Final Exam review

Voltage and Current
mega (M) = 10^6
kilo (k) = 10^3
milli (m) = 10^{-3}
micro (μ) = 10^{-6}
nano (n) = 10^{-9}
pico (p) = 10^{-12}

i = dq/dt = Q/t
v = dw/dq = W/Q
$q_e = 1.6 \times 10^{-19}$C
Current is through and voltage is across

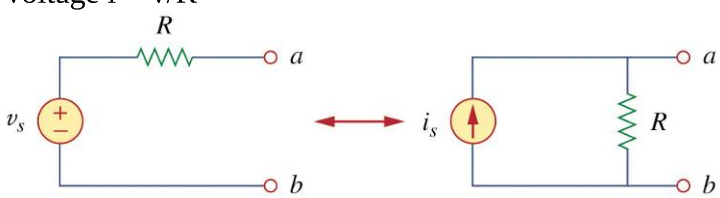
Resistance and Ohm's law
$R = \frac{\rho \ell}{A}$
v = iR
Short Circuit: R = 0 Ω, V = 0 V
Open Circuit: R = ∞ Ω, I = 0 A
Passive component sign convention (current enters the + voltage polarity on a resistor)

Power and Energy
Battery life (h) = $\frac{\text{Capacity (Ah)}}{\text{Discharge rate (A)}}$
$P = \frac{W}{t}$
P = vi = i^2R = v^2/R
Cost ($) = Energy (kwh) × Price ($/kwh)
1 hp = 746 W
$\eta = \frac{P_{out}}{P_{in}}$
$\eta_{total} = \eta_1 \times \eta_2 \times \eta_3$

Series Circuits, Voltage Divider, and KVL
Branch: A single circuit element
Node: A point of connection between two or more branches
Loop: A closed path in a circuit
Series: Two elements that exclusively share a node (carry the same current through)
KVL: $\Sigma v_n = 0$ around a loop
N resistors in series: $R_{eq} = \Sigma R_n$
Same value in series: $R_{eq} = N R_n$
Voltage divider for resistors in series: $v_x = (R_x/R_{eq})\, v_{series}$
Subscript notation: $V_{ab} = V_a - V_b, V_a = V_a - 0(ground)$

Parallel Circuits, Current Divider, KCL, and Meters
Parallel: Two or more elements that share the same two nodes (have the same voltage across)
KCL: $\Sigma\, i_n = 0$ at a node
N resistors in parallel: $1/R_{eq} = \Sigma\, 1/R_n$
Same value in parallel: $R_{eq} = R_n/N$
Current divider for resistors in parallel: $i_x = (R_{eq}/R_x)\, i_{parallel}$
Voltmeter: Measures voltage between two nodes, acts like an open circuit
Ammeter: Measures current in a branch, acts like a short circuit
Ohmmeter: Measures equivalent resistance between two nodes, contains its own source

Current Sources and Source Transformation
Voltage source is a plus and minus sign inside a circle with the held voltage (V) near it.
Series voltage sources are added using KVL.
Current source is an arrow inside a circle with the held current (A) value near it.
Parallel current sources are added using KCL.
An equivalent circuit for a voltage source in series with a resistor is a current source with a resistor in parallel. The voltage converts to a current using Ohm's Law $v = i\,R$ or current is found from the voltage $i = v/R$



Nodal Analysis
1. Identify nodes
2. Select a reference node (ground)
3. Identify known node voltages
4. Label all other nodes
5. Assume current direction in resistors
6. Express unknown currents in terms of node voltages
7. Apply KCL to all non-reference nodes
8. Solve simultaneous equations (if doing by hand, multiply by Lowest Common Denominator)

Thévenin Equivalent and Maximum Power Transfer
Thévenin equivalent
1. Remove the load (open circuit)
2. V_{TH} = open circuit voltage from the circuit where the load was
3. R_{TH} = equivalent resistance at the terminals with the independent sources turned off ("zeroized")
4. Redraw circuit as a voltage source (V_{TH}) in series with a resistor (R_{TH})

Maximum power is transferred to a load when $R_L = R_{TH}$.
Maximum power delivered to this load is $P_{max} = (V_{TH})^2/(4R_{TH})$

Capacitors and Inductors

Capacitor	Inductor
$q = C v$	
$i = C\, dv/dt$	$v = L\, di/dt$
$w = ½\, Cv^2$	$w = ½\, Li^2$
In DC, capacitor is an open	In DC, inductor is a short
Voltage is continuous in a capacitor	Current is continuous in an inductor
Combine opposite of resistor equations	Combine same as resistor equations
Power = 0 W	Power = 0 W

Step Transient Response

Capacitor

$\tau = R_{eq}C$

$v(t) = v(\infty) + (v(0) - v(\infty))e^{-t/\tau}$

Sinusoids and Phasors

$v(t) = V_m \sin(\omega t + \theta_v)$

$i(t) = I_m \sin(\omega t + \theta_i)$

$\omega = 2\pi f$

$\theta = \frac{\Delta t}{T} 360$

$X_{rms} = \frac{X_m}{\sqrt{2}}$

$j = \sqrt{-1}, j^2 = -1, \frac{1}{j} = -j$

Leading phasor is determined by looking at the angle between the phasors that is less than 180° and determining which phasor will cross the positive x-axis first.

Leading sinusoid peaks first in time.

Rectangular complex: $\mathbf{C} = x + jy$ $\quad x = r\cos\theta \quad y = r\sin\theta$

Polar complex: $\mathbf{C} = r\,\angle\theta$ $\quad r = \sqrt{x^2 + y^2} \quad \theta = \tan^{-1}\frac{y}{x}$

Phasor: $v(t) = V_m \cos(\omega t + \theta_v) \Rightarrow \mathbf{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v$

Complex add: $\mathbf{C}_1 + \mathbf{C}_2 = x_1 + jy_1 + x_2 + jy_2 = (x_1 + x_2) + j(y_1 + y_2)$

Complex multiply: $\mathbf{C}_1 \times \mathbf{C}_2 = r_1\angle\theta_1 \times r_2\angle\theta_2 = (r_1 \times r_2)\angle(\theta_1 + \theta_2)$

$\mathbf{C}^* = r\angle-\theta = x - jy$

Impedance

$\mathbf{Z} = \mathbf{V}/\mathbf{I} = R = j\omega L = -j/(\omega C)$

$\mathbf{Z} = R + jX$; R = resistance, X = reactance

$\mathbf{Z}_R = R\angle 0 \qquad \mathbf{Z}_L = \omega L\angle 90 \qquad \mathbf{Z}_C = \frac{1}{\omega C}\angle-90$

Inductor current lags, capacitor current leads (ELI the ICE man)

AC Circuit Analysis

AC circuit analysis uses the same tools and techniques as DC circuit analysis

Instead of using V, I, and R, AC analysis uses phasors for **V** and **I** and the complex impedance **Z**.

AC analysis steps

1. Convert all components to phasors and impedances.
2. Solve using DC analysis methods.
3. Convert back to time domain if needed.

Maximum Power Transfer

Maximum power is transferred to a load when $\mathbf{Z}_L = \mathbf{Z}_{TH}^*$.

Maximum power delivered to this load is $P_{max} = |\mathbf{V}_{TH}|^2/(4R_{TH})$

AC Power

Instantaneous power: $p(t) = v(t)\, i(t)$

Average/Real power: $P = V_{rms}\, I_{rms} \cos\theta_{vi} = |I_{rms}|^2\, R$

Apparent power: $S = V_{rms}\, I_{rms} = \sqrt{P^2 + Q^2}$

Complex power: $\mathbf{S} = \mathbf{V_{rms}}\, \mathbf{I_{rms}^*} = S\angle\theta_{vi} = P + j\,Q = |I_{rms}|^2\, \mathbf{Z} = |V_{rms}|^2/\mathbf{Z}^*$

Reactive power: $Q = V_{rms}\, I_{rms} \sin\theta_{vi} = |I_{rms}|^2\, X$

Power factor: $F_p = \cos\theta_{vi} = P/S$

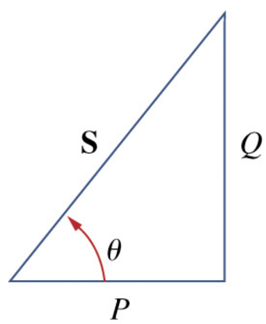
Power triangle

Apparent Power (VA)

$$S = |\mathbf{S}| = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$

Complex Power

$$\mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^* = V_{\text{rms}} I_{\text{rms}} \angle(\theta_v - \theta_i)$$



Reactive Power (VAR)

$$Q = \operatorname{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

Real Power (W)

$$P = \operatorname{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

Power Factor Correction

Eliminate Q_L by placing capacitor in parallel with the load

$X = \frac{V^2}{Q}$ $\quad$ $C = \frac{1}{\omega X}$ $\quad$ $L = \frac{X}{\omega}$

Resonance

At resonance $X_L = X_C$

$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{\omega}{2\pi}$

$BW = f_2 - f_1$

$f_{1,2} = f_r \pm \frac{BW}{2}$

$$Q = \frac{f_r}{BW} = \frac{X}{R} = \frac{\omega L}{R} = \frac{1}{\omega RC}$$

Filters

Low pass and High pass cutoff frequency: $f_c = \frac{1}{\omega\tau}$ $\quad$ $\tau = RC$ or $\tau = \frac{L}{R}$

Transformers

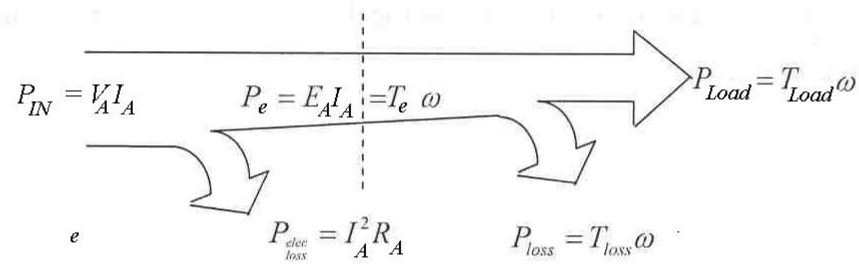
$a = \frac{N_{pri}}{N_{sec}} = \frac{\mathbf{E}_{pri}}{\mathbf{E}_{sec}} = \frac{\mathbf{I}_{sec}}{\mathbf{I}_{pri}}$ $\quad$ $S_{in} = S_{out}$ for an ideal transformer

$\mathbf{Z}_{pri,refl} = a^2 \mathbf{Z}_{sec}$

Magnetism and Linear Machines
$\mathbf{B}$ is magnetic flux density in Tesla (T)
$\mathbf{F} = q\, \mathbf{u} \times \mathbf{B}$
$\mathbf{F} = I\, \mathbf{L} \times \mathbf{B}$
$E_a = \mathbf{B} \cdot \mathbf{L} \times \mathbf{u}$; induced voltage
$V_a = I_a R_a + E_a$

Permanent Magnet DC Machine
$E_a = K_v\, \omega_m$
$\tau_e = K_v\, I_a$

DC motor power flow diagram



$\tau_{dev} = \tau_{load} + \tau_{loss}$
Parts of a motor: Rotor, Stator, Armature, Commutator, Brushes, Field (Poles)

3-Phase AC Circuits
Y-connected positive sequence phase voltages: $\mathbf{V}_{AN} = V_{rms}\angle 0°$ $\mathbf{V}_{BN} = V_{rms}\angle -120°$ $\mathbf{V}_{CN} = V_{rms}\angle 120°$
The neutral line has no current or voltage in a balanced three phase system, so it is not required. It can be included as a connection without elements for reducing a three phase system to a single phase system.
Conversion to line voltages: $\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{an}\angle 30°$ $\mathbf{V}_{bc} = \sqrt{3}\mathbf{V}_{bn}\angle 30°$ $\mathbf{V}_{ca} = \sqrt{3}\mathbf{V}_{cn}\angle 30°$

Equivalent Impedance: $\mathbf{Z}_\Delta = 3\mathbf{Z}_Y$

Convert to line currents from Δ-phase currents: $\mathbf{I}_a = \sqrt{3}\mathbf{I}_{ab}\angle -30°$ $\mathbf{I}_b = \sqrt{3}\mathbf{I}_{bc}\angle -30°$ $\mathbf{I}_c = \sqrt{3}\mathbf{I}_{ca}\angle -30°$

If the system has line impedances, convert the circuit to a Y-Y system, isolate a single phase, and solve using single phase AC techniques.

3-Phase AC Power
Instantaneous power: $p(t) = 3V_{rms}I_{rms}\cos\theta$ $q(t) = 3V_{rms}I_{rms}\sin\theta$

Power per phase: $\text{Real(Watts)} = P_\emptyset = |V_{rms}||I_{rms}|\cos\theta$
$\text{Reactive(VARs)} = Q_\emptyset = |V_{rms}||I_{rms}|\sin\theta$ $\text{Apparent(VAs)} = S_\emptyset = |V_{rms}||I_{rms}|$

3-phase power: $\text{Real(Watts)} = P_{3\emptyset} = 3P_\emptyset$ $\text{Reactive(VARs)} = Q_{3\emptyset} = 3Q_\emptyset$
$\text{Apparent(VAs)} = S_{3\emptyset} = 3S_\emptyset$

3-phase power from line quantities: Real(Watts) $= \mathrm{P}_{3\emptyset} = \sqrt{3}|\mathrm{V_L}||\mathrm{I_L}|\cos\theta$
Reactive(VARs) $= \mathrm{Q}_{3\emptyset} = \sqrt{3}|\mathrm{V_L}||\mathrm{I_L}|\sin\theta$ Apparent(VAs) $= \mathrm{S}_{3\emptyset} = \sqrt{3}|\mathrm{V_L}||\mathrm{I_L}|$

Total complex power in volt-amperes(VAs):

$$\mathbf{S} = \mathbf{S}_{3\emptyset} = 3\mathbf{S}_{\emptyset} = 3(\mathrm{P}_{\emptyset} + j\mathrm{Q}_{\emptyset}) = 3\mathbf{V}_{\mathrm{rms}}\mathbf{I}_{\mathrm{rms}}^{*} = 3|\mathbf{I}_{\mathrm{rms}}|^2\mathbf{Z} = \frac{3|\mathbf{V}_{\mathrm{rms}}|^2}{\mathbf{Z}^*}$$

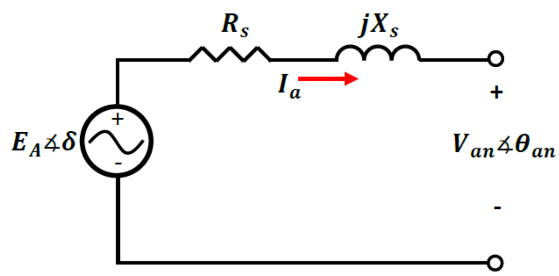
The complex power angle is equal to the angle of the impedance.

Power Factor Correction
Eliminate the Reactive power seen by the source by placing a reactive component in parallel with the load.

$$\mathrm{Q}_{\emptyset} = \frac{|\mathbf{V}_{\mathrm{rms}}|^2}{X} = -|\mathbf{V}_{\mathrm{rms}}|^2\omega\mathrm{C} = \frac{|\mathbf{V}_{\mathrm{rms}}|^2}{\omega\mathrm{L}}$$

AC Synchronous Machines
Rotor spins, has slip rings and field windings, and is DC (direct current) with a constant magnetic field.

Stator is stationary, has 3-phase windings, and is AC (alternating current) with a rotating magnetic field.



$\omega_e = 2\pi f_e = \frac{P}{2}\omega_m$, where P is the number of poles
$N_m = \frac{120 f_e}{P}$, where N_m is the rotational velocity of the prime mover in rpm.

Excitation voltage:

$$\mathbf{E}_a = \mathbf{V}_{\mathrm{an}} + \mathbf{I}_{\mathrm{a}}(\mathrm{R}_s + j\mathrm{X}_s)$$

Torque from the prime mover $\tau_{dev} = \frac{\mathrm{P}_{mech}}{\omega_m}$