CHAPTER

# 3

# Banking Services

*The entire essence of America is the hope to first make money—then make money with money—then make lots of money with lots of money.*

**Paul Erdman, Business and Financial Author**


**3-1** Checking Accounts
**3-2** Reconcile a Bank Statement
**3-3** Savings Accounts
**3-4** Explore Compound Interest
**3-5** Compound Interest Formula
**3-6** Continuous Compounding
**3-7** Future Value of Investments
**3-8** Present Value of Investments


**What does Paul Erdman mean in this quotation?**

**What do you think?**

Most people feel that having money is a prerequisite to living the "American dream." Once you have money, you can save it and invest it to earn more money.

**TEACHING RESOURCES**

**Instructor's Resource CD**

**Exam *View*® CD, Ch. 3**

**eHomework, Ch. 3**

**www.cengage.com/school/math/financialalgebra**

Chapters 1 and 2 introduced you to the business world. In that world, people take risks and make investments in businesses with the hope of making money. Where do people keep the money they earn? They keep it in checking accounts and savings accounts in banks. The money in their checking accounts is used to pay bills, and the money in their savings accounts is actually another form of investment. In Chapter 3, this less risky form of investment is examined. Although savings accounts may already be familiar, Chapter 3 answers questions about unknown factors of savings accounts. How safe is your money in a bank? Where do banks get the money they pay you in interest? Can you get rich from the interest? What requirements are involved in opening an account? Together, Chapters 1, 2, and 3 give an inside look at the different degrees of risk and reward inherent in investing money in different ways.

# Really?

Most people are familiar with the United States Secret Service as the group that guards the President. Its officers are frequently seen on television surrounding the President as he tends to the affairs of the country.

What most people do not realize is that the Secret Service, established in 1865, was created to help the United States government combat the widespread counterfeiting of U.S. currency at the time. Counterfeiting, one of the oldest crimes in history, had become a national problem. It is estimated that approximately $\frac{1}{3}$ to $\frac{1}{2}$ of the nation's currency in circulation at that time was counterfeit.

The problem, although not as severe, still exists today. Modern printing and scanning equipment makes counterfeiting easier, and the government has instituted changes in currency to make it harder to counterfeit. Although most citizens have no intentions of counterfeiting U.S. currency, Americans have a responsibility to learn about counterfeiting, because they may receive a counterfeit bill one day. If a counterfeit bill is received, try to recall where it was acquired. Contact the nearest Secret Service office. The bill will be taken and no compensation will be returned to you. If a counterfeit bill is deposited in a bank account, you will lose the bill and the credit for the value of the deposit. Go to the Federal Reserve Bank website and read tips for spotting counterfeit currency. The penalty for trying to pass a counterfeit bill is a fine or imprisonment.

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## CHAPTER OVERVIEW

The first three chapters focus on risk and reward. The degree of risk and the degree of reward depends upon the venture undertaken as well as the investor's comfort level with monetary risks. Use this discussion as a springboard for this chapter on banking services. Ask students about the perceived risks and rewards of savings accounts and checking accounts.

## REALLY? REALLY!

Counterfeiting has always been troublesome for the government, for banks, and for the consumer. This feature introduces students to the federal agency that was instituted to combat counterfeiting in its early years. Students are encouraged to go to the Federal Reserve website to read more about how the counterfeiting problem can affect them and what they can do about it.

# Really!



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*There have been three great inventions since the beginning of time: fire, the wheel, and central banking.*

**Will Rogers,** Actor and Columnist

# 3-1 Checking Accounts

## Objectives

- Understand how checking accounts work.
- Complete a check register.

## Key Terms

- checking account
- check
- electronic funds transfer (EFT)
- payee
- drawer
- check clearing
- deposit slip
- direct deposit
- hold
- endorse
- canceled
- insufficient funds
- overdraft protection
- automated teller machine (ATM)
- personal identification number (PIN)
- maintenance fee
- interest
- single account
- joint account
- check register
- debit
- credit

### EXAMINE THE QUESTION

Access can be direct or indirect. Direct access to money in a bank account can be made by physically going to the bank or to the ATM. Indirect access can be attained through writing checks, using a debit card, or making any electronic transactions. In the latter case, the actual bills are not handled by the consumer.

### CLASS DISCUSSION

Ask if anyone has a checking account. If there are some students who do, ask them how they use the account. What are the responsibilities of having a checking account? Why do they have one? What are the ways that they access the money in that account?

Why might a person need overdraft protection? Some people say that it is a form of a loan from the bank. Explain why they might think that.

Why is it important to use the check register?

## How do people gain access to money they keep in the bank?

Consumers can have savings, checking, and loan accounts in a variety of different banks. A survey reported that most consumers consider their primary bank to be the one where they have their main checking account even when they use banking services at other banks. A **checking account** is an account at a bank that allows a customer to deposit money, make withdrawals, and make transfers from the funds on deposit.

A **check** is a written order used to tell a bank to pay money (transfer funds) from an account to the check holder. Payments can be made by writing a paper check or by making an electronic funds transfer. An **electronic funds transfer (EFT)** is the process of moving funds electronically from an account in one bank to an account in another bank. An EFT is often referred to as an *electronic check* or *e-check*. Because the transfer is electronic, the processing time is very short. Both the paper and electronic forms of a check are written to a **payee**, the receiver of the transferred funds. The account owner of the check is the **drawer**. Both the payee and the drawer can be a person, persons, or a company. The checking account needs to have enough money in it to cover the amount of a check in order for the check to *clear*, that is, to be paid by the bank. This process is known as **check clearing**.

You can make deposits using a **deposit slip**. Often **direct deposit** is used to deposit payroll or government checks directly into an account. The validity and financial worthiness of deposits must be verified before the bank will allow customers to draw on the funds. If you would like to receive cash back when you deposit a check, there must be sufficient funds already in the checking account. A **hold** is put on the checking



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account in the amount of the cash received. When the deposit is cleared, the hold is lifted and all of the money in the account is available.

When cashing a check, the payee must **endorse** the check either in writing, by stamp, or electronically. Once the money is paid to the payee, the check is **canceled**.

If a check is written for an amount that cannot be paid out of the account, the check is returned, or dishonored. This means that there are **insufficient funds** in the account and the payee will not receive the money. Banks charge a fee for processing returned checks. Some banks offer customers **overdraft protection** plans that pay a check even though there are not enough funds in the account. There is a fee for this service and the money must be repaid.

Most banks offer **automated teller machines (ATMs)** that give customers 24-hour access to banking services such as deposits and withdrawals. You need a bank card and a **personal identification number (PIN)** to use an ATM. Usually there is no charge if you use one of your bank's ATMs. If you use another ATM, there may be a fee by the bank that owns the ATM and your bank as well.

There are many types of checking accounts, the names of which vary from bank to bank. Each has a different name and different benefits and requirements. Some banks offer free checking while others have accounts that have a monthly **maintenance fee**. Some banks pay **interest** on their checking accounts, which is a percentage of the money that is in the account over a given period of time. Some popular checking accounts are listed and explained below.

- **Basic checking accounts** are the most widely used types of checking accounts. Customers can move money in and out of the account by making deposits and writing checks to pay bills or access money. Many of these accounts do not pay interest.
- **Interest-bearing checking accounts** pay customers interest, usually on a monthly basis, on the money that is in the account. A minimum balance is often required and a fee is charged if the account balance drops below that minimum.
- **Free checking accounts** require no minimum balance and charge no maintenance fees. The Federal Truth in Savings Act guarantees such accounts are available.
- **Joint checking accounts** are accounts owned by more than one person. All owners have equal access to the money in the account.
- **Express checking accounts** are accounts for people who want to avoid going to a traditional bank. Express accounts are often accessed electronically via telephone, computer, or ATM. Some banks charge a fee when an Express account owner uses the services of bank personnel.
- **NOW accounts** stand for *negotiable order of withdrawal*. These are free checking accounts that have interest payments attached to them.
- **Lifeline checking accounts** are available in many states for low-income consumers. Fees and minimum balances are low or nonexistent. Lifeline accounts are required by law in many states.

Bank accounts can be owned by an individual or a group of individuals or a business. In a **single account**, only one person can make withdrawals. These are also called *individual* or *sole owner* accounts. **Joint accounts** have more than one person listed as the owner. Any person listed on a joint account can make withdrawals.

### TEACH

Students need to have an understanding of checking account terminology. The highlighted words in the text are critical and will be used throughout the lesson. Ask students to have a discussion about checking accounts with their parents. What kind of account do their parents have? Are there any fees? Are there any minimum balances? What is the bank's name for the type of account they own? Have a discussion with the students about what they have learned from their parents.



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## EXAMPLE 1

The purpose of this exercise is to make sure that students know the mathematical difference between a deposit and a withdrawal in a checking account and how each affects the balance. Have students make a list of situations in which a deposit might be made (a birthday check, a paycheck, and so on) and when a withdrawal might be made (a utility bill, tuition, and ATM fee).

## CHECK YOUR UNDERSTANDING

***Answer*** $x + b + 2c - d$

To obtain an expression for the new balance, begin with the old balance, add the deposits, $x + b + 2c$, and subtract the withdrawal, $d$.

# Skills and Strategies

Here you will learn how to deposit money into a checking account and to track the transactions in the account on a monthly basis.

## EXAMPLE 1

Allison currently has a balance of $2,300 in her checking account. She deposits a $425.33 paycheck, a $20 rebate check, and a personal check for $550 into her checking account. She wants to receive $200 in cash. How much will she have in her account after the transaction?

**SOLUTION** Allison must fill out a deposit slip and hand it to the bank teller along with her endorsed checks. Although deposit slips vary from bank to bank, there is usually a line for cash deposits and a few lines for individual check deposits and for cash received. Allison is not making a cash deposit, so the cash line is blank. She lists the three checks on the deposit slip separately. In order for Allison to get $200 back from this transaction, she must have at least that amount already in her account.

DEPOSIT TICKET

Allison Megham
7 Tome Way
Philadelphia, PA 19255

DATE *June 4*

DEPOSITS MAY NOT BE AVAILABLE FOR IMMEDIATE WITHDRAWAL

*Allison Megham*

SIGN HERE FOR CASH RECEIVED (IF REQUIRED)*

ROME
FINANCIAL BANK

⑆04 2000⑆⑆9⑆3⑆ 51?⑆⑆5⑆00?⑈

| | DOLLARS | CENTS |
|---|---|---|
| CASH | | |
| LIST CHECKS SINGLY | *425* | *33* |
| | *20* | *00* |
| | *550* | *00* |
| | | |
| SUB TOTAL | *995* | *33* |
| LESS CASH RECEIVED | *200* | *00* |
| TOTAL | | |

Add the check amounts.

$$\begin{array}{r} \$425.33 \\ 20.00 \\ +550.00 \\ \hline \$995.33 \end{array}$$

Subtract the cash received. $-200.00$

Total on deposit slip $\$795.33$

Allison's current balance is $2,300.

Add current balance and deposit amount. $2{,}300 + 795.33 = 3{,}095.33$

Allison's new balance is $3,095.33.

### CHECK YOUR UNDERSTANDING

Lizzy has a total of $x$ dollars in her checking account. She makes a deposit of $b$ dollar in cash and two checks each worth $c$ dollars. She would like $d$ dollars in cash from this transaction. She has enough to cover the cash received in her account. Express her new checking account balance after the transaction as an algebraic expression.



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## Check Registers

You should keep a record of all transactions in your checking account, including checks written, deposits made, fees paid, ATM withdrawals, and so on. This record is a **check register**. The record can be handwritten or electronic. It tracks the **debits** (withdrawals) and **credits** (deposits) of a checking account.

### EXAMPLE 2

Nick has a checking account with the Park Slope Savings Bank. He writes both paper and electronic checks. For each transaction, Nick enters the necessary information: check number, date, type of transaction, and amount. He uses E to indicate an electronic transaction. Determine the balance in his account after the Star Cable Co. check is written.

| NUMBER OR CODE | DATE | TRANSACTION DESCRIPTION | PAYMENT AMOUNT | | ✓ | FEE | DEPOSIT AMOUNT | | $ BAL 3,672.27 |
|---|---|---|---|---|---|---|---|---|---|
| 3271 | 5/5 | Dewitt Auto Body (Car Repair) | $ 1,721 | 00 | | | | | |
| 3272 | 5/7 | Kate's Guitar Hut (Strings) | 32 | 50 | | | | | |
| | 5/9 | Deposit (Paycheck) | | | | | 821 | 53 | |
| E | 5/10 | Verizon Wireless | 101 | 50 | | | | | |
| E | 5/10 | Star Cable Co. | 138 | 90 | | | | | |

**SOLUTION** Perform the calculations needed as shown below. The balance in Nick's register is $2,499.90.

| NUMBER OR CODE | DATE | TRANSACTION DESCRIPTION | PAYMENT AMOUNT | | ✓ | FEE | DEPOSIT AMOUNT | | $ BAL 3,672.27 |
|---|---|---|---|---|---|---|---|---|---|
| 3271 | 5/5 | Dewitt Auto Body (Car Repair) | $ 1,721 | 00 | | | | | - 1,721.00 |
| | | | | | | | | | 1,951.27 |
| 3272 | 5/7 | Kate's Guitar Hut (Strings) | 32 | 50 | | | | | - 32.50 |
| | | | | | | | | | 1,918.77 |
| | 5/9 | Deposit (Paycheck) | | | | | 821 | 53 | + 821.53 |
| | | | | | | | | | 2,740.30 |
| E | 5/10 | Verizon Wireless | 101 | 50 | | | | | - 101.50 |
| | | | | | | | | | 2,638.80 |
| E | 5/10 | Star Cable Co. | 138 | 90 | | | | | - 138.90 |
| | | | | | | | | | 2,499.90 |

### CHECK YOUR UNDERSTANDING

Nick writes a check to his friend James Sloan on May 11 for $150.32. What should he write in the check register and what should the new balance be?

### EXTEND YOUR UNDERSTANDING

Would the final balance change if Nick had paid the cable bill before the wireless bill? Explain.

### EXAMPLE 2

Many students may never have seen a check register and do not know how to use one. Make copies of a blank form. It can be downloaded at www.cengage.com/school/math/financialalgebra.

Work through the check register in the solution to Example 2 line by line, performing the calculations needed to attain the correct balance.

### CHECK YOUR UNDERSTANDING

***Answer*** 3273; 5/11; James Sloan; $150.32; $2,349.58

### EXTEND YOUR UNDERSTANDING

***Answer*** No

2,740.30 – 138.90 = 2,601.40; 2,601.40 – 101.50 also equals 2,499.90.



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# Applications

*There have been three great inventions since the beginning of time: fire, the wheel, and central banking.*

**Will Rogers,** Actor and Columnist

**1.** How might the quote apply to what has been outlined in this lesson? See margin.

**2.** Jackie deposited a $865.98 paycheck, a $623 stock dividend check, a $60 rebate check, and $130 cash into her checking account. Her original account balance was $278.91. Assuming the checks clear, how much was in her account after the deposit was made? $1,957.89

**3.** Rich has $t$ dollars in his checking account. On June 3, he deposited $w$, $h$, and $v$ dollars, and cashed a check for $k$ dollars. Write an algebraic expression that represents the amount of money in his account after the transactions. $t + w + h + v - k$

**4.** John cashed a check for $630. The teller gave him three fifty-dollar bills, eighteen twenty-dollar bills, and $t$ ten-dollar bills. Determine the value of $t$. $t = 12$

**5.** Gary and Ann have a joint checking account. Their balance at the beginning of October was $9,145.87. During the month they made deposits totaling $2,783.71, wrote checks totaling $4,871.90, paid a maintenance fee of $12, and earned $11.15 in interest on the account. What was the balance at the end of the month? $7,056.83

**6.** Anna has a checking account at Garden City Bank. Her balance at the beginning of February was $5,195.65. During the month, she made deposits totaling $6,873.22, wrote checks totaling $c$ dollars, was charged a maintenance fee of $15, and earned $6.05 in interest. Her balance at the end of the month was $4,200.00. What is the value of $c$? $c = \$7{,}859.92$

**7.** Queens Meadow Bank charges a monthly maintenance fee of $13 and a check writing fee of $0.07 per check. Last year, Mark wrote 289 checks from his account at Queens Meadow. What was the total of all fees he paid on that account last year? $176.23

**8.** Joby had $421.56 in her checking account when she deposited $g$ twenty-dollar bills and $k$ quarters. Write an expression that represents the amount of money in her account after the deposit. $421.59 + 20g - 0.25k$

**9.** Neka cashed a check for $245. The teller gave him two fifty-dollar bills, six twenty-dollar bills and $f$ five-dollar bills. Determine the value of $f$. $f = 5$

**10.** Olivia cashed a check for $113. The teller gave her four twenty-dollar bills, $x$ ten-dollar bills, and three one-dollar bills. Find the value of $x$. 3

**11.** Hector had $y$ dollars in his savings account. He made a deposit of twenty-dollar bills and dollar coins. He had four times as many dollar coins as he had twenty-dollar bills and the total of his twenty-dollar bills was $60. Write an expression for the balance in Hector's account after the deposit. $y + 72$

## TEACH

**Exercises 2 and 3**
Notice that Exercises 2 and 3 are related. In Exercise 2, students are asked to find a numerical balance. In Exercise 3, students will mimic the process used in Exercise 2 but with variables. This balance will be represented as a variable expression. When reviewing these two exercises, make a connection between them.

**Exercises 5 and 6**
Get students into the habit of sorting checking account transactions by credits and debits. This will help organize the given information in both exercises.

## ANSWERS

1. Will Rogers indicates that banking is as essential to your daily life as fire and the wheel.



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**12.** On September 1, Chris Eugene made the following band equipment purchases at Leslie's Music Store. Calculate her total bill. Complete a check for the correct amount. Print a copy of the check from www.cengage.com/school/math/financialalgebra. See additional answers.

| DESCRIPTION | CATALOG NUMBER | LIST PRICE | QUANTITY | TOTAL |
|---|---|---|---|---|
| Speaker Cabinets | RS101 | $400.00 | 2 | |
| Speaker Cabinets | RG306 | $611.00 | 2 | |
| Horns | BG42 | $190.00 | 2 | |
| Audio Console | LS101 | $1,079.00 | 1 | |
| Power Amplifier | NG107 | $416.00 | 5 | |
| Microphones | RKG-1972 | $141.92 | 8 | |
| Microphone Stands | 1957-210 | $32.50 | 8 | |
| | | | TOTAL | |
| | | | 13% DISCOUNT | |
| | | | SALE PRICE | |
| | | | 8% SALES TAX | |
| | | | TOTAL COST | |

**13.** Create a check register for the transactions listed. There is a $2.25 fee for each ATM use. See additional answers.

**a.** Your balance on 10/29 is $237.47
**b.** You write check 115 on 10/29 for $18.00 to Fox High School.
**c.** You deposit a paycheck for $162.75 on 10/30.
**d.** You deposit a $25 check for your birthday on 11/4.
**e.** On 11/5, you go to a sporting event and run out of money. You use the ATM in the lobby to get $15 for snacks.
**f.** Your credit card bill is due on 11/10, so on 11/7 you write check 116 to Credit USA for $51.16.
**g.** Your sister repays you $20 on 11/10. You deposit it.
**h.** You withdraw $25 from the ATM to buy flowers on 11/12.
**i.** You deposit your paycheck for $165.65 on 11/16.
**j.** Your deposit a late birthday check for $35 on 11/17.

**14.** Ridgewood Savings Bank charges a $27 per check overdraft protection fee. On July 8, Nancy had $1,400 in her account. Over the next four days, the following checks arrived for payment at her bank: July 9, $1,380.15; July 10, $670 and $95.67; July 11, $130; and July 12, $87.60. How much will she pay in overdraft protection fees? How much will she owe the bank after July 12? $108; $1,071.42

**15.** 123 Savings and Loan charges a monthly fee of $8 on checking accounts and an overdraft protection fee of $33. Neela's check register showed she had a balance of $456 when she wrote a check for $312. Three days later she realized her check register had an error and she actually only had $256. So she transferred $250 into her checking account. The next day, her monthly account statement was sent to her. What was the balance on her statement? $153

## TEACH

### Exercise 12

When reviewing this exercise, ask students if it would have made a difference if the 13% discount line and the 8% sales tax line had been switched on the equipment bill. Although it makes no difference in the total cost, this will be counterintuitive to many students. Show them that the switch will result in the same answer.

### Exercise 13

Once again, it is important that students have a command of a credit (deposit) and a debit (a withdrawal). You might want to have students identify each before assigning this problem.



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## TEACH

### Exercise 16

Review the ramifications of a voided check. Students need to realize that the destroyed check must be accounted for in the register so that when the user returns to balance the account he/she will have a record of that check. In part f of the problem, students will need to know that once the check is canceled, the very next check in the numerical sequence is recorded in the register and used.

## ANSWERS

17. a. 623
    b. 629
    c. 630
    d. 71.10
    e. –500.00
    f. 1,292.80
    g. –51.12
    h. 1,241.68
    i. –25.00
    j. 1,216.68
    k. +650.00
    l. 1,866.68
    m. –200.00
    n. 1,666.68
    p. –90.00
    q. 1,576.68
    r. –49.00
    s. 1,527.68
    t. –65.00
    u. 1,462.68
    v. –300.00
    w. 1,162.68
    x. +400.00
    y. 1,562.68
    z. 1,191.02

**16.** Create a check register for the transactions listed. Download a blank check register from www.cengage.com/school/math/financialalgebra.

**a.** Your balance on 12/15 is $2,546.50. See additional answers.

**b.** On 12/16, you write check 2345 for $54 to Kings Park High School Student Activities.

**c.** On 12/17, you deposit your paycheck in the amount of $324.20.

**d.** Your grandparents send you a holiday check for $100 which you deposit into your account on 12/20.

**e.** On 12/22 you write three checks: 2346 to Best Buy in the amount of $326.89, 2347 to Macy's in the amount of $231.88, and 2348 to Target in the amount of $123.51.

**f.** On 12/24, you go to the Apple Store. As you are writing the check for $301.67, you make a mistake and must void that check. You pay with the next available check in your checkbook.

**g.** On 12/26, you return a holiday gift. The store gives you $98. You deposit that into your checking account.

**h.** On 12/28, you write an e-check to Allstate Insurance Company in the amount of $876.00 to pay your car insurance.

**i.** On 12/29, you withdraw $200 from an ATM. There is a $1.50 charge for using the ATM.

**17.** Download a copy of the check register shown below from www.cengage.com/school/math/financialalgebra. Complete items a through y. See margin.

| PLEASE BE SURE TO DEDUCT CHANGES THAT AFFECT YOUR ACCOUNT | | | SUBTRACTIONS | | | ADDITIONS | | BALANCE FORWARD | |
|---|---|---|---|---|---|---|---|---|---|
| ITEM NO. FOR TRANSACTION CODE | DATE | DESCRIPTION OF TRANSACTION | AMOUNT OF PAYMENT OR WITHDRAWAL | ✓ | OTHER | AMOUNT OF DEPOSIT OR INTEREST | | 1,863 | 90 |
| 621 | 10/3 | TO Telephone Co. | d. | | | | | − 71 | 10 |
| | | FOR Dec. Bill | | | | | | 1,792 | 80 |
| 622 | 10/7 | TO Banner Reality | 500 | 00 | | | | e. | |
| | | FOR Rent | | | | | | f. | |
| a. | 10/8 | TO Electric Co. | 51 | 12 | | | | g. | |
| | | FOR Dec. Bill | | | | | | h. | |
| 624 | 10/10 | TO Cathy Santoro | 25 | 00 | | | | i. | |
| | | FOR Piano Lesson | | | | | | j. | |
| | 10/15 | TO Deposit | | | | 650 | 00 | k. | |
| | | FOR | | | | | | l. | |
| 625 | 10/16 | TO Don's Day Camp | 200 | 00 | | | | m. | |
| | | FOR Kid's Summer Camp | | | | | | n. | |
| 626 | 10/18 | TO Ed's Sporting Goods | 90 | 00 | | | | p. | |
| | | FOR Winter Coat | | | | | | q. | |
| 627 | 10/21 | TO Maple Place Garage | 49 | 00 | | | | r. | |
| | | FOR antifreeze & hose | | | | | | s. | |
| 628 | 10/22 | TO Dr. Moe Goldstein | 65 | 00 | | | | t. | |
| | | FOR Check-up | | | | | | u. | |
| b. | 11/4 | TO Hicksville H.M.O. | 300 | 00 | | | | v. | |
| | | FOR yearly premium | | | | | | w. | |
| | 11/5 | TO Deposit | | | | 400 | 00 | x. | |
| | | FOR | | | | | | y. | |
| c. | 11/9 | TO State Insurance Co. | 371 | 66 | | | | − 371 | 66 |
| | | FOR Auto Insurance | | | | | | z. | |



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*My problem lies in reconciling my gross habits with my net income.*

**Errol Flynn,** Actor

# Reconcile a Bank Statement 3-2

## Key Terms

- account number
- bank statement
- statement period
- starting balance
- ending balance
- outstanding deposits
- outstanding checks
- balancing
- reconciling

## Objectives

- Reconcile a checking account with a bank statement by hand and by using a spreadsheet.

## How do checking account users make sure that their records are correct?

A customer keeps a record of all transactions concerning a checking account in a paper or electronic check register. The bank also keeps a record of all transactions. Every month, the bank makes available a statement listing all of the transactions and balances for the account. The bank statement contains important information related to the account.

- The **account number** appears on all checks, deposit slips, and paper and electronic bank statements.
- The **bank statement** includes all transactions that have occurred for a period of approximately one month. The **statement period** indicates the dates in which the transactions occurred.
- The **starting balance** is the amount of money in a checking account at the beginning of a statement period.
- The **ending balance** is the amount of money in a checking account at the end of a statement period.
- The deposits section shows the money that was put into the account during the statement period. Deposits that do not appear on the statement are **outstanding deposits**.
- Checks that do not appear on the statement are **outstanding checks**.

Whether using paper or electronic statements, you should verify the bank's records to make sure no mistakes have been made. This process is called **balancing** a checkbook or **reconciling** a bank statement. Most bank statements include a checking account summary which guides the user through the reconciling process. Check registers contain a column to place a check mark for cleared items to assist in balancing.

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### EXAMINE THE QUESTION

There is a difference between keeping accurate records and checking that records are accurately kept. This question addresses both. Students will be introduced to the check register and the bank statement in this unit. Both should be accurate accountings of their transactions. But, both should be checked for accuracy.



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## CLASS DISCUSSION

Discuss the importance of keeping accurate records and reconciling the check register on a monthly basis. In particular, focus on the problems that could arise if the balance you think you have in the account is more than the actual balance the bank knows you have.

## TEACH

This lesson contains a great deal of terminology which must be introduced to students, many of whom may not have a checking account. Make sure that students have a firm understanding of these terms. You may want to make a word wall that lists these terms so that you can refer back to them as the lesson progresses.

## EXAMPLE 1

Since reconciling a bank statement will be new to most students, it is important that you walk students through the steps necessary to complete this example. Students should know how the register and the bank statement interact with each other in order to determine if the account records balance. They also need to know what to do when the amounts do not balance.

One common error made in a check register is the transposition of numbers. For example, a check in the amount of $179 might be written as $197. Transposition errors always result in a difference that is divisible by 9. Have students make a transposition error then test this fact. Ask students how knowing that the difference is a multiple of 9 can be helpful.

# Skills and Strategies

Here you will learn to reconcile a bank statement and a check register.

## EXAMPLE 1

Below is a bank statement and check register for Michael Biak's checking account. What steps are needed to reconcile Michael's bank statement?

PLEASE BE SURE TO DEDUCT CHANGES THAT AFFECT YOUR ACCOUNT

| ITEM NO. FOR TRANSACTION CODE | DATE | DESCRIPTION OF TRANSACTION | SUBTRACTIONS: AMOUNT OF PAYMENT OR WITHDRAWAL | ✓ | OTHER | ADDITIONS: AMOUNT OF DEPOSIT OR INTEREST | BALANCE FORWARD |
|---|---|---|---|---|---|---|---|
| | | | | | | | 748 95 |
| 1763 | 10/13 | TO Deepdale Country Club<br>FOR Swimming lessons | 50 00 | | | | − 50 00<br>698 95 |
| 1764 | 10/13 | TO Joe's Sporting Goods<br>FOR Tennis Racket | 48 00 | ✓ | | | − 48 00<br>650 95 |
| 1765 | 10/14 | TO Ellio's Pizzeria<br>FOR Pizza Party | 19 50 | | | | − 19 50<br>631 45 |
| 1766 | 10/15 | TO Bethpage Auto Parts<br>FOR Air Filter | 9 00 | ✓ | | | − 9 00<br>622 45 |
| | 10/15 | TO Deposit<br>FOR | | ✓ | | 100 00 | + 100 00<br>722 45 |
| 1767 | 10/16 | TO Maple Place Garage<br>FOR Inspection | 18 00 | | | | − 18 00<br>704 45 |
| 1768 | 10/18 | TO Ticket Man<br>FOR Concert Tickets | 46 50 | | | | − 46 50<br>657 95 |
| 1769 | 10/21 | TO Caruso's Restaurant<br>FOR Dinner | 74 64 | | | | − 74 64<br>583 31 |
| 1770 | 10/22 | TO Mickel's Home Center<br>FOR Tool Chest | 158 08 | | | | − 158 08<br>425 23 |
| 1771 | 11/4 | TO Aunt Bella's Restaurant<br>FOR Dinner | 29 10 | | | | − 29 10<br>396 13 |
| | 11/5 | TO Deposit<br>FOR | | | | 35 00 | + 35 00<br>431 13 |
| 1772 | 11/9 | TO Living Color Lab<br>FOR Film Developing | 15 00 | | | | − 15 00<br>416 13 |
| | 11/11 | TO Deposit<br>FOR | | | | 100 00 | + 100 00<br>516 13 |
| | 12/1 | TO Deposit<br>FOR | | | | 125 00 | + 125 00<br>641 13 |

Michael Biak
17 Breeze Way
Lake City, FL 32025

ACCOUNT NUMBER: 7843390
STATEMENT PERIOD: 11/01 - 11/30

STARTING BALANCE ⟶ **$ 791.95**

| DATE | DESCRIPTION | CHECK NUMBER | TRANSACTION AMOUNT | BALANCE |
|---|---|---|---|---|
| 11/05 | DEPOSIT | | 35.00 | $ 826.95 |
| 11/11 | DEPOSIT | | 100.00 | $ 926.95 |
| 11/13 | W/D | 1770 | 158.08 | $ 768.87 |
| 11/17 | W/D | 1768 | 46.50 | $ 722.37 |
| 11/19 | W/D | 1769 | 74.64 | $ 647.73 |
| 11/27 | W/D | 1765 | 19.50 | $ 628.23 |

**ENDING BALANCE ⟶ $ 628.23**



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**SOLUTION** Compare the entries in Michael's check register with the bank statement. The entries marked with a ✓ appeared on a previous month's statement. Enter a check mark in the check register for each deposit and check listed on the monthly statement.

Some of the entries in his check register are not on his bank statement. List any outstanding checks or other withdrawals and find the total. Then list any outstanding deposits and find the total.

| Outstanding Withdrawals | |
|---|---|
| ITEM | AMOUNT |
| 1763 | 50.00 |
| 1767 | 18.00 |
| 1771 | 29.10 |
| 1772 | 15.00 |
| TOTAL | 112.10 |

| Outstanding Deposits | |
|---|---|
| DATE | AMOUNT |
| 12/1 | 125.00 |
| | |
| | |
| | |
| TOTAL | 125.00 |

Then complete the following steps.

| | |
|---|---|
| Statement ending balance | $628.24 |
| Total deposits outstanding | + $125.00 |
| Total withdrawals outstanding | – $112.10 |
| Revised statement balance | $641.14 |
| Check register balance | $641.14 |

The revised statement balance equals the last balance in the check register, so the statement is reconciled.

If the balances are not equal, then there is an error. To find errors, check the arithmetic in the check register and on the statement. Be sure all fees, transaction charges, and interest have been included.

**CHECK YOUR UNDERSTANDING**

Name some reasons why a check may not have cleared during the monthly cycle and appear on the bank statement.

**EXAMPLE 2**

Use algebraic formulas and statements to model the check register balancing process.

**SOLUTION** Represent each line in the account summary with a variable.

| | |
|---|---|
| Statement ending balance | $a$ |
| Total deposits outstanding | $b$ |
| Total withdrawals outstanding | $c$ |
| Revised statement balance | $d$ |
| Check register balance | $r$ |

The revised statement balance equals the statement balance plus the total outstanding deposits, $b$, minus the total withdrawals outstanding, $c$.

$$d = a + b - c$$

If the revised statement balance, $d$, equals the check register balance, $r$, the statement is reconciled.

**CHECK YOUR UNDERSTANDING**

***Answer*** Sample answer: Many people and businesses hold on to checks and do not deposit or cash them immediately. If checks are written towards the end of a cycle, they will probably appear on the next monthly statement.

**EXAMPLE 2**

Some students may be able to jump right into the solution while others might have some difficulty writing the algebraic expression. For the latter, offer them an example using numbers, and then replace those numbers with the variables.



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## CHECK YOUR UNDERSTANDING

***Answer*** Yes
Let $a = \$885.84$, $b = \$825$, $c = \$632.84$, $r = \$1{,}078$. Then $d = a + b - c = \$1{,}078$. Both $d$ and $r$ equal \$1,078, so the check register is balanced.

### CHECK YOUR UNDERSTANDING

Nancy has a balance of \$1,078 in her check register. The balance on her bank account statement is \$885.84. Not reported on her bank statement are deposits of \$575 and \$250 and two checks for \$195 and \$437.84. Is her check register balanced? Explain.

## EXAMPLE 3

Have students work through the computations first before assigning cell locations and writing the formulas. This will assist them in transferring from the arithmetic view to the algebraic view.

### EXAMPLE 3

Marina and Brian have a joint checking account. They have a balance of \$3,839.25 in the check register. The balance on the bank statement is \$3,450.10. Not reported on the statement are deposits of \$2,000, \$135.67, \$254.77, and \$188.76 and four checks for \$567.89, \$23.83, \$598.33, and \$1,000. Reconcile the bank statement using a spreadsheet.

**SOLUTION** Enter the outstanding deposits in cells A3 to A9.

Enter the outstanding checks in cells B3 to B9.

| | A | B | C |
|---|---|---|---|
| 1 | | | |
| 2 | Outstanding Deposits | Outstanding Checks | |
| 3 | 2,000.00 | 567.89 | |
| 4 | 135.67 | 23.83 | |
| 5 | 254.77 | 598.33 | |
| 6 | 188.76 | 1,000.00 | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | 2,579.20 | 2,190.05 | Total |
| 11 | | | |
| 12 | Check register balance | | 3,839.25 |
| 13 | Statement ending balance | | 3,450.10 |
| 14 | Revised statement balance | | 3,839.25 |
| 15 | | | |
| 16 | Statement is reconciled. | | |

Cell A10 calculates the total amount of the outstanding deposits and cell B10 calculates the total amount of the outstanding checks. The cell formula for the total of the outstanding deposits in A10 is =sum(A3:A9).

In the Check Your Understanding, you will be asked to write the cell formula for the total outstanding checks.

Enter the check register balance in cell C12.

Enter the statement ending balance in cell C13.

Cell C14 calculates the revised statement balance, which is the sum of the statement ending balance and total outstanding deposits minus the total outstanding checks. The formula is =C13+A10−B10.

You can make the spreadsheet check to see if the revised statement balance equals the check register balance. Use an IF statement in the form =IF(test, output if true, output if false).

The test portion of the statement must contain a mathematical equation or inequality. The spreadsheet uses the values in the cells to test the truth of the statement. If the statement is true, the first output will be printed. If the statement is false, the second output will be printed.

In the spreadsheet, cell A16 contains the IF statement, =IF(C12=C14, "Statement is reconciled.", "Statement is not reconciled."). Cell A16 states the statement is reconciled.

## CHECK YOUR UNDERSTANDING

***Answer*** Although formulas vary based on the spreadsheet being used, most spreadsheet programs would use the following formula to determine the sum: =sum(B3:B9)

### CHECK YOUR UNDERSTANDING

Write a formula to calculate the sum of the outstanding checks.



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# Applications

*My problem lies in reconciling my gross habits with my net income.*

**Errol Flynn**, Actor

**1.** How might the quote apply to this lesson? See margin.

**2.** Rona filled out this information on her monthly statement. Find Rona's revised statement balance. Does her account reconcile? $864.52; yes

| Checking Account Summary | |
|---|---|
| Ending Balance | $ 725.71 |
| Deposits | + $ 610.00 |
| Checks Outstanding | − $ 471.19 |
| Revised Statement Balance | |
| Check Register Balance | $ 864.52 |

**3.** Ken filled out this information on the back of his bank statement. Find Ken's revised statement balance. Does his account reconcile? $181.95; no

| Checking Account Summary | |
|---|---|
| Ending Balance | $ 197.10 |
| Deposits | + $ 600.00 |
| Checks Outstanding | − $615.15 |
| Revised Statement Balance | |
| Check Register Balance | $ 210.10 |

**4.** Hannah wants to write a general formula and a comparison statement that she can use each month when she reconciles her checking account. Use the Checking Account Summary at the right to write a formula and a statement for Hannah. $B + D - C = S$; If $S = R$, the account is reconciled.

| Checking Account Summary | |
|---|---|
| Ending Balance | $B$ |
| Deposits | $D$ |
| Checks Outstanding | $C$ |
| Revised Statement Balance | $S$ |
| Check Register Balance | $R$ |

**5.** Jill has not been able to maintain the $1,000 minimum balance required to avoid fees on her checking account. She wants to switch to a different account with a fee of $0.20 per check and a $12.50 monthly maintenance fee. Jill wants to estimate the fees for her new account. Below is a summary of the checks she has written from May to August.

| Month | Number of Checks on Statement |
|---|---|
| May | 14 |
| June | 19 |
| July | 23 |
| August | 24 |

**a.** What is the mean number of checks Jill wrote per month during the last four months? 20

**b.** Based on the mean, estimate how much Jill expects to pay in per-check fees each month after she switches to the new account. $4.00

**c.** Estimate the total monthly fees Jill will pay each month for the new checking account. $16.50

## TEACH

### Exercise 2 and 3

These similar exercises test the students' understanding of the components needed to be in place if a check register is to balance. As an extension to these exercises, you might want to make up similar ones in which students have to find different amounts (ending balance, deposits, checks outstanding) when given all of the other values in a balanced check register.

### Exercise 5

Ask students if using the median number of checks might change this problem.

## ANSWERS

1. Errol Flynn use the word reconcile in the same way that it is used in a financial context. With a checking account, the problem lies in reconciling or balancing the check register amount with the revised statement amount. Errol Flynn's problem is in balancing his personal gross (monetary) habits such as making expensive purchases with the money he actually has coming in.



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## TEACH

**Exercise 6**
Students need to first locate or determine the amounts in question. They then should follow the reconciling process as outlined in Example 1.

## ANSWERS

7. Let $x$ = the number of checks Donna writes each month; let $F$ = fee charged; $F = 9.75 + 0.15x$
8. no; adding $75 will correct that he subtracted $75 when he should not have. He will also need to add another $75 for the original deposit.

**6.** Use Tina Weaver's monthly statement and check register to reconcile her account.

**a.** What is the ending balance on the statement? $1,434.19
**b.** What is the total of the outstanding deposits? $700.00
**c.** What is the total of the outstanding withdrawals? $89.00
**d.** What is the revised statement balance? $2,045.19
**e.** What is the balance of the check register? $2,045.19
**f.** Does the account reconcile? yes

Tina Weaver
41 Slider Lane
Greenwich, CT 06830

ACCOUNT NUMBER: 766666600A
STATEMENT PERIOD: 3/01 - 3/31

STARTING BALANCE → **$ 871.50**

| DATE | DESCRIPTION | CHECK NUMBER | TRANSACTION AMOUNT | BALANCE |
|---|---|---|---|---|
| 3/3 | W/D | 395 | $ 79.00 | $ 792.50 |
| 3/4 | DEPOSIT | | $ 600.00 | $ 1,392.50 |
| 3/10 | W/D | 396 | $ 51.10 | $ 1,341.40 |
| 3/14 | W/D | 393 | $ 12.00 | $ 1,329.40 |
| 3/19 | W/D | 394 | $ 133.81 | $ 1,195.59 |
| 3/24 | DEPOSIT | | $ 250.00 | $ 1,445.59 |
| 3/30 | W/D | 398 | $ 11.40 | $ 1,434.19 |

**ENDING BALANCE → $1,434.19**

| NUMBER OR CODE | DATE | TRANSACTION DESCRIPTION | PAYMENT AMOUNT | ✓ | FEE | DEPOSIT AMOUNT | $ BALANCE |
|---|---|---|---|---|---|---|---|
| 393 | 2/20 | Conn Telephone Co. | $ 76 80 | ✓ | | | 871.50 |
| 393 | 2/21 | Rod's Bike Shop | 12 00 | | | | 859.50 |
| 394 | 3/1 | Window's Restaurant | 133 81 | | | | 725.69 |
| 395 | 3/2 | Centsible Bank | 79 00 | | | | 646.69 |
| | 3/4 | Deposit | | | | 600 00 | 1,246.69 |
| 396 | 3/4 | Spear's Department Store | 51 10 | | | | 1,195.59 |
| 397 | 3/15 | Mary Lewis | 50 00 | | | | 1,145.59 |
| 398 | 3/21 | Sea Cliff Records | 11 40 | | | | 1,134.19 |
| | 3/23 | Deposit | | | | 250 00 | 1,384.19 |
| 399 | 4/2 | Ciangiola Motors | 39 00 | | | | 1,345.19 |
| | 4/2 | Deposit | | | | 700 00 | 2,045.19 |

**7.** Donna has a checking account that charges $0.15 for each check written and a monthly service charge of $9.75. Write a formula that Donna can use each month to find the fees she will be charged. Identify any variable you use in the formula. See margin.

**8.** Mason discovered that when he recorded a deposit of $75 two weeks ago, he mistakenly subtracted it from the running total in his check register. He decided that he would write a new entry after his most recent entry and add $75. Will this correct his mistake? Explain. See margin.



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**9.** When Payne removed his bank statement from the envelope, it got caught on a staple and a corner was ripped from the page. Now he cannot read his ending balance. Explain the computations he can do to find his ending balance. See margin.

Payne Johnston
1234 Main Street
Miami, FL 33299

ACCOUNT NUMBER: 99887766D
STATEMENT PERIOD: 1/1 - 1/31

STARTING BALANCE → $754.33

| DATE | DESCRIPTION | CHECK NUMBER | TRANSACTION AMOUNT | BALANCE |
|---|---|---|---|---|
| 1/08 | W/D | 5502 | 121.28 | $ 633.05 |
| 1/11 | W/D | 5501 | 140.00 | $ 493.05 |
| 1/15 | DEPOSIT | | 998.15 | $ 1,491.20 |
| 1/24 | W/D | 5504 | 107.78 | $ 1,383.42 |
| 1/27 | W/D | 5503 | 12.00 | $ 1,371.42 |
| 1/30 | W/D | 5506 | 58.70 | $ 1,3 |

ENDING BALANCE

**10.** Use Allison Shannon's bank statement and check register to reconcile her account. See margin.

Allison Shannon
3 Honey Drive
Dallas, TX 75372

ACCOUNT NUMBER: 76574709A
STATEMENT PERIOD: 12/01 - 12/31

STARTING BALANCE → $1,685.91

| DATE | DESCRIPTION | CHECK NUMBER | TRANSACTION AMOUNT | BALANCE |
|---|---|---|---|---|
| 12/08 | W/D | 1502 | 147.28 | $ 1,538.63 |
| 12/10 | W/D | 1501 | 130.00 | $ 1,408.63 |
| 12/15 | DEPOSIT | | 749.00 | $ 2,157.63 |
| 12/23 | W/D | 1504 | 250.00 | $ 1,907.63 |
| 12/27 | W/D | 1503 | 72.00 | $ 1,835.63 |
| 12/29 | W/D | 1506 | 26.00 | $ 1,809.63 |

ENDING BALANCE → **$1,809.63**

PLEASE BE SURE TO DEDUCT CHANGES THAT AFFECT YOUR ACCOUNT

| ITEM NO. FOR TRANSACTION CODE | DATE | DESCRIPTION OF TRANSACTION | SUBTRACTIONS: AMOUNT OF PAYMENT OR WITHDRAWAL | ✓ | OTHER | ADDITIONS: AMOUNT OF DEPOSIT OR INTEREST | BALANCE FORWARD |
|---|---|---|---|---|---|---|---|
| | | | | | | | 1,685 91 |
| 1500 | 11/20 | TO Girl Scouts<br>FOR Cookies | 32 00 | | | | − 32 00<br>1,653 91 |
| 1501 | 11/30 | TO Bank of Seaford<br>FOR Loan Payment | 130 00 | ✓ | | | − 130 00<br>1,523 91 |
| 1502 | 12/2 | TO Lacy's Department Store<br>FOR Radio | 147 28 | ✓ | | | − 147 28<br>1,376 63 |
| 1503 | 12/11 | TO Charge Tix<br>FOR Concert Tickets | 72 00 | ✓ | | | − 72 00<br>1304 63 |
| | 12/15 | TO Deposit<br>FOR | | ✓ | | 749 00 | + 749 00<br>2,053 63 |
| 1504 | 12/16 | TO FLASHCARD<br>FOR Monthly payment | 250 00 | ✓ | | | − 250 00<br>1,803 63 |
| 1505 | 12/17 | TO Red Cross<br>FOR Donation | 100 00 | | | | − 100 00<br>1,703 63 |
| 1506 | 12/18 | TO Daily Newspaper<br>FOR Subscription | 26 00 | ✓ | | | − 26 00<br>1,677 63 |
| | 1/5 | TO Deposit<br>FOR | | | | 150 00 | + 150 00<br>1,827 63 |

## TEACH

### Exercise 10

Although it is customary to mark off each transaction that is listed on the statement with a check mark, do not let students write in the text. Tell students to put a piece of blank paper over the Balance Forward column in the register and make their marks on that paper. This way, they will be able to easily account for the deposits and withdrawals that are outstanding.

## ANSWERS

9. On the statement you can still see that the balance on 1/27 was $1,371.42 and the check written on 1/30 was for $58.70. Subtract to find the ending balance. 1,371.41 – $58.70 = 1.312.71
10. outstanding deposit: $150; outstanding check: $132; 1,827.63 – 150 + 132 = 1,809.63; which reconciles with the statement balance.



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## TEACH

**Exercise 11**
Review the spreadsheet commands with students before assigning this problem.

**11.** Fill in the missing balances in Raymond Marshall's check register. Use the spreadsheet from www.cengage.com/school/math/financialalgebra to determine if Raymond's checking account reconciles with his statement. See additional answers.

Raymond Marshall
34 2630 Street
Oaks, NY 11004

ACCOUNT NUMBER: 6732281
STATEMENT PERIOD: 1/01 - 1/30

STARTING BALANCE → **$ 653.30**

| DATE | DESCRIPTION | CHECK NUMBER | TRANSACTION AMOUNT | BALANCE |
|---|---|---|---|---|
| 1/12 | W/D | 1776 | 28.00 | $ 625.30 |
| 1/13 | W/D | 1778 | 56.73 | $ 568.57 |
| 1/13 | W/D | 1777 | 120.00 | $ 448.57 |
| 1/14 | DEPOSIT | | 1,000.00 | $1,448.57 |
| 1/17 | W/D | 1774 | 70.00 | $1,378.57 |

**ENDING BALANCE → $1,378.57**

PLEASE BE SURE TO DEDUCT CHANGES THAT AFFECT YOUR ACCOUNT

| ITEM NO. FOR TRANSACTION CODE | DATE | DESCRIPTION OF TRANSACTION | SUBTRACTIONS: AMOUNT OF PAYMENT OR WITHDRAWAL | ✓ | OTHER | ADDITIONS: AMOUNT OF DEPOSIT OR INTEREST | BALANCE FORWARD |
|---|---|---|---|---|---|---|---|
| | | | | | | | 728 30 |
| 1773 | 12/28 | TO Galaxy Theather<br>FOR Tickets | 75 00 | ✓ | | | − 75 00 |
| 1774 | 12/30 | TO American Electric Company<br>FOR Electric Bill | 70 00 | | | | − 70 00 |
| 1775 | 12/30 | TO Hillsdake Water Co.<br>FOR Water Bill | 38 50 | | | | − 38 50 |
| 1776 | 1/2 | TO Barbara's Restaurant<br>FOR Dinner | 28 00 | | | | − 28 00 |
| 1777 | 1/3 | TO Platter Records<br>FOR Compact Disc | 120 00 | | | | − 120 00 |
| 1778 | 1/9 | TO A1 Gas Co.<br>FOR Gas Bill | 56 73 | | | | − 56 73 |
| 1779 | 1/12 | TO Al and Jean Adams<br>FOR Wedding Gift | 100 00 | | | | − 100 00 |
| 1780 | 1/12 | TO Greene College<br>FOR Fees | 85 00 | | | | − 85 00 |
| | 1/14 | TO Deposit<br>FOR | | | | 1,000 00 | + 1,000 00 |
| 1780 | 1/25 | TO Rob Gerver<br>FOR Typing Fee | 80 00 | | | | − 80 00 |
| | 2/1 | TO Deposit<br>FOR Salary | | | | 950 00 | + 950 00 |

**12.** When comparing his check register to his bank statement, Donté found that he had failed to record deposits of $55.65, $103.50, and $25.00. What is the total of these amounts and how will he use this information to reconcile his account? $184.15; these are the outstanding deposits.

**13.** Alisha has a February starting balance of $678.98 in her checking account. During the month, she made deposits that totaled $d$ dollars and wrote checks that totaled $c$ dollars. Let $E$ = her ending balance on February 28. Write an inequality using $E$ and the starting balance to show the relationship of her starting and ending balances for each condition.

**a.** if $d > c$ $E > 678.98$

**b.** if $d < c$ $E < 678.98$



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*Anything that we can do to raise personal savings is very much in the interest of this country.*

**Alan Greenspan,** Economist

# Savings Accounts 3-3

## Key Terms

- savings account
- interest
- interest rate
- principal
- simple interest
- simple interest formula
- statement savings
- minimum balance
- money market account
- certificate of deposit (CD)
- maturity

## Objectives

- Learn the basic vocabulary of savings accounts.
- Compute simple interest using the simple interest formula.

## What types of savings accounts do banks offer customers?

Most banks offer savings accounts, money market accounts, certificates of deposit (CDs), loans, life insurance policies, safe deposit boxes, and credit and debit cards, as well as checking accounts. Banks provide these services so they can attract customers and make a profit.

A **savings account** is an account in which the bank pays **interest** for the use of the money deposited in the account. The money on deposit with a bank is used by the bank to give loans. The people who borrow the money from a bank must pay it back with interest. The interest they pay is greater than the interest the bank pays for use of a customer's money. This way, the bank is able to pay depositors interest and still make a profit.

Interest is based on **interest rate** and **principal**, or balance. There are two classification for interest: *simple interest* and *compound interest*. Compound interest is discussed in the next lesson. **Simple interest**, explored in this lesson, is calculated on the principal only.

**Simple Interest Formula**

$I = prt$ where $I$ = interest
$p$ = principal
$r$ = annual interest rate expressed as a decimal
$t$ = number of years

Is there risk in putting money into a savings account? The Federal Deposit Insurance Corporation (FDIC) guarantees the safety of money in a bank by insuring each depositor for up to a specified amount. In 2008 the amount was $250,000 per depositor per bank depending on the type of account. It is important that all customers be aware if this amount is changed. If the bank fails, the money is replaced by the federal government.

### EXAMINE THE QUESTION

Most students are familiar with savings accounts in general, yet they know little about the specifics. Most students can't even give a ballpark estimate of current savings account interest rates. Discuss with them what banking services their families have used. Ask them where the money that banks lend out for loans comes from.

### CLASS DISCUSSION

Which banking services are you familiar with? What do you know about them?

Explain that compound interest will use the interest earned to increase the principal.

Explain direct deposit to students. Discuss advantages of direct deposit. (Money starts earning interest immediately, there is no check to get lost or stolen, and so on). Are there any disadvantages to direct deposit?



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Banks offer many forms of savings accounts. The most common type of savings account is a **statement savings** account where you receive a monthly statement showing all activity, including deposits, withdrawals, interest earned, and any fees. Some savings accounts require a **minimum balance**. With certain accounts, a fee is charged for each month the balance falls below the minimum amount. Some banks do not pay interest if a savings account falls below the minimum balance.

Another popular account is a **money market account**. It has a higher interest rate, but usually requires a greater initial deposit and a greater minimum balance requirement. You may be limited to a certain number of transactions per month in a money market account.

A **certificate of deposit (CD)** is a certificate that states you have a specific sum on deposit and guarantees the payment of a fixed interest rate until **maturity**, which is a specified date in the future. Maturity dates can be from seven days to ten years. Usually, the longer the term is, the higher the interest rate is. You cannot make deposits to or withdrawals from a CD without a penalty.

## Skills and Strategies

All banks report interest rates as annual rates. When choosing a savings account at a bank, or which bank to use, compare the interest rates. Also, consider penalties, fees, minimum balances, and other banking services.

### EXAMPLE 1

Grace wants to deposit $5,000 in a certificate of deposit for a period of two years. She is comparing interest rates quoted by three local banks and one online bank. Write the interest rates in ascending order. Which bank pays the highest interest for this two-year CD?

First State Bank: $4\frac{1}{4}\%$ E-Save Bank: $4\frac{3}{8}\%$

Johnson City Trust: 4.22% Land Savings Bank: 4.3%

**SOLUTION** Numbers in ascending order are written from least to greatest. Convert the fractions to decimals and compare.

To convert a fraction to an equivalent decimal, divide the numerator by the denominator.

$4\frac{1}{4}\% = 4.25\%$ $4\frac{3}{8}\% = 4.375\%$

Add zeroes so they all have the same number of decimal places.

4.250% 4.375% 4.220% 4.300%

Then write the original numbers in order from least to greatest.

$4.22\%, \quad 4\frac{1}{4}\%, \quad 4.3\%, \quad 4\frac{3}{8}\%$

E-Save Bank pays the highest interest on this two-year CD.

### CHECK YOUR UNDERSTANDING

Write the following five interest rates in descending order (greatest to least):

$5.51\%, \ 5\frac{1}{2}\%, \ 5\frac{5}{8}\%, \ 5.099\%, \ 5.6\%$

---

**TEACH**

Students have seen the simple interest formula before. Remind them that they need to change percents to equivalent decimals before substituting interest rates into the formula. You can review some of the more difficult percent conversions with fractions on the board. Remind them to write the formula in every problem in which they are using it.

**EXAMPLE 1**

Sometimes students enter division problems on the calculator in the incorrect order. Give students practice in entering division problems on the calculator, when they are given with the dividend/divisor notation or the divisor )dividend notation.

Filling in the extra zeroes on the right makes it easier to compare the numbers, because they will have the same number of decimal places and the decimal points can be lined up. All students may not need to do this, but it is a good method for students who need it.

**CHECK YOUR UNDERSTANDING**

***Answer*** $5\frac{5}{8}\%$, 5.6%, 5.51%, $5\frac{1}{2}\%$, 5.099%

Students should write the final answers using the original forms of the percents.



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## EXAMPLE 2

Raoul's savings account must have at least \$500, or he is charged a \$4 fee. His balance was \$716.23, when he withdrew \$225. What was his balance?

**SOLUTION**

Subtract the withdrawal. $716.23 - 225.00 = 491.23$

Compare to the minimum balance. $491.23 < 500$

Subtract the penalty. $491.23 - 4.00 = 487.23$

Raoul's balance after the withdrawal and penalty is \$487.23.

### CHECK YOUR UNDERSTANDING

Mae has \$891 in her account. A \$7 fee is charged each month the balance is below \$750. She withdraws \$315. If she makes no deposits or withdrawals for the next $x$ months, express her balance algebraically.

## EXAMPLE 3

Mitchell deposits \$1,200 in an account that pays 4.5% simple interest. He keeps the money in the account for three years without any deposits or withdrawals. How much is in the account after three years?

**SOLUTION** Use the simple interest formula, $I = prt$. The interest rate is given as a percent, but you need to express it as a decimal.

Substitute to find the interest. $I = (1{,}200)(0.045)(3) = 162$

Add the interest and the principal. $162 + 1{,}200 = 1{,}362$

The balance after three years is \$1,362.

### CHECK YOUR UNDERSTANDING

How much simple interest is earned on \$4,000 in $3\frac{1}{2}$ years at an interest rate of 5.2%?

## EXAMPLE 4

How much simple interest does \$2,000 earn in 7 months at an interest rate of 5%?

**SOLUTION** Use the simple interest formula, $I = prt$. Convert 5% to a decimal and 7 months to years.

$r = 5\% = 0.05$ $t = 7 \text{ months} = \frac{7}{12} \text{ years}$

Substitute and simplify. Round. $I = (2{,}000)(0.05)\left(\frac{7}{12}\right) \approx 58.33$

The account earns \$58.33.

### CHECK YOUR UNDERSTANDING

How much simple interest would \$800 earn in 300 days in a non-leap year at an interest rate of 5.71%? Round to the nearest cent.

### EXAMPLE 2

Point out that some banks do not give interest on accounts that fall below a minimum balance, and that it is important for them to know their bank's policies.

### CHECK YOUR UNDERSTANDING

***Answer*** $576 - 7x$

### EXAMPLE 3

Remind students that it is always a good idea to write the formula before starting these exercises. It makes the substitution easier, and allows a teacher to pinpoint an error that led to an incorrect answer.

### CHECK YOUR UNDERSTANDING

***Answer*** \$728

Give students additional practice on the board representing fractions of a year in months, and months as fractions of a year.

### EXAMPLE 4

Students can ignore the fact that the months can have 28, 29, 30, or 31 days in them.

### CHECK YOUR UNDERSTANDING

***Answer*** \$37.55

This problem extends the fraction of a year notion to include days as the units. In a non-leap year, 300 days is 300/365 of a year.



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## EXAMPLE 5

Manipulating literal equations is important in algebra, and Examples 5, 6, and 7 give the students practice with it.

## CHECK YOUR UNDERSTANDING

***Answer*** $4,615.38

Remind students that there are no additional withdrawals or deposits to this account over the two years.

## EXAMPLE 6

Give students practice in manipulating literal equations that include addition and subtraction.

## CHECK YOUR UNDERSTANDING

***Answer*** approximately 9 years

## EXAMPLE 7

Review the notation differences between equivalent quantities 0.04 and 4%. Point out that these expressions are not equivalent to 0.04%.

## CHECK YOUR UNDERSTANDING

***Answer*** 16%

Some students may assume that earning $200 means "at least $200", and can write any percent over 16%.

## EXAMPLE 5

How much principal must be deposited to earn $1,000 simple interest in 2 years at a rate of 5%?

**SOLUTION** Use the simple interest formula and solve for $p$.

Divide each side by $rt$ and simplify. $\frac{I}{rt} = \frac{prt}{rt} \rightarrow \frac{I}{rt} = p$

Substitute and simplify. $p = \frac{1{,}000}{(0.05)(2)} = 10{,}000$

A principal of $10,000 must be deposited.

### CHECK YOUR UNDERSTANDING

How much principal must be deposited in a two-year simple interest account that pays $3\frac{1}{4}\%$ interest to earn $300 in interest?

## EXAMPLE 6

Derek has a bank account that pays 4.1% simple interest. The balance is $910. When will the account grow to $1,000?

**SOLUTION** Find the interest, $I = 1{,}000 - 910 = 90$.

Use the formula and solve for $t$. $t = \frac{I}{pr}$

Substitute and simplify. Round. $t = \frac{90}{(1{,}000)(0.041)} = 2.2$ years

Convert time to months. $t = (2.2)(12) = 26.4$

Derek's account will grow to $1,000 in approximately 27 months.

### CHECK YOUR UNDERSTANDING

How long will it take $10,000 to double at 11% simple interest?

## EXAMPLE 7

Kerry invests $5,000 in a simple interest account for 5 years. What interest rate must the account pay so there is $6,000 at the end of 5 years?

**SOLUTION** Subtract to find the interest, $I = 6{,}000 - 5{,}000 = 1{,}000$.

Use the formula and solve for $r$. $r = \frac{I}{pt}$

Substitute and simplify. $r = \frac{1{,}000}{(5{,}000)(5)} = 0.04 = 4\%$

The account must pay 4% annual simple interest.

### CHECK YOUR UNDERSTANDING

Marcos deposited $500 into a 2.5-year simple interest account. He wants to earn $200 interest. What interest rate must the account pay?



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## Applications

*Anything that we can do to raise personal savings is very much in the interest of this country.*

**Alan Greenspan,** Economist

**1.** How might those words apply to what has been outlined in this lesson? What "play on words" do you notice in Greenspan's quote? See margin.

**2.** Arrange the following interest rates in ascending order: 3.4%, 3.039%, $3\frac{3}{16}$%, 3.499%, $3\frac{1}{2}$%. 3.039%, $3\frac{3}{16}$%, 3.4%, 3.499%, $3\frac{1}{2}$%

**3.** Josh has a savings account at a bank that charges a $10 fee for every month his balance falls below $1,000. His account has a balance of $1,203.44 and he withdraws $300. What will his balance be in six months if he makes no deposits or withdrawals? $843.44

**4.** Linda's savings account has fallen below the $1,000 minimum balance required to receive interest. It is currently $871.43. The monthly fee charged by the bank for falling below the minimum is $x$ dollars. Express algebraically how you compute the number of months it will take Linda's account to reach a zero balance if she makes no deposits. Explain. If $x = 9$, how many months will it take? See margin.

**5.** John, Paul, and George are having a disagreement over interest rates. John says that $6\frac{3}{4}$% can be expressed as 6.75%. George thinks that $6\frac{3}{4}$% can be expressed as 0.0675. Paul remembers converting percents to equivalent decimals and thinks it can be expressed as 0.0675%. Who is correct, and who is incorrect? Explain. See margin.

**6.** Beth and Mark would like to put some savings in the bank. They most likely will not need this money for 4 years, so Beth wants to put it in a four-year CD. Mark wants to put the money in a passbook savings account. What is the advantage of a CD? What is the disadvantage? See margin.

**7.** Find the simple interest on a $2,350 principal deposited for six years at a rate of 4.77%. $672.57

**8.** Ryan deposits $775 in an account that pays 4.24% simple interest for four years. Brian deposits $775 in an account that pays 4.24% simple interest for one year.

**a.** What is Ryan's interest after the four years? $131.44
**b.** What is Ryan's balance after four years? $906.44
**c.** How much interest did Ryan's account earn the first year? $32.86
**d.** How much interest did Ryan's account earn the fourth year? $32.86
**e.** What is Brian's interest after the first year? $32.86
**f.** What is Brian's balance after the first year? $807.86
**g.** Suppose Brian withdraws all of the principal and interest after the first year and deposits it into another one-year account at the same rate, what is his interest for the second year? Round to the nearest cent. $34.25
**h.** Compare the interest Brian earns with the interest Ryan earns for the second year. Who earned more interest? Explain. Ryan earned more; he earned interest on his interest since he opened up a new account.

### TEACH

**Exercise 2**
When students need to convert percents to fractions, have them convert the fractional part to a decimal first, and then move the decimal point.

**Exercise 8**
This exercise plants the seeds for the underlying concept of compound interest, which is in the next lesson. Notice that if Brian re-deposits his principal and interest each year, he is compounding his interest.

### ANSWERS

1. In addition to the pun on 'interest', Greenspan thinks that savings help in several ways. First, they provide citizens with a financial cushion. They also give banks more money to lend for people to buy homes, cars, and so on.
4. $\frac{871.43}{x}$; 97 months; although the quotient is 96.825, it is not until the 97th month that the balance will reach zero.
5. John and George are correct. Paul is incorrect—when the percent is changed to an equivalent decimal, the percent sign is dropped.
6. The advantage is a higher rate of interest. The disadvantage is the CD has a penalty if the money is withdrawn before maturity.



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## TEACH

**Exercise 9**
Remind students that if the number of months is not a counting number, they need to round up.

**Exercises 15 and 18**
Remind students that if they have trouble with formulating literal algebraic expressions, they can solve the same problem with numbers first, to see how the variables should be manipulated.

**Exercises 16 and 17**
Point out that there are no deposits made to these accounts, and that might be unrealistic. Tell them they will learn how to handle long-term accounts with deposits being made in a later lesson.

## ANSWERS

9. a. $268
   b. $179.38
   c. $1,211.51
   d. about 20 months
   e. about 22 months
   f. 5.56%
   g. $4,545.45
   h. $\frac{x}{0.03p}$

**9.** Use the simple interest formula to find the missing entries in the table. Round monetary amounts to the nearest cent. See margin.

| Interest | Principal | Rate (to the nearest hundredth of a percent) | Time |
|---|---|---|---|
| **a.** | $2,000 | 3.35% | 4 years |
| **b.** | $3,500 | 4.1% | 15 months |
| **c.** | $20,100 | 5.5% | 400 days |
| $100 | $700 | 8.8% | **d.** |
| $250 | $3,000 | $4\frac{3}{4}$% | **e.** |
| $500 | $3,000 | **f.** | 3 years |
| $500 | **g.** | 4.4% | 30 months |
| $x$ | $p$ | 3% | **h.** |

**10.** How much simple interest does $2,560 earn in 17 months at a rate of $5\frac{1}{8}$%? Round to the nearest cent. $185.87

**11.** How long does it take $450 to double at a simple interest rate of 14%? approximately 86 months

**12.** How long does it take $450 to double at a simple interest rate of 100%? one year

**13.** What interest rate is needed for $9,500 to earn $900 in 19 months? Round to the nearest hundredth of a percent. 5.98%

**14.** Assume $20,000 is deposited into a savings account. Bedford Bank offers an annual rate of 4% simple interest for five years. Slick Bank offers a rate of 20% simple interest for one year. Which earns more interest? Neither; they are the same.

**15.** Assume $\$x$ is deposited into a savings account. Blank Bank offers an annual rate of $r$% for $y$ years. Thank Bank offers a rate of $ry$% for one year. Which earns more interest? Neither; they are the same.

**16.** A couple is planning a savings account for a newborn baby. They start with $3,450 received in newborn baby gifts. If no deposits or withdrawals are made, what is the balance of the account if it earns simple interest at 5% interest for 18 years? $6,555

**17.** Ron estimates that it will cost $400,000 to send his daughter to a private college in 18 years. He currently has $90,000 to deposit in an account. What simple interest rate must his account have to reach a balance of $400,000 in 18 years? Round to the nearest percent. 19%

**18.** Zoe creates a spreadsheet to make simple interest calculations. The user inputs values for the principal, rate, and time in years in row 2. Write each formula.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **Interest** | **Principal** | **Rate** | **Time in Years** | **Time in Months** |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

**a.** For A2 to compute the interest. =B2*C2*D2
**b.** For B2 to compute the principal. =A2/(C2*D2)
**c.** For C2 to compute the interest rate. =A2/(B2*D2)
**d.** For D2 to compute time in years, given the interest, rate, and the principal. =A2/(B2*C2)
**e.** For E2 to compute the time in months, given the time in years. =12*D2



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*In the old days a man who saved money was a miser; nowadays he's a wonder.*

Author Unknown

# Explore Compound Interest 3-4

## Key Terms

- compound interest
- annual compounding
- semiannual compounding
- quarterly compounding
- daily compounding
- crediting

## Objectives

- Understand the concept of getting interest on your interest.
- Compute compound interest using a table.

## What is compound interest?

When opening up any bank account, the annual interest rate is of major concern to most consumers. However, it is not enough to just know the interest rate. *How* the interest is computed should also be known.

Principal is used to compute interest. For simple interest, only the original principal is used to compute annual interest. Principal increases each time interest is added to the account. Sometimes, interest is computed using the new principal. That is, the account earns interest on the interest. **Compound interest** is money earned on the money deposited plus previous interest. This is not the case for simple interest. For simple interest, only the original principal is used to compute annual interest.

Interest can be compounded in different ways.

- **Annual compounding** is interest compounded once each year.
- **Semiannual compounding** is interest compounded twice per year, or every six months.
- **Quarterly compounding** is interest compounded four times per year, or every three months.
- **Daily compounding** is interest compounded every day. There are 365 days in a year and 366 days in a leap year.

The most common form of compounding is daily compounding. The bank pays interest every single day, based on that day's principal. The bank, however, does not add the interest every day. They keep a record of interest earned and add it into the account monthly or quarterly. This is called **crediting** an account. Compounding daily and crediting monthly is the most common procedure used by banks today.

### EXAMINE THE QUESTION

When interest is added to principal, the principal increases, and the resulting interest for the next period increases.

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## Skills and Strategies

Here you will get an understanding of how compound interest works. This will help you see how interest accumulates. As you read the examples, compare the results from each to the examples before it.

### EXAMPLE 1

How much interest would $1,000 earn in one year at a rate of 6%, compounded annually? What would be the new balance?

**SOLUTION** The first year of interest compounded annually can be found using the simple interest formula.

| | |
|---|---|
| Convert 6% to a decimal. | $r = 6\% = 0.06$ |
| Use the simple interest formula. | $I = prt$ |
| Substitute. | $I = 1{,}000 \times 0.06 \times 1 = 60$ |

The account would earn $60 interest, and the balance after one year is $1,060.

**CHECK YOUR UNDERSTANDING**

How much would $x$ dollars earn in one year at a rate of 4.4% compounded annually?

### EXAMPLE 2

Maria deposits $1,000 in a savings account that pays 6% interest, compounded semiannually. What is her balance after one year?

**SOLUTION** Accounts that pay interest semiannually have the interest added on twice each year (every six months).

| | |
|---|---|
| Convert 6% to a decimal. | $r = 6\% = 0.06$ |
| Convert 6 months to years. | $t = 6 \text{ months} = 0.5 \text{ years}$ |
| Use the simple interest formula. | $I = prt$ |
| Substitute and simplify. | $I = 1{,}000 \times 0.06 \times 0.5 = 30$ |
| Add the interest to the principal. | $1{,}000 + 30 = 1{,}030$ |

This new principal, $1,030, is used to compute the next six months' interest.

| | |
|---|---|
| Use the simple interest formula. | $I = prt$ |
| Substitute and simplify. | $I = 1{,}030 \times 0.06 \times 0.5 = 30.90$ |
| Add the interest to the principal. | $1{,}030 + 30.90 = 1{,}060.90$ |

Maria's balance is $1,060.90 after one year. Compare this to the balance from Example 1.

**CLASS DISCUSSION**

Point out that the interest for a subsequent interest period would be based on this adjusted principal—that the higher amount is used for the following interest period. It may sound obvious to some, but isn't necessarily to someone just being exposed to compound interest.

Could interest be compounded every hour? How many hours are in a year? Could interest be compounded every minute?

**TEACH**

Students will use the simple interest formula to make compound interest calculations. They will do a day-by-day analysis of a daily-compounded account so they can really see the interest being computed on a higher principal each day. After understanding these examples, students will be ready to learn the compound interest formula in the next lesson.

**EXAMPLE 1**

Since interest rates are reported for an annual interest period, yearly compounding is equivalent to simple interest for one year. However, point out that for the second year, under simple interest, the principal in this problem would still be $1,000, and under yearly compounding, it would be $1,060.

**CHECK YOUR UNDERSTANDING**

***Answer*** 0.044$x$ dollars

**EXAMPLE 2**

This problem uses the same numbers as Example 1 for comparison purposes. The only difference is the compounding period, so have students compare the answers to the two examples.



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**CHECK YOUR UNDERSTANDING**

Alex deposits $4,000 in a savings account that pays 5% interest, compounded semiannually. What is his balance after one year?

**EXAMPLE 3**

How much interest does $1,000 earn in three months at an interest rate of 6%, compounded quarterly? What is the balance after three months?

**SOLUTION** Accounts that pay interest quarterly earn interest every three months.

| | |
|---|---|
| Convert 6% to a decimal. | $r = 6\% = 0.06$ |
| Convert 1 quarter to years. | $t = 1 \text{ quarter} = 3 \text{ months} = 0.25 \text{ years}$ |
| Use the simple interest formula. | $I = prt$ |
| Substitute and simplify. | $I = 1{,}000 \times 0.06 \times 0.25 = 15$ |
| Add the interest to the principal. | $1{,}000 + 15 = 1{,}015$ |

The first quarter earns $15 interest, so the principal after one quarter is $1,015.

**CHECK YOUR UNDERSTANDING**

How much does $3,000 earn in six months at an interest rate of 4%, compounded quarterly?

**EXAMPLE 4**

How much interest does $1,000 earn in one day at an interest rate of 6%, compounded daily? What is the balance after a day?

**SOLUTION** Accounts that pay interest daily earn interest every day. There are 365 days in a year.

| | |
|---|---|
| Convert 6% to a decimal. | $r = 6\% = 0.06$ |
| Convert 1 day to years. | $t = 1 \text{ day} = \frac{1}{365} \text{ years}$ |
| Use the simple interest formula. | $I = prt$ |
| Substitute. | $I = 1{,}000 \times 0.06 \times \frac{1}{365} \approx 0.16$ |
| Add the interest to the principal. | $1{,}000 + 0.16 = 1{,}000.16$ |

The first day's interest is approximately 16 cents, so the new balance is $1,000.16. This larger principal is used to compute the next day's interest.

**CHECK YOUR UNDERSTANDING**

How much interest does $x$ dollars earn in one day at an interest rate of 5%, compounded daily? Express the answer algebraically.

**CHECK YOUR UNDERSTANDING**

***Answer*** $4,202.50

**EXAMPLE 3**

Again the numbers from the previous two examples are used, with a change in the compounding method. You can have students compute the next three quarters' interest. Then they can compare the annual interest for the three examples.

**CHECK YOUR UNDERSTANDING**

***Answer*** 3,060.30

**EXAMPLE 4**

Ask students how they would compute the interest for the entire year under daily compounding. Make sure they understand the drudgery of using this method repeatedly. Tell them that a better method will be introduced in the next lesson.

**CHECK YOUR UNDERSTANDING**

***Answer*** $(x)(0.05)\left(\frac{1}{365}\right) = \frac{0.05x}{365}$

Encourage students to simplify the form of their answers.



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## EXAMPLE 5

The calendar tables allow students to see what is going on with daily compounding. The formula, which will be introduced in the next lesson, isn't as intuitive, so it is important that students understand the process. Also, the formula cannot be used if there are sporadic deposits and withdrawals.

## CHECK YOUR UNDERSTANDING

***Answer*** $976.29

A student could do this at the board so all table entries, and not just the final answer, can be scrutinized by students.

## EXAMPLE 5

Jennifer has a bank account that compounds interest daily at a rate of 3.2%. On July 11, the principal is $1,234.98. She withdraws $200 for a car repair. She receives a $34 check from her health insurance company and deposits it. On July 12, she deposits her $345.77 paycheck. What is her balance at the end of the day on July 12?

| Date | July 11 | July 12 |
|---|---|---|
| Opening balance | $1,234.98 | |
| Deposit (+) | $34.00 | $345.77 |
| Withdrawal (−) | $200.00 | |
| Principal used to compute interest | | |
| Day's interest rounded to the nearest cent | | |
| Ending balance | | |

**SOLUTION** Organize the information in a table like the three-column table that is shown.

| Date | July 11 | July 12 |
|---|---|---|
| Opening Balance | $1,234.98 | |
| Deposit (+) | $34.00 | $345.77 |
| Withdrawal (−) | $200.00 | |
| Principal used to compute interest | $1,068.98 | |
| Day's interest rounded to the nearest cent | $0.09 | |
| Ending balance | $1,069.07 | |

For July 11, the principal used to compute interest is computed by adding the $34 deposit and subtracting the $200 withdrawal.

The day's interest is the daily compounded interest.

To find the ending balance, add the July 11 interest to the principal used to compute interest to the nearest cent.

| Date | July 11 | July 12 |
|---|---|---|
| Opening Balance | $1,234.98 | $1,069.07 |
| Deposit (+) | $34.00 | $345.77 |
| Withdrawal (−) | $200.00 | — |
| Principal used to compute interest | $1,068.98 | $1,414.84 |
| Day's interest rounded to the nearest cent | $0.09 | $0.12 |
| Ending balance | $1,069.07 | $1,414.96 |

The opening balance for July 12 is the same as the ending balance from July 11.

The July 12 deposit must be added to the opening balance before the interest for July 12 is computed.

Compute interest to the nearest cent.

Add the interest to the principal used to compute interest, to compute the ending balance.

Jennifer's balance is $1,414.96 at the end of the day on July 12.

### CHECK YOUR UNDERSTANDING

On January 7, Joelle opened a savings account with $900. It earned 3% interest, compounded daily. On January 8, she deposited her first paycheck of $76.22. What was her balance at the end of the day on January 8?



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## Applications

*In the old days a man who saved money was a miser; nowadays he's a wonder.*

Author Unknown

**1.** How might those words apply to what you learned in this lesson? See margin.

**2.** Jerome deposits \$3,700 in a certificate of deposit that pays $6\frac{1}{2}\%$ interest, compounded annually. How much interest does Jerome earn in one year? \$240.50

**3.** Sally deposits \$4,000 in a certificate of deposit that pays $6\frac{3}{4}\%$ simple interest. What is her balance after one year? \$4,270

**4.** Pierre deposits \$9,000 in a certificate of deposit that pays 8% interest, compounded semiannually. How much interest does the account earn in the first six months? What is the balance after six months? \$360; \$9,360

**5.** Kevin has $x$ dollars in an account that pays 2.2% interest, compounded quarterly. Express his balance after one quarter algebraically. $x + 0.0055x$

**6.** Regina deposits \$3,500 in a savings account that pays $7\frac{1}{2}\%$ interest, compounded semiannually.

**a.** How much interest does the account earn in the first six months? \$131.25

**b.** What is the balance at the end of the first six months? \$3,631.25

**c.** How much interest does the account earn in the second six months? \$136.17

**d.** What is the balance at the end of the year? \$3,767.42

**e.** How much interest does the account earn the first year? \$267.42

**f.** How much interest would \$3,500 earn in one year at $7\frac{1}{2}\%$ interest, compounded annually? \$262.50

**g.** How much more interest does Regina earn at an interest rate of $7\frac{1}{2}\%$ compounded semiannually than compounded annually? \$4.92

**7.** Liam deposits \$3,500 in a saving account that pays $7\frac{1}{2}\%$ interest, compounded quarterly.

**a.** Find the first quarter's interest. \$65.63

**b.** Find the first quarter's ending balance. \$3,565.63

**c.** Find the second quarter's interest. \$66.86

**d.** Find the second quarter's ending balance. \$3,632.49

**e.** Find the third quarter's interest. \$68.11

**f.** Find the third quarter's ending balance. \$3,700.60

**g.** Find the fourth quarter's interest. \$69.39

**h.** What is the balance at the end of one year? \$3,769.99

**i.** How much interest does the account earn in the first year? \$269.99

**8.** Janine opens a savings account with a deposit of \$720. The account pays 3.4% interest, compounded daily. What is the first day's interest? Round to the nearest cent. \$0.07

**9.** Laura deposits \$2,000 in an account that has an annual interest rate of 3.96%, compounded monthly. How much interest will she earn at the end of 1 month? \$6.60

### TEACH

**Exercises 2 and 3**
Note that exercise 2 asks for the interest and exercise 3 asks for the balance. Remind students not to skim math problems—every word is important.

**Exercises 4–7**
It is too much work to compute an entire year's interest using these methods of compounding.

**Exercise 8**
Ask students how much work it would be to compute an entire year's interest under daily compounding! This motivates the need for a formula, which is introduced in the next lesson.

### ANSWERS

1. Compound interest is better than simple interest, but it won't make you rich. Time is very influential in making savings grow, so the earlier an account is started, the longer the money earns interest.



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10. a. $0
b. $4,550.00
c. $4,550.50
d. $0.50
e. $4,550.50
f. $4,550.50
g. $300.00
h. $4,850.50
i. $0.53
j. $4,851.03
k. $4,851.03
l. $900.00
m. $3,951.03
n. $0.43
p. $3,951.46

11. a. $6,000.00
b. $0
c. $0
d. $6,000.00
e. $0.57
f. $6,000.57
g. $6,000.57
h. $500.00
i. $0
j. $6,500.57
k. $0.61
l. $6,501.18
m. $6,501.18
n. $0
p. $2,500.00
q. $4,001.18
r. $0.38
s. $4,001.56

14. a. $P + D$

b. $\dfrac{0.02(P + D)}{365}$

c. $P + D + \dfrac{0.02(P + D)}{365}$

d. $P + D + \dfrac{0.02(P + D)}{365}$

e. $P + D - W + \dfrac{0.02(P + D)}{365}$

f. $0.02\left(\dfrac{P + D - W + 0.02\left(\dfrac{P + D}{365}\right)}{365}\right)$

g. $(P + D - W) + \dfrac{0.02(P + D)}{365}$

**10.** Jacob opens a savings account in a non-leap year on August 10 with a $4,550 deposit. The account pays 4% interest, compounded daily. On August 11 he deposits $300, and on August 12 he withdraws $900. Find the missing amounts in the table. See margin.

| Date | Aug. 10 | Aug. 11 | Aug. 12 |
|---|---|---|---|
| Opening balance | **a.** | **f.** | **k.** |
| Deposit | **b.** | **g.** | ----- |
| Withdrawal | ----- | ----- | **l.** |
| Principal used to compute interest | **c.** | **h.** | **m.** |
| Day's interest rounded to nearest cent | **d.** | **i.** | **n.** |
| Ending balance | **e.** | **j.** | **p.** |

**11.** On December 18 of a leap year, Stacy opened a savings account by depositing $6,000. The account pays 3.45% interest, compounded daily. On December 19 she deposited $500, and on December 20 she withdrew $2,500. Find the missing amounts in the table. Round to the nearest cent. What is her opening balance on December 21? See margin; $4,001.56

| Date | Dec. 18 | Dec. 19 | Dec. 20 |
|---|---|---|---|
| Opening balance | **a.** | **g.** | **m.** |
| Deposit | **b.** | **h.** | **n.** |
| Withdrawal | **c.** | **i.** | **p.** |
| Principal used to compute interest | **d.** | **j.** | **q.** |
| Day's interest rounded to nearest cent | **e.** | **k.** | **r.** |
| Ending balance | **f.** | **l.** | **s.** |

**12.** On May 29, Rocky had an opening balance of $x$ dollars in an account that pays 3% interest, compounded daily. He deposits $y$ dollars. Express his ending balance on May 30 algebraically. $x + y + \dfrac{0.03(x + y)}{365}$

**13.** Linda has $d$ dollars in an account that pays 3.4% interest, compounded weekly. She withdraws $w$ dollars. Express her first week's interest algebraically. $\dfrac{0.034(d - w)}{52}$

**14.** The table represents the compound interest calculations for an account that pays 2% interest compounded daily. Represent a–g algebraically. See margin.

| Date | Feb. 2 | Feb. 3 |
|---|---|---|
| Opening balance | $P$ | **d.** |
| Deposit | $D$ | ------ |
| Withdrawal | ----- | $W$ |
| Principal used to compute interest | **a.** | **e.** |
| Interest | **b.** | **f.** |
| Ending balance | **c.** | **g.** |

**15.** One day before the end of the month, George had an opening balance of $m$ dollars in an account that pays 2.25% interest compounded monthly. On the last day of the month, he made a deposit equal to twice his opening balance. Express his ending balance on the last day of the month algebraically. $\dfrac{3m + 0.0225(3m)}{12}$



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*To make a million, start with $900,000.*
**Morton Shulman,** Politician, Businessman, and Television Personality

# Compound Interest Formula 3-5

## Key Terms

- **compound interest formula**
- **annual percentage rate (APR)**
- **annual percentage yield (APY)**

## Objectives

- Become familiar with the derivation of the compound interest formula.
- Make computations using the compound interest formula.

## What are the advantages of using the compound interest formula?

Julio deposited $10,000 in a five-year CD, with the intention of using the money for his son's college education. The account pays 5.2% interest compounded daily. There will be no deposits or withdrawals during the five years. Julio wants to know how much the $10,000 will grow to by the end of the five years. Imagine if he set up a daily compound interest table as in the last lesson. There are over 1,800 days in five years, so the table would get quite tedious. It is not practical to solve this problem one day at a time.

Calculating compound interest using the simple interest formula is tedious when there are numerous periods. The power of mathematics can turn this long procedure into a relatively small amount of work. Numerical examples and algebra can be combined to uncover a pattern that leads to a formula that finds compound interest. The **compound interest formula** relates principal, interest rate, the number of times interest is compounded per year, and the number of years the money will be on deposit, and the ending balance. The formula is used for any type of compounding: annually, semiannually, monthly, weekly, daily, and so on.

In Lesson 3-3, you used the annual interest rate to compute interest. Banks call this the **annual percentage rate (APR)**. Most banks advertise the **annual percentage yield (APY)** since it is higher than the APR for accounts compounded more than once per year. The bank takes the dollar amount of interest you earn under the compounding to create the APY. The APY is the simple interest rate that would be required to give the same dollar amount of interest that the compounding gave. Therefore, annual percentage yield (APY) is an annual rate of interest that takes into account the effect of compounding.

### EXAMINE THE QUESTION

Exercises 8–14 from the previous lesson illustrate how tedious weekly and

(continued on next page)

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daily compound interest calculations can get. Ask students to imagine how difficult it would be for a bank to do this for thousands of customers, before there were computers! Explain that someone programmed a formula into the computers, and that they are about to learn that formula.

### CLASS DISCUSSION

Remind students about the calendar computations from the last lesson. Have them approximate how long it would take them to do the computations for five years!

### TEACH

Students will get to see how the compound interest formula can be derived. They will have a chance to look for patterns and generalize to discover the formula.

### EXAMPLE 1

Elicit patterns the students see when the example is finished.

## Skills and Strategies

Here you will solve some compound interest problems and then look for a pattern to derive the compound interest formula.

### EXAMPLE 1

Jose opens a savings account with principal $P$ dollars that pays 5% interest, compounded quarterly. What will his ending balance be after one year?

**SOLUTION 1** Find the first quarter's interest, where $p = P$, $r = 0.05$, and $t = \frac{1}{4}$.

Use the simple interest formula. $I = prt$

Substitute. $I = (P)(0.05)\left(\frac{1}{4}\right)$

Simplify. $I = \frac{0.05}{4}P$

Let $B_1$ represent the first quarter's ending balance, the sum of $P$ and the first quarter's interest.

Principal + Interest $B_1 = P + \frac{0.05}{4}P$

Factor out $P$. $B_1 = P\left(1 + \frac{0.05}{4}\right)$

To get the second quarter's ending balance, follow the same procedure with the new balance $B_1$.

Principal + Interest $B_2 = B_1 + \frac{0.05}{4}B_1$

Factor out $B_1$. $B_2 = B_1\left(1 + \frac{0.05}{4}\right)$

Substitute $P\left(1 + \frac{0.05}{4}\right)$ for $B_1$. $B_2 = p\left(1 + \frac{0.05}{4}\right)\left(1 + \frac{0.05}{4}\right)$

Write in exponential form. $B_2 = P\left(1 + \frac{0.05}{4}\right)^2$

To get the third quarter's ending balance, follow the same procedure with the new balance $B_2$.

Principal + Interest $B_3 = B_2 + \frac{0.05}{4}B_2$

Factor out $B_2$. $B_3 = B_2\left(1 + \frac{0.05}{4}\right)$

Substitute $P\left(1 + \frac{0.05}{4}\right)^2$ for $B_2$. $B_3 = P\left(1 + \frac{0.05}{4}\right)^2\left(1 + \frac{0.05}{4}\right)$

Write in exponential form. $B_3 = P\left(1 + \frac{0.05}{4}\right)^3$



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To get the fourth quarter's ending balance, follow the same procedure with the new balance $B_3$.

Factor out $B_4$. $$B_4 = B_3 + \frac{0.05}{4} B_3 = B_3\left(1 + \frac{0.05}{4}\right)$$

Substitute $P\left(1 + \frac{0.05}{4}\right)^3$ for $B_3$. $$B_4 = P\left(1 + \frac{0.05}{4}\right)^3\left(1 + \frac{0.05}{4}\right)$$

Ending balance after one year $$B_4 = P\left(1 + \frac{0.05}{4}\right)^4$$

This is the balance after one year. Examine the formula for patterns.

**CHECK YOUR UNDERSTANDING**

Rico deposits \$800 at 3.87% interest, compounded quarterly. What is his ending balance after one year? Round to the nearest cent.

## EXAMPLE 2

If you deposit $P$ dollars for one year at 5% compounded daily, express the ending balance algebraically.

**SOLUTION** Use the formula from Example 1 and make adjustments for daily compounding. When the interest was compounded quarterly, there was a denominator of 4 and an exponent of 4 in the formula.

$$B_4 = P\left(1 + \frac{0.05}{4}\right)^4$$

With daily compounding, these entries are replaced with 365. Rewrite the formula.

Ending balance after one year $$B = P\left(1 + \frac{0.05}{365}\right)^{365}$$

This is the ending balance expressed algebraically.

**CHECK YOUR UNDERSTANDING**

Nancy deposits \$1,200 into an account that pays 3% interest, compounded monthly. What is her ending balance after one year? Round to the nearest cent.

**EXTEND YOUR UNDERSTANDING**

Nancy receives two offers in the mail from other banks. One is an account that pays 2.78% compounded daily. The other account pays 3.25% compounded quarterly. Would either of these accounts provide Nancy with a better return than her current account? If so, which account?

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### CHECK YOUR UNDERSTANDING

***Answer*** \$831.41

Show how parentheses must be placed when the expression is entered on a calculator. Make sure students enter the entire expression into the calculator, and do not compute separate parts of the expression, round them, and then make subsequent calculations.

### EXAMPLE 2

Give other examples of weekly, hourly, monthly compounding using the formula, so students get used to adjusting the formula.

### CHECK YOUR UNDERSTANDING

***Answer*** \$1,236.50

Have students compare this to the balance under simple interest, which is \$1,236.

### EXTEND YOUR UNDERSTANDING

***Answer*** yes; the account that pays 3.25% compounded quarterly



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## EXAMPLE 3

Point out that the product representing the exponent must be put in parentheses for the calculator to compute this correctly. Have students write calculator keystroke sequences given the formula for practice.

## CHECK YOUR UNDERSTANDING

***Answer*** $2,743.45

Look at the calculator entries of students who got incorrect answers. Look for parentheses placement errors.

## EXTEND YOUR UNDERSTANDING

***Answer***

$2{,}000\left(1+\frac{0.035}{365}\right)^{365k}$

# Compound Interest Formula

Examples 1 and 2 involved accounts for one year. The exponent and the denominator in those formulas are the number of times the interest is compounded *in one year*. You can leave your money in for more than one year. The formula used to compute the ending balance includes the variable $t$, where $t$ is the number of years.

**Compound Interest Formula**

$$B = p\left(1+\frac{r}{n}\right)^{nt}$$

where $B$ = ending balance
$p$ = principal or original balance
$r$ = interest rate expressed as a decimal
$n$ = number of times interest is compounded annually
$t$ = number of years

## EXAMPLE 3

Marie deposits $1,650 for three years at 3% interest, compounded daily. What is her ending balance?

**SOLUTION** Use the compound interest formula. The values for the variables are $p = 1{,}650$, $r = 0.03$, $n = 365$, and $t = 3$.

Substitute the values for Marie's account.

$$B = 1{,}650\left(1+\frac{0.03}{365}\right)^{365(3)}$$

Use your calculator to enter the expression. Enter the entire expression; try not to do it in separate terms. The keystrokes are:

1650(1+0.03/365)^(365×3) ENTER

```
1650(1+0.03/365)
^(365*3)
        1805.380891
```

Marie's ending balance, to the nearest cent, is $1,805.38.

### CHECK YOUR UNDERSTANDING

Kate deposits $2,350 in an account that earns interest at a rate of 3.1%, compounded monthly. What is her ending balance after five years? Round to the nearest cent.

### EXTEND YOUR UNDERSTANDING

Write an algebraic expression for the ending balance after $k$ years of an account that starts with a balance of $2,000 and earns interest at a rate of 3.5%, compounded daily.



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## EXAMPLE 4

Sharon deposits $8,000 in a one year CD at 3.2% interest, compounded daily. What is Sharon's annual percentage yield (APY) to the nearest hundredth of a percent?

**SOLUTION** Find the APY using the compound interest formula and the simple interest formula.

Use the compound interest formula. $B = p\left(1 + \frac{r}{n}\right)^{nt}$

Substitute. $B = 8{,}000\left(1 + \frac{0.032}{365}\right)^{365\times 1}$

Simplify. $B = 8{,}260.13$

Subtract the principal from the new balance.

$$I = 8{,}260.13 - 8{,}000 = 260.13$$

Use the simple interest formula. $I = prt$

Solve for $r$. $r = \frac{I}{pt}$

Substitute. $r = \frac{260.13}{8{,}000 \times 1}$

Simplify. $r \approx 0.0325 = 3.25\%$

The annual percentage yield is 3.25%.

APY can also be found by using the formula $APY = \left(1 + \frac{r}{n}\right)^{n} - 1$, where $r$ is the interest rate and $n$ is the number of times interest is compounded per year.

Use the *APY* formula. $APY = \left(1 + \frac{r}{n}\right)^{n} - 1$

Substitute. $APY = \left(1 + \frac{0.032}{365}\right)^{365} - 1$

Simplify. $APY \approx 0.0325 = 3.25\%$

The annual percentage yield is 3.25%, which is the same as the previous answer.

### CHECK YOUR UNDERSTANDING

Barbara deposits $3,000 in a one year CD at 4.1% interest, compounded daily. What is the APY to the nearest hundredth of a percent?

### EXTEND YOUR UNDERSTANDING

Consider an amount $x$ deposited into a CD at 2.4% interest compounded daily, and the same amount deposited into a CD at the same rate that compounds monthly. Explain why, after 1 year, the balance on a CD that compounds daily is greater than the CD that compounded monthly.

### EXAMPLE 4

The annual percentage yield gives the simple interest rate that would have earned Sharon the same dollar amount of interest, if she began with the same $2,350. The APY is more than the APR since the APY doesn't give interest on the interest like compounding. Since the principals used to compute interest when compounding are not used in simple interest, the APY must be higher to compensate for the lower principal used throughout the process.

### CHECK YOUR UNDERSTANDING

***Answer*** 4.18%

### EXTEND YOUR UNDERSTANDING

***Answer*** Because there are more compounding periods for the interest to be earned on already accumulated interest, the balance grows quicker.



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## Applications

*To make a million, start with $900,000.*
**Morton Shulman,** Politician, Businessman, and Television Personality

### TEACH

**Exercises 1–11**
Remind students to write the formula for each problem, and identify the value for each variable in the formula. This way, if they enter an incorrect keystroke sequence into the calculator, you can troubleshoot where their difficulties began. You can also use it to give partial credit on a graded assignment or exam.

### ANSWERS

1. Bank interest on its own will not make you rich; the interest rates are much smaller than possible returns on business investments. However, there is much less risk.

**1.** How might these words apply to what is in this lesson? See margin.

**2.** Jimmy invests $4,000 in an account that pays 5% annual interest, compounded semiannually. What is his balance, to the nearest cent, at the end of 10 years? $6,554.47

**3.** On Olga's 16th birthday, her uncle invested $2,000 in an account that was locked into a 4.75% interest rate, compounded monthly. How much will Olga have in the account when she turns 18? Round to the nearest cent. $2,198.91

**4.** Samantha deposits $1,500 into the Park Street Bank. The account pays 4.12% annual interest, compounded daily. To the nearest cent, how much is in the account at the end of three non-leap years? $1,697.33

**5.** Joanne deposits $4,300 into a one-year CD at a rate of 4.3%, compounded daily.
**a.** What is her ending balance after the year? $4,488.92
**b.** How much interest does she earn? $188.92
**c.** What is her annual percentage yield to the nearest hundredth of a percent? 4.39%

**6.** Mike deposits $5,000 in a three-year CD account that yields 3.5% interest, compounded weekly. What is his ending balance at the end of three years? $5,553.36

**7.** Rob deposits $1,000 in a savings account at New York State Bank that pays 4.4% interest, compounded monthly.
**a.** How much is in his account at the end of one year? $1,044.90
**b.** What is the APY for this account to the nearest hundredth of a percent? 4.49%

**8.** How much more does $1,000 earn in eight years, compounded daily at 5%, than $1,000 over eight years at 5%, compounded semiannually? $7.27

**9.** If $3,000 is invested at an interest rate of 4.8%, compounded hourly for two years, what is the ending balance? $3,302.28

**10.** Mike and Julie receive $20,000 in gifts from friends and relatives for their wedding. They deposit the money into an account that pays 4.75% interest, compounded daily.
**a.** Will their money double in fourteen years? no
**b.** Will their money double in fifteen years? yes

**11.** Lindsay invests $80 in an account that pays 5% annual interest, compounded monthly. Michele invests $60 in an account that pays 8% annual interest, compounded weekly.
**a.** Whose balance is greater after one year? Lindsay's
**b.** Whose balance is greater after twelve years? Michele's



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**12.** Investigate the difference between compounding annually and simple interest for parts a–j.

**a.** Find the simple interest for a one-year CD for $5,000 at a 6% interest rate. $300

**b.** Find the interest for a one-year CD for $5,000 at an interest rate of 6%, compounded annually. $300

**c.** Compare the results from parts a and b. The interest is the same.

**d.** Find the simple interest for a three-year CD for $5,000 at an interest rate of 6%. $900

**e.** Find the interest for a three-year CD for $5,000 at an interest rate of 6%, compounded annually. $955.08

**f.** Compare the results from parts d and e. See margin.

**g.** Find the simple interest for a six-year CD for $5,000 at an interest rate of 4%. $1,200

**h.** Find the interest for a six-year CD for $5,000 at an interest rate of 4%, compounded annually. $1,326.60

**i.** Compare the results from parts g and h. See margin.

**j.** Is interest compounded annually the same as simple interest? Explain. See margin.

**13.** Rodney invests a sum of money, $P$, into an account that earns interest at a rate of $r$, compounded yearly. Gerald invests half that amount into an account that pays twice Rodney's interest rate. Which of the accounts will have the higher ending balance after one year? Explain. See margin.

**14.** Island Bank is advertising a special 6.55% APR for CDs. Manny takes out a one-year CD for $40,000. The interest is compounded daily. Find the annual percentage yield for Manny's account to the nearest hundredth of a percent. 6.77%

**15.** Businesses deposit large sums of money into bank accounts. Imagine an account with 10 million dollars in it.

**a.** How much would the account earn in one year of simple interest at a rate of 5.12%? $512,000

**b.** How much would the account earn in one year at 5.12% if the interest was compounded daily? $525,296.00

**c.** How much more interest is earned by interest compounded daily compared to simple interest? $13,296

**16.** An elite private college receives large donations from successful alumni. The account that holds these donations has $955,000,000 currently.

**a.** How much would the account earn in one year of simple interest at a rate of 5.33%? $50,901,500

**b.** How much would the account earn in one year at 5.33% if the interest was compounded daily? Round to the nearest cent. $52,278,530.93

**c.** How much more interest is earned by compounded daily as compared to simple interest? $1,377,030.93

**d.** If the money is used to pay full scholarships, and the price of tuition is $61,000 per year to attend, how many more students can receive full four-year scholarships if the interest was compounded daily rather than using simple interest? 22

**Exercises 15 and 16**
Students get a glimpse into the world of high finance. Seldom do they get a chance to see the large amount of interest that can be earned on savings accounts with high principals.

## ANSWERS

12f. The annual compounded interest earned $55.08 more than the simple interest.

12i. The annual compounded interest earned $126.60 more than the simple interest.

12j. No; they are the same for one year. For anything longer, compounded interest grows faster than simple interest.

13. Rodney's account balance will always be greater; $p\left(1 + \frac{r}{1}\right)^t > 0.5p\left(1 + \frac{2r}{1}\right)^t$, or $p(1 + r) > p(0.5 + r)$



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*The infinite! No other question has ever moved so profoundly the spirit of man.*

**David Hilbert,** Mathematician

# 3-6 Continuous Compounding

## Objectives

- Compute interest on an account that is continuously compounded.

## Key Terms

- **limit**
- **finite**
- **infinite**
- **continuous compounding**
- **exponential base *(e)***
- **continuous compound interest formula**

### EXAMINE THE QUESTION

Pose the idea of compounding daily, hourly, every minute, every second, and so on. Can you compound every half-second? Every millisecond? Microsecond?

At the end of the lesson introduction, students are asked about depositing $1,000 at 100% interest compounded continuously for one year. As they offer their estimates for the balance after one year, write their guesses on the board. Students may not realize that the frequent interest calculations counteract the smaller amounts of interest being added on each period.

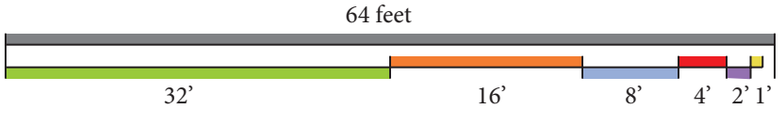
### CLASS DISCUSSION

You can have a student actually walk across the classroom, walking half of the remaining distances each time. They are really adding an infinite number of "half of the remaining distances." Yet they reach the other side of the room, proving that this infinite sum has a finite answer. You can point out that all infinite sums do not necessarily have a finite answer; for example, 1 + 2 + 3 + 4 + ... does not have a finite sum.

## How can interest be compounded continuously?

Compounding interest daily makes money grow more quickly than simple interest. It is possible to compound interest every hour, every minute, even every second! There are over 31 million seconds in a year. The compound interest formula works with seconds just as it did for compounding daily. There are one million microseconds in one second! It works even if interest is compounded every microsecond!

How do millions of compounds affect the ending balance after a year? To understand this, you need to learn about **limits**. Imagine you want to walk all the way across a 64-foot wide room. The length 64 feet is a **finite** distance—it can be represented by a real number. To do this, you first must walk halfway across the room, or 32 feet. To continue the walk, you must cover half of the remaining 32 feet, which is 16 feet. Then you must cover half of the remaining 16 feet, which is 8 feet. Next, you must cover half of the remaining 8 feet, which is 4 feet. Then, you need to cover half of the remaining 4 feet, which is 2 feet. Next, you need to cover half of the remaining 2 feet, which is 1 foot. Then you need to cover $\frac{1}{2}$ foot, then $\frac{1}{4}$ foot, then $\frac{1}{8}$ foot, and so on. The distances walked so far are shown in the diagram below.




$$32 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

What do the three dots at the end of the expression mean? Because there will always be some distance between you and the wall, no matter how small, you will always have a positive number to take half of. You will be taking half of the remaining distance infinitely many times! The expression will never end. Yet, you know you can touch the wall you were walking towards. And you know it is 64 feet away. Conclusion? You can add an **infinite** amount of numbers and get a finite sum!



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The infinite sum shown adds to 64. If you stopped adding at any time, you would not reach the sum of 64. The limit of the sum is 64 since every addition gets the sum closer to 64. The sum will never reach 64.

Now think about compound interest. Rather than compounding every minute, or every microsecond, imagine compounding infinitely many times each year. This is called **continuous compounding**. Will it make you rich? Consider: If you deposited $1,000 at 100% interest, compounded continuously, what would your ending balance be after one year?

Notice the extremely high interest rate. Before reading Skills and Strategies, write down your best guess for this balance. Compare your guess to the guesses of your classmates.

**TEACH**

The notion of limit is usually introduced in a precalculus class. Students may have some experience with asymptotes and infinity. A calculator is a helpful tool to illustrate patterns in sequences that depict properties of infinity.

## Skills and Strategies

The question just posed will be answered through the following series of examples. Be sure to compare your guess to the correct answer.

### EXAMPLE 1

Given the quadratic function $f(x) = x^2 + 3x + 5$, as the values of $x$ increase to infinity, what happens to the values of $f(x)$?

**SOLUTION** Use your calculator. Find the value of $f(x)$ for each of the increasing values of $x$ in the table.

| $x$ | $f(x)$ |
|---|---|
| 100 | 10,305 |
| 1,000 | 1,003,005 |
| 90,000 | 8,100,270,005 |
| 900,000 | $> 8(10)^{11}$ |
| 8,000,000 | $> 6(10)^{13}$ |
| 50,000,000 | $> 2(10)^{15}$ |

As $x$ approaches infinity, the value of $f(x)$ increases without bound. Therefore, $f(x)$ has no limit.

**CHECK YOUR UNDERSTANDING**

As the values of $x$ increase towards infinity, what happens to the values of $g(x) = -5x + 1$?

**EXAMPLE 1**

Students should notice that due to the squaring, multiplication by 3, and addition, that the numbers will strictly increase, and there will be no limit.

**CHECK YOUR UNDERSTANDING**

***Answer*** They decrease.

### EXAMPLE 2

Given the function $f(x) = \dfrac{6x - 1}{3x + 2}$, as the values of $x$ increase to infinity, what happens to the values of $f(x)$?

**SOLUTION** Set up a table with increasing values of $x$. The pattern in the table shows that as $x$ approaches infinity, $f(x)$ approaches 2. It keeps getting closer to 2; it, never reaches 2. You can say, "The limit of $f(x)$, as $x$ approaches infinity, is 2," written

$$\lim_{x \to \infty} f(x) = 2$$

Lim is an abbreviation for limit. The arrow represents "approaching." The symbol for infinity is $\infty$.

| $x$ | $f(x)$ |
|---|---|
| 100 | 1.983443709 |
| 1,000 | 1.998334444 |
| 90,000 | 1.999981482 |
| 900,000 | 1.999998148 |
| 8,000,000 | 1.999999792 |
| 50,000,000 | 1.999999967 |
| 2,000,000,000 | 1.999999999 |

**CHECK YOUR UNDERSTANDING**

If $f(x) = \dfrac{1}{x}$, use a table and your calculator to find $\lim_{x \to \infty} f(x)$.

**CHECK YOUR UNDERSTANDING**

***Answer*** 0



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### EXAMPLE 3

Students should make a conjecture before using their calculators.

### CHECK YOUR UNDERSTANDING

***Answer*** 1

First ask students for the limits of $3^x$, $4^x$, $5^x$, and so on before $1^x$.

### EXAMPLE 4

You can preface this example by asking the students for the limits as $x$ increases for $\frac{1}{x}$, and then for $\left(1+\frac{1}{x}\right)$.

Compare the designation of this limit as $e$ to the designation of $\pi$ for the ratio of a circle's circumference to its diameter. That's how important $e$ is in mathematics!

### CHECK YOUR UNDERSTANDING

***Answer*** 1.0513

You can vary the numerator in the fraction of the expression and have students see what happens to the limits.

## EXAMPLE 3

Given the function $f(x) = 2^x$, find $\lim_{x\to\infty} f(x)$.

**SOLUTION** You can use a table or your mathematical intuition. As the values of $x$ increase, the values of $f(x)$ increase without bound, so the limit is undefined.

**CHECK YOUR UNDERSTANDING**

Given the function $f(x) = 1^x$, find $\lim_{x\to\infty} f(x)$.

## EXAMPLE 4

If $f(x) = (1 + \frac{1}{x})^x$, find $\lim_{x\to\infty} f(x)$.

**SOLUTION** You can do this intuitively in gradual steps.

From Check Your Understanding for Example 3 $\lim_{x\to\infty} 1^x = 1$

From Check Your Understanding for Example 2 $\lim_{x\to\infty}\left(\frac{1}{x}\right) = 0$

As a result $\lim_{x\to\infty}\left(1+\frac{1}{x}\right) = 1$

Keep in mind the expression in parentheses is always a little greater than 1. Because $\left(1+\frac{1}{x}\right)$ is greater than 1, the expression $\left(1+\frac{1}{x}\right)^x$ has a "battle" going on within it as $x$ approaches infinity. As $x$ approaches infinity, the expression in parentheses decreases. Simultaneously, the exponent increases. For this reason, it is difficult to tell what happens to the entire expression. You cannot use mathematical intuition to find the limit.

| $x$ | $f(x)$ to nine decimal places |
|---|---|
| 100 | 2.704813829 |
| 1,000 | 2.716923932 |
| 90,000 | 2.718266724 |
| 900,000 | 2.718280046 |
| 8,000,000 | 2.718281659 |
| 50,000,000 | 2.718281801 |
| 2,000,000,000 | 2.718281828 |

Use a table. The pattern in the table shows that as $x$ approaches infinity, $f(x)$ approaches a number around 2.718... This number is very special in mathematics. It is called the **exponential base**, and is abbreviated ***e***. Look for a key labeled $e$ on your calculator. Press it.

$$e \approx 2.718281828$$

The exponential base $e$ is an irrational number. It is a non-terminating, non-repeating decimal. Your calculator shows only the first few decimal places of $e$.

Therefore $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

**CHECK YOUR UNDERSTANDING**

Use a table and your calculator to find $\lim_{x\to\infty}\left(1+\frac{0.05}{x}\right)^x$, rounded to five decimal places.



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## EXAMPLE 5

If you deposited \$1,000 at 100% interest, compounded continuously, what would your ending balance be after one year?

**SOLUTION** This is the original question posed in the beginning of the lesson. Compounding continuously requires taking a limit as the number of compounds approaches infinity.

$$\lim_{x \to \infty} 1{,}000\left(1 + \frac{1}{x}\right)^x = 1{,}000e \approx 1{,}000(2.7182818) = 2{,}718.28$$

Therefore, \$1,000 at 100% interest, compounded continuously would grow to \$2,718.28 in one year. You may have originally thought that, with 100% interest, and an infinite amount of compounds, that the \$1,000 would grow tremendously. Keep in mind that, as the exponent $x$ increases, the fraction $\frac{1}{x}$ in the parentheses decreases, somewhat counteracting, or "battling" the exponent. Think of the result as a "compromise" of this battle.

### CHECK YOUR UNDERSTANDING

The irrational, exponential base $e$ is so important in mathematics that it has a single-letter abbreviation, $e$, and has its own key on the calculator. When you studied circles, you studied another important irrational number that has a single-letter designation and its own key on the calculator. The number was $\pi$. Recall that $\pi = 3.141592654$. Use the $e$ and $\pi$ keys on your calculator to find the difference between $e^{\pi}$ and $\pi^e$. Round to the nearest thousandth.

## EXAMPLE 6

If you deposit \$1,000 at 4.3% interest, compounded continuously, what would your ending balance be to the nearest cent after five years?

**SOLUTION** Using 4.3% instead of 100% changes the limit expression to $\lim_{x \to \infty} 1{,}000\left(1 + \frac{0.043}{x}\right)^x$. Use the following formula.

**Continuous Compound Interest Formula**

$B = pe^{rt}$ where $B$ = ending balance
$p$ = principal
$e$ = exponential base
$r$ = interest rate expressed as a decimal
$t$ = number of years

Substitute. $B = 1{,}000e^{0.043(5)}$

Calculate. $B = 1{,}239.86$

The ending balance would be \$1,239.86.

### CHECK YOUR UNDERSTANDING

Craig deposits \$5,000 at 5.12% interest, compounded continuously for four years. What would his ending balance be to the nearest cent?

### EXAMPLE 5

This answers the question posed at the beginning of the lesson. Look back at the student guesses and see who is closest. Ask students with high guesses what they didn't account for.

### CHECK YOUR UNDERSTANDING

***Answer*** 0.682

If any students have encountered the golden ratio, point out that it also has a letter designation, the Greek letter phi, $\varphi$.

### EXAMPLE 6

The formula makes it easy to find the balance. Using the limit expression would require increasing substitutions into the calculator.

### CHECK YOUR UNDERSTANDING

***Answer*** \$6,136.40



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# Applications

*The infinite! No other question has ever moved so profoundly the spirit of man.*

**David Hilbert**, Mathematician

**1.** How might these words apply to this lesson? See margin.

**2.** A bank representative studies compound interest, so she can better serve customers. She analyzes what happens when $2,000 earns interest several different ways at a rate of 4% for 3 years.
- **a.** Find the interest if it is computed using simple interest. $240
- **b.** Find the interest if it is compounded annually. $249.73
- **c.** Find the interest if it is compounded semiannually. $252.32
- **d.** Find the interest if it is compounded quarterly. $253.65
- **e.** Find the interest if it is compounded monthly. $254.54
- **f.** Find the interest if it is compounded daily. $254.98
- **g.** Find the interest if it is compounded hourly. $254.99
- **h.** Find the interest if it is compounded every minute. $254.99
- **i.** Find the interest if it is compounded continuously. $254.99
- **j.** What is the difference in interest between simple interest and interest compounded continuously? $14.99

**3.** Ed computes the ending balance for an account he is considering. The principal is $20,000, and the interest rate is 5.39%, compounded continuously for four years. He uses the formula $B = pe^{rt}$ and substitutes directly on his calculator. Look at the keystrokes he entered.

20,000e^(.0539)(4)

He presses ENTER and sees this display.

20000e^(.0539)(4) = 84430.32472

Ed's knowledge of compound interest leads him to believe that this answer is extremely unreasonable. To turn $20,000 into over $84,000 in just four years at 5% interest seems incorrect to him.
- **a.** Find the correct ending balance. $24,812.12
- **b.** Explain what part of Ed's keystroke sequence is incorrect. See margin.

**4.** Find the interest earned on a $50,000 deposited for six years at $4\frac{1}{8}\%$ interest, compounded continuously. $14,040.97

**5.** Whitney deposits $9,000 for two years. She compares two different banks. State Bank will pay her 4.1% interest, compounded monthly. Kings Savings will pay her 4.01% interest, compounded continuously.
- **a.** How much interest does State Bank pay? $767.74
- **b.** How much interest does Kings Savings pay? $751.53
- **c.** Which bank pays higher interest? How much higher? State Bank; $16.21
- **d.** What other factors might affect Whitney's choice besides interest? See margin.

## TEACH

So far in this chapter, students have used the simple interest formula and the compound interest formula. Now they also need to use the continuous compounding formula. Spiral the homework assignments so students have to do all three types for a few days.

### Exercises 1–5

The use of parentheses in the continuous compounding formula is not as tricky as the compound interest formula, but students do need to make sure the entire exponent is in parentheses.

## ANSWERS

1. People have a difficult time conceiving of infinity. The concept of infinite time, space, and number triggers the imagination and often boggles the mind.

3b. The exponent on $e$ is only 0.0539 since the parentheses were closed before the 4 was included. Ed's expression raised $e$ to the 0.0539 power, multiplied it by 20,000, and multiplied that result by 4.

5d. Sample answers: How close is the bank to her home? Will she still be living in the same place in two years? Does she have other banking business at that bank? What are the hours? How customer-friendly is the service?



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**6.** Interest rates fluctuate with the economy. In the 1980s, the highest CD interest rate was over 16%. By 2009, the highest CD interest rates were approximately 5%.

**a.** If $1,000 is invested at 16% interest, compounded continuously, for five years, what is the ending balance? $2,225.54

**b.** If $1,000 is invested at 5% interest, compounded continuously, for five years, what is the ending balance? $1,284.03

**c.** What is the difference between the two ending balances? $941.51

**7.** Find the interest earned on a $30,000 deposit for six months at $4\frac{1}{2}\%$ interest, compounded continuously. $682.65

**8.** Caroline is opening a CD to save for college. She is considering a 3-year CD or a $3\frac{1}{2}$-year CD since she starts college around that time. She needs to be able to have the money to make tuition payments on time, and she does not want to have to withdraw money early from the CD and face a penalty. She has $19,400 to deposit.

**a.** How much interest would she earn at 4.2% compounded monthly for three years? Round to the nearest cent. $2,600.23

**b.** How much interest would she earn at 4.2% compounded monthly for $3\frac{1}{2}$ years? Round to the nearest cent. $3,066.30

**c.** Caroline decides on a college after opening the $3\frac{1}{2}$-year CD, and the college needs the first tuition payment a month before the CD matures. Caroline must withdraw money from the CD early, after 3 years and 5 months. She faces two penalties. First, the interest rate for the last five months of the CD was lowered to 2%. Additionally, there was a $250 penalty. Find the interest on the last five months of the CD. Round to the nearest cent. $183.95

**d.** Find the total interest on the $3\frac{1}{2}$ year CD after 3 years and 5 months. $2,784.18

**e.** The interest is reduced by subtracting the $250 penalty. What does the account earn for the 3 years and 5 months? $2,534.18

**f.** Find the balance on the CD after she withdraws $12,000 after 3 years and five months. $9,934.18

**g.** The final month of the CD receives 2% interest. What is the final month's interest? Round to the nearest cent. $16.56

**h.** What is the total interest for the $3\frac{1}{2}$ year CD? $2,550.74

**i.** Would Caroline have been better off with the 3-year CD? Explain? Yes, with the penalty and reduced interest, the 3-year earned more.

**9.** Samuel wants to deposit $4,000 and keep that money in the bank without deposits or withdrawals for three years. He compares two different options. Option 1 will pay 3.8% interest, compounded quarterly. Option 2 will pay 3.5% interest, compounded continuously.

**a.** How much interest does Option 1 pay? $480.60

**b.** How much interest does Option 2 pay? $442.84

**10.** Write an algebraic expression for the interest earned on a $15,000 deposit for $t$ months at 2.75% interest, compounded continuously. $15{,}000e^{0.0275\left(\frac{t}{12}\right)}$

## TEACH

### Exercise 6

Have students compare the interest rates from the 1980s to today's rates. Point out that loan rates were also much higher in the 1980s than they are today.

### Exercise 8

Committing money into a CD is something that needs to be undertaken with much thought about future financial expenses.



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*It is never too early to encourage long-term savings.*
**Ron Lewis,** Politician

# 3-7 Future Value of Investments

## Objectives

- Calculate the future value of a periodic deposit investment.
- Graph the future value function.
- Interpret the graph of the future value function.

## Key Terms

- **future value of a single deposit investment**
- **periodic investment**
- **biweekly**
- **future value of a periodic deposit investment**

### EXAMINE THE QUESTION

Many people deposit money in a savings account without much thought as to what the balance will be at some date in the future. Their only concern is that the balance grows over time. But some people want to know what needs to be deposited into an account now or at regular intervals so there is a certain amount of money after a fixed period of time.

### CLASS DISCUSSION

In Lessons 3-7 and 3-8, students will learn about four different investment situations: future value of single deposit investments, future value of periodic deposit investments, present value of single deposit investments, and present value of periodic deposit investments. Students should know how to identify the correct situation.

## How can you effectively plan for the future balance in an account?

Suppose you open an account that pays interest. You make no further contributions. You just leave your money alone and let compound interest work its magic. The balance your account grows to at some point in the future is called the **future value of a single deposit investment**. To calculate the future balance, use the compound interest formula $B = P\left(1 + \frac{r}{n}\right)^{nt}$, where $B$ is the balance at the end of a time period in years $t$, $P$ is the original principal, $r$ is the interest rate expressed as a decimal, and $n$ is the number of times the interest is compounded in one year.

Many people add money to their savings accounts on a regular basis. **Periodic investments** are the same deposits made at regular intervals, such as yearly, monthly, biweekly, weekly, or even daily. Suppose Enrique gets paid every other week and has $200 directly deposited into his savings account. He wants to know how much he will have in the account after 5 years. In this case, Enrique makes an initial deposit of $200 and continues to make deposits biweekly for five years. **Biweekly** means every two weeks and is a common schedule for paychecks. Because he will get 26 biweekly paychecks per year, he makes a total of 130 periodic direct deposits ($26 \times 5$) each in the amount of $200. Had the account offered no interest, he would have at least ($130 \times 200$), or $26,000 at the end of the five-year period. Banks offer compound interest, so Enrique needs a different formula to calculate his balance at the end of five years.

**Future Value of a Periodic Deposit Investment**

$$B = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}}$$

where $B$ = balance at end of investment period
$P$ = periodic deposit amount
$r$ = annual interest rate expressed as a decimal
$n$ = number of times interest is compounded annually
$t$ = length of investment in years



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## Skills and Strategies

Here you will learn how to calculate the balance in an account in which periodic investments have been made at a given compound interest rate.

### EXAMPLE 1

Rich and Laura are both 45 years old. They open an account at the Rhinebeck Savings Bank with the hope that it will gain enough interest by their retirement at the age of 65. They deposit $5,000 each year into an account that pays 4.5% interest, compounded annually. What is the account balance when Rich and Laura retire?

**SOLUTION** You are looking to determine a balance at some point in the future, so this is a future value problem. Because $5,000 is deposited each year for 20 years, this is a periodic investment.

Use the formula for the future value of a periodic investment.

$$B = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}}$$

Substitute.

$$B = \frac{5{,}000\left(\left(1 + \frac{0.045}{1}\right)^{1(20)} - 1\right)}{\frac{0.045}{1}}$$

Calculate to the nearest cent.

$$B \approx 156{,}857.11$$

The account balance will be $156,857.11 when Rich and Laura retire.

### CHECK YOUR UNDERSTANDING

How much more would Rich and Laura have in their account if they decide to hold off retirement for an extra year?

### EXTEND YOUR UNDERSTANDING

Carefully examine the solution to Example 1. During the computation of the numerator, is the 1 being subtracted from the 20? Explain your reasoning.

### EXAMPLE 2

How much interest will Rich and Laura earn over the 20-year period?

**SOLUTION** The balance at the end of 20 years was $156,857.11. Rich and Laura deposited $5,000 into the account every year for 20 years.

Find the total amount deposited.

$$5{,}000 \times 20 = 100{,}000$$

Subtract.

$$156{,}857.11 - 100{,}000 = 56{,}857.11$$

Rich and Laura will earn $56,857.11 in interest.

### TEACH

It is important that students have a working knowledge of the two formulas presented. Those formulas are algebraically manipulated in Lesson 3-8 in order to find a present rather than future balance. Ask students why someone might make a single deposit into an account and no further deposits (for example, a trust fund from the estate of a grandparent). Then, ask them to cite a situation where an account might be open and like amounts deposited at regular intervals (direct deposit accounts as employee deductions).

### EXAMPLE 1

The formulas in Lessons 3-7 and 3-8 require the use of a calculator. Therefore, students need to know the correct keystroke sequences and where parentheses are necessary for calculator input.

### CHECK YOUR UNDERSTANDING

***Answer*** $12,058.57

Determine the amount that will be in the account after 21 years ($168,915.68) and subtract from that the answer to Example 1.

### EXTEND YOUR UNDERSTANDING

***Answer*** No

Using the order of operations, $(1 + 0.045)^{20}$ will be computed first. 1 is subtracted from that value, not from the exponent of 20.

### EXAMPLE 2

Students must take into consideration what Rich and Laura deposited over the 20-year period. Subtracting that amount from the 20-year balance yields the interest made.



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**CHECK YOUR UNDERSTANDING**

***Answer*** Rich and Laura will earn $7,058.57 more by retiring one year later.

**EXAMPLE 3**

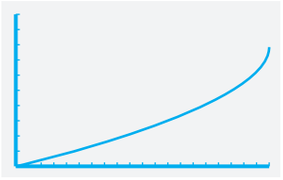
The formula remains the same but the value of $n$ used twice in the formula is now 12 (to represent 12 monthly deposits/compounds per year).

**CHECK YOUR UNDERSTANDING**

***Answer*** Not necessarily. It depends on the principal, rate, and length of time opened.

**CHECK YOUR UNDERSTANDING**

***Answer***



**CHECK YOUR UNDERSTANDING**

Use Example 1 Check Your Understanding. How much more interest would Rich and Laura earn by retiring after 21 years?

**EXAMPLE 3**

Linda and Rob open an online savings account that has a 3.6% annual interest rate, compounded monthly. If they deposit $1,200 every month, how much will be in the account after 10 years?

**SOLUTION**

Use the formula for the future value of a periodic investment.

$$B = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}}$$

Substitute.

$$B = \frac{1{,}200\left(\left(1 + \frac{0.036}{12}\right)^{12(10)} - 1\right)}{\frac{0.036}{12}}$$

Calculate to the nearest cent.

$$B \approx 173{,}022.87$$

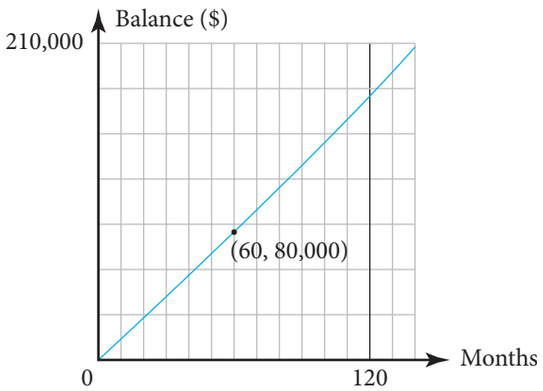
Linda and Rob will have $173,022.87 in the account after 10 years.

**CHECK YOUR UNDERSTANDING**

Would opening an account at a higher interest rate for fewer years have assured Linda and Rob at least the same final balance?

**EXAMPLE 4**

Construct a graph of the future value function that represents Linda and Rob's account for each month. Use the graph to approximate the balance after 5 years.




**SOLUTION** Let $x$ represent each of the monthly interest periods. The minimum value of $x$ is 0 and corresponds with the opening of the account. The maximum value of $x$ is 120, because Linda and Rob make deposits for 120 months (10 years × 12 months). Use a graphing calculator to graph the future value function.

$$B = \frac{1{,}200\left(\left(1 + \frac{0.036}{12}\right)^{x} - 1\right)}{\frac{0.036}{12}}$$

In 5 years, the balance will be approximately $80,000.

**CHECK YOUR UNDERSTANDING**

Construct a graph for Rich and Laura's situation in Example 1.



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## Applications

*It is never too early to encourage long-term savings.*
**Ron Lewis,** Politician

**1.** How might those words apply to what has been outlined in this lesson? See margin.

**2.** Suppose that $1,000 is deposited into an account that yields 5% interest, compounded annually. How much money will be in that account at the end of 4 years? $1,215.51

**3.** Arianna deposits $500 in an account that pays 3% interest, compounded semiannually. How much is in the account at the end of two years? $530.68

**4.** When Derrick turned 15, his grandparents put $10,000 into an account that yielded 4% interest, compounded quarterly. When Derrick turns 18, his grandparents will give him the money to use toward his college education. How much does Derrick receive from his grandparents on his 18th birthday? $11,268.25

**5.** Barbara wants to restore her '66 Mustang in 4 years. She puts $200 into an account every month that pays 4.5% interest, compounded monthly. How much is in the account after 4 years? $10,496.77

**6.** Robbie opens an account at a local bank by depositing $100. The account pays 2.4% interest, compounded weekly. He deposits $100 every week for three years.
- **a.** How much is in the account after three years? $16,171.46
- **b.** Write the future value function if $x$ represents the number of weeks. See margin.
- **c.** Use a graphing calculator to graph the future value function. See margin.
- **d.** Using the graph, what is the approximate balance after 2 years? $11,000

**7.** Suppose $600 is deposited into an account every quarter. The account earns 5% interest, compounded quarterly.
- **a.** What is the future value of the account after 5 years? $13,537.79
- **b.** Write the future value function if $x$ represents the number of quarters. See margin.
- **c.** Use a graphing calculator to graph the future value function. See margin.
- **d.** Using the graph, what is the approximate balance after 3 years? $7,500

**8.** When Abram was born, his parents put $2,000 into an account that yielded 3.5% interest, compounded semiannually. When he turns 16, his parents will give him the money to buy a car. How much will Abram receive on his 16th birthday? $3,483.43

### TEACH

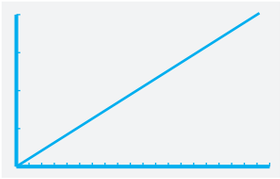
**Exercise 4**
Alert students that the elapsed time of this account is not a number given in the problem statement but rather needs to be determined by using the time that elapsed from when Derrick turned 15 to his 18th birthday.

### ANSWERS

1. Answers will vary but should include some mention of the fact that the longer an amount is in a savings account the more it will accrue in interest. Therefore, the earlier the start of the account, the better.

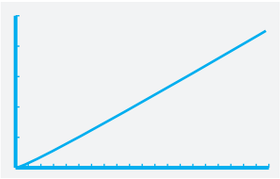
6b. $B = \dfrac{100\left(\left(1 + \frac{0.024}{52}\right)^x - 1\right)}{\frac{0.024}{52}}$

6c.



7b. $B = \dfrac{600\left(\left(1 + \frac{0.05}{4}\right)^x - 1\right)}{\frac{0.05}{4}}$

7c.





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## TEACH

**Exercise 9**
Ask students to predict who will have the most money in the account after 20 years and why. Then, after completing all the parts of the problem, ask if their predictions were backed by the math and the graph.

**Exercise 11**
This is an excellent guided discovery exercise. Here, students discover Einstein's Rule of 72. This problem is suitable as a stand-alone homework assignment or an in-class activity.

## ANSWERS

9g. $B = \dfrac{100\left(\left(1 + \dfrac{0.05}{12}\right)^{x} - 1\right)}{\dfrac{0.05}{12}}$

9h. $B = \dfrac{80\left(\left(1 + \dfrac{0.08}{12}\right)^{x} - 1\right)}{\dfrac{0.08}{12}}$

9j. Sydney's balance starts out higher than Benny's but by about the 160th month (a little over 13 years), Benny's amount begins to overtake Sydney's.

**9.** Sydney invests \$100 every month into an account that pays 5% annual interest, compounded monthly. Benny invests \$80 every month into an account that pays 8% annual interest rate, compounded monthly.
- **a.** Determine the amount in Sydney's account after 10 years. \$15,528.23
- **b.** Determine the amount in Benny's account after 10 years. \$14,635.68
- **c.** Who had more money in the account after 10 years? Sydney
- **d.** Determine the amount in Sydney's account after 20 years. \$41,103.37
- **e.** Determine the amount in Benny's account after 20 years. \$47,121.63
- **f.** Who had more money in the account after 20 years? Benny
- **g.** Write the future value function for Sydney's account. See margin.
- **h.** Write the future value function for Benny's account. See margin.
- **i.** Graph Benny and Sydney's future value function on the same axes. See additional answers.
- **j.** Explain what the graph indicates. See margin.

**10.** You are constructing a future value spreadsheet. Users will be asked to enter the periodic investment in cell A3, the interest rate as an equivalent decimal in cell A4, the time in years in cell A5, and the number of times per year the interest is compounded in cell A6. Cell A8 will contain the future value of the periodic investment. Write the formula that will display this value in A8.
=(A3*((1+A4/A6)^(A5*A6)–1))/(A4/A6)

**11.** Albert Einstein said that compound interest was ". . .the most powerful thing I have ever witnessed." Work through the following exercises to discover a pattern Einstein discovered which is now known as the Rule of 72.
- **a.** Suppose that you invest \$2,000 at a 1% annual interest rate. Use your calculator to input different values for $t$ in the compound interest formula. What whole number value of $t$ will yield an amount closest to twice the initial deposit? 70 years
- **b.** Suppose that you invest \$4,000 at a 2% annual interest rate. Use your calculator to input different values for $t$ in the compound interest formula. What whole number value of $t$ will yield an amount closest to twice the initial deposit? 35 years
- **c.** Suppose that you invest \$20,000 at a 6% annual interest rate. Use your calculator to input different values for $t$ in the compound interest formula. What whole number value of $t$ will yield an amount closest to twice the initial deposit? 12 years
- **d.** Albert Einstein noticed a very interesting pattern when an initial deposit doubles. In each of the three examples above, multiply the value of $t$ that you determined times the percentage amount. For example, in a. multiply $t$ by 1. What do you notice? You get a number close to 72.
- **e.** Einstein called this the Rule of 72 because for any initial deposit and for any interest percentage, 72 ÷ (percentage) will give you the approximate number of years it will take for the initial deposit to double in value. Einstein also said that "If people really understood the Rule of 72 they would never put their money in banks." Suppose that a 10-year-old has \$500 to invest. She puts it in her savings account that has a 1.75% annual interest rate. How old will she be when the money doubles? 51 years old



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*Before you can really start setting financial goals, you need to determine where you stand financially.*

**David Bach,** Financial Consultant

# Present Value of Investments 3-8

## Key Terms

- present value
- present value of a single deposit investment
- present value of a periodic deposit investment

## Objectives

- Calculate the present value of a single deposit investment.
- Calculate the present value of a periodic deposit investment.

## How can you determine what you need to invest now to reach a financial goal?

Everyone has future plans. Those plans may be more defined for some people than others. Look ahead to the future. What might you need to save for? An education? A car? A house? A family? While you don't know what the expense for these items will be in the future, you can probably be assured that they will cost more than they do now. Perhaps a college tuition that now costs $25,000 per year might be $30,000 per year five years from now.

You need to start now to plan for large expenses in the future. Planning for a large expense in the future requires financial planning for that expense in the present. It helps to know how much you need to save now or on a regular basis in order to meet your future financial goal.

**Present value** is the current value of a deposit that is made in the present time. You can determine the **present value of a single deposit investment**, meaning you can calculate how much a one-time deposit should earn at a specific interest rate in order to have a certain amount of money saved for a future savings goal.

You can also determine how much to save on a regular basis at a specific interest rate to meet that future goal by finding the **present value of a periodic deposit investment**. In both cases, you determine what you need to save now in order to have enough money in your account later on to meet a given expense.

### EXAMINE THE QUESTION

Many people make financial plans that include a fixed expenditure in the future. Perhaps they want to save for college, buy a home, or go on a vacation. This entails planning now by making decisions about money that should be deposited on a one-time or periodic basis. Ask students what large "purchases" they might foresee in their futures.

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## CLASS DISCUSSION

Up to this point, students have been solving for a future balance. In this lesson, they have an amount in mind they will need at some point in the future and are asked to determine how much should be deposited once in the present time, or multiple times on a periodic basis. It is important that they understand the difference between what is asked in this lesson and what was asked in the previous lesson.

## TEACH

You might want to set up the following diagram to assist students:

| | Single | Periodic |
|---|---|---|
| Present | | |
| Future | | |

Within each of the empty cells, enter the appropriate formulas. They have already studied future values, so those formulas can be entered now as a review of the last lesson. In this lesson, they will learn about the two formulas that are used for present value exercises. Those formulas can be entered during the review of Example 1 and Example 3. As students read a problem, they need to find the row and column that match the problem statement. This will get them into the habit of defining the situation in terms of the type of deposit and what balance is being determined.

## EXAMPLE 1

This is a present value of a single deposit investment problem. Have students enter the formula into the chart that was created (see TEACH). Make sure they understand why the formula is entered in the designated location.

## CHECK YOUR UNDERSTANDING

***Answer*** Using the same formula, it would take approximately 14 years.

# Skills and Strategies

Using algebra, the present value formulas are derived from the future value formulas that you studied in the previous lessons.

## EXAMPLE 1

Mr. and Mrs. Johnson know that in 6 years, their daughter Ann will attend State College. She will need about $20,000 for the first year's tuition. How much should the Johnsons deposit into an account that yields 5% interest, compounded annually, in order to have that amount? Round your answer to the nearest thousand dollars.

**SOLUTION** Use the formula for the future value of a single deposit investment, where $B$ = ending balance, $P$ = principal or original balance, $r$ = interest rate expressed as a decimal, $n$ = number of times interest is compounded annually, and $t$ = number of years.

Solve the formula for $P$.

$$B = P\left(1 + \frac{r}{n}\right)^{nt}$$

Divide each side by $\left(1 + \frac{r}{n}\right)^{nt}$.

$$\frac{B}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{P\left(1 + \frac{r}{n}\right)^{nt}}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Simplify.

$$\frac{B}{\left(1 + \frac{r}{n}\right)^{nt}} = P$$

Rewrite the previous equation so that you have a new formula.

**Present Value of a Single Deposit Investment**

$$P = \frac{B}{\left(1 + \frac{r}{n}\right)^{nt}}$$

where $B$ = ending balance
$P$ = principal or original balance (present value)
$r$ = interest rate expressed as a decimal
$n$ = number of times interest is compounded annually
$t$ = number of years

Substitute 20,000 for $B$, 0.05 for $r$, 1 for $n$, and 6 for $t$.

$$P = \frac{20{,}000}{\left(1 + \frac{0.05}{1}\right)^{1(6)}}$$

Simplify.

$$P = \frac{20{,}000}{(1 + 0.05)^6}$$

Calculate.

$$P \approx 14{,}924.31$$

The Johnsons should deposit approximately $15,000 into the account.

### CHECK YOUR UNDERSTANDING

How many years would it take for $10,000 to grow to $20,000 in the same account?



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## EXAMPLE 2

Ritika just graduated from college. She wants $100,000 in her savings account after 10 years. How much must she deposit in that account now at a 3.8% interest rate, compounded daily, in order to meet that goal? Round up to the nearest dollar.

**SOLUTION** Use the formula for the present value of a single deposit investment. Let $B = 100$, $r = 0.038$, $t = 10$, and $n = 365$.

$$P = \frac{B}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Substitute. $$P = \frac{100{,}000}{\left(1 + \frac{0.038}{365}\right)^{365(10)}}$$

Calculate. $$P = 68{,}387.49$$

Ritika must deposit approximately $68,388.

### CHECK YOUR UNDERSTANDING

How does the equation from Example 2 change if the interest is compounded weekly?

## EXAMPLE 3

Nick wants to install central air conditioning in his home in 3 years. He estimates the total cost to be $15,000. How much must he deposit monthly into an account that pays 4% interest, compounded monthly, in order to have enough money? Round up to the nearest hundred dollars.

**SOLUTION** Use the formula for the future value of a periodic deposit investment, where $B$ = ending balance, $P$ = periodic deposit amount, $r$ = interest rate expressed as a decimal, $n$ = number of times interest compounded annually, and $t$ = number of years.

Solve the formula for $P$. $$B = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}}$$

Multiply each side by $\frac{r}{n}$. $$B \times \frac{r}{n} = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\frac{r}{n}} \times \frac{r}{n}$$

Simplify. $$B \times \frac{r}{n} = P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)$$

Divide each side by $\left(1 + \frac{r}{n}\right)^{nt} - 1$. $$\frac{B \times \frac{r}{n}}{\left(1 + \frac{r}{n}\right)^{nt} - 1} = \frac{P\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{\left(1 + \frac{r}{n}\right)^{nt} - 1}$$

Simplify. $$\frac{B \times \frac{r}{n}}{\left(1 + \frac{r}{n}\right)^{nt} - 1} = P$$

### EXAMPLE 2

Example 2 represents a present value of a single deposit investment problem in which $n = 365$.

### CHECK YOUR UNDERSTANDING

***Answer*** Everything remains the same except $n = 52$.

### EXAMPLE 3

This is a present value of a periodic investment problem since Nick will be making a deposit monthly. Have students enter the formula in the chart that was created (see TEACH). Make sure they understand why this formula is entered in the designated location.



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**CHECK YOUR UNDERSTANDING**

***Answer*** $P = \dfrac{x\left(\frac{r}{2}\right)}{\left(1+\frac{r}{2}\right)^{2y}-1}$

**EXAMPLE 4**

You may need to assist students in identifying the independent variable. Because the number of months is given in the example, the independent variable, *x*, replaces *nt* in the formula.

**CHECK YOUR UNDERSTANDING**

***Answer*** Using the graph found in Example 3, the approximate values are: $16,000; $1,300; and $500.

Rewrite the previous equation so that you have a new formula.

**Present Value of a Periodic Deposit Investment**

$$P = \frac{B \times \frac{r}{n}}{\left(1+\frac{r}{n}\right)^{nt}-1}$$

where $B$ = ending balance
$P$ = principal or original balance
$r$ = interest rate expressed as a decimal
$n$ = number of times interest is compounded annually
$t$ = number of years

Substitute. $P = \dfrac{15{,}000 \times \frac{0.04}{12}}{\left(1+\frac{0.04}{12}\right)^{12(3)}-1}$

Calculate. $P \approx 392.86$

Nick must deposit about $400 every month.

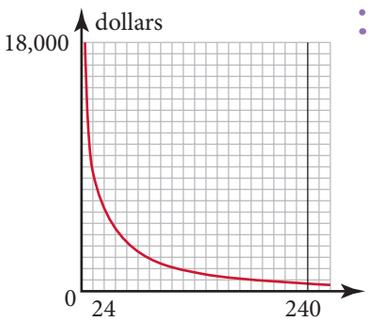
**CHECK YOUR UNDERSTANDING**

Write the formula to find the present value of an *x*-dollar balance that is reached by periodic investments made semiannually for *y* years at an interest rate of *r*.

**EXAMPLE 4**

Randy wants to have saved a total of $200,000 by some point in the future. He is willing to set up a direct deposit account with a 4.5% APR, compounded monthly, but is unsure of how much to periodically deposit for varying lengths of time. Graph a present value function to show the present values for Randy's situation from 12 months to 240 months.

**SOLUTION** Let *x* represent the number of months. Begin with a one year investment. The minimum value of *x* is 12. The maximum value is 240.




Use the present value of a periodic investment formula. $P = \dfrac{B \times \frac{r}{n}}{\left(1+\frac{r}{n}\right)^{nt}-1}$

Substitute 200,000 for *B*, 0.045 for *r*, 12 for *n*, and *x* for *nt*. $P = \dfrac{200{,}000 \times \frac{0.045}{12}}{\left(1+\frac{0.045}{12}\right)^{x}-1}$

Graph the function.

The present value decreases as the number of months increases.

**CHECK YOUR UNDERSTANDING**

Use the graph to estimate how much to deposit each month for 1 year, 10 years, and 20 years.



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## Applications

*Before you can really start setting financial goals, you need to determine where you stand financially.*

**David Bach,** Financial Consultant

**1.** How might those words apply to what has been outlined in this lesson? See margin.

**2.** Complete the table to find the single deposit investment amounts.

| Future Value | Interest Rate | Interest Periods | Deposit |
|---|---|---|---|
| $1,000 | 4%, compounded annually | 3 years | **a.** |
| $2,500 | 3%, compounded semiannually | 5 years | **b.** |
| $10,000 | 5%, compounded quarterly | 10 years | **c.** |
| $50,000 | 2.75%, compounded monthly | 8 years | **d.** |

See margin.

**3.** Complete the table to find the periodic deposit investment amounts.

| Future Value | Interest Rate | Interest Periods | Deposit |
|---|---|---|---|
| $50,000 | 2%, compounded annually | 8 years | **a.** |
| $25,000 | 1.5%, compounded semiannually | 4 years | **b.** |
| $100,000 | 3.75%, compounded quarterly | 10 years | **c.** |
| $1,000,000 | 4%, compounded monthly | 20 years | **d.** |

See margin.

**4.** Bob wants $50,000 at the end of 7 years in order to buy a car. If his bank pays 4.2% interest, compounded annually, how much must he deposit each year in order to reach his goal? $6,292.16

**5.** Grandpa Joe wants to open an account for his grandchildren that he hopes will have $80,000 in it after 20 years. How much must he deposit now into an account that yields 2.75% interest, compounded monthly, so he can be assured of reaching his goal? $46,185.04

**6.** Mary wants to go on a $10,000 vacation in 6 months. She has a bank account that pays 4.25% interest, compounded monthly. How much must she deposit each month to afford the vacation? $1,651.97

**7.** Janine is 21 years old. She opens an account that pays 4.4% interest, compounded monthly. She sets a goal of saving $10,000 by the time she is 24 years old. How much must she deposit each month? $260.36

**8.** Suni needs to repay her school loan in 4 years. How much must she semiannually deposit into an account that pays 3.9% interest, compounded semiannually, to have $100,000 to repay the loan? $11,671.58

**9.** Rich needs $50,000 for a down payment on a home in 5 years. How much must he deposit into an account that pays 6% interest, compounded quarterly, in order to meet his goal? $37,123.52

**10.** Marcy wants to have $75,000 saved sometime in the future. How much must she deposit into an account that pays 3.1% interest, compounded monthly? Use a graphing calculator to graph the present value function. See margin.

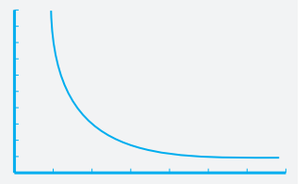
### TEACH

**Exercises 2–10**
In each of these exercises a financial goal has been set. Students need to look for verbal clues that alert them to whether or not the problem is a single deposit or a periodic deposit.

**Exercise 10**
This exercise does not give students a fixed time by which the balance must be $75,000. Therefore finding a good viewing window might take a few trials. Encourage students to select maximum $x$-values systematically, increasing the max until the graph can be seen crossing $y = 75{,}000$.

### ANSWERS

1. Answers will vary but should include the fact that to look ahead to a future value savings, it is necessary to carefully examine what you can afford to save in the present.
2. a. $889.00
   b. $2,154.17
   c. $6,084.13
   d. $40,136.04
3. a. $5,825.49/year
   b. $3,043.89/6 months
   c. $2,072.04/quarter
   d. $2,726.47/month
10.





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CHAPTER

# 3 Assessment

## Real Numbers

## You Write the Story!!

### CHAPTER 3 ASSESSMENT

### REAL NUMBERS

**You Write the Story**

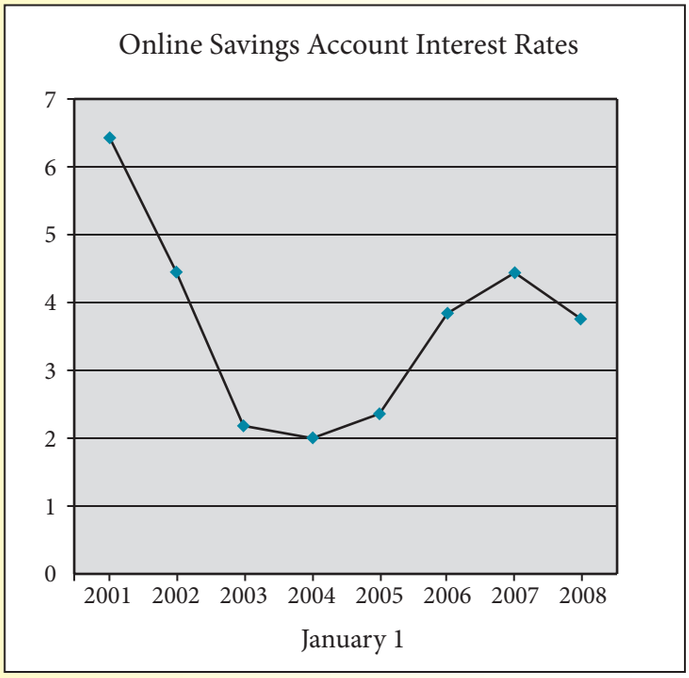
Have the students list the points as ordered pairs. Then show them how to enter them and draw a time plot on their graphing calculators.

Have them go online and find out the interest rates for the years past 2008 to update the graph.

### REALITY CHECK

Reality Check projects are a terrific form of alternative assessment. They give students an additional avenue to show what they've learned, so their grades are not solely based on tests.

Examine the line graph below. It depicts the average online savings account interest rates at the beginning of January for 8 years. Write a short newspaper-type article centered on the graph. Use the Internet if you need additional information and background to help you explain the graph. An electronic copy of the graph is at www.cengage.com/school/math/financialalgebra. Copy and paste it into the article.




## Reality Check

**1.** Go to the FDIC website. Find information on how a person can be insured at one bank for more than $250,000. Use the e-mail or phone contacts to ask questions. Speak to a representative at a bank to ask further questions. Create five different hypothetical families, the accounts they have, and how much of each account is insured. Prepare examples on a poster to present to the class.



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2. Interview a bank representative about trust accounts. Find out what the abbreviations POD, ATF, and ITF mean. Prepare questions about FDIC insurance limits and beneficiaries. Ask for any brochures they offer about trust accounts. Prepare a report or a poster on trust accounts to present to the class.

3. Go to a bank or bank website to find three different types of checking accounts. Compare and contrast the accounts offered by the same bank. What are the benefits of each? What are the drawbacks of each? Who might be better served by each type of checking account? Explain which account might be best for your financial situation.

4. While the law states that free checking accounts cannot have minimum balances or per check fees, there are other fees and penalties that are allowable. Research the allowable fees and penalties on checking accounts. Make a list and explain the purpose and cost of each.

5. Visit two local banks. Speak to a bank representative at each bank. Prepare a list of services to compare. What are the CD rates at each bank? What are the penalties for withdrawing money from a CD before it is due? What are the minimum balances for different types of accounts? What are the fees for insufficient funds? What are the different types of checking accounts they offer? What are the fees and requirements for these accounts? What are the hours of service? Think of other questions to ask. Prepare the findings in a report.

6. Interest rates have historically fluctuated with the economy. Go online and/or use the library to find interest rates over the past 50 years. Make a graph to display the information.

7. Each year, there are contests in schools all over the nation to see how many decimal places of the number $\pi$ students can memorize. The records are amazing! Go online and find out the decimal representation of $e$ to as many decimal places as possible. Talk to the teacher about having a memorization contest in class. Research how experts memorize long sequences of digits. Visit a few local businesses to see if they would be willing to donate a prize for the contest. Ask the school newspaper to cover the contest. Emcee the contest in class.

8. Some employers allow employees to have money deducted from their accounts and automatically placed into a savings account. Interview three adults working in different professions. Ask them about employer-sponsored savings plans. Prepare a report on the findings.

9. Visit a local bank. Get brochures they offer about their services. If the brochures are two-sided, take two of each so you can cut them out and paste them onto a poster board. Pick several services to highlight. Cut out the portions of the brochures that explain each service. Give each service an original, short title, and print out your title. Organize the titles and descriptions of the banking services onto a poster board.

10. The Rule of 72 is a method for quickly estimating how many years it will take principal to double, assuming the interest was compounded. Go to the library and/or use the Internet to research the Rule of 72 beyond what was presented in Lesson 3-7. Prepare some examples to illustrate the rule. Discuss the history and the use of the rule. Display your research on a poster board.

Projects can be presented to the class on any schedule that works for your program. It may be too time-consuming for every student to present their project for every chapter.

Remind students who are personally visiting local businesses or community members that they are representing the school, and need to be cordial and patient. These projects deliberately have students taking little field trips, so they don't conduct everything online.

Point out that they need to thank any person who helps them in completing their Reality Check projects. A letter is the most personal way to do this.

You can offer students extra projects for extra credit.



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## Dollars and Sense — Your Financial News Update

**Go to www.cengage.com/school/math/financialalgebra where you will find a link to a website containing current issues about banking. Try one of the activities.**

## Really? Really! REVISITED

**REALLY? REALLY! REVISITED**

The Publications Department of the Federal Reserve Bank offers materials on counterfeiting that can be accessed online. You can have students request some of the materials.

The Secret Service drastically reduced the occurrences of counterfeit money since the Civil War. Nevertheless, the problem still exists. Look at the following table.

**U.S. Counterfeiting 1999–2005**

| Year | Dollar value of currency that was found after being in circulation, in millions | Dollar value of currency that was found before getting into circulation, in millions |
|---|---|---|
| 1999 | 39.2 | 13.7 |
| 2000 | 39.7 | 20.9 |
| 2001 | 47.5 | 12.6 |
| 2002 | 42.9 | 9.7 |
| 2003 | 36.6 | 10.7 |
| 2004 | 43.6 | 10.3 |
| 2005 | 56.2 | 12.7 |

Source: U.S. Secret Service; Board of Governors of the Federal Reserve System; U.S. Department of the Treasury

1. Create a line graph for each column of data. Let the horizontal axis represent the year, and let the vertical axis represent dollars. Put both graphs on the same set of axes, in different colors. See additional answers.
2. Look at the graphs you created in Exercise 1. Do you think the pattern of catching counterfeit bills before and after circulation follows the same pattern of increases and decreases? The patterns are not identical. Changes in one category are not the same in direction or magnitude. See additional answers for graph.



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## Applications

**1.** Go to www.cengage.com/school/math/financialalgebra and download a blank check register. Complete all of the necessary information in the check register. See additional answers.

**a.** The balance on December 10 is $3,900.50.

**b.** On December 11 check #1223 is written for $84 to North Shore High School Drama Club.

**c.** On December 12 a paycheck in the amount of $240.80 is deposited.

**d.** On December 13 a birthday check for $100 is received from grandparents. The check is deposited that afternoon.

**e.** On December 17 three checks are written while holiday shopping. One is to Best Buy in the amount of $480.21, one is to Target in the amount of $140.58, and one is to Aeropostale in the amount of $215.60.

**f.** Staples sells computers. On December 20 a laptop is purchased for $1,250. A mistake is made on the first check, and the check must be voided. A correct check for the right amount is then written with the next available check.

**g.** On December 22 a gift is returned to Barnes and Noble. The $120 amount is deposited into the checking account.

**h.** On December 24, $300 is withdrawn from an ATM for food at a holiday party. The company that owns the ATM charges $1.50 fee for the transaction, and the customer's bank charges a $2.50 fee for the transaction. The fees are taken directly out of the checking account.

**i.** On December 28 a check for $521 is written to Len's Auto Body Shop to repair a dent in the fender of a car.

**j.** On December 29 a check is written to AMTRAK for $150.80 to visit a cousin in Washington, D.C. for New Year's Eve.

**2.** Use the check register from Exercise 1. It is now one month later, and the checking account statement has arrived. Does the account balance? See margin.

| Checking Account Statement | | | | |
|---|---|---|---|---|
| **Date** | **Description** | **Check #** | **Amount** | **Balance** |
| 12/12 | Deposit | | $240.80 | $4,141.30 |
| 12/13 | Deposit | | $100.00 | $4,241.30 |
| 12/19 | W/D | 1223 | $ 84.00 | $4,157.30 |
| 12/19 | W/D | 1226 | $215.60 | $3,941.70 |
| 12/20 | W/D | 1225 | $140.58 | $3,801.12 |
| 12/21 | W/D | 1224 | $480.21 | $2,320.91 |
| 12/24 | ATM Withdrawal | | $300.00 | $3,020.91 |
| 12/24 | ATM Fee | | $ 1.50 | $3,019.41 |
| 12/24 | ATM Fee | | $ 2.50 | $3,016.91 |
| 01/15 | W/D | 1229 | $521.00 | $2,495.91 |
| | | | Ending Balance: | $2,495.91 |

| | |
|---|---|
| Ending balance from statement | **a.** |
| Deposits outstanding | **b.** |
| Checks outstanding | **c.** |
| Revised statement balance | **d.** |
| Balance from checkbook | **e.** |

**ANSWERS**

2a. $2,495.91
b. $120
c. $1,400.80
d. $1,215.11
e. $1,215.11



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**ANSWERS**

7b. All of Matt's money is insured. The FDIC insures up to $250,000 at one bank, if the accounts are the same type.

9. a. $0
b. $5,200.00
c. $5,200.00
d. $0.57
e. $5,200.57
f. $5,200.57
g. $700.00
h. $5,900.57
i. $0.65
j. $5,901.22
k. $5,901.22
l. $500.00
m. $5,401.22
n. $0.59
p. $5,401.81

**3.** Find the simple interest on a $2,219 principal, deposited for six years at a rate of 5.11%. $680.35

**4.** Ruth has a savings account at a bank that charges a $3.50 fee for every month her balance falls below $1,500. Her account has $1,722 and then she withdraws $400. What is her balance in five months if her account balance never reaches $1,500? $1,304.50

**5.** Nine months ago Alexa deposited $7,000 in a three-year CD. She has received $224.16 in interest. She withdraws $1,000. This is before the CD matures, so she pays a $250 penalty. What is her balance after the withdrawal? $5,974.16

**6.** Ralph deposited $910 in an account that pays 5.2% simple interest, for $3\frac{1}{2}$ years.

**a.** How much interest did the account earn? $165.62
**b.** What is the ending balance? $1,075.62
**c.** How much interest did the account earn the first year? $47.32
**d.** How much interest did the account earn the third year? $47.32

**7.** Matt has two single accounts at Midtown Bank. One account has a balance of $74,112.09 and the other has a balance of $77,239.01.

**a.** What is the sum of Matt's balances? $151,351.10
**b.** Is all of Matt's money insured by the FDIC? Explain. See margin.

**8.** Rhonda deposits $5,600 in a savings account that pays $4\frac{1}{2}\%$ interest, compounded semiannually.

**a.** How much interest does the account earn in the first six months? $126
**b.** What is the ending balance after six months? $5,726
**c.** How much interest does the account earn in the second six months? $128.84
**d.** What is the balance after one year? $5,854.84
**e.** How much interest does the account earn the first year? $254.84

**9.** Rebecca opened a savings account on March 20, with a $5,200 deposit. The account pays 3.99% interest, compounded daily. On March 21 she made a $700 deposit, and on March 22 she made a $500 withdrawal. Use this information to find the missing amounts. See margin.

| Date | March 20 | March 21 | March 22 |
|---|---|---|---|
| Opening balance | **a.** | **f.** | **k.** |
| Deposit | **b.** | **g.** | ----- |
| Withdrawal | ----- | ----- | **l.** |
| Principal used to compute interest | **c.** | **h.** | **m.** |
| Interest | **d.** | **i.** | **n.** |
| Ending balance | **e.** | **j.** | **p.** |

**10.** Nick deposited $3,000 in a three-year CD account that pays 4.08% interest, compounded weekly. What is the ending balance? $3,390.46

**11.** How much more would $10,000 earn in three years compounded daily at 4.33%, than compounded semiannually at 4.33%? $15.69



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**12.** Austin deposits \$2,250 into a one-year CD at an interest rate of 5.3%, compounded daily.

**a.** What is the ending balance after the year? \$2,372.46

**b.** How much interest did the account earn during the year? \$122.46

**c.** What is the annual percentage yield? Round to the nearest hundredth of a percent. 5.44%

**13.** Find the interest earned on a \$25,000 deposit for $2\frac{1}{2}$ years at 4.7% interest, compounded continuously. \$3,117.04

**14.** Examine each of the following situations, labeled I, II, and III. Identify which of the three cases below applies. Do not solve the problems.

**I.** future value of a single deposit investment
**II.** future value of a periodic deposit investment
**III.** present value of a periodic deposit investment

**a.** You want to save for a new car that you will buy when you graduate college in 4 years. How much will you be able to afford if you deposit \$1,000 per quarter in an account that compounds interest at a rate of 4.1% quarterly? future value of a periodic investment

**b.** You deposit \$3,000 into an account that yields 3.22% interest compounded semiannually. How much will you have in the account in 5 years? future value of a single deposit investment

**c.** You want to put a \$40,000 down payment on a store front for a new business that you plan on opening in 5 years. How much should you deposit monthly into an account with an APR of 3.75%, compounded monthly? present value of a periodic investment

**15.** Santos deposited \$1,800 in an account that yields 2.7% interest, compounded semiannually. How much is in the account after 54 months? \$2,030.89

**16.** Stephanie signed up for a direct deposit transfer into her savings account from her checking account. Every month \$150 is withdrawn from her checking account. The interest in this account is at 2.6% compounded monthly. How much will be in the account at the end of $6\frac{1}{2}$ years? \$12,731.79

**17.** Jazmine needs \$30,000 to pay off a loan at the end of 5 years. How much must she deposit monthly into a savings account that yields 3% interest, compounded monthly? \$464.06

**18.** Use a table of increasing values of $x$ to find each of the following limits. If no limit exists, say the limit is undefined.

**a.** $\lim_{x \to \infty} f(x)$ if $f(x) = \dfrac{9x - 1}{3x - 5}$ 3

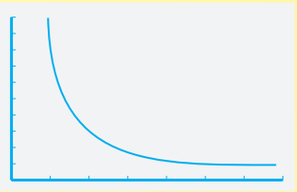
**b.** $\lim_{x \to \infty} g(x)$ if $g(x) = \dfrac{3x^2 + 9x}{4x + 1}$ undefined

**c.** $\lim_{x \to \infty} h(x)$ if $h(x) = \dfrac{7x}{x^2 - 41}$ 0

**19.** Tom wants to have \$50,000 saved sometime in the future. How much must he deposit every month into an account that pays 2.8% interest, compounded monthly. Use a graphing calculator to graph the present value function. See margin.

**ANSWERS**

19.





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