# Final Exam Solutions

## 16-311: Introduction to Robotics

## 2019

Name: _____

Andrew ID: _____

Team Number: _____

- You will have 1 hour and 30 minutes to complete this exam

- There are 10 sections on 30 pages. Make sure you have all of the pages. Write your Andrew ID on all the sections and keep your work in that section (they will be graded separately). There are blank pages throughout the sections.

- There are 86 points available on the exam. This test will count for 25% of your final grade.

- When making drawings - be precise. Rounded edges should look rounded, sharp edges should look sharp, sizes should be close to scale. Neatness counts.

- Show your work. Partial credit may apply. Likewise, justify algebraically your work to ensure full credit, where applicable.

- It should be very clear what your final answer is, circle it if necessary.

- You may need to make certain assumptions to answer a problem. State them (e.g. what is optimal).

- You are allowed one handwritten two-sided reference sheet for the exam. No cell phones, laptops, calculators, neighbors, etc. allowed.

- Good luck and you can do it.

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# 1   Vision

Andrew ID: _____

1. Convolve the following section of an image with the given mask. The edges have been padded with 0s for you. [2 points]

| 1 | 0 | 0.5 | 1 | 0 | 0 |
|---|---|-----|---|---|---|

Figure 1: Image.

| 0.3 | 0.4 | 0.3 |
|-----|-----|-----|

Figure 2: Mask.

| 0 | 0.45 | 0.5 | 0.55 | 0.3 | 0 |
|---|------|-----|------|-----|---|

Figure 3: Your solution.

2. Come up with any 3x3 mask that would be able to detect only vertical edges. [2 points]

| -1 | 0 | 1 |
|----|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |

Figure 4: Your mask.

Or something of similar form. Or [-1 2 -1; -1 2 -1; -1 2 -1] or something of similar form.

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# 2  Machine Learning

Andrew ID: _____

1. Label the following items in the neural net diagram below: input layer, output layer, hidden layer. Be clear with what you are referring to. [2 points]
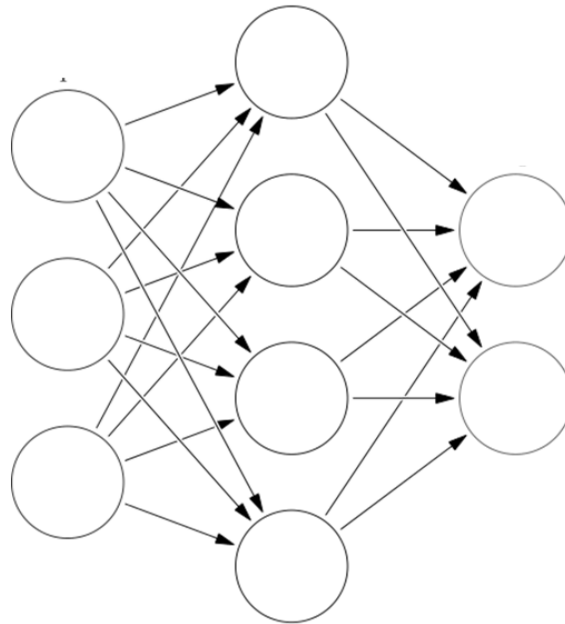


Figure 5: Neural Net Diagram.

3 nodes in left column are input. 4 nodes in center are hidden. Two nodes on right are output.

2. In one to two sentences, what is the difference between supervised and unsupervised learning? [2 points]

Supervised learning uses labeled data. Unsupervised earning uses unlabeled data.

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# 3 Control

Andrew ID: _____

You have a self-balancing robot that uses PID to control its tilt based on sensor information from an IMU (sensor that detects orientation). Diagnose the following issues. Explain what is likely wrong about the tuning and how you would fix it. [2 points]

1. The robot takes a reasonable amount of time to settle. Then it is stable for a few seconds. Then it begins to oscillate again with bigger and bigger oscillations. [2 points]

   Ki is too high and is causing ramp up. To fix this, reduce Ki.

2. The robot take a reasonable amount of time to rise but an extremely long time to settle. [2 points]

   The system is underdamped. Either d is too low or p is too high. To fix this, increase d or decrease p. Watch what happens to the rise time, may need to alter the other value as well to keep rise time.

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# 4    Path Planning

Andrew ID: _____

1. Including start and goal, number the following grid based on the wavefront planner using 4-point connectivity. The robot will go to the next lowest number from the grid space it is currently on, so make sure your numbering allows for this. [2 points]
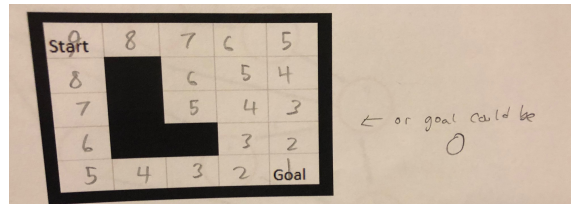


Figure 6: Configuration space.

2. In the following configuration space, draw the shortest path from start to goal with respect to the L2 distance traveled. [3 points]
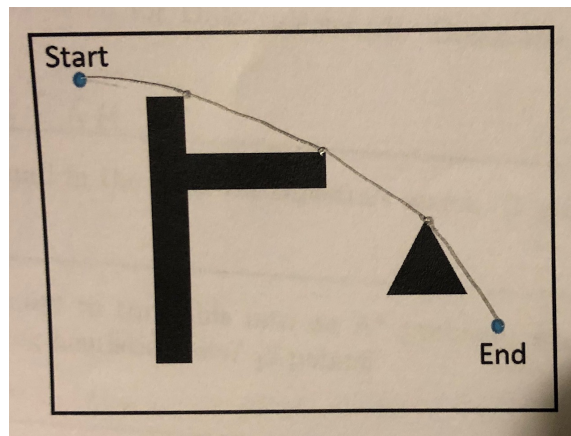


Figure 7: Configuration space.

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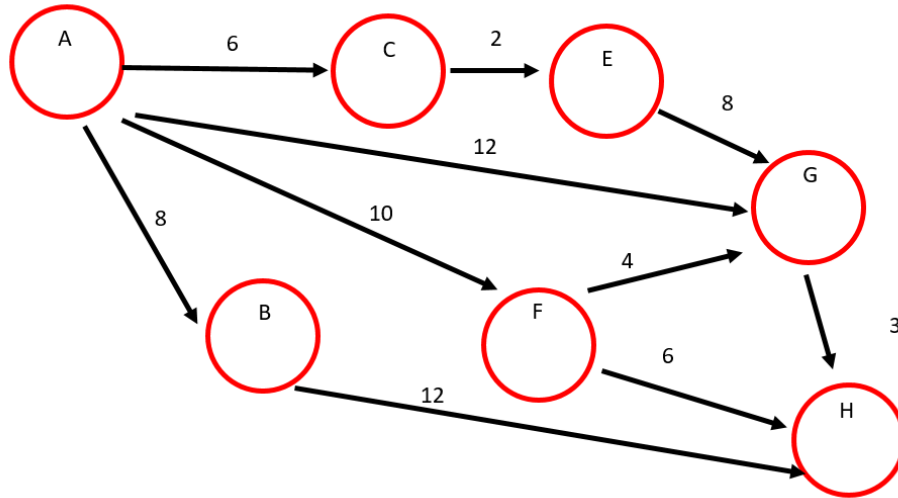# 5 Graph Search

Andrew ID: _____



Figure 8: A* graph. You will plan a path from Node A to Node B

1. List the nodes expanded for Dijkstra's search. Break ties alphabetically. [2 points]

   <u>ACBEFGH</u>

2. List the nodes found in the path via Dijkstra's search. [1 point]

   <u>AGH</u>

3. If we instead wanted to turn a problem like this into an A* problem, what would be one method of assigning heuristic costs? [2 points]

   <u>L2 distance from node in question to goal, etc.</u>

4. Which of the following would be an acceptable heuristic cost for node E? Circle all that apply. [2 points]

   1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, (Not 13, 14, 15, 16)

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# 6  Localization

Andrew ID: _____

1. For two fair 6-sided dice, what is the probability of rolling both ones? [1 point]

   1/36

2. For two fair 6-sided dice, what is the probability of rolling at least a single one? [1 point]

   11/36

3. Bayes' Theorem is used to find the conditional property of event A given B when we know the conditional probability of B given A, and the independent probabilities of both events A and B on their own. Write the formula below: [2 points]

   $P(A|B) =$ P(B—A)P(A)/P(B)

4. What are the two models that we combined for the Localization Lab (Lab 6)? [2 points]

   Sensor model and motion model (aka observation and transition model).

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# 7 Vehicle Design and Human Robot Interaction

Andrew ID: _____

1. Name one pro and one con of each of the following vehicles: [4 points]



Figure 9: Vehicle 1 (left). Vehicle 2 (right).

Pro Vehicle 1: Easier to make, higher ground clearance, lower resistance to turning

Con Vehicle 1: Higher surface pressure

Pro Vehicle 2: Lower surface pressure, higher stability

Con Vehicle 2: More losses, louder, less efficient

2. The horizontal bar is fixed. How many degrees of freedom does the following closed chain have? [1 point]
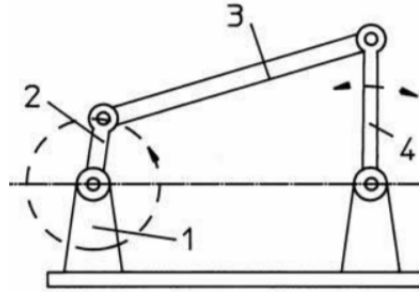
<u>1</u>



Figure 10: 4 bar linkage.

3. Describe one example of good HRI and one example of bad HRI for your USAR robot.[2 points]

<u>Any reasonable answer</u>

Choose one of the following two questions to answer. If you answer both, the first will be graded. [2 points]

4. David Kohanbash and Joey Wood came in to discuss the DARPA Subterranean challenge. what is the purpose of this program? Answer in one sentence

   From DARPA: "The DARPA Subterranean (SubT) Challenge aims to develop innovative technologies that would augment operations underground. The SubT Challenge will explore new approaches to rapidly map, navigate, search, and exploit complex underground environments, including human-made tunnel systems, urban underground, and natural cave networks."

5. Reuben Aaronson also discussed his on work with manipulation. How did Reuben try to make manipulation easier for the user? Answer in one sentence.

   Reuben first implemented trajectory prediciton based on movement and then trajectory prediction based on where the person was looking

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# 8 Forward Kinematics

Andrew ID: _____

1. We have a PRR arm as shown below. The first joint is a prismatic joint starting the origin and continuing up along the y axis. It is of length $s_1$ and it is limited from 5 to 10 grid spaces. The second and third joints are revolute joints with angles defined relative to the previous link where 0 is in line with the previous link and angles increasing counter-clockwise. The second link is of length 7. The third link is of length 5. The revolute joints ($\theta_2$ and $\theta_3$) are constrained between 0 and 180 degrees. The angle of the end effector is with respect to the positive x axis.
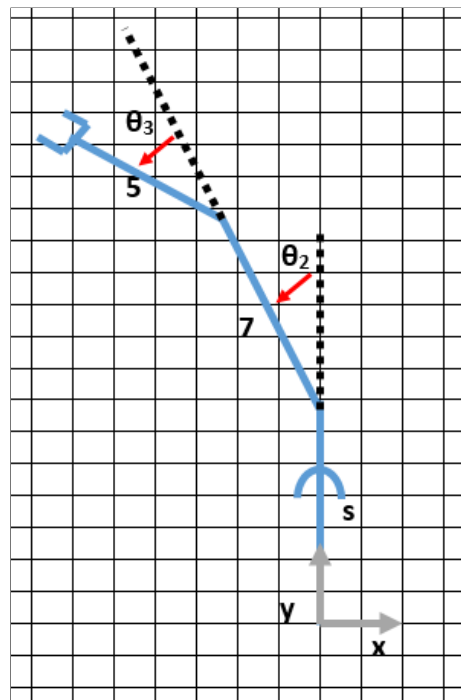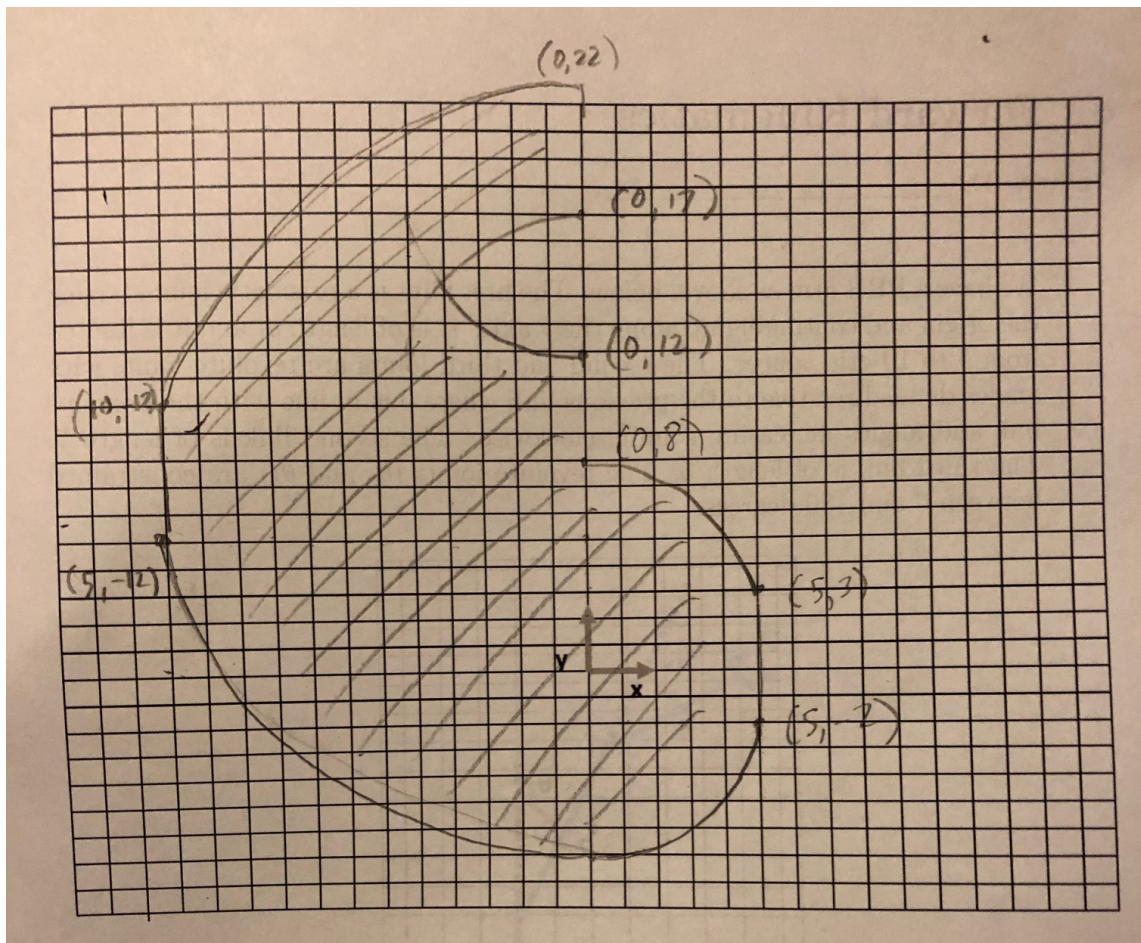


Figure 11: PRR arm.

Draw the shape of the $x$ and $y$ coordinates that the end effector can reach with the given joint limits. Be clear. Corners should be accurate to the grid lines. [6 points]

(0,22)

(0,17)

(0,12)

(0,8)

(10,12)

(5,-12)

(5,3)

(5,-3)

y

x

2. In terms of $s$, $\theta_2$ and $\theta_3$, write an equation for the $x$, $y$ and $\theta$ of the end effector. [6 points]

$x =$ -7sin(th2)-5sin(th2+th3)

$y =$ 7cos(th2)+5cos(th2+th3(+s

$\theta =$ pi/2+th2+th3

3. Write down the homogeneous transformation matrix to describe the following relative transformation: [1 point]



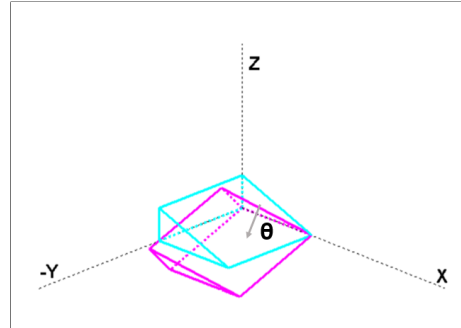Figure 12: Transformation 1.

$H_1 = [1\ 0\ 0\ 0;\ 0\ \cos(\text{th})\ -\sin(\text{th})\ 0;\ 0\ \sin(\text{th})\ \cos(\text{th})\ 0;\ 0\ 0\ 0\ 1]$

4. Write down the homogeneous transformation matrix to describe the following relative transformation: [1 point]
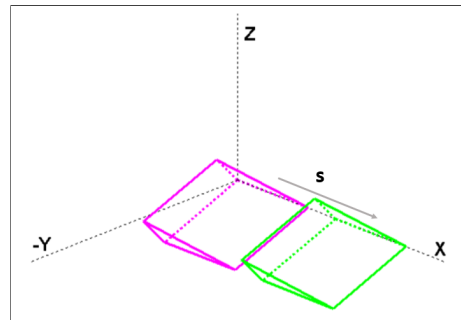


Figure 13: Transformation 2.

$H_2 = [1\ 0\ 0\ \text{s};\ 0\ 1\ 0\ 0;\ 0\ 0\ 1\ 0;\ 0\ 0\ 0\ 1]$

5. How would we combine the above matricies to describe the first transformation followed by the second? Write your answer in terms of $H_1$ and $H_2$. [1 point]

$H_{both} = \text{H1H2}$

# 9 Inverse Kinematics

Andrew ID: _____

We will use the same PRR arm for this question. For your convenience, the description and image is copied below:

We have a PRR arm as shown below. The first joint is a prismatic joint starting the origin and continuing up along the y axis. It is of length $s_1$ and it is limited from 5 to 10 cm. The second and third joints are revolute joints with angles defined relative to the previous link where 0 is in line with the previous link and angles increasing counter-clockwise. The second link is of length 7. The third link is of length 5. The revolute joints ($\theta_2$ and $\theta_3$) are constrained between 0 and 180 degrees. The angle of the end effector is with respect to the positive x axis.
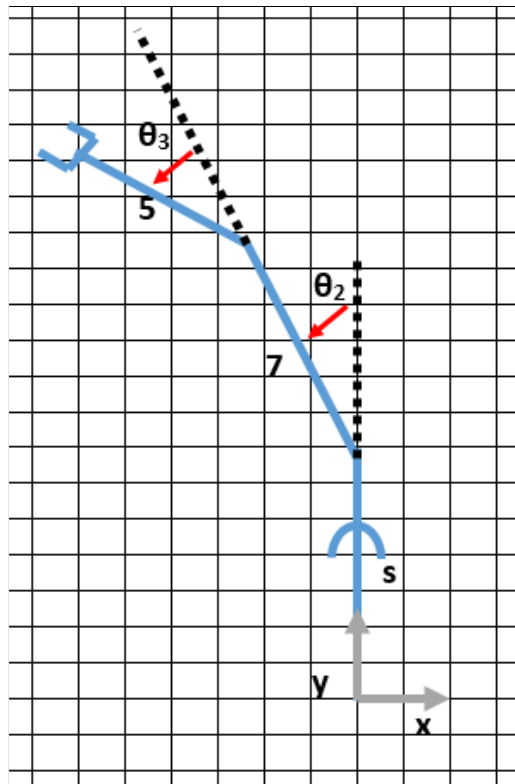


Figure 14: PRR arm.

1. How many solutions are there to place the end effector at (0,22)? [1 point]

   <u>1</u>

2. How many solutions are there to place the end effector at (7,1)? [1 point]

   <u>0</u>

3. What is one possible solution for (-8,13)? You do not have to simplify your answer, but each variable should be written in the following format. [6 points]

   Answer: Let s = some number. We will use l so that it is easier to check the student's solutions. Let $y' = y - l$. Let $thf = atan2(8, y') = acos(y'/(sqrt(8^2 + y^2)) = asin(8/(\sqrt{(8^2+y^2)})) = -atan2(-8, y)$. From LOC, $alpha = acos(\frac{8^2+y'^2-5^2-7^2}{2*5*7})$. From LOC, $beta = acos(\frac{5^2-8^2-y'^2-7^2}{2*7*\sqrt{(8^2+y^2)}})$ or, by LOS, $beta = asin(5*sin(alpha)/\sqrt{(8^2+y^2)})$

   $s = \underline{l}$

   $\theta_2 = $ <u>thf-beta</u>

   $\theta_3 = $ <u>pi-alpha</u>

4. Now we have a RPR arm with angles described from the horizontal. The prismatic link must always be vertical. Find $\theta_1$, $s$, and $\theta_3$ in terms of $(x, y,$ and $\theta)$ of the end effector: [6 points]



Figure 15: RPR arm.

$\theta_1 = \underline{\text{acos}((\text{x-5cos(th)})/7)}$

$s = \underline{\text{y-7sin(th1)-5sin(th3)}}$

$\theta_3 = \underline{\text{th}}$
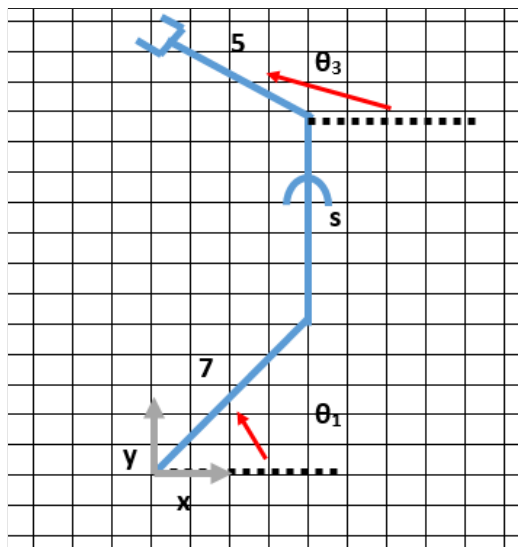
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# 10    Nonholonomic Constraints
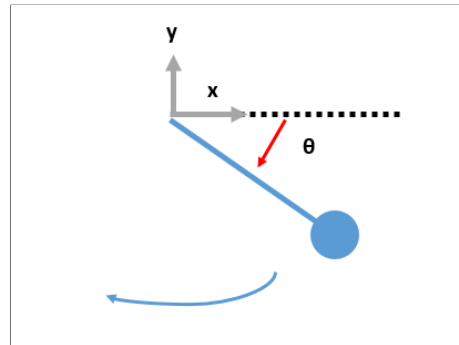
Andrew ID: _____



Figure 16: Pendulum.

1. For a simple pendulum, what is the constraint on the velocity of the weight? Use $q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ as the state vector, the pendulum weight as the reference point, and the depicted definition of $\theta$. [1 point]

   $w_1 = \underline{[\text{costh -sinth } 0]}$

2. Is this constraint a holonomic constraint or a nonholonomic constraint? Explain in one sentence. [2 points]

   Holonomic. It can be integrated. A constraint on velocity and position.
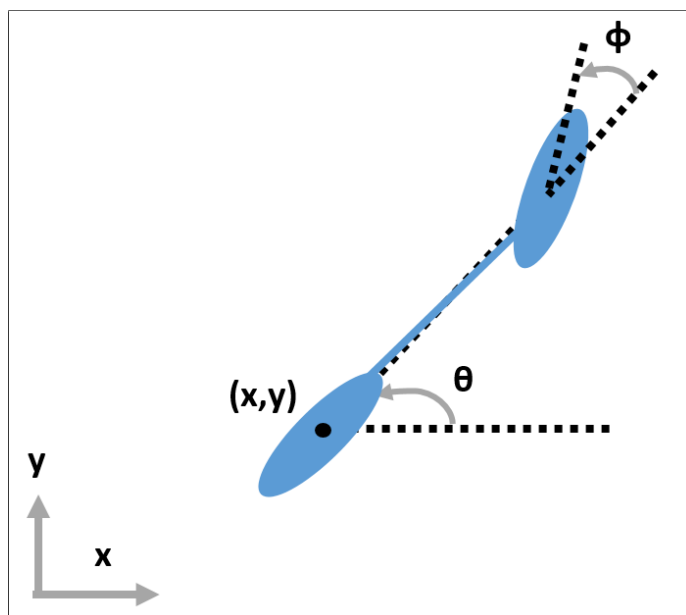
Figure 17: Bicycle.

3. For a bicycle, what are the constraints on the vehicle's velocity? Use $q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix}$ as the state vector and the depicted reference point and angle definitions. Indicate any additional variables that you name in the image. [4 points]

$w_1 = $ [-sinth costh 0 0]          $w_2 =$[-sin(th+phi cos(th+phi) lcos(phi) 0]

4. What are the allowable motions? [4 points]

$g_1 = $ [0; 0; 0; 1]          $g_2 = $ [costh; sinth; tan(phi)/l; 0]

5. What is the formula for Lie Bracket? [1 point]

[g1, g2]=partial(g2)g1-partial(g1)g2

6. What is the Lie Bracket of $g_1$ and $g_2$? Show your work. [4 points]

[0; 0; sec(phi)*sec(phi)/l; 0]

7. What does the result of this Lie bracket mean? Explain in one sentence. [2 points]

The cart can position at any angle.

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END OF EXAM.