Fall 2016 Math 251 Final Exam Name_____

Instructions:
Show ALL work.
Simplify wherever possible (unless told otherwise).
Clearly indicate your final answer.

| Problem Number | Points Possible | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 30 | |
| 3 | 30 | |
| 4 | 30 | |
| 5 | 40 | |
| Subtotal | 150 | |
| Extra credit | 10 | |
| Total | 150 | |

1) Indicate whether each of the following statements are true or false (no explanation necessary):
(20 points)

a) $\frac{d}{dx}\left(5^x\right) = x5^{x-1}$

b) For a continuous function *f*, if $f'(2) = 0$ and $f''(2) = -1$, then 2 must be a local maximum of *f*.

c) For a continuous function *f*, if $f'(2) = 0$ and $f''(2) = 0$, then 2 must be neither a local minimum nor a local maximum of *f*.

d) $\frac{d}{dx}\left(\ln\left(e^{-x}\right)\right) = -1$

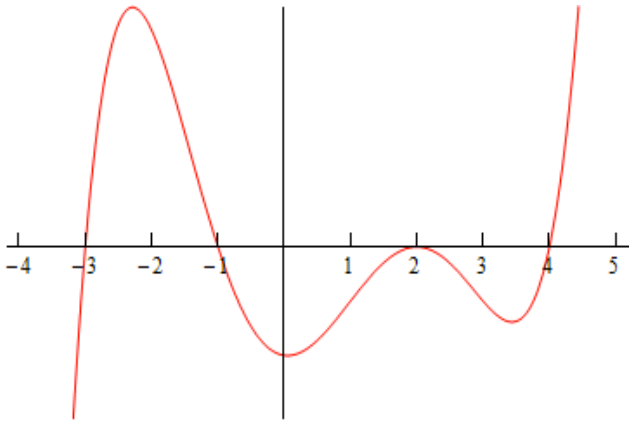
e) $\frac{d}{dx}\left(\ln 2 - e^{5^2+2} + \frac{1}{\cos\ e}\right) = 0$

f) $\frac{d}{dx}\left(\ln\left(\cos x\right)\right) = \frac{1}{-\sin x}$

g) Brian Jinxed the Golden State Warriors' first game of the season with a dumb midterm T/F question.

2) Graphs (30 points)

a) Pictured below is the graph of a function $g$ (assume it continues in both directions forever). Use it to answer the questions below about $g'$ and $g''$.

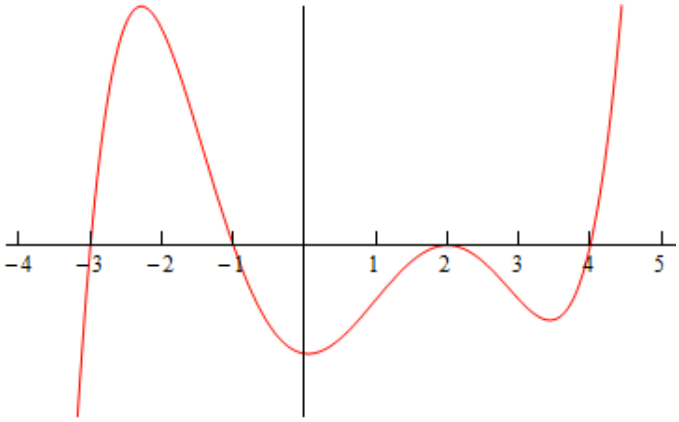



i) On what intervals is $g'$ positive?

ii) On what intervals (approximately) is $g''$ positive?

iii) Rank the following from least to greatest. Include an estimate for the first 3, and the sign of the 4th. $g(-1), g'(-3), g'(1), g''(2)$

b) Pictured below is the graph $f'$ (assume it continues in both directions forever). Use it to estimate the answers to the questions below **about *f***. (careful, the question are not about $f'$)




i) On what interval(s) is the function $f$ increasing?

ii) On what interval(s) is the function $f$ concave up?

iii) At which point(s) does the function $f$ have a local maximum?

iv) At which points(s) does the function $f$ have an inflection point?

3) Hard Derivatives: Find the derivative specified below. (no need to simplify) (30 points)

a) $\frac{d}{dx}\left(\frac{2}{x^2 e^{x\sin x}}+\frac{3}{\sqrt{x^5}}\right)$

Hint: It's not pretty, but it's not as bad if you rewrite it before taking derivative

b) $\frac{d}{dx}\left(\operatorname{arcsec}\left(\frac{1}{\sqrt{x^4-x^2}}\right)\right)$

Hint: $\frac{d}{dx}\left(\operatorname{arcsec} x\right)=\frac{1}{\sqrt{x^4-x^2}}$

c) $\frac{d}{dx}\left(x^{\ln(x+1)}\right)$

Hint: You'll need logarithmic differentiation.

d) Find $\frac{dy}{dx}$ for $x^3 + y^3 - 3xy = y$

e) Find $\frac{dy}{dx}$ for $x^3 y^3 = y^y$

Hint: logs are your friend.

4) Critical Values: (30 points)

a) Let $f(x) = \sqrt[3]{x^2 - 4}$. Find all critical values of *f*. There should be 3 CV's.

b) Classify (as relative min, relative max or neither) these critical values **using the 1st derivative test**. If you couldn't answer part a, make up 3 CV's and use them for b and c.

**c)** Classify (as relative min, relative max or neither) one of these critical values using the **2nd derivative test.**

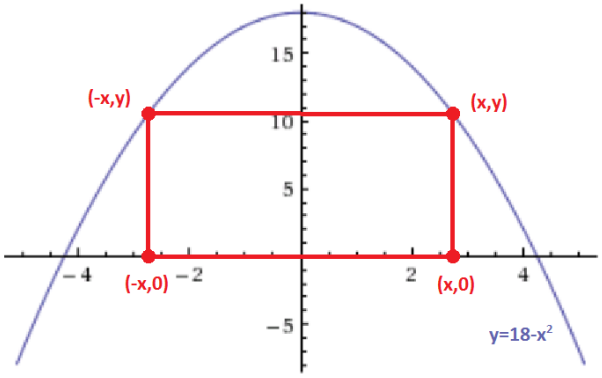
5) Word Problems: **Pick 4 of the following 5.** To minimize my work, make it clear which one you don't want me to grade. (40 points)

a) At noon, ship A is 9 miles west of ship B. Ship A is traveling **east** 3 mi/hr. Ship B is traveling south 2 mil hr. How fast is the distance between them changing at 2pm?

b) I need to create a box with a square base and no top. The material for the base costs \$2 per square foot, while the material for the sides only costs \$1 per square foot. If I have \$60 to spend, what is the length of one side of the base of the largest volume box that I can build?

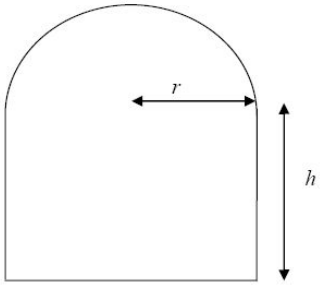
c) If two resistors with resistances $R_1$ and $R_2$ are connected in parallel, then the total resistance, R, is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If $R_1$ and $R_2$ are increasing at rates of 9 and 18, respectively, at what rate is R increasing when $R_1$ and $R_2$ are 3 and 6, respectively?

d) Find the value of $x$ in the picture below that maximizes the area of the rectangle inscribed under the parabola $y = 18 - x^2$.




e) A Norman Window has the shape of a rectangle surmounted by a semicircle (see below). If the perimeter of the window is 16 meters, find the radius that maximizes the area of the window (if you happen to have the answer memorized, you still need to show work).

Hint: The area of a circle is $\pi r^2$, and the circumference of a circle is $2\pi r$



EXTRA CREDIT (5 points each) – Determine the derivatives below:

1) $\frac{d}{dx}\left(x^{\left(x^x\right)}\right)$

Hint: You'll need to use logarithmic differentiation, and you'll need to take the log of both sides twice.

2) $\frac{d}{dx}\left(\left(x^x\right)^x\right)$

Hint: Logarithmic differentiation again, but you only need to take the log of both sides once.