

**ME309****Homework #5****Final project: Design problem**

Select **one** of the analysis problems listed below to solve. Your solution, along with a description of your analysis process, should be handed in as a final report. For the report, **follow the format shown in Table 1.**

**Possible Problems / Projects:**

1. Transient thermal-stress study
2. Prediction of stress intensity factors
3. Bar element finite element code
4. Truss bridge design, analysis and testing (2 students per team)
5. Your own design project

**Resources:**

1. References provided and listed below
2. ANSYS on-line manuals  
[http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp\\_E\\_CH1.html#aVL8sq1fcldm](http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp_E_CH1.html#aVL8sq1fcldm)
3. ANSYS verification manual  
[http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp\\_V\\_VMT\\_OC.html](http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp_V_VMT_OC.html)
4. ANSYS tutorials  
<http://www.mece.ualberta.ca/tutorials/ansys/index.html>
5. All course materials, tutorials, notes, and modules
6. Teaching staff, project coaches and classmates

Final projects are due on Friday, June 05, 12pm. They can be dropped off at the box in front of Durand 217. Late projects will not be accepted.

**Table 1: Table of Contents for an Analysis Report  
(guidelines for Homework #5 Report):**

**1. Problem Description:** Description of the objectives of the analysis. Describe the failure criteria or engineering requirements against which the analysis will be compared. Include a physical description of the part to be analyzed. The overall dimensions, material, loading conditions, and description of the operation or application of the part should be included. Obviously, a sketch of the part is helpful.

**2. FEA Code:** Brief summary of the finite element program and computer system used for the analysis (this section might be omitted for an in-house report, but not here).

**3. Model Description and Assumptions:** Include plots of the finite element model and a description of types of elements used, boundary conditions, applied loads and relevant engineering assumptions.

**4. Results and Analysis:** Include the important results (displacement, mode shape, thermal, and/or stress contour plots). A discussion should be accompanied these plots, describing the behavior of the model and how it relates to the actual expected behavior of the part. Include tables showing the stresses and displacements (if structure analysis) for critical sections of the model. Include hand calculations, theoretical solutions and/or experimental results supporting the finite element results. A brief discussion of these calculations along with references should be included. For long results or discussions that detract from the flow of the main report, include parts of this section as an appendix to the main body of the report.

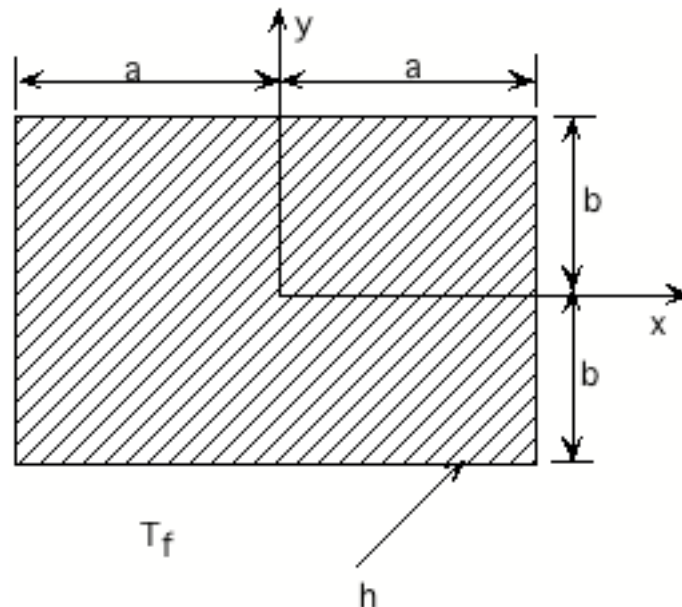
**5. Conclusions and recommendations:** Describe what was learned from the analysis and what conclusions can be drawn. Summarize the results in conjunction with the failure criteria or engineering requirements. If the analysis shows an inadequate design, recommendation for design modifications would be included in this section.

The importance of thorough documentation (and judicious use of appendices) cannot be overemphasized. First, documentation is required to support the design or analysis decisions resulting from the finite element analysis. Second, and of equal importance, the process of preparing the report forces you to check all aspects of your analysis. Even if your work situation does not require a formal report, it is strongly recommended that you go through the process described above as a means of checking your analysis. Reports should include enough detail that an experienced analyst could completely reproduce your results from reading the report alone.

### 1. Transient thermal-stress study.

A long metal bar of rectangular cross-section is initially at a temperature  $T_o$  and is then suddenly quenched in a large mass of fluid at temperature  $T_f$ . The material conductivity is anisotropic, having different X and Y directional properties and the surface convection coefficient is  $h$ . The bar is made out of steel.

- Determine using finite element analysis the temperature distribution in the slab after 3 seconds if the block is totally immersed in the fluid. Compare with an analytic solution (see reference below).
- Determine the stresses in the bar at three seconds, assuming all elastic behavior.
- Determine, using finite element analysis, the temperature distribution in the slab after 3 seconds if the immersed in the fluid along its bottom edge. All other boundary surfaces can be assumed to be adiabatic.
- Determine the stresses in the bar in (c) at three seconds, assuming all elastic behavior. Also, determine the curvature (distortion) of the bar about the x-axis, where curvature is the maximum deflection in the y-direction for this case. The bar extends 10 in. in the z-direction.



**Given:**  $k_x=20$  Btu/hr-ft-°F,  $k_y=k_z= 5.0$  Btu/hr-ft-°F,  $\rho=400$  lb/ft<sup>3</sup>,  $c=0.009009$  Btu/lb-°F,  $T_o=500^\circ\text{F}$ ,  $T_f=100^\circ\text{F}$ ,  $h=240$  Btu/hr-ft<sup>2</sup>-°F, with  $E_{\text{steel}}=26,875,000$  psi,  $\nu_{\text{steel}}=0.27$ ,  $\alpha_{\text{steel}}=6.5\text{e-}6/\text{degree F}$ ,  $a=2$  in= $0.166666$  ft,  $b=1$  in= $0.083333$  ft.

**References:** Schneider, P.J., "Conduction Heat Transfer," Addison-Wesley Publishing Co., Inc., Reading, Mass., 2nd Printing, 1957, Pg. 261, Example 10-7.

[http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp\\_G\\_TheTOC.html](http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp_G_TheTOC.html)

113.1

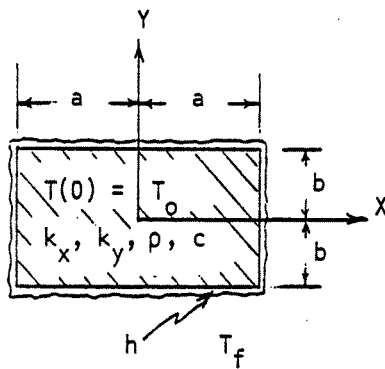
## VERIFICATION PROBLEM NO. 113

**TITLE:** Transient Temperature Distribution in an Orthotropic Metal Bar.

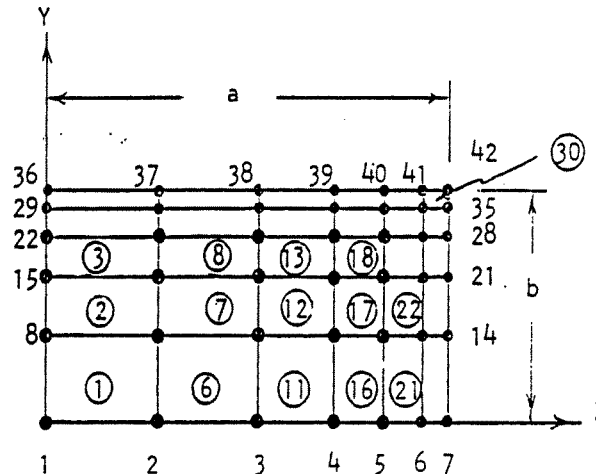
**TYPE:** Heat Transfer analysis (KAN=-1), conducting elements (STIF55).

**REFERENCE:** Schneider (Ref. 14), Page 261, Example 10-7.

**PROBLEM:** A long metal bar of rectangular cross-section is initially at a temperature  $T_0$  and is then suddenly quenched in a large mass of fluid at temperature  $T_f$ . The material conductivity is orthotropic, having different X and Y directional properties. If the surface convection coefficient is  $h$ , determine the temperature distribution in the slab after 3 seconds.



Problem Sketch



Finite Element Model

**GIVEN:**  $k_x = 20$  BTU/hr-ft-°F,  $k_y = 3.6036$  BTU/hr-ft-°F,  $\rho = 400$  lb/ft<sup>3</sup>,  
 $c = 0.009009$  BTU/lb-°F,  $T_0 = 500^\circ\text{F}$ ,  $T_f = 100^\circ\text{F}$ ,  $h = 240$  BTU/hr-ft<sup>2</sup>-°F,  
 $a = 2$  in = 0.166666 ft,  $b = 1$  in = 0.083333 ft.

**NON-UNIFORM GRID:** The grid spacing is automatically calculated by the program based upon input requiring that the spacing between the last two nodes be 1/3 the spacing between the first two nodes, in each direction.

**MODELING HINTS:** A non-uniform grid (based on a geometric progression) is used in both X and Y directions. The transient time step optimization procedures are used. The initial integration time step ( $0.00083333/20 = 0.000041666$  hr) is based on  $\approx \delta^2/4\alpha$ , where  $\delta$  is the shortest element length (0.0089 ft) and  $\alpha$  is the thermal diffusivity ( $k/\rho c = 1.0$  ft<sup>2</sup>/hr).

113.2

## VERIFICATION PROBLEM NO. 113 (Continued)

## INPUT DATA LISTING:

```

/PREP7
/TITLE, VM113, TRANSIENT TEMP. DIST. IN AN ORTHOTROPIC METAL BAR
C*** CONDUCTION HEAT TRANSFER, SCHNEIDER, 2ND. PRINTING, PAGE 261, EX. 10-7
KAN, -1
ET, 1, 55
KXX, 1, 20
KYY, 1, 3.6036
DENS, 1, 400
C, 1, .009009
N, 1
N, 7, .1666666666
FILL, . . . . . 333333333
NGEN, 6, 7, 1, 7, 1, . . . 0166666 . . . 333333333
E, 2, 9, 8, 1
EGEN, 5, 7, 1
EGEN, 6, 1, 1, 5
ITER, -20, 1
TIME, .00083333
TUNIF, 500
KBC, 1
CV, 36, 37, 240, 100, 41
, 7, 14, 240, 100, 35, 7
AFWRITE
FINISH
/INPUT, 27
FINISH

```

## SOLUTION COMPARISON:

Time = 0.0008333 hr (3 sec):

	Node 1	Node 7	Node 36	Node 42
	T, °F	T, °F	T, °F	T, °F
Theory*	459.5	279.0	202.0	150.8
ANSYS	464.0	286.0	200.8	154.2
Difference**	≈ None	≈ None	≈ None	≈ None

\* Based on graphical estimates.

\*\* The accuracy is within the graphical read-out range.

EXAMPLE 10-7. A long metal bar of rectangular cross section  $4'' \times 2''$  ( $x_1 = \delta_1 = 2''$ ,  $y_1 = \delta_2 = 1''$ ) is heated to a uniform temperature of  $500^\circ\text{F}$ , and then suddenly quenched in a large mass of fluid at  $100^\circ\text{F}$ . The bar material

is anisotropic, with thermal conductivities in the  $x$ - and  $y$ -directions of  $k_x = 20$  and  $k_y = 5$  Btu/hr-ft- $^\circ\text{F}$ , and thermal diffusivities of  $\alpha_x = 5.55$  and  $\alpha_y = 1.00$  ft<sup>2</sup>/hr. If the unit surface conductance during the quenching process is estimated to be a uniform  $240$  Btu/hr-ft<sup>2</sup>- $^\circ\text{F}$ , then: (a) What is the central temperature in the bar after 3 sec of quenching? (b) What is the surface temperature  $t_1$  at the center of its short face ( $x = \delta_1$ ,  $y = 0$ ), the center of its long face ( $x = 0$ ,  $y = \delta_2$ ), and at its edge ( $x = \delta_1$ ,  $y = \delta_2$ )?

*Solution.* In this case the two-dimensional temperatures are computed as products of one-dimensional solutions, as in Article 10-10, and the anisotropic property dealt with by computing separate Nusselt numbers and Fourier moduli in the two directions as

$$(1/N_u)_x = \frac{k_x}{h\delta_1} = \frac{20 \times 12}{240 \times 2} = 0.50,$$

$$(1/N_u)_y = \frac{k_y}{h\delta_2} = \frac{5 \times 12}{240 \times 1} = 0.25,$$

and

$$\Theta_x = \frac{\alpha_x \theta}{\delta_1^2} = \frac{5.55(3/3600)}{(2/12)^2} = 0.166,$$

$$\Theta_y = \frac{\alpha_y \theta}{\delta_2^2} = \frac{1.00(3/3600)}{(1/12)^2} = 0.120.$$

(a) Since the  $\Theta$ 's are less than 0.2, we will have to use the short-time charts. Entering the infinite-plate curves of Fig. 10-11(c), we find that the central temperature for an infinite plate in the  $y$ -direction from the ordinate corresponding to  $(1/N_u)_x$  and  $\Theta_x$

$$\left(\frac{T_0}{T_i}\right)_x = 1 - 0.028N_{ux} = 0.944,$$

and in like fashion for an infinite plate in the  $x$ -direction,

$$\left(\frac{T_0}{T_i}\right)_y = 1 - 0.042 = 0.958.$$

In this way

$$\frac{t_0 - 100}{500 - 100} = 0.944 \times 0.958 = 0.904,$$

or  $t_0 = 462^\circ\text{F}$ .

(b) For the surface temperature  $t_{x,y} = t_{1,0}$ , we use the product of  $T_1/T_i$  for an infinite plate in the  $y$ -direction and  $T_0/T_i$  for an infinite plate in the  $x$ -direction. Entering the curves for  $x/\delta_1 = 1$  in Fig. 10-11(c), we find

$$\left(\frac{T_1}{T_i}\right)_x = 1 - 0.530 = 0.470,$$

so that

$$\frac{t_{1,0} - 100}{500 - 100} = 0.470 \times 0.958 = 0.450,$$

or  $t_{1,0} = 280^\circ\text{F}$ .

10-15] SEMI-INFINITE SOLID 263

By the same procedure for the surface temperature  $t_{x,y} = t_{0,y}$ , we find

$$\left(\frac{T_1}{T_i}\right)_y = 1 - 0.670 = 0.330$$

so that

$$\frac{t_{0,y} - 100}{500 - 100} = 0.944 \times 0.330 = 0.312,$$

or  $t_{0,y} = 225^\circ\text{F}$ .

For the edge temperature  $t_{x,y} = t_{1,y}$ , we use the product of the surface solutions,

$$\frac{t_{1,y} - 100}{500 - 100} = 0.470 \times 0.330 = 0.155,$$

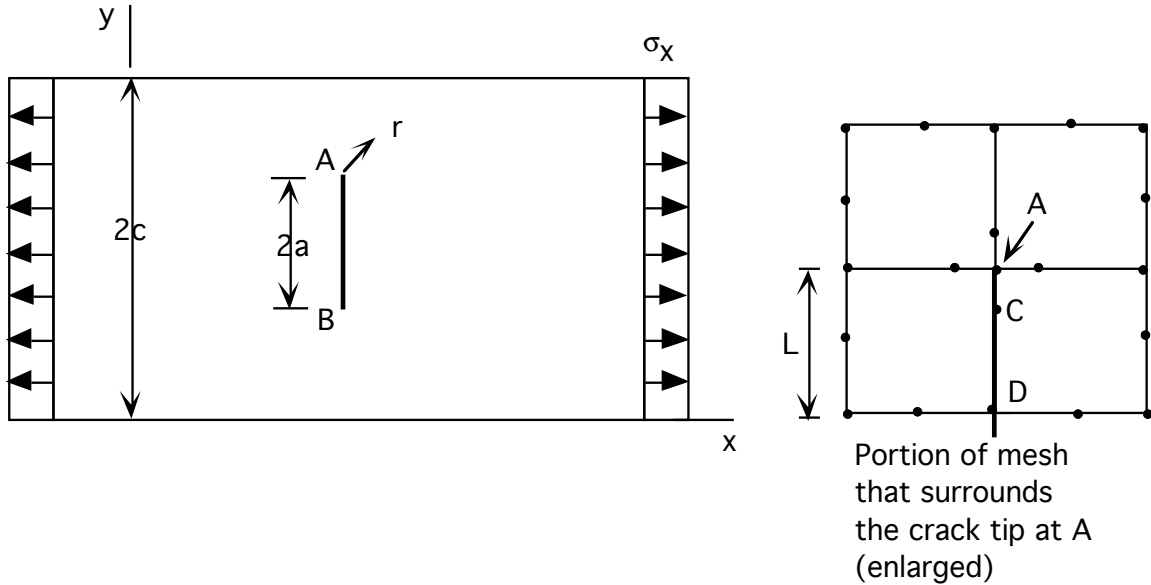
whereby  $t_{1,y} = 162^\circ\text{F}$ .

Schneider

## 2. Prediction of stress intensity factors.

[http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp\\_G\\_StrTOC.html](http://www1.ansys.com/customer/content/documentation/80/ansys/Hlp_G_StrTOC.html)

The sketch shows a central crack of length  $2a$  in a flat strip of material whose width is  $2c$ .



If side nodes of isoparametric elements are moved to quarter points in the manner shown, stresses vary as  $r^{-0.5}$  along certain radial lines. The  $r^{-0.5}$  variation is consistent with the theory of linear fracture mechanics. The mode I stress intensity factor  $K_I$  can be computed as:

$$K_I = \frac{2G}{\kappa + 1} \left( \frac{\pi}{2L} \right)^{0.5} [4\Delta_C - \Delta_D]$$

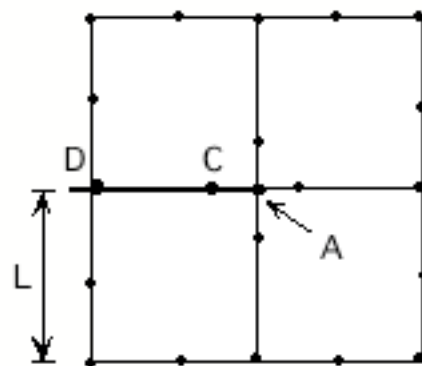
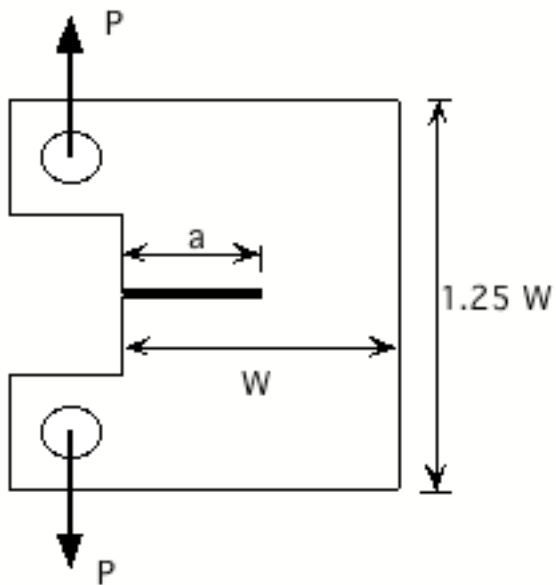
where  $G$  is the shear modulus,  $\kappa = (3 - \nu)/(1 + \nu)$  for plane stress conditions, or  $\kappa = 3 - 4\nu$  for plane strain conditions, and  $\Delta_C$  and  $\Delta_D$  are the amounts of crack opening at  $C$  and  $D$ . A handbook gives a formula for  $K_I$ :

$$K_I = \sigma_x \sqrt{\pi a} \frac{1 - 0.5(a/c) + 0.326(a/c)^2}{[1 - a/c]^{0.5}}$$

- (a) Assign convenient dimensions for the center cracked specimen, complete the FEA model, do computations, and compare the computed and formula values of  $K_I$ . Also, use alternative methods for calculation of  $K_I$  if the software provides them.
- (b) Repeat (a) for the compact specimen (figure below). A handbook gives a formula for  $K_I$  for a compact specimen (as shown below) :

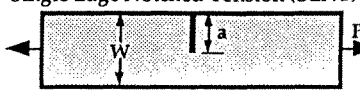
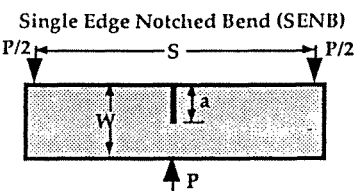
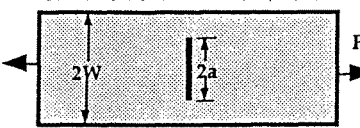
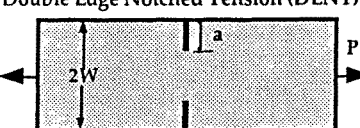
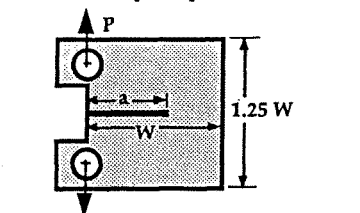
$$K_I = \frac{P}{B\sqrt{W}} \left[ \frac{2 + \frac{a}{W}}{\left(1 - \frac{a}{W}\right)^{3/2}} \right] \left[ 0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.60\left(\frac{a}{W}\right)^4 \right]$$

where B is the specimen thickness.



Portion of mesh  
that surrounds  
the crack tip at A  
(enlarged)

### Some Useful Solutions:

GEOMETRY	$f(a/W)^*$
<b>Single Edge Notched Tension (SENT)</b> 	$\frac{\sqrt{2 \tan \frac{\pi a}{2W}}}{\cos \frac{\pi a}{2W}} \left[ 0.752 + 2.02 \left( \frac{a}{W} \right) + 0.37 \left( 1 - \sin \frac{\pi a}{2W} \right)^3 \right]$
<b>Single Edge Notched Bend (SENB)</b> 	$\frac{3 \frac{S}{W} \sqrt{\frac{a}{W}}}{2 \left( 1 + 2 \frac{a}{W} \right) \left( 1 - \frac{a}{W} \right)^{3/2}} \left[ 1.99 - \frac{a}{W} \left( 1 - \frac{a}{W} \right) \left\{ 2.15 - 3.93 \left( \frac{a}{W} \right) + 2.7 \left( \frac{a}{W} \right)^2 \right\} \right]$
<b>Center Cracked Tension (CCT)</b> 	$\sqrt{\frac{\pi a}{4W} \sec \frac{\pi a}{2W}} \left[ 1 - 0.025 \left( \frac{a}{W} \right)^2 + 0.06 \left( \frac{a}{W} \right)^4 \right]$
<b>Double Edge Notched Tension (DENT)</b> 	$\frac{\sqrt{\frac{\pi a}{2W}}}{\sqrt{1 - \frac{a}{W}}} \left[ 1.122 - 0.561 \left( \frac{a}{W} \right) - 0.205 \left( \frac{a}{W} \right)^2 + 0.471 \left( \frac{a}{W} \right)^3 + 0.190 \left( \frac{a}{W} \right)^4 \right]$
<b>Compact Specimen</b> 	$\frac{2 + \frac{a}{W}}{\left( 1 - \frac{a}{W} \right)^{3/2}} \left[ 0.886 + 4.64 \left( \frac{a}{W} \right) - 13.32 \left( \frac{a}{W} \right)^2 + 14.72 \left( \frac{a}{W} \right)^3 - 5.60 \left( \frac{a}{W} \right)^4 \right]$

$*K_I = \frac{P}{B \sqrt{W}} f(a/W)$  where B is the specimen thickness.

Use Principle of Superposition to find additional solutions:

eg. Through Crack under Internal Pressure

$$\begin{array}{c} \uparrow \uparrow \uparrow \\ \downarrow \downarrow \downarrow \end{array} \quad (a) = \begin{array}{c} \uparrow \uparrow \uparrow \\ \downarrow \downarrow \downarrow \end{array} \quad (b) - \begin{array}{c} \uparrow \uparrow \uparrow \\ \downarrow \downarrow \downarrow \end{array} \quad (c) \quad \text{ie} \quad K_{(a)} = K_{(b)} - K_{(c)} \\
 = \sigma \sqrt{\pi a} - 0 \\
 = p \sqrt{\pi a}
 \end{array}$$

specimens and models apart and using a special etching process on the cut surfaces it is possible to reveal the interior regions which have yielded and thus obtain information regarding the flow of metal at the points of stress concentration.<sup>42</sup>

#### 64. Photoelastic Method of Stress Measurements.—

There are many stress analysis problems in which the deformation is essentially parallel to a plane. These are called *two-dimensional problems*. Illustrations are the bending of beams of a narrow rectangular cross section, bending of girders, arches, gear teeth, or, more generally, plates of any shape but of constant thickness acted on by forces or couples in the plane of the plate. Their shapes may be such that the stress distributions are very difficult to determine analytically and for such cases the *photoelastic* method has proved very

useful. In this method models cut out of a plate of an isotropic transparent material such as glass, celluloid or bakelite are used. It is well known that under the action of stresses these materials become *doubly refracting* and if a beam of *polarized light* is passed through a transparent model under stress, a colored picture may be obtained from which the stress distribution can be found.<sup>43</sup>

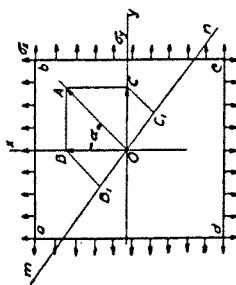


FIG. 217.

The application of the method in investigating stresses in machine parts was made by Dietrich and Lehr, V. D. I., Vol. 76, 1932. See also H. Kayser, "Bautechnik," 1936, and A. V. de Forest and Greer Ellis, Journal of the Aeronautical Sciences, Vol. 7, p. 205, 1940.

<sup>42</sup> See paper by A. Fry, Krupp'sche Monatshefte, 1921; also Stahl u. Eisen, 1921.

<sup>43</sup> The phenomenon of double refraction due to stressing was discovered by D. Brewster, Phil. Trans. Roy. Soc., 1816. It was further studied by F. E. Neumann, Berlin Abh., 1841, and by J. C. Maxwell, Edinburgh Roy. Soc. Trans., Vol. 20, 1853, and his Scientific Papers, Vol. 1, p. 30. The application of this phenomenon to the solution of engineering problems was started by C. Wilson, Phil. Mag. (Ser. 5), Vol. 32 (1891), and further developed by A. Mesnager, Annales des Ponts et Chaussées, 1901 and 1913, and E. G. Coker, General Electric

In Fig. 217 *abcd* represents a transparent plate of uniform thickness and *O* the point of intersection with the plate of a beam of polarized light perpendicular to the plate. Suppose that *OA* represents the plane of vibration of the light and that the length *OA* = *a* represents the amplitude of this vibration. If the vibration is considered to be simple harmonic, the displacements may be represented by the equation:

$$s = a \cos pt, \quad (a)$$

where *p* is proportional to the frequency of vibration, which depends on the color of the light.

Imagine now that the stresses  $\sigma_x$  and  $\sigma_y$ , different in magnitude, are applied to the edges of the plate. Due to the difference in stresses the optical properties of the plate also become different in the two perpendicular directions. Let  $v_x$  and  $v_y$  denote the velocities of light in the planes *ox* and *oy* respectively. The simple vibration in the plane *OA* is resolved into two components with amplitudes  $\overline{OB} = a \cos \alpha$  and  $\overline{OC} = a \sin \alpha$  in the planes *ox* and *oy* respectively, and the corresponding displacements are

$$x = a \cos \alpha \cos pt; \quad y = a \sin \alpha \cos pt. \quad (b)$$

If *h* is the thickness of the plate, the intervals of time necessary for the two component vibrations to cross the plate are

$$t_1 = \frac{h}{v_x} \quad \text{and} \quad t_2 = \frac{h}{v_y}, \quad (c)$$

and vibrations (b) after crossing the plate are given by the equations:

$$x_1 = a \cos \alpha \cos p(t - t_1); \quad y_1 = a \sin \alpha \cos p(t - t_2). \quad (d)$$

Co. Magazine, 1920, and Journal of Franklin Institute, 1925. For further development of the photoelastic method see the paper by Henry Favre, Schweizerische Bauzeitung, Vol. 20 (1927), p. 291; see also his dissertation: Sur une nouvelle méthode optique de détermination des tensions intérieures, Paris, 1929. The use of monochromatic light, so called "Fringe Method," was introduced by Z. Tuzi, "Inst. Phys. and Chem. Research," Vol. 8, p. 247, 1928.

These components have the phase difference  $p(t_2 - t_1)$ , due to the difference in velocities. Experiments show that the difference in the velocities of light is proportional to the difference in the stresses; hence

$$t_2 - t_1 = \frac{h}{v_v} - \frac{h}{v_z} = \frac{h(v_z - v_v)}{v_z v_v} \\ = \frac{h(v_z - v_v)}{v^2} \quad (\text{approximately}) = k(\sigma_z - \sigma_v), \quad (e)$$

where  $v$  is the velocity of light when the stresses are zero, and  $k$  is a numerical factor which depends on the physical properties of the material of the plate. We see that the difference of the two principal stresses can be found by measuring the difference in phase of the two vibrations. This can be done by bringing them into interference in the same plane. For this purpose a Nicol prism (called the *analyser*) is placed behind the plate in such a position as to permit the passage of vibrations in the plane  $mn$  perpendicular to the plane  $OA$  only. The components of the vibrations ( $d$ ), which pass through the prism, have the amplitudes  $\overline{OB}_1 = \overline{OB} \sin \alpha = (a/2) \sin 2\alpha$  and  $\overline{OC}_1 = \overline{OC} \cos \alpha = (a/2) \sin 2\alpha$ . The resultant vibration in the plane  $mn$  is therefore

$$\frac{a}{2} \sin 2\alpha \cos p(t - t_1) - \frac{a}{2} \sin 2\alpha \cos p(t - t_2) \\ = \left( a \sin 2\alpha \sin p \frac{t_1 - t_2}{2} \right) \sin p \left( t - \frac{t_1 + t_2}{2} \right). \quad (f)$$

This is a simple harmonic vibration, whose amplitude is proportional to  $\sin p[(t_1 - t_2)/2]$ ; hence the intensity of the light is a function of the difference in phase  $p(t_1 - t_2)$ . If the stresses  $\sigma_z$  and  $\sigma_v$  are equal,  $t_1$  and  $t_2$  are also equal, the amplitude of the resultant vibration ( $f$ ) is zero and we have darkness. There will be darkness also whenever the difference in stresses is such that

$$p \frac{t_1 - t_2}{2} = n\pi, \quad (g)$$

where  $n$  is an integer. The maximum intensity of light is obtained when the difference in stresses is such that

$$\sin p \frac{t_1 - t_2}{2} = \pm 1.$$

Imagine that instead of the element  $abcd$ , Fig. 217, we have a strip of a transparent material under simple tension. By gradually increasing the tensile stress we obtain a dark picture of the strip on the screen each time equation (g) is fulfilled. In this manner we can establish experimentally for a given material of a given thickness the stress corresponding to the interval between two consecutive dark pictures of the specimen. For instance, for one kind of "phenolite" plate, 1 mm. thick, this stress was found<sup>4</sup> to be 1,620 lbs. per sq. in. Hence for a plate  $1/4$  in. thick, the corresponding stress will be  $1,620/6.35 = 255$  lbs. per sq. in. With this information we can determine the stress in a strip under tension by counting the number of intervals between the consecutive dark images occurring during the gradual loading of the specimen. If we use a strip in pure bending, we obtain a picture such as is shown in Fig. 218. The parallel dark fringes indicate

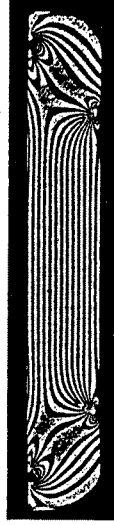


FIG. 218.

that in the portion of the strip at a considerable distance from the points of application of the loads the stress distribution is the same in all vertical cross sections. By counting the number of fringes we can determine the magnitudes of the stresses, as the stress difference between two consecutive fringes is the same as the stress difference between two con-

<sup>4</sup>Z. Tuzi, Sci. Papers, Inst. Phys. Chem. Research, Tokyo, Vol. 12, 1929, p. 247.

secutive dark images in simple tension. By watching the strip while the load is applied gradually, we may see how the number of dark fringes increases with increase of load. The new ones always appear at the top and the bottom of the strip and gradually move toward the neutral plane so that the fringes become more and more closely packed. The stress at any point is then obtained by counting the number of fringes which pass over the point.

The method of counting the number of dark fringes passing a chosen point can be used also for any plane stress distribution. As it is seen from our previous discussion, this number gives generally the difference between the two principal stresses at the point. For a complete determination of the stress at the point it remains then to find the directions of the principal stresses and their sum. Equation (f) shows that the intensity of the light passing through the analyzer is proportional to  $\sin 2\alpha$ , where  $\alpha$  is the angle between the plane of polarization and the plane of one of the principal stresses, Fig. 217. If these two planes coincide,  $\sin 2\alpha$  is zero and we obtain a dark spot on the screen. Hence in examining a stressed transparent model in polarized light we observe not merely the dark fringes discussed before but also dark lines connecting the points at which one of the principal stress directions coincides with the plane of polarization. By rotating both Nicol prisms, polarizer and analyzer, and marking dark lines on the image of the stressed plate for various directions of the plane of polarization, we obtain the system of so-called *isoclinic lines* which join together points with the same directions of principal stresses. Having these lines, we can draw the lines which are tangential at each point to the principal axes of stress. These latter lines are called the *trajectories* of the principal stresses, see p. 123, Part I. Thus the directions of the principal stresses at each point of the plate can be obtained experimentally.

The sum of the principal stresses can also be obtained experimentally by measuring the change  $\Delta h$  in the thickness

$h$  of the plate due to the stresses  $\sigma_x$  and  $\sigma_y$ ,<sup>46</sup> and using the known relation

$$\Delta h = \frac{\mu h}{E} (\sigma_x + \sigma_y). \quad (h)$$

Having the difference of the two principal stresses from the photo-elastic test and their sum from expression (h), we can readily calculate the magnitude of the principal stresses. The fringes obtained in a plate with fillets submitted to the action of pure bending are shown as an illustration in Fig. 219.

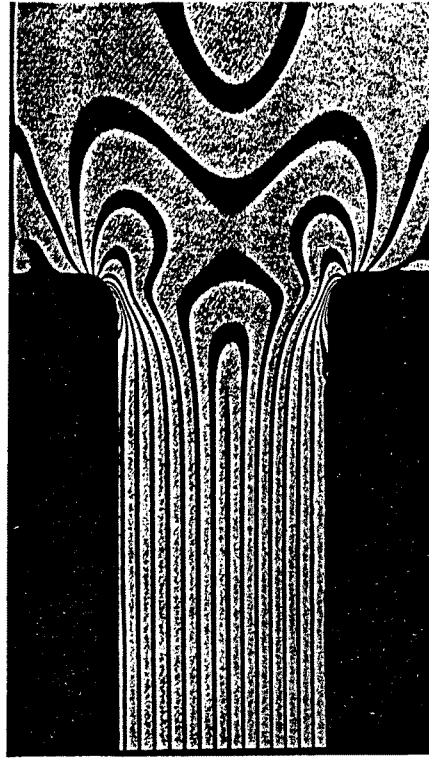


FIG. 219.

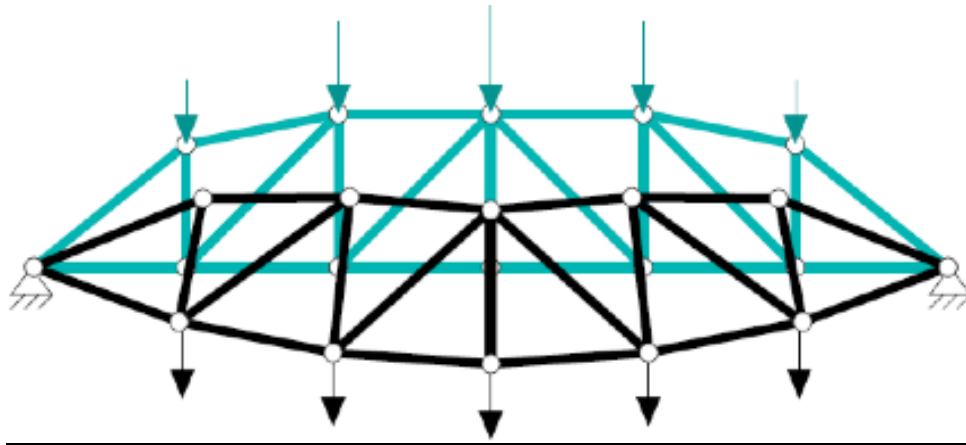
From the fact that the fringes are crowded at the fillets it may be concluded that a considerable stress concentration takes place at those points.

<sup>46</sup> This method was suggested by A. Mesnager, loc. cit., p. 346. The necessary lateral extensometer was developed and successfully used by A. M. Wahl, see paper by R. E. Peterson and A. M. Wahl, *Journal of Appl. Mech.*, Vol. 2, 1935, p. 1.

In the previous discussion of the photo-elastic stress analysis it was always assumed that we were dealing with two-dimensional problems. More recently considerable efforts have been made to expand the photo-elastic method on three-dimensional problems and some promising results have already

**3. Bar element finite element code.**

Write a finite element code that will accept information on a planar truss structure made of up to seven bar elements (each with a different area), and constraints and loads, and will solve (using the finite element formulation developed in class) for the reaction loads, and the bar forces and stresses. Validation of the code with hand calculations should be included, along with a listing of the code that includes comments. This should be original code in a standard programming language (e.g., Fortran, C, C++), not in an applications language.



#### 4. Truss bridge design, analysis and testing (2 students per team).

**Technical Overview:** Your team is to

- design, analyze, construct and test a bridge according to specifications below,
- report on your design and testing results in a formal report (1 report per team).

**Technical Details:**

• **Materials:**

- no more than seven (7) 3-foot lengths of balsa wood,
- a bottle of wood glue,
- a sheet of wax paper,
- straight pins,
- some simple fixturing.

• **Tools:** You may use any tools you like for fabrication. Finite element analysis using ANSYS should be the tool you use for predicting failure load and weight of structure.

• **Bridge Specifications/Requirements:**

<i>Design Specifications</i>	
Span	24 cm
Width	3.5 cm
Maximum Height	10.0 cm
Maximum Load	~66 N (as applied by the road-bed <sup>1</sup> )
Minimum Factor of Safety	2
Maximum Cost	\$10 (team expenditures)
Material	Balsa (limited supply)
Dimensions of Cross Section of Balsa	3.175mm X 3.175mm
Max Internal Compressive Load	69 N <sup>2</sup>
Max Internal Tensile Load	735 N
Total Weight of Structure	As light as possible to sustain the required load, with the required safety factor (of 2.0)

<sup>1</sup> The road bed will be 3cm tall, 3 cm wide, and 21 cm long. The four hooks will be placed at 6.69cm, 9.23cm, 11.77cm, and 14.31cm. These are at one inch intervals, offset from the center joints as shown in the diagram in class.

<sup>2</sup> The compressive yield strength typically ranges from 6.9 MPa to 9 MPa for balsa wood along the grain. This data was taken from matweb. <http://www.matweb.com/search/SpecificMaterial.asp?bassnum=PTSIA>

**5. Your own project.**

Propose your own project. Remember, this project should be of a reasonable size (most folks underestimate how hard it is to model something) and should include verification of the solution. Verification might be via an experiment or a theoretical solution.

Make sure to discuss the scope of your proposed project (including how your solutions will be verified) with the teaching staff before starting to work on it. Submit a brief problem description to [ekuhl@stanford.edu](mailto:ekuhl@stanford.edu) or [n0e@stanford.edu](mailto:n0e@stanford.edu) before you start working on it to make sure that the problem is not too difficult!