola (eccentricity > 1)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center: (h, k)

4- Circle (eccentricity = 0)

$$(x - h)^2 + (y - k)^2 = r^2$$

Center: (h, k)

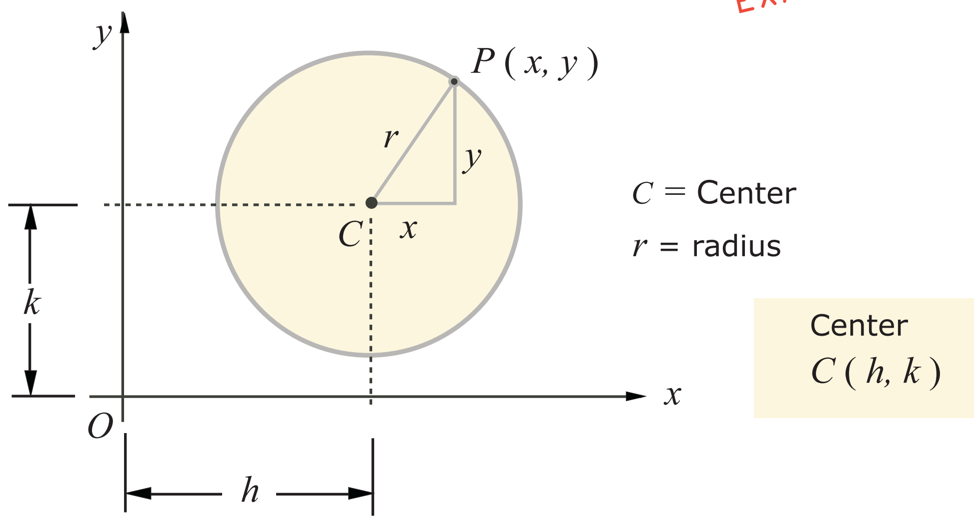
radius:

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

FUNDAMENTALS OF ENGINEERING

EQUATION OF A CIRCLE & SPHERE

FE EXAM



The standard form of the equation for a *CIRCLE*:

NCEES-RH PAGE-26

$$(x - h)^2 + (y - k)^2 = r^2$$

Center at (h, k)

r = radius of circle

The standard form of the equation for a *SPHERE*:

NCEES-RH PAGE-21

$$(x - h)^2 + (y - k)^2 + (z - m)^2 = r^2$$

Center at (h, k, m)

r = radius of sphere

A sphere is defined as the set of all points in three-dimensional Euclidean space $\mathbf{R}^3$ that are located at a distance r from a given point C. Here r is the *radius* and C is the *center* of the sphere.

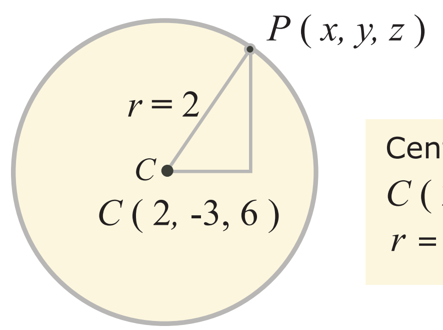
FUNDAMENTALS OF ENGINEERING

EQUATION OF A SPHERE

Problem: (Equation of a sphere)

FE
EXAM

SPHERE



Center
C (2, -3, 6)
r = 2 (radius)

$$(x - h)^2 + (y - k)^2 + (z - m)^2 = r^2$$

Center at (h, k, m), r = radius

NCEES-RH
PAGE-21

The equation of a sphere with center at (2, -3, 6) and a radius of r = 2 is most nearly:

(A) $(x + 2)^2 + (y - 3)^2 + (z - 6)^2 = 2$

(B) $(x - 2)^2 + (y + 3)^2 + (z + 6)^2 = 2$

(C) $(x + 2)^2 + (y + 3)^2 + (z - 6)^2 = 4$

(D) $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 4$

FIRST ORDER LINEAR

ORDINARY DIFFERENTIAL EQUATIONS

$$\frac{dy}{dt} = a\,y - b$$

FE EXAM

(a) Find the general solution of this ODE
(b) Find the particular solution when t = 0 then $y = y_o$

Solution:

$$\frac{dy}{dt} = a\,y - b$$

$$\frac{dy}{dt} = a(y - b/a)$$

$$\frac{dy}{(y - b/a)} = a\,dt$$

$$\int \frac{dy}{(y - b/a)} = \int a\,dt$$

$$\ln|y - b/a| = a\,t + C_1$$

$$\ln|y - b/a| = a\,t + C_1$$

$$|y - b/a| = e^{(a\,t + C_1)}$$

$$|y - b/a| = e^{a\,t} \cdot e^{C_1}$$

$$|y - b/a| = e^{a\,t} \cdot C_2$$

$$y - b/a = \pm C_2 e^{a\,t}$$

$$\boxed{y = b/a + C\,e^{a\,t}}$$

General Solution

C: Arbitrary constant

Because of the arbitrary constant C we have infinitely many solutions.

Particular solution is also called the **Initial Value Problem:**

$$\left.\begin{array}{l} t = 0 \\ y = y_o \end{array}\right\} \quad C = y_o - b/a$$

$$\boxed{y = b/a + (y_o - b/a)\,e^{a\,t}}$$

Particular Solution

FUNDAMENTALS OF ENGINEERING

ORDINARY DIFFERENTIAL EQUATIONS

MATHEMATICS

Problem: (1)

$$\frac{dy}{dt} + 7y = 0 \qquad y(0) = 1$$

An ordinary differential equation and the boundary condition is given above. The general solution is most nearly:

(A) e^{-t}

(B) e^{7t}

(C) e^{-7t}

(D) $7.e^{-7t}$

Problem: (2)

$$\frac{dy}{dt} - 6y = 0 \qquad y(0) = 1$$

An ordinary differential equation and the boundary condition is given above. The general solution is most nearly:

(A) e^{-t}

(B) e^{6t}

(C) e^{-6t}

(D) $6.e^{-6t}$

ABSOLUTE ERROR & RELATIVE ERROR

The accuracy of a computation is very important in numerical analysis. There are two ways to express the size of the error in a computed result:

(a) Absolute Error
(b) Relative Error

$$\text{ABSOLUTE ERROR} = |\text{ True Value} - \text{Approximate Value }|$$

$$\text{RELATIVE ERROR} = \frac{\text{Absolute Error}}{|\text{ True Value }|}$$

Example:

True Value = 10/3
Approximate Value = 3.333

(a) Determine the absolute error
(b) Determine the relative error
(c) Find the significant digits

Solution:

$$\begin{aligned}\text{ABSOLUTE ERROR} &= |\text{True Value} - \text{Approximate Value }| \\ &= 10/3 - 3.333 \\ &= 0.000333\ldots \\ &= 1 / 3000\end{aligned}$$

$$\text{RELATIVE ERROR} = \frac{\text{Absolute Error}}{|\text{True Value }|}$$

$$= \frac{(1/3000)}{(10/3)}$$

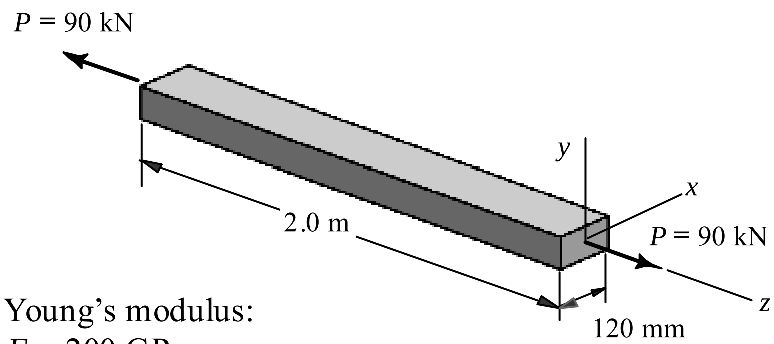
$$= 1/10{,}000$$

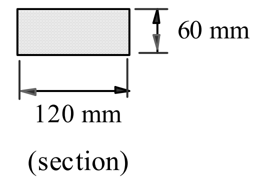
Here, the number of significant digits is 4.

STRENGTH OF MATERIALS

AXIAL STRESS & STRAIN / HOOKE'S LAW

FE & PE EXAM





Young's modulus:
$E = 200$ GPa
Poisson's ratio:
$\nu = 0.33$

$E = 200\,\text{GPa}$

A rectangular steel plate is axially loaded as shown. Using the listed dimensions and material properties, answer the following questions:

(1) Total elongation in micrometers (μm) parallel to the applied load:

(A) 150
(B) 145
(C) 130
(D) 125

$\delta_z = ?$

(2) Change in dimension in micrometers (μm) parallel to the x axis:

(A) - 1.25
(B) - 1.95
(C) - 2.48
(D) - 3.25

$\delta_x = ?$

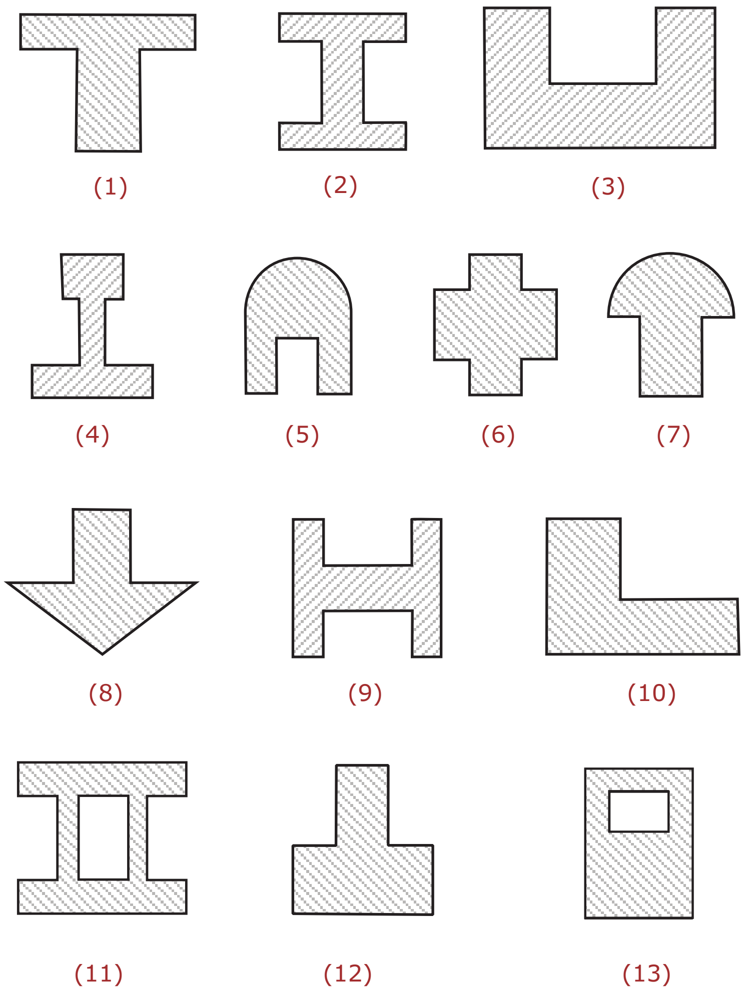
(3) Change in dimension in micrometers (μm) parallel to the y axis:

(A) - 1.10
(B) - 1.24
(C) - 1.86
(D) - 2.36

$\delta_y = ?$

THE BEAUTY OF COMPOSITE AREAS

CENTROIDS & MOMENTS OF INERTIA



LOOK FOR AXES/AXIS OF SYMMETRY

VISUALLY LOCATING CENTROIDS

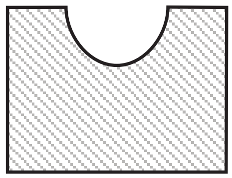
PARALLEL AXIS THEOREM (PAT)

ANALYTICAL COMPUTATIONS

CHECK YOUR ANSWERS USING SOFTWARE

CENT-555
ZEYTINCI
SPRING 2014

LOCATING CENTROIDS VISUALLY



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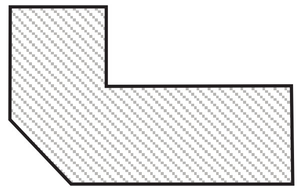

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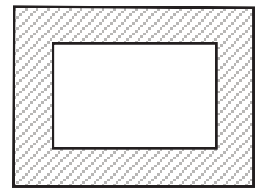

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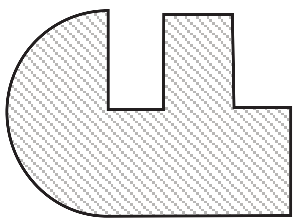

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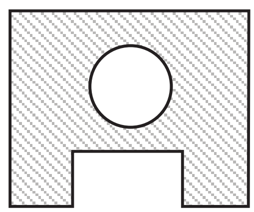

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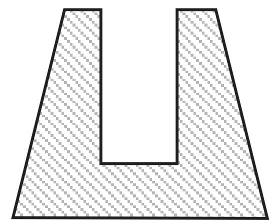
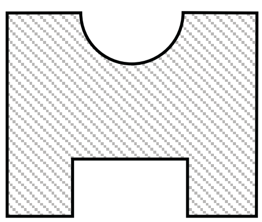
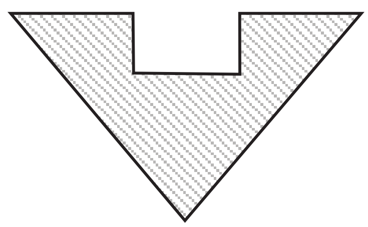


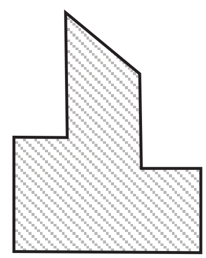
(19)



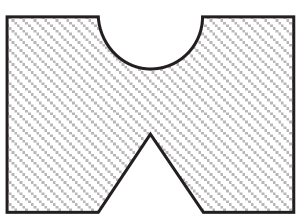
(20)



(21)



(22)



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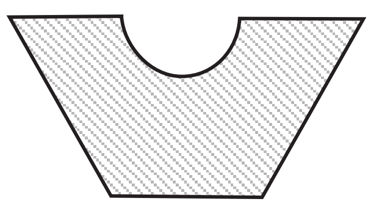

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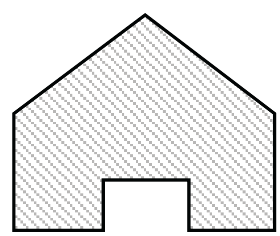


(25)

CENTROIDS / MOMENTS OF INERTIA

$$\bar{x} = \frac{\int_A x\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A y\, dA}{\int_A dA}$$

Moments of Inertia (*I*)

$$I_x = \int_A y^2\, dA \qquad I_y = \int_A x^2\, dA \qquad J_o = I_x + I_y$$

J_o = Polar Moment of Inertia

Parallel Axis Theorem (PAT)

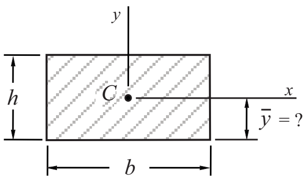
$$I_{cx} = \Sigma I_o + \Sigma A d^2$$
$$I_{cy} = \Sigma I_o + \Sigma A d^2$$

Radius of Gyration (*r*)

$$r_x = \sqrt{\frac{I_x}{A}} \qquad r_y = \sqrt{\frac{I_y}{A}} \qquad r_o = \sqrt{\frac{J_o}{A}}$$

BASIC FORMULAS

Problem: (Rectangle)



C: Centroid
Origin of axes at C

(a) Area of the rectangle (A = ?)
(b) Centroid of the rectangle ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Formulas:

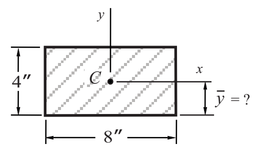
$$A = bh$$

$$\bar{y} = \frac{h}{2}$$

$$I_{cx} = \frac{b\,h^3}{12}$$

$$I_{cy} = \frac{h\,b^3}{12}$$

Example:



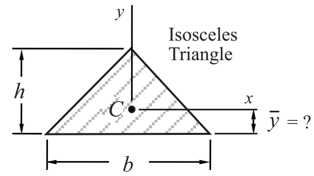
$$A = bh = (8)(4) = 32 \text{ in.}^2$$

$$\bar{y} = \frac{h}{2} = \frac{4}{2} = 2 \text{ in.}$$

$$I_{cx} = \frac{b\,h^3}{12} = \frac{(8)(4)^3}{12} = 42.67 \text{ in.}^4$$

$$I_{cy} = \frac{h\,b^3}{12} = \frac{(4)(8)^3}{12} = 170.67 \text{ in.}^4$$

Problem: (Triangular Area)



C : Centroid
x, y : Centroidal axes

(a) Area of the triangle (A = ?)
(b) Centroid of the triangle ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Formulas:

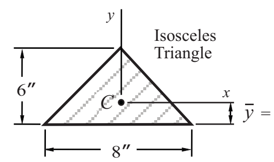
$$A = \frac{bh}{2}$$

$$\bar{y} = \frac{h}{3}$$

$$I_{cx} = \frac{b\,h^3}{36}$$

$$I_{cy} = \frac{h\,b^3}{48}$$

Example:



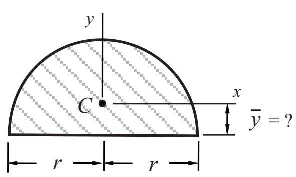
$$A = \frac{bh}{2} = \frac{(8)(6)}{2} = 24 \text{ in.}^2$$

$$\bar{y} = \frac{h}{3} = \frac{6}{3} = 2 \text{ in.}$$

$$I_{cx} = \frac{b\,h^3}{36} = \frac{(8)(6)^3}{36} = 48.00 \text{ in.}^4$$

$$I_{cy} = \frac{h\,b^3}{48} = \frac{(6)(8)^3}{48} = 64.00 \text{ in.}^4$$

Problem: (Half Circle)



(a) Area of the semicircle (A = ?)
(b) Centroid of the semicircle ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Solution:

$$A = \frac{\pi r^2}{2}$$

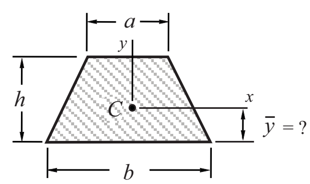
$$\bar{y} = \frac{4r}{3\pi}$$

$$I_{cx} = \frac{r^4}{72\pi} \cdot \left(9\,\pi^2 - 64\right)$$

$$I_{cx} \approx 0.109757 \cdot r^4$$

$$I_{cy} = \frac{\pi r^4}{8}$$

Problem: (Trapezoidal Area)



C : Centroid
x, y : Centroidal axes

Isosceles
Trapezoid

(a) Area of the trapezoid (A = ?)
(b) Centroid of the trapezoid ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Formulas

$$A = \frac{h}{2}\left(a + b\right)$$

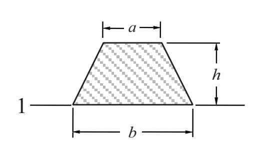
$$\bar{y} = \frac{h}{3}\,\frac{\left(2a + b\right)}{\left(a + b\right)}$$

Isosceles
Trapezoid

$$I_{cx} = \frac{h^3}{36} \cdot \frac{\left(a^2 + 4ab + b^2\right)}{\left(a + b\right)}$$

$$I_{cy} = \frac{h\left(a + b\right)\left(a^2 + b^2\right)}{48}$$

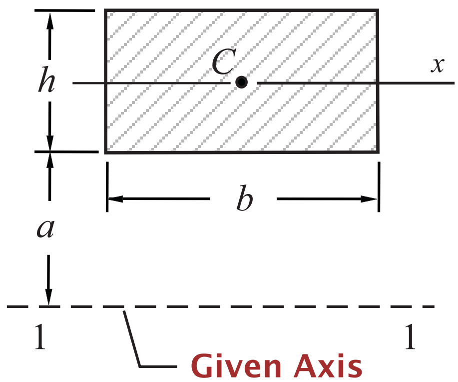
$$I_{1\text{-}1} = \frac{h^3\left(3\,a + b\right)}{12}$$



Isosceles
Trapezoid

CENT-88
ZEYTINCI
FALL 2015

PARALLEL AXIS THEOREM



PAT | **Parallel Axis Theorem**

(a) Moment of inertia about the x axis (I_{cx} = ?)
(b) Moment of inertia about the (1-1) axis ($I_{1\text{-}1}$ = ?)

Formulas:

$$I_{cx} = \frac{b\,h^3}{12}$$

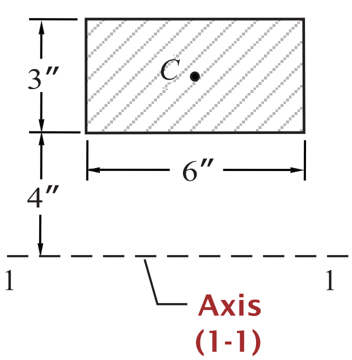
$$I_{1\text{-}1} = I_{cx} + A \cdot d^2$$ **PAT**

ONE OF THE MOST IMPORTANT PRINCIPLES IN STRUCTURAL MECHANICS

I_{cx} = Moment of inertia about the centroidal x axis
$I_{1\text{-}1}$ = Moment of inertia about axis (1-1)
A = Area ($A = b.h$)
d = Distance from the centroid to the axis (1-1)

$$d = a + h/2$$

Example:



$$I_{cx} = \frac{b\,h^3}{12} = \frac{(6)(3)^3}{12} = 13.50 \text{ in.}^4$$

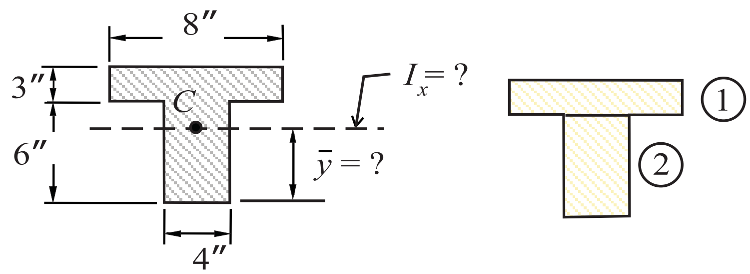
$$A = bh = (6)(3) = 18 \text{ in.}^2$$

$$d = 4 + 3/2 = 5.5 \text{ in.}$$

$$I_{1\text{-}1} = I_{cx} + A.d^2 = 13.50 + (18)(5.5)^2 = 558 \text{ in.}^4$$

$$I_{1\text{-}1} = 558 \text{ in.}^4$$

Centroid / Moments of Inertia



Centroid Calculations

	A_i	y_i	$A_i\, y_i$
	in.^{2}	in.	in.^{3}
1	24.00	7.5	180.00
2	24.00	3.0	72.00
Σ	**48.00**	—	**252.00**

$$\bar{y} = \frac{\Sigma A_i\, y_i}{\Sigma A_i} = \frac{252}{48} = 5.25 \text{ in.}$$

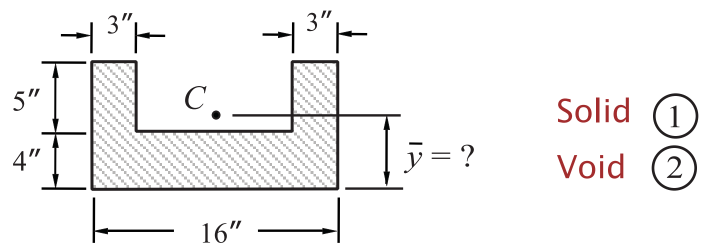
Moments of Inertia Calculations

	I_o	A_i	d_i	$A_i d_i^2$
	in.^{4}	in.^{2}	in.	in.^{4}
1	18.0	24.00	2.25	121.5
2	72.0	24.00	2.25	121.5
Σ	**90.0**		—	**243.0**

$$I_{cx} = \Sigma I_o + \Sigma A_i \,.\, d_i^2 = 90 + 243 = 333 \text{ in.}^4$$

$$I_{cy} = I_{y1} + I_{y2} = \frac{3 \times 8^3}{12} + \frac{6 \times 4^3}{12} = 128 + 32 = 160 \text{ in.}^4$$

Centroid / Moments of Inertia



Centroid Calculations

	A_i	y_i	$A_i\, y_i$
	in.^{2}	in.	in.^{3}
1	144.00	4.5	648.00
2	-50.00	6.5	-325.00
Σ	**94.00**	—	**323.00**

$$\bar{y} = \frac{\Sigma A_i\, y_i}{\Sigma A_i} = \frac{323}{94} = 3.44 \text{ in.}$$

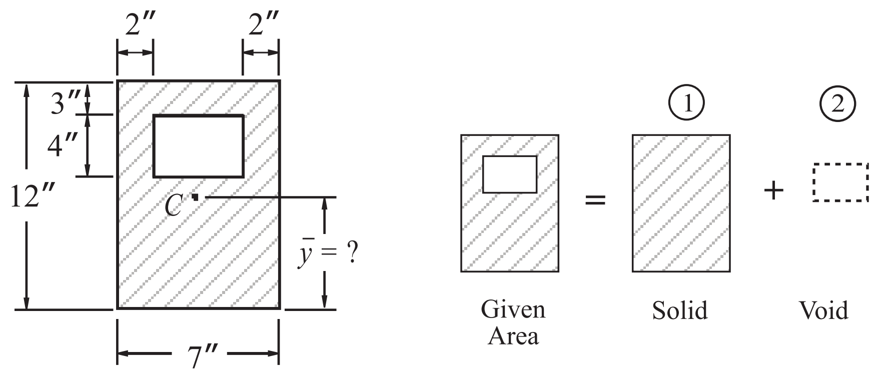
Moments of Inertia

	I_o	A_i	d_i	$A_i d_i^2$
	in.^{4}	in.^{2}	in.	in.^{4}
1	972.00	144.00	1.06	162.97
2	-104.17	-50.00	3.06	-469.35
Σ	**867.83**		—	**-306.38**

$$I_{cx} = \Sigma I_o + \Sigma A_i \, . \, d_i^2 = 867.83 - 306.38 = 561.45 \text{ in.}^4$$

$$I_{cy} = I_{y1} + I_{y2} = \frac{9 \times 16^3}{12} - \frac{5 \times 10^3}{12} = 3072 - 416.7 = 2655.3 \text{ in.}^4$$

Centroid / Moments of Inertia



Centroid Calculations

	A_i	y_i	$A_i\, y_i$
	in.^{2}	in.	in.^{3}
1	84.00	6.0	504.00
2	-12.00	7.0	-84.00
Σ	**72.00**	—	**420.00**

$$\bar{y} = \frac{\Sigma A_i\, y_i}{\Sigma A_i} = \frac{420}{72} = 5.83 \text{ in.}$$

Moments of Inertia Calculations

	I_o	A_i	d_i	$A_i d_i^2$
	in.^{4}	in.^{2}	in.	in.^{4}
1	1008	84.00	0.17	2.43
2	-16.00	-12.00	1.17	-16.43
Σ	**992.0**		—	**-14.00**

$$I_{cx} = \Sigma I_o + \Sigma A_i \,.\, d_i^2 = 992 - 14 = 978 \text{ in.}^4$$

$$I_{cy} = I_{y1} + I_{y2} = \frac{12 \text{ x} 7^3}{12} - \frac{4 \text{ x } 3^3}{12} = 343 - 9 = 334 \text{ in.}^4$$

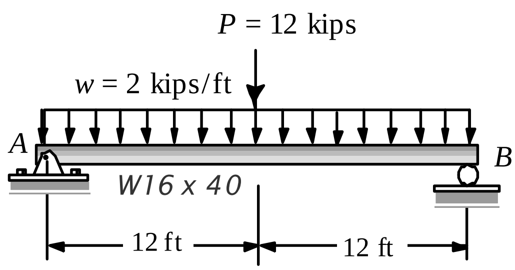
PROFESSIONAL ENGINEERING (PE)

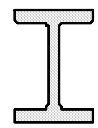
FUNDAMENTALS OF ENGINEERING (FE)

MECHANICS OF MATERIALS

FULL SOLUTION
NEXT PAGE

Problem: (Deflection of Beams)





W16 x 40

E = 29,000 ksi

A determinate beam is loaded as shown. Knowing that the beam weight is included in the uniform load, answer the following questions:

(1) The maximum deflection (inches) is most nearly:

(A) 0.98
(B) 1.15
(C) 1.39
(D) 1.85

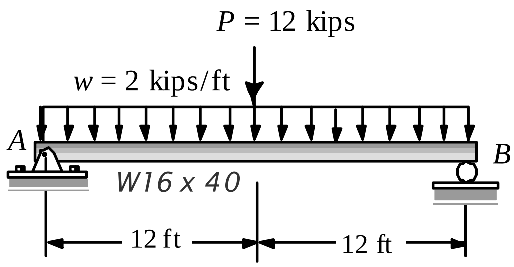
δ_{max} = ?

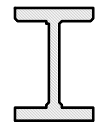
(2) The slope (radians) at support A is most nearly:

(A) 0.0055
(B) 0.0152
(C) 0.0880
(D) 0.1250

θ_A = ?

Problem: (Beam Deflections)



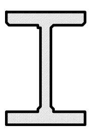


W 16 x 40

E = 29,000 ksi

For the simple beam shown the beam weight is included in the uniform load. Determine the maximum deflection and the slope at A (in radians).

Solution: We will use NCEES-Reference Handbook, page 155 and 81.



W 16 x 40

$I = 518 \text{ in}^4$

FOR DEFLECTIONS : (12^3)

FOR SLOPES : (12^2)

Finding the maximum deflection:

The maximum deflection will be at the midpoint of the span. For quick calculations when using US unit systems, architects and engineers use conversion factors like (12^3) and (12^2). For DEFLECTIONS this conversion factor is (12^3) and for SLOPES the conversion factor will be (12^2).

$$\delta_{max} = \frac{5}{384}\frac{wL^4}{EI} + \frac{1}{48}\frac{PL^3}{EI} = \frac{5}{384}\frac{(2.0)\,(24)^4}{(29{,}000)\,(518)}(12^3) + \frac{1}{48}\frac{(12)\,(24)^3}{(29{,}000)\,(518)}(12^3)$$

$$= 0.994 + 0.397 = \underline{1.391 \text{ inches}}$$

Finding the slope at support A:

$$\theta_A = \frac{wL^3}{24EI} + \frac{PL^2}{16\,EI} = \frac{1}{24}\frac{(2.0)\,(24)^3}{(29{,}000)\,(518)}(12^2) + \frac{1}{16}\frac{(12)\,(24)^2}{(29{,}000)\,(518)}(12^2)$$

= 0.01104 + 0.00414 = 0.01518 Radians

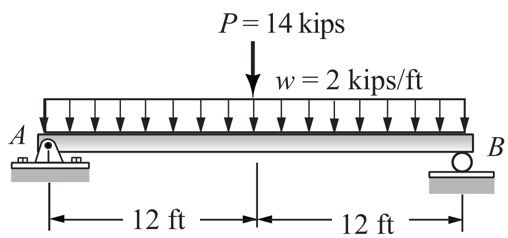
= <u>0.01518 Radians</u>

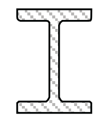
DEF-216
ZEYTINCI
FALL 2014

MECHANICS OF SOLIDS

DESIGN OF STEEL STRUCTURES

BEAM DEFLECTIONS





section
W16 x 57

NCEES-RH
PAGE-158
W-SHAPES

$E = 29{,}000$ ksi

A determinate beam is loaded as shown. Knowing that the beam weight is included in the uniform load, answer the following questions:

(1) The maximum moment (ft.kips) in the beam is most nearly:

(A) 264
(B) 204
(C) 144
(D) 228

$M_{max} = ?$

(2) The maximum deflection (inches) is most nearly:

(A) 1.00
(B) 0.42
(C) 0.67
(D) 1.14

$\delta_{max} = ?$

NCEES-RH
PAGE-84
DEFLECTIONS

(3) The slope (degrees) at the left support is most nearly

(A) 0.413
(B) 0.622
(C) 0.752
(D) 0.982

$\theta_A = ?$

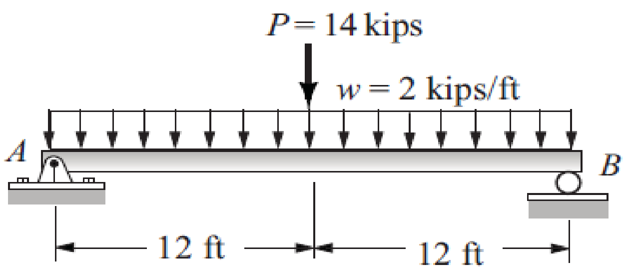
ANSWERS

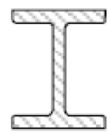
MECHANICS OF SOLIDS

DESIGN OF STEEL STRUCTURES

BEAM DEFLECTIONS

Problem: ***DEF-280*** ***Solution in MS Excel***





SECTION: **W16X57**

E = 29000 ksi

P = 14 kips

w = 2 kips/ft

L = 24 ft

ID	Quantity	Symbol	Value	Unit
37	x - Moment of Inertia	I_x	758.00	in^4

w = 0.166666667 kips/in

L = 288 in

M_P = 84 ft·kips

M_w = 144 ft·kips

M_{max} = **228** ft·kips

$$M_P = \frac{PL}{4}$$

$$M_w = \frac{wL^2}{8}$$

δ_P = 0.317 in

δ_w = 0.679 in

δ_{max} = **1.00** in

$$\delta_P = \frac{1}{48}\frac{PL^3}{EI}$$

$$\delta_w = \frac{5}{384}\frac{wL^4}{EI}$$

θ_P = 0.00330 rad

θ_w = 0.00755 rad

θ_A = *0.0108* rad

θ_A = ***0.622*** deg

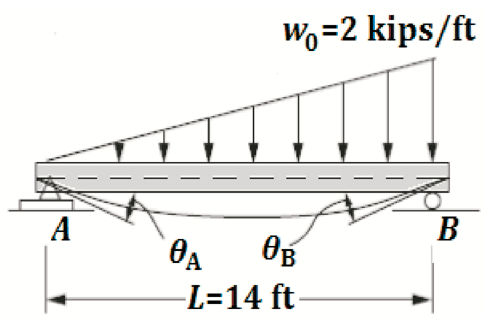
$$\theta_P = \frac{1}{16}\frac{PL^2}{EI}$$

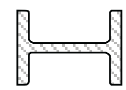
$$\theta_w = \frac{1}{24}\frac{wL^3}{EI}$$

MECHANICS OF SOLIDS

DESIGN OF STEEL STRUCTURES

BEAM DEFLECTIONS





section

W16 x 57

NCEES-RH
PAGE-158
W-SHAPES

$E = 29{,}000$ ksi

A determinate beam is loaded as shown. Knowing that the beam weight is included in the uniform load, answer the following questions:

(1) The maximum deflection (inches) is most nearly:

(A) 0.82
(B) 0.53
(C) 0.69
(D) 0.98

$\delta_{max} = ?$

(2) The slope (degrees) at the left support (A) is most nearly:

(A) -0.704
(B) -0.633
(C) -0.521
(D) -0.312

$\theta_A = ?$

NCEES-RH
PAGE-84
DEFLECTIONS

(3) The slope (degrees) at the right support (B) is most nearly:

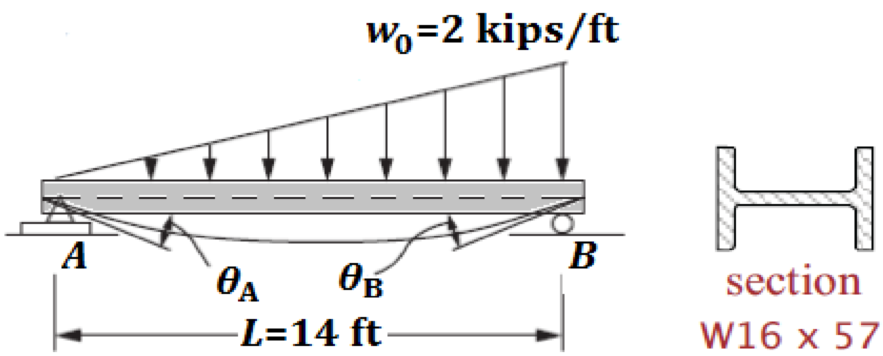
(A) 0.711
(B) 0.659
(C) 0.982
(D) 0.805

$\theta_B = ?$

ANSWERS

MECHANICS OF SOLIDS
DESIGN OF STEEL STRUCTURES
BEAM DEFLECTIONS

Problem: ***DEF-282*** ***Solution in MS Excel***



SECTION:	**W16X57**	
E =	29000	ksi
w_0 =	2	kips/ft
L =	14	ft

ID	Quantity	Symbol	Value	Unit
41	y - Moment of Inertia	I_y	43.10	in^4

w_0 = 0.166666667 kips/in
L = 168 in

δ_{max} = 0.69 in

$$\delta_{max} = 0.00652 \frac{w_0 L^4}{EI}$$

at x = 7.27 ft

$$\text{at } x = 0.5193L$$

θ_A = -0.01229 rad
θ_A = -0.704 deg

$$\theta_A = \frac{-7w_0 L^3}{360EI}$$

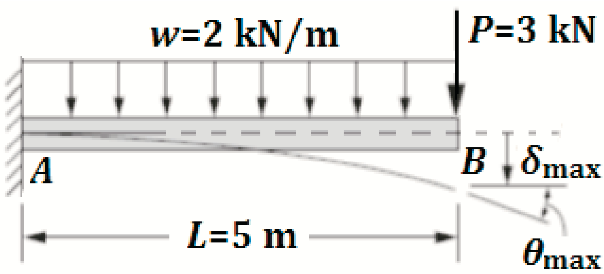
θ_B = 0.01405 rad
θ_B = 0.805 rad

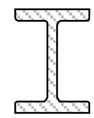
$$\theta_B = \frac{w_0 L^3}{45EI}$$

MECHANICS OF SOLIDS

DESIGN OF STEEL STRUCTURES

BEAM DEFLECTIONS





section
W460x128

AISC DATABASE
(METRIC UNITS)

$E = 200$ GPa

A determinate beam is loaded as shown. Knowing that the beam weight is included in the uniform load, answer the following questions:

(1) The maximum moment (kN.m) in the beam is most nearly:

(A) 25
(B) 40
(C) 60
(D) 72

$M_{max} = ?$

(2) The maximum deflection (mm) is most nearly:

(A) 2.2
(B) 1.8
(C) 2.7
(D) 1.4

$\delta_{max} = ?$

NCEES-RH
PAGE-84
DEFLECTIONS

(3) The slope (degrees) at the right edge is most nearly

(A) 0.052
(B) 0.028
(C) 0.024
(D) 0.036

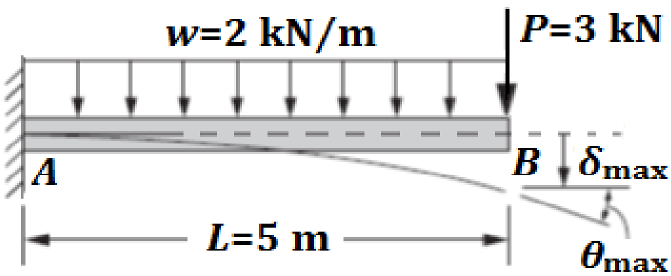
$\theta_{max} = ?$

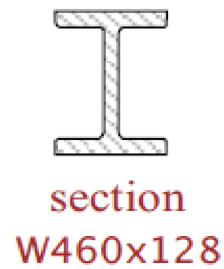
ANSWERS

DEF-284
VPLEVRIS
SPRING 2016

MECHANICS OF SOLIDS
DESIGN OF STEEL STRUCTURES
BEAM DEFLECTIONS

Problem: ***DEF-284*** ***Solution in MS Excel***





SECTION: **W460X128**

E	=	200	GPa
P	=	3	kN
w	=	2	kN/m
L	=	5	m

ID	Quantity	Symbol	Value	Unit
36	x - Moment of Inertia	I_x	63700.00	cm^4

E = 200000000 kPa

I_x = 0.000637 m^4

M_P = 15 kN·m

M_w = 25 kN·m

M_{max} = 40 kN·m

$$M_P = PL$$

$$M_w = \frac{wL^2}{2}$$

δ_P = 0.00098 m

δ_w = 0.00123 m

δ_{max} = 0.00221 m

δ_{max} = 2.2 mm

$$\delta_P = \frac{PL^3}{3EI}$$

$$\delta_w = \frac{wL^4}{8EI}$$

θ_P = 0.00029 rad

θ_w = 0.00033 rad

θ_{max} = 0.00062 rad

θ_{max} = 0.036 deg

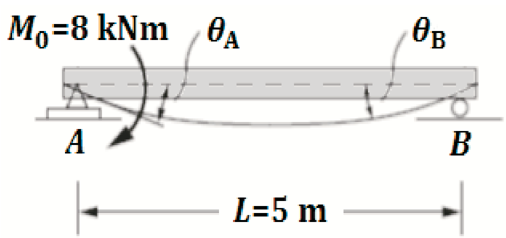
$$\theta_P = \frac{PL^2}{2EI}$$

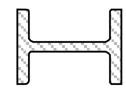
$$\theta_w = \frac{wL^3}{6EI}$$

MECHANICS OF SOLIDS

DESIGN OF STEEL STRUCTURES

BEAM DEFLECTIONS





section
W530x109

AISC DATABASE
(METRIC UNITS)

$E = 200$ GPa

A determinate beam is loaded as shown. Without taking into account the beam weight, answer the following questions:

(1) The maximum deflection (mm) is most nearly:

(A) 1.4
(B) 2.5
(C) 2.2
(D) 1.7

$\delta_{max} = ?$

(2) The slope (degrees) at the left support (A) is most nearly:

(A) -0.092
(B) -0.130
(C) -0.122
(D) -0.103

$\theta_A = ?$

NCEES-RH
PAGE-84
DEFLECTIONS

(3) The slope (degrees) at the right support (B) is most nearly:

(A) 0.032
(B) 0.043
(C) 0.051
(D) 0.065

$\theta_B = ?$

ANSWERS

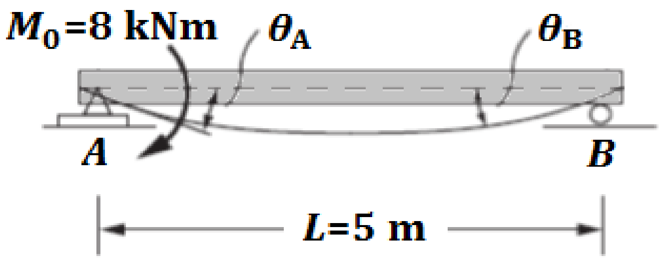
DEF-286
VPLEVRIS
SPRING 2016

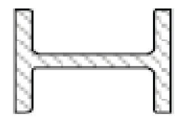
MECHANICS OF SOLIDS

DESIGN OF STEEL STRUCTURES

BEAM DEFLECTIONS

Problem: *DEF-286* *Solution in MS Excel*





SECTION: **W530X109**

E = 200 GPa

M_0 = 8 kNm (clockwise)

L = 5 m

ID	Quantity	Symbol	Value	Unit
40	y - Moment of Inertia	I_y	2940.00	cm^4

E = 200000000 kPa

I_y = 0.0000294 m^4

δ_{max} = 0.00218 m

δ_{max} = 2.2 mm

$$\delta_{max} = \frac{M_0 L^2}{\sqrt{243} EI}$$

θ_A = -0.00227 rad

θ_A = -0.130 deg

θ_B = 0.00113 rad

θ_B = 0.065 deg

$$\theta_A = \frac{-M_0 L}{3EI}$$

$$\theta_B = \frac{M_0 L}{6EI}$$

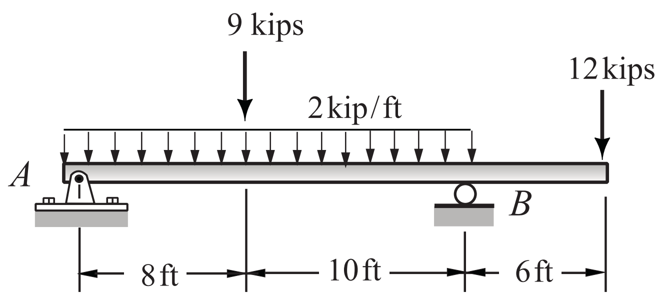
FUNDAMENTALS OF ENGINEERING

SHEAR FORCE AND BENDING MOMENT

STATICS & STRENGTH OF MATERIALS

Problem:

FE/PE EXAMS



Support A : Pin
Support B : Roller

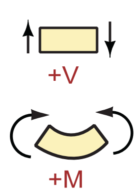
An overhanging beam is loaded as shown. Using the given dimensions and support conditions, answer the following:

(1) The vertical support reaction (kips) at B is most nearly:

(A) 33.5
(B) 38.0
(C) 42.5
(D) 47.0

$B_y = ?$

(2) The absolute maximum bending moment (k-ft) in the beam is most nearly:



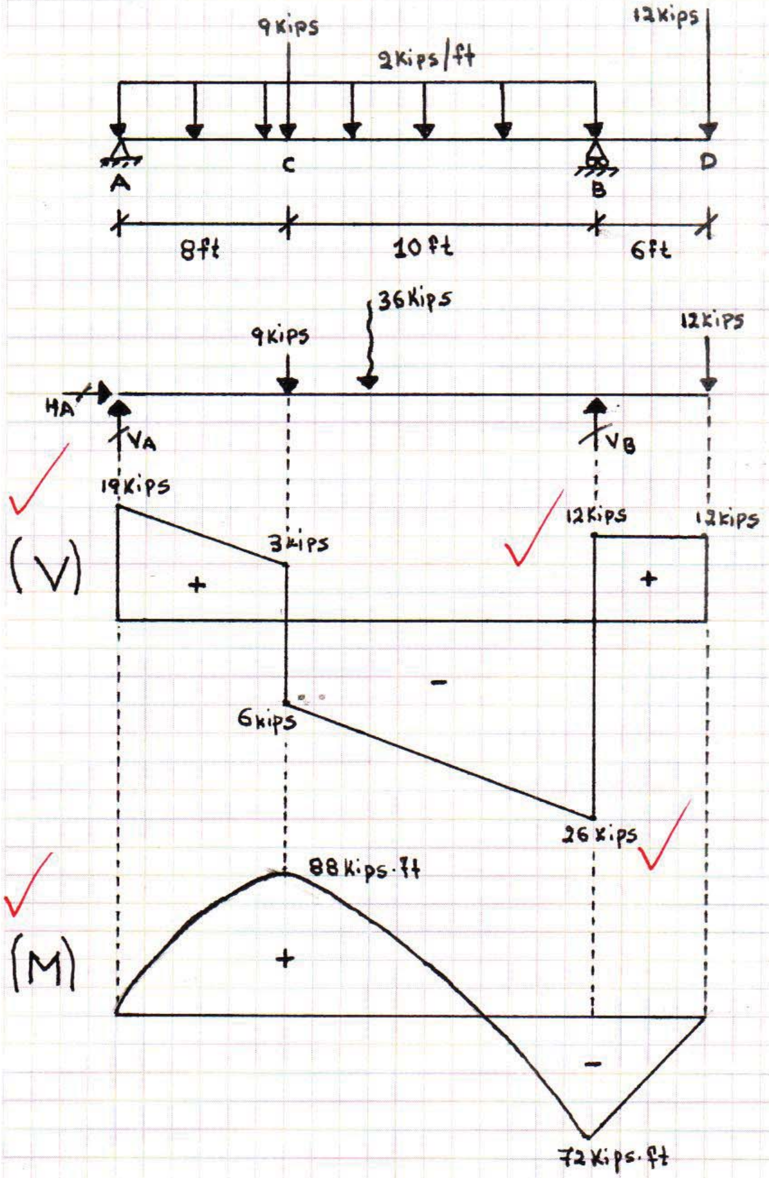
(A) 88.0
(B) 94.5
(C) 72.0
(D) 108.0

$M_{max} = ?$

ANSWERS

BM-626
VPLEVRIS
SPRING 2016

Solution:



$\Sigma F_x = 0 \Rightarrow H_A = 0$ (1)

$\curvearrowright + \; \Sigma M_A = 0 \Rightarrow 2 \times 18 \times \frac{18}{2} + 9 \times 18 + 12 \times 24 - V_B \times 18 = 0 \Rightarrow \boxed{V_B = 38\ Kips}$ ✓ (2)

$\curvearrowright + \; \Sigma M_B = 0 \Rightarrow 12 \times 6 - 2 \times 18 \times \frac{18}{2} - 9 \times 10 + V_A \times 18 = 0 \Rightarrow \boxed{V_A = 19\ kips}$ ✓ (3)

Check: $V_A + V_B \stackrel{?}{=} 9 + 12 + 2 \times 18 \Rightarrow$
$\Rightarrow 57 Kips \stackrel{?}{=} 57 Kips$ OK

$M_C = \text{Area of } V(AC) = \frac{8}{2} \times (19+3) \Rightarrow$
$\Rightarrow \boxed{M_C = 88\ Kips \times ft}$ ✓

$M_B = \text{Area of } V(BD) = -12 \times 6 \Rightarrow$
$\Rightarrow \boxed{M_B = -72\ Kips \times ft}$ ✓

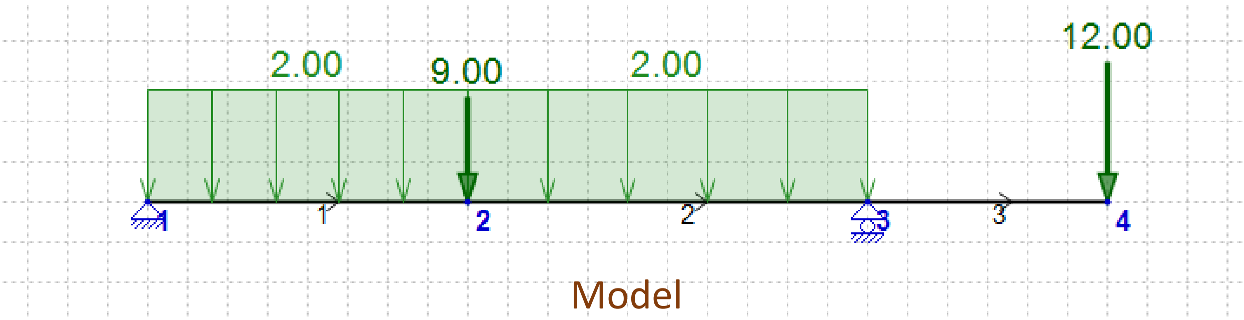
Great Job Theofilos!

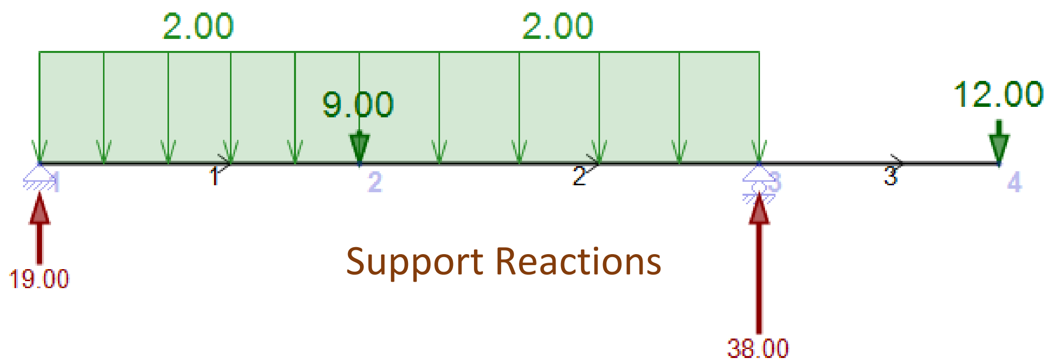
V. Plevris

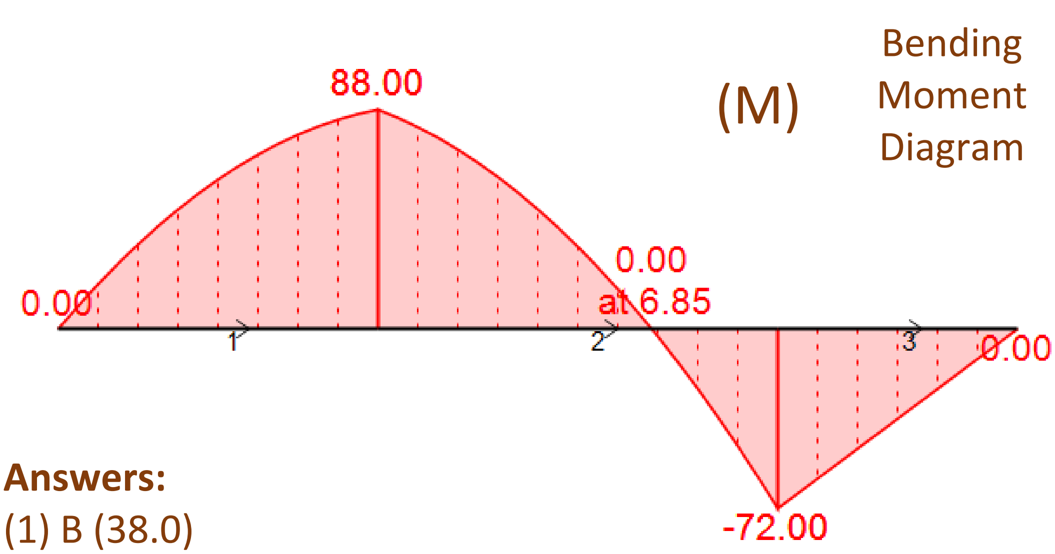
ANSWERS

1. (B) $V_B = 38$ Kips ✓
2. (A) $M_{max} = 88$ Kips·ft ✓

Solution by Dr. Vagelis Plevris







Answers:

(1) B (38.0)

(2) A (88.0)

BM-626
VPLEVRIS
SPRING 2016

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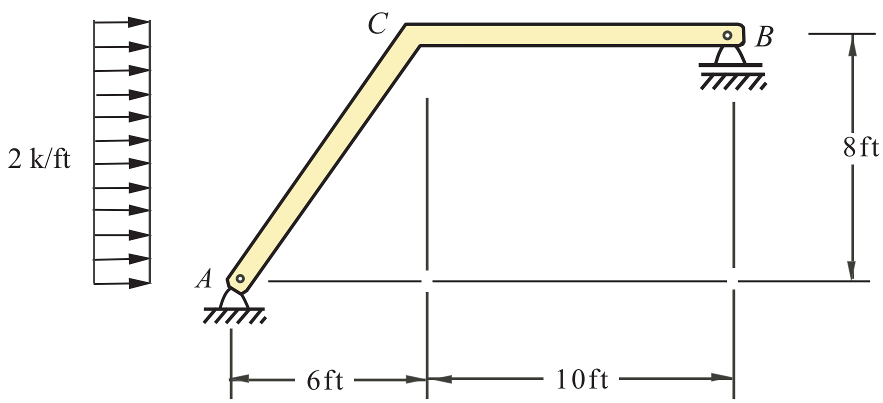
FUNDAMENTALS OF ENGINEERING

MECHANICS OF SOLIDS

Internal Forces in Determinate Frames

Problem:



Support A : Pin
Support B : Roller

The dimensions, loading and support conditions of a determinate frame are given as shown in the figure:

(1) the magnitude of the *support reaction* (kips) at point *B*

(A) 8.0
(B) 6.4
(C) 5.5
(D) 4.0

(2) the magnitude of the *axial force* (kips) at point *A*

(A) 18.4
(B) 14.6
(C) 12.8
(D) 10.6

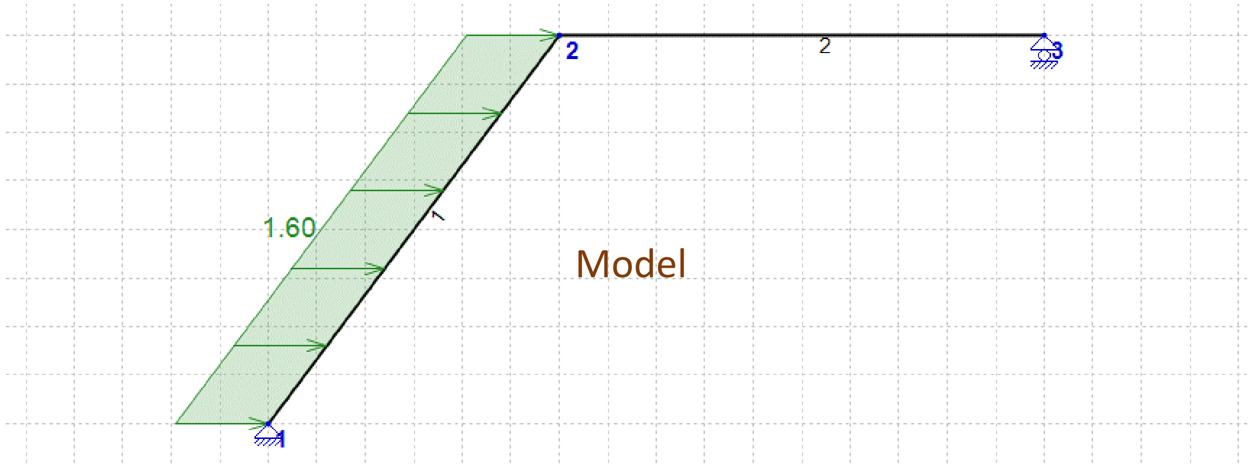
(3) the magnitude of the *shear force* (kips) at point *A* is most nearly:

(A) 8.6
(B) 10.4
(C) 12.8
(D) 16.5

ANSWERS

FR-235
VPLEVRIS
SPRING 2016

Solution by Dr. Vagelis Plevris



Model Equilibrium

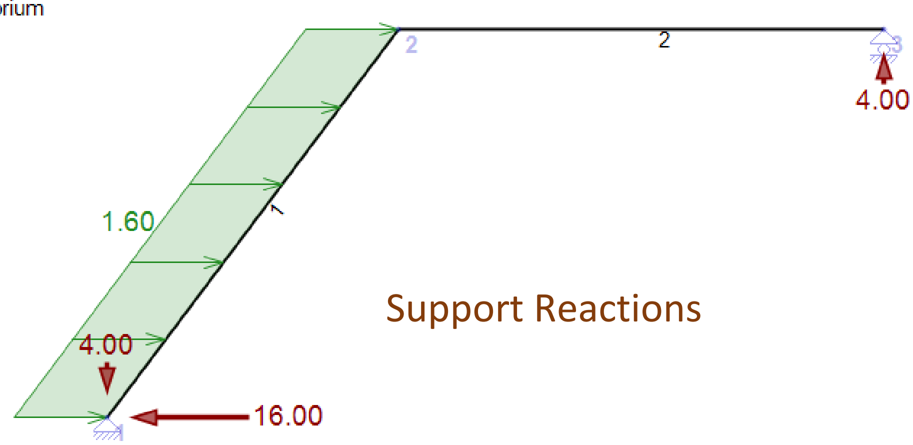
ΣFX= 0.00
ΣFY= 0.00
ΣM = 0.00

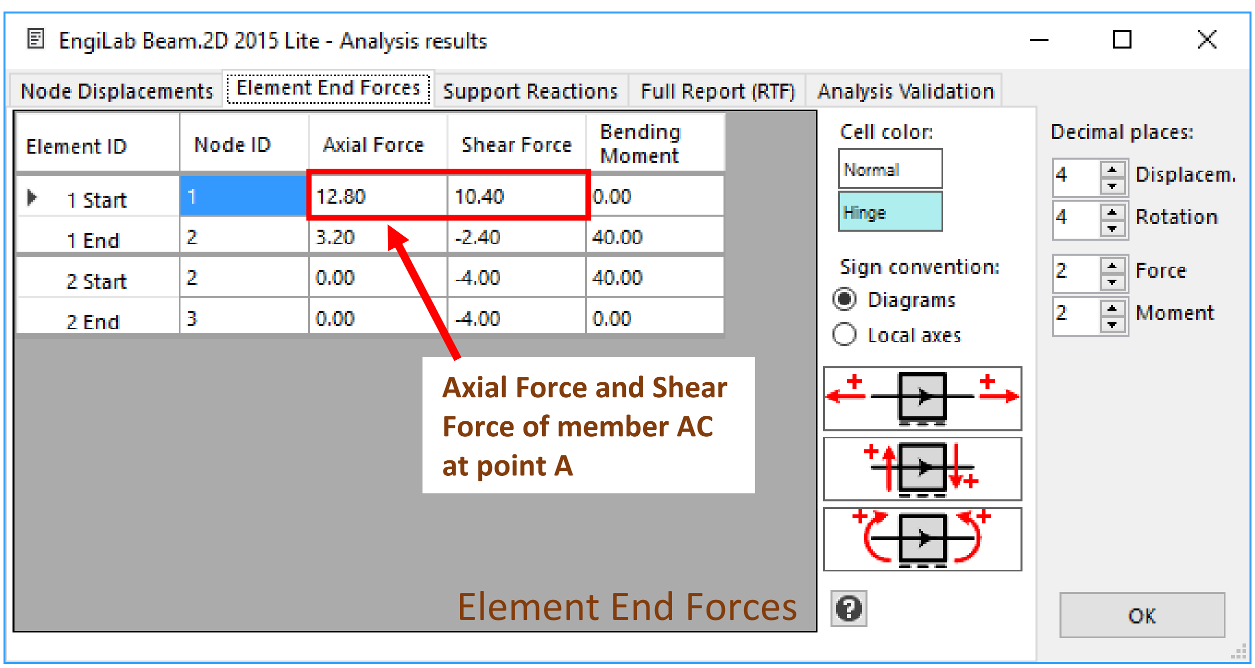
Answers:

(1) D (4.0)

(2) C (12.8)

(3) B (10.4)





Element ID	Node ID	Axial Force	Shear Force	Bending Moment
1 Start	1	12.80	10.40	0.00
1 End	2	3.20	-2.40	40.00
2 Start	2	0.00	-4.00	40.00
2 End	3	0.00	-4.00	0.00

FR-235
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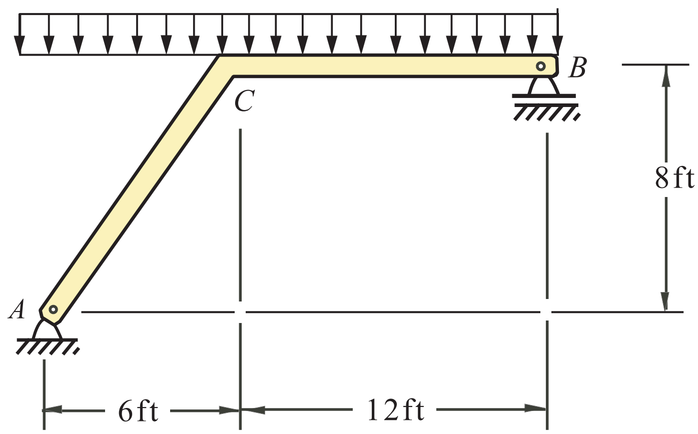
FUNDAMENTALS OF ENGINEERING

MECHANICS OF SOLIDS

Internal Forces in Determinate Frames

Problem:



Support *A* : Pin
Support *B* : Roller

The dimensions, loading and support conditions of a determinate frame are given as shown in the figure:

(1) the magnitude of the *support reaction* (kips) at point *B*

(A) 24
(B) 18
(C) 14
(D) 12

(2) the magnitude of the *axial force* (kips) at point *A*

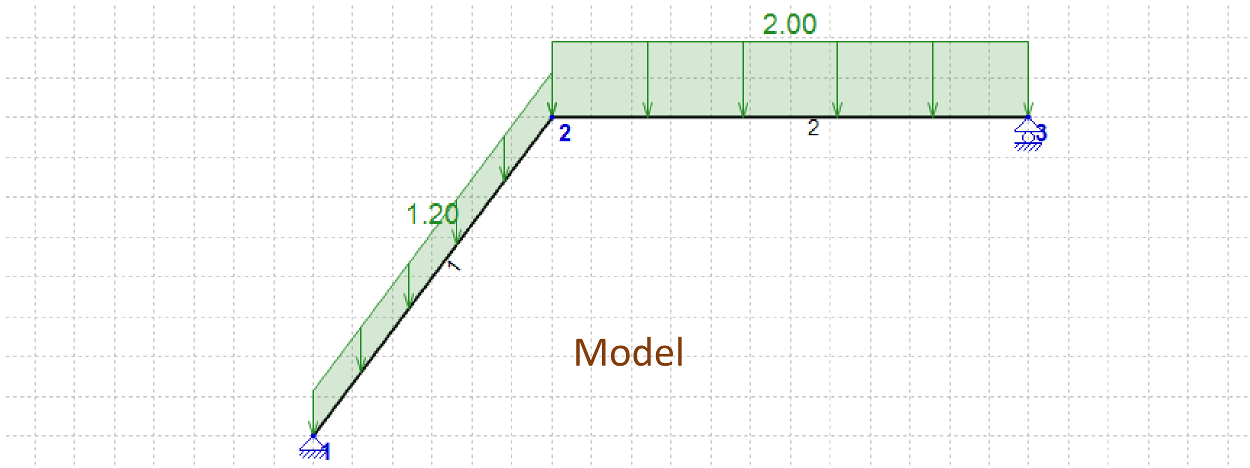
(A) 16.4
(B) 14.4
(C) 12.8
(D) 10.6

(3) the magnitude of the *max. bending moment* (ft-kips) in member *CB* is most nearly:

(A) 68.6
(B) 72.4
(C) 81.0
(D) 96.5

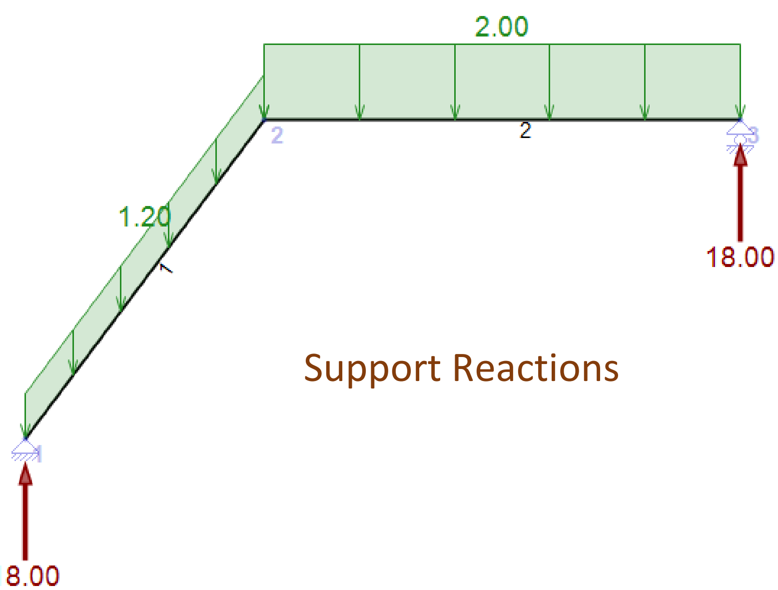
ANSWERS

Solution by Dr. Vagelis Plevris



Model Equilibrium

ΣFX= 0.00
ΣFY= 0.00
ΣM = 0.00

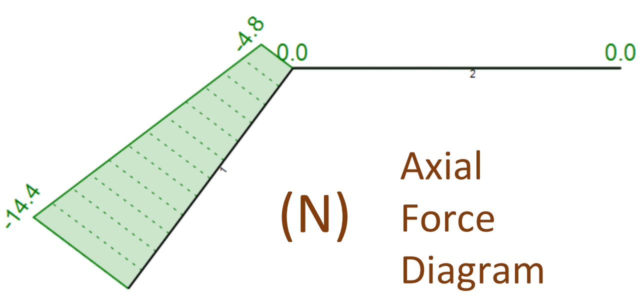


Answers:

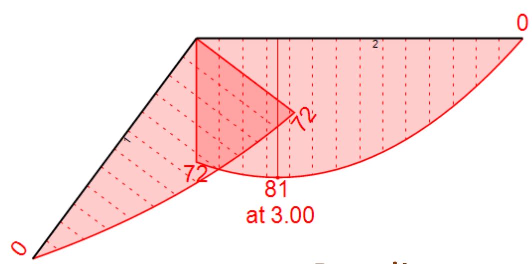
(1) B (18)

(2) B (14.4)

(3) C (81.0)



(N) Axial Force Diagram



(M) Bending Moment Diagram

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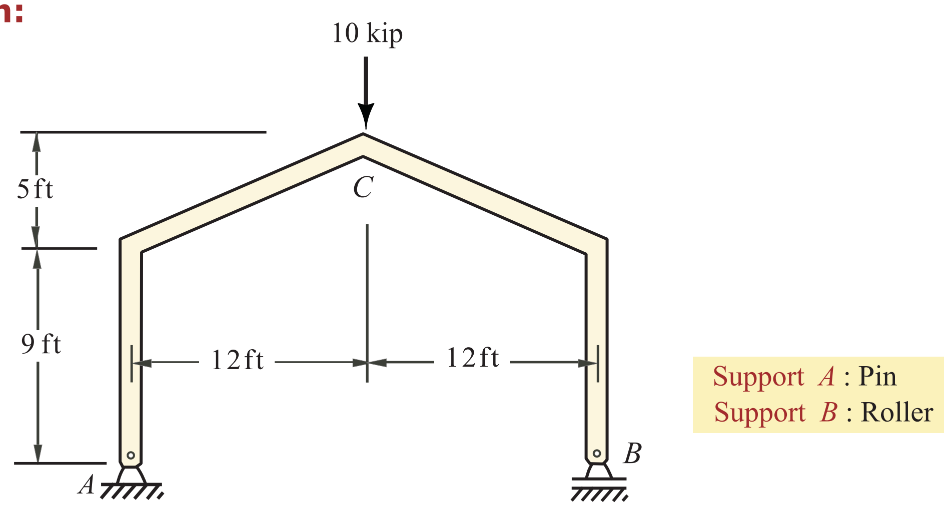
FUNDAMENTALS OF ENGINEERING

MECHANICS OF SOLIDS

Shear Force & Bending Moment Diagrams

Problem:



The dimensions and support conditions of a determinate frame are given as shown. Assuming that the vertical load P=10 kip is applied at C, answer the following questions:

(1) the *bending moment diagram* of this frame is composed of:

(A) Two triangles
(B) Two rectangles
(C) One triangle and two rectangles
(D) One rectangle and one triangle

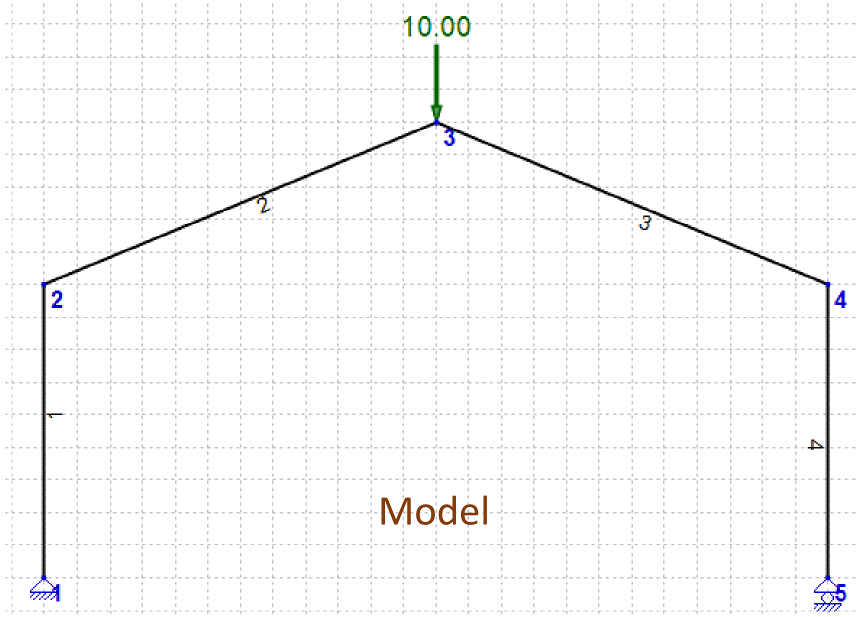
(2) the *shear force diagram* of this frame is composed of:

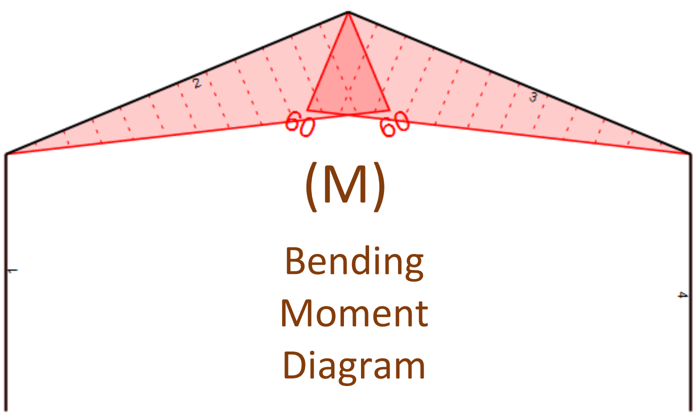
(A) Two triangles
(B) Two rectangles
(C) One triangle and two rectangles
(D) One rectangle and one triangle

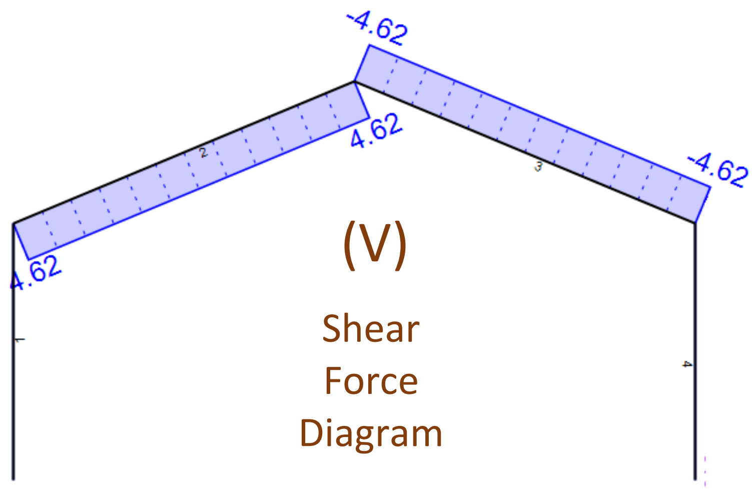
ANSWERS

FR-284
VPLEVRIS
SPRING 2016

Solution by Dr. Vagelis Plevris







Answers:

(1) A (Two triangles)

(2) B (Two rectangles)

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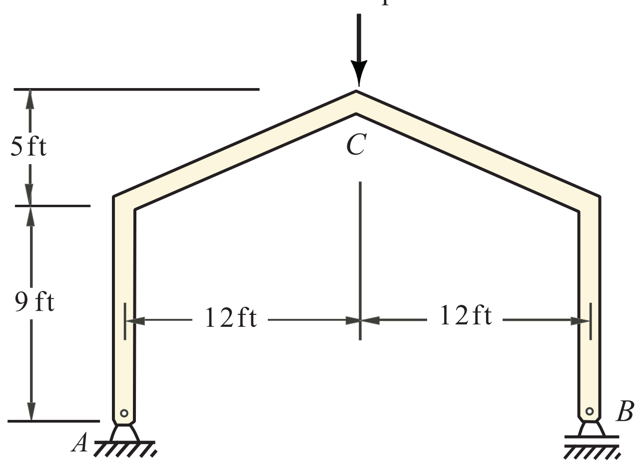
FUNDAMENTALS OF ENGINEERING

MECHANICS OF SOLIDS

Internal Forces in Determinate Frames

Problem:



Support *A* : Pin
Support *B* : Roller

The dimensions and support conditions of a determinate frame are given as shown. Assuming that the vertical load P=10 kip is applied at C, answer the following questions:

(1) the magnitude of the *bending moment* (ft-kips) at point C is most nearly:

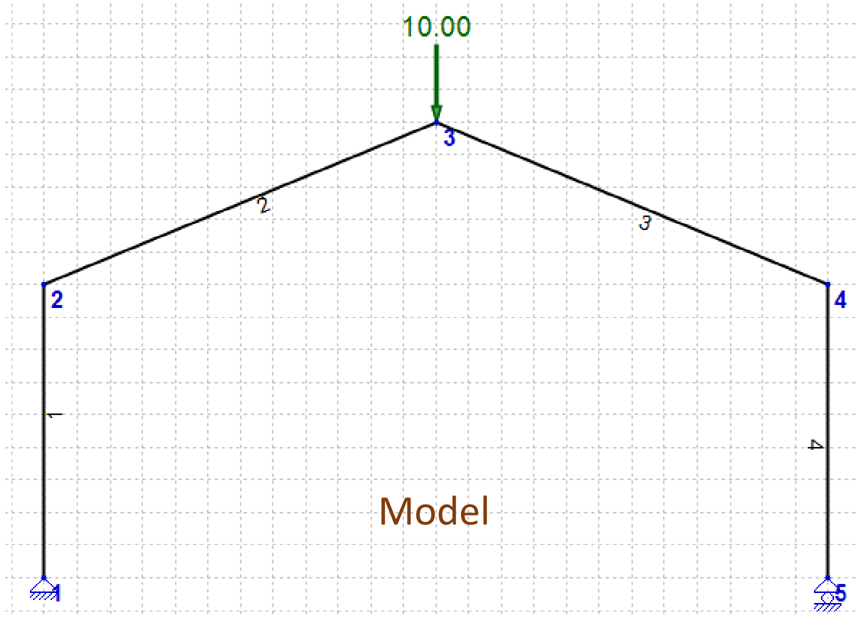
(A) 25
(B) 35
(C) 50
(D) 60

(2) the magnitude of the *shear force* (kips) in the inclined members is most nearly:

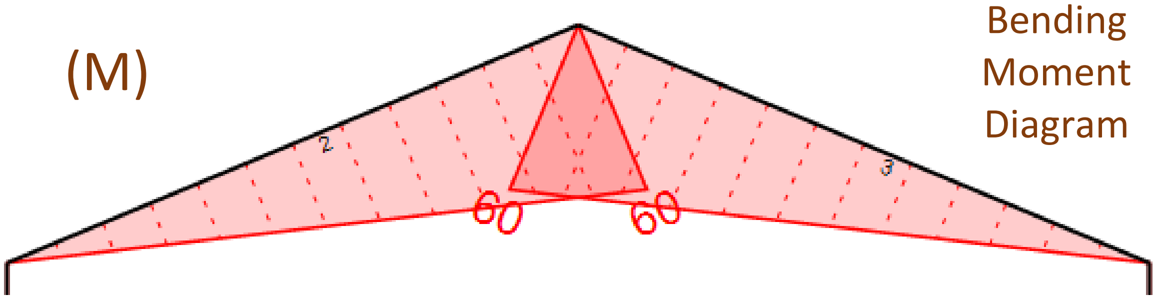
(A) 3.4
(B) 4.6
(C) 5.8
(D) 6.6

ANSWERS

Solution by Dr. Vagelis Plevris

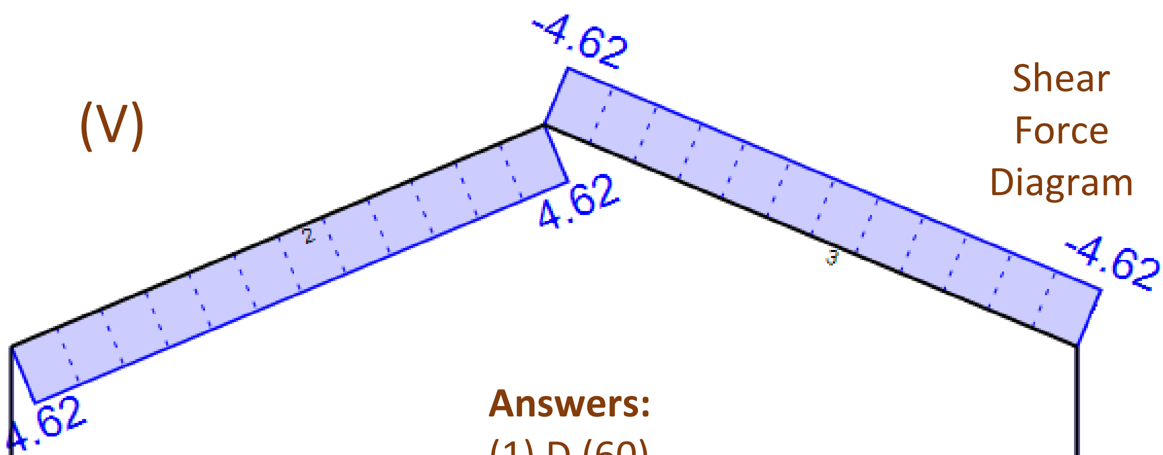


(M)



Bending Moment Diagram

(V)



Shear Force Diagram

Answers:

(1) D (60)

(2) B (4.6)

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SPRING 2016

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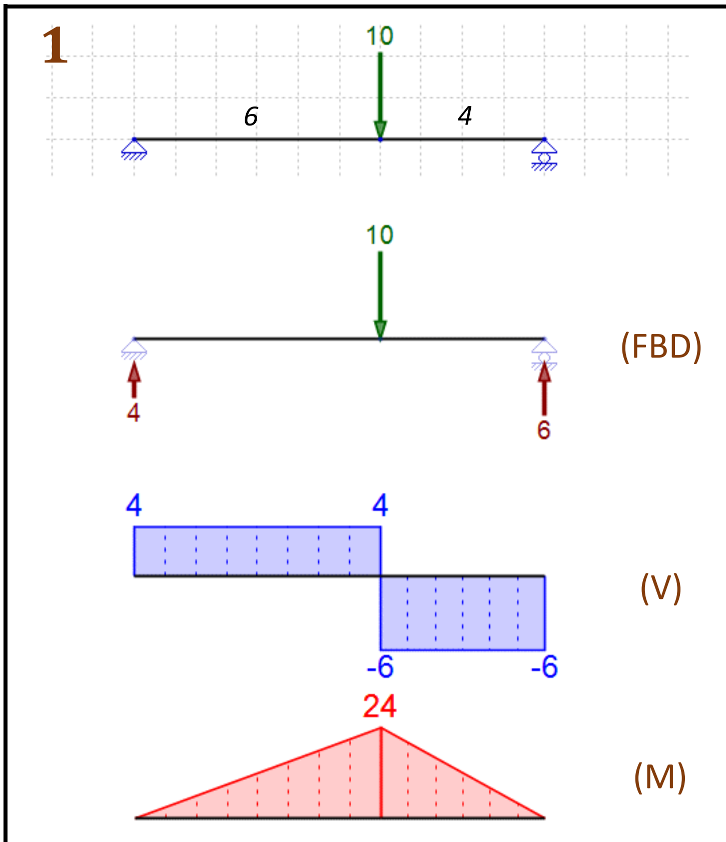
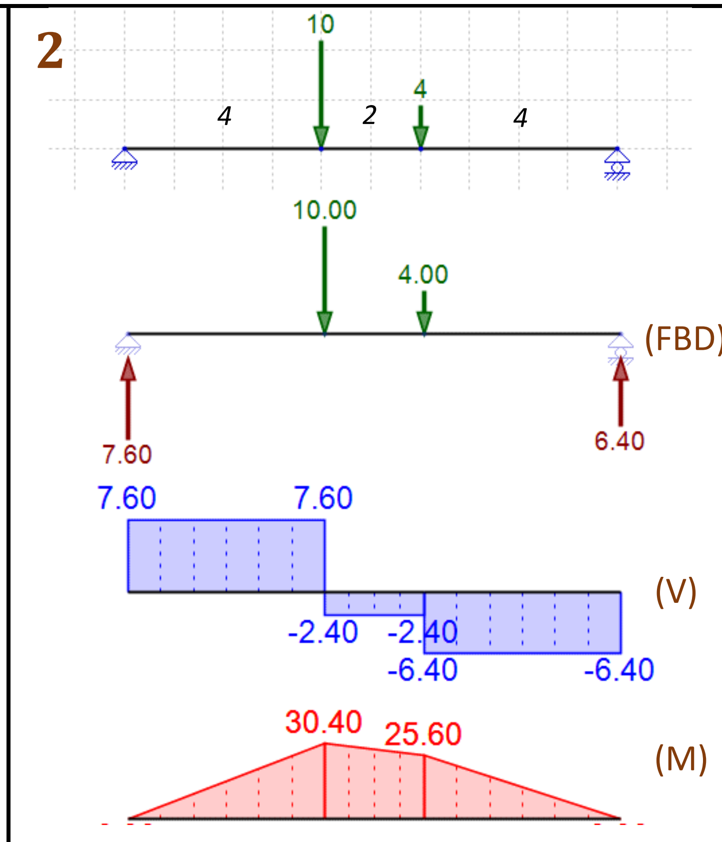
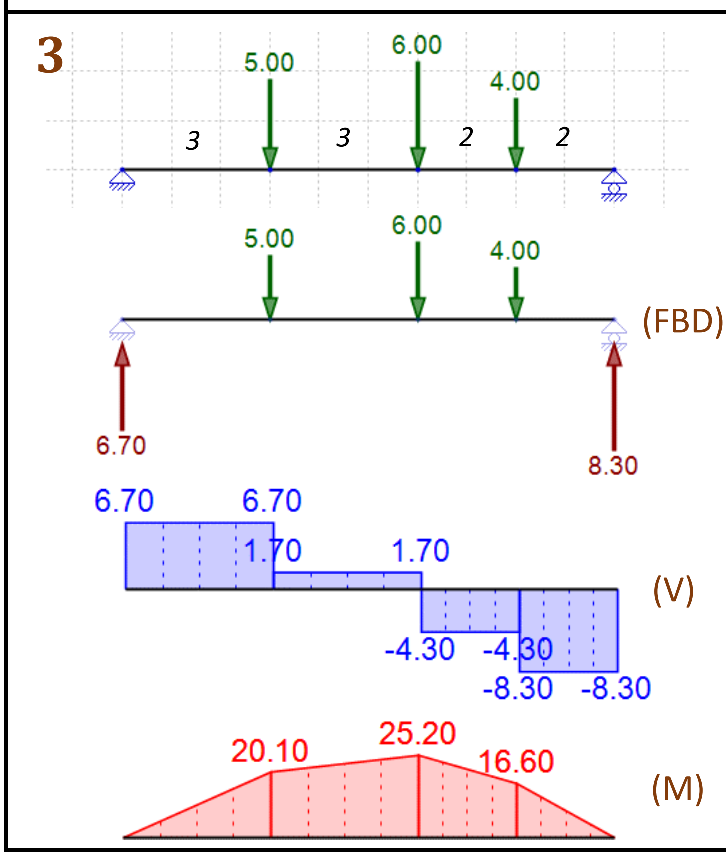
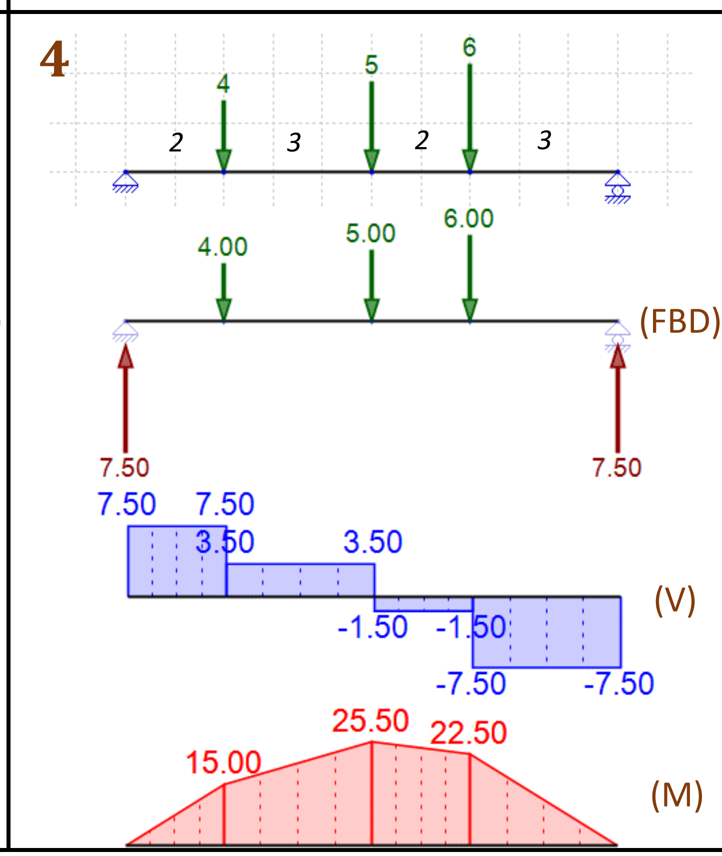
Shear Force & Bending Moment Diagrams

Simple Beams









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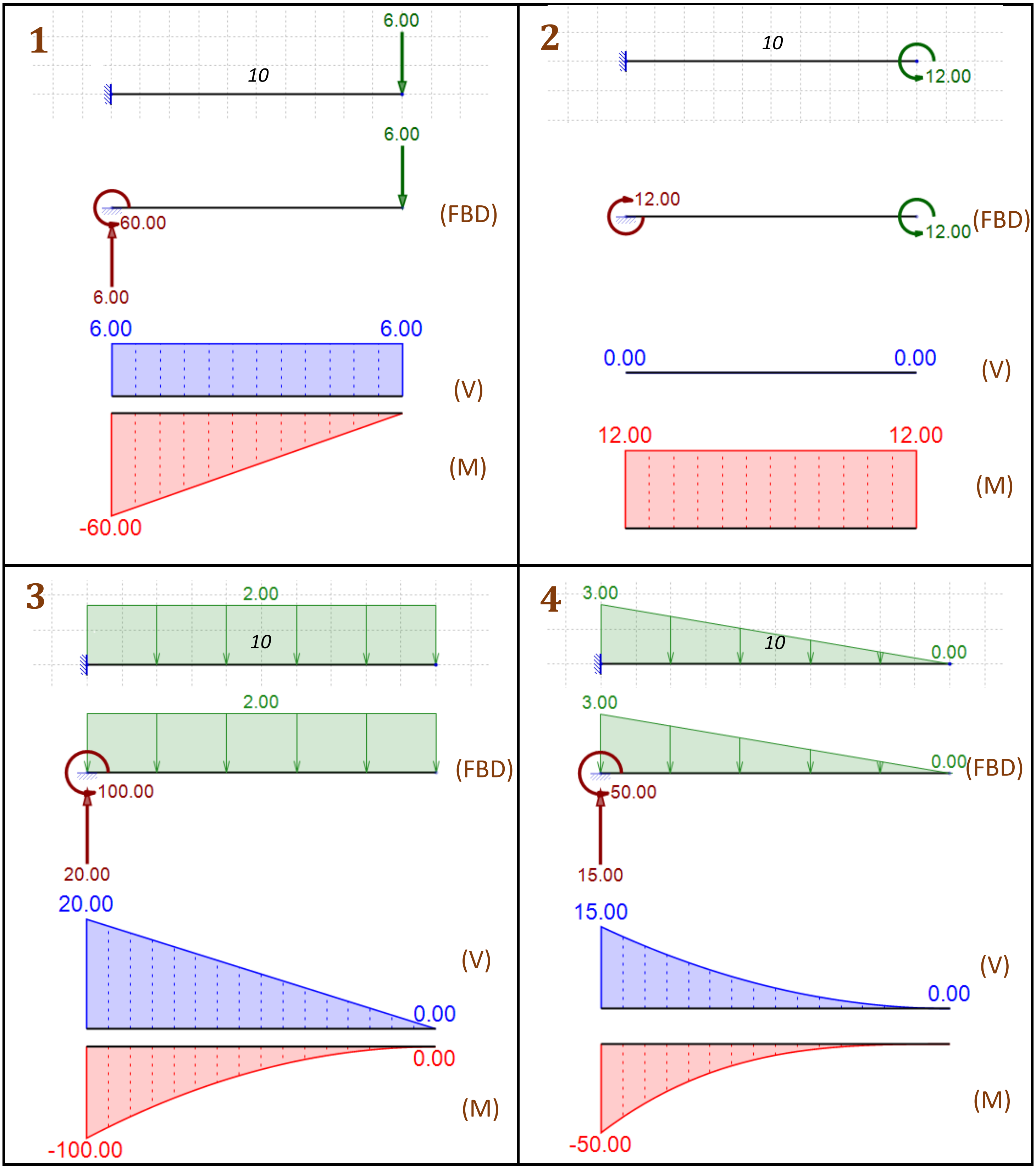
Shear Force & Bending Moment Diagrams

Cantilever Beams



MVN-802
VPLEVRIS
SPRING 2016

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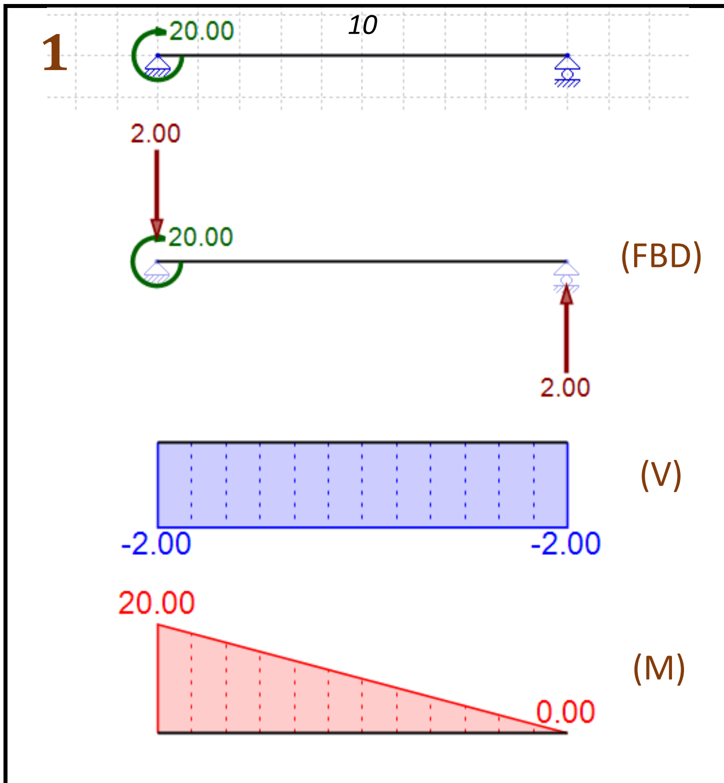
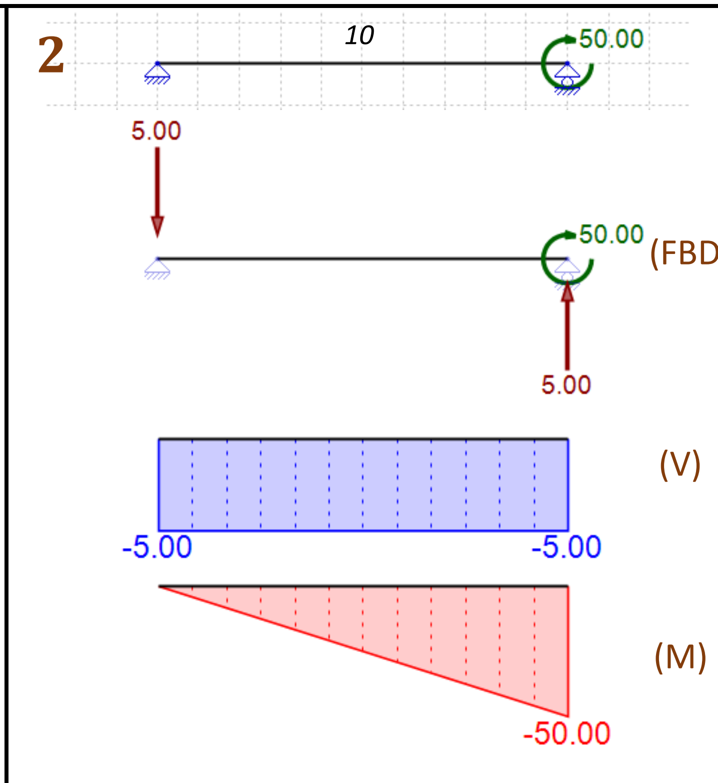
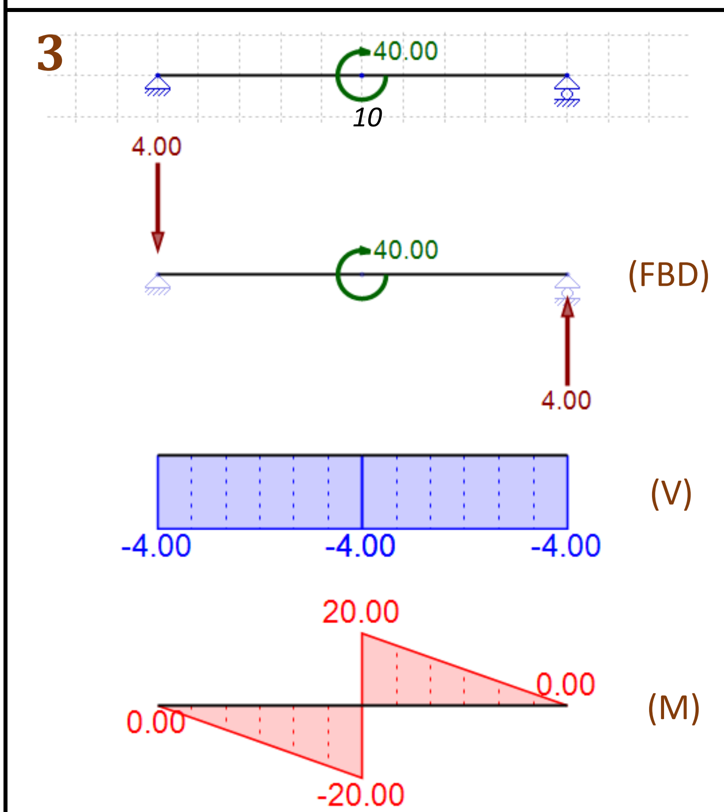
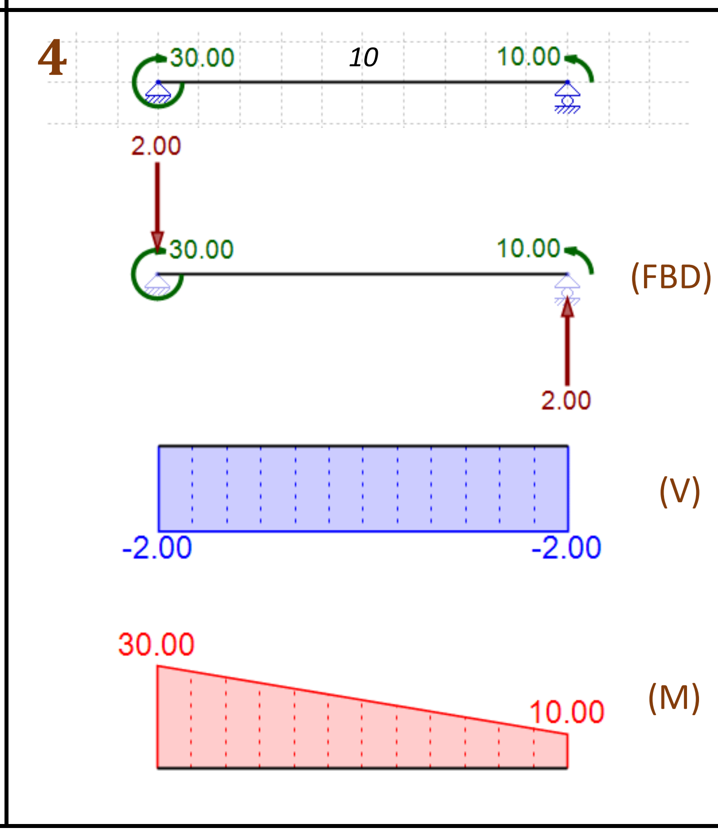
Shear Force & Bending Moment Diagrams

Simple Beams with Moments







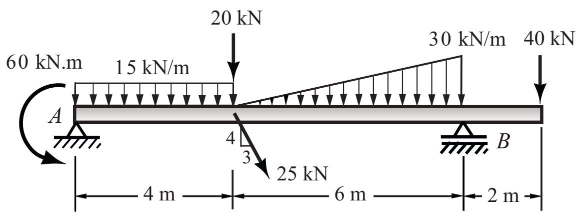


MVN-804
VPLEVRIS
SPRING 2016

Software BEAM.2D by ENGILAB - www.engilab.com

HOW TO CALCULATE SUPPORT REACTIONS OF DETERMINATE BEAMS?

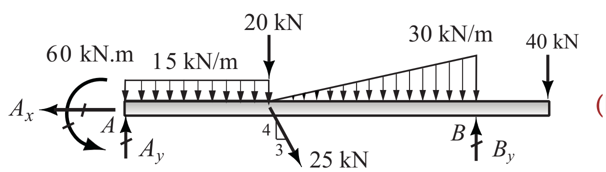
Problem: (Determinate Beams /Support Reactions)



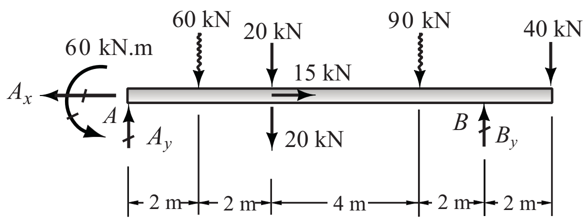
Find the support reactions.

Support A : Hinge
Support B : Roller

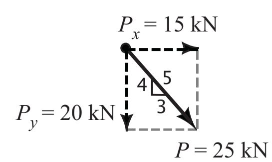
Solution:



(FBD)



How to find the components of the P = 25 kN inclined load?



Horizontal Equilibrium Equation: ($\Sigma F_x = 0$)

$\xrightarrow{+} \Sigma F_x = 0$ $\quad -A_x + 15 \text{ kN} = 0 \quad$ $A_x = 15$ kN ←

$P_x = P\,(3/5) = 25\,(3/5) = 15$ kN
$P_y = P\,(4/5) = 25\,(4/5) = 20$ kN

Taking moment about point *A*

$\circlearrowright^{+} \; \Sigma M_A = 0$

$$-60 + (60)(2m) + (20+20)(4m) + (90)(8m)+(40)(12m) - 10.B_y = 0$$

$10.B_y = -60 + 120 + 160 + 720 + 480$ solving for B_y : $B_y = 142$ kN ↑

Taking moment about point *B*

$\circlearrowright^{+} \; \Sigma M_B = 0$

$$60 + (60)(8m) + (20+20)(6m) + (90)(2m) - (40)(2m) - 10.A_y = 0$$

$10.A_y = 60 + 480 + 240 + 180 - 80$ solving for A_y : $A_y = 88$ kN ↑

Check:

$+\uparrow \Sigma F_y = 0$

$$A_y + B_y \overset{?}{=} 60 + 20 + 20 + 90 + 40$$
$$88 + 142 \overset{?}{=} 230$$
$$\underline{230 = 230} \quad \text{O.K.}$$

Answers

$A_x = 15$ kN ←
$A_y = 88$ kN ↑
$B_y = 142$ kN ↑