Feedback and Side-Information in Information Theory

Anant Sahai and Sekhar Tatikonda

UC Berkeley and Yale
sahai@eecs.berkeley.edu and sekhar.tatikonda@yale.edu

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Our Collaborators

- Vivek Borkar (TIFR)
- Cheng Chang (Berkeley)
- Giacomo Como (Torino)
- Stark Draper (Wisconsin)
- Nicola Elia (Iowa)
- Alek Kavcic (Hawaii)
- Jialing Liu (Iowa)
- Sanjoy Mitter (MIT)
- Hari Palaiyanur (Berkeley)
- Tunc Simsek (Mathworks)
- Shaohua Yang (Marvell)
- Serdar Yuksel (Queens)

Outline

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- Guide overall system architecture
- Develop *nonasymptotic* algorithms that are *robustly implementable*.

Feedback and side-information

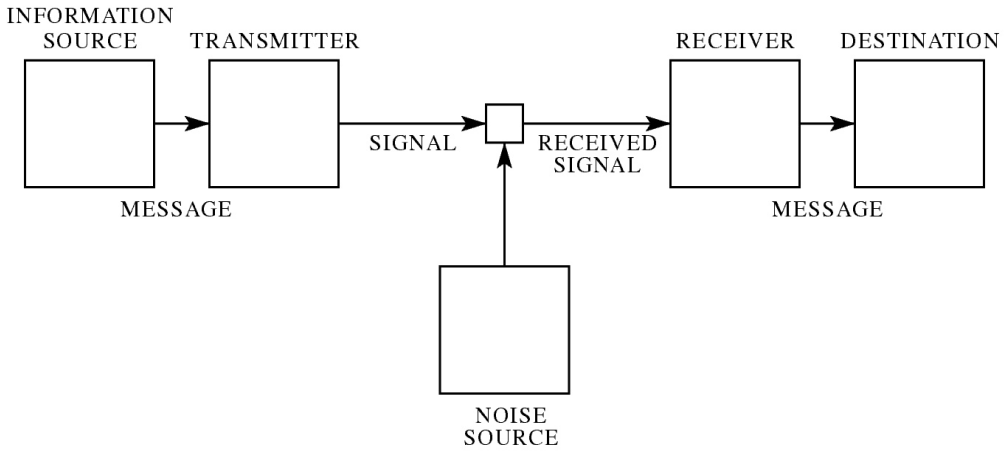

Fig. 1 — Schematic diagram of a general communication system.

- Communication is idealized as a one-way flow of information.

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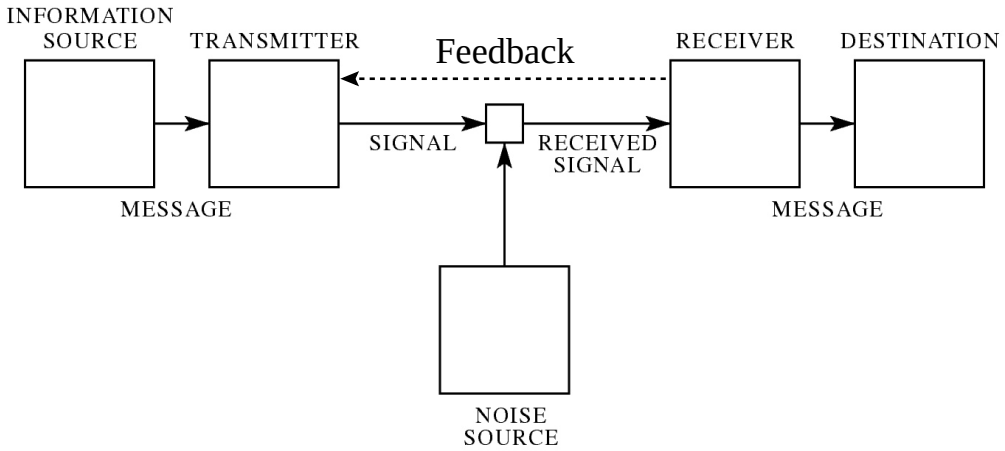

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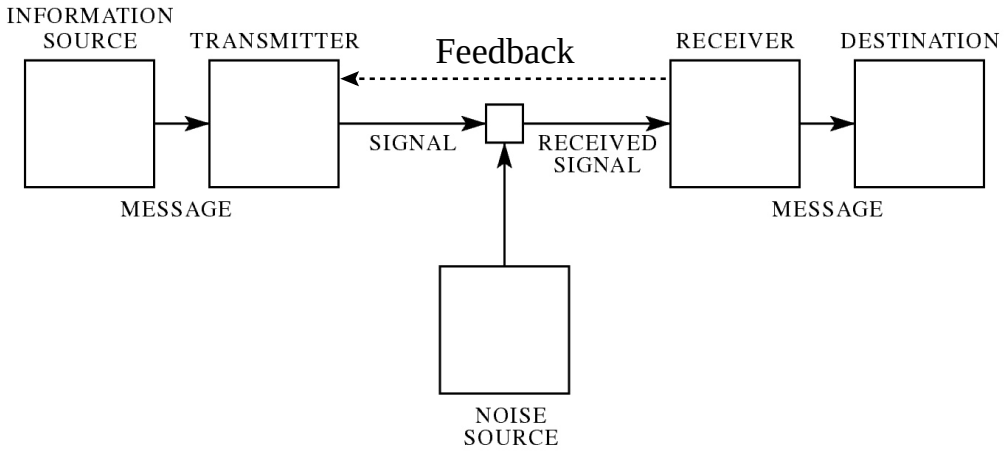

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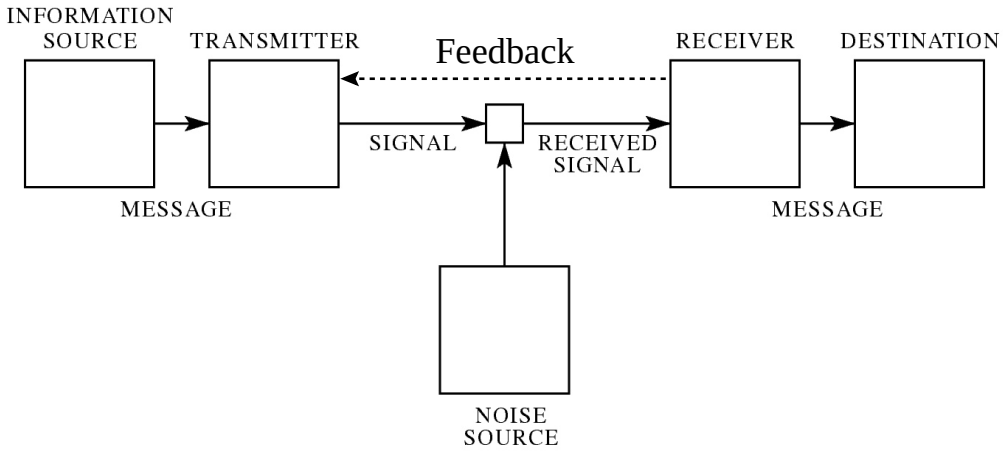

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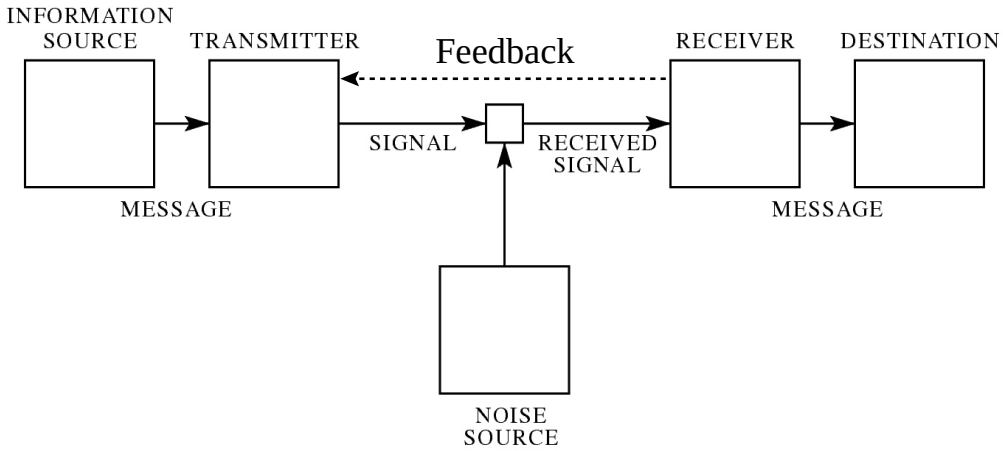

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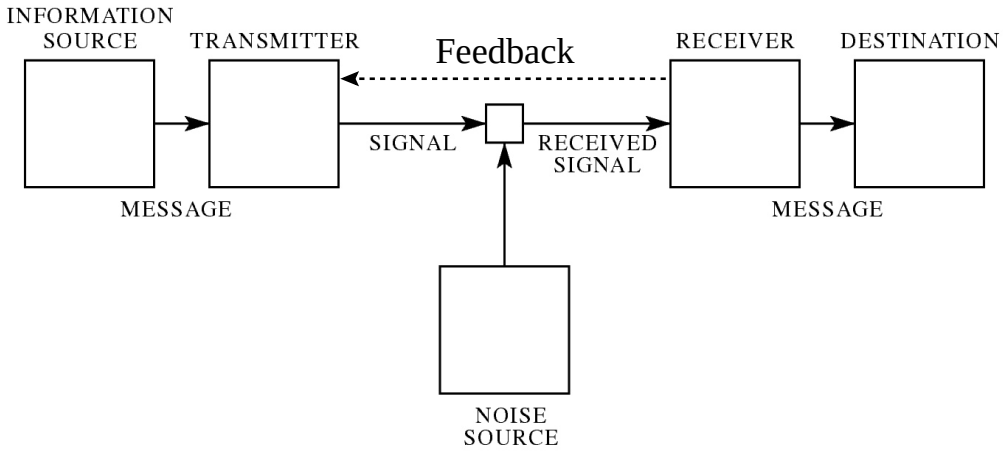

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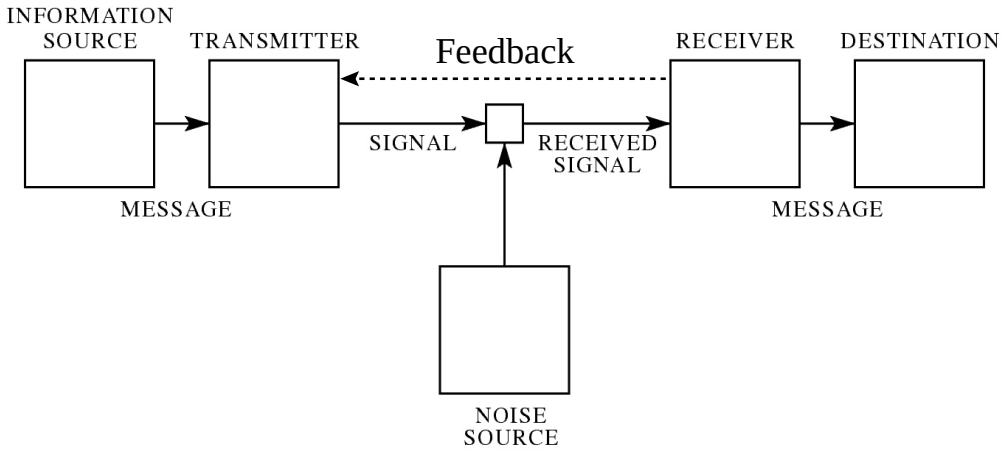

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  - Improve QoS: delay, P_e, rate.
  - Simplify implementation.

Reason for hope: Schalkwijk/Kailath ('68) scheme

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Y_t &= X_t + V_t \text{ where } \{V_t\} \text{ iid } N(0,1) \\
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  - Probability exponentially small in MMSE error standard deviation.
- Double-exponential decay of P_e and very simple scheme!

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- What is wrong with $I(X_1^n; Y_1^n)$?
  - Consider X, Y disconnected. Y is just random noise. $C = 0$
  - Use $X_i = Y_{i-1}$ as encoding. $I(X_1^n; Y_1^n) = (n-1)H(Y) > 0$.

It gets worse

"Feedback communications was an area of intense activity in 1968...A number of authors had shown constructive, even simple, schemes using noiseless feedback to achieve Shannon-like behavior...The situation in 1973 is dramatically different...The subject itself seems to be a burned out case...

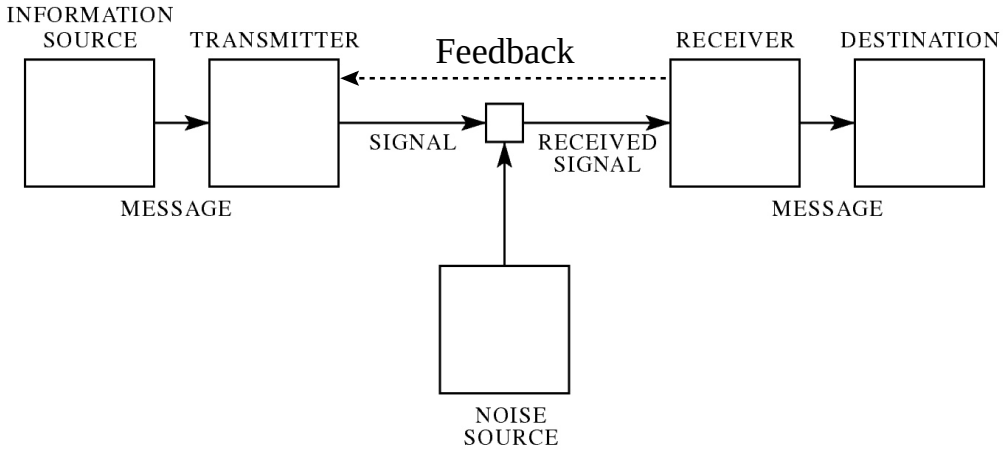
In extending the simple noiseless feedback model to allow for more realistic situations, such as noisy feedback channels, bandlimited channels, and peak power constraints, theorists discovered a certain "brittleness" or sensitivity in their previous results..."

Robert Lucky (1973)

Outline

1. Motivation and background
2. Feedback, delay, and error probability: memoryless channels
   - Idealized cases with “magical” feedback
     - ★ Block-oriented results
     - ★ Fixed-delay with hard deadlines
     - ★ Fixed-delay with soft deadlines
   - Unreliable/Noisy feedback
3. Feedback and memory
4. Conclusions and open problems

Shannon's Prophecy:



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- Delay is the most basic price of reliability

"[The duality between source and channel coding] can be pursued further and is related to a duality between past and future and the notions of control and knowledge. Thus we may have knowledge of the past and cannot control it; we may control the future but have no knowledge of it." — *Claude Shannon '59*

Review of block coding

- Long block codes are the traditional info theory approach
  - Source: $X_1^n \rightarrow B_1^{Rn} \rightarrow \widehat{X}_1^n$
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- Source coding:

$$\begin{aligned} E_b(R) &= \min_{Q:H(Q)\geq R} D(Q||P) \\ &= \sup_{\rho\geq 0} \rho R - E_0(\rho) \\ E_0(\rho) &= \ln[\sum_x P(x)^{\frac{1}{1+\rho}}]^{(1+\rho)} \end{aligned}$$

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- Generic channels: (Haroutunian '77)

$$E^+(R) \quad = \quad \inf_{G:C(G)<R} \max_{\vec{q}} D\left(G||P|\vec{q}\right)$$

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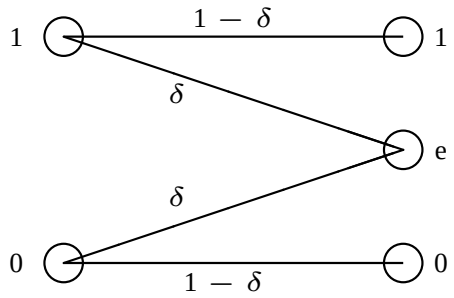
- Block error exponents: $P_e \propto \exp(-nE(R))$
- Channel "sphere-packing" bound: (Haroutunian '68, Blahut '74)

$$\begin{aligned} E_{sp}(R) &= \max_{\vec{q}} \min_{G:I(\vec{q},G)\leq R} D\left(G||P|\vec{q}\right) \\ &= \sup_{\rho\geq 0} E_0(\rho) - \rho R \\ E_0(\rho) &= \max_{\vec{q}} -\ln \sum_y \left[\sum_x q_x p_{y|x}^{\frac{1}{1+\rho}} \right]^{(1+\rho)} \end{aligned}$$

My favorite example: the BEC



- Simple capacity $1 - \delta$ bits per channel use
- With perfect feedback, simple to achieve: retransmit until it gets through
  - Time till success: Geometric$(1 - \delta)$
  - Expected time to get through: $\frac{1}{1-\delta}$

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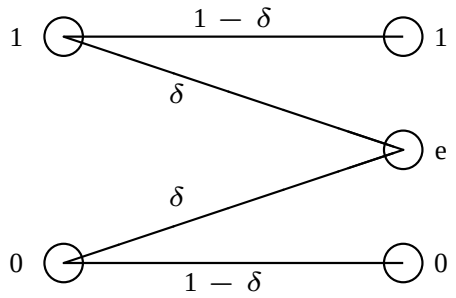

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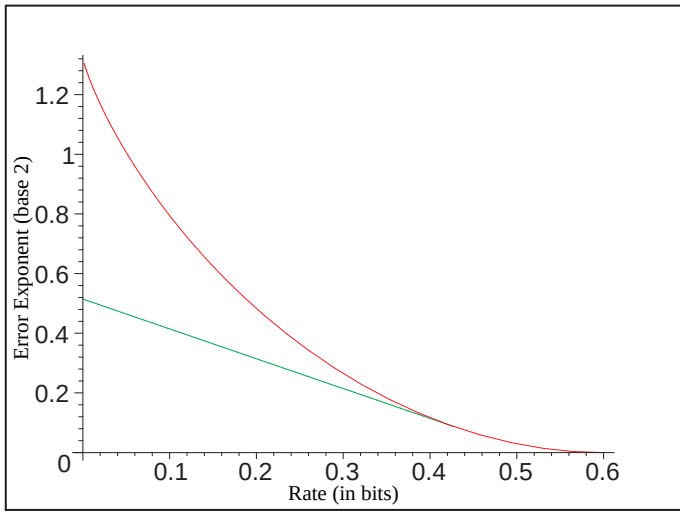

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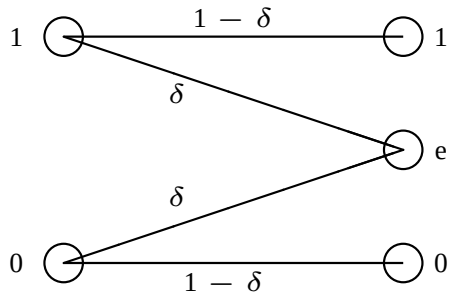

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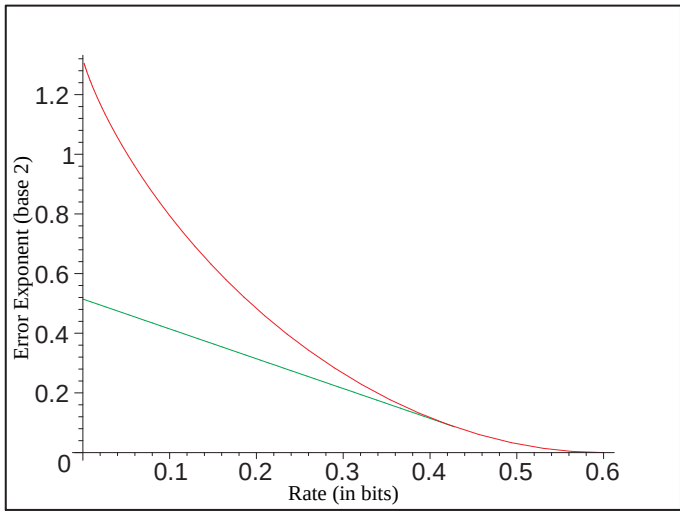

- Classical bounds
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- What happens with feedback?

BEC with feedback and fixed *blocks*

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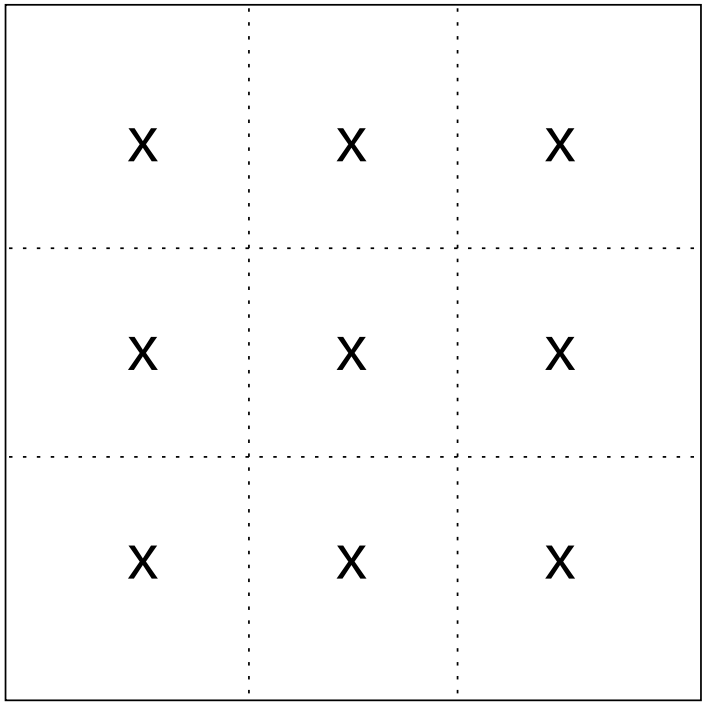
- Dobrushin-62 showed that this type of behavior is common: $E^+(R) = E_{sp}(R)$ for symmetric channels.

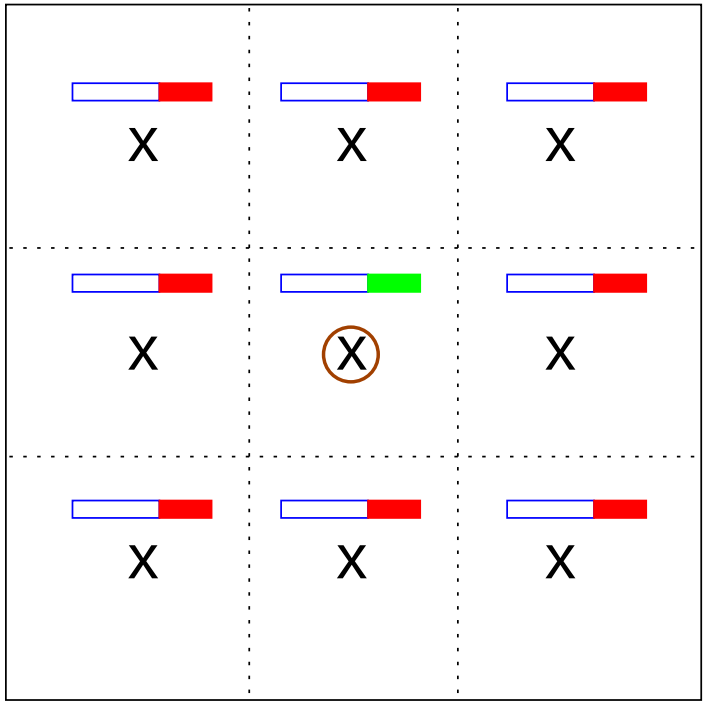
Review: Fixed blocks



- Hard decision regions cover space

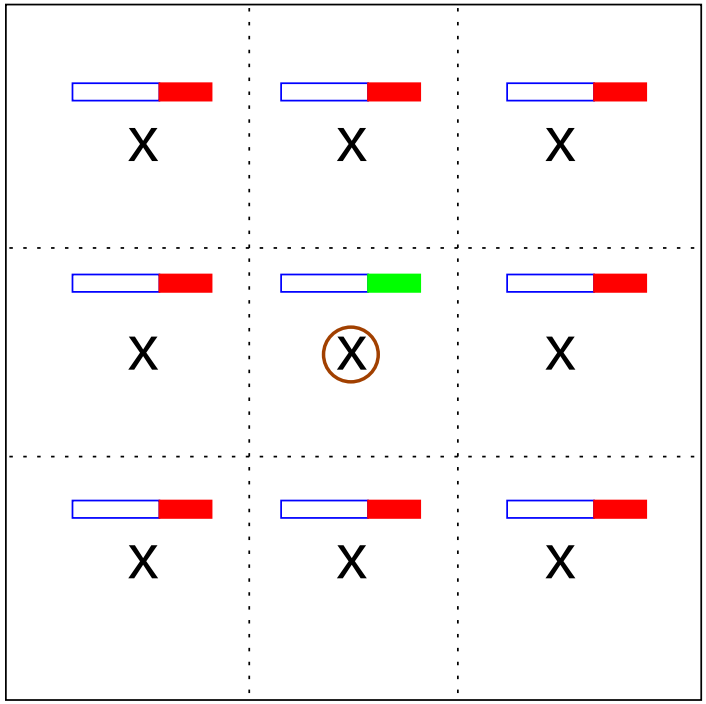
- Feedback is pointless

Review: Fixed blocks, Soft deadlines



- Hard decisions regions but check hash signatures

Review: Fixed blocks, Soft deadlines



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- Can interpret as expected block-length

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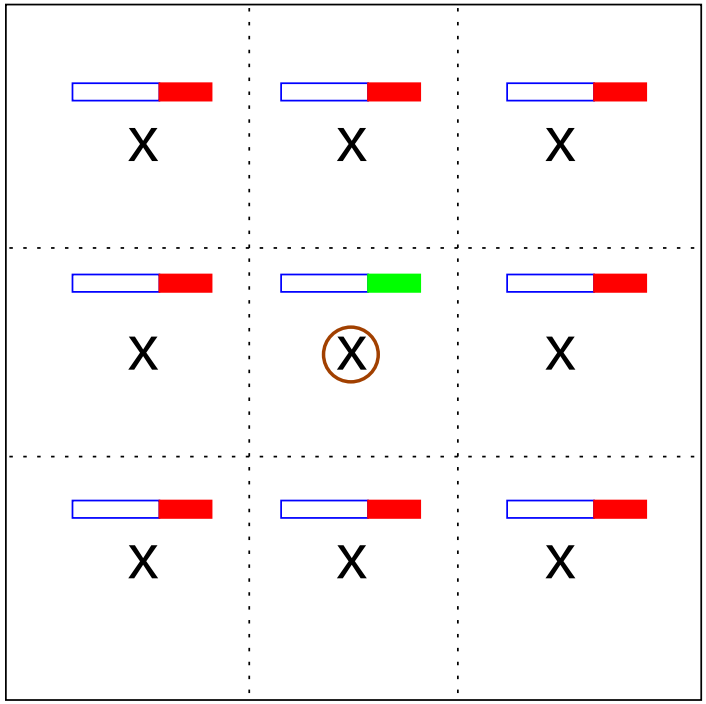

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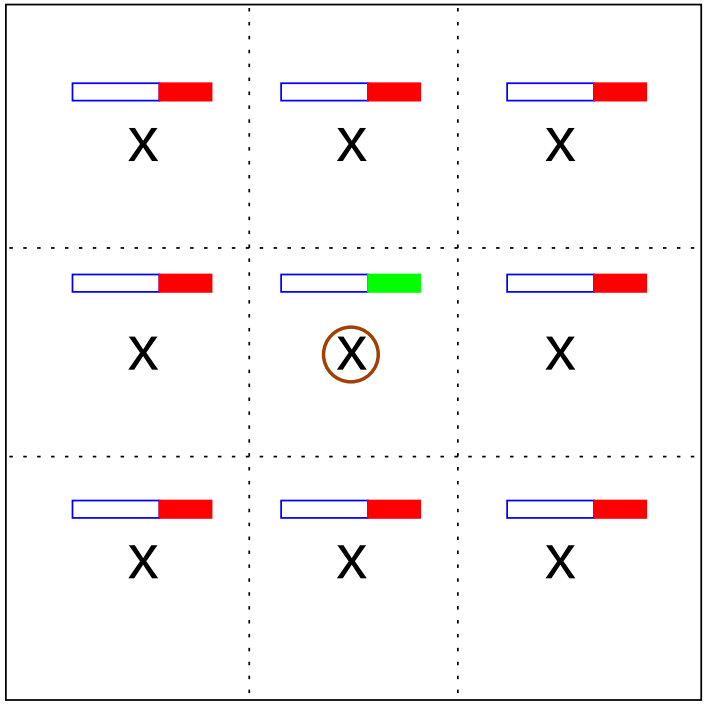

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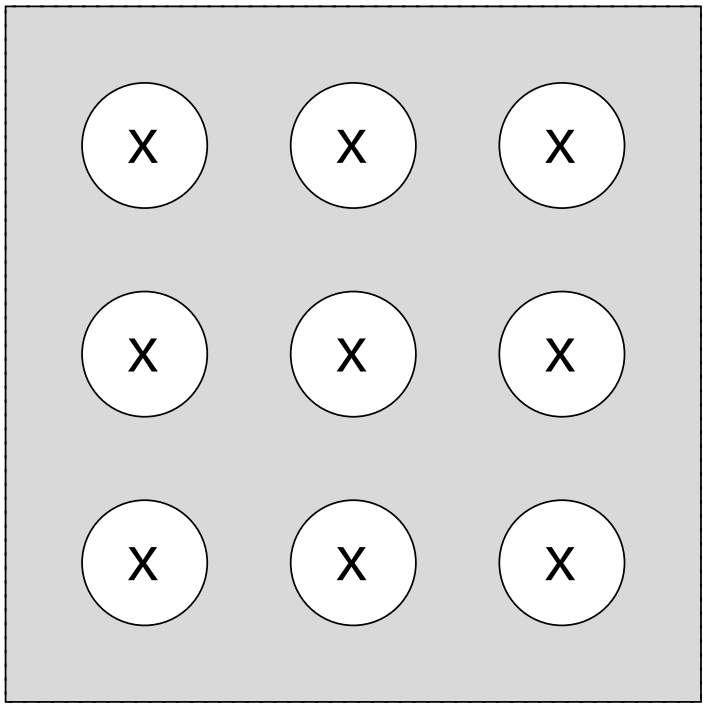
- Linear slope -1 for error exponent

Review: Fixed blocks, Soft deadlines: Forney-68



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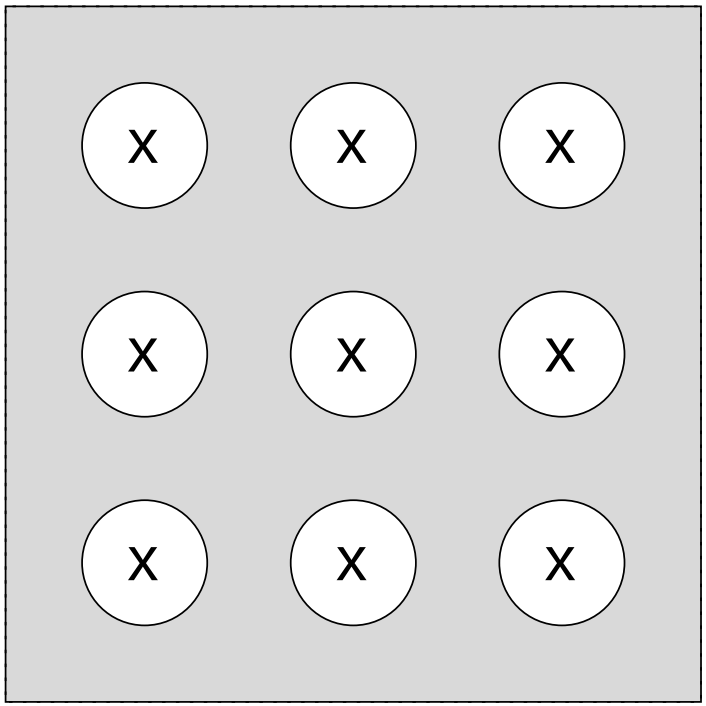

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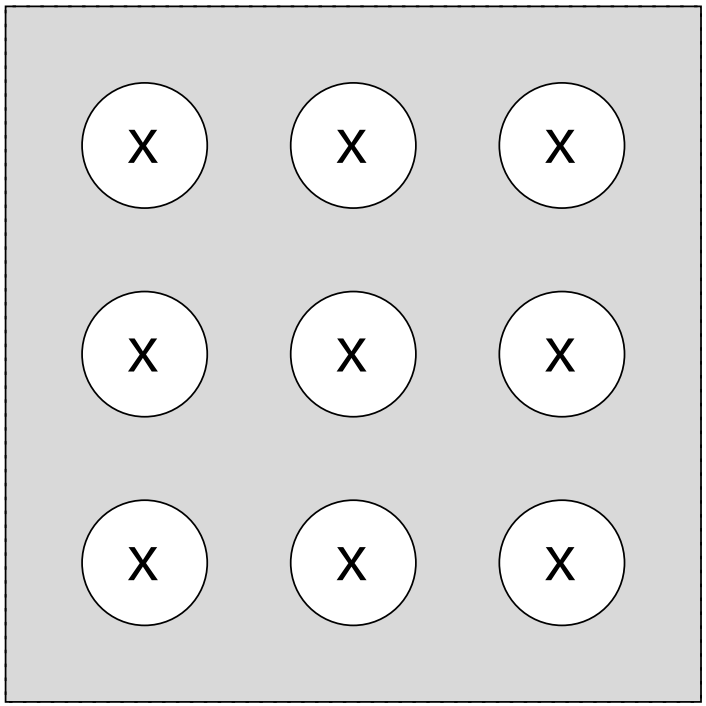

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- Can interpret as expected block-length
- Decision regions catch the typical sets only
- Better error exponents at lower rates

Review: Fixed blocks, Soft deadlines: Burnashev-76

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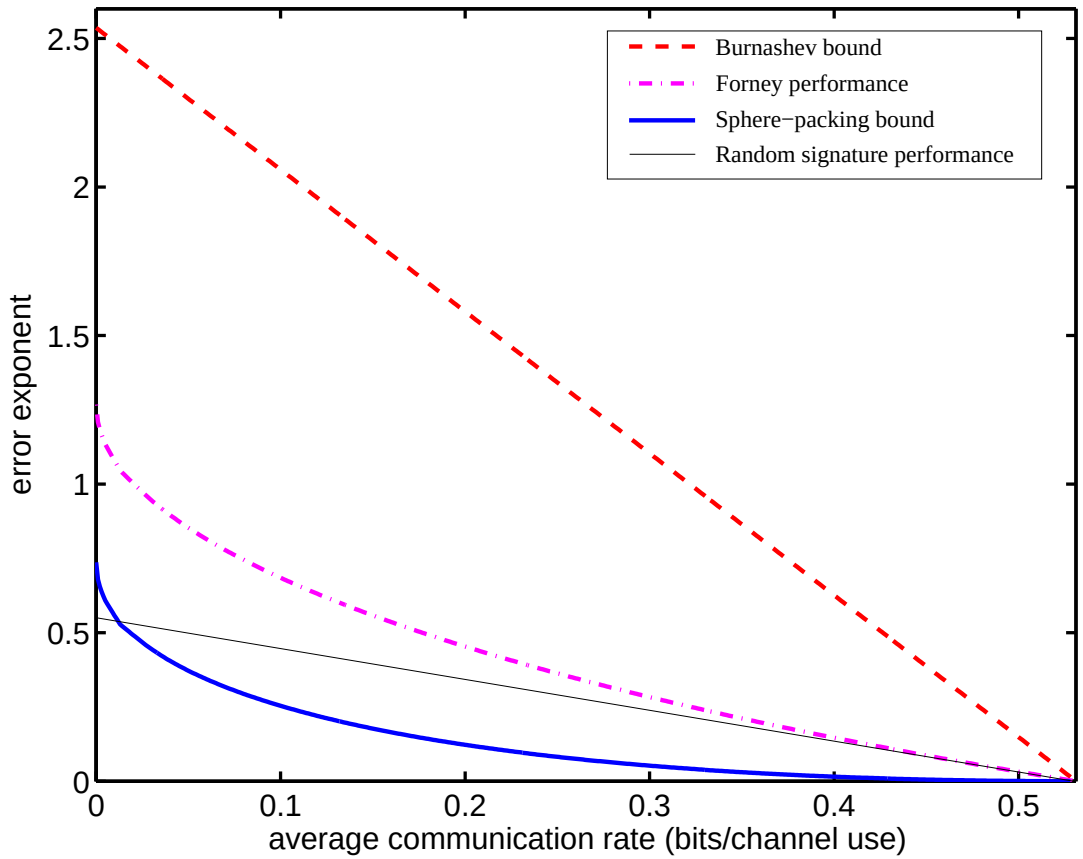

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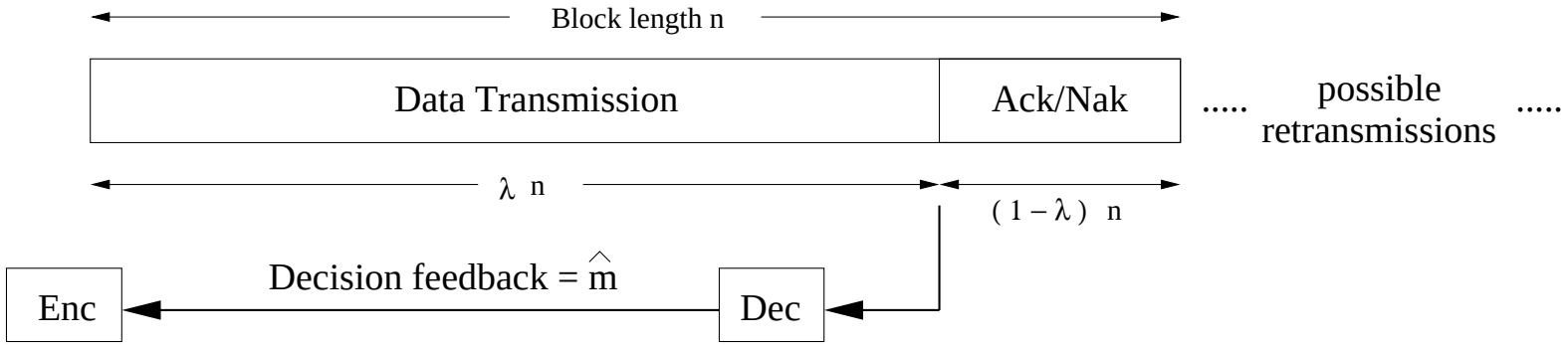

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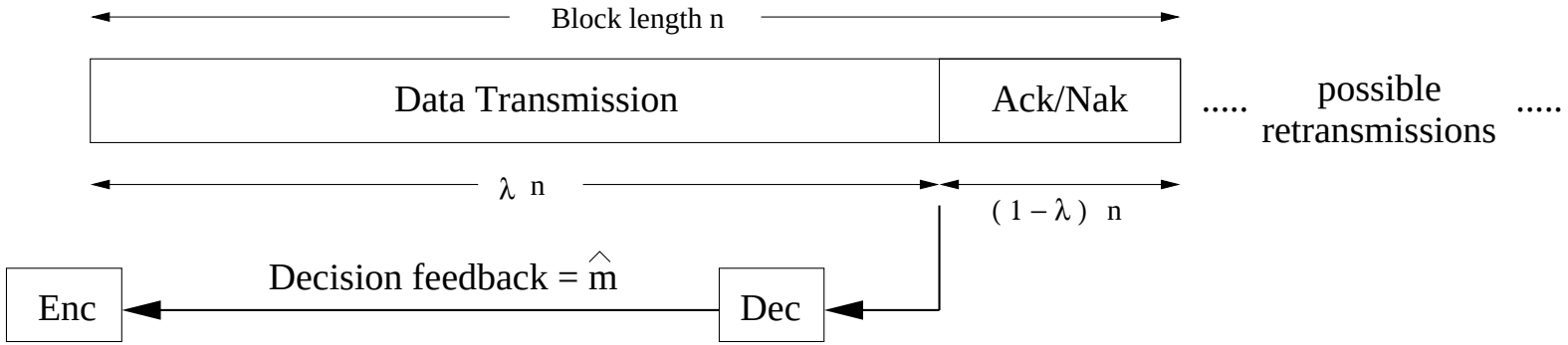

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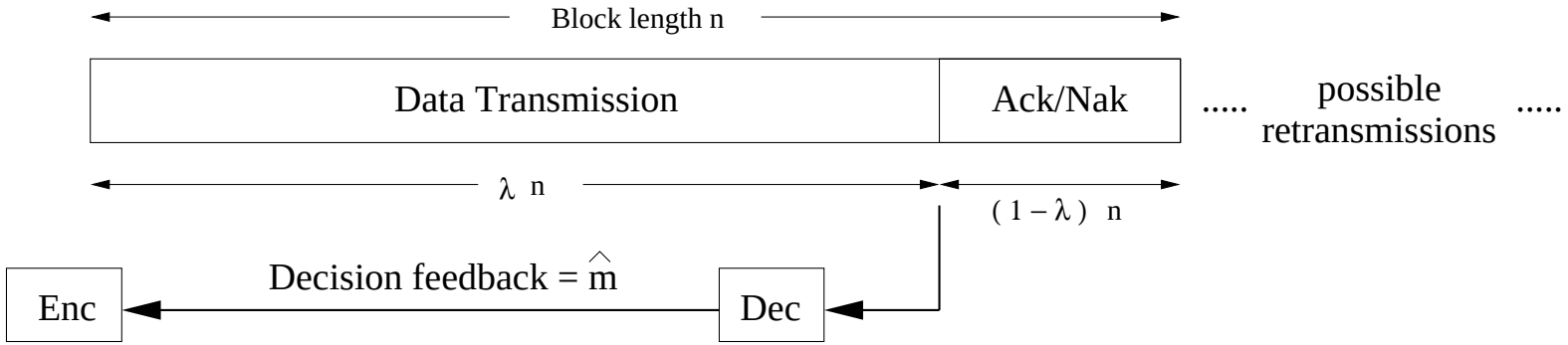

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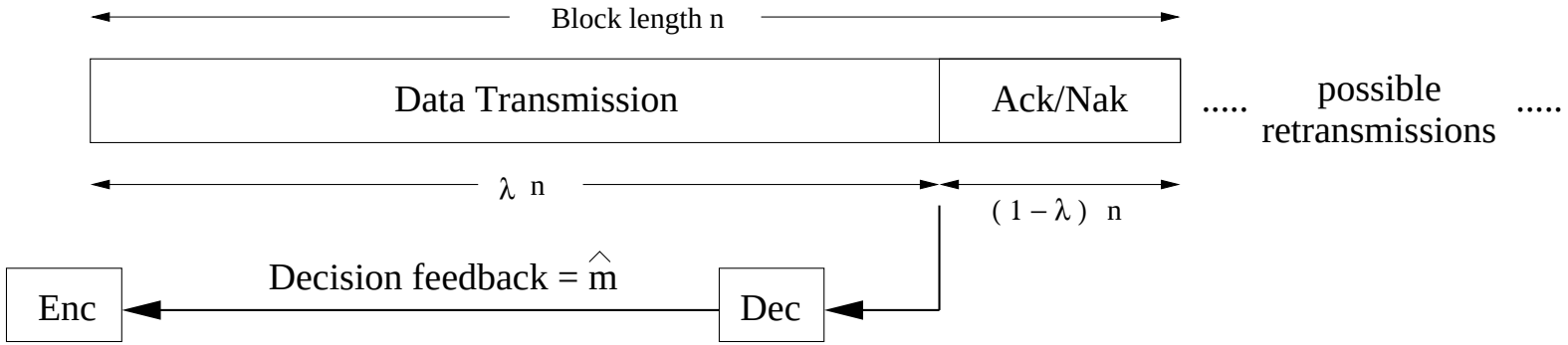

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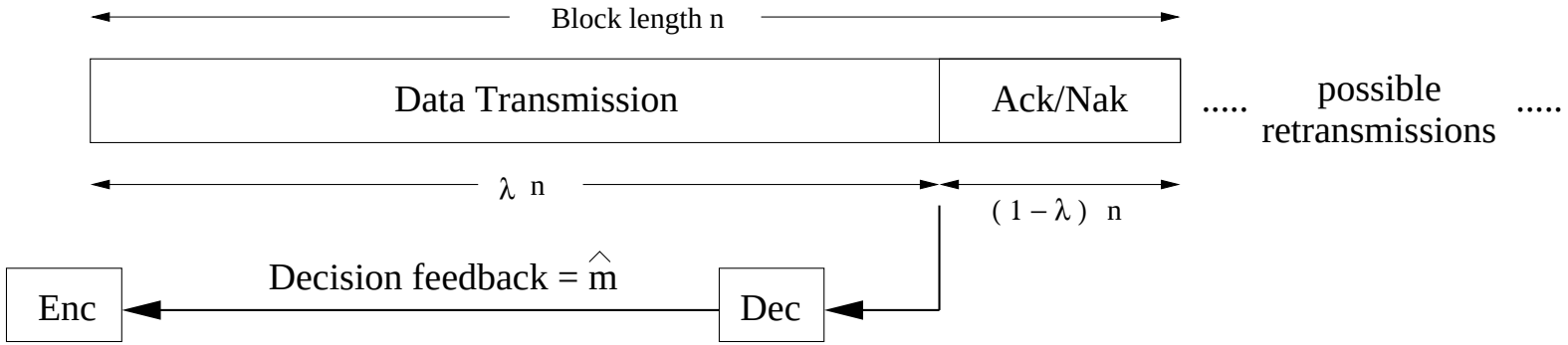

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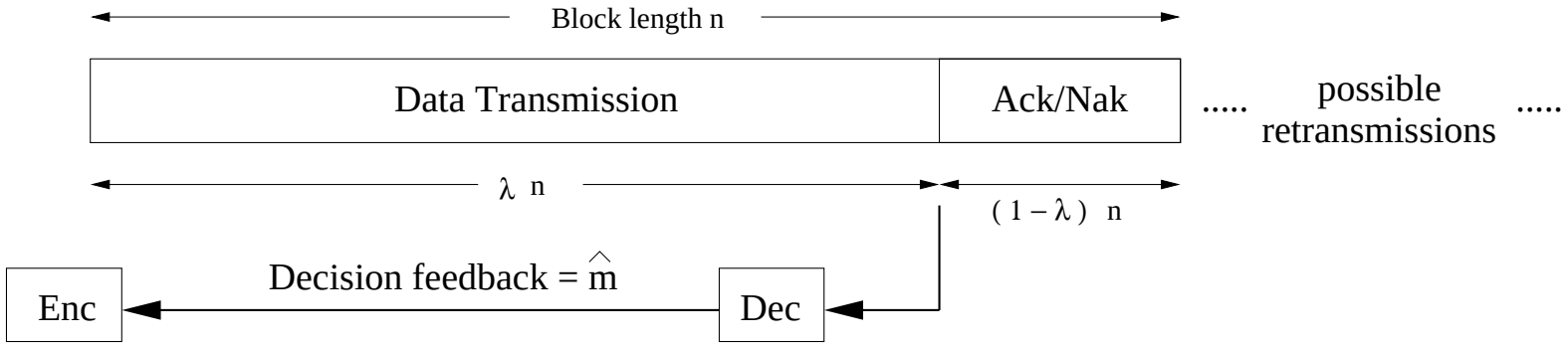

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- Moral: **Collective** reward/punishment is good for reliability

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2. Feedback, delay, and error probability: memoryless channels
   - Block-oriented results
   - Idealized case: fixed-delay with hard deadlines
     - ★ The BEC example
     - ★ Without feedback
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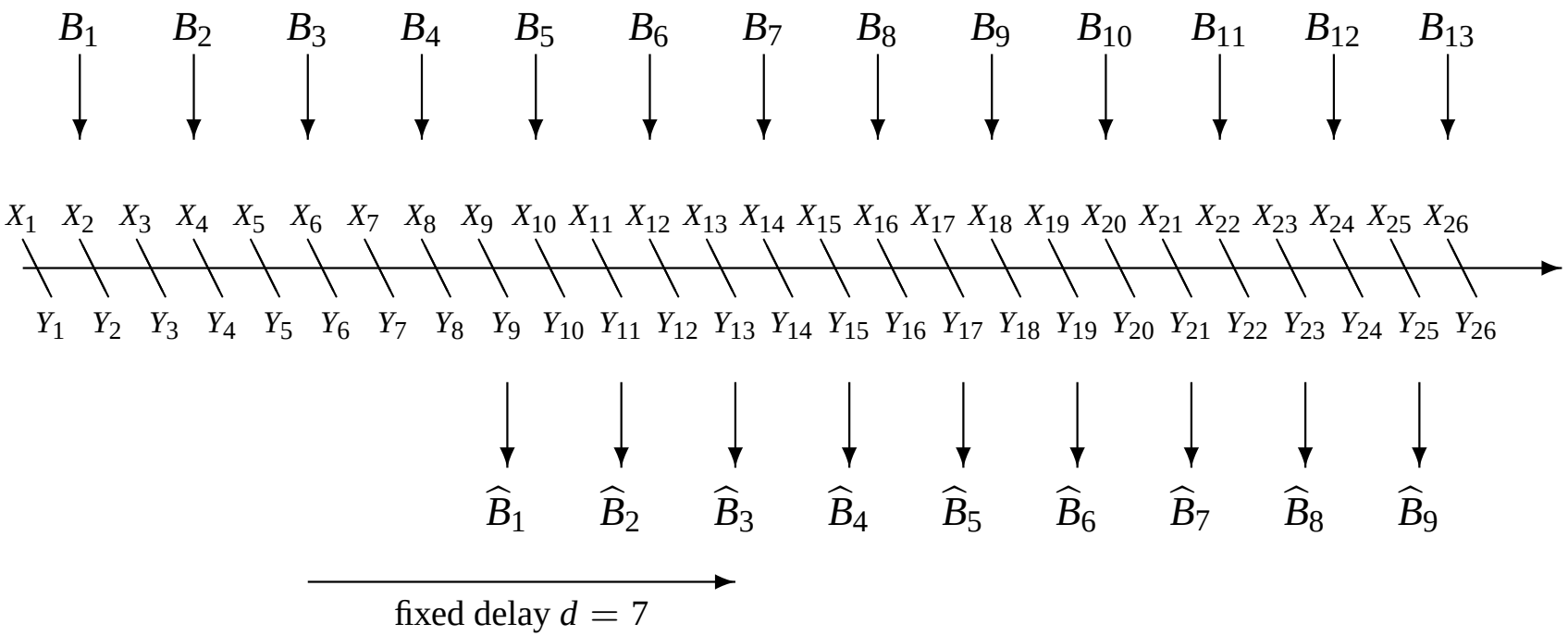
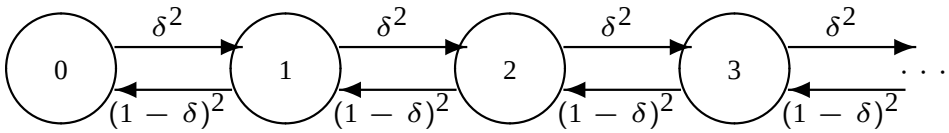
What do we mean by fixed delay?



- "Hard" deadlines: must commit to each bit
- "Soft" deadlines: allow saying "I don't know"

BEC with feedback and fixed *delay*

- $R = \frac{1}{2}$ example:



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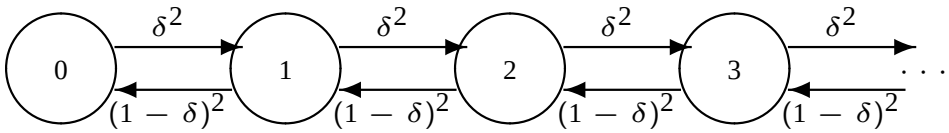

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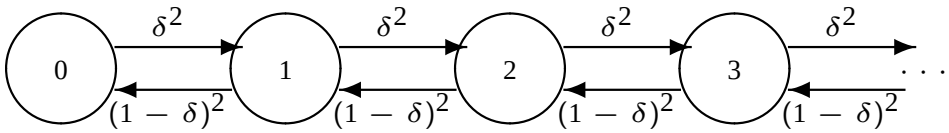

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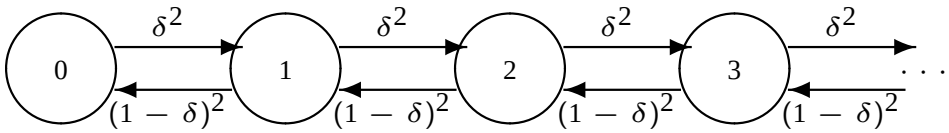

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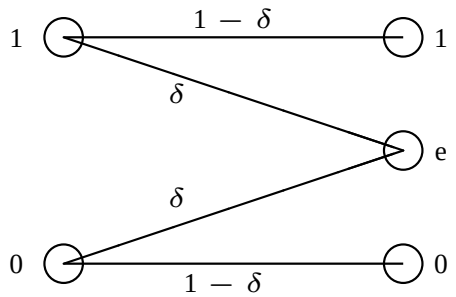
Block-coding is misleading!

The BEC: understanding why?



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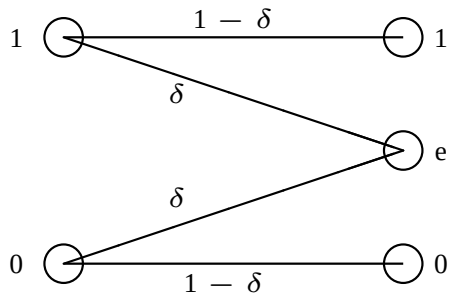

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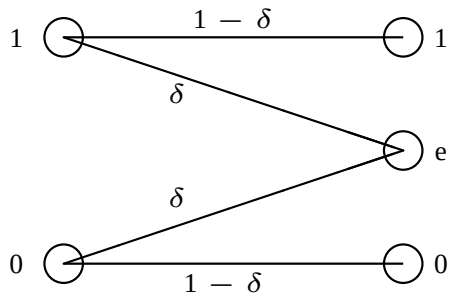

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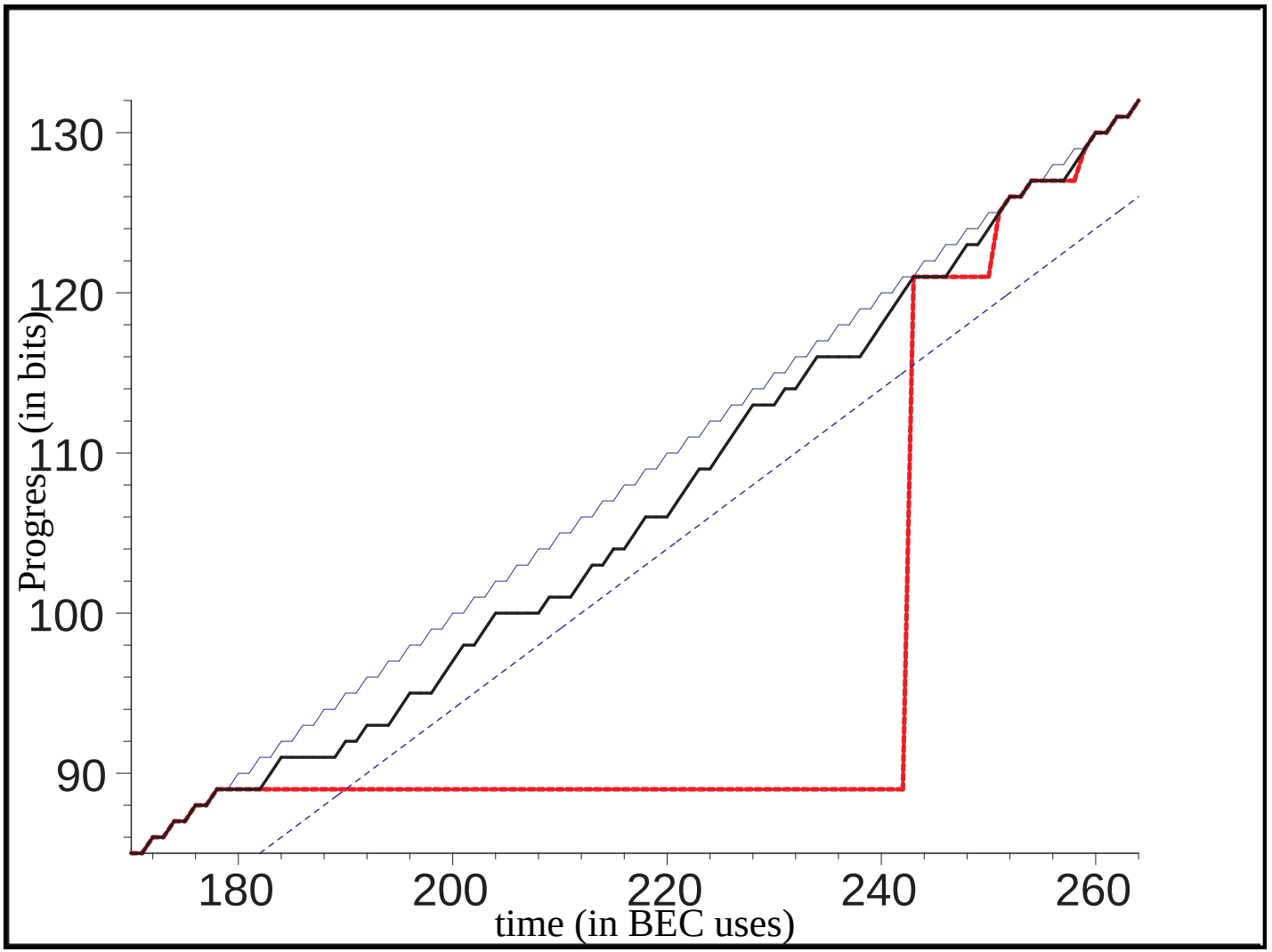
- Would behave at least as well if the service times were independent and dominated by a geometric.

So where is this boost coming from?



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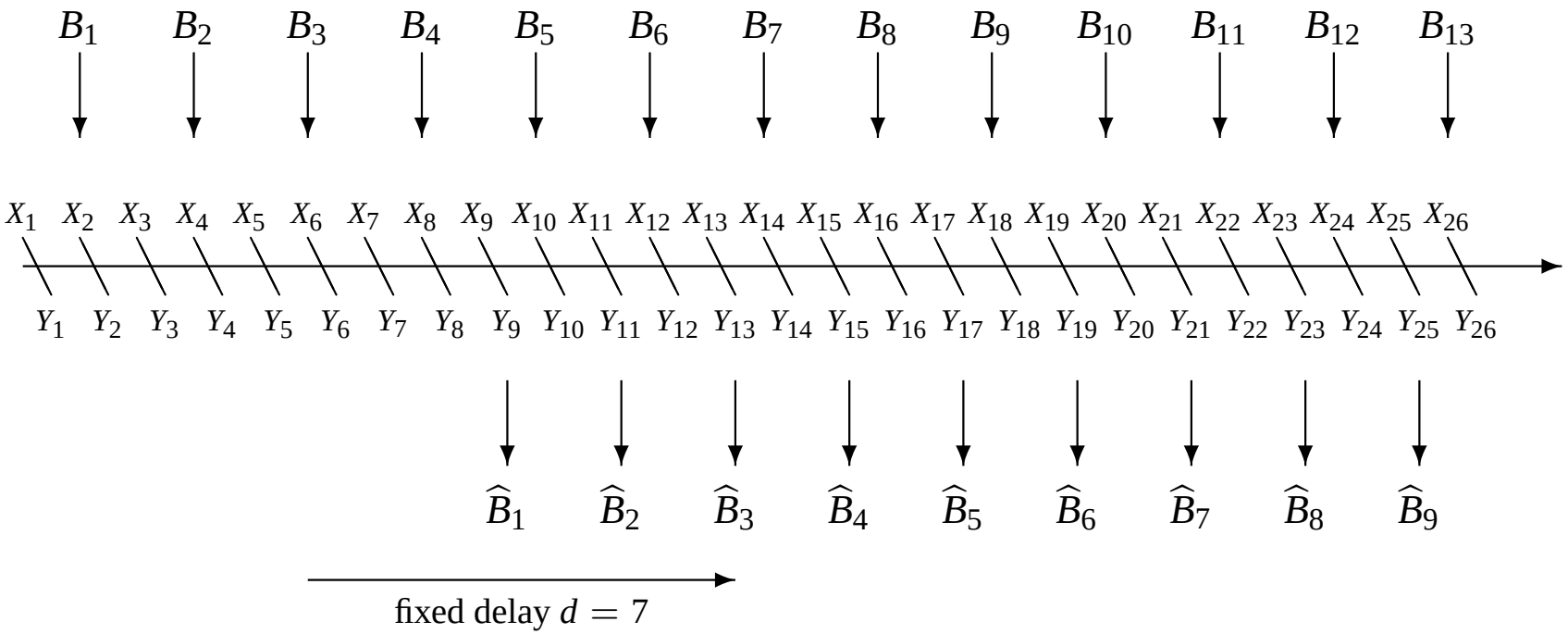

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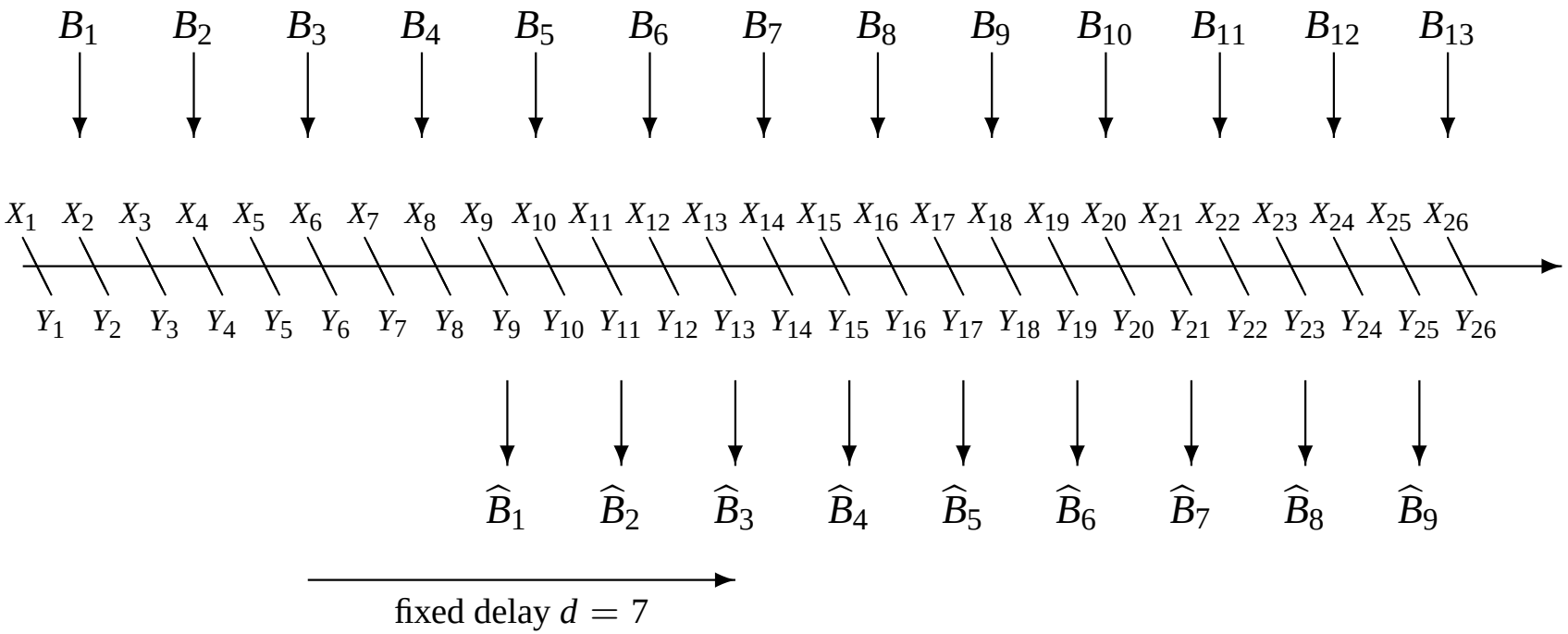
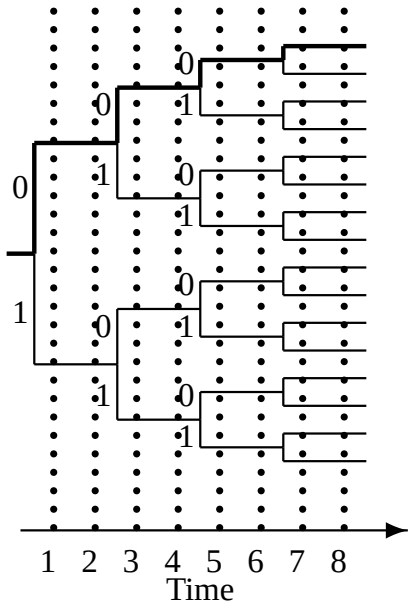


- Can generally achieve $E_r(R)$ with delay using convolutional codes.
- Pinsker (1967: PPI 3.4.44-55) claimed that the block-exponents continued to govern the non-block case *with and without feedback*.

Nonblock codes without feedback



Infinite binary tree, with iid random labels:

- Choose a path through the tree based on data bits
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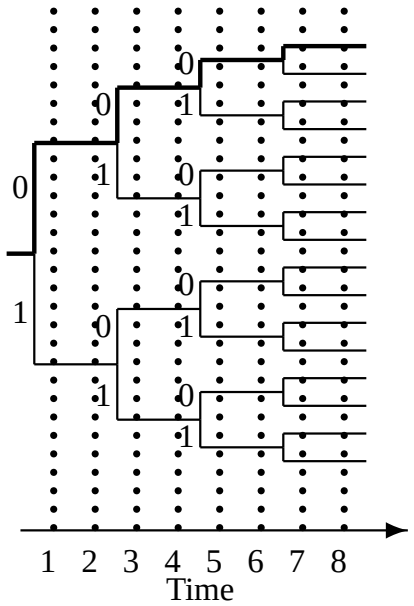

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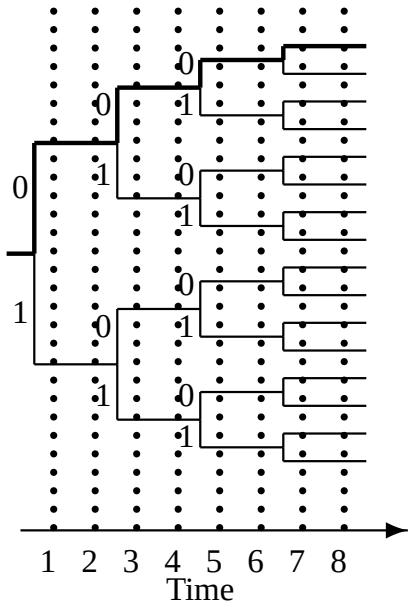

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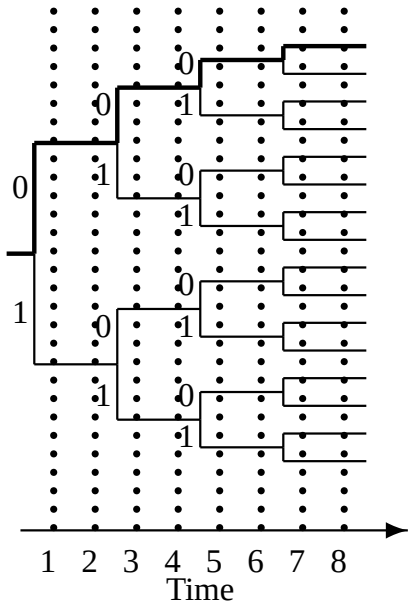

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- **Achieves** $P_e(d) \leq K \exp(-E_r(R)d)$ **for every** d **for all** $R < C$

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- *Same probability of error with delay: achieves* $E_r(R)$

Pinsker's bounding construction explained

- Without feedback: $E^+(R)$ continues to be a bound.
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  - Use it to construct a block-code with blocksize $n \gg d$
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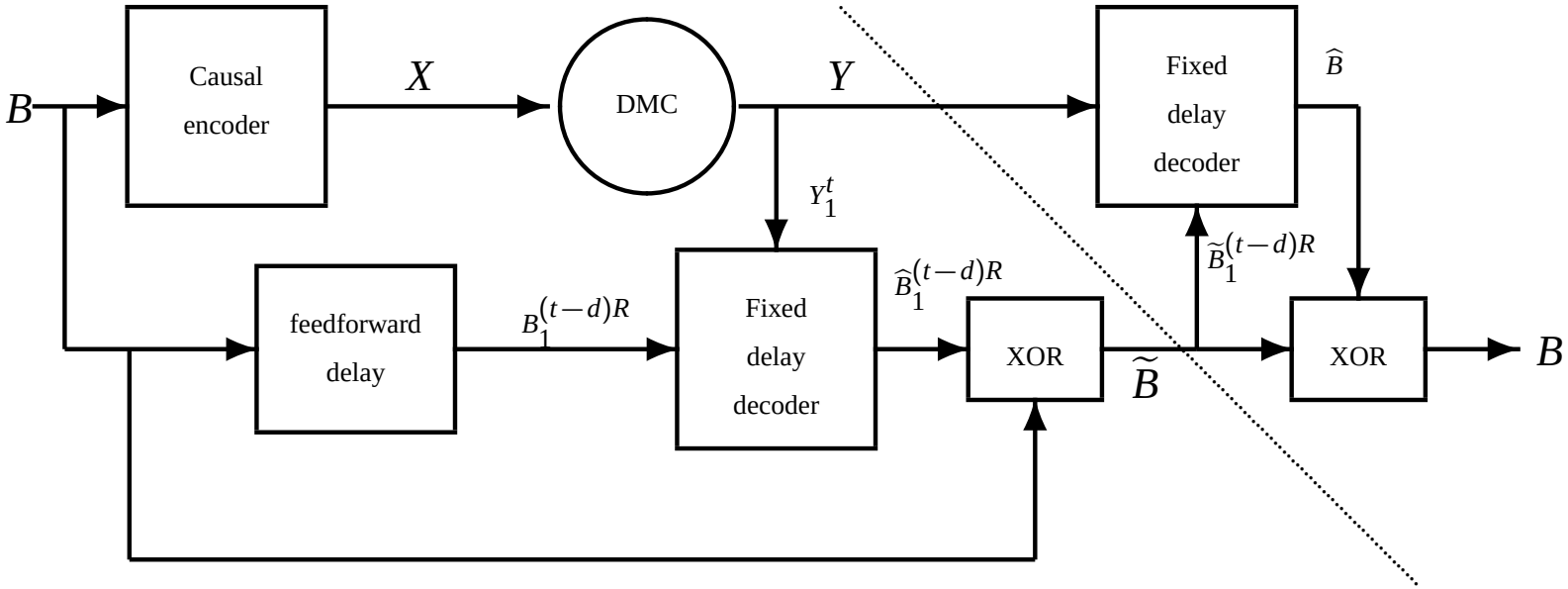

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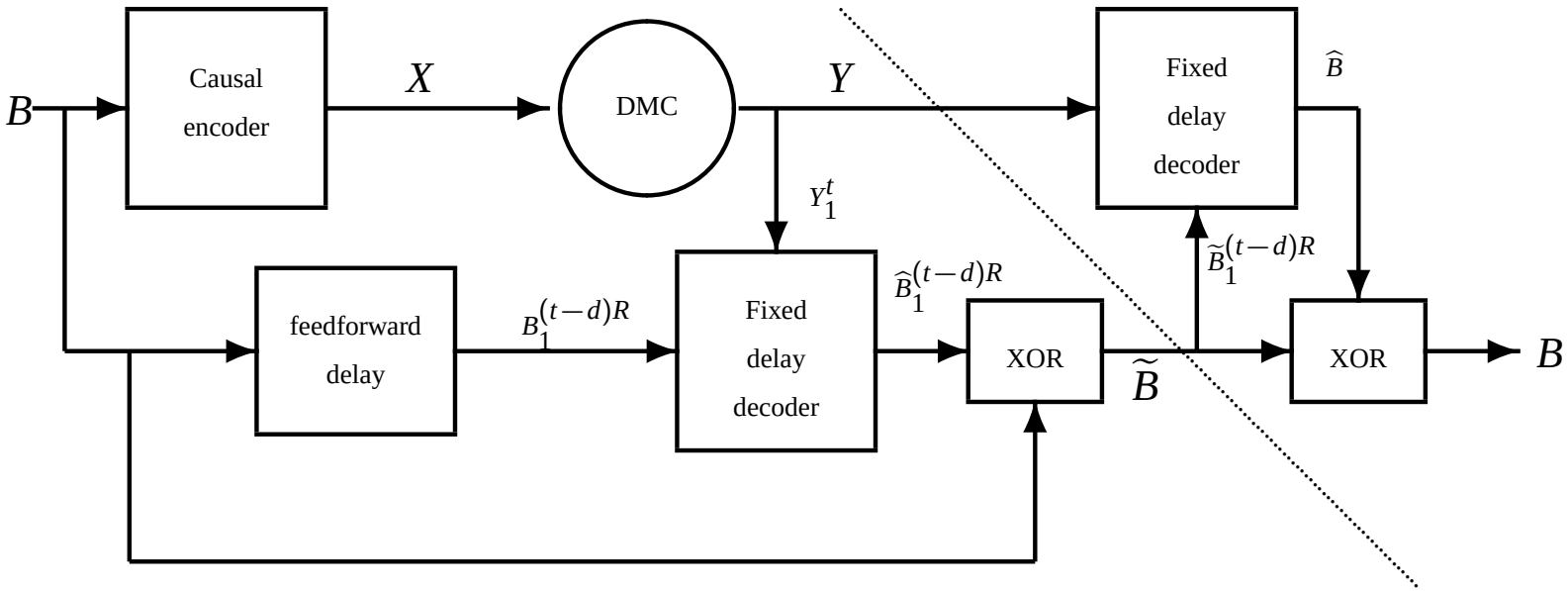

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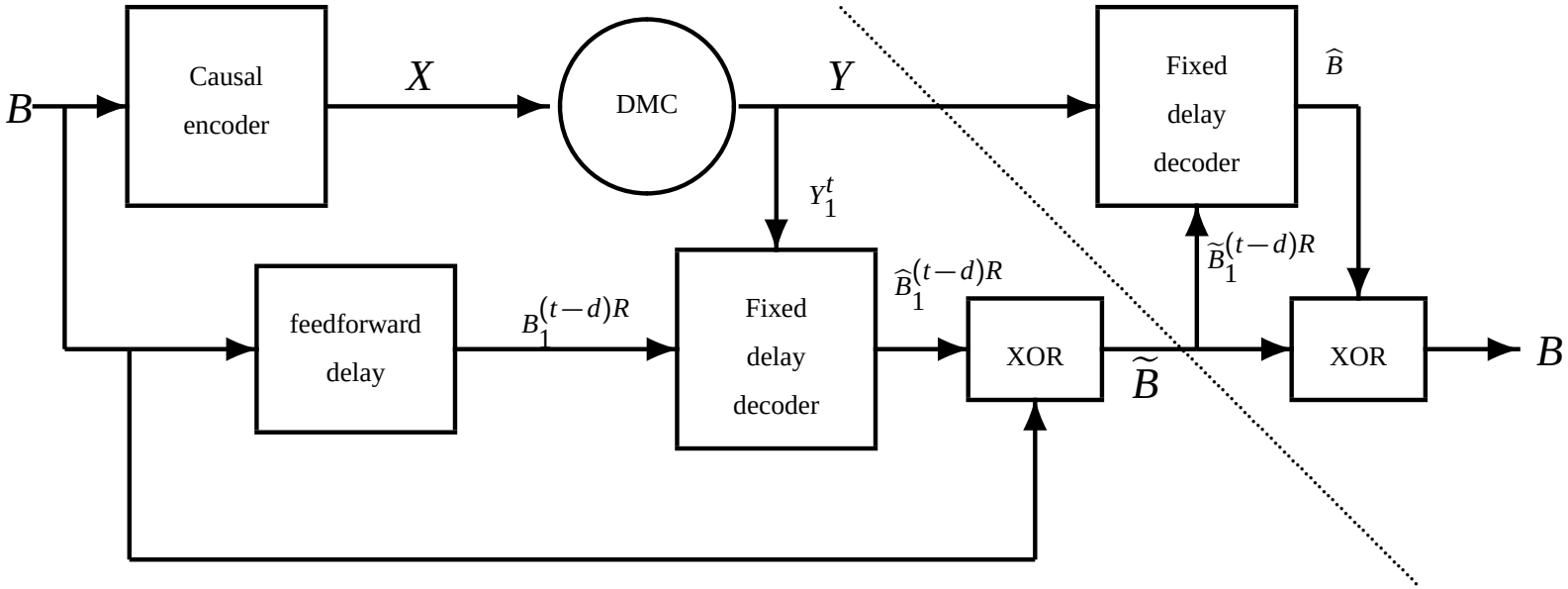
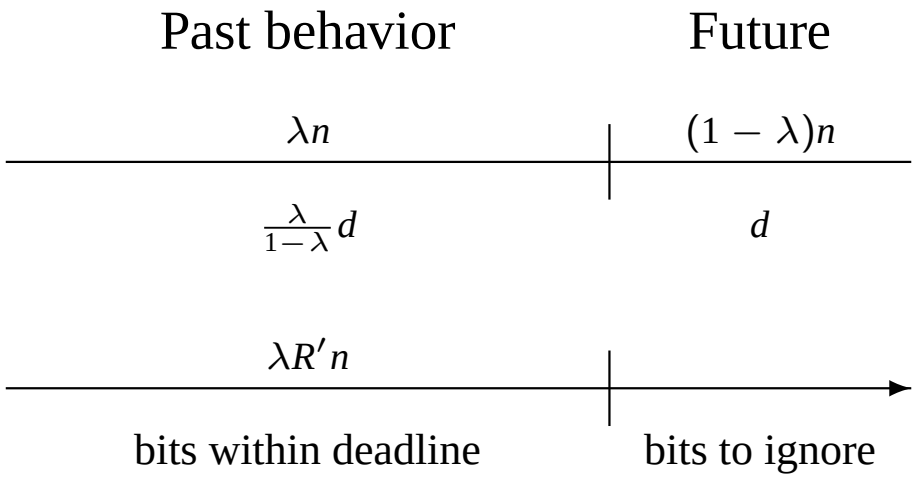
  - Apply a change of measure argument



Using E^+ to bound α^* in general



- The block error probability is like $e^{-\alpha(1-\lambda)n}$ which cannot exceed the Haroutunian bound $e^{-E^+(\lambda R)n}$

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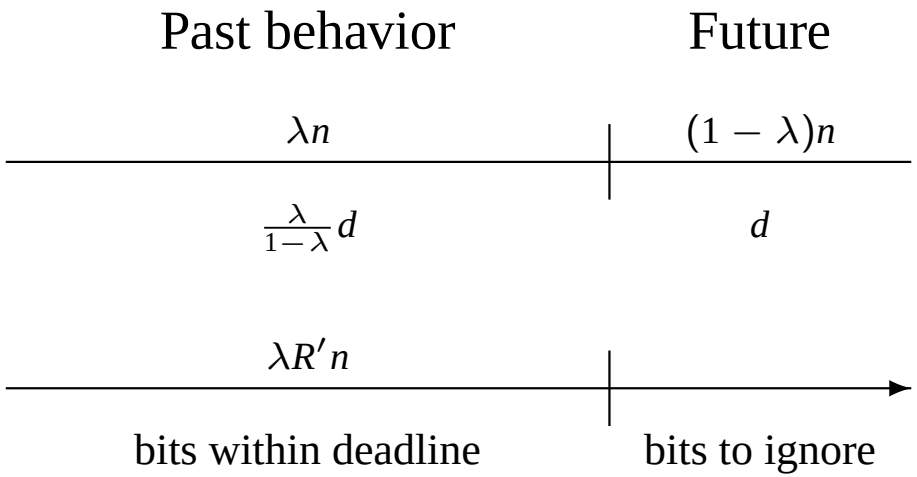

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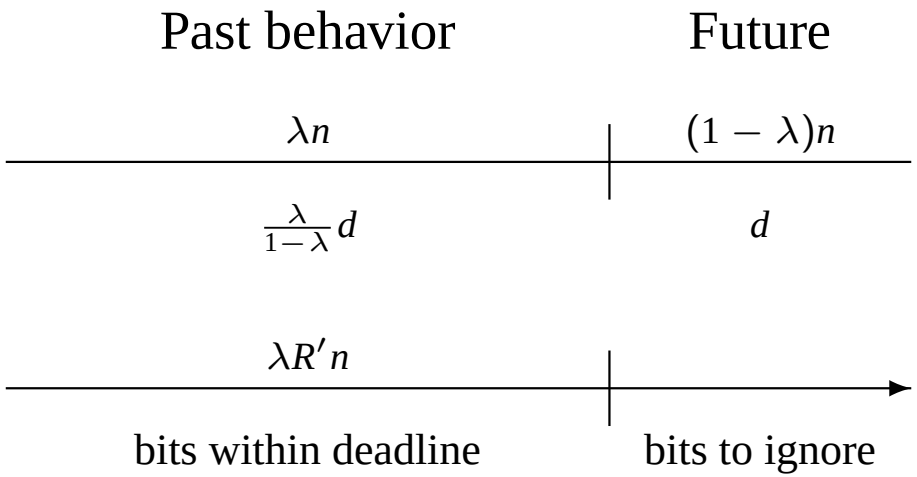

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- The error events involve *both* the past and the future.

Uncertainty focusing bound for symmetric DMCs

Minimize over λ for symmetric DMCs to sweep out frontier by varying $\rho > 0$:

$$\begin{aligned} R(\rho) &= \frac{E_0(\rho)}{\rho} \\ E_a^+(\rho) &= E_0(\rho) \end{aligned}$$

Using the Gallager function:

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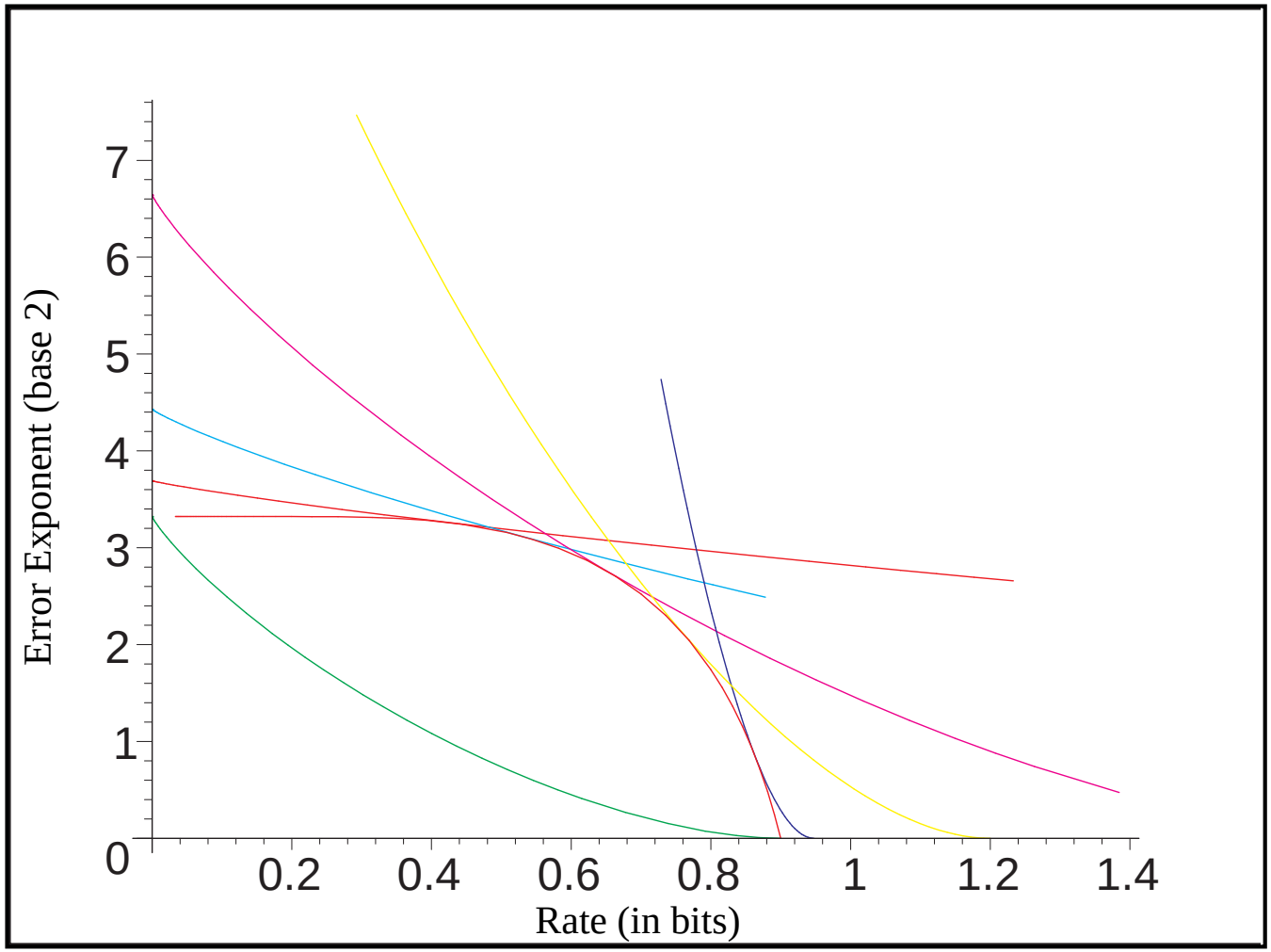
Same form as Viterbi's "convolutional coding bound" for constraint-lengths, but a lot more fundamental!

Upper bound tight for the BEC with feedback



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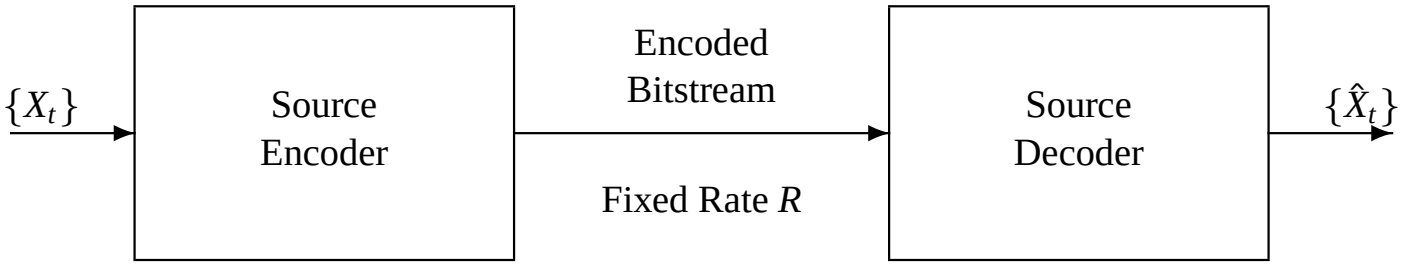
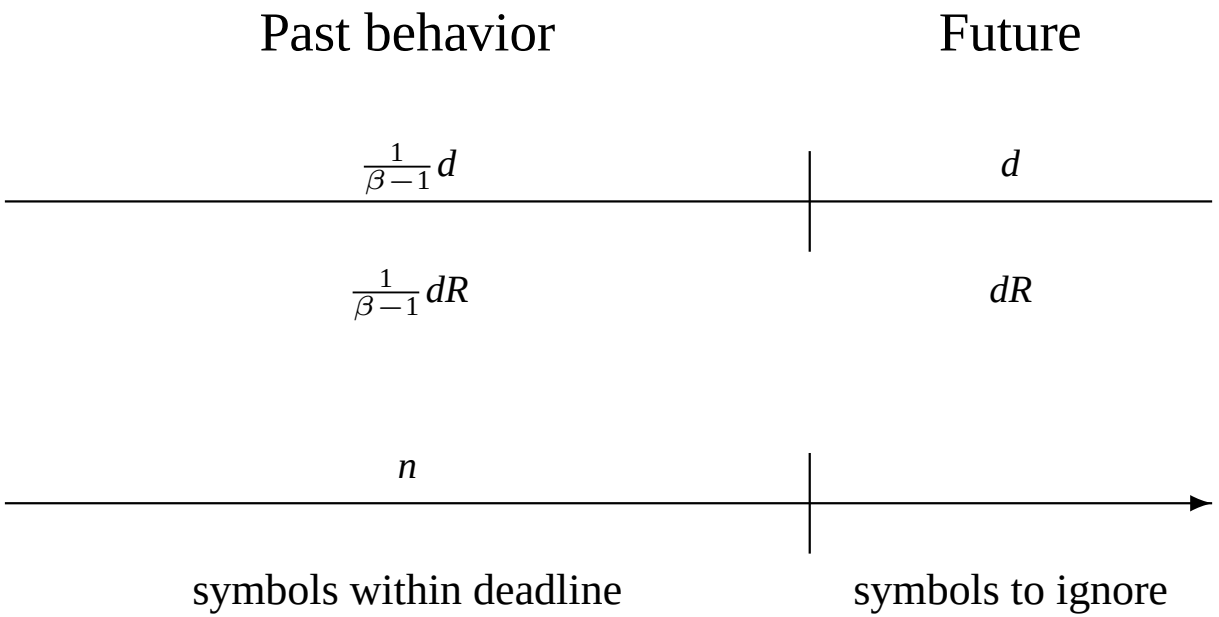
The source coding problem



- *Assume* $\{X_t\}$ *iid*
- Application-level interface
  - Symbol error probability: $P_e = P(X_t \neq \hat{X}_t)$
  - End-to-end latency: d (measured in source timescale)
- Channel-code interface: fixed rate R (assumed noiseless)

Using E_b to bound E_s in general



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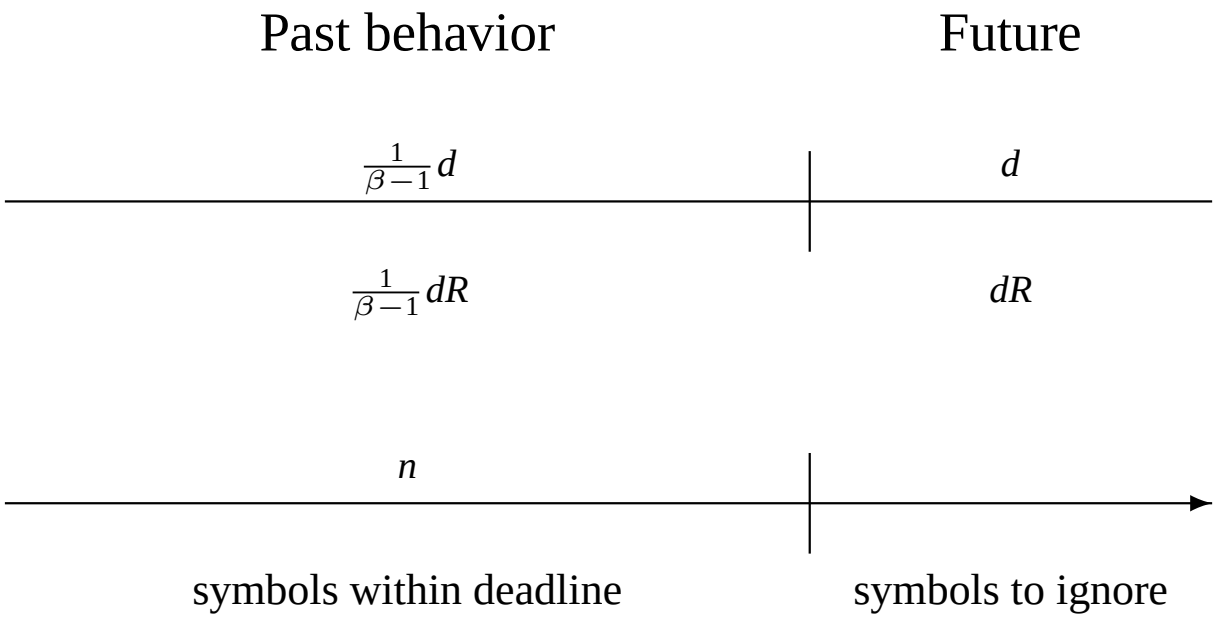

The error probability is bounded by $K\exp(-dE_s(R))$ which cannot exceed the block-coding bound $\exp(-nE_b(\beta R))$

$$E_s(R) \leq \frac{E_b(\beta R)}{\beta - 1}$$

Only the past matters!

Achieving the focusing bound

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\begin{aligned}
R(\rho) &= \frac{E_0(\rho)}{\rho} \\
E_s(\rho) &= E_0(\rho)
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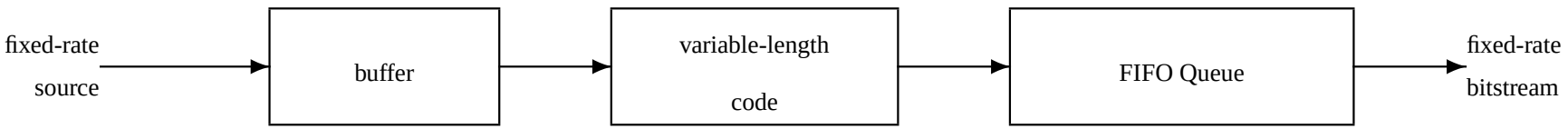

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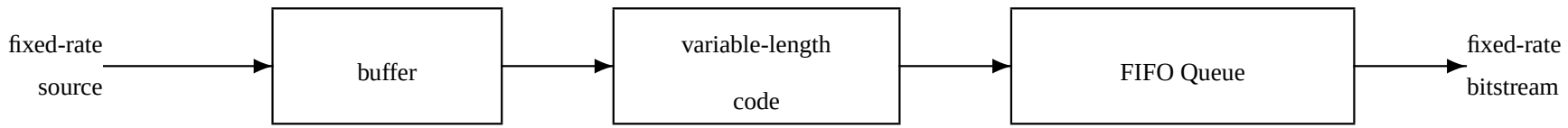

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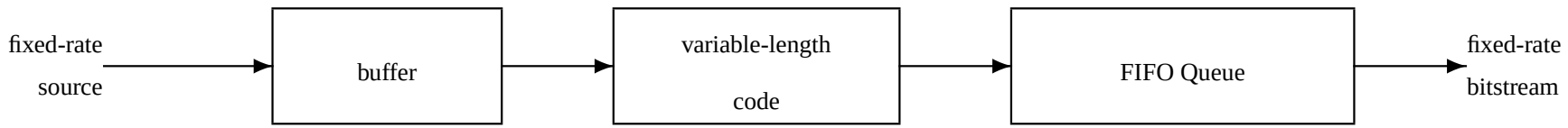

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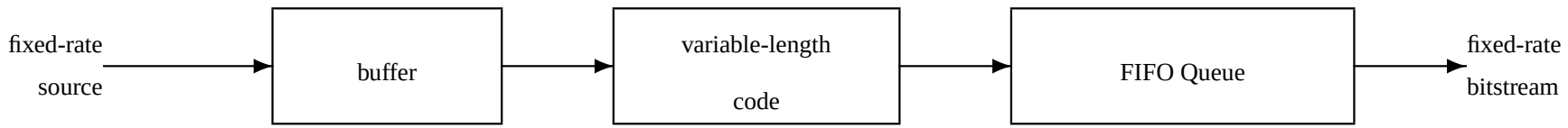

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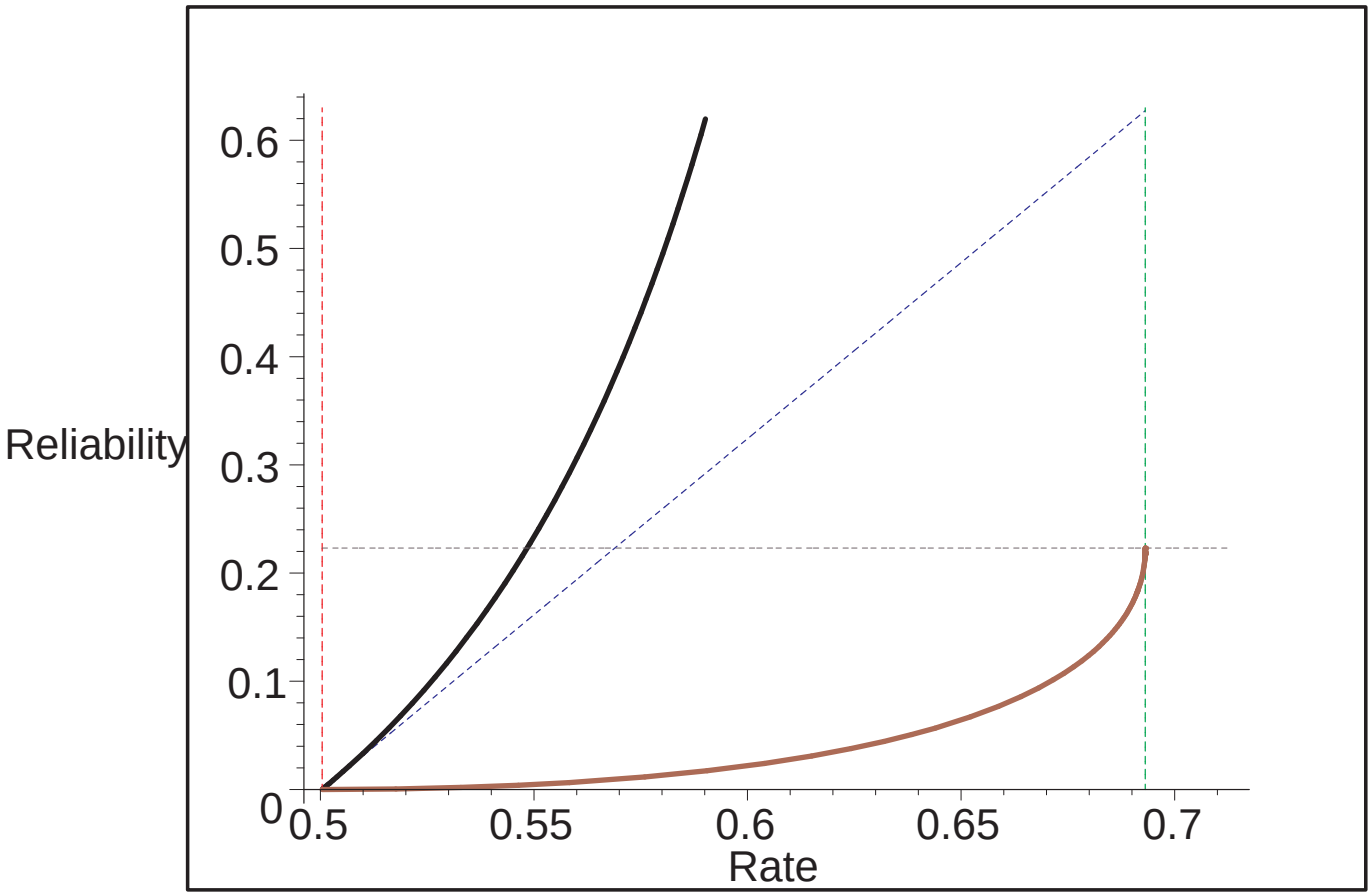
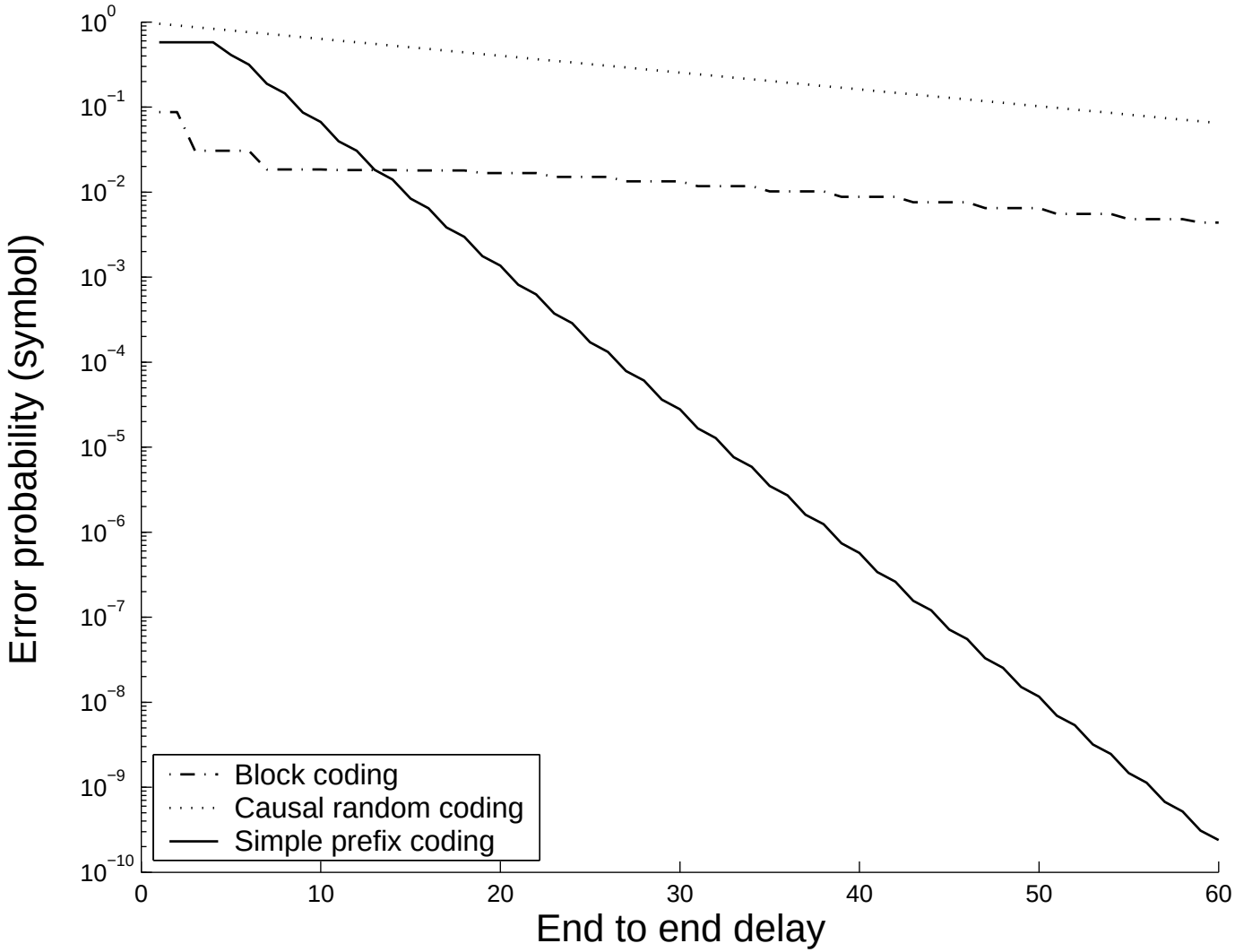
- R translates queue length into delay.

A simple example: related to Jelinek-68

- Unfair coin tosses $P(H) = 0.2$

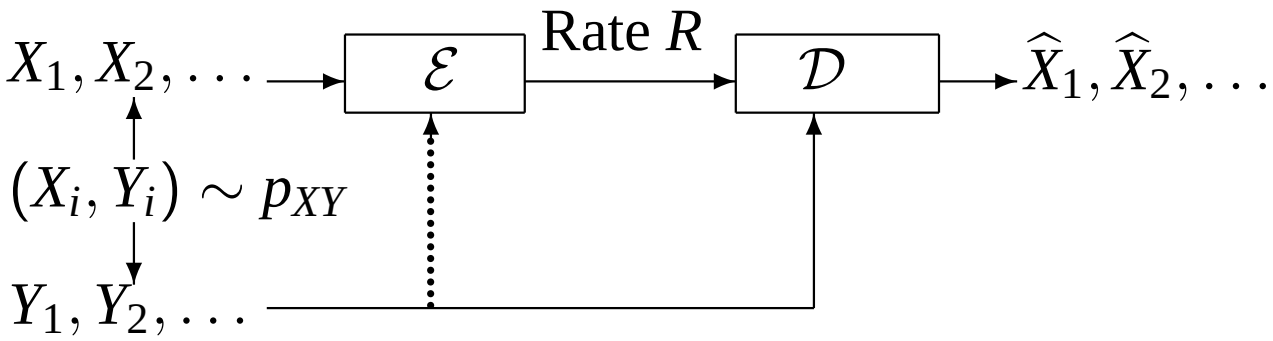


How far till Asymptopia?



Simple example with a ternary source and a specific code ($n = 2$).

Side-information at the decoder



- Encoder may be ignorant of the side-information. (Slepian-Wolf)
- If $P_{X,Y}$ symmetric with uniform marginals, can do no better than $E_b(R)$ with delay. (analogous to Pinsker's argument)

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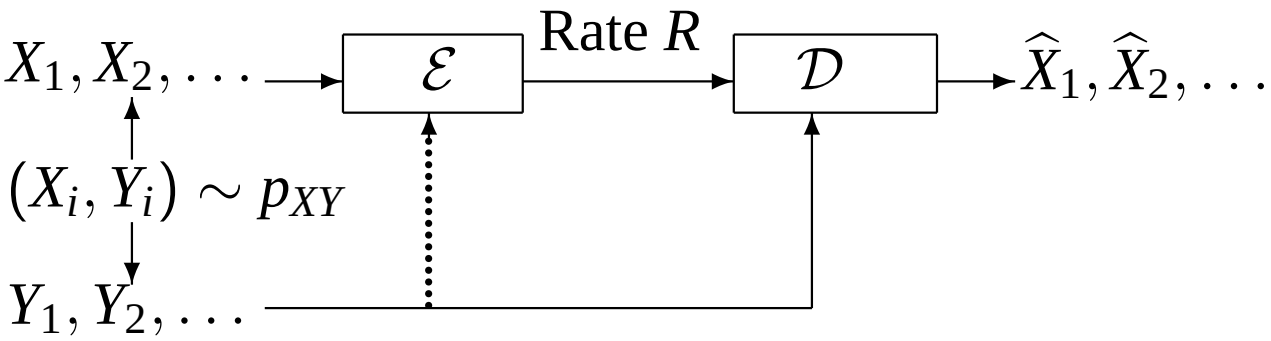

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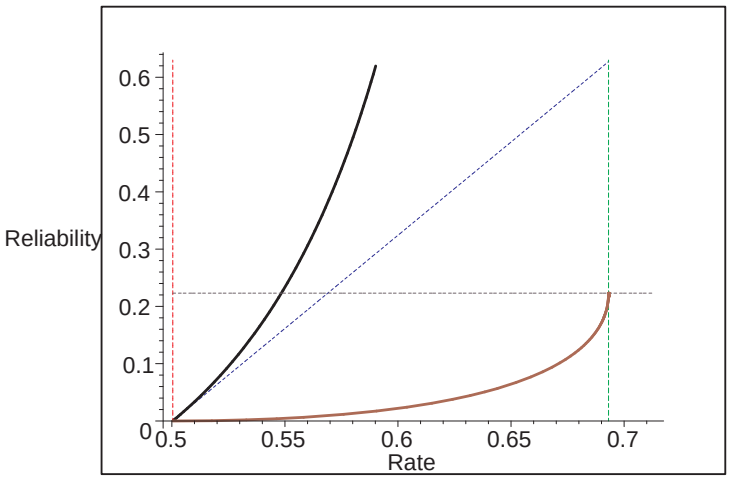

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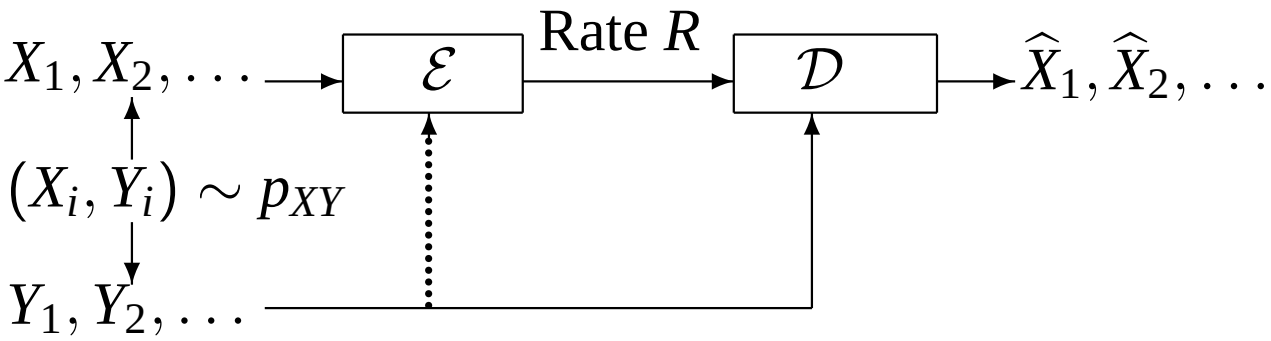

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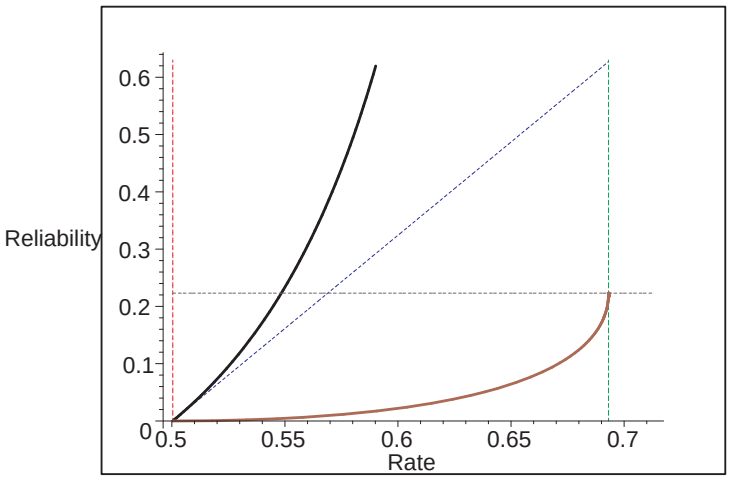
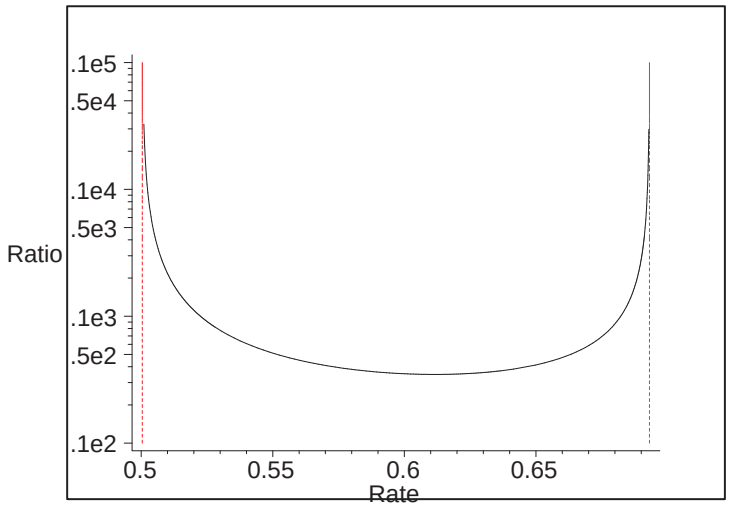
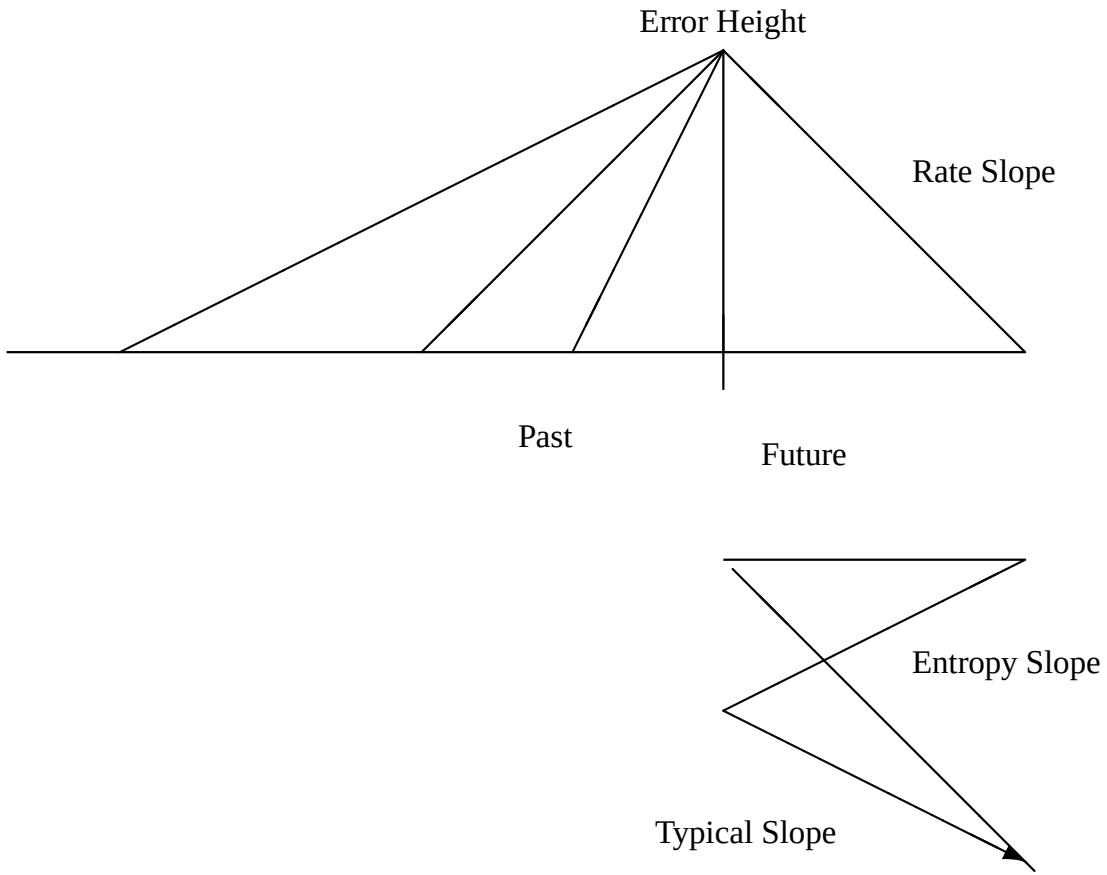




Intuition: Long vs. Large deviations



- Shorter deviation periods must be larger.
- Smaller deviations must be over longer periods.

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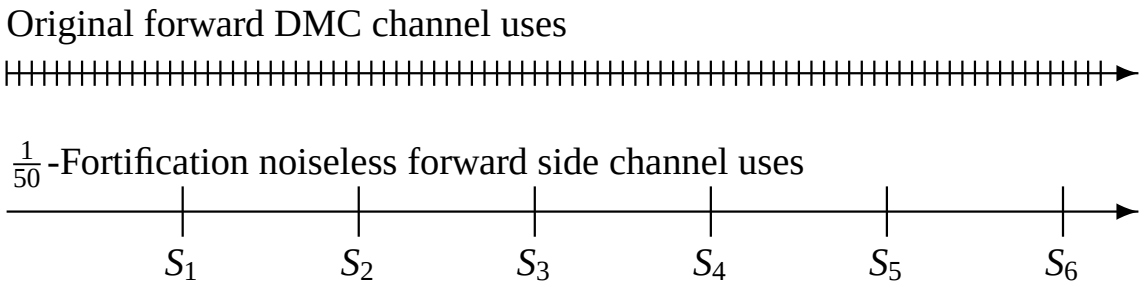
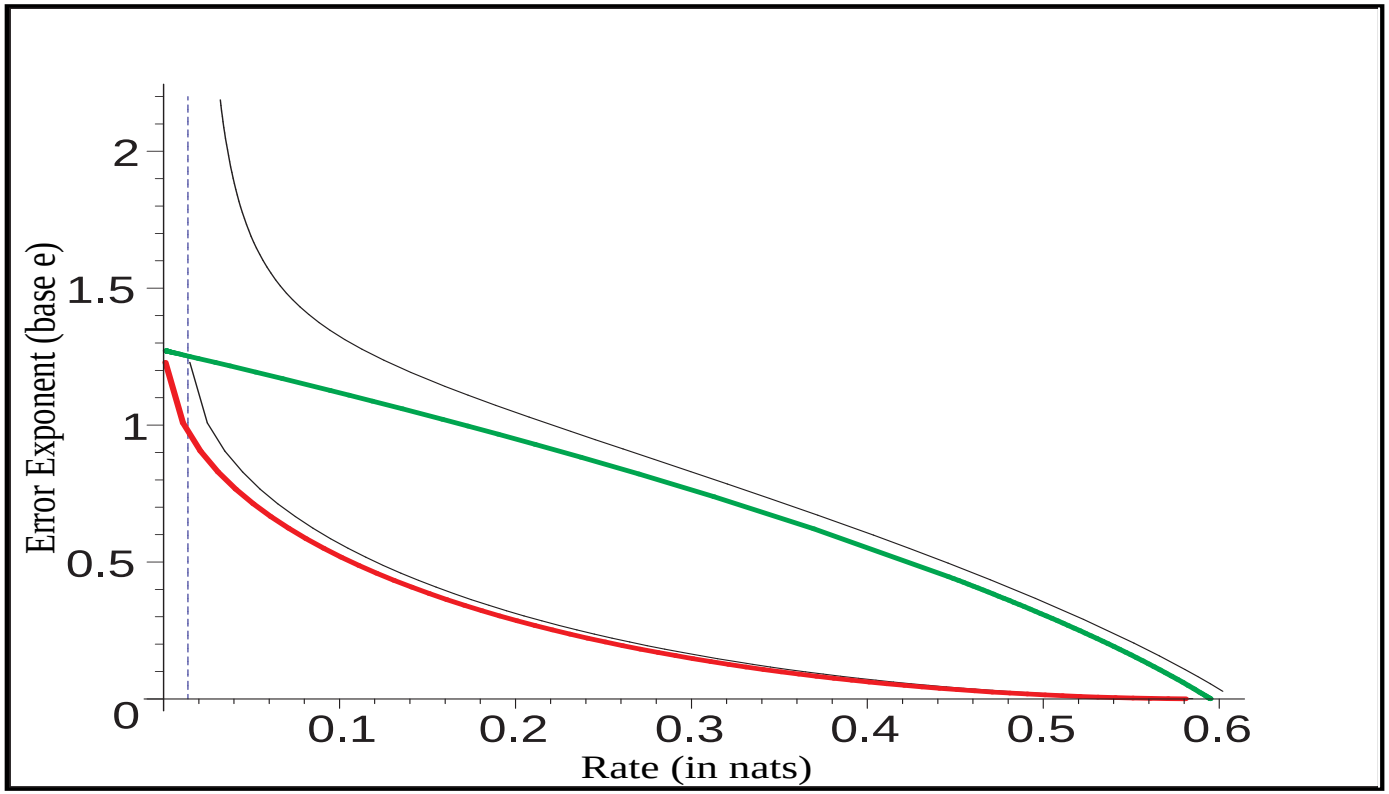
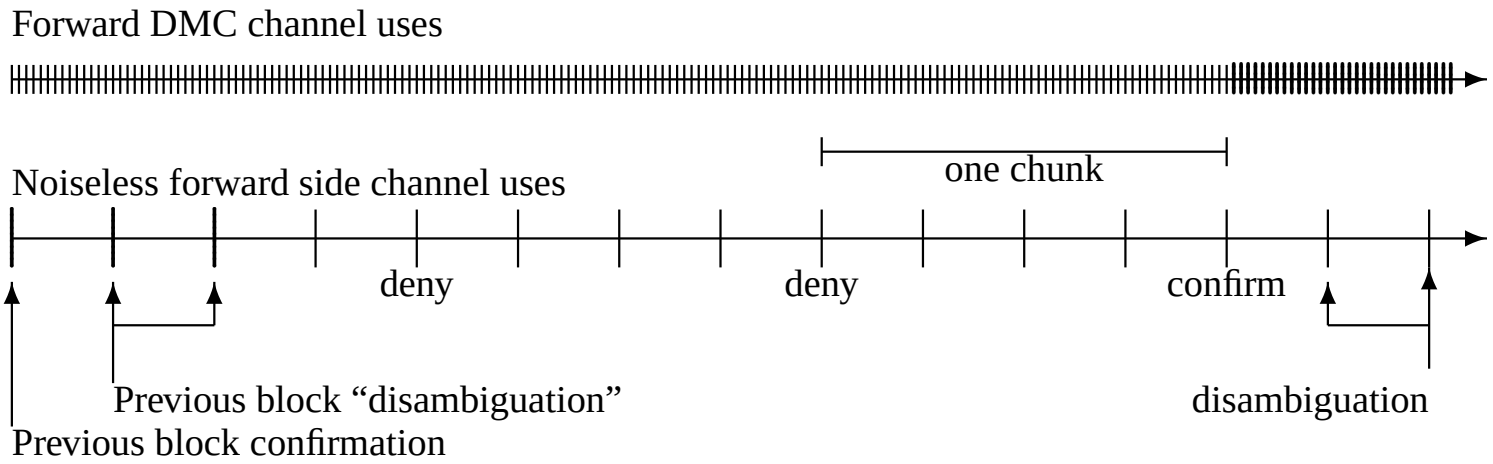
A spoonful of “sugar” helps the bits get across.





Harnessing the power of “flow control”

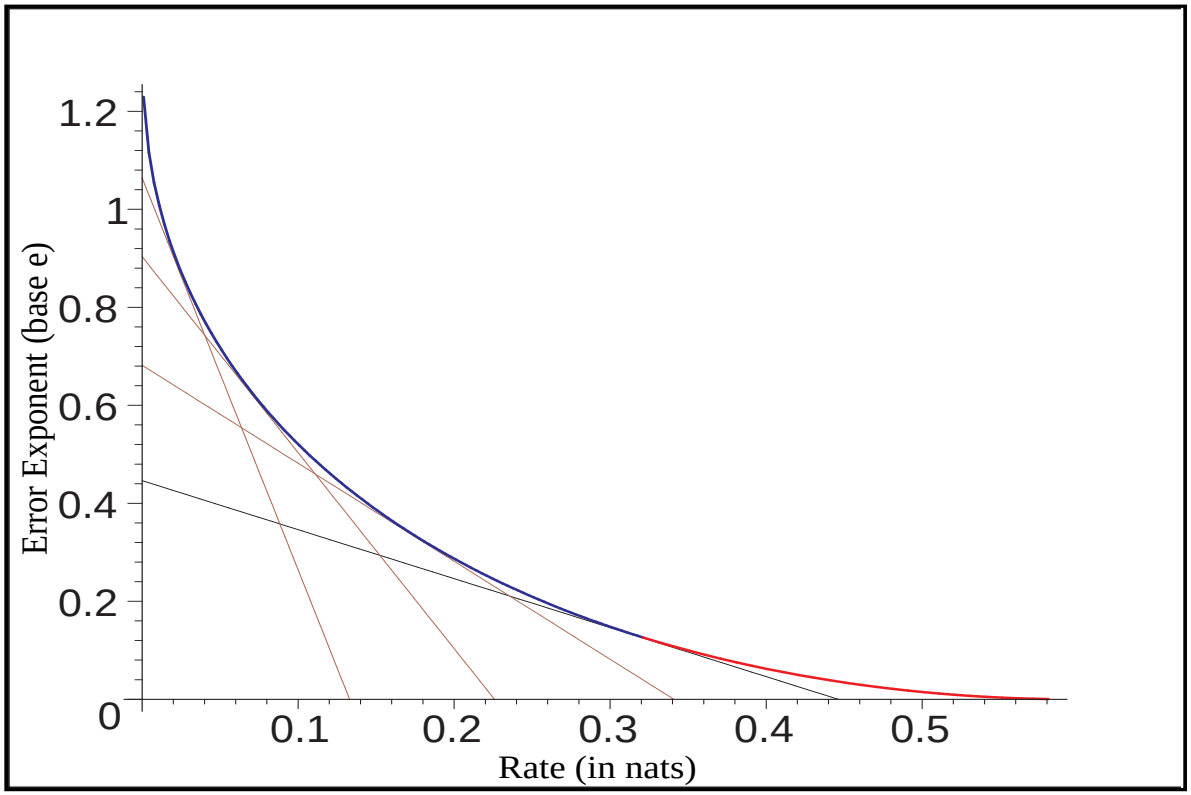


1. Group bits into miniblocks of size nR. ($n \ll d$)
2. Transmit using an ∞-length random codebook.
3. Use the “sugar” to tell decoder when it’s done.

No decoding errors, just queuing plus transmission delays.

Why this works: operational interpretation of $E_0(\rho)$

Variable block transmission time T can be bounded by a constant plus a geometric random variable.



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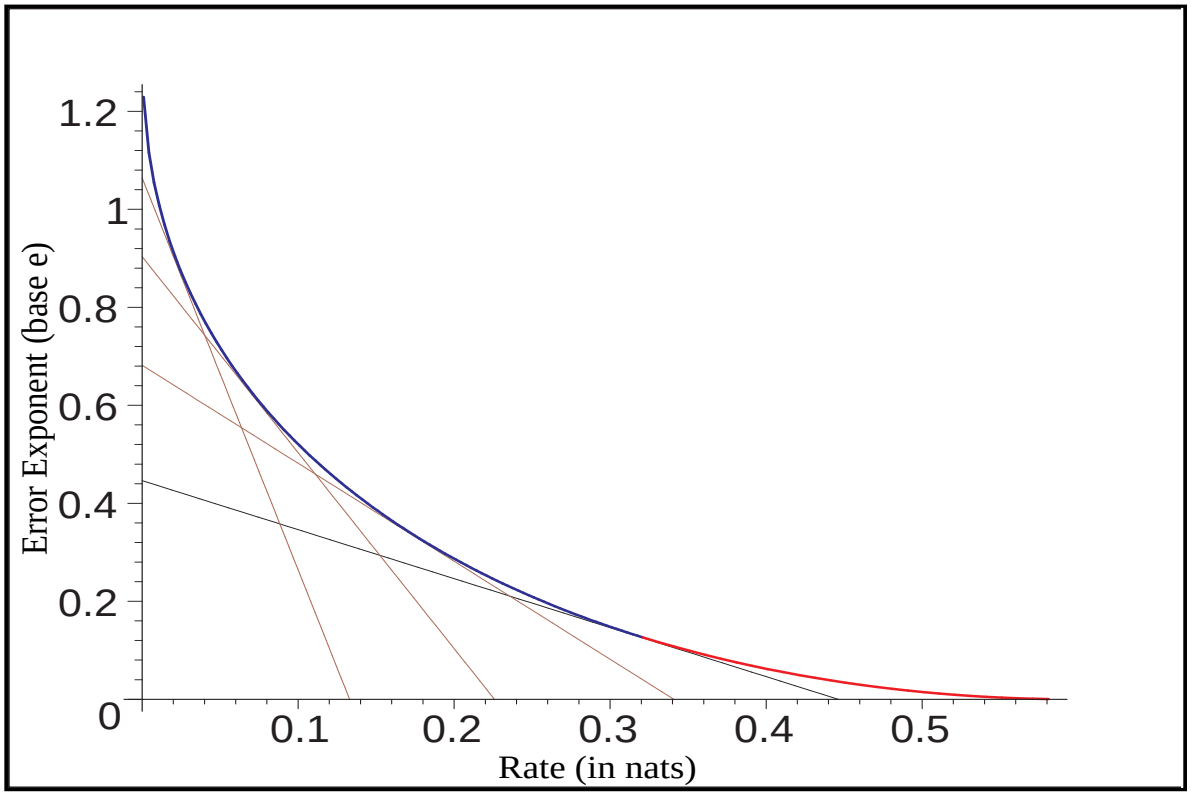
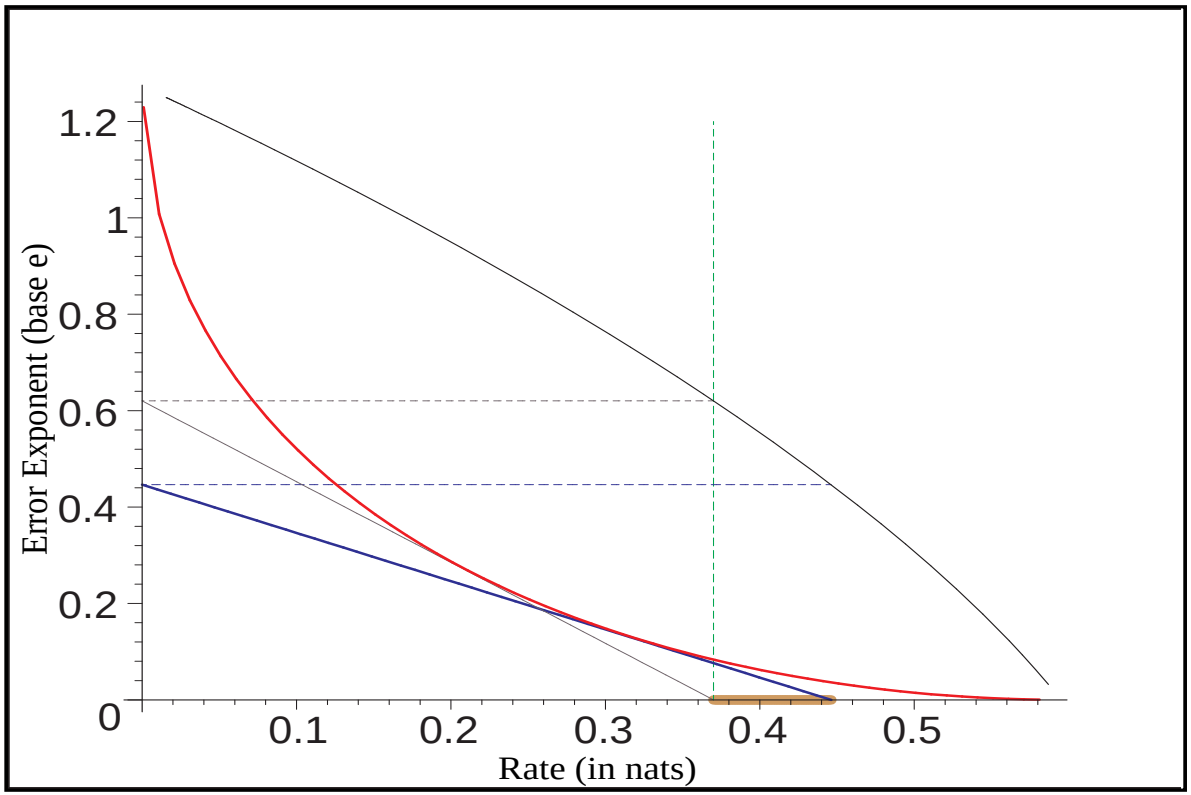


Need to do list-decoding at low rates.

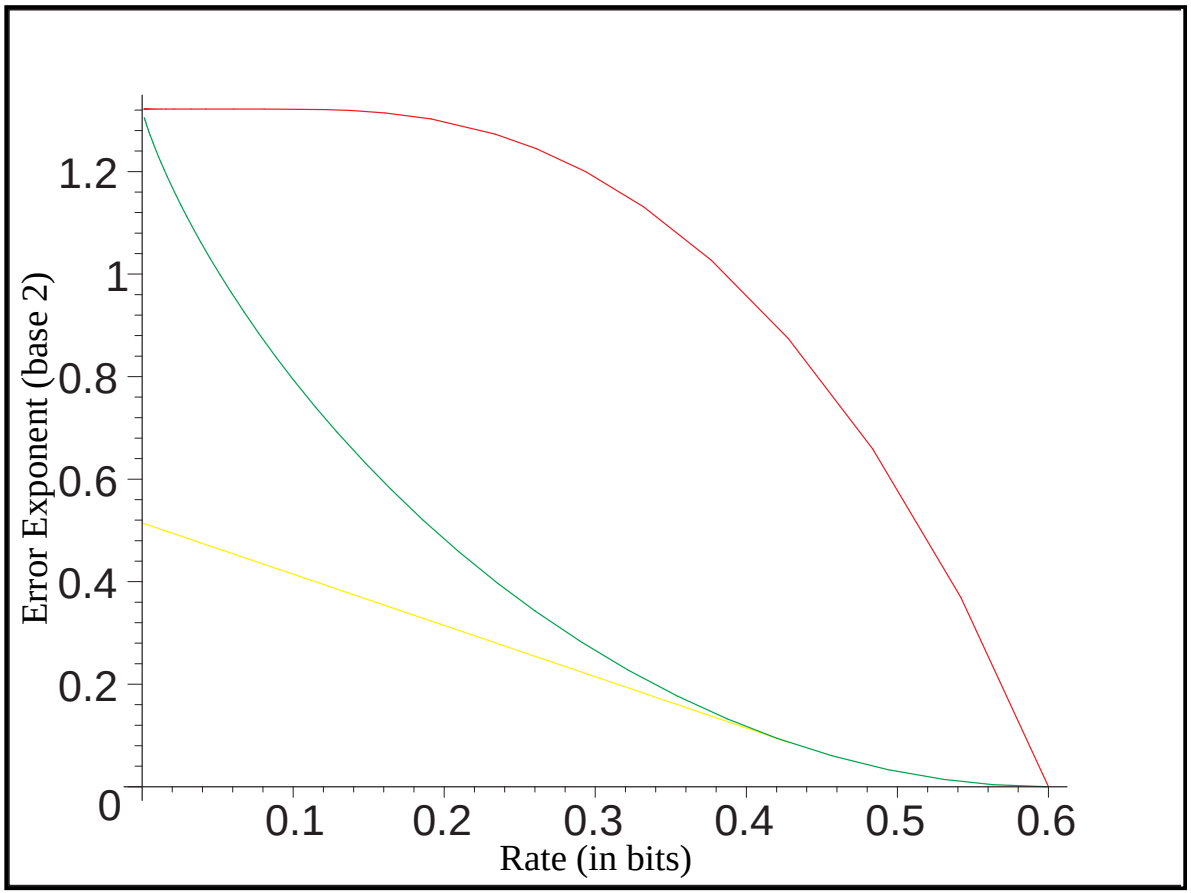
Reduces to the low-rate erasure case

Pick $R < R' < C$ and aim for $E^+(R') = E_0(\rho')$ exponent.



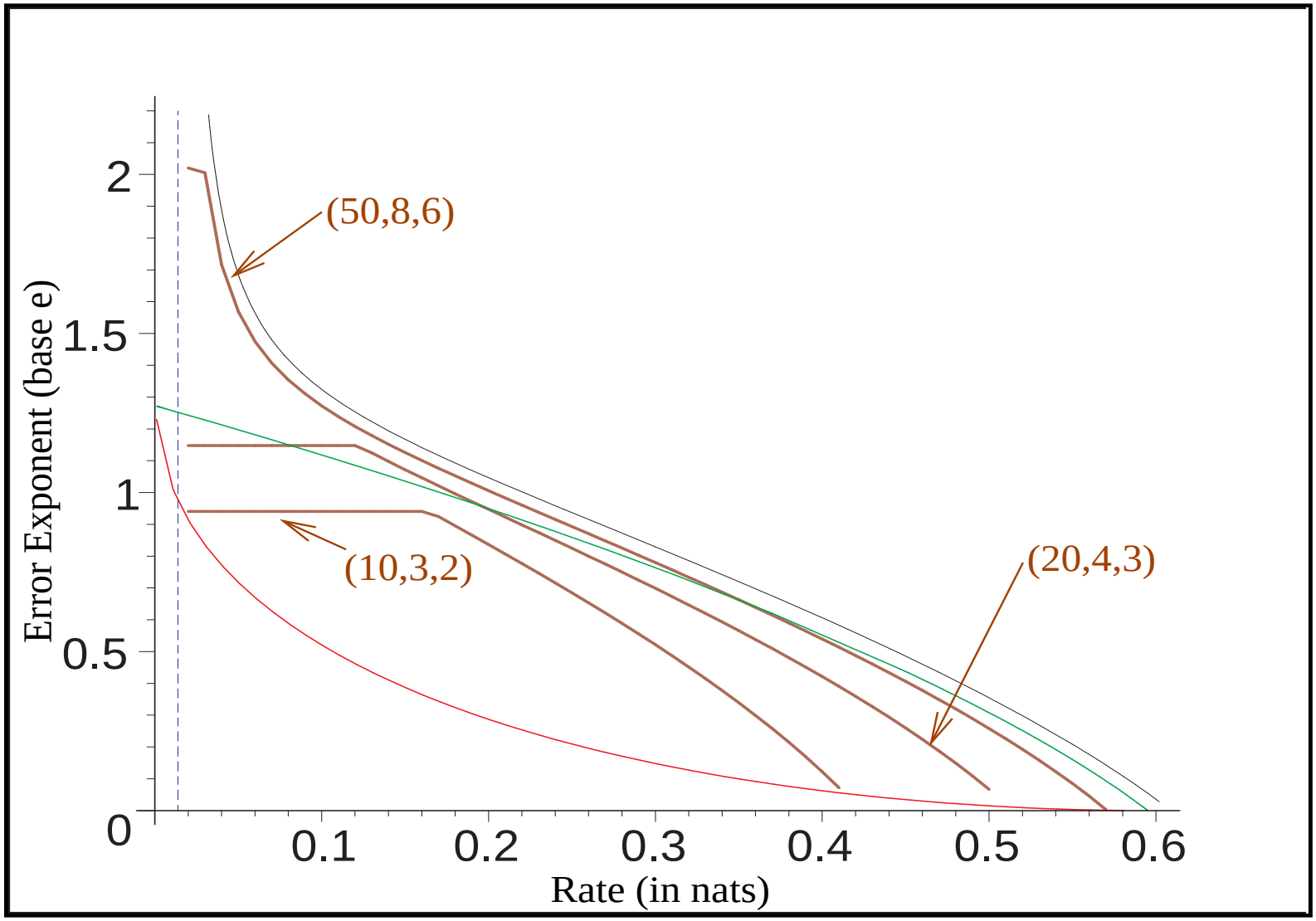
If n large, effective point-message rate $(n(1 - \frac{R}{R'}))^{-1}$ is small.

But low-rate erasure exponent $\approx -\log \delta$

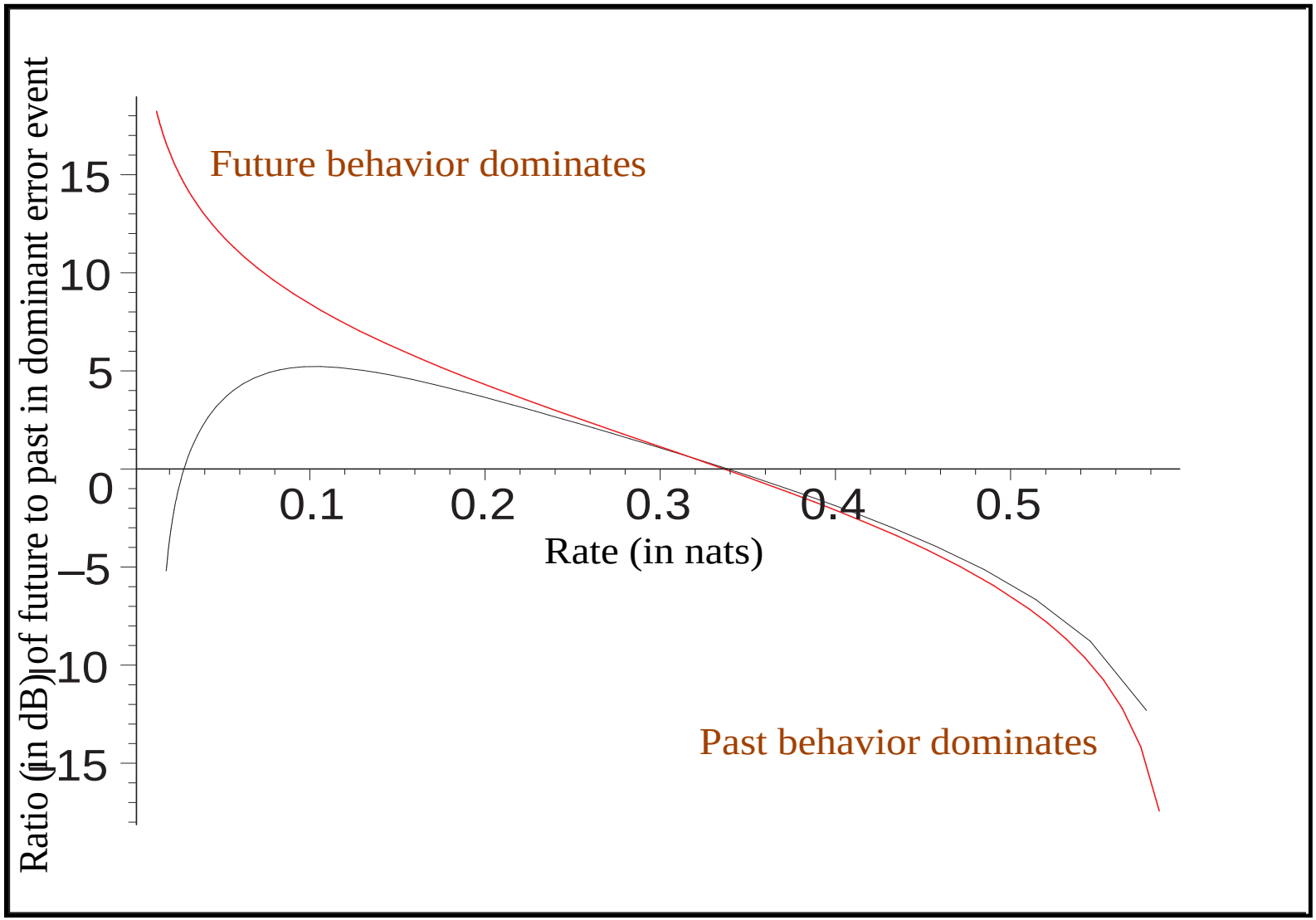


$-\log \delta = E_0(\rho')$ in our context.

Approaching the focusing bound



The dominant error events: past vs future



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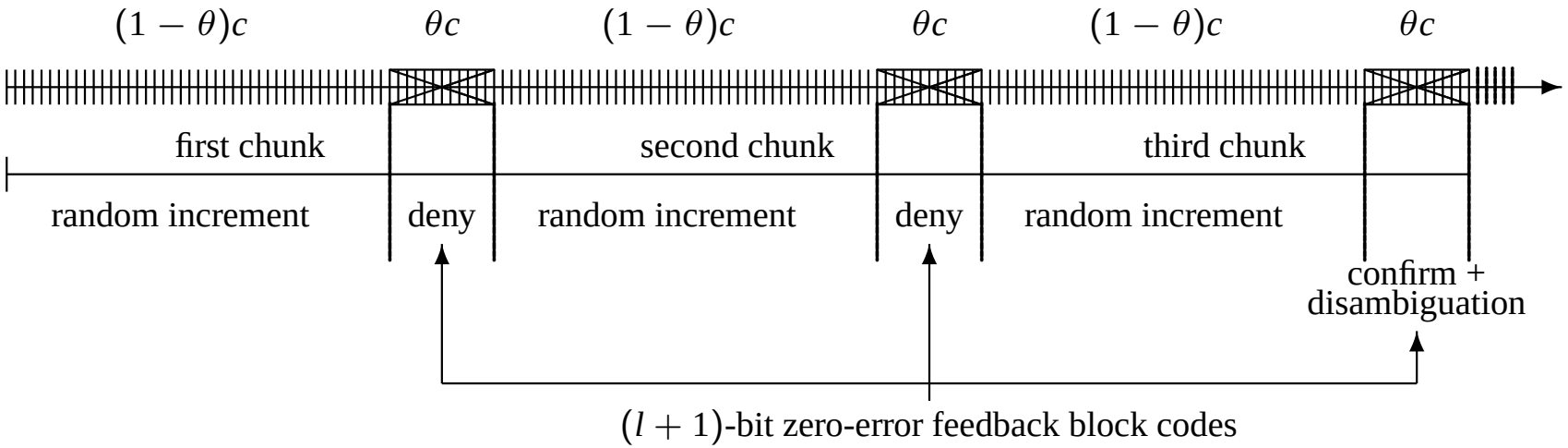
Channels with positive zero-error capacity

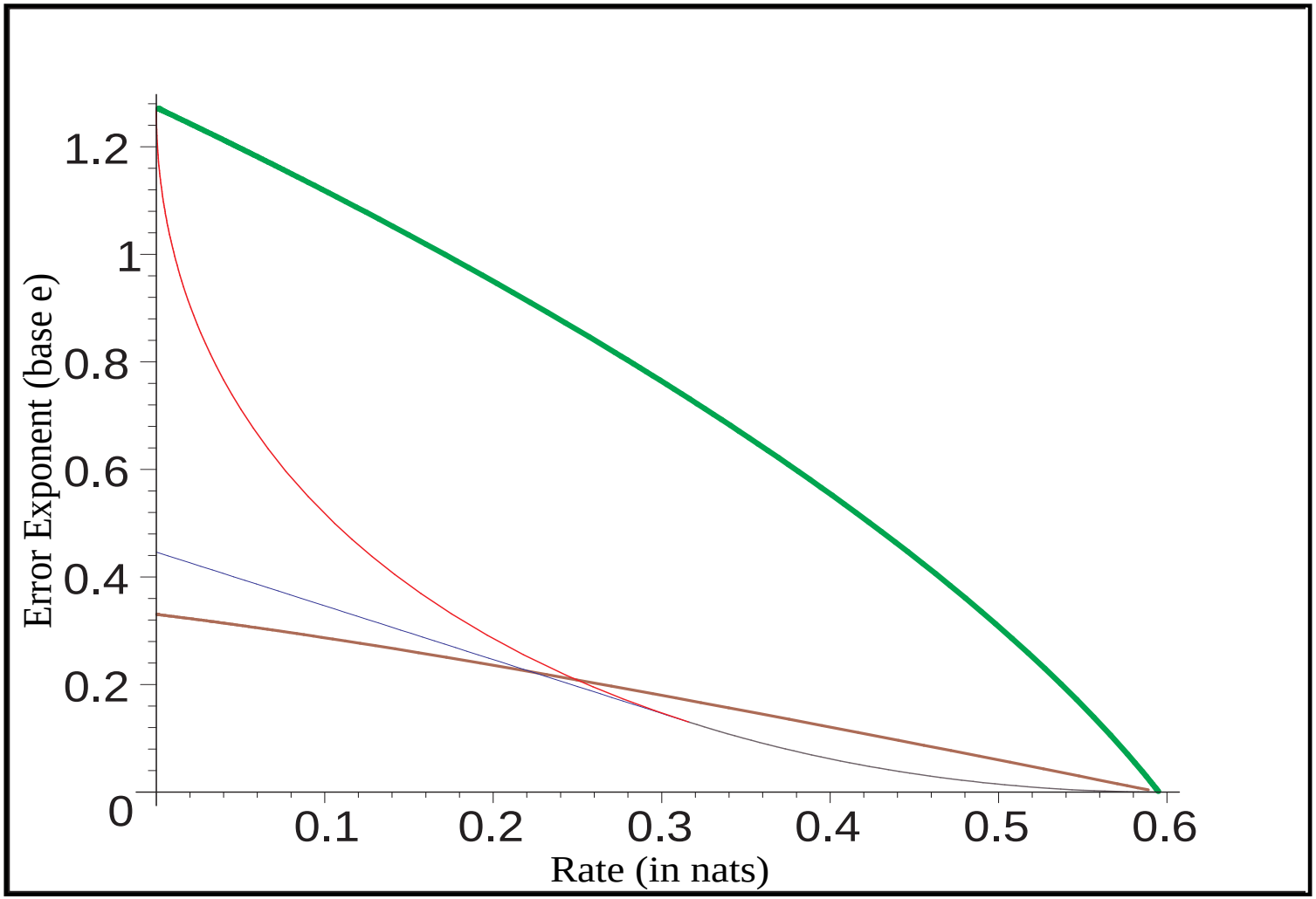


Throw away a fraction θ of channel uses for flow-control overhead

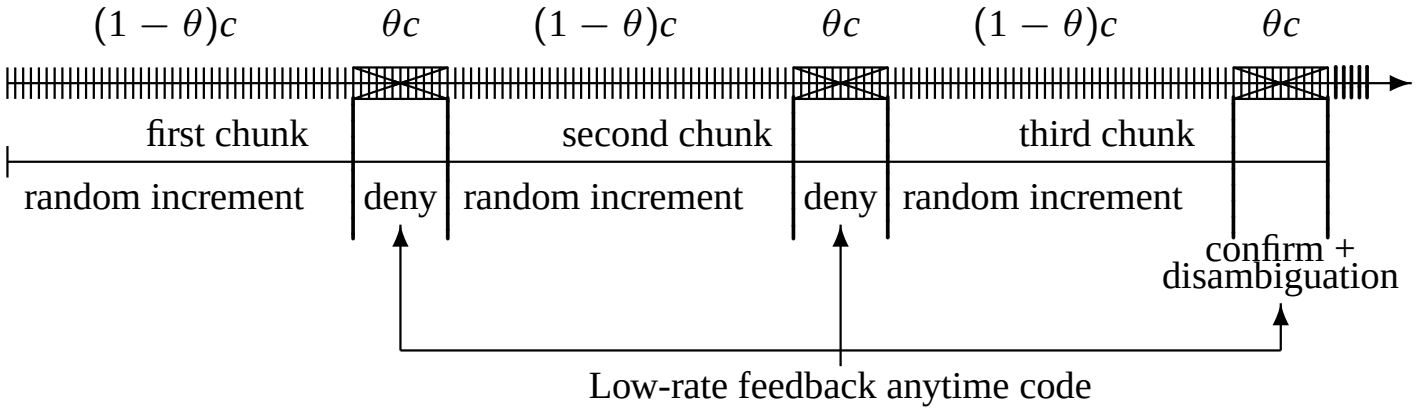
Asymptotically achieves the focusing bound as $\theta \to 0$.

Can do well even without "sugar"

Time-share flow-control and data and optimize fraction θ for flow-control.

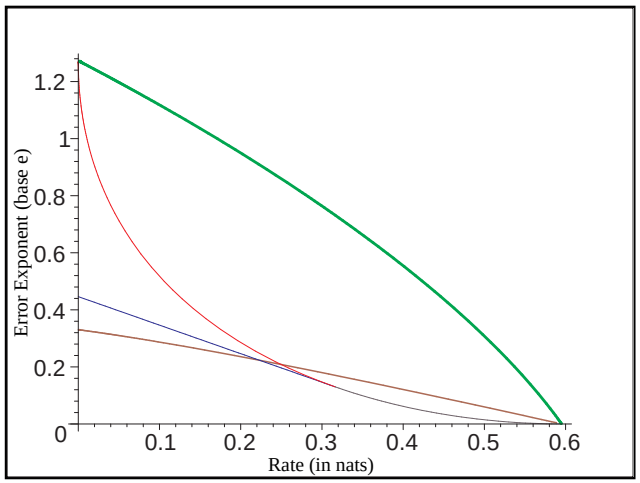


Optimize fraction θ for flow-control



Flow-control encoded with "∞-length" convolutional code.

- Flow-control exponent $\approx \theta E_0(1)$
- Data exponent $\approx (1-\theta)E_0(\rho)$
- Data rate $\approx (1-\theta)\frac{E_0(\rho)}{\rho}$



$$\theta^* = \frac{E_0(\rho)}{E_0(1)+E_0(\rho)}$$

$$E_0^*(\rho) = \frac{E_0(\rho)E_0(1)}{E_0(\rho)+E_0(1)}$$

$$R^*(\rho) = \frac{E_0^*(\rho)}{\rho}$$

$$E^*(\rho) = E_0^*(\rho)$$

Dealing with channel uncertainty

- Compound DMC
  - Fix input distribution.
  - Assume only $I(X;Y) \geq C$ is known.
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$$E_r(R) \geq \frac{(C-R)^2}{8/e^2 + 4[\ln|\mathcal{Y}|]^2}$$

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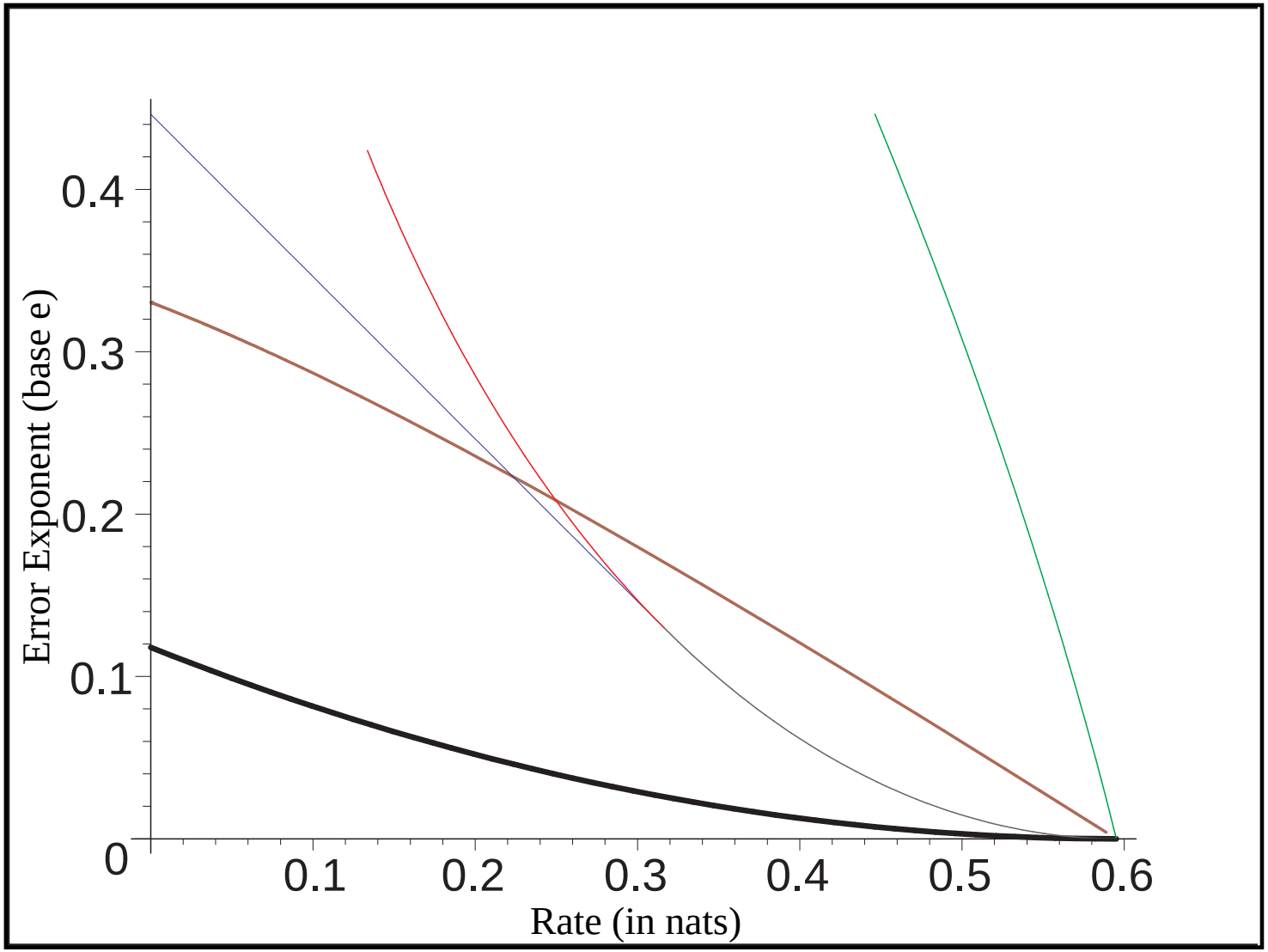
- Can translate to delay and focusing-bound.

Universal bounds illustrated



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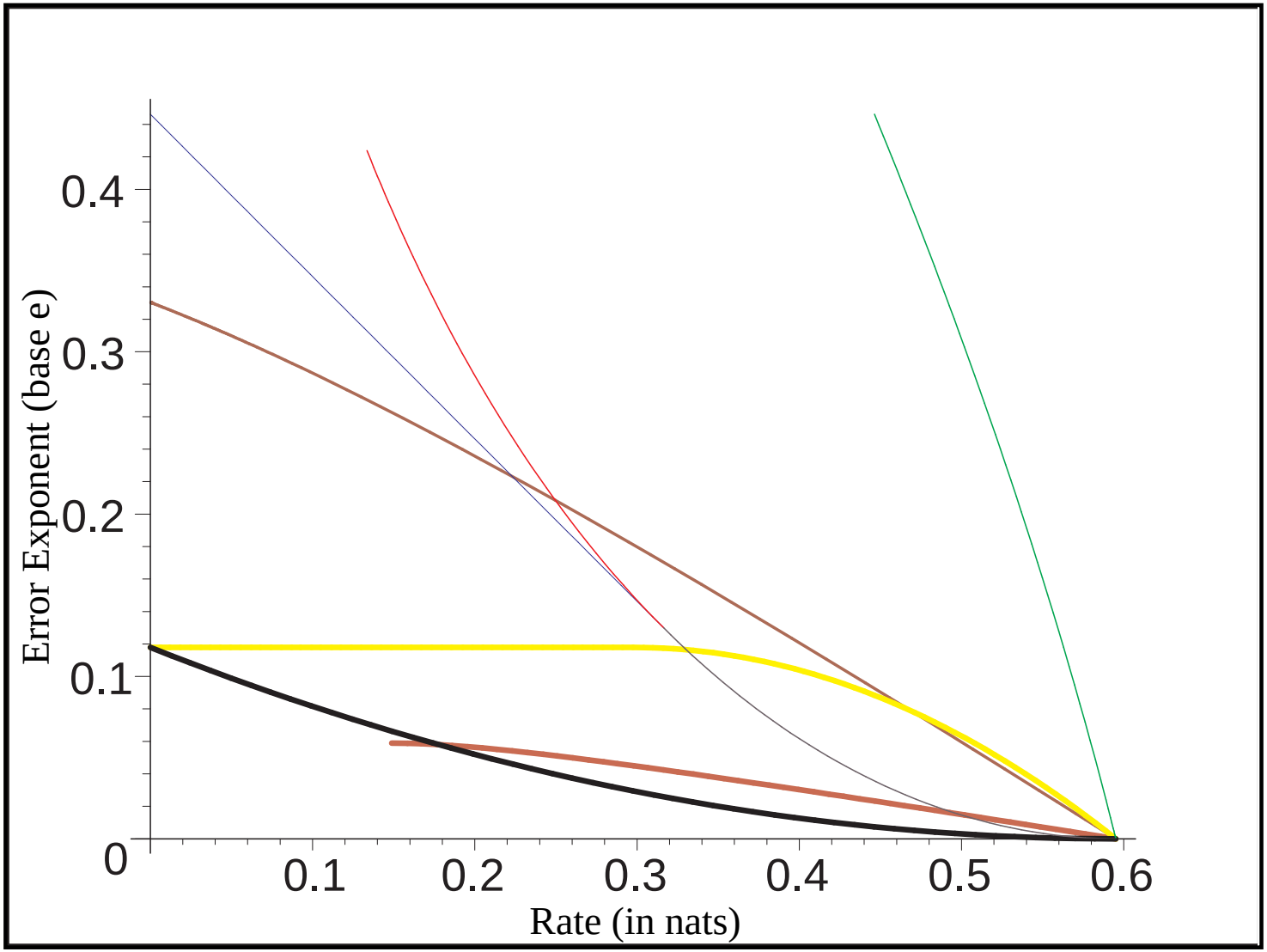

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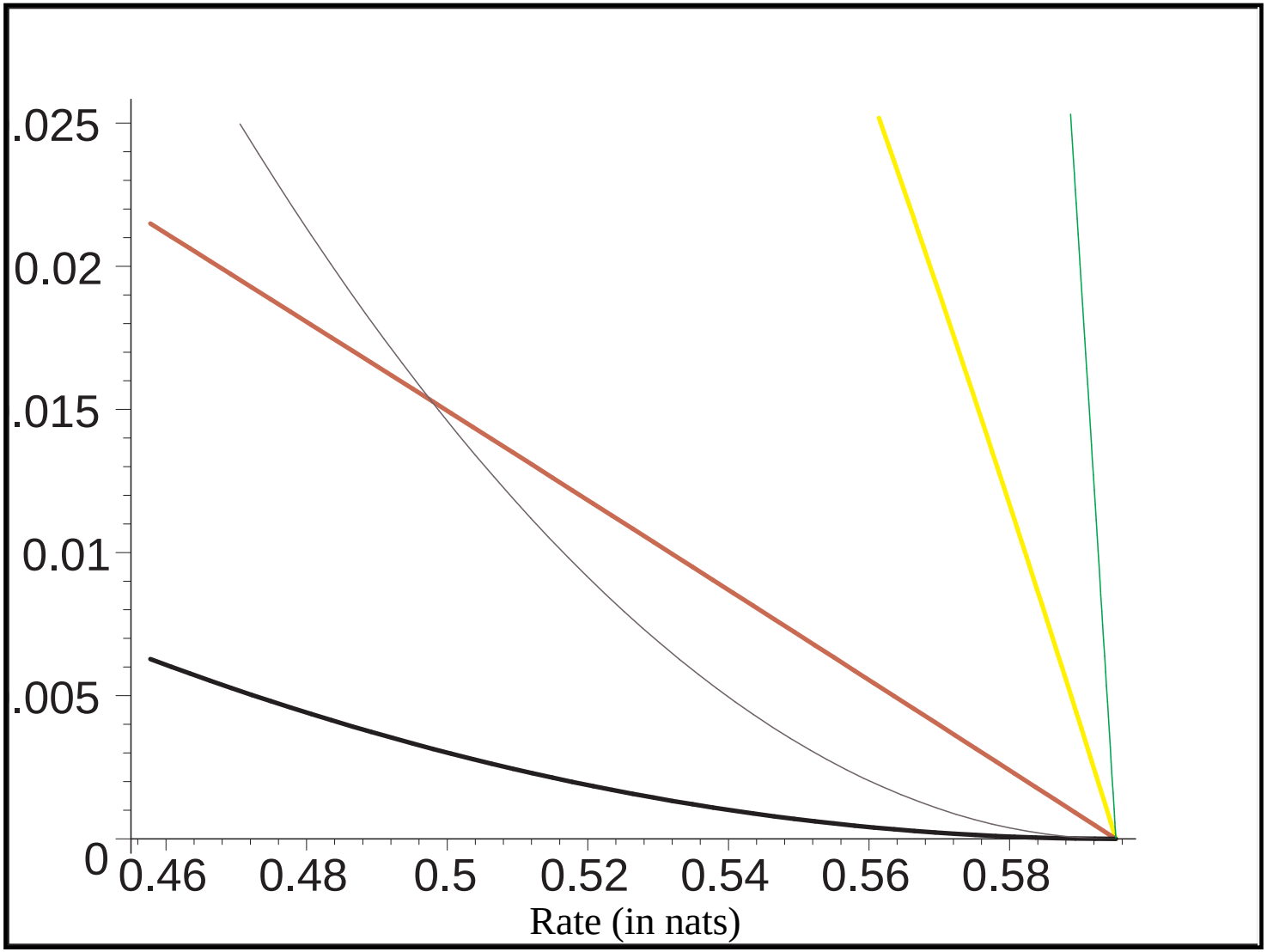

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Hard deadline coding final comments

- For known channels, computation per channel use does not depend on probability of error = *infinite computational exponent*
- The code is “anytime” in that it is delay universal — application can pick what latency is desired.
- Queuing delay dominates at all rates.
- Even with unknown channels, get strictly positive slope at capacity.

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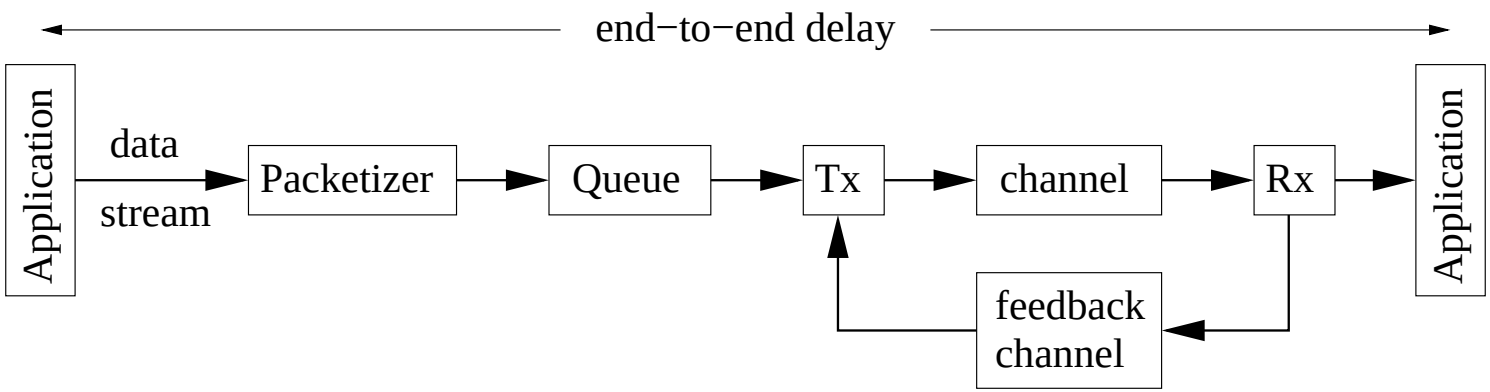
A streaming data perspective on soft deadlines



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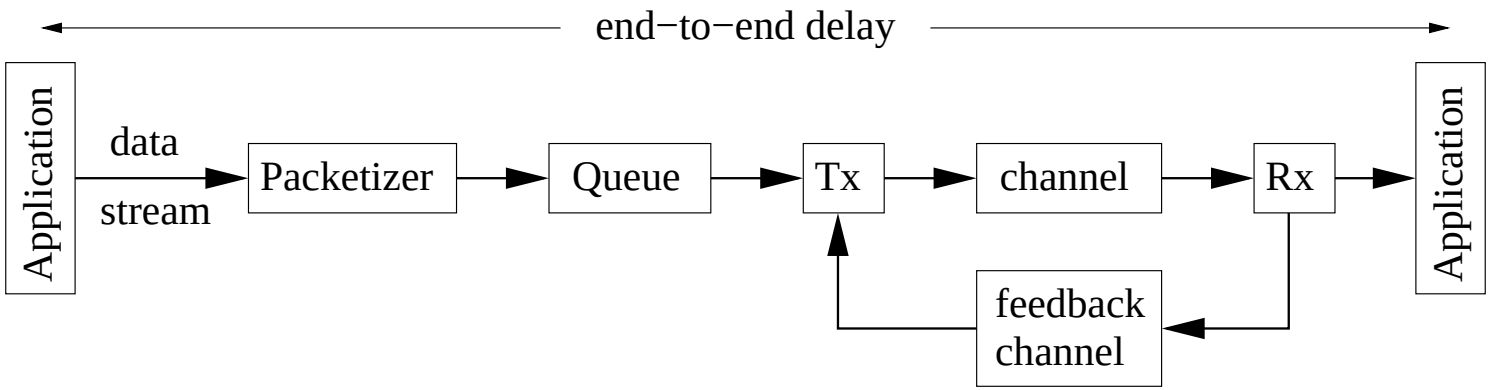

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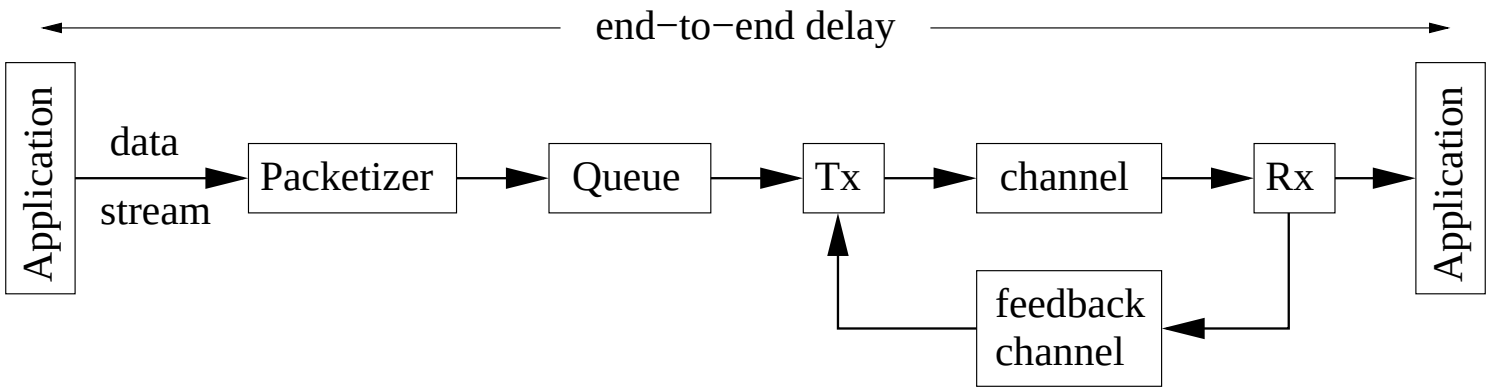
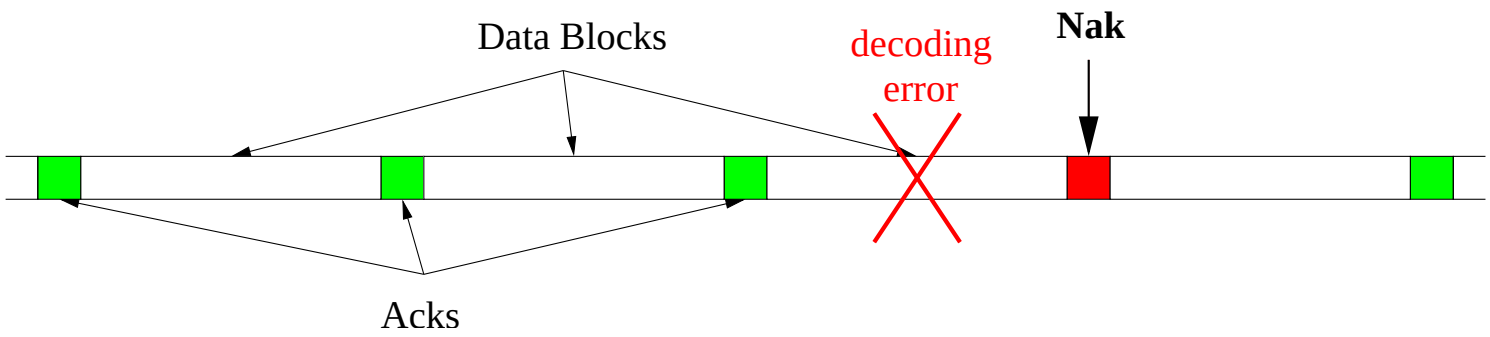


- Queue is optional. Needed if retransmissions are required
- If retransmissions are rare, then **expected** end-to-end delay is dominated by the Tx to Rx delay.

An opportunity presents itself



- Erasures are rare so most messages are confirmed.

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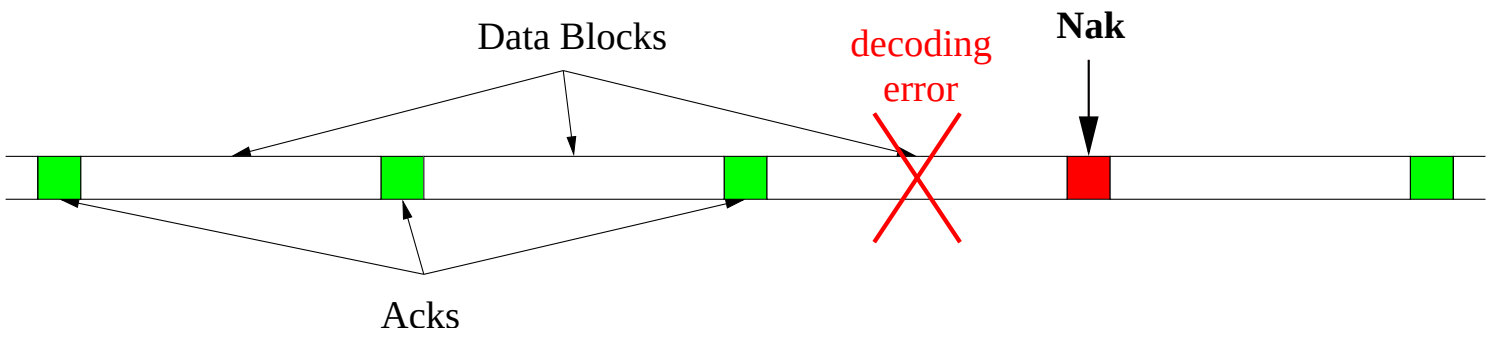

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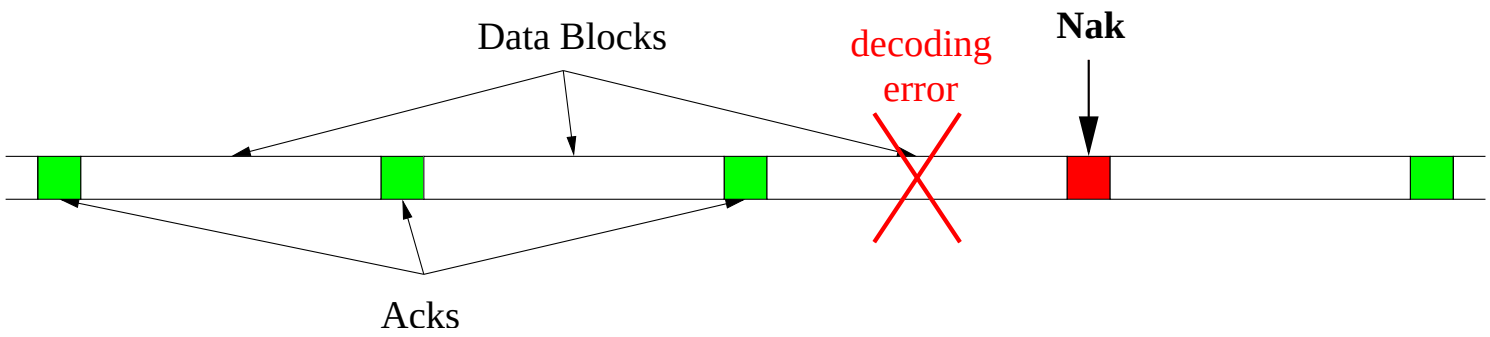

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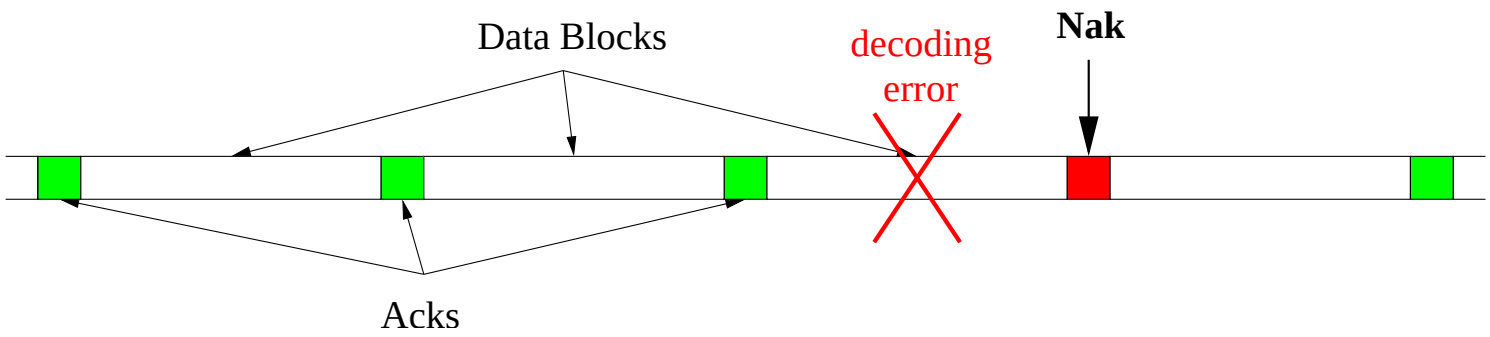

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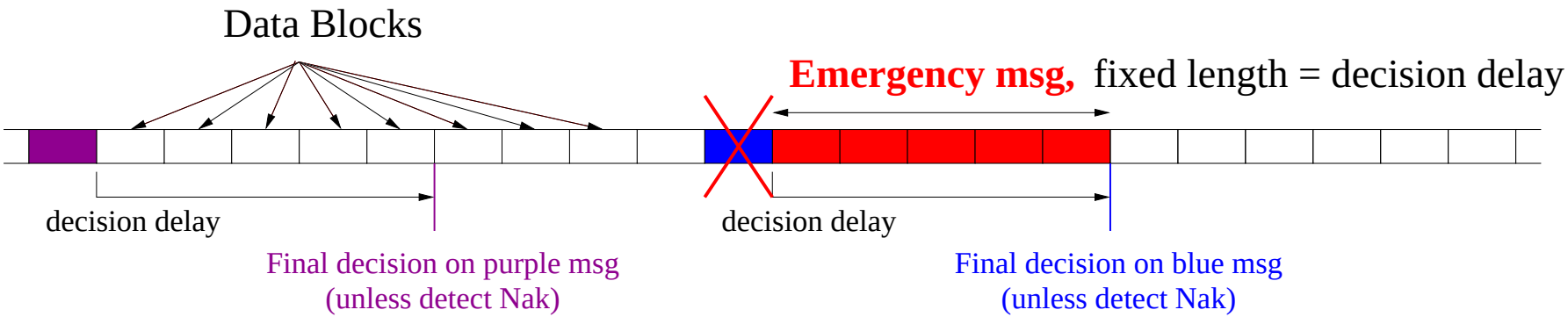
- What if we only sent NAKs when needed?
- Have a special message for this purpose.

Sliding blocks with collective punishment only (Kudryashov-79)



- Make packet size n much smaller than soft deadline d.

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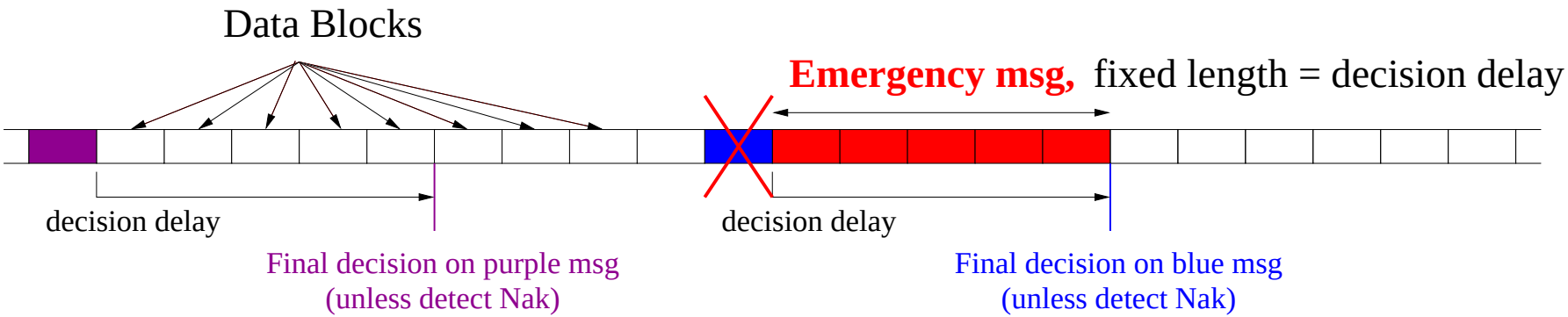

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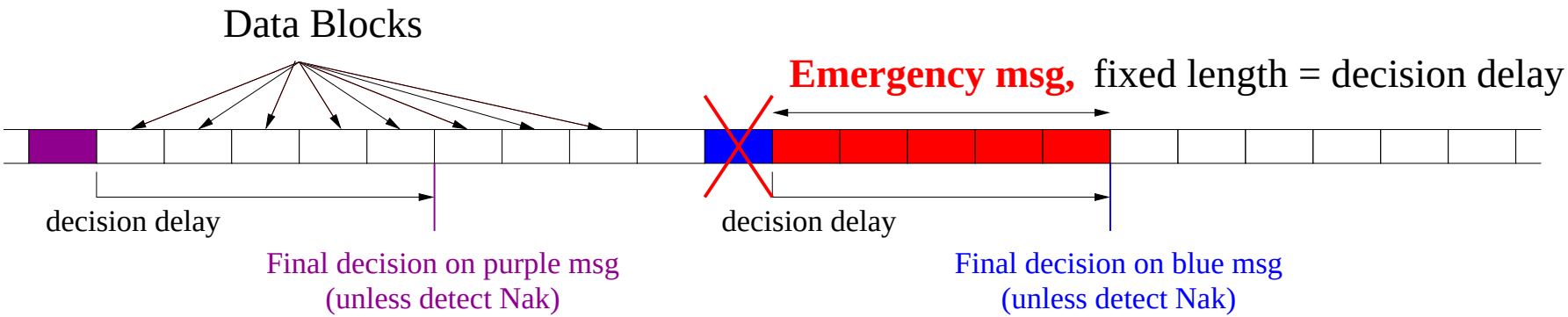
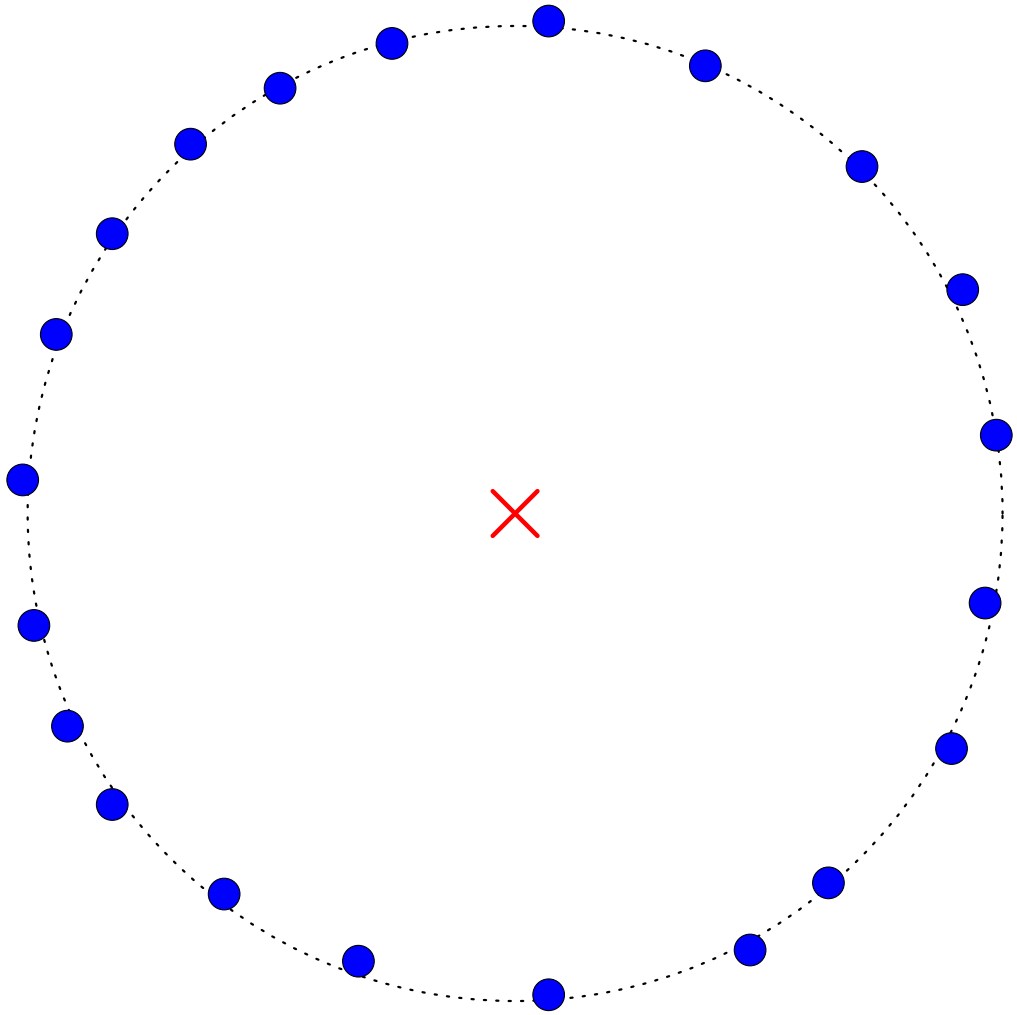


- Make packet size n much smaller than soft deadline d.
- A NAK collectively denies the past $\frac{d}{n} - 1$ packets
- Error only if $\frac{d}{n} - 1$ NAKs are all missed

Unequal error protection required in codebook



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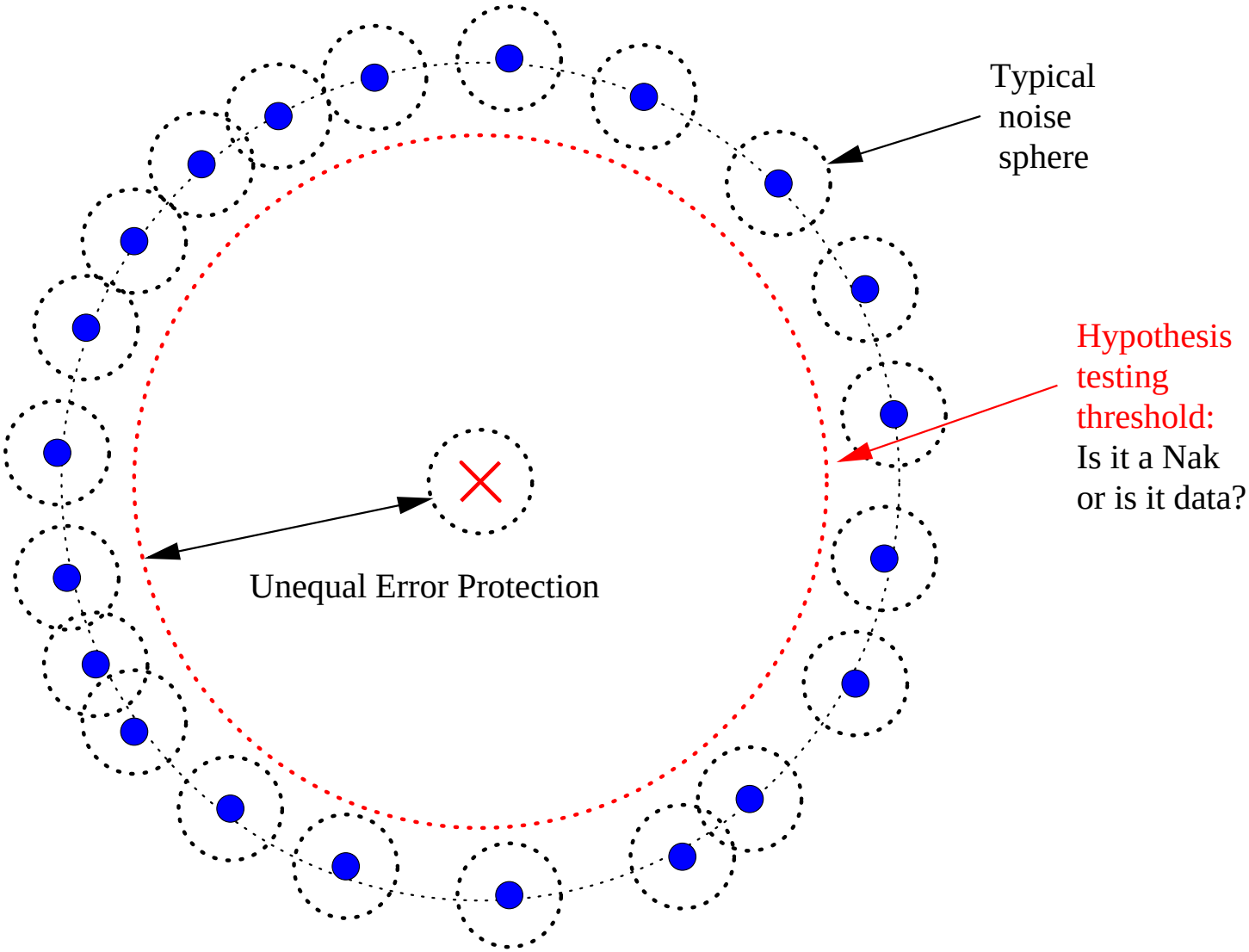
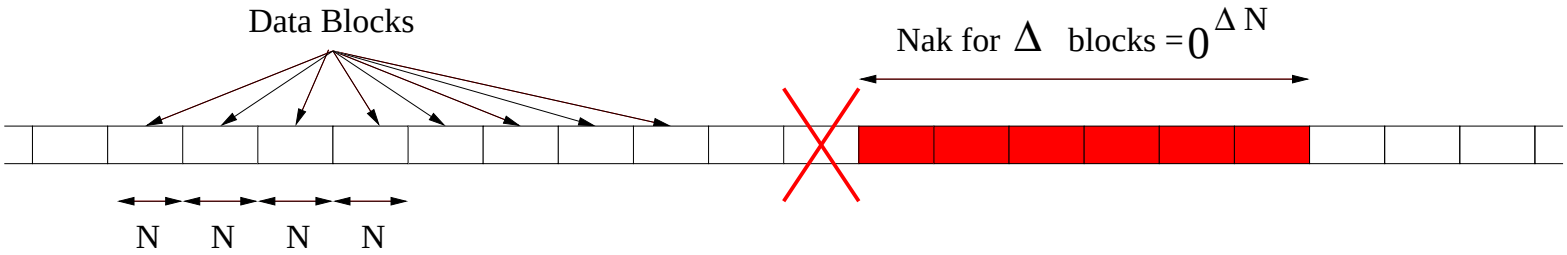
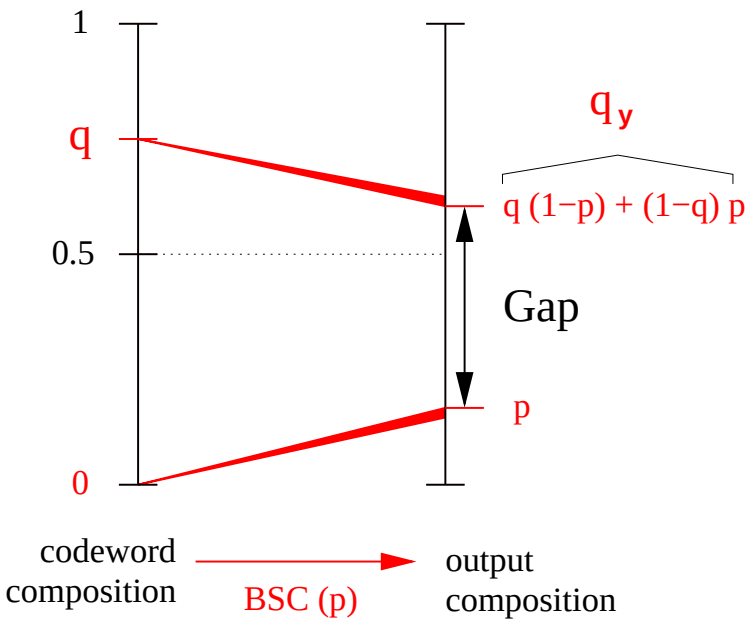


Specialize to BSC case





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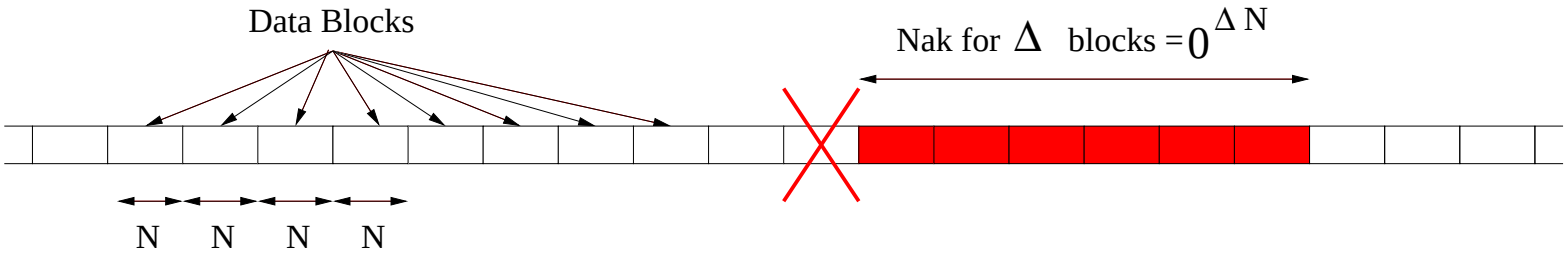

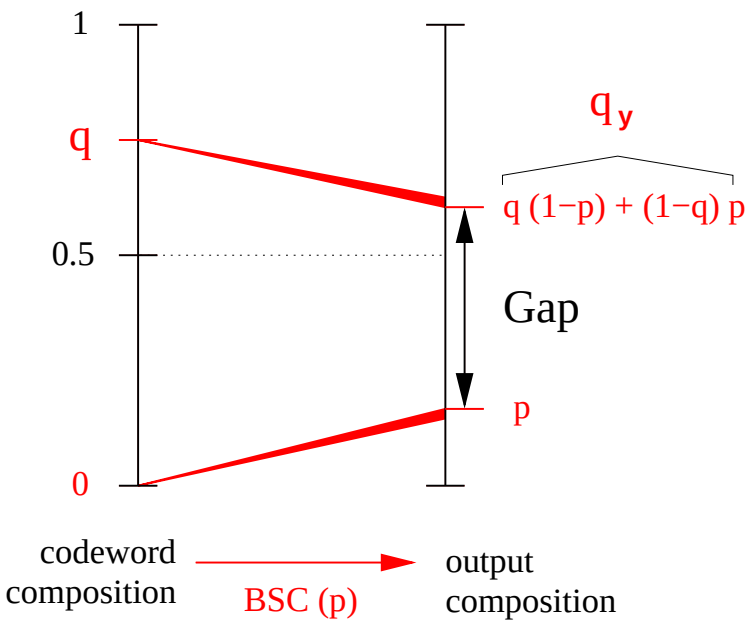

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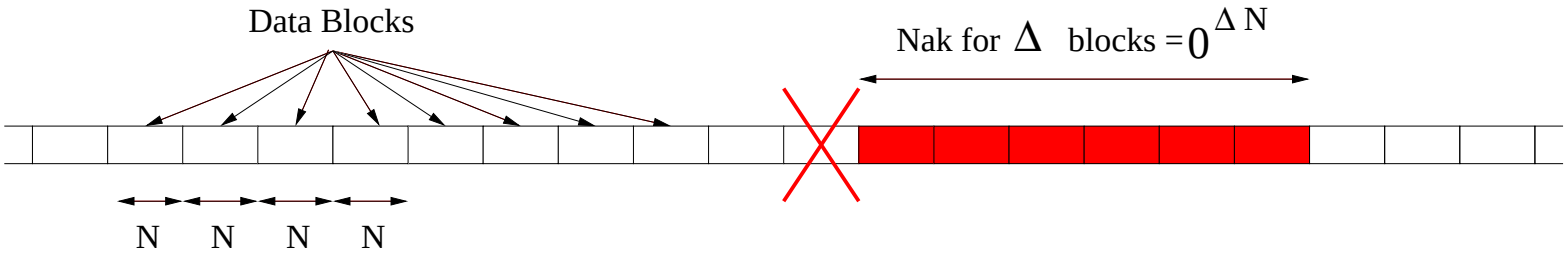

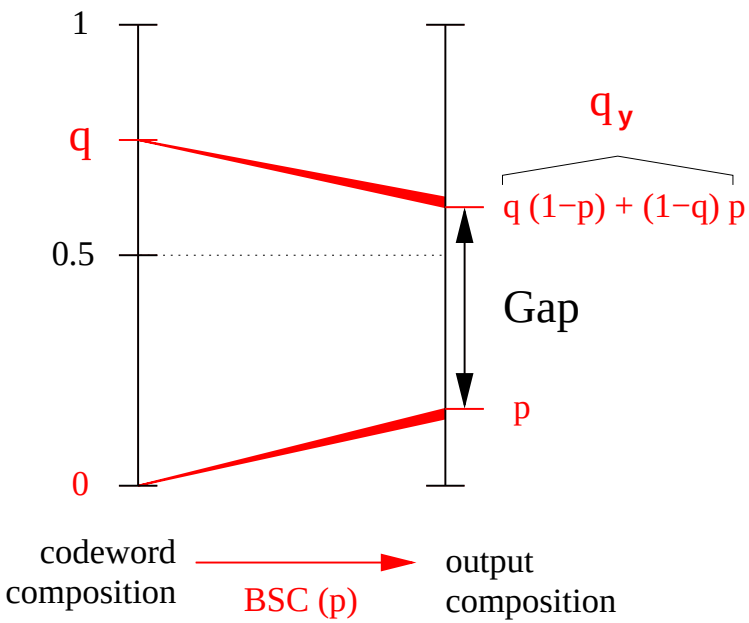

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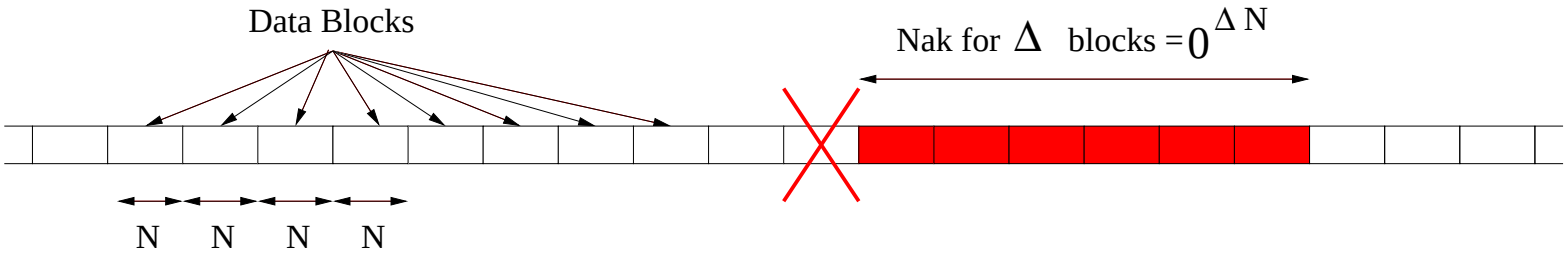

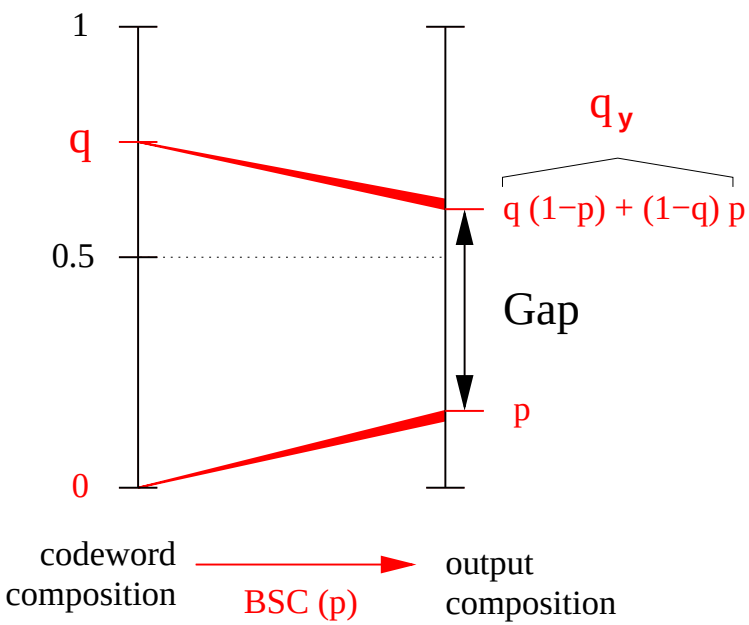

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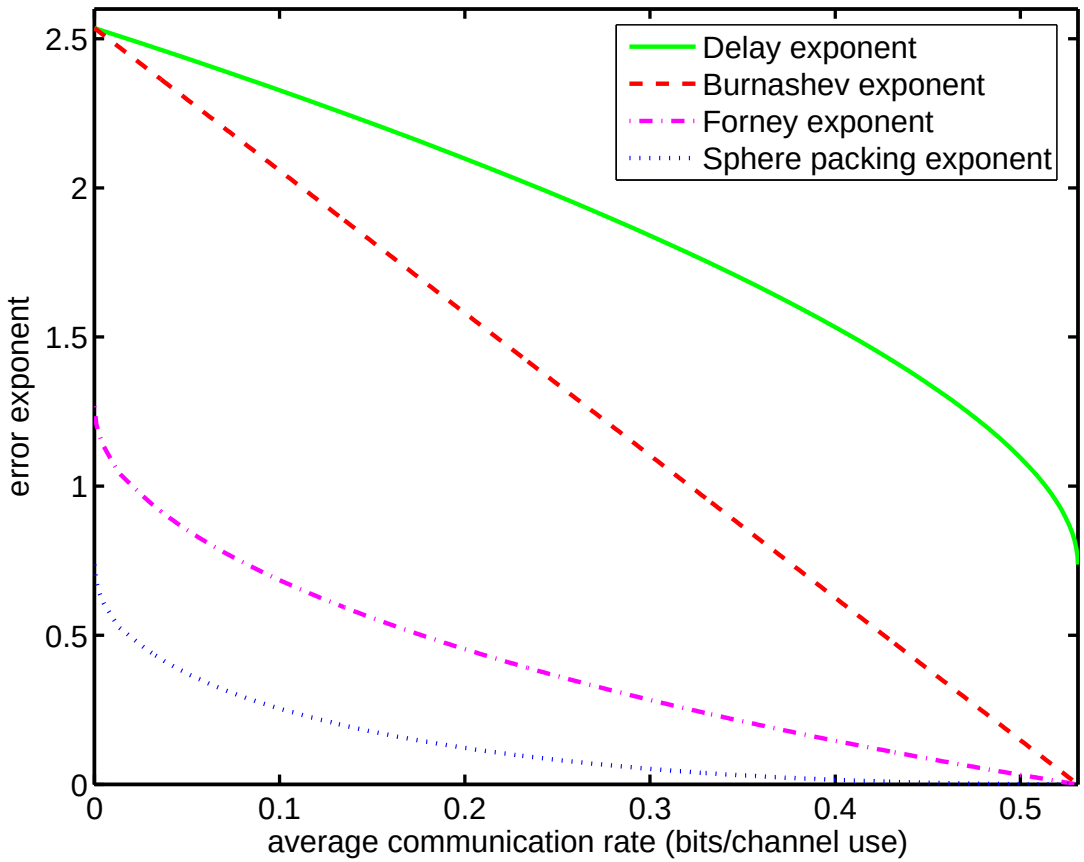

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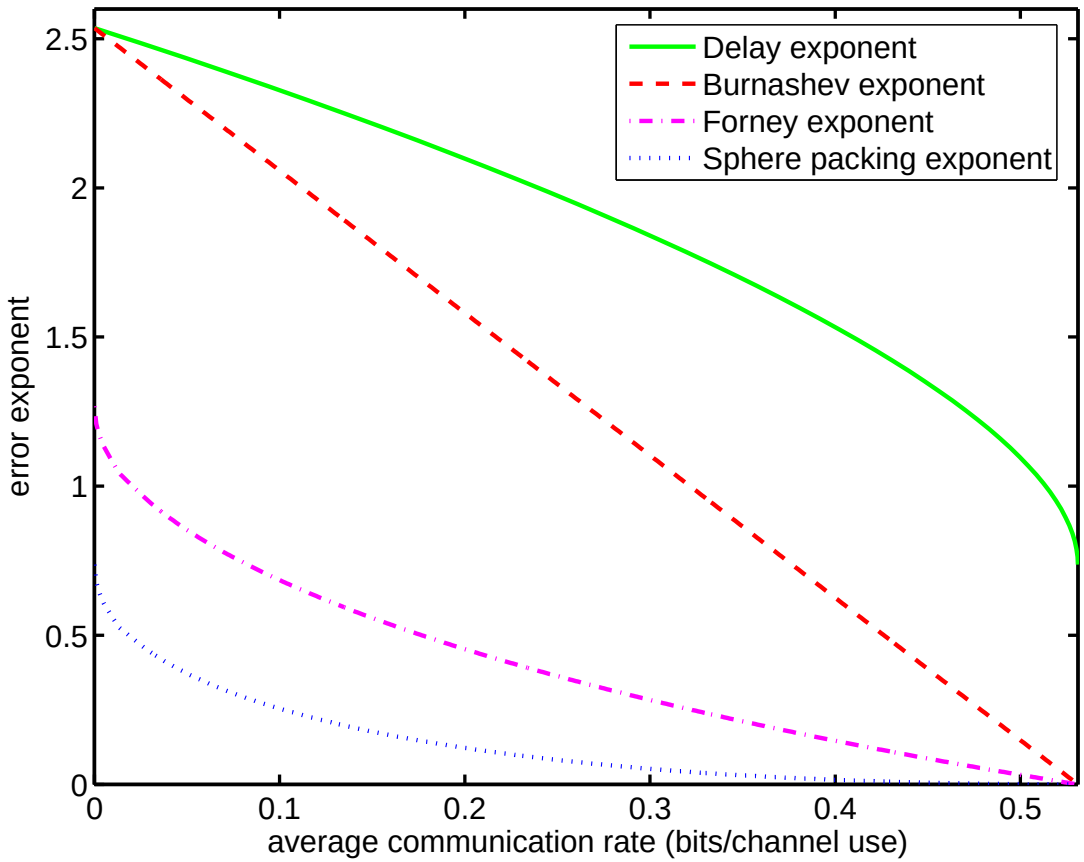

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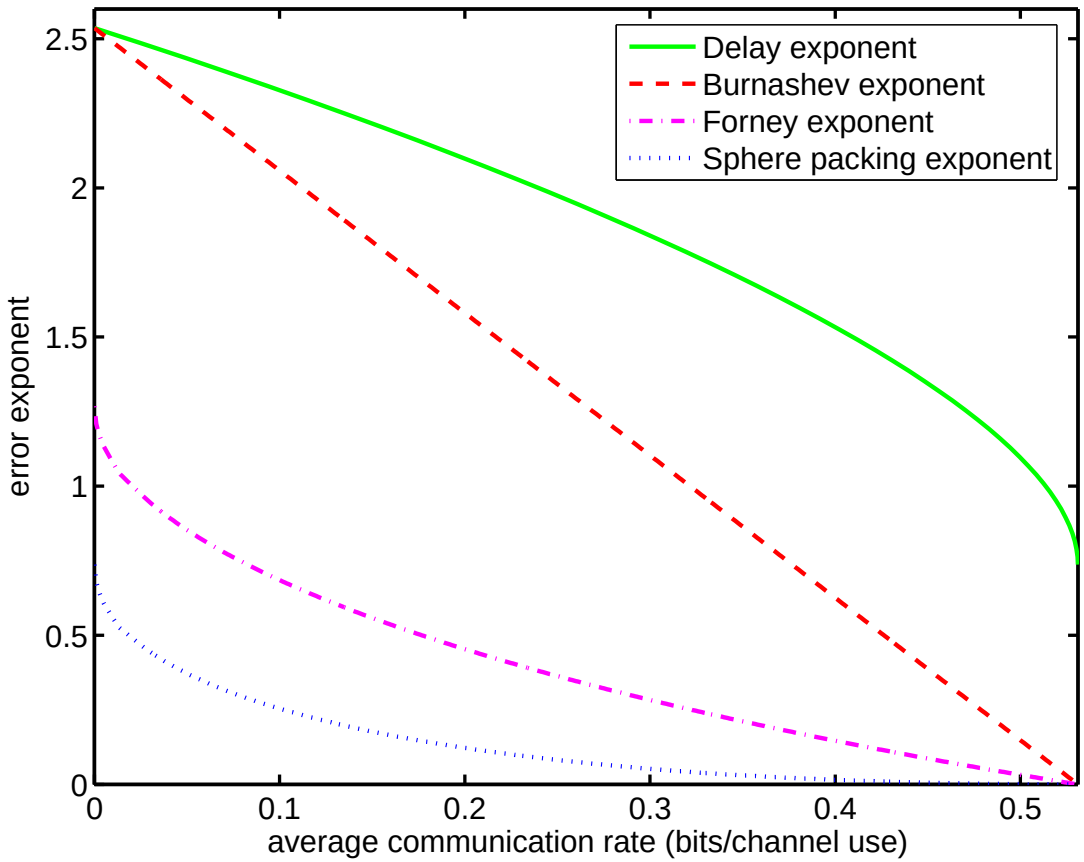

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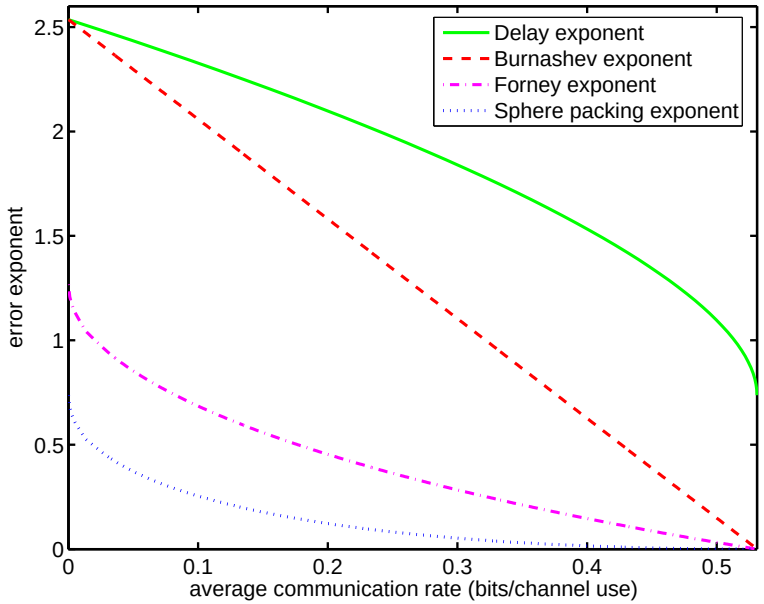
- Matches Horstein-63 exponent.

Matches the “hallucination bound”



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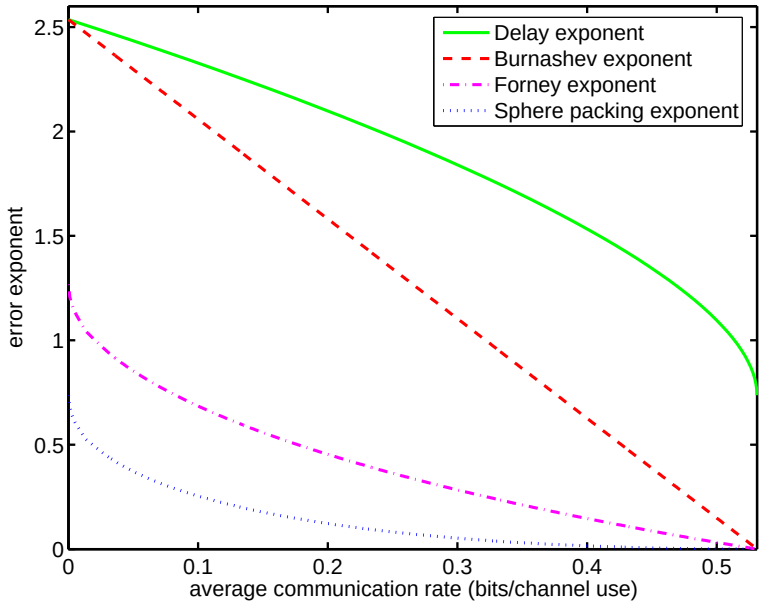

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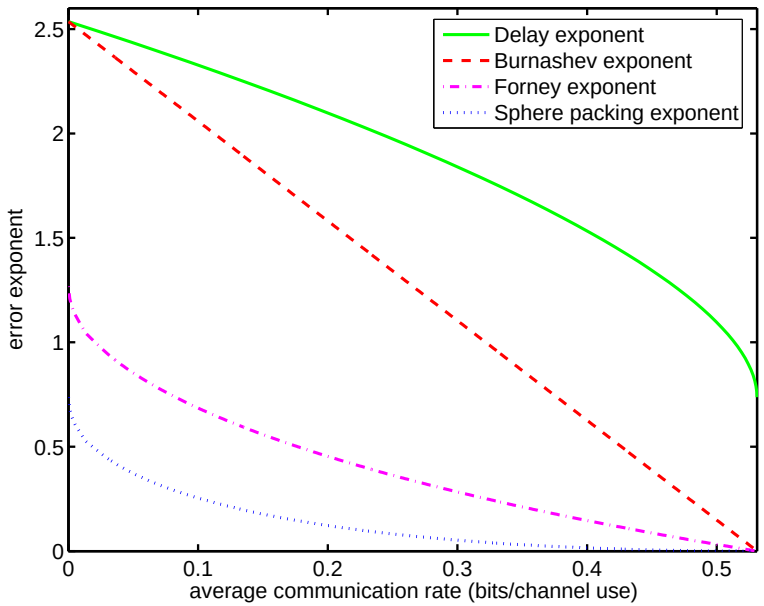

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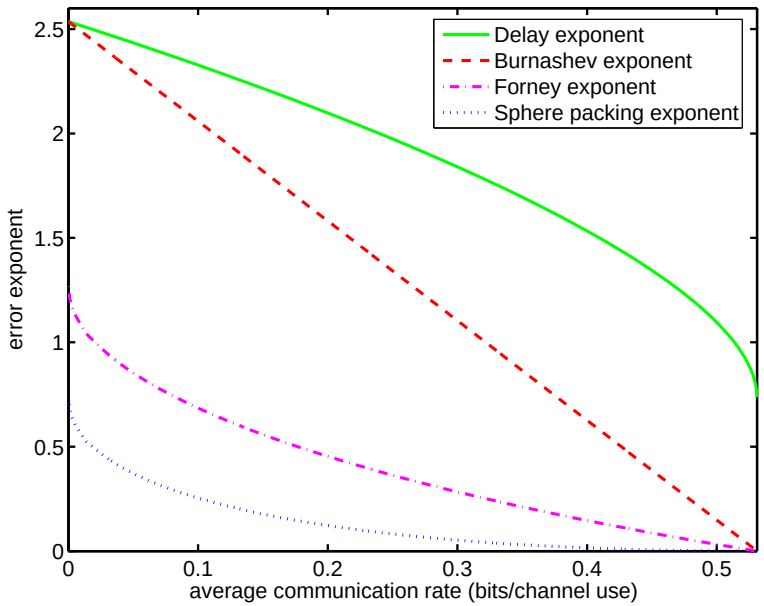

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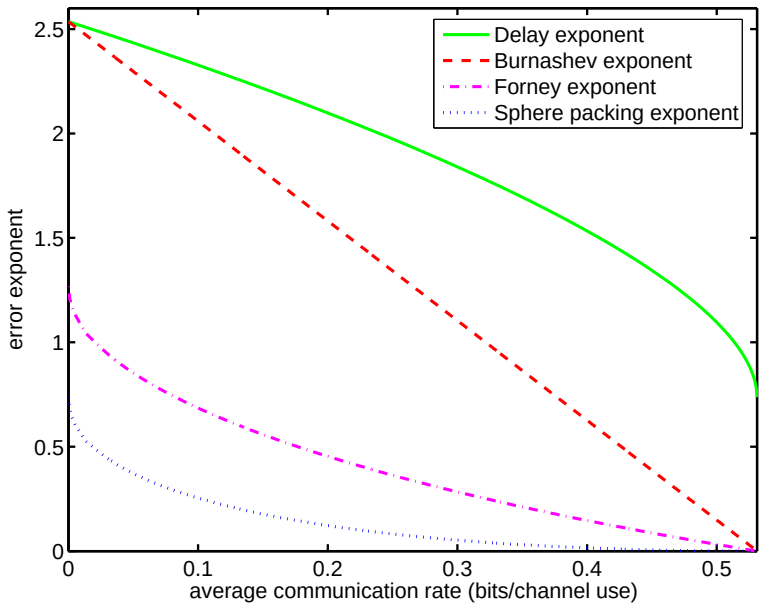

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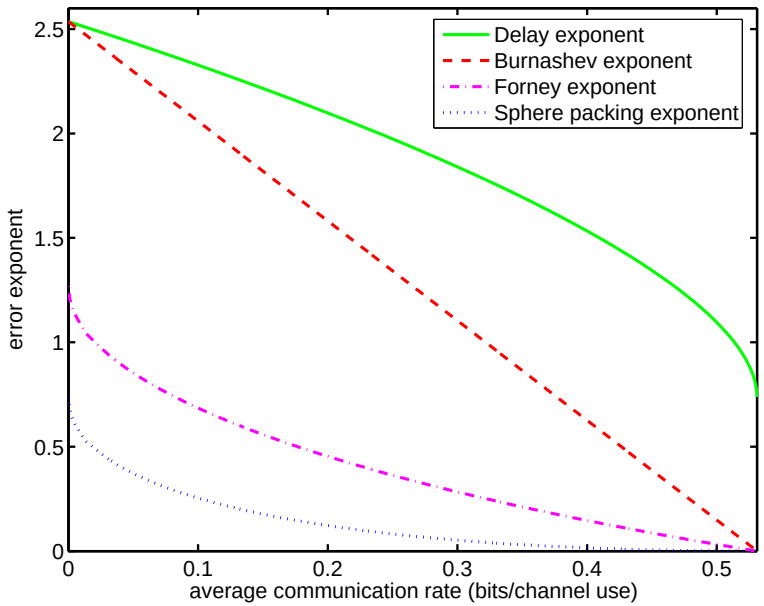

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The goal: show "robustness" of gains due to feedback.

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  - Increases average power slightly.

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Packet-erasure channel

- Forward packets either delivered or erased
- Feedback limitations:
  - Delayed but noiseless
  - Over a separate low-rate packet-erasure channel
  - Over a shared packet-erasure channel

Example: short round-trip-time k

First approach: treat as k parallel unit-delay channels

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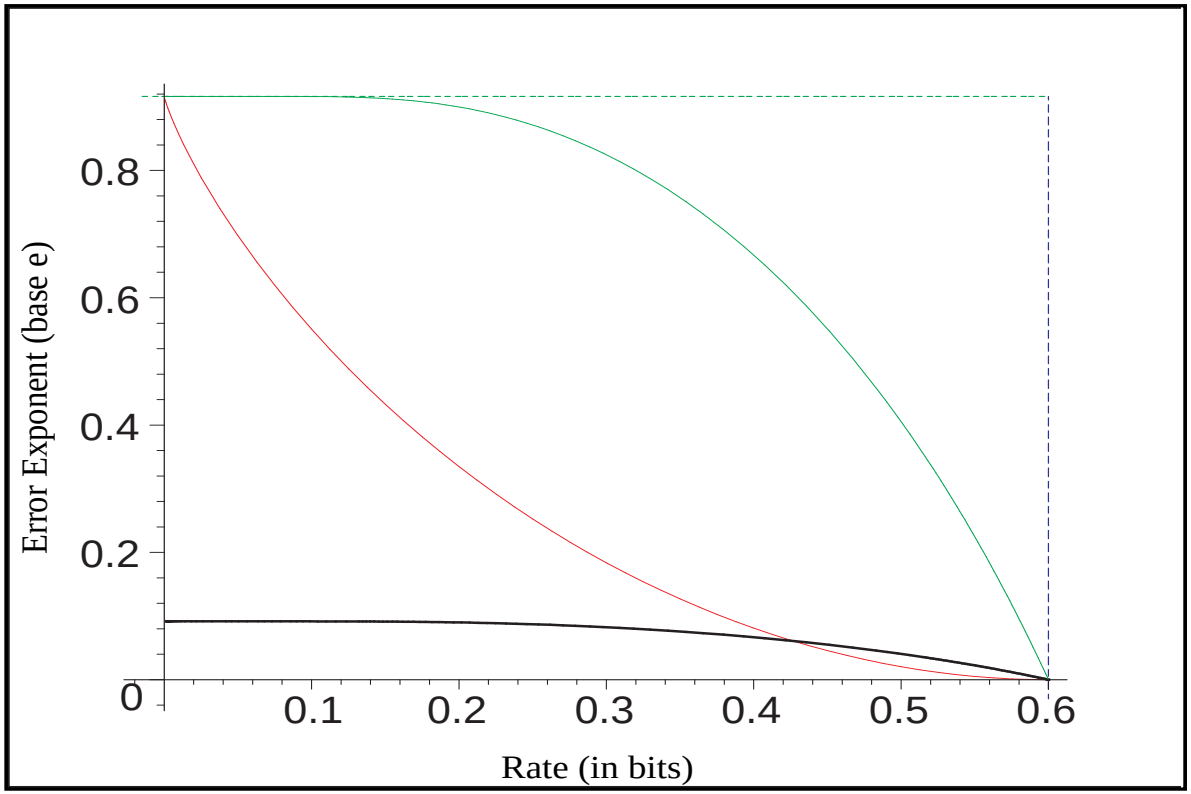

Serious penalty for "dividing the channel"

Example: unreliable feedback

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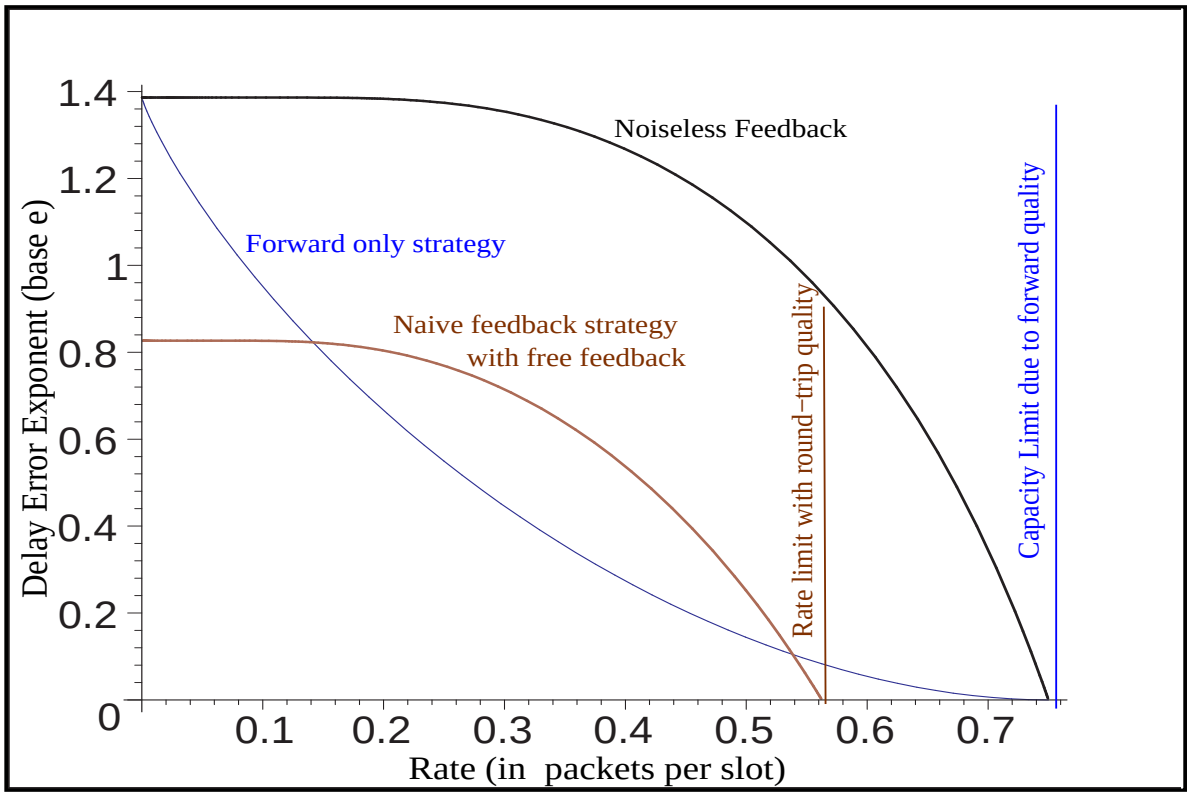
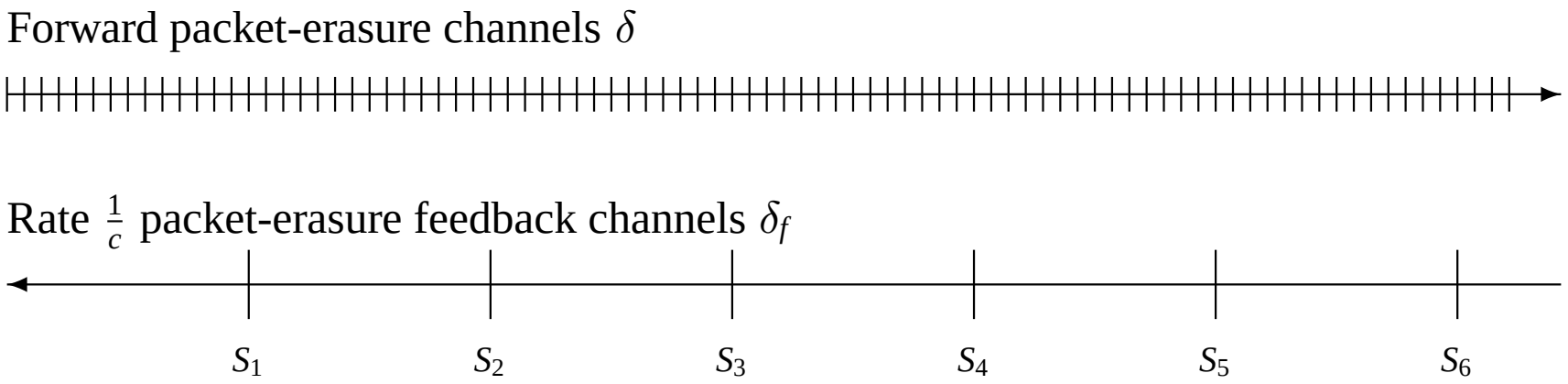


Capacity penalty for treating both losses symmetrically.

Unreliable feedback picture



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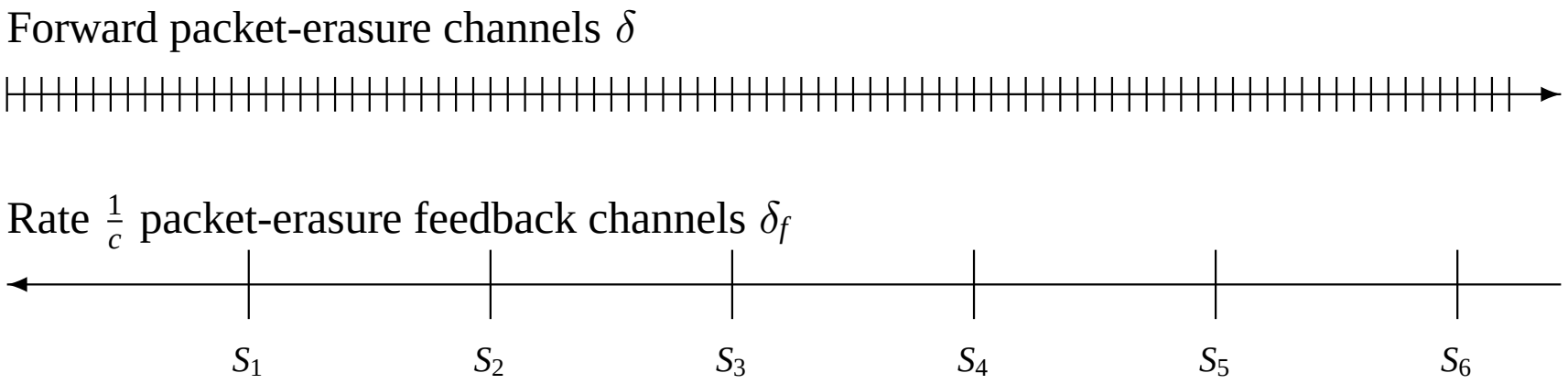

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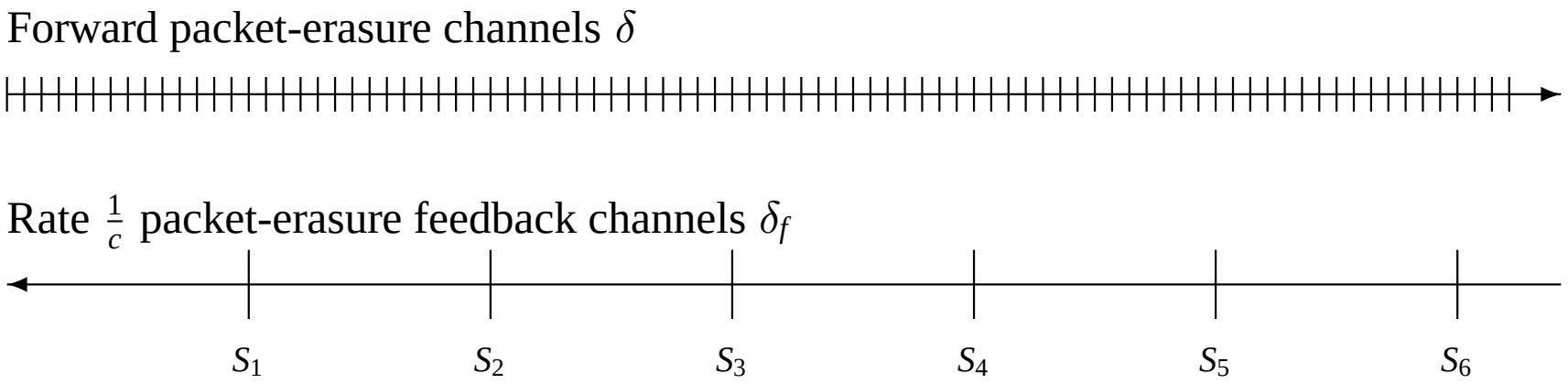

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- Assume target latency $d \gg c, k$.

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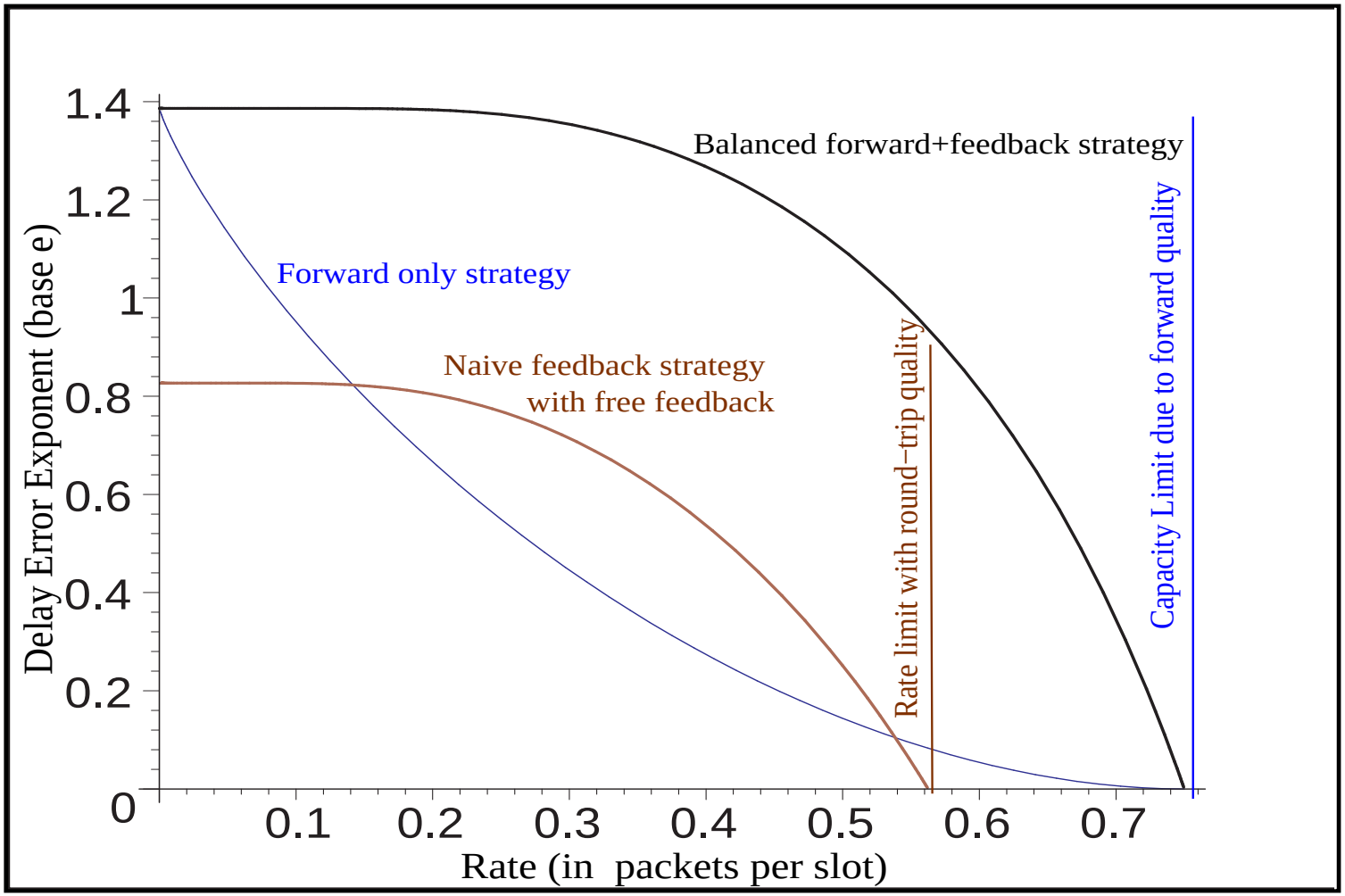
- Add one header bit to forward packets (Massey)
  - 0: first message block
  - 1: second message block
  - 0: third message block
  - $\vdots$

Performance with unreliable feedback channel



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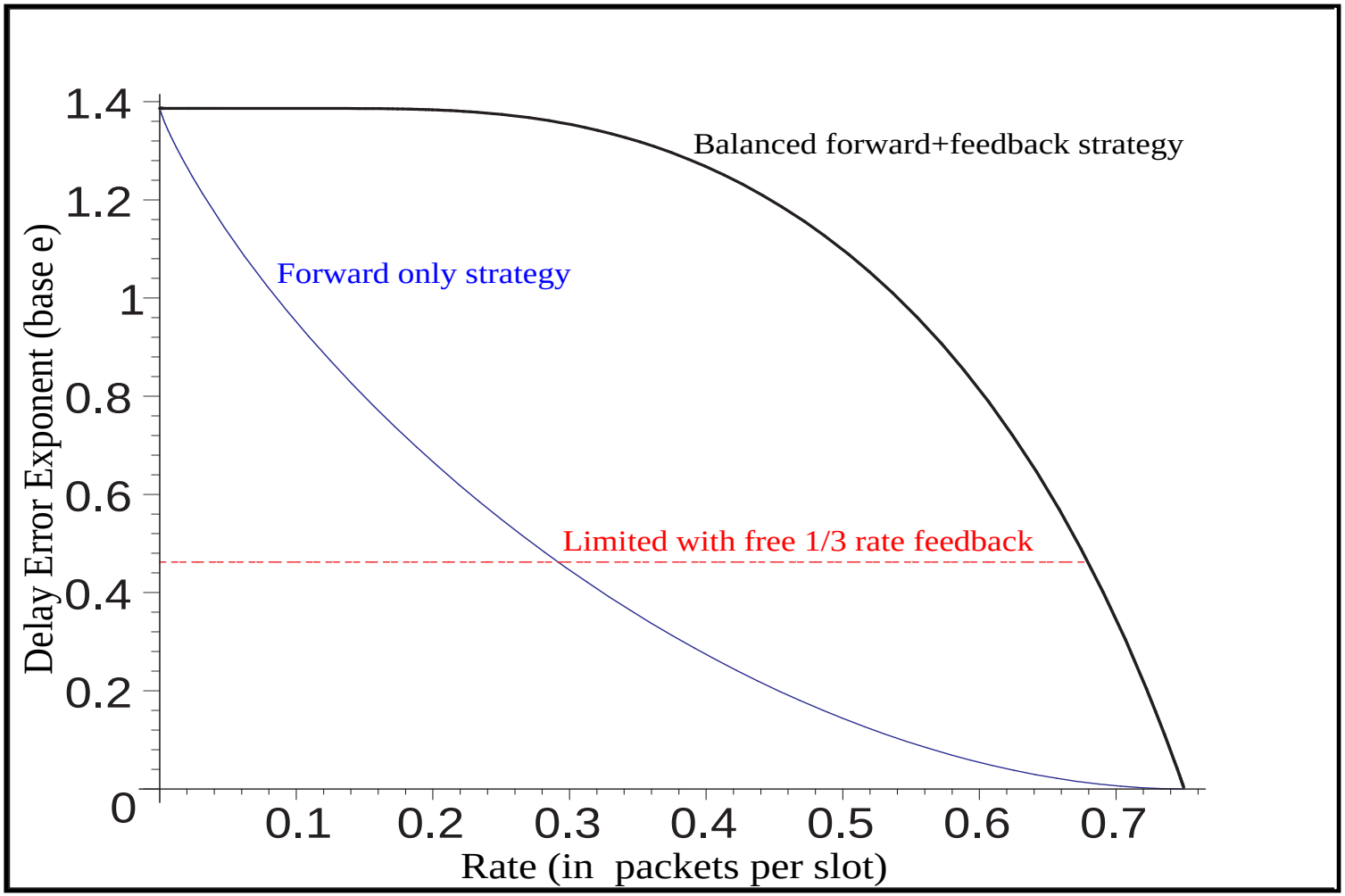
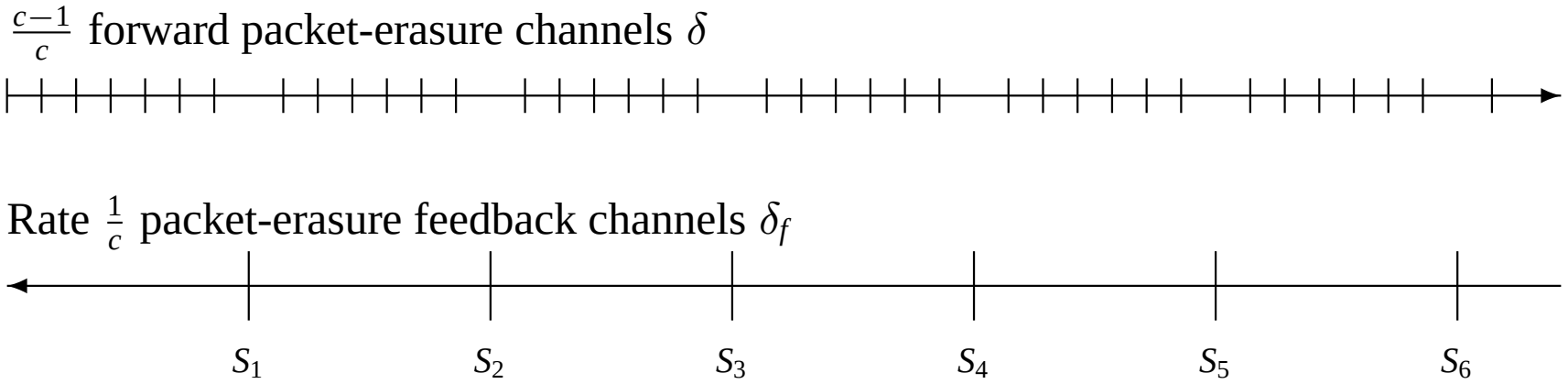
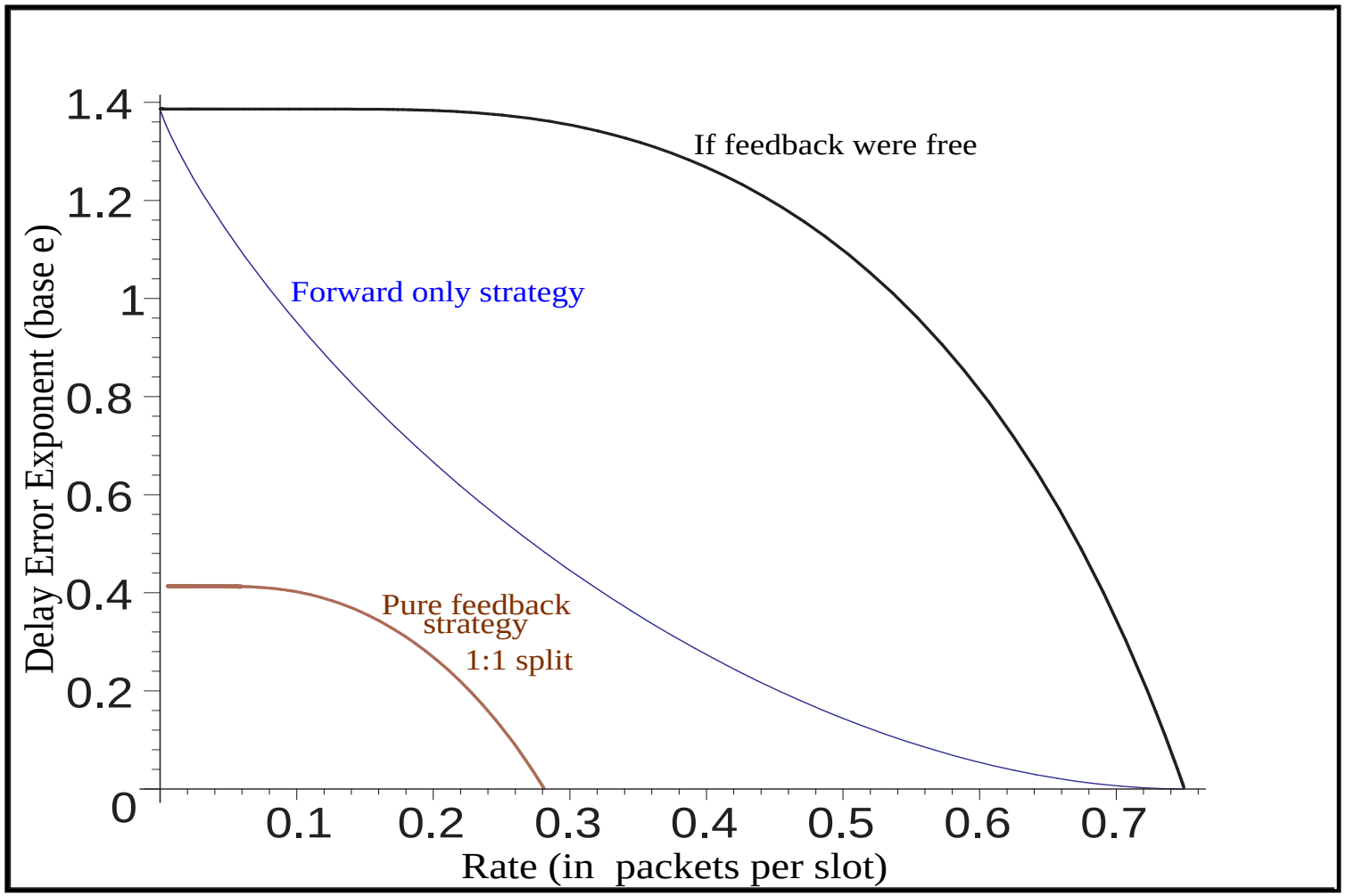


Shared channel picture



- *Both* users share a single physical channel
- Half-duplex constraint: only one can use at a time
- Assume we must schedule them in advance
- Same erasure probability in both directions

Shared unreliable channel used for feedback



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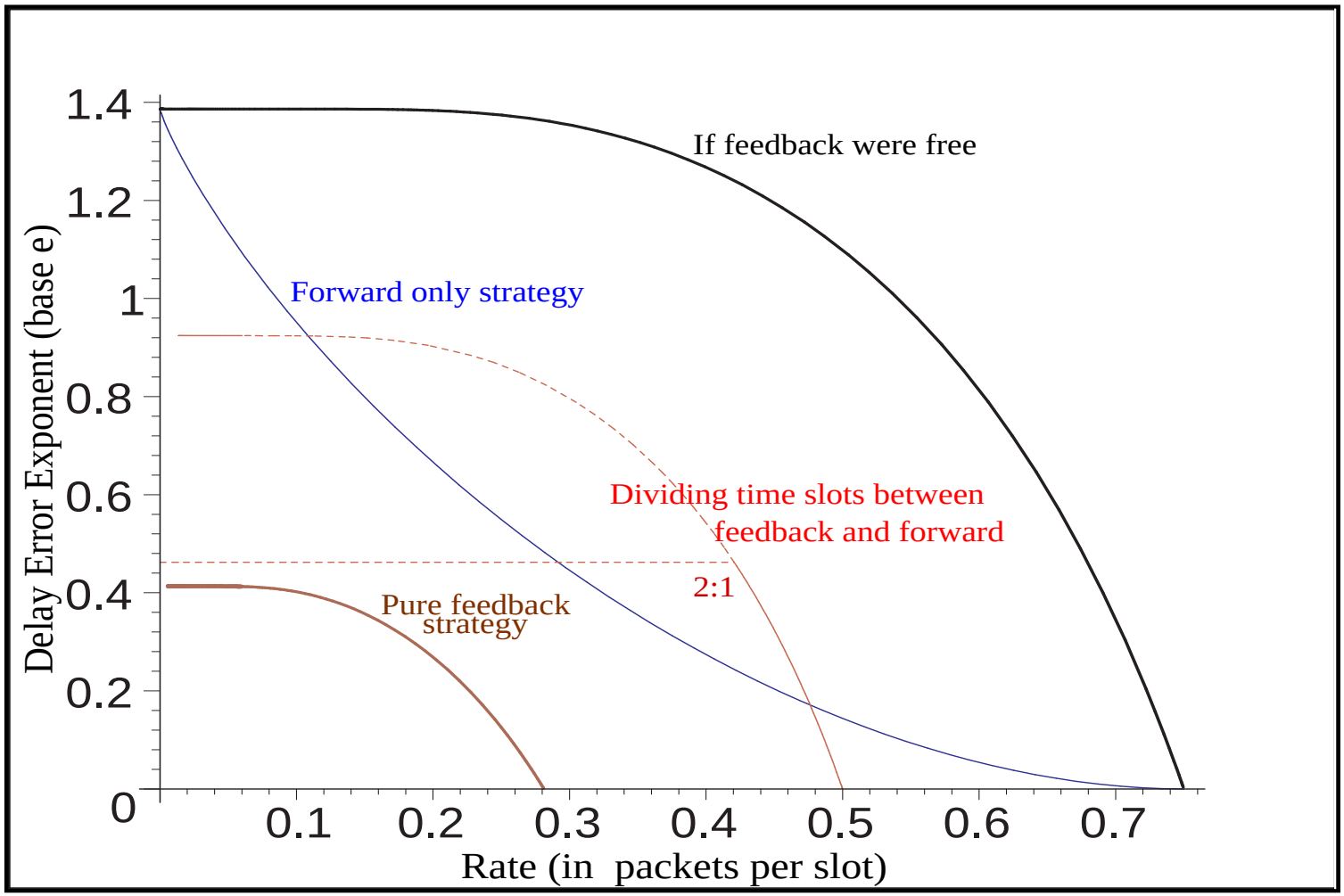

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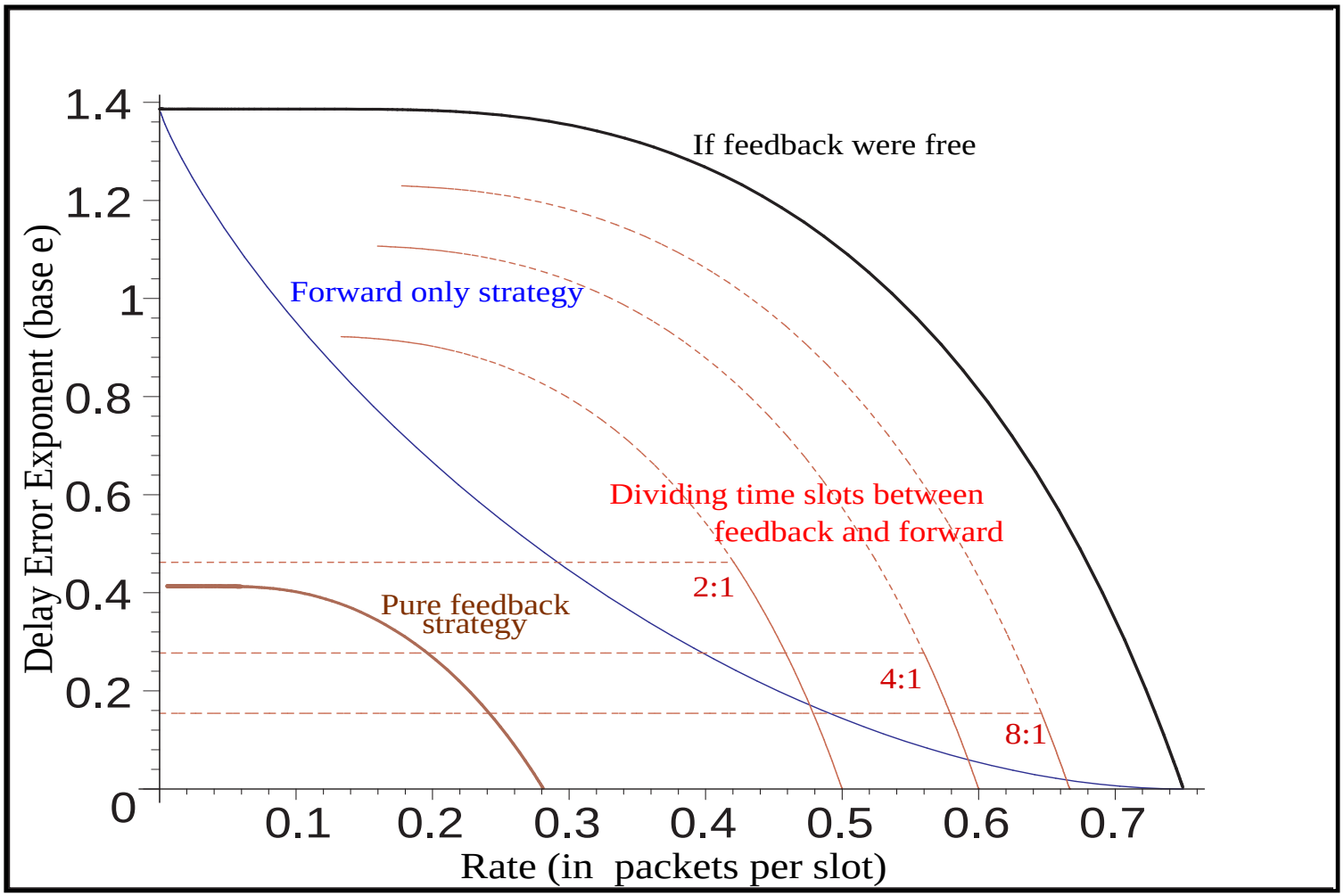

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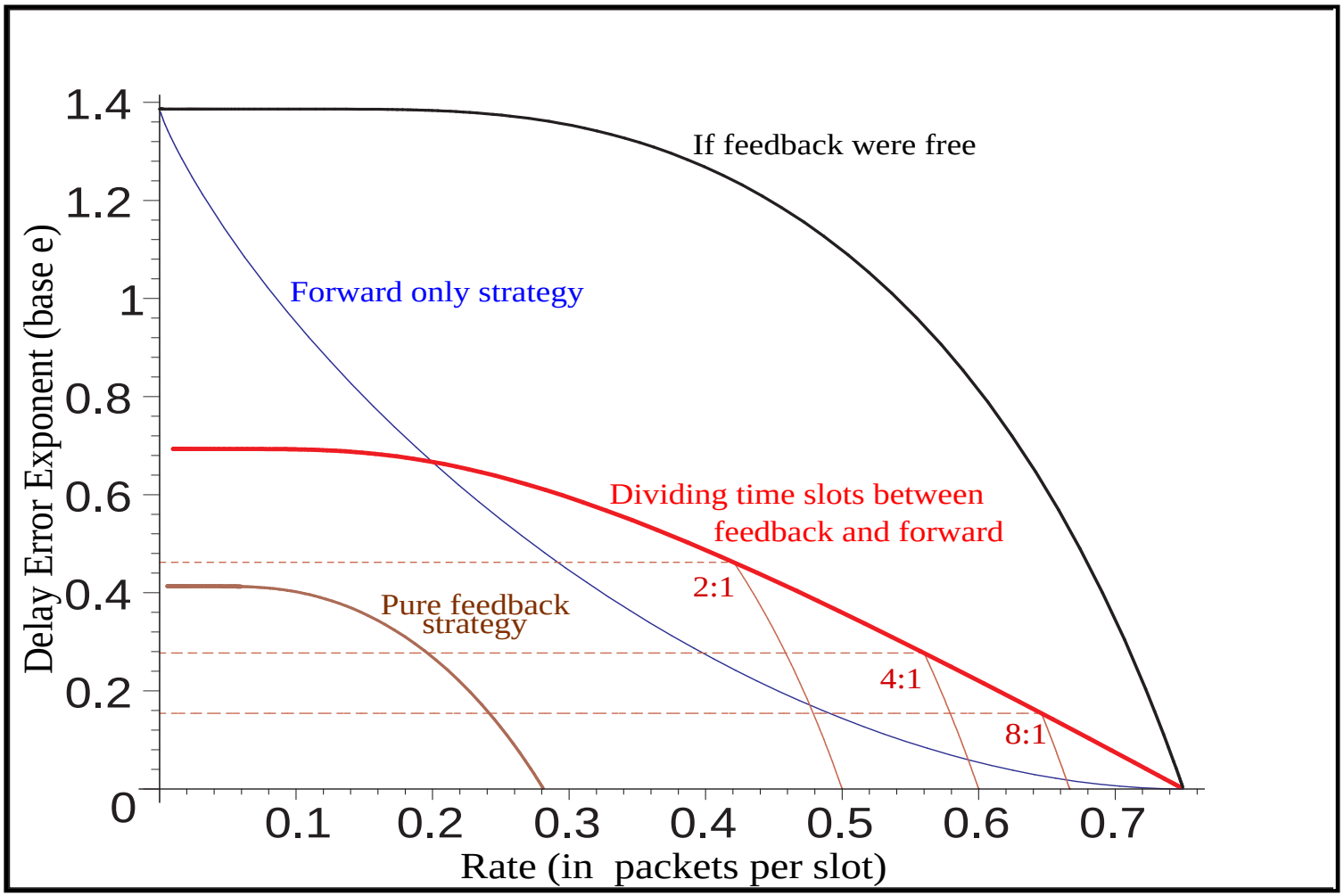

Like focusing bound using $E_0'(\rho) = (-\ln(\delta_f)^{-1} + E_0^{-1}(\rho))^{-1}$

Summary of results:

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- It is worth allocating slots for feedback *even if this means taking them away from the forward channel.*

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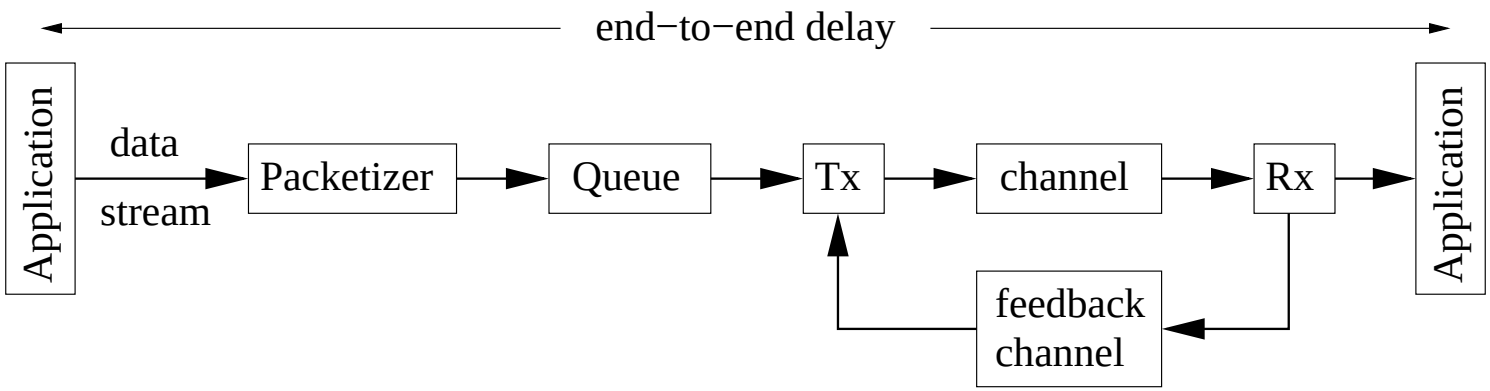

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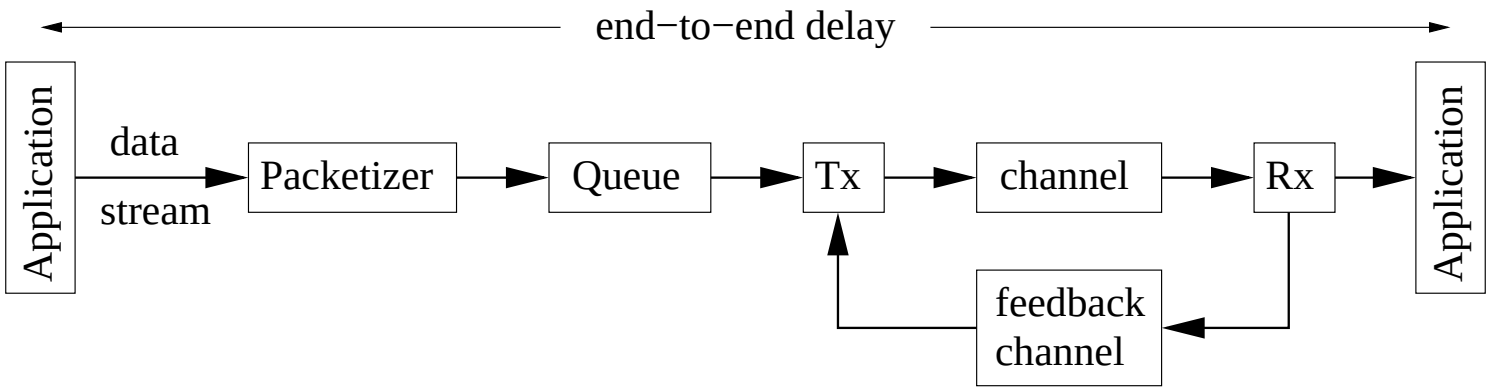

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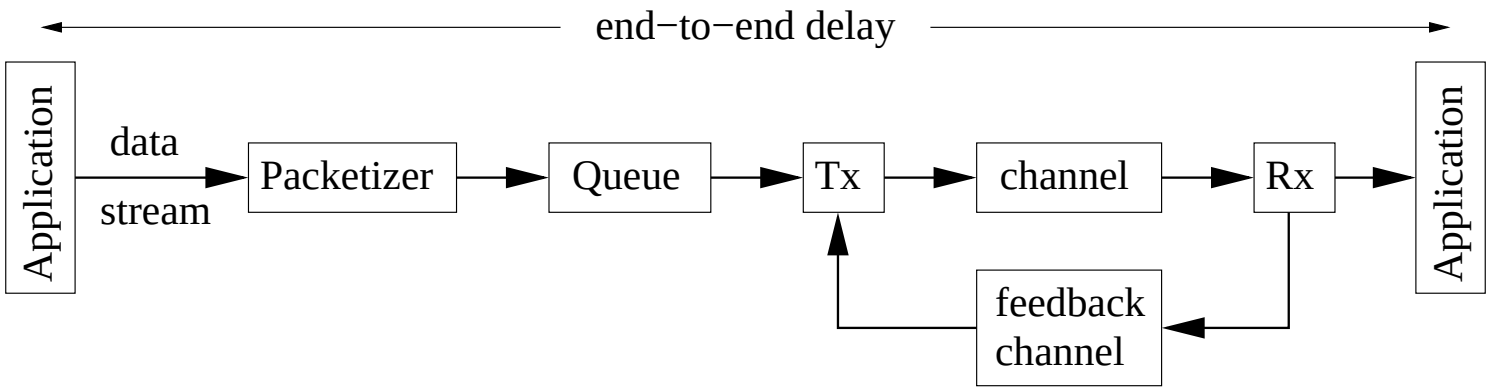

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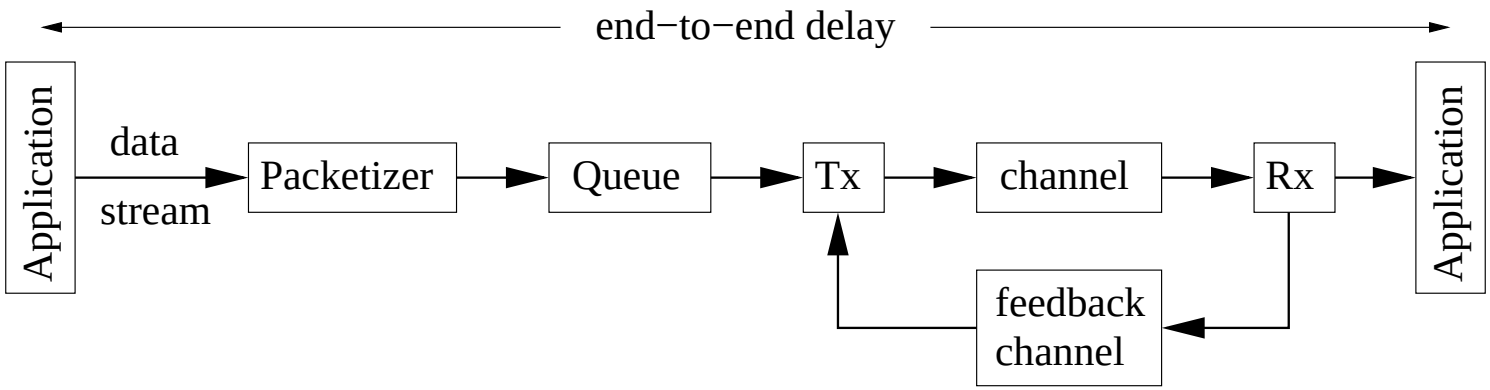

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  - Unstable process, but it can be tracked using an *anytime* code.

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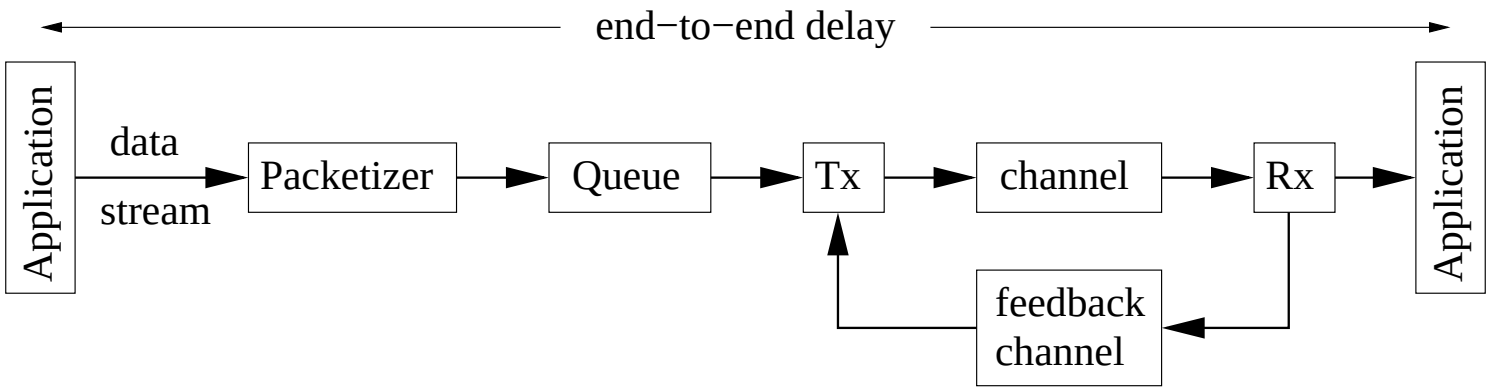

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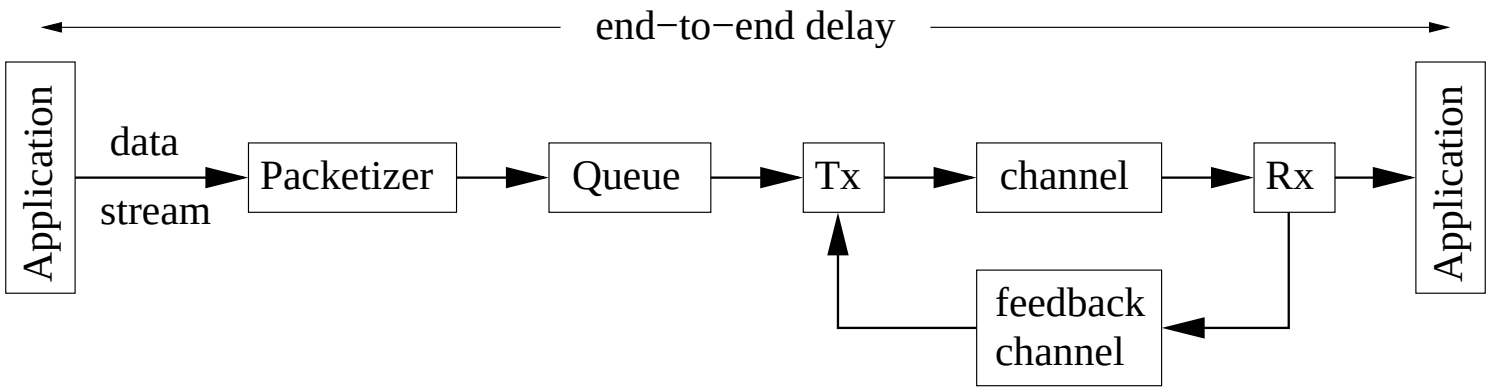
Noisy feedback: two distinct issues



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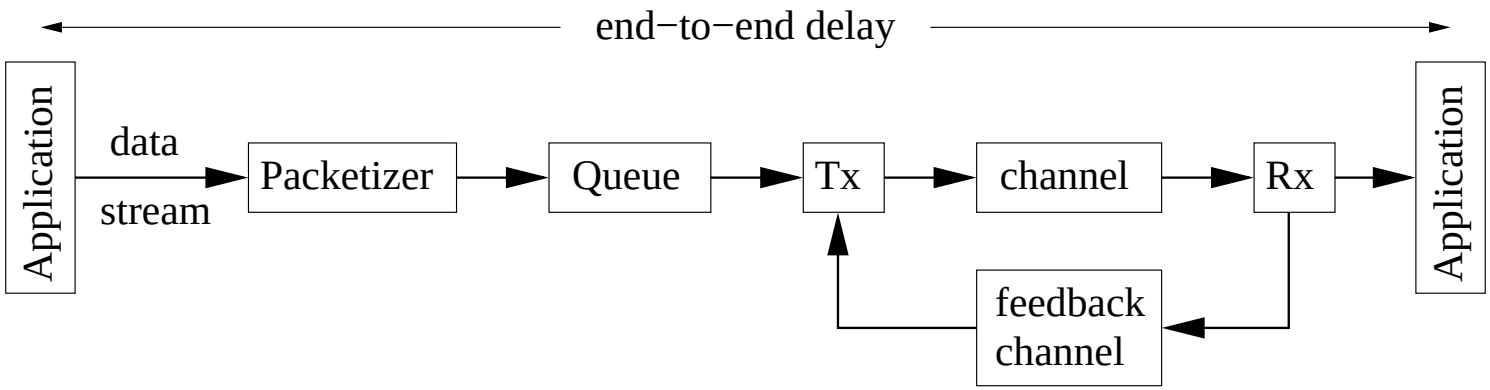

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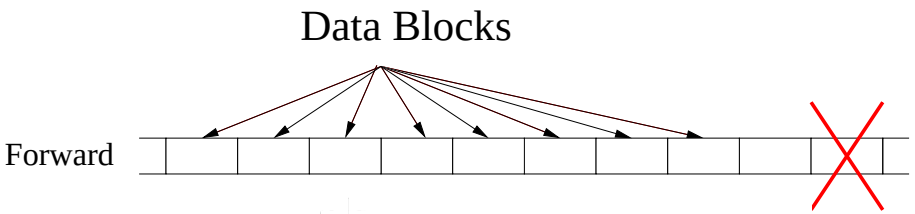
- How to NAK?

What is needed to NAK?



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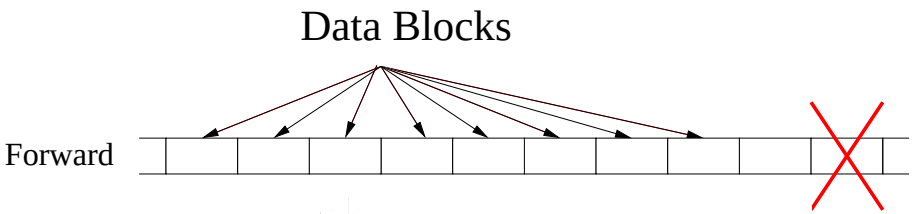

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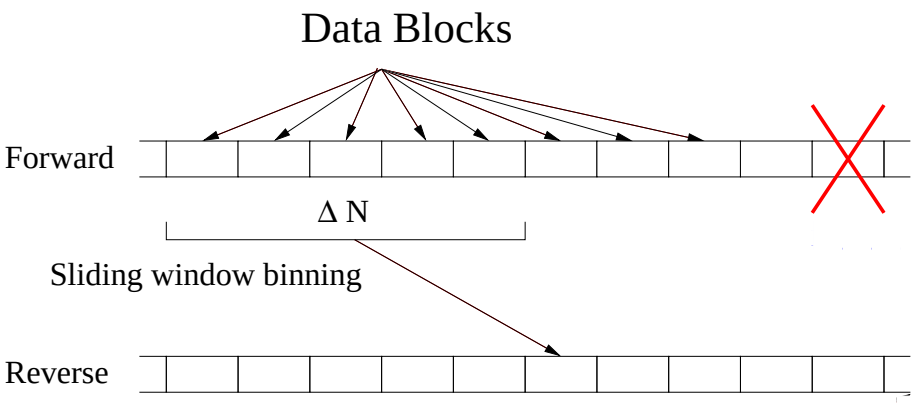

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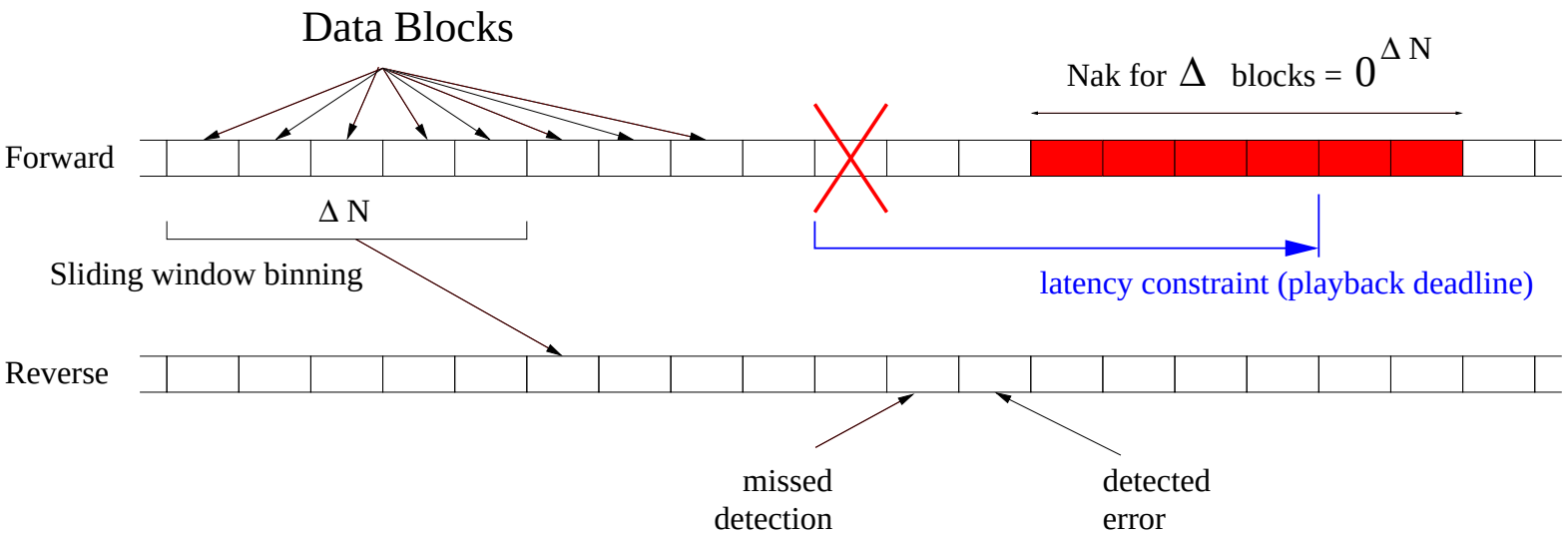

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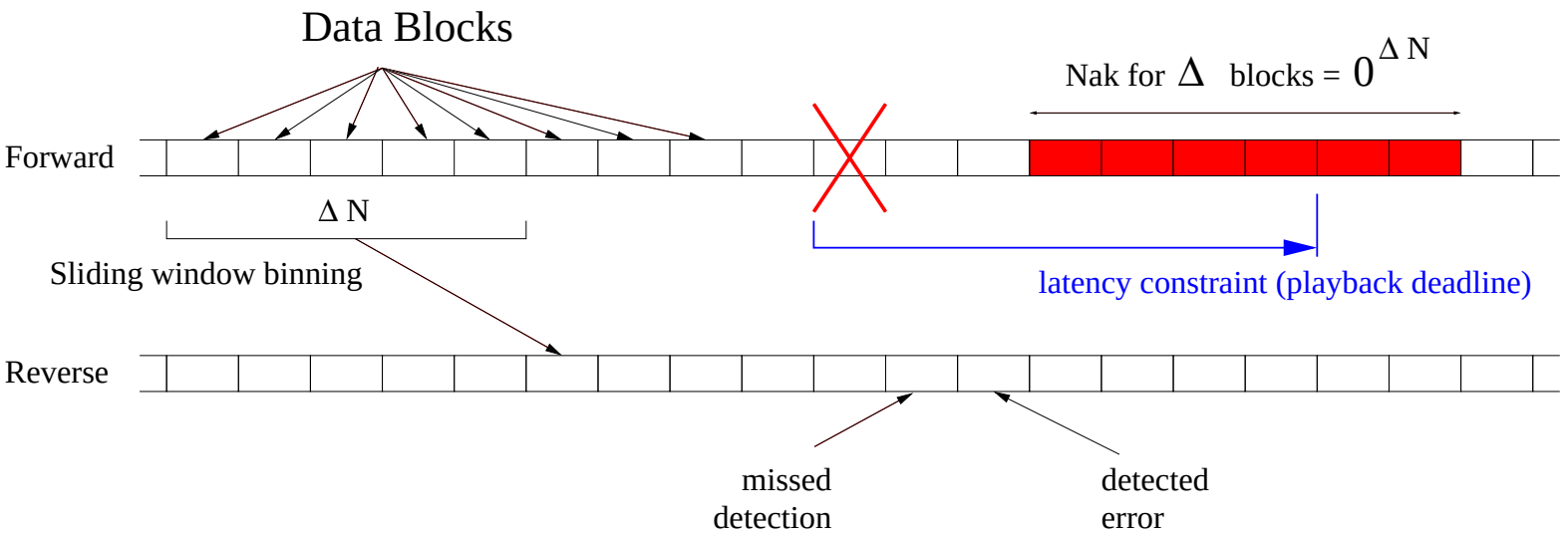

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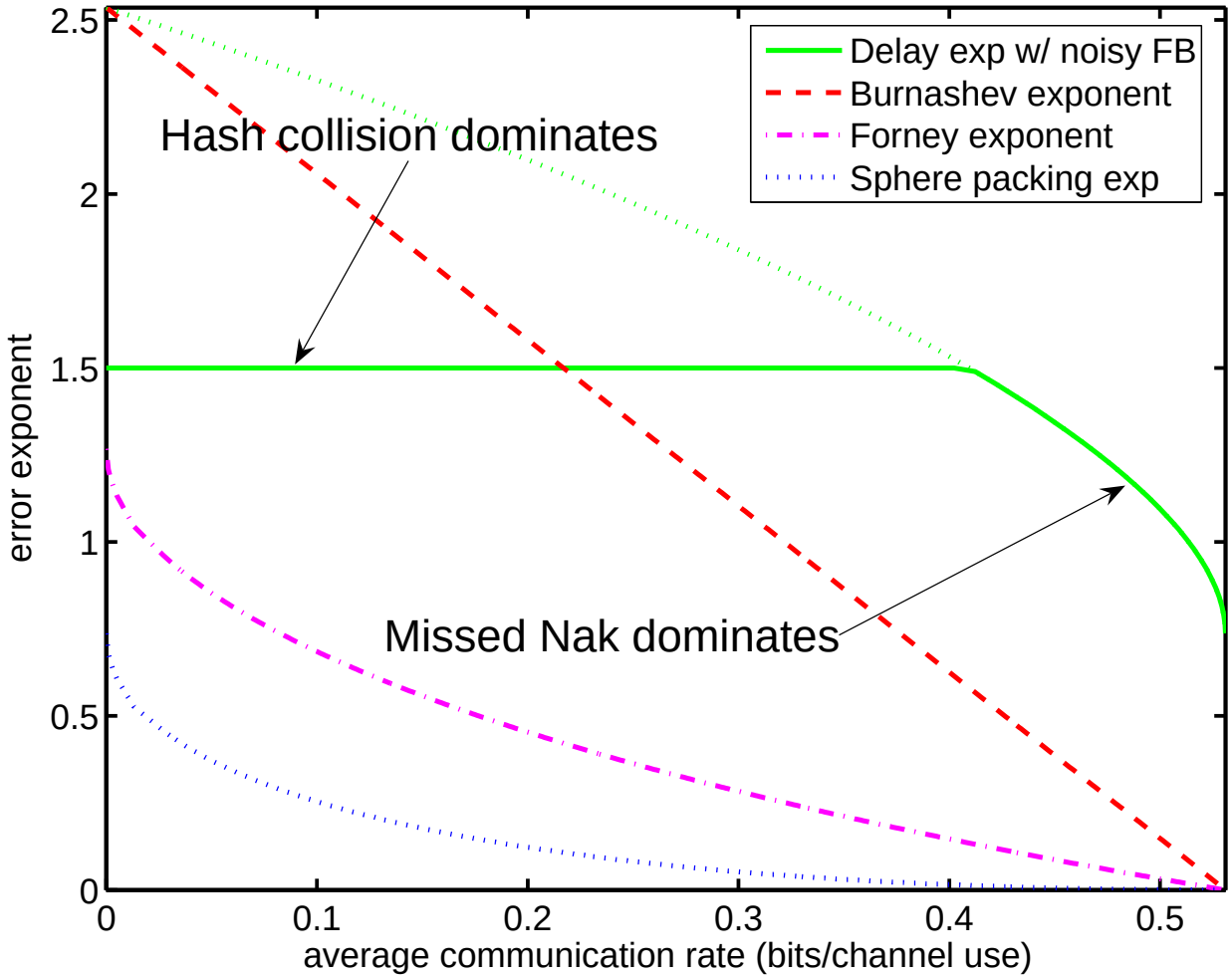
- If $C_{fb} > D(q_y||p)$, no loss in net exponent!

Feedback capacity acts as a ceiling to reliability



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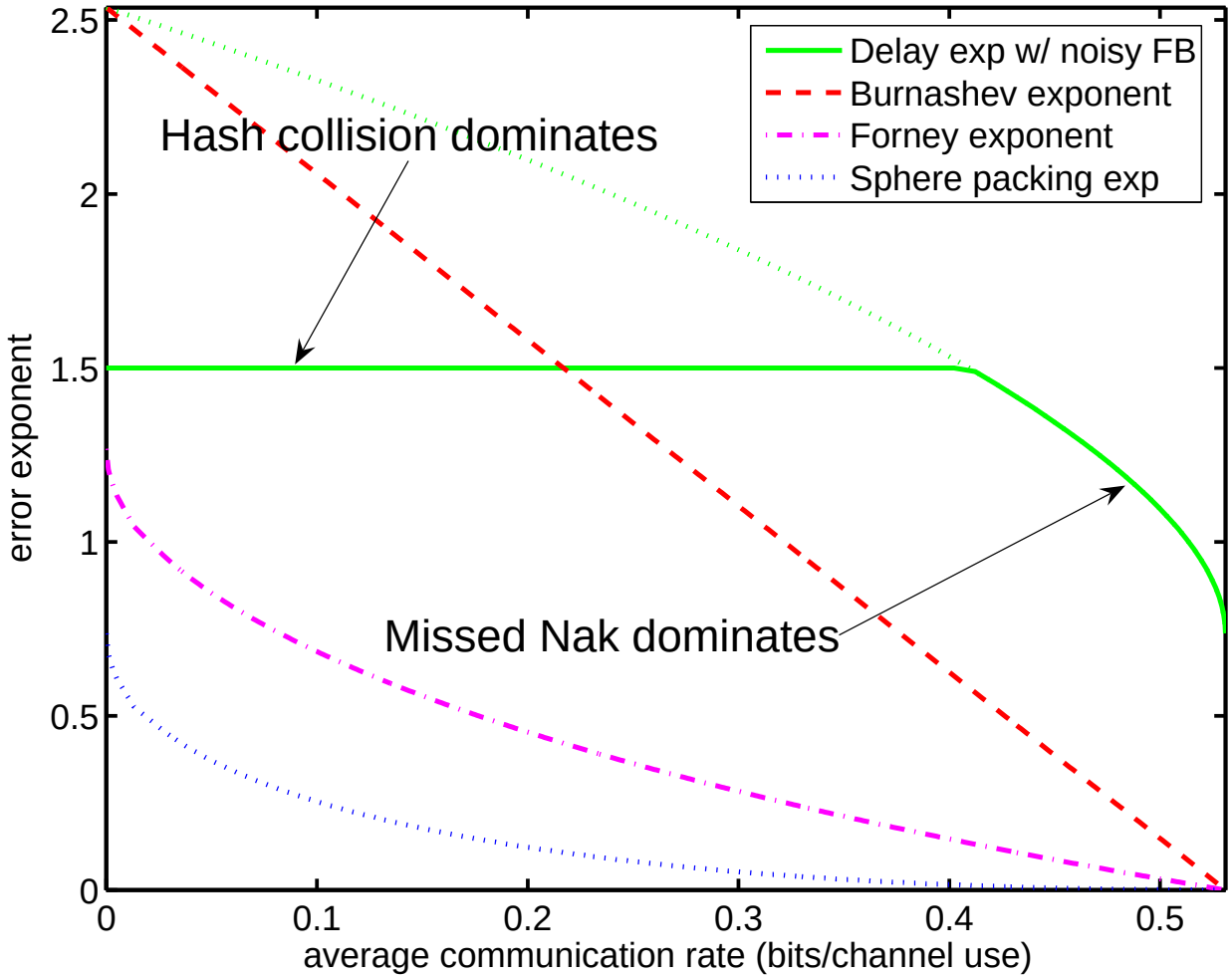

- **Feedback reliability gains are robust to noisy feedback.**
- Open problem: does this also hold in the general hard deadline case?

Summary of reliability for *symmetric* channels

	Hard Deadlines			**Soft Deadlines** (undetected errors only)		
	Bound	*Achievable*	*Robust*	*Bound*	*Achievable*	*Robust*
Block	Sphere-packing	Yes	trivial	Burnashev	Yes	Mild*
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- * entries are our contributions with **bold** for those in this talk.
- Trivial robustness is when feedback is not used at all.
- Partial achievability of the focusing bound is:
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- For *asymmetric* channels, Haroutunian vs sphere-packing style gaps exist between bounds and achievable codes everywhere except *Block*, *No-FB*.
- Ultimately, there should be a continuum between perfect feedback and no feedback, as well as between hard and soft deadlines.