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1. Let *R* be the region between the graph of $y = e^{-2x}$ and the *x*-axis forx≥3 . The area of *R* is

(A) $\frac{1}{2e^6}$ ✔

(B) $\frac{1}{e^6}$

(C) $\frac{2}{e^6}$

(D) $\frac{\pi}{2e^6}$

(E) infinite

---

2. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,\ 6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

(B) $y = 5 + x^3$ ✔

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

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3. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

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(A) $\int_1^2 \sqrt{1+\frac{1}{x^2}}\, dx$ ✔

(B) $\int_1^2 \left(1+\frac{1}{x^2}\right)\, dx$

(C) $\int_1^2 \sqrt{1+e^{2x}}\, dx$

(D) $\int_1^2 \sqrt{1+(\ln x)^2}\, dx$

(E) $\int_1^2 \left(1+(\ln x)^2\right)\, dx$

4.

x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	–1

The table above gives selected values for a differentiable function *f* and its first derivative. Using a left Riemann sum with 3 subintervals of equal length, which of the following is an approximation of the length of the graph of *f* on the interval $[1,\ 7]$?

(A) 6

(B) 34

(C) $2\sqrt{3}+2\sqrt{2}+2$

(D) $2\sqrt{5}+2\sqrt{2}+2$ ✔

(E) $2\sqrt{5}+4\sqrt{2}+2$

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5.

x	$f(x)$	$f'(x)$
0	2	5
4	-3	11

The function *f* has a continuous derivative. The table above gives values of *f* and its derivative for *x*=0 and *x*=4 . If $\int_0^4 f(x)dx = 8$, what is the value of $\int_0^4 xf'(x)dx$?

(A) −20 ✔

(B) −13

(C) −12

(D) −7

(E) 36

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6. $\int \frac{7x}{(2x-3)(x+2)}\, dx =$

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(A) $\frac{3}{2}\ln|2x-3|+2\ln|x+2|+C$ ✔

(B) $3\ln|2x-3|+2\ln|x+2|+C$

(C) $3\ln|2x-3|-2\ln|x+2|+C$

(D) $-\frac{6}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$

(E) $-\frac{3}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$

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7. $\int \frac{1}{x^2-7x+10}\,dx=$

(A) $\ln|(x-2)(x-5)|+C$

(B) $\frac{1}{3}\ln|(x-2)(x-5)|+C$

(C) $\frac{1}{3}\ln\left|\frac{2x-7}{(x-2)(x-5)}\right|+C$

(D) $\frac{1}{3}\ln\left|\frac{x-2}{x-5}\right|+C$

(E) $\frac{1}{3}\ln\left|\frac{x-5}{x-2}\right|+C$ ✔

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8. For $0<P<100$, which of the following is an antiderivative of $\frac{1}{100P-P^2}$?

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(A) $\frac{1}{100}\ln(P) - \frac{1}{100}\ln(100 - P)$ ✔

(B) $\frac{1}{100}\ln(P) + \frac{1}{100}\ln(100 - P)$

(C) $100\ \ln(P) - 100\ \ln(100 - P)$

(D) $\ln\left(100P - P^2\right)$

(E) $\frac{1}{50P^2 - \frac{P^3}{3}}$

---

9. $\displaystyle\int \frac{8}{x^2 - 4}dx =$

(A) $4\tan^{-1}\left(\frac{x}{2}\right) + c$

(B) $8\ln\left|x^2 - 4\right| + c$

(C) $2\ln\left|\frac{x-2}{x+2}\right| + c$ ✔

(D) $2\ln\left|\frac{x+2}{x-2}\right| + c$

(E) $2\ln|x + 2| + 2\ln|x - 2| + c$

---

10. $\displaystyle\int \frac{1 + 3x}{(1 - x)(3x - 5)}dx =$

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(A) $2\ ln\ |1 - x| - 3\ ln\ |3x - 5| + C$ ✔

(B) $2\ ln\ |1 - x| - 27\ ln\ |3x - 5| + C$

(C) $-2\ ln\ |1 - x| - 3\ ln\ |3x - 5| + C$

(D) $-2\ ln\ |1 - x| - 9\ ln\ |3x - 5| + C$

---

11. Given that *y*(1) = −3 and $\frac{dy}{dx} = 2x + y$, what is the approximation for *y*(2) if Euler's method is used with a step size of 0.5, starting at *x* = 1 ?

(A) −5

(B) −4.25

(C) −4

(D) −3.75 ✔

(E) −3.5

---

12. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5 ?

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(A) 3

(B) 5

(C) 6 ✓

(D) 10

(E) 12

13.

x	2	2.2	2.4
$f'(x)$	-0.5	-0.3	-0.1

Let y = f(x) be the solution to the differential equation $\frac{dy}{dx} = f'(x)$ with initial condition f(2) = 3. Selected values of f' are given in the table above. What is the approximation for f(2.4) if Euler's method is used, starting at x = 2 with two steps of equal size?

(A) 2.80

(B) 2.82

(C) 2.84 ✓

(D) 2.92

(E) 3.16

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14. The number of moose in a national park is modeled by the function *M* that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where *t* is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?

(A) 50

(B) 200 ✔

(C) 500

(D) 1000

(E) 2000

---

15. The number of students in a cafeteria is modeled by the function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{2000}P(200 - P)$, where *t* is the time in seconds and *P(0)* = 25. What is the greatest rate of change, in students per second, of the number of students in the cafeteria?

(A) 5 ✔

(B) 25

(C) 100

(D) 200

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16. A population of wolves is modeled by the function *P* and grows according to the logistic differential equation $\frac{dP}{dt} = 5P\left(1 - \frac{P}{5000}\right)$, where *t* is the time in years and *P(0)* = 1000. Which of the following statements are true?

1.
$\lim_{t \to \infty} P(t) = 5000$

2.
$\frac{dP}{dt} is, positive, for, t > 0.$

3.
$\frac{d^2P}{dt^2} is, positive, for, t > 0.$

(A) I only

(B) II only

(C) I and II only ✔

(D) I and III only

(E) I, II, and III

---

17. The function *N* satisfies the logistic differential equation $\frac{dN}{dt} = \frac{N}{10}\left(1 - \frac{N}{850}\right)$, where N(0) = 105.Which of
the following statements is false?

(A) $\lim_{x \to \infty} N(t) = 850$

(B) $\frac{dN}{dt}$ has a maximum value when N = 105 ✔

(C) $\frac{d^2N}{dt^2} = 0$ when N = 425

(D) When N > 425, $\frac{dN}{dt} > 0\, and\, \frac{d^2N}{dt^2} < 0$

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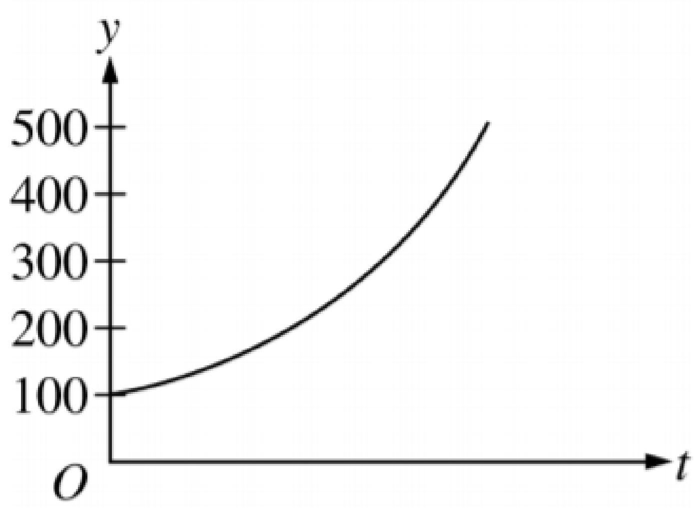
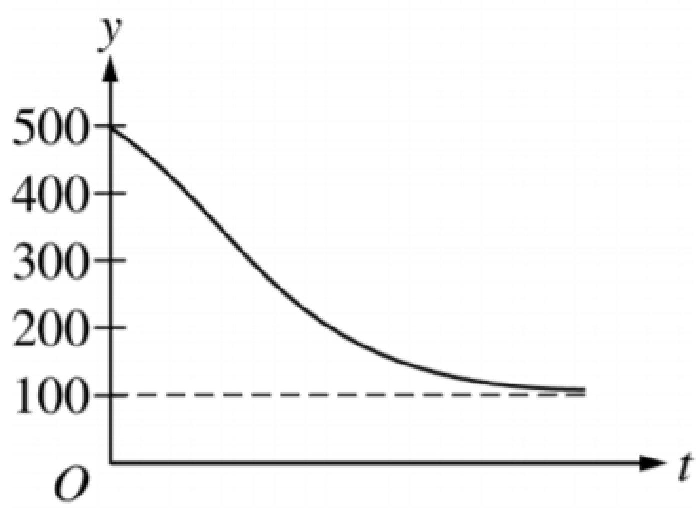
18. Which of the following graphs is the solution to the logistic differential equation $\frac{dy}{dt} = \frac{y}{5}\left(1 - \frac{y}{500}\right)$ with the initial condition $y(0) = 100$?

(A)



(B)



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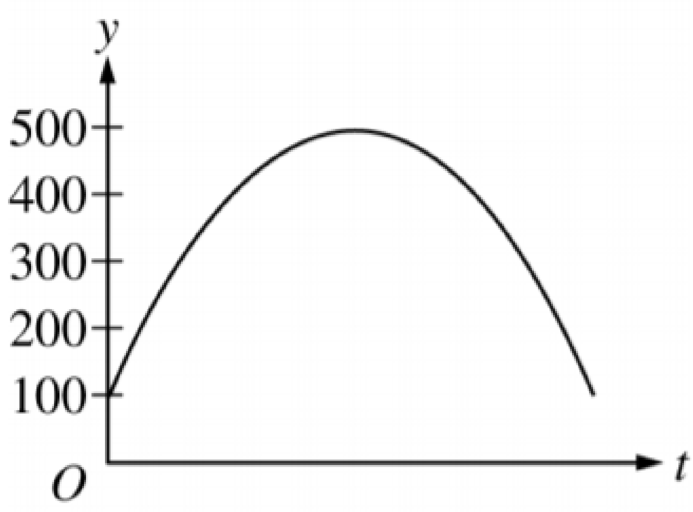
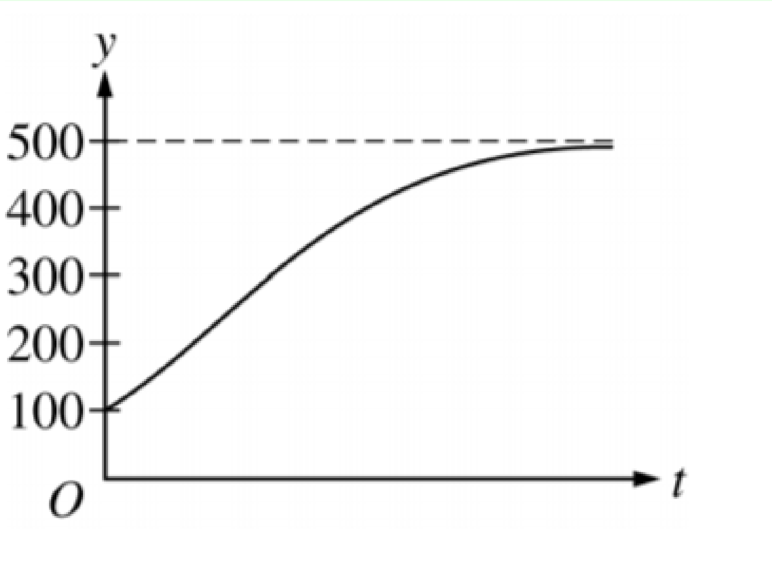
(C)



(D)



19. Let *k* be a positive constant. Which of the following is a logistic differential equation?

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(A) $\frac{dy}{dt} = kt$

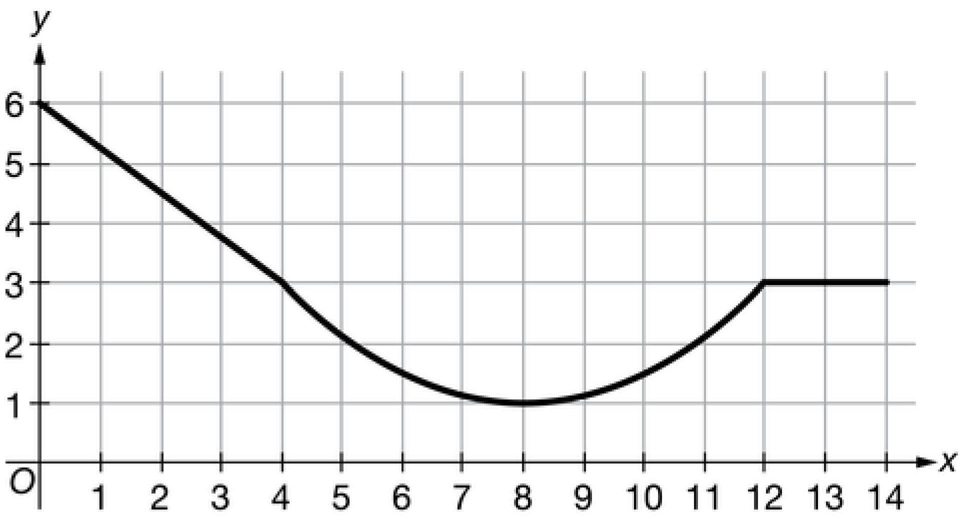
(B) $\frac{dy}{dt} = ky$

(C) $\frac{dy}{dt} = kt\,(1-t)$

(D) $\frac{dy}{dt} = ky\,(1-t)$

(E) $\frac{dy}{dt} = ky\,(1-y)$ ✔

20.



Graph of f

$$f(x) = \begin{cases} 6 - \frac{3}{4}x & \text{for } 0 \le x < 4 \\ 1 + \frac{1}{8}(x-8)^2 & \text{for } 4 \le x \le 12 \\ 3 & \text{for } 12 < x \le 14 \end{cases}$$

A skateboard track consists of a straight ramp followed by a curved section and a horizontal ledge. The track is modeled by the piecewise-defined function f above, and the graph of f is shown in the figure above. Which of the following expressions gives the total length of the track from $x = 0$ to $x = 14$?

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(A) $2+\int_0^{12}\sqrt{1+\left(-\frac{3}{4}+\frac{1}{4}(x-8)\right)^2}\,dx$

(B) $2+\int_0^{12}\left(\sqrt{1+\left(-\frac{3}{4}\right)^2}+\sqrt{1+\frac{1}{16}(x-8)^2}\right)dx$

(C) $7+\int_4^{12}\sqrt{1+\left(1+\frac{1}{8}(x-8)^2\right)^2}\,dx$

(D) $7+\int_4^{12}\sqrt{1+\frac{1}{16}(x-8)^2}\,dx$ ✔

---

21. The rate of change, $\frac{dP}{dt}$, of the number of people entering a movie theater is modeled by a logistic differential equation. The capacity of the theater is 500 people. At a certain time, the number of people in the theater is 100 and is increasing at the rate of 50 per minute. Which of the following differential equations could describe this situation?

(A) $\frac{dP}{dt}=\frac{1}{8}(500-P)$

(B) $\frac{dP}{dt}=\frac{1}{50}P(500-P)$

(C) $\frac{dP}{dt}=\frac{1}{800}P(500-P)$ ✔

(D) $\frac{dP}{dt}=\frac{1}{1200}P(500+P)$

---

22. If $\int_1^x f(t)\,dt=\frac{20x}{\sqrt{4x^2+21}}-4$, then $\int_1^\infty f(t)\,dt$ is

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(A) 6 ✓

(B) 1

(C) -3

(D) -4

(E) divergent

---

23. $\int_1^{\infty} \frac{x^2}{(x^3+2)^2}\, dx$ is

(A) $-\frac{1}{9}$

(B) $\frac{1}{9}$ ✓

(C) $\frac{1}{3}$

(D) 1

(E) divergent

---

24. $\int_1^{\infty} xe^{-x^2}\, dx$ is

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A $-\frac{1}{e}$

B $\frac{1}{2e}$ ✔

C $\frac{1}{e}$

D $\frac{2}{e}$

E divergent

---

25. $\int_0^3 \frac{dx}{(1-x)^2}$

A $-\frac{3}{2}$

B $-\frac{1}{2}$

C $\frac{1}{2}$

D $\frac{3}{2}$

E divergent ✔

---

26. $\int_1^\infty \frac{1}{x^p} dx \; and \int_0^1 \frac{1}{x^p} dx$ both diverge when p =

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(A) 2

(B) 1 ✓

(C) $\frac{1}{2}$

(D) 0

(E) -1

---

27. Which of the following statements about the integral $\int_0^{\pi} \sec^2 x dx$ is true

(A) The integral is equal to 0.

(B) The integral is equal to 2/3.

(C) The integral diverges because $\lim_{x\to\frac{\pi}{2}} \sec^2 x$ does not exist.

(D) The integral diverges because $\lim_{x\to\frac{\pi}{2}} \tan x$ does not exist. ✓

---

28. What are all values of *p* for which $\int_1^{\infty} \frac{1}{x^{3p+1}} dx$ converges?

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(A) $p < 0$

(B) $p > -1/3$

(C) $p > 0$ ✔

(D) $p > 1$

---

29. Which of the following are equivalent to $\int_2^4 \frac{2x+5}{5-x} dx$?

1. $\frac{\int_2^4 (2x+5) dx}{\int_2^4 (5-x) dx}$
2. $\int_2^4 \left(-2 + \frac{15}{5-x}\right) dx$
3. $\int_1^3 \left(\frac{15}{u} - 2\right) du$

(A) I only

(B) II only

(C) III only

(D) II and III only ✔

---

30. Which of the following is equivalent to $\int \frac{1}{x^2-16} dx$?

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(A) $\int \frac{1}{u} du$, where $u = x^2 - 16$

(B) $\frac{1}{2x} \int \frac{1}{u} du$, where $u = x^2 - 16$

(C) $\frac{1}{8} \int \frac{1}{x-4} dx - \frac{1}{8} \int \frac{1}{x+4} dx$ ✔

(D) $\int \frac{1}{x-4} dx + \int \frac{1}{x+4} dx$

---

31. Which of the following is equivalent to $\int_3^5 x \ln x dx$?

(A) $\int_{\ln 3}^{\ln 5} u du$

(B) $\int_3^5 x dx \cdot \int_3^5 \ln x dx$

(C) $\frac{1}{2} x^2 \ln x \Big|_3^5 - \int_3^5 \frac{1}{2} x dx$ ✔

(D) $\frac{1}{2} x^2 \ln x \Big|_3^5 + \int_3^5 \frac{1}{2} x dx$

---

32. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = y - 10x^2$ with the initial condition $f(0) = 3$. What is the approximation for $f(0.4)$ if Euler's method is used, starting at $x = 0$ with steps of size 0.2 ?

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(A) 4.120

(B) 4.200

(C) 4.240 ✔

(D) 4.768

---

33. $\int_0^4 \frac{1}{\sqrt{x}\,(1+\sqrt{x})}dx$ is

(A) $\ln 3$

(B) $\ln 5$

(C) $2\ln 3$ ✔

(D) divergent

34.

x	1	2	3	4
$f(x)$	-2	1	6	3
$f'(x)$	2	4	-1	-4

The function f has a continuous second derivative. The table above gives values of f and its derivative, f', at selected values of x. What is the value of $\int_1^2 x\,f''(x)\,dx$?

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(A) 3 ✓

(B) 4

(C) 7

(D) 9

35. Which of the following expressions is equal to $\int_0^2 \frac{17x+4}{3x^2-7x-6}dx$?

(A) $\int_0^2 \frac{2}{x-3}dx + \int_0^2 \frac{5}{3x+2}dx$

(B) $\int_0^2 \frac{5}{x-3}dx + \int_0^2 \frac{2}{3x+2}dx$ ✓

(C) $\int_0^2 \frac{4}{x-3}dx + \int_0^2 \frac{17x}{3x+2}dx$

(D) $\int_0^2 \frac{17x}{x-3}dx + \int_0^2 \frac{4}{3x+2}dx$