Fibonacci Sequence & the Golden Ratio

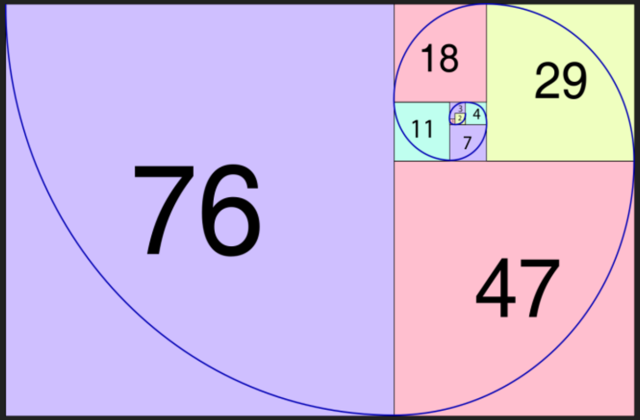
The Perfect Patterns of Nature

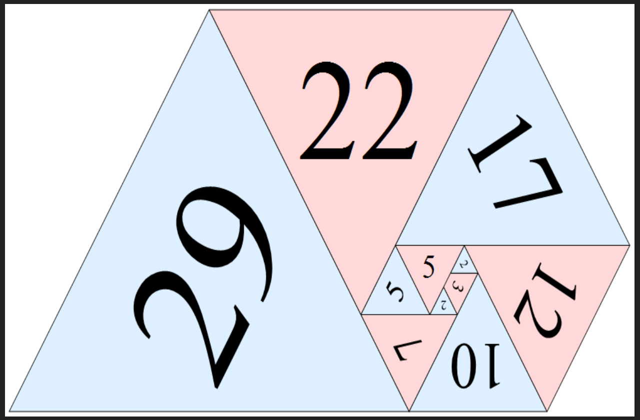
What is the Fibonacci Sequence?

- An integer sequence whereby each number is the sum of the two preceding numbers.

- It is a set of numbers that starts with a one or a zero, followed by a one, and proceeds based on the rule that each number (called a **Fibonacci** number) is equal to the sum of the preceding two numbers.

Example: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...





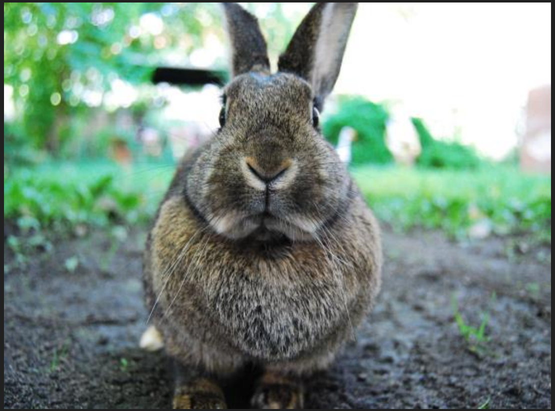
Who Discovered the Sequence?

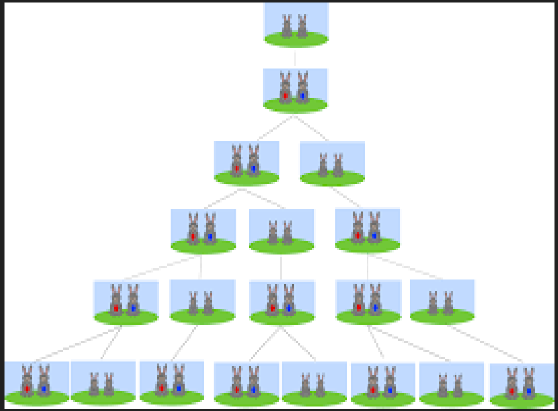
- Leonardo Pisano
  - Also known as Leonardo of Pisa, which means, “from Pisa.”
  - He was also known as Fibonacci which means “son of Bonacci.”
- Background
  - Growing up in North Africa, Fibonacci learned the more efficient Hindu-Arabic arithmetic system (1, 2, 3, 4…).
  - He published his knowledge in 1202 in a famous book called “Liber Abaci.”
  - This book showed how much better the Hindu-Arabic arithmetic system was compared to the Roman Numeral System.

How Was the Sequence Discovered?

- The Fibonacci sequence was the outcome of a mathematical problem about rabbit breeding which can be found in the Liber Abaci.

- The question: In optimal conditions, how many pairs of rabbits can be produced from a single pair of rabbits (one male and one female) in one year, assuming that every month each male and female rabbit gives birth to a new pair of rabbits, and the new pair of rabbits itself starts giving birth to additional pairs of rabbits after the first month of their birth?





The Breakdown...

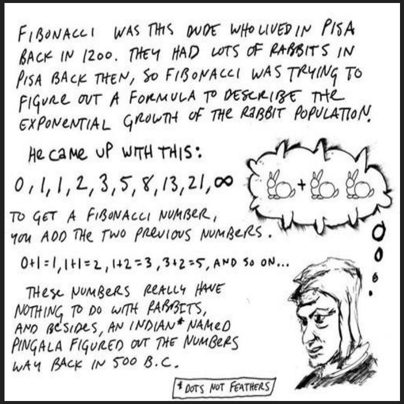
- Month 1 – Rabbits cannot reproduce until they are at least one month old, so at this point, only **one pair** remains.

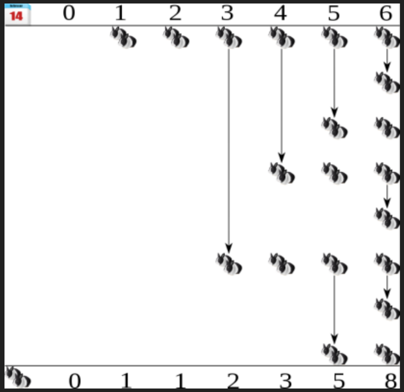
- Month 2 – The female gives birth, leaving **two pairs** of rabbits.

- Month 3 – The original pair of rabbits produce another pair of newborns while their earlier offspring grow into adulthood, resulting in **three pairs** of rabbits

- Month 4 – The two pairs of mature rabbits will each give birth to another pair of babies, resulting in **five pairs** of rabbits.

The order pattern then becomes: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 and to infinity. Each number is the sum of the previous two. The ratio between the numbers (1.618034) is frequently called the golden radio or the golden number.

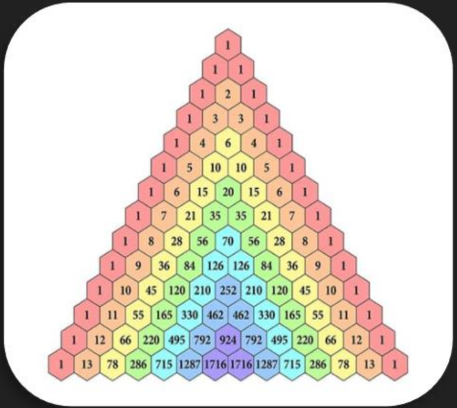


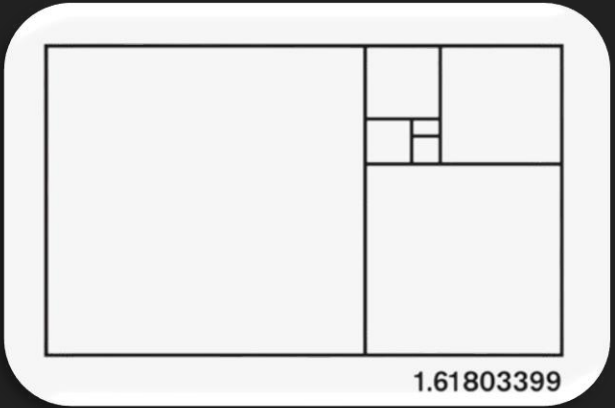


The Golden Ratio Explained

- The Fibonacci sequence ties directly into the Golden Ratio because if you take any two successive numbers, their ratio is very close to the golden ratio. As the numbers get higher, the ratio becomes even closer to 1.618.

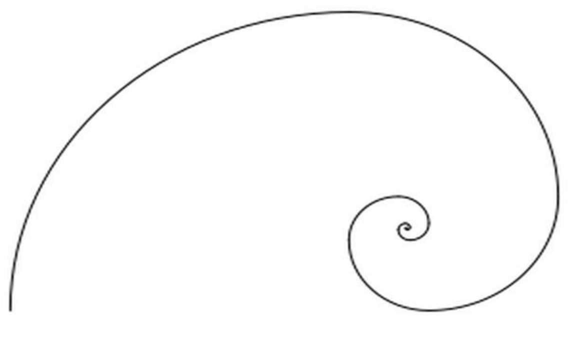
- For example, the ratio of 3 to 5 is 1.666. But the ratio of 13 to 21 is 1.625. Getting even higher, the ratio of 144 to 233 is 1.618. These numbers are all successive numbers in the Fibonacci Sequence.

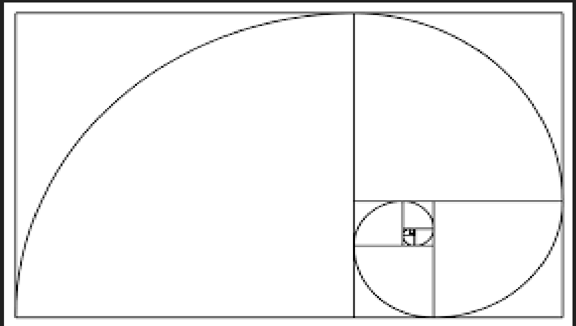


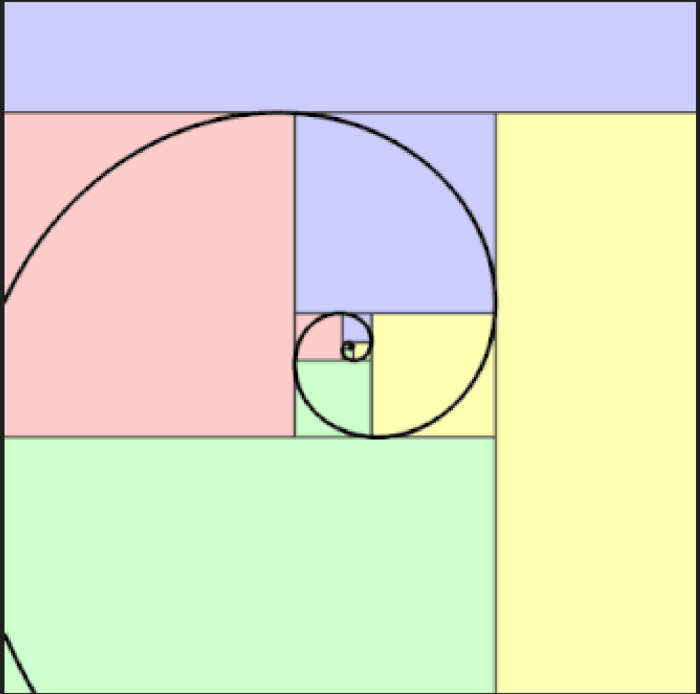


The Golden Spiral

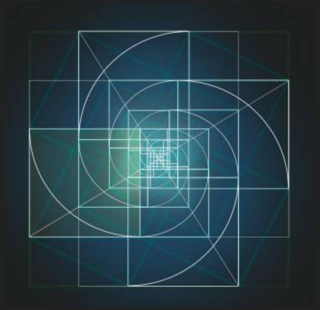
The golden spiral is another aspect of these numbers that can be created by increasing the spiral's radius by the golden proportion every 90 degrees.



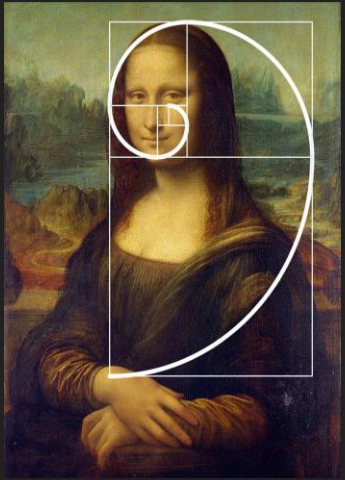




The Golden Rectangle

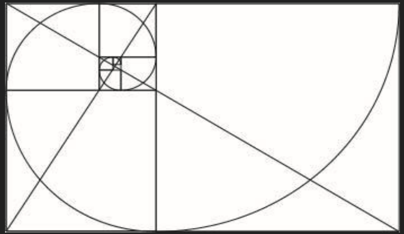


- Fibonacci numbers can be applied to the proportions of a rectangle, called the golden rectangle.



- This is known as one of the most visually satisfying of all geometric forms and appearance of the golden ratio in art.

- The golden rectangle is also related to the golden spiral, which is created by making adjacent squares of Fibonacci dimensions.



Fibonacci Sequence in Nature

Plants use a Fibonacci spiral form because they are constantly trying to grow by staying secure. A spiral shape causes plants to condense themselves and take up less space, causing it to be stronger and more durable against the elements.

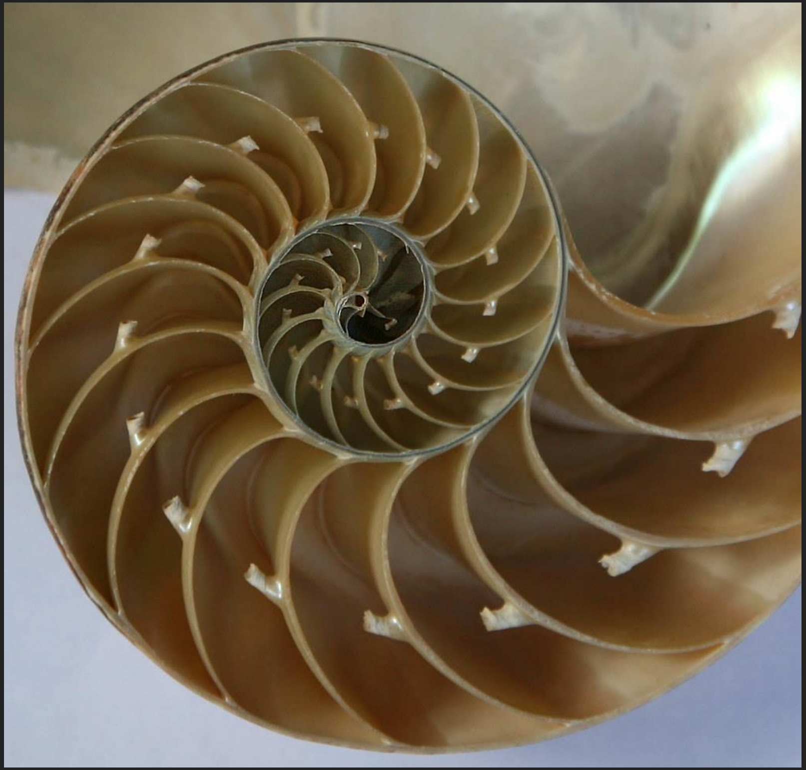
Oftentimes, the seeds of a flower are produced at the center and migrate outward to fill the space, like in dandelions or sunflowers.

You can also recognize the Fibonacci sequence in plants like ferns before they open up!

Acorns are another great example.

Pinecones, too.

And seashells!



Fibonacci Sequence in you Kitchen

You can see the pattern in items like broccoli and cabbage.

As well as berries and pineapple.

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Extensions and Interdisciplinary Connections

In an interdisciplinary manner, Fibonacci's sequences are strongly connected to most content areas.

Fibonacci in Language Arts

According to Greg Pincus – Writer Guy, a "Fib" is a six-line, 20 syllable poem in which each line gets its syllable count from following the Fibonacci sequence. This means the six lines have a syllable count of 1, 1, 2, 3, 5, and 8 respectively. Some would say the first number of the Fibonacci sequence is actually a zero... so imagine every Fib starting with a beat of silence.

Fibs do not have any set rhyme scheme, though lines can rhyme if you'd like. (He writes) For the one syllable lines, I try never to use articles (a, an, the) or conjunctions (and, or, but).

http://www.gregpincus.com/how-to-write-a-fib.html

The Poetry Foundation

The Poetry Foundation's article about Fibonacci poetry points to its popularity. To read more about "Fibs", check out this link: https://www.poetryfoundation.org/articles/68971/1-1-2-3-5-8-fun

This is one example of a poem from the article:

One
Small,
Precise,
Poetic,
Spiraling mixture:
Math plus poetry yields the Fib.

This poem was written by a math teacher:

Word Crunching

I
wrote
a poem
on a page
but then each line grew
by the word sum of the previous two
until I started to worry at all these words coming with such frequency
because, as you can see, it can be easy to run out of space when a poem gets all Fibonacci sequency.

https://unbound.co.uk/books/brian-bilston

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Finding Fibonacci

The famous Fibonacci sequence is found in natural items including trees, flowers, fruits, both aquatic and terrestrial animals, and even weather events. The attached PowerPoint guides the learner through a brief history of Fibonacci himself, examples of some naturally occurring objects, and a snapshot of how the sequence influences poets, composers, architects, scientists, and of course, mathematicians.

With just a little bit of background knowledge, we hope you will adventure out to your backyard or green space, to see what natural elements you can find that may include the "Golden Ratio."

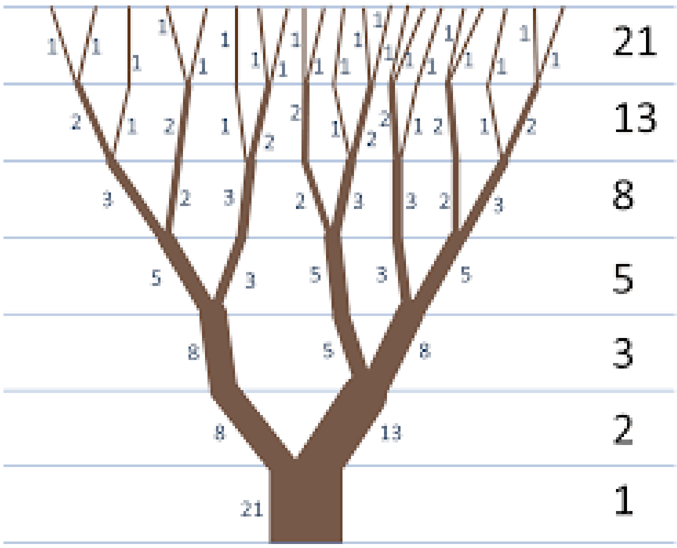
We would love to see your examples! We are collecting images to appear in an upcoming Duke Farms exhibit. If you're interested in this opportunity, you can email photos to Kate Reilly, Manager of Education at kreilly@dukefarms.org

For Educators

This is a "get outside" activity for your students that is aligned directly to NJ Learning Standards in science, mathematics, and almost all content areas. If you have questions on how to apply it to your own curriculum, please email Kate Reilly, Manager of Education at kreilly@dukefarms.org

Fibonacci in Art

Note how the number sequences are observed in this diagram of a tree and how it is then transformed to a piece of 3D textile artwork.



Source: Cambridge Mathematics

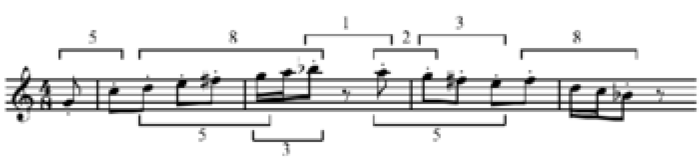
There's Fibonacci numbers everywhere in this tree: in the number of rows you do at each stage; in the size of each branch; and in the number of branches at each horizontal cross-section. There is a ratio of successive Fibonacci numbers that gives the artwork a pleasing dimension.

Source: Botanica Mathematic, *a textile taxonomy of mathematical plant forms*

Fibonacci in Music

Fibonacci numbers have also influenced musical composition around the world.

This is one example:



Fibonacci intervals (counting in semitones) are evident in Hungarian composer, Bela Bartok's, Sonata for Two Pianos and Percussion, 3rd mov. (1937).

https://library.harvard.edu/onlineexhibits/solti/twentieth/bartok-sonata/index.html

Fibonacci in Architecture

Krishnendra Shekhawat, the author of the article, “Why golden triangle is used so often by architects: A mathematical approach” (*Alexandria Engineering Journal – Egypt,* Volume 54, Issue 2, June 2015, Pages 213-222) includes an assortment of architectural connections to Fibonacci.

Examples include:

- The ϕ or golden rectangle has been found in the natural world through human proportions and through growth patterns of many living plants, animals, and insects. Basically, it has been always considered that ϕ is the most pleasing proportion to human eyes.

- The presence of ϕ in the design of the Pyramids represents that the Egyptians were aware of the number. A Greek sculptor and mathematician, Phidias (490–430 BC), was first to study and apply Phi, to the design of sculptures for the Parthenon (example of Doric architecture, the main temple of the goddess Athena)

- Around 1200 AD, Leonardo Fibonacci (1170–1250 AD), an Italian born mathematician found ϕ in a numerical series (known as Fibonacci series) and named it *divine proportion*, due to which, Fibonacci series can be used to construct the golden rectangle.

- The design of Notre Dame in Paris, which was built in between 1163 and 1250, appears to have golden rectangle in number of its key proportions.

ϕ has been favorite to many key architects and designers in history. For more information, check these resources:

- Palladio – *I Quattro Libri dell’Architettura*
- Le Corbusier -*The Modular ad Modular 2*
- Leonardo Da Vinci - *De Divina Proportione*

Sample Learning Standards

Exploring topics related to Fibonacci has a broad range of multi-disciplinary content applications. Listed below are some K-12 ideas:

New Jersey Learning Standards Mathematics

4. OA Operations and Algebraic Thinking

Generate and analyze patterns: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

New Jersey learning Standards in Visual Arts

1.1.5.D.1 Identify elements of art and principles of design that are evident in everyday life. (*This includes natural features.)*

Visual awareness stems from acute observational skills and interest in visual objects, spaces, and the relationship of objects to the world. 1.3.2.D.5 Create works of art that are based on observations of the physical world and that illustrate how art is part of everyday life, using a variety of art mediums and art media.

Next Generation Science Standards

Interdependent Relationships in Ecosystems

2-LS2-1 Structure and Function: The shape and stability of structures of natural and designed objects are related to their function(s).

2-LS4-1 Connections to Nature of Science Scientific Knowledge is Based on Empirical Evidence: Scientists look for patterns and order when making observations about the world. (2-LS4-1)

K-2 Engineering Design

K-2 – ETS1-2 Crosscutting Concepts Structure and Function: The shape and stability of structures of natural and designed objects are related to their function(s).

4-LS1-1. Construct an argument that plants and animals have internal and external structures that function to support survival, growth, behavior, and reproduction

New Jersey Social Studies Learning Standards

6.2 World History/Global Studies: All students will acquire the knowledge and skills to think analytically and systematically about how past interactions about people, cultures and the environment affect issues across time and cultures. Such knowledge and skills enable students to make informed decisions as socially and ethically responsible world citizens of the 21st century.

Era - Expanding Exchanges and Encounters (500CE – 1450 CE)