FIFTH EDITION

PHARMACEUTICAL CALCULATIONS

MARIA GLAUCIA TEIXEIRA
JOEL L. ZATZ

WILEY

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To Alfred (Al), my life's partner and soulmate,
for his infinite friendship and
genuine contributions to this work.

To my mother, Julia, the strongest woman
I have ever known, for her example of tenacity
and wise choices in life.

Maria Glaucia Teixeira

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PREFACE

TO THE STUDENT

A goal of this new edition of *Pharmaceutical Calculations* is to update the material to current changes in pharmacy practice, continuing with an active learning approach. It draws upon previous experiences in the classroom, especially those over the 11 years since the previous edition, working with pharmacy students with diverse backgrounds ranging from pre-pharmacy college to master degrees in scientific fields. The active learning approach means that the student is required to solve problems themselves and think about what they are doing, rather than just demonstrating solution techniques. To glance ahead at the solutions and answers to the problems defeats the process. Too often, students come in to say that they understand and follow the solutions when the instructor does problems in class, but don't know where to start when presented problems with the book closed. We think, as a student, you will understand and follow the solutions provided as they involve simple logic and mathematical processes that you've learned since high school or earlier. The key to active learning is putting in the time, doing the work, and thinking about what you are doing. Treat the practice questions at the end of each chapter as your quality control check or just to test your learned skills. And always do all the problems before glancing at the answers.

Almost every topic contains some problems to test your understanding. Read the information in the topic and work on the question(s) in the space provided. We have found it helpful to use an index card or a folded sheet of paper to cover everything below the line where the solutions and answers are provided. When you're done, check your answer. If you are correct, move on to the next topic. If not, try the problem again before consulting the solution. You may uncover your error and be able to do it right the second time. If you still can't get it, you may need to go back to a few topics to see if there is something you missed. If all else fails, look at the solution. You may also want to consult the solution if you struggled to get the right answer and wondered if a more direct approach is available.

We assume that you can perform the usual arithmetic operations of addition, subtraction, multiplication, and division and that you can solve simple algebraic and exponential equations. A quick review of certain techniques and practice questions are provided in Chapter 1 of this edition in case you have gotten rusty and need to brush up. You will find out that there are multiple ways to solve pharmaceutical calculations – there is no one "correct" way. With practice, you will find the approach that makes the most sense to you.

Some topics will be easy for you and you'll zoom through the text. Difficult spots will take more time; in any case, stick with it and take as much time as you need to work through the material until you understand it and can handle the problems. Happy problem solving!

NEW IN THIS EDITION

New practice problems (NAPLEX-patterned) have been added, reflecting real-practice situations a pharmacist and/or other health care professionals will encounter when working at various settings. These new problems also prepare the pharmacist for NAPLEX exams by stimulating critical thinking and encouraging logical reasoning and attention to detailed information, sometimes available in a real-world situation (or provided in a question) but not needed for the mathematical solution of a problem.

New features of this edition also include the following:

- The main focus across the 12 chapters is on reflecting traditional teaching of pharmaceutical calculations applicable to daily pharmacy practice activities and required for successful licensing.
- Chapters provided in logical sequence such that each builds knowledge upon previously learned topics.
- New format providing numbered sections and subsections that relate to each chapter, facilitating retrieval of topics.
- Updated content to every chapter.
- New chapter dedicated to practical calculations used in contemporary compounding.
- New appendices.
- Solutions and answers for all problems.

OTHER FEATURES IN THIS EDITION

Since its first edition, it has been the main goal of this textbook to provide a level consistent with the requirements of students being presented to the applications of basic mathematical techniques to the pharmaceutical/health fields, often part of the early professional curriculum. We have thus retained some and added other important introductory topics and descriptions that are frequently needed while students are introduced to pharmaceutical dosage forms, drug delivery systems, and learning basic compounding skills. These topics are not strictly needed to solve pharmaceutical calculation problems, but are included nonetheless because experience has shown that students often need this background.

Some features retained and enhanced from the fourth edition are as follows:

- Chapter objectives informing the students what they will master upon successful completion of each chapter.
- Introduction of topics in a programmed manner, allowing self-study.
- Various practice problems that were designed to offer opportunities for creative thought and application of the content of each section.
- Appendices providing basic information and useful references.

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CHAPTER 1

REVIEW OF BASIC MATHEMATICAL PRINCIPLES

Learning Objectives *After completing this chapter the student should be able to:*

1. Recall the skills of basic mathematical operations required to work in the health field.
2. Use estimation as a means of preventing errors.
3. Perform mathematical operations containing units.
4. Compare two quantities (ratio).
5. Apply ratio, proportion, and dimensional analysis in problem solving.

Pharmacists, nurses, doctors, and most health-related professionals perform basic calculations as a daily practice. While working in a variety of settings, pharmacists, for example, need to calculate doses and determine the number of dosage units required to fill prescriptions accurately, must determine the quantities of pharmaceutical ingredients required to compound formulas, and perform calculations related to dose adjustments for disease state management, and so on. The correct drug, strength, and amount of each medication prescribed that is dispensed in pharmacies must be finally checked by the pharmacist, who is legally accountable for an incorrect dose or dispensing of a wrong drug. The fact that most pharmaceuticals are prefabricated and not prepared inside the pharmacy does not lessen the pharmacist's responsibility.

Modern drugs are effective, potent, and therefore potentially toxic if not taken correctly. An overdose may be fatal. Knowing "how to" calculate the amount of each drug and "how to" combine them is not sufficient. Of course, dispensing a subpotent dose is not satisfactory either. The drug(s) given will probably not elicit the desired therapeutic effect and will therefore be of no benefit to the patient. Clearly, the only satisfactory approach is one that is completely free of error. Absolute accuracy is any health professional's goal. Since our goal when performing calculations is the correct answer, it is logical to suppose that any rational approach to a problem that results in the correct answer is acceptable. While this is true, some approaches are more coherent and practical than others. In this text we strive to use a method that requires as few steps as possible and that with which you will feel comfortable. Usually, the simplest, most direct pathway to the solution allows less opportunity for error in computation than does one that is more complicated.

In this chapter, we will review some techniques basic to all types of calculations. To help you regain the basic mathematical operations required to work in the health field, we will briefly review significant figures, rounding off, fractions, exponents, power-of-10 notation, and estimation, and will make sure that you can solve simple algebraic

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expressions. We will go over how units participate in arithmetic operations and how we can take advantage of units in our calculations. Finally, we will review dimensional analysis, ratio, and proportion.

You will probably find that you are already familiar with all or most of these techniques. After this refreshing, you will make rapid progress through the self-study format of this text. If you need further review or instruction, that will be provided.

1.1. SIGNIFICANT FIGURES

Significant figures are digits that have practical consequences in pharmacy. Sometimes, in a calculated dose at a clinical setting, or in a weighed or measured amount at a compounding pharmacy, zeros are significant; other times they just designate the order of magnitude of the other digits indicating the location of the decimal point. Since the majority of medications currently prescribed are manufactured products, significant figures have minor significance to the counter pharmacist, if no compounding is involved on a daily basis. For the compounding pharmacist, however, all weighing and measuring will have a degree of accuracy that is only approximate, due to the many sources of error related to the type and limitations of the instrument used, room temperature, personal skills, attentiveness, and so on.

While compounding pharmacists must achieve the highest accuracy possible with their equipment, one could never claim to have weighed 5 mg of a solid substance on a torsion balance with sensitivity of 10 mg, or that 33.45 mL of a liquid was measured in a 50 mL graduate with only 1 mL graduations. Consequently, when writing quantities, the numbers should contain only the digits that are *significant* within the precision of the instrument. However, when performing calculations, all digits should be retained until the end. The final result will then be rounded so that the accuracy is implied by the number of significant figures.

The following illustrate the practical meaning of significant figures:

(a) If 0.0125 g is weighed, the zeros are not significant and only indicate the location of the decimal point.

(b) For a measured weight of 1250.0 g, the last zero may or may not be significant, depending on the method of measurement. The zero will not be significant if indicating the decimal point; alternatively, it may indicate that the weight is closer to 1249 or 1251 g, in which case the zero is significant.

(c) For some recorded measurements, the last significant figure is "approximate," while all preceding figures are "accurate." For example, in a measured volume of 398.0 mL, all digits are significant but it is accurate to the nearest 0.1 mL, which means the measurement falls between 397.5 and 398.5, or that the measurement was made within ± 0.05 mL. In 39.86 mL, the 6 is approximate, with the true volume being between 39.855 and 39.865 mL. This means that 39.8 mL is accurate to the nearest 0.01 mL, or that the measurement was made within ± 0.005 mL.

(d) It is thus possible to calculate the maximum error incurred in every measurement. Using the examples above, we would have

$$\frac{0.05\ \text{mL}}{398.0} \times 100\% = 0.013\%$$

$$\frac{0.005\ \text{mL}}{39.86} \times 100\% = 0.013\%$$

(e) When establishing the number of significant figures in mathematical operations, use the following practical rules:

- The result of addition and subtraction should contain the same number of decimal places as the component with the fewest decimal places. For example, 12.5 g + 10.65 g + 8.30 g = 31.45 g = 31.5 g.
- The result of products and quotients should have no more significant figures than the component with the smallest number of significant figures. For example, 2.466 mg/dose × 15 doses = 36.99 = 37 mg.

Now practice with the following:

(a) What is the maximum percentage error experienced in the measurement of 248.0 mL? 12.60 g?

(b) Determine the number of significant figures.

Amounts weighed	Number of significant figures
i. 4.58 g **ii.** 4.580 **iii.** 0.0458 **iv.** 0.0046	

(c) Which of the following has the greatest degree of accuracy (solve as in 3.5 mL ± 0.05 mL, accurate to the nearest 0.1 mL)?

15.7 mL ±

15.70 mL ±

15.700 mL ±

(d) Using significant figure practical rules, calculate the following:

5.5 g + 12.35 g + 4.40 g =

2.533 mg/day × 5 days =

Answers

(a) 0.02%; 0.04%

(b) **(i)** 3, **(ii)** 4, **(iii)** 3, **(iv)** 2 significant figures

(c) 15.700 mL has the greatest degree of accuracy

(d) 22.3 g; 12.7 or 13 mg

SOLUTIONS

(a) The zero in 248.0 mL is a significant figure, implying that the measurement was made within the limits 247.95 and 248.05 mL. The possible error is then calculated as

$$\frac{0.05\ \text{mL}}{248.0} \times 100\% = 0.02\%$$

Applying the same reasoning for 12.60 g, the maximum error is $\frac{0.005\ \text{mL}}{12.60} \times 100\% = 0.0397\% = 0.04\%$

(b) 3, 4, 3, 2 significant figures, respectively.

(c) 15.7 mL = 15.7 mL ± 0.05 mL, accurate to the nearest 0.1mL.

15.70 mL = 15.70 mL ± 0.005 mL, accurate to the nearest 0.01 mL.

15.700 mL = 15.700 mL ± 0.0005 mL, accurate to the nearest 0.001 mL. (The last measurement has the greatest degree of accuracy.)

(d) 5.5 g + 12.35 g + 4.40 g = 22.25 g = 22.3 g

2.533 mg/day × 5 days = 12.665 = 12.7 mg = 13 mg

1.2. ROUNDING OFF

The number of decimal places to which a medical calculation can be precisely calculated is determined by the number of significant figures. As mentioned earlier, when performing calculations, all figures should be retained until the end, when rounding off is performed. Rounding off is based on the last decimal place. If it is ≥5, the preceding decimal place is rounded up to the next digit, for example, 2.356 = 2.36. If it is <5, the preceding decimal place is left as it is, for example, 2.33 = 2.3.

Practice rounding off with the following measurements:

(a) 2.344 g = ________________

(b) 1.5 × 2.1114 mg = ________________

(c) 5.246 mL (using a pipette calibrated 1/10 mL) = ________________

(d) How many powder charts (individual doses) would a compounding pharmacist be able to prepare from a 10.5 g mixture of powders, if each chart should contain 1.33 g?

Answers

(a) 2.34 g

(b) 3.2 mg

(c) 5.3 mL

(d) 7.9 charts = 7 powder charts (doses must be accurate) and 0.9 g waste

SOLUTIONS

(a) In 2.344 the last decimal place is <5, so it is rounded to 2.34 g.

(b) $1.5 \times 2.1114\ mg = 3.167 = 3.2\ mg$, with only one decimal point because 1.5 is the least precise number in the operation (limiting number of significant digits) and it has only one place after the decimal.

(c) In 5.246 the last decimal place is >5, so it would be rounded to 5.25. However, the measuring tool is calibrated in 1/10 mL, which means only fractions as large as 0.1 of a milliliter can be measured accurately so 5.3 mL should be measured.

(d) $10.5\ g \times (chart/1.33\ \ g) = 7.8947 = 7.9$ charts (10.5 is the least precise number in the operation). Since each dose must be complete, there is enough powder mixture to dispense only seven charts with accurate doses and 0.9 g will be discarded as waste.

1.3. FRACTIONS

Most math skills required in health care fields require handling fractions, which measure a portion or part of a whole number and are usually written as ***common*** fractions or ***decimal*** fractions.

A *common fraction*, frequently referred simply as a fraction, can be exemplified as 1/5 or $\frac{1}{5}$, 3/16 or $\frac{3}{16}$, and so on. The first or upper number, the *numerator*, identifies the number of parts with which we are concerned, while the second or lower number, the *denominator*, indicates the number of aliquot parts into which *the numerator* is divided. When the numerator is divided by the denominator, the result is called the *quotient.*

When trying to perform pharmaceutical calculations with common fractions, you will find some principles and rules very handy:

(a) If the denominator of a fraction is 1, the value corresponds to the number in the numerator, for example, $\frac{3}{1} = 3$.

(b) If the value in the numerator of a fraction is lower than the one in the denominator, then the value of the fraction is <1. If the numerator and denominator are the same, the value of the fraction is =1. Numerator with greater value than the denominator will generate values >1.

Examples: $\frac{3}{5} < 1; \quad \frac{3}{3} = 1; \quad \frac{3}{2} > 1$

(c) If both the numerator and denominator are multiplied or divided by the same number, the value of a fraction does not change. By the other hand, the value will increase if a number is multiplied by the numerator and will decrease if multiplied by the denominator. The opposite occurs if either the nominator or the denominator is divided independently.

Examples: $\frac{3 \times 2}{4 \times 2} = \frac{6}{8} = 0.75$

$$\frac{3 \times 4}{4} = \frac{12}{4} > \frac{3}{4} \quad \text{as} \quad 3 > 0.75$$

$$\frac{3}{4 \times 4} = \frac{3}{16} < \frac{3}{4} \quad \text{as} \quad 0.188 < 0.75$$

(d) Practical consequences of the principles described above, which will reduce computing errors, are as follows:

- **(i)** The ability to reduce fractions to the *lowest common denominator*, or the smallest number divisible by all denominators in consideration.

(ii) The possibility to *reduce a fraction to lower terms* when recording a final result or during a series of calculations.

Ex.1. Reducing $\frac{1}{5}, \frac{3}{4}, \frac{2}{3}$ to a common denomination:

By serial testing it can be found that 60 is the smallest number divisible by 5, 4, and 3, then

$$\left.\begin{aligned} \frac{1}{5} &= \frac{1 \times 12}{5 \times 12} = \frac{12}{60} \\ \frac{3}{4} &= \frac{3 \times 15}{4 \times 15} = \frac{45}{60} \\ \frac{2}{3} &= \frac{2 \times 20}{3 \times 20} = \frac{40}{60} \end{aligned}\right\}, \quad \text{all have the same denominator}$$

Ex.2. Reducing $\frac{12}{480}$ to the lowest term:

$$\frac{12}{480} = \frac{12 \div 12}{480 \div 12} = \frac{1}{40}$$

A *decimal fraction* is a fraction with a denominator of 10 or any other power of 10. Usually, the numerator and the decimal point are satisfactory to express a decimal fraction, for example, $\frac{1}{10} = 0.1$ and $\frac{25}{1000} = 0.025$.

Consequences of this definition include the following:

(a) Moving the decimal point one place to the *right* multiplies a number by 10, two places by 100, and so on. Similarly, moving the decimal point one place to the *left* divides the number by 10 and so on.

Examples: $0.1 \times 10 = 1$
$0.025 \div 100 = 0.00025$

(b) A decimal fraction may be changed to a common fraction, and if desired, reduced to lowest terms.

Example: $0.125 = \frac{125}{1000} = \frac{1}{8}$

(c) If the numerator of a common fraction is divided by the denominator, a decimal fraction will be obtained. However, this sometimes leads to an endless decimal fraction.

Examples: $\frac{1}{5} = 1 \div 5 = 0.2$
$\frac{1}{6} = 1 \div 6 = 0.166\overline{6}$

Decimals, in health care settings, are usually rounded to the nearest tenth or hundredth. For example, $10 \div 6 = 1.6666$. This value may be rounded to the nearest tenth, which is 1.7, or to the nearest hundredth, 1.67. It will depend on the accuracy of the instrument being used or the type of medication. For example, a 1 mL tuberculin syringe is calibrated in hundredths of a milliliter, so for a 0.468 mL dose to be measured, it will be rounded off to 0.47 mL. A 3 mL syringe is calibrated in tenths of a milliliter, so volumes must be rounded off to the nearest tenth of a milliliter, for example, 2.468 mL, is rounded off to 2.5 mL.

Now practice with the following problems:

(a) The dose of a drug is $1/200$ gram. How many doses would a compounding pharmacist be able to prepare with $2/5$ grams?

(b) Alesse™-28, a combination hormone medication used as oral contraceptive, contains $1/10,000$ g levonorgestrel and $2/100,000$ g of ethinyl estradiol per tablet. How many *milligrams* of each drug are present in each tablet?

(c) A compounded capsule is required to contain $5/8$ gram of ingredient W, $1/4$ gram of ingredient X, $1/100$ gram of ingredient Y, and enough of ingredient Z to make a total of 1500 mg. The pharmacist is asked to prepare six capsules. How many total grams of ingredient Z will be required to fill the prescription?

(d) If a patient received the following doses of a drug, what is the total amount of drug received by the patient?

Four doses each containing 0.25 mg

Three doses each containing 0.5 mg

Two doses each containing 1.25 mg

One dose containing 1.5 mg

(e) How many 0.00025 g doses can be prepared with 0.150 g of a pure drug?

(f) A pharmacist had 5 g of promethazine HCl. How many grams were left after he compounded the following prescriptions?

℞$_1$: 6 suppositories each containing 10 mg

℞$_2$: 6 suppositories each containing 25 mg

℞$_3$: 10 suppositories each containing 40 mg

Answers

(a) 80 doses

(b) 0.1 mg levonorgestrel and 0.02 mg ethinyl estradiol in each tablet.

(c) $3.69\ \text{g} = 3.7\ \text{g}$

(d) 6.5 mg

(e) 600 doses

(f) $4.39\ \text{g} = 4.4\ \text{g}$

SOLUTIONS

(a) $\frac{2}{5} \times \frac{200}{1} = \frac{400}{5} = 80$ doses or $\frac{0.4 \times 200}{1} = 80$ doses

(b) $\frac{1}{10,000} = 0.0001\ \text{g} = 0.1\ \text{mg}$ levonorgestrel

$\frac{2}{100,000} = 0.00002\ \text{g} = 0.02\ \text{mg}$ ethinyl estradiol

(c) $\frac{5}{8} + \frac{1}{4} + \frac{1}{100} + Z = 1.5 \rightarrow \frac{250 + 100 + 4 + 400\,Z}{400} = 1.5$

$354 + 400\,Z = 600 \rightarrow Z = 0.615\ \text{g/caps}$

$\frac{6\ \text{caps}}{\text{Rx}} = 6 \times 0.615 = 3.69\ \text{g total}$

(d) $(4 \times 0.25) + (3 \times 0.5) + (2 \times 1.25) + (1 \times 1.5)$

$= 1 + 1.5 + 2.5 + 1.5 = 6.5\ \text{mg}$

(e) $\frac{0.150\ \text{g}}{0.00025\ \text{g/dose}} = 600$ doses or $\frac{1.5 \times 10^{-1}\ \text{g}}{2.5 \times 10^{-4}\ \text{g/dose}} = 0.6 \times \left[10^{-1+4}\right] = 0.6 \times 10^{3} = 600$ doses

(f) $5\ \text{g} - (6 \times 0.01\ \text{g}) + (6 \times 0.025\ \text{g}) + (10 \times 0.04\ \text{g})$

$= 5 - (0.06 + 0.15 + 0.4) = 5 - 0.61 = 4.39\ \text{g}$

Or, $0.06 + 0.15 + 0.40 = 0.61 \rightarrow 5 - 0.61 = 4.39\ \text{g}$

1.4. EXPONENTS AND POWERS

An exponent, power, or index is a mathematical representation that indicates the number of times a number is to be multiplied by itself. For example, $2^4 = 2 \times 2 \times 2 \times 2 = 16$ can be read as "2 to the *power* of 4." Exponents make it easier to write and read instead of several multiplications. Almost any number can be multiplied by itself as many times as desired: $a^n = a \times a \times \cdots \times a$. Exceptions include the exponent being 1, which means just the number itself (e.g., $5^1 = 5$) and the exponent being 0 (zero), when the result is 1 (e.g., $5^0 = 1$). Exponents may also be negative, in which case it is solved by the operation that is inverse to multiplication, that is, division. In other words, a negative exponent indicates the number of times *the number 1* will be divided by the number. For example, 4^{-3} can be solved as $1 \div 4 \div 4 \div 4 = 0.0156$ or represented and solved as $\frac{1}{4^3} = \frac{1}{64} = 0.0156$.

Scientific notation, also called *power-of-10 notation*, is particularly important in the health care field because of the use of the metric system, which is based on a power of 10 and also because many drugs are used at extremely diluted concentrations. Powers of 10 are very useful to write down very large or very small numbers, eliminating the need to use lots of zeros. Similarly, negative powers of 10 have special uses in all health care fields. When performing calculations remember that powers of 10 indicate how many places one needs to move the decimal point to the right or to the left (negative power of 10). This is also how we make conversions within the metric system. Metric conversions will be discussed in more detail in Chapter 2.

Some typical examples of scientific notation:

$10^3 = 10$ to the third power, 10 to the power of 3, or 10 cubed $= 10 \times 10 \times 10 = 1000$

$4000 = 4 \times 1000 = 4 \times 10^3$

$10{,}000 = 10 \times 1000 = 10^4$

$0.0005 = 5 \times 10^{-4}$

Let us also review some simple conversions of powers of 10 to ordinary numbers:

$$1.25 \times 10^3 = 1.25 \times (10 \times 10 \times 10) = 1.25 \times 1{,}000 = 1250$$

This calculation would be easier if you would just move the decimal point three places to the right: $1.25 \rightarrow 12.5 \rightarrow 125. \rightarrow 1250$

$$5.1 \times 10^{-4} = 5.1 \times \left(\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}\right) = 5.1 \times \frac{1}{10{,}000}$$
$$= 5.1 \times 0.0001 = 0.00051$$

Again, it is easier to just move the decimal point four places to the left: $5.1 \rightarrow 0.51 \rightarrow 0.051 \rightarrow 0.0051 \rightarrow 0.00051$

Now it is your turn to practice.

(a) Write 5.26×10^3 as an ordinary number.

(b) Write 2.35×10^{-4} as an ordinary number.

(c) Circle below what corresponds to 8.725×10^{-3}?

(i) 8725
(ii) 87.25
(iii) 0.8725
(iv) 0.008725

(d) Write 510,000 in scientific notation (power-of-10).

(e) Write 0.0004506 in scientific notation.

(f) What is 1 trillion as a power of 10?

Answers

(a) 5260
(b) 0.000235
(c) 0.008725
(d) 5.1×10^5
(e) 4.506×10^{-4}
(f) 10^{12}

SOLUTIONS

(a) $5.26 \times 10^3 = 5.26 \times (10 \times 10 \times 10) = 5.26 \times 1000 = 5260$ or, moving the decimal point three places to the right: $52.6 \rightarrow 526 \rightarrow 5260$

(b) $$2.35 \times 10^{-4} = 2.35 \times \left(\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}\right) = 2.35 \times 1/10,000$$
$$= 2.35 \times 0.0001 = 0.000235$$

Again, it is easier to just move the decimal point four places to the left:

$2.35 \rightarrow 0.235 \rightarrow 0.0235 \rightarrow 0.00235 \rightarrow 0.000235$

(c) $8.725 \times 10^{-3} =$ 0.008725 by moving the decimal point three places to the left.
(d) 510,000 as a power of 10 is 5.1×10^5
(e) 0.0004506 in scientific notation is 4.506×10^{-4}
(f) 1 trillion $= 1 \times 1000 \times 1000 \times 1000 \times 1000 = 10^{12}$

If you completed these successfully, you are familiar with exponents and power-of-10 expressions and may proceed to reviewing estimation. If you had difficulty or feel a bit unsure of yourself, go to the end of this chapter and continue for a more thorough review by doing the practice problems.

1.5. ESTIMATION

Because medical errors are one of the leading causes of death and injury in medical practice, it is a good idea for members of a health care team to check all results when performing calculations. One might think that this is unnecessary, since calculators are habitually used and typically reliable. However, we tend to take their results for granted, without thinking about making an error in entering data, which is liable to go unnoticed. Additionally, sometimes we find a wrong answer by use of a wrong method and an inattentive, mechanical verification of our calculations may not reveal the error.

For patient's safety sake, it is necessary to check every calculation in some way, to make sure that the result is reasonable.

One kind of check is particularly useful in preventing errors of large magnitude such as *misplacement of the decimal point*. This kind of error will not likely be missed if we check it against an initial *estimation* of what the result should be. For example, how much would cost a compounding pharmacist to prepare 325 capsules of a drug at $1.80/100?

By estimation:

$$300 \times \frac{2}{100} = \$6$$

Solving:

$$325 \text{ caps} \times \frac{\$1.80}{100 \text{ caps}} = \$5.85$$

Analyzing the estimated amount, it is easy to verify the order of magnitude and spot a 10-fold error (not \$ 0.58 or \$58.50).

Estimation using *rounded values* involves rounding all values to one figure. The figure is kept as it appears in the original number if the figure following it is 4 or less. The single figure is promoted to the next higher number if it is followed by a 5 or higher number.

For example,

4.27 rounded to one figure is 4

0.37 rounded to one figure is 0.4

3508 rounded to one figure is 4000

0.00949 rounded to one figure is 0.009

Now it is your turn to practice estimation by rounding the following to 1 significant figure:

A. 72

B. 0.08294

C. 0.452

D. 0.75

E. 820

Answers

A. 70
B. 0.08
C. 0.5
D. 0.8
E. 800

Estimation of the answer before attempting to obtain the exact solution to a problem is found by rounding off the quantities involved in the calculation to one significant figure and then computing the result. After solving the problem, compare the exact solution with the estimate. Unless they are reasonably close to each other, both should be recalculated. Unfortunately, it is necessary to know how to do the problem in order to come up with an estimate. It is therefore possible to "solve" a problem incorrectly and to have that wrong answer check against the estimate. Estimation is helpful in preventing errors and will give an idea of the order of magnitude of a calculated value but is not infallible. For example, A formula for 42 capsules calls for 180 mg of sucrose. To estimate the amount of sucrose per capsule, round 42 capsules to 40 capsules and 180 mg to 200 mg:

$$\frac{200\text{ mg}}{40\text{ caps}} = \frac{5\text{ mg}}{\text{caps}}$$

The exact answer is 4.28 mg per capsule.

Now try these two problems.

A certain tablet contains 32.5 mg of phenobarbital. Estimate the number of milligrams of phenobarbital in 24 tablets.

—————————————————

Answer
600 mg

SOLUTION

$$\frac{30\text{ mg}}{\text{tablet}} \times 20\text{ tablets} = 600\text{ mg}$$

The exact answer is 780 mg. You may think that 600 mg is rather a poor estimate, but it is good enough to tell you that your answer is in the ballpark. Certainly, if you were to solve the problem and come up with an answer of 78 mg or 7800 mg, you would realize that an error had been made.

A liquid costs \$3.27 per pint. Estimate the cost of 418 pints.

—————————————————

Answer
\$1200

SOLUTION

$$\frac{\$3}{\text{pt}} \times 400\text{ pt} = \$1200$$

The exact answer is \$1366.86.

1.6. UNITS

In compounding and filling prescriptions, the pharmacist deals with measured quantities. The magnitude of each such quantity is expressed as the product of a number and a unit. The unit name specifies the scale of measurement. Changing the scale of measurement (unit) causes the multiplying number to change as well. Thus, in describing any measured quantity, it is *always* necessary to specify the unit used. The unit is an essential part of the designation of a value that is either measured directly or calculated from measured data. *Units must not be permitted to drop off or fade away during calculations*.

In most manufactured products containing one or more drugs, the units are metric, for example, benzonatate 100 mg/capsule, azithromycin 250 mg/tablet. However, other drug strengths are expressed in terms of units of activity or potency according to specific biologic assay (e.g., heparin, insulin, bacitracin, Vitamins A, D, and E). Thus, it is important to keep in mind that units of activity (also known as USP units or units of potency) and international units are *not* equivalent. Calculations involving units of potency will be discussed in detail later in Chapter 10.

Some basic rules we need to know when working with units, according to the SI (Système International d'Unités = International System of Units) Guide:

- Unit symbols are unaltered in the plural, for example, 1 mg and 10 mg.
- Unit symbols are not followed by a period unless at the end of a sentence. However, in common usage, "in." is used for inch. This is probably used so as not to be confused with the word "in".
- Unit symbols are printed in lowercase letters, except
  - **(a)** if the symbol or the first letter of the symbol is an uppercase letter when the name of the unit is derived from the name of a person; examples: m (meter), s (second), V (volt), Pa (Pascal);
  - **(b)** the recommended symbol for the *liter* in the United States is L, which was adopted by the CGPM (Conférence Générale des Poids et Mesures = General Conference on Weights and Measures) in order to avoid the risk of confusion between the letter l and the number 1. Thus, although both l and L are internationally accepted symbols for the liter, to avoid this risk the symbol to be used in the United States is L. The script letter ℓ is not an approved symbol for the liter.
- Because acceptable units generally have internationally recognized symbols and names, it is not permissible to use abbreviations for their unit symbols or names, such as sec (for either s or second), sq. mm (for either mm^2 or square millimeter), cc (for either cm^3 or cubic centimeter), mins (for either min or minutes), hrs (for either h or hours), and lit (for either L or liter). Although the values of quantities are normally expressed using symbols for numbers and symbols for units, if for some reason the name of a unit is more appropriate than the unit symbol, the name of the unit should be spelled out in full.
- Prefix names and symbols are printed in roman (upright) type regardless of the type used in the surrounding text, and are attached to unit symbols without a space between the prefix name or symbol and the unit name or symbol. This last rule also applies to prefixes attached to unit names. Examples: mL (milliliter), μg (microgram), pm (picometer), Gb (gigabite), and THz (terahertz).
- In the expression for the value of a quantity, the unit symbol is placed after the numerical value and a space is left between the numerical value and the unit symbol, for example 10 mL, not 10mL.
- When the value of a quantity is used as an adjective but the meaning has any ambiguity, the words should be rearranged accordingly. For example, "the samples were placed in 22 mL vials" should be replaced with "the samples were placed in vials of volume 22 mL." When unit names are spelled out, the normal rules of English apply. Thus, for example, "a roll of 35 millimeter film" is acceptable.
- Contrary to the recommendation by the SI regarding the symbol % as an internationally recognized symbol for the number 0.01, in order to prevent calculation mistakes within the health care fields it is common to use terms such as "percentage by weight, % (by

weight), % (W/W)," "percentage by volume, % (by volume), % (V/V)." Similarly, in the medical fields it is acceptable to use ppm, ppb, and ppt for part per million, part per billion, and part per trillion, though these terms are rarely used in clinical practice.

Along this text you will recognize that computations involving units will frequently require some knowledge of different systems of measurement and intersystem conversions, which will be reviewed in Chapter 2 and Appendix 1. This is because sometimes you find that the units in which a measured quantity is expressed are not convenient for the results you are searching. For example, an American traveling in France wants to buy 2 lb of beef at the neighborhood butchery. Before going to the store around the corner, the traveler may wish to convert this amount into the equivalent metric unit (grams or kilograms), used in that country. To perform the conversion, it is necessary to know that

$$1 \text{ lb} = 454 \text{ g}$$

One advantage of performing a calculation involving units is that they may be multiplied and divided in much the same way as numbers or algebraic symbols. If the same unit appears in both the numerator and denominator, they will cancel each other. For example,

$$2 \text{ lb} \times \frac{454 \text{ g}}{1 \text{ lb}} = 908 \text{ g}$$

The traveler can go to the butcher and ask for 900 g of beef to get approximately the 2 lb needed.

Perform the conversions indicated:

A. $3 \text{ dL} \times \dfrac{100 \text{ mL}}{1 \text{ dL}} =$

B. $154 \text{ lb} \times \dfrac{1 \text{ kg}}{2.2 \text{ lb}} =$

C. $250 \text{ mg} \times \dfrac{1 \text{ tablet}}{50 \text{ mg}} =$

Answers

A. 300 mL

B. 70 kg

C. 5 tablets

1.7. RATIO

Ratio is a relationship between two values, similar to fractions, the relationship of a part to the whole. It provides a comparison between two like quantities and may be expressed in several different ways (quotient, fraction, percentage, or decimal).