Financial and Workplace Mathematics 110

Curriculum

Implemented September 2012

Department of Education and
Early Childhood Development
2014

Acknowledgements

The New Brunswick Department of Education and Early Childhood Development gratefully acknowledges the contributions of the following groups and individuals toward the development of the *New Brunswick Financial and Workplace Mathematics 110* Curriculum Guide:

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education: *The Common Curriculum Framework for Grade 10-12 Mathematics*, January 2008. *New Brunswick Financial and Workplace Mathematics 110* curriculum is based on the Outcomes and Achievement Indicators of *WNCP Apprenticeship and Workplace Mathematics Grade 11*. Reproduced (and/or adapted) by permission. All rights reserved.

- Newfoundland and Labrador Department of Education, Prince Edward Island Department of Education and Early Childhood Development.

- The NB High School Mathematics Curriculum Development Advisory Committee of Bev Amos, Roddie Duguay, Suzanne Gaskin, Nicole Giberson, Karen Glynn, Beverlee Gonzales, Ron Manuel, Stacey Leger, Jane Pearson, Elaine Sherrard, Alyssa Sankey (UNB), Mahin Salmani (UNB), Maureen Tingley (UNB),

- The NB Grade 11 Curriculum Development Writing Team of Brice Betts, Richard Brown, Yvonne Caverhill, Mary Clarke, Cindy Doucet, Nancy Everett, Cindy Grasse, Nancy Hodnett, Bradley Lynch, Sheridan Mawhinney, Sean Newlands, Yvan Pelletier, Parise Plourde, Tony Smith, and Jeff Taylor.

- Martha McClure, Learning Specialist, 9-12 Mathematics and Science, NB Department of Education

- The Mathematics Learning Specialists, Numeracy Leads, and Mathematics teachers of New Brunswick who provided input and feedback throughout the development and implementation of this document.

Table of Contents

Table of Contents con't

Curriculum Overview for Grades 10-12 Mathematics

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol* has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others.

The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics. The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals for Mathematically Literate Students

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Diverse Cultural Perspectives

Students come from a diversity of cultures, have a diversity of experiences and attend schools in a variety of settings including urban, rural and isolated communities. To address the diversity of knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students, a variety of teaching and assessment strategies are required in the classroom. These strategies must go beyond the incidental inclusion of topics and objects unique to a particular culture.

For many First Nations students, studies have shown a more holistic worldview of the environment in which they live (Banks and Banks 1993). This means that students look for connections and learn best when mathematics is contextualized and not taught as discrete components. Traditionally in Indigenous culture, learning takes place through active participation and little emphasis is placed on the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is important that teachers understand and respond to both verbal and non-verbal cues to optimize student learning and mathematical understandings.

Instructional strategies appropriate for a given cultural or other group may not apply to all students from that group, and may apply to students beyond that group. Teaching for diversity will support higher achievement in mathematics for all students.

Adapting to the Needs of All Learners

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Universal Design for Learning

The New Brunswick Department of Education and Early Childhood Development‘s definition of inclusion states that every child has the right to expect that his or her learning outcomes, instruction, assessment, interventions, accommodations, modifications, supports, adaptations, additional resources and learning environment will be designed to respect his or her learning style, needs and strengths.

Universal Design for Learning is a "…framework for guiding educational practice that provides flexibility in the ways information is presented, in the ways students respond or demonstrate knowledge and skills, and in the ways students are engaged." It also "...reduces barriers in instruction, provides appropriate accommodations, supports, and challenges, and maintains high achievement expectations for all students, including students with disabilities and students who are limited English proficient" (CAST, 2011).

In an effort to build on the established practice of differentiation in education, the Department of Education and Early Childhood Development supports *Universal Design for Learning* for all students. New Brunswick curricula are created with universal design for learning principles in mind. Outcomes are written so that students may access and represent their learning in a variety of ways, through a variety of modes. Three tenets of universal design inform the design of this curriculum. Teachers are encouraged to follow these principles as they plan and evaluate learning experiences for their students:

- **Multiple means of representation:** provide diverse learners options for acquiring information and knowledge
- **Multiple means of action and expression:** provide learners options for demonstrating what they know
- **Multiple means of engagement:** tap into learners' interests, offer appropriate challenges, and increase motivation

For further information on *Universal Design for Learning*, view online information at http://www.cast.org/

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **patterns**, **relationships, spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12,… can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain (Steen, 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to already learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Many important properties in mathematics do not change when conditions change. Examples of constancy include:

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- the theoretical probability of an event.

Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

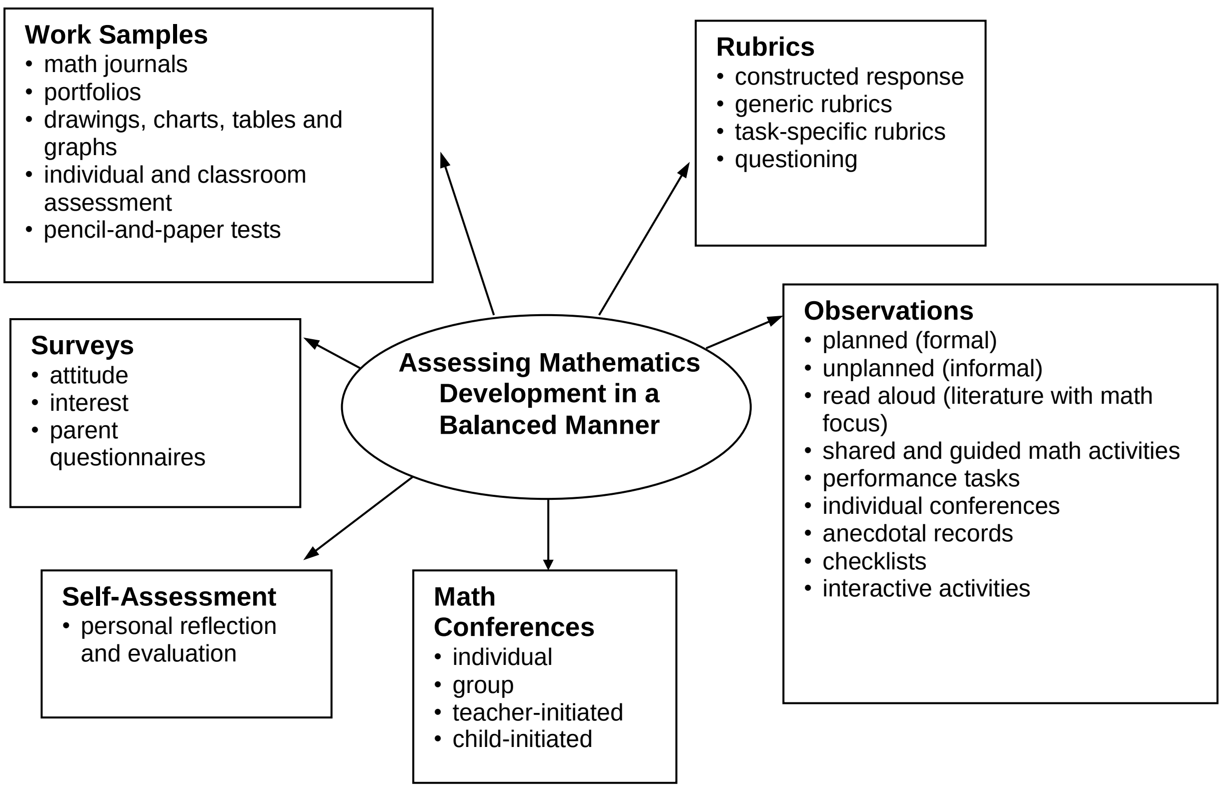
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

GRADE TOPICS	10 11 12
The topics of study vary in the courses for grades 10–12 mathematics. Topics in the pathways include: • *Algebra* • *Financial Mathematics* • *Geometry* • *Logical Reasoning* • *Mathematics Research Project* • *Measurement* • *Number* • *Permutations, Combinations and Binomial Theorem* • *Probability* • *Relations and Functions* • *Statistics* • *Trigonometry*	**GENERAL OUTCOMES** **SPECIFIC OUTCOMES** **ACHIEVEMENT INDICATORS**

NATURE OF MATHEMATICS

Change
Constancy
Number Sense
Patterns
Relationships
Spatial Sense
Uncertainty

MATHEMATICAL PROCESSES:
Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.
Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- connect mathematical ideas to other concepts in mathematics, to everyday
- experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).
- develop mathematical reasoning (Reasoning: R)

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you...?* or *How could you ...?,* the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*" (Rubenstein, 2001).

Mental mathematics "*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense.

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking.

Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if ….*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

ESSENTIAL GRADUATION LEARNINGS

Graduates from the public schools of Atlantic Canada will be able to demonstrate knowledge, skills, and attitudes in the following essential graduation learnings. These learnings are supported through the outcomes described in this curriculum document.

Aesthetic Expression
Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship
Graduates will be able to assess social, cultural, economic, and environmental interdependence in a local and global context.

Communication
Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) as well as mathematical and scientific concepts and symbols to think, learn, and communicate effectively.

Personal Development
Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving
Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language, mathematical, and scientific concepts.

Technological Competence
Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems

PATHWAYS AND TOPICS

The Common Curriculum Framework for Grades 10–12 Mathematics on which the New Brunswick Grades 10-12 Mathematics curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. In New Brunswick all Grade 10 students share a common curriculum covered in two courses: *Geometry, Measurement and Finance 10* and *Number, Relations and Functions 10*. Starting in Grade 11, three pathways are available: *Finance and Workplace*, *Foundations of Mathematics*, and *Pre-Calculus*.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. Students are encouraged to cross pathways to follow their interests and to keep their options open. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all three pathways are to provide prerequisite attitudes, knowledge, skills and understandings for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

Financial and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, statistics and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, statistics and probability.

Pre-calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Students develop a function tool kit including quadratic, polynomial, absolute value, radical, rational, exponential, logarithmic and trigonometric functions. They also explore systems of equations and inequalities, degrees and radians, the unit circle, identities, limits, derivatives of functions and their applications, and integrals.

Outcomes and Achievement Indicators

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each course.

Specific Curriculum Outcomes (SCO) are statements that identify the specific knowledge, skills and understanding that student are required to attain by the end of a given course. The word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome. The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome. The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.

Instructional Focus

Each pathway in *The Common Curriculum Framework for Grades 10–12 Mathematics* is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful. Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning, assessment as learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.

Mathematics K-9

Grade 10

- *2 x 90 hr courses; required to pass both*
- *May be taken in any order or in the same semester*

Geometry, Measurement and Finance 10

Number, Relations and Functions 10

Grade 11

- *3 x 90 hr courses offered in 3 pathways*
- *Students are required to pass at least one of "Financial and Workplace Mathematics 11" **or** "Foundations of Mathematics 11".*
- *Pre-requisite Grade 10 course(s) must be passed before taking Grade 11 courses.*

Financial and Workplace Mathematics 110

Pre-requisite:
Geometry, Measurement and Finance 10

Foundations of Mathematics 110

Pre-requisites:
*Geometry, Measurement and Finance 1 **and** Number, Relations and Functions 10*

Pre-Calculus 110

Pre-requisite or Co-requisite:
Foundations of Mathematics 110

Grade 12

- *5 x 90 hr courses offered in 3 pathways*
- *Pre-requisite Grade 11 or Grade 12 course must be passed before taking Grade 12 courses.*

Financial and Workplace Mathematics 120

Pre-requisite:
*Financial and Workplace Mathematics 110 **or** Foundations of Mathematics 110*

Foundations of Mathematics 120

Pre-requisite: Foundations of Mathematics 110

Pre-Calculus A 120

Pre-requisite: Pre-Calculus 110

Pre-Calculus B 120

Pre-requisite or Co-requisite:
Pre-Calculus A 120

Calculus 120

Pre-requisites:
*Pre-Calculus A 120 **and** Pre-Calculus B 120*

SUMMARY

The Conceptual Framework for Grades 10–12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in grades 10–12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The heading of each page gives the General Curriculum Outcome (GCO), and Specific Curriculum Outcome (SCO). The key for the mathematical processes follows. A Scope and Sequence is then provided which relates the SCO to previous and next course SCO's. For each SCO, Elaboration, Achievement Indicators, Suggested Instructional Strategies, and Suggested Activities for Instruction and Assessment are provided. For each section, the *Guiding Questions* should be considered.

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

Mathematical Processes

[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Math and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Scope and Sequence

Previous Grade or Course SCO's	Current Grade SCO	Following Grade or Course SCO's

Elaboration

Describes the "big ideas" to be learned and how they relate to work in previous Grades

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Achievement Indicators

Describes observable indicators of whether students have met the specific outcome

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

Suggested Instructional Strategies

General approach and strategies suggested for teaching this outcome

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Suggested Activities for Instruction and Assessment

Some suggestions of specific activities and questions that can be used for both instruction and assessment.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

Financial and Workplace Mathematics 110

Specific Curriculum Outcomes

SCO **G1: Solve problems that involve two and three right triangles**. [CN, PS, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Geometry

G1: Solve problems that involve two and three right triangles

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
A1: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income. currency exchange, interest and finance charges. *(GMF10)* **G2:** Demonstrate an understanding of the Pythagorean theorem by: identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems. *(GMF10)* **G3:** Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems. *(GMF10)* **G4:** Solve problems that involve angle relationships between parallel, perpendicular and transversal lines. *(GMF10)* **G5:** Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: drawing, replicating and constructing, bisecting, and solving problems. *(GMF10)*	**G1:** Solve problems that involve two and three right triangles.	**G1:** Solve problems by using the sine law and cosine law, excluding the ambiguous case. *(FWM12)* **G2:** Solve problems that involve triangles, quadrilaterals, regular polygons. *(FWM12)*

ELABORATION

Students have background knowledge of right angle triangles, Pythagorean theorem, basic trig ratios, similar triangles, and angles of elevation and depression, from grades 8-10.

Students will revisit these concepts and apply them to solve open-ended problems by combining sides and using methods that relate back to the right triangle methods of trigonometry ratios, and Pythagorean Theorem. Students will further explore these concepts and extend them when they solve multiple triangles and explore 3-D situations.

ACHIEVEMENT INDICATORS

- Identify all of the right triangles in a given illustration for a context.
- Determine if a solution to a problem that involves two or three right triangles is reasonable.
- Sketch a representation of a given description of a problem in a 2-D or 3-D context.
- Solve a contextual problem that involves angles of elevation or angles of depression.
- Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.

SCO **G1: Solve problems that involve two and three right triangles**. [CN, PS, T, V]

Suggested Instructional Strategies

- Discuss how the 3,4,5 ratio found in some right triangles is used in carpentry and other trades.
- Work with students to create the understanding that the smallest side of a triangle is opposite the smallest angle and similarly the largest side is opposite the largest angle, while working on contextual problems.
- Have students recognize situations and problems where there is more than one method of obtaining the solution and where the Pythagoras Theorem and Trig Ratios may be used.
- Have students apply trigonometry concepts used in the workplace and/or take students into the workplace where these skills are being applied.
- Incorporate questions using both the SI and Imperial systems of measurement into questions
- Take advantage of free electronic resources produced by the Construction Sector of Canada at www.csc-ca.org. One of the resources on this site, "Build on Talents", is an effective resource for this outcome.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Have students check the classroom dimensions, and other classroom items for 90^{o} angles using the $3, 4, 5$ method.

Act Invite a Trades Person into your classroom to discuss and demonstrate how skills in Trigonometry are used in their work. This could also be the Industrial Arts Teacher in your school or a local community member.

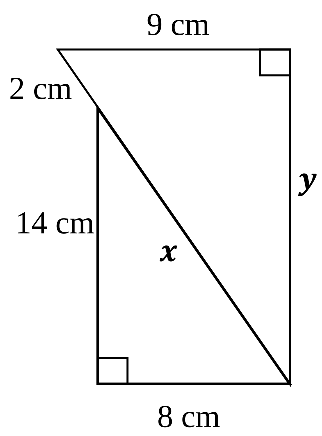
Act Have students perform an experiment to calculate the height of a tree (or whatever is available – perhaps a high point within the school). They will use their previous knowledge of similar triangles to solve a small triangle formed by themselves and a mirror placed on the floor or ground. By measuring the distance between the mirror and object and the equal angle of elevation in each triangle they will be able to solve for the height of the tree or similar tall object.

Q To calculate the height of a tree, Marie measures the angle of elevation from a point A along the ground to the top of the tree to be $34°$. She measures her distance to be $8\ m$ from the base of the tree. How tall is the tree? $Answer\text{: } 5.4\ m$

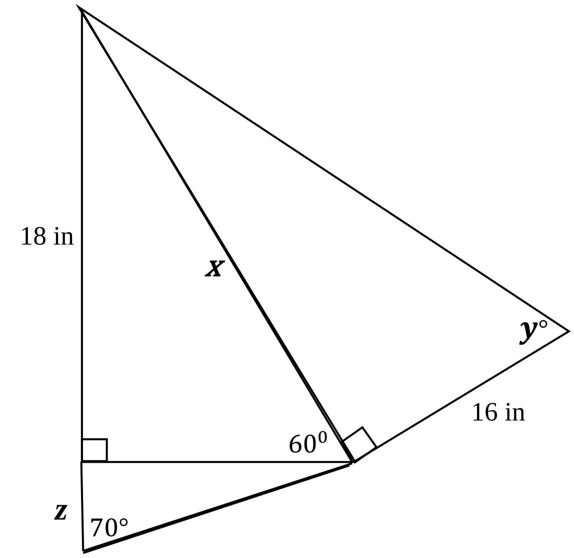
SCO **G1: Solve problems that involve two and three right triangles**. [CN, PS, T, V]

Q Solve for x, y and z in the following diagrams.

a)



b)



Answers: a) $x = 16.12 cm, y = 15.72\ cm$ b) $x = 20.8\ in, y = 52.4°, z = 3.8\ in$

SCO **G2: Solve problems that involve scale.** [PS, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

G2: Solve problems that involve scale.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
G3: Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems. *(GMF10)* **G5:** Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: drawing, replicating and constructing, bisecting, and solving problems. *(GMF10)*	**G2:** Solve problems that involve scale.	**G3.** Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations. *(FWM 12)*

ELABORATION

Students were introduced to 2-D scale diagrams grade 9, as related similar triangles. In Grade 10 this concept was developed in the context of trigonometric ratios. They also developed their skills in proportional reasoning and use of algebraic formulas, and in using both Imperial and SI units of measure.

In this outcome students will draw a variety of 3-D objects to scale and will use ratios and equations to draw and construct diagrams and models. A **scale statement** is a ratio that compares the size of the model to the original object. The **scale factor** is the number by which all measurements of the model or object are multiplied to obtain the measurements of the original or enlargement or reduction.

Questions and activities in this outcome should reflect both Imperial and SI units of measure.

ACHIEVEMENT INDICATORS

- Describe contexts in which a scale representation is used.
- Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model.
- Build a model of a 3-D object, given the scale.
- Draw, with and without technology, a scale diagram of a given figure.
- Solve a contextual problem that involves scale.

SCO **G2: Solve problems that involve scale.** [PS, R, T, V]

Suggested Instructional Strategies

- Revisit the concept of scale factor and how it is applied to all scale figures.
- Investigate where scale diagrams are used in specific occupations (fashion, graphics, interior design, surveying, drafting, etc).
- Brainstorm and/or gather scaled objects, maps, models that are found in the school or homes and discuss the related scale factor.
- Revisit Imperial and SI systems and conversions and implement both systems in questions. Scales in both measurement systems are usually provided on maps.
- Gather simple models and manipulatives for students to draw.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Create a scale drawing of your bedroom. Choose an appropriate scale and note it on your drawing. Include the location of windows, doors, closets, etc. Websites to examine scale on maps http://atlas.nrcan.gc.ca.

Q A scale for a mini car toy collection is $1:67$. The dimensions of the toy car are: $3.4 \times 2.5 \times 1.5\ cm$. Determine the actual dimensions of the vehicle.
$Answer$: $Multiply\ each\ dimension\ by\ 67.\ The\ actual\ dimensions\ are\ 227.8 \times 167.5 \times 100.5\ cm\ \ or\ \ 2.278 \times 1.675 \times 1.005 m.$

Q Gideon has a doghouse that has a length of 5 feet and a width of 3 feet. His neighbour Leah likes the design of Gideon's doghouse but has a smaller dog and wants her doghouse to have a length of 4 feet.

a) What scale factor would Leah use to find the dimensions of her doghouse?
$Answer$: $80\%\ \ or\ 1.0: 0.8$

b) What would the width of Leah's doghouse be?
$Answer$: $80\%\ of\ 3 = 2.4\ ft$

SCO: **G3: Model and draw 3-D objects and their views.** [CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

G3: Model and draw 3-D objects and their views.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
M5: Solve problems using SI and imperial units that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres. *(GMF10)*	**G3:** Model and draw 3-D objects and their views.	**G3:** Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations. *(FWM12)*

ELABORATION

Students have been exposed to some 3-D concepts. In grade 8, students had to draw and construct nets for 3-D objects, and draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms. In grade 9, students worked with isometric dot paper to draw 2-D shapes and found surface areas of 3-D shapes. In grade 10, students found the surface area and volume of 3-D objects.

For this outcome students will draw various views of 3-D objects to scale. **Orthographic drawings** present a 2-D view of a 3-D object, as a front, top and side view of the object. **Isometric drawings** present a view of a 3-D object in which all horizontal edges of the object are drawn at a $30°$ angle, all vertical edges of the object are drawn vertically, and all lines are drawn to scale.

Students will also learn to create **one point perspective drawings** in which the objects appear proportionally smaller with distance. Objects in the background will appear to join at a **vanishing point** in the distance. The point of perspective from above, below, or from one side or another will determine the direction to which the drawing decreases with distance.

This material is new to the math curriculum but is integral to studies in visual arts, engineering and industrial arts, in which the ability to visualize and draw from different perspectives without technology is an important skill. It is important to approach this outcome as cross curricular, and to acknowledge and build on students' previous knowledge and skills from other courses and experiences.

SCO: **G3: Model and draw 3-D objects and their views.** [CN, R, V]

ACHIEVEMENT INDICATORS

- Draw a 2-D representation of a given 3-D object.
- Draw, using isometric dot paper, a given 3-D object.
- Draw to scale top, front and side views of a given 3-D object.
- Construct a model of a 3-D object, given the top, front and side views.
- Draw a 3-D object, given the top, front and side views.
- Determine if given views of a 3-D object represent a given object, and explain the reasoning.
- Identify the point of perspective of a given one-point perspective drawing of a 3-D object.
- Draw a one-point perspective view of a given 3-D object.

Suggested Instructional Strategies

- Have students create shapes with linking cubes or other building tools and then experiment with drawing the objects on graph, isometric and plain paper.
- Gather simple objects (shelves, boxes, etc) to have in the classroom for students to use for drawings.
- Access internet resources that explain and demonstrate how to draw from various perspectives. For example, http://www.mr-d-n-t.co.uk/isometric.htm provides instructions and some exercises on isometric drawing, and on YouTube there are multiple drawing lessons available if you search “orthographic drawing”, “isometric drawing” and “one-point perspective”.
- Collaborate with technology, industrial art, graphic design, and visual art teachers (at the high school or *NBCC*) to develop cross-curricular activities and resources.
- Students should demonstrate the ability to draw various perspectives by hand but can also explore electronic tools such as the isometric drawing tool available at: http://illuminations.nctm.org/ActivityDetail.aspx?ID=125.
- As an extension have students research and attempt 2-point perspective drawings.

SCO: **G3: Model and draw 3-D objects and their views.** [CN, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Have students create a simple 3-D figure with linking cubes. Without showing the figure to other students have them draw the top, bottom, and side views of the figure and label them. Have partners share only their drawings, each recreating the 3-D figure with linking blocks. In pairs have students compare their creation with the original, and work together to make corrections to the drawings or figures to ensure they match.

Q Sue built the birdhouse pictured below. The front measures $23 \times 15\ cm$, the back $19\ \times 15\ cm$. The side panel is $13\ cm$ wide. The roof measures $18 \times 15\ cm$. The bird hole has a diameter of $3\ cm$ and it's centre is $5\ cm$ from the top and $7.5\ cm$ from each side.

a) Determine the measurements of the base.
b) Draw the component parts of the diagram using a scale of $1{:}\,5$.

Q Cody is designing a flyer for a local furniture store. He wants to include a perspective view of a dining table. Sketch what his drawing could look like.

SCO	**G4: Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.** [CN, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

G4: Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
	G4: Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.	**G3:** Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations. *(FWM12)*

ELABORATION

This is a continuation of outcome G3 in which students will create a set of isometric drawings as **exploded view diagrams**. These diagrams show the parts slightly separated, expose the hidden parts, and show the relative position and orientation of the parts with respect to each other. They are often used to illustrate the sequence of steps required to assemble an object.

This material is new to the math curriculum but is related to studies in visual arts, engineering and industrial arts, in which the ability to visualize and draw exploded views without technology is an important skill. It is important to approach this outcome as cross curricular, and to acknowledge and build on students' previous knowledge and skills from other courses and experiences.

ACHIEVEMENT INDICATORS

- Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object.
- Sketch an exploded view of a 3-D object to represent the components.
- Draw the components of a 3-D object to scale.
- Sketch a 2-D representation of a 3-D object, given its exploded view.

Suggested Instructional Strategies

- Start the exploration of exploded views with a simple cube, and then go on to explore more difficult shapes.
- Do as many hands-on activities as possible. Work both from diagram to object, and from object to diagram.
- Access internet resources that show exploded views as part of the assembly instructions for various products. For example, furniture companies such as IKEA provide online assembly instructions which include exploded views.
- Access internet resources that demonstrate how to draw exploded views. For example, http://www.mr-d-n-t.co.uk/exploded-view.htm shows exploded views of objects, and YouTube provide links to examples of exploded views, and lessons on how to draw exploded views (e.g., Sketch-A-Day 377 Quick Exploded View)

SCO	**G4: Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.** [CN, V]

- Collaborate with technology, industrial art, graphic design, and visual art teachers (at the high school or *NBCC*) to develop cross-curricular activities and resources.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

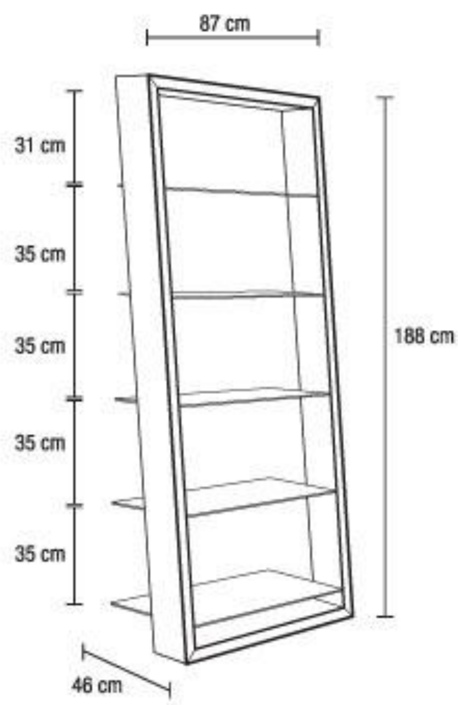
Act Present students with exploded diagrams gathered from assembly directions for items like furniture, appliances or computers, or from the internet, and have a "guess what it is" quiz.

Q Create the instruction pamphlet for an object of your choice. Your pamphlet needs to include the exploded view and the written directions for assembling the object. Have the object available for hands-on demonstration or testing.

Q Lee is a woodworking teacher at NBCC. One project that his students are required to do is to create a components diagram and an exploded diagram to help the students understand how to cut and assemble the project.

a) Draw the components of the shelving unit using a scale factor of 10.

b) Draw the exploded diagram showing how the pieces of the shelving unit fit together.



SCO **N1**: **Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.** [C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Number

N1: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
G1: Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.*(GMF10)*	**N1:** Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.	**N1:** Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. *(FWM12)*

ELABORATION

Puzzles and games provide opportunities to explore patterns and to link spatial and numerical concepts. In grade 10, students focused on playing and analyzing puzzles and games that involved spatial reasoning. They discussed strategies used to solve a puzzle or win a game. In grade 11, they will extend these skills to games and puzzles which involve numerical reasoning.

Students will use familiar problem solving strategies to explain and verify a strategy to solve the puzzle or win the game. All puzzles/games should utilize mathematical concepts that students are familiar with from previous courses so that the emphasis is placed on the more complex skill of reasoning.

This outcome provides tremendous opportunity to differentiate as students tackle games or puzzles appropriate to their current level of ability and understanding. Teachers should try games in advance as the difficulty and the instructions to games or puzzles are not always clear.

It is intended that this outcome be integrated throughout the course giving students a related activity every week or two as time permits.

ACHIEVEMENT INDICATORS

- Determine, explain and verify a strategy to solve a puzzle or to win a game. For example,
  - guess and check
  - look for a pattern
  - make a systematic list
  - draw or model
  - eliminate possibilities
  - simplify the original problem
  - work backward
- Develop alternative approaches to solving puzzles.
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

SCO	**N1**: **Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.** [C, CN, PS, R]

- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Suggested Instructional Strategies

- Have students look for numerical or other patterns and then develop a strategy to fit these patterns.
- Have students develop a game for classmates to play.
- Using a known game, change a rule or parameter and explain how it affects the outcome of the game.
- Find a game online and critique the quality of the game.
- Plan to have a “games and puzzles day” every other week. Students should switch partners periodically to provide opportunities for new strategies to be shared.
- Have students keep a games and puzzles journal that they are required to write in every “games and puzzles day”. Have them reflect on the strategies they used to solve the puzzle or win the game. The following gives an example of how the journal could be set up.

Games Journal			
Date	Game	Win or Lose OR Score	Explain your strategies

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Solve the following puzzles:

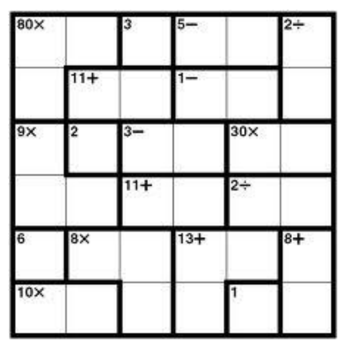
a)

	2	4	
1			3
4			2
	1	3	

b)

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

c)



Act The internet is a good source form numerical reasoning problems or games. Here are a few examples of useful sites:
http://samgine.com/free/number-puzzles/
http://www.fibonicci.com/numeracy/number-sequences-test/medium/
http://www.mindjolt.com
http://education.jlab.org/nim/index.html
http://dtai.cs.kuleuven.be/projects/ALP/newsletter/archive_93_96/humour/index-num.html
www.combinationlock.com
http://pbskids.org/

SCO **N2: Analyze costs and benefits of renting, leasing and buying.** [CN, PS, R, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

N2: Analyze costs and benefits of renting, leasing and buying.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
A1: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income. currency exchange, interest and finance charges.*(GMF10)* **N1:** Solve problems that involve unit pricing and currency exchange, using proportional reasoning. *(GMF10)* **N3:** Demonstrate an understanding of financial institution services used to access and manage finances. *(GMF10)* **N4:** Demonstrate an understanding of compound interest. *(GMF10)* **N5:** Demonstrate an understanding of credit options, including: credit cards, and loans. *(GMF10)*	**N2:** Analyze costs and benefits of renting, leasing and buying.	

ELABORATION

In Grade 10, students solved problems involving simple and compound interest, and the computation of finance charges. In this outcome they will use this knowledge to explore the costs of renting, leasing and buying.

This outcome is not covered in the core resource for this Grade 11 course, but is covered in the core resource for *Financial and Workplace 120* (*Math at Work 12* Ch.4.1-4.2, renting and leasing vehicles) and for *Foundations of Mathematics 120* (*Foundations of Mathematics 12* Ch.2.4, renting and leasing in other situations).

Assets or property are items that are owned or partially owned. For example: vehicles, i-phones, laptops or real estate.

Appreciation is the increase in the value of an asset over time. This increase can occur for a number of reasons including increased demand or weakening supply, or as a result of changes in inflation or interest rates. **Depreciation** is the decrease in the value of an asset over time.

Renting and **leasing** are similar, but differ in duration. A **rental** is a short-term agreement or contract under which capital property is rented from one person to another on an hourly, daily, weekly or monthly basis with rates tending to decrease the longer the rental period. On the other hand, a **lease** is a long-term agreement or contract, under which capital property is rented from one person to another for a fixed period of time (usually one year or more) at a specified rate. However, these terms are often used interchangeably.

SCO **N2: Analyze costs and benefits of renting, leasing and buying.** [CN, PS, R, T]

ACHIEVEMENT INDICATORS

- Identify and describe examples of assets that appreciate or depreciate.
- Compare, using examples, renting, leasing and buying.
- Justify, for a specific set of circumstances, if renting, buying or leasing would be advantageous.
- Solve a problem involving renting, leasing or buying that requires the manipulation of a formula.
- Solve, using technology, a contextual problem that involves cost-and-benefit analysis.

Suggested Instructional Strategies

- Bring in a manager from a car dealership or a bank officer to speak to the class about the advantages and disadvantages of renting, leasing and buying.
- Have students browse the classified ads, and make predictions as to which properties might appreciate or depreciate the most in the next few years. Discuss the reasons for these predictions.
- When discussing depreciation, choose examples which are pertinent to teenagers. Get students to choose their favorite vehicle and research the depreciation rate of the car. Use a website such as www.ehow.com/list_6923399_depreciation-rules-canada.html, or www.canadianblackbook.com for research on depreciation.
- Using a graphing calculator, explore the decay curve that is representative of an automobile's depreciation.
- Use www.smbtn.com/books/gb79.pdf as a reference when comparing advantages and disadvantages of renting, leasing and buying.
- The site http://www.handsonbanking.org/en/ provides free curriculum resources for a variety of financial mathematics topics.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Choose three different vehicles that you may want to purchase some day. Research the depreciation on each model of car using a website such as www.ehow.com/list_6923399_depreciation-rules-canada.html, or www.canadianblackbook.com.

a) Which model of car depreciates fastest?

b) Which model of car depreciates slowest?

c) For each model of car determine the depreciation rate after 1, 2, and 3 years.

d) What affects the rate at which a car depreciates?

Q Sara is thinking of buying a new TV. She went to one store and found an excellent deal on the lease of a Toshiba 32" LCD HDTV. The weekly lease payment is $12/week, plus the one-time cost of product of $10.93 and a one-time cost of TPC (total protection coverage) of $1.07. On her brother's suggestion, Sara checked another shop and found the same TV on sale there for $349.99.

a) If Sara decides to lease the TV from the first shop rather than buy it at the other shop, how many months will it be before she exceeds the purchase price?

$Answer\text{: } Leasing \ = \$24 + \$12x \qquad Buying\ \$349.99 + 13\%\ tax \ = \ \$395.49$

$\therefore 395.49 = \ 24 + 12x \qquad 371.49 = 12x \qquad 30.95 = x$

$After\ 31\ weeks\ or\ {\sim}7\ months\ the\ leasing\ price\ will\ exceed\ the\ purchase\ price.$

b) Do you think this would be the right choice? Why or why not?

Q Rhonda has just graduated from community college and now wants to move out of her parents' house. She worked part time while she studied and saved $5000 over the past three years. She found a nice apartment that is renting for $480/month and requires a $400 damage deposit. Using her savings, for how long will Rhonda be able to rent her apartment?

$Answer\text{: } She\ will\ be\ able\ to\ rent\ for\ 9\ months.$

Q Joanne is going away to university. She has saved up some money from part-time jobs and plans to buy or lease a new vehicle. She has found a compact car that she likes and is trying to decide whether she should lease or buy, based on the following information from the dealership's website:

Pricing details	**Finance (60 months)**	**Lease (60 months)**	**Cash**
	$13 995	$13 995	$13 995
Selected Savings and Offers	- $750	- $750	- $750
Selected Accessories	$0.00	$0.00	$0.00
	Monthly Payment	**Monthly Payment**	**Cash Price**
	$276.92	**$218.75**	**$13 245**
		Lease end value $4618.35	

a) What is the total price for each option?

b) What are the advantages and disadvantages of each option.

c) If you were Joanne, which option would you choose and why?

$Answer\text{:}$

Option	Total Price	Advantages	Disadvantages
Financing	$16,615.20	Not paying for extra mileage Own vehicle after 60 months	Higher monthly payments Higher final cost with interest
Leasing	$13,125.00	Cheapest monthly payment	Mileage charge Still owe $4618.35 if wanting to own at end of 60 months
Cash	$13,295.00 + tax	Own outright Least expensive option	Need to have money up front

SCO	**N3**: **Analyze an investment portfolio in terms of interest rate, rate of return and total return.** [ME, PS, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

N3 Analyze an investment portfolio in terms of interest rate, rate of return and total return.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
N3: Demonstrate an understanding of financial institution services used to access and manage finances. *(GMF10)* **N4:** Demonstrate an understanding of compound interest. *(GMF10)* **N5:** Demonstrate an understanding of credit options, including: credit cards, and loans. *(GMF10)*	**N3:** Analyze an investment portfolio in terms of interest rate, rate of return and total return.	

ELABORATION

In grade 10, students investigated services offered by financial institutions, and were introduced to simple and compound interest, with a focus on credit options. For this outcome students will apply this knowledge to investment opportunities.

Investments are covered in the core resource with reference to budgets, compound interest, investing and borrowing (*Math at Work 11* Ch.5.2-5.4). Other resources that extend this to investment portfolios are also available in text form or online, including the core resources for *Foundations of Mathematics 110 and 120* in which this outcome is addressed in some detail (*NB Foundations of Mathematics 11* Ch.8.5-8.6, or *Foundations of Mathematics 12* Ch.1.5-1.6 – same material).

The focus for this outcome is on understanding and comparing the effects of simple and compound interest on future values of investments, and on analyzing, comparing and designing **investment portfolios** to meet specific financial goals.

An investment portfolio is comprised of all the different investments that an individual or organization holds. Investments can include **shares**, **bonds**, or **investment certificates**.

When you buy shares (also called stocks or equities) you become a part owner in a company. This gives you the right to a portion of the company's earnings and may entitle you to vote at the shareholder meetings. Compared to other types of investment, shares can be riskier but can potentially offer higher returns. The value of the shares are dependent on the success of the company.

When you buy a bond you are lending money to a government or company for a certain period of time. In return, you receive a fixed rate of interest and your money back at the end of the term. Company bonds offer better rates of return than investments such as **Guaranteed Investment Certificates** but this is because if the company fails you may not get back all the money you originally paid, so there is more risk involved. Government bonds such as Canada Savings Bonds are more secure. **Mutual funds** are a mix of stocks and bonds.

Investment certificates are offered at banks at a higher rate of interest than a chequing account, and are easier to access that stocks or bonds.

SCO **N3**: **Analyze an investment portfolio in terms of interest rate, rate of return and total return.** [ME, PS, R, T]

ACHIEVEMENT INDICATORS

- Determine and compare the strengths and weaknesses of two or more portfolios.
- Determine, using technology, the total value of an investment when there are regular contributions to the principal.
- Graph and compare the total value of an investment with and without regular contributions.
- Apply the Rule of 72 to solve investment problems, and explain the limitations of the rule.
- Determine, using technology, possible investment strategies to achieve a financial goal.
- Explain the advantages and disadvantages of long-term and short-term investment options.
- Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term.
- Solve an investment problem.

Suggested Instructional Strategies

- Go to the *New Brunswick Securities Commission* website http://investinknowingmore.ca/educationprograms.html for links to:
    - Download PDF of *Make it Count: An Instructors Guide for Youth Money Management* (with associated budgeting app, Parent's Guide, Make it Count website) http://csa-acvm.ca/investortools.aspx?id=87
    - the NB Financial Education Network http://investinknowingmore.ca/FinancialEducationNetwork.html which lists other free resources from groups across NB.
- The *Canadian Securities Administrators* site http://csa-acvm.ca/investortools.aspx?id=1005 offers a wealth of information (in English and French) on investing, including free downloadable PDF brochures on investing: *Investing basics: Getting started, Investments at a Glance, Understanding Mutual Funds.*
- Have a competition within the class. Group students and give each a set amount to invest in a portfolio. Every day, give the students an opportunity to check, buy and sell stocks. The group whose portfolio is worth the most at the end of a specified time period wins. You may wish to use the website www.wallstreetsurvivor.com to do this activity.
- Periodically, provide a copy of a newspaper which carries a "World Markets" section (e.g. the Globe and Mail) and have student report on what they read.
- Go to www.getsmarteraboutmoney.ca which is a Canadian website, to access lesson plans, videos for students, and reference materials that support this curriculum.
- Invite local stockbrokers and/or wealth managers to speak to the class.
- Using a T1-83 graphing calculator, have students explore the TVM solver application for various interest rates, amortization periods, principal amounts etc.

SCO **N3**: **Analyze an investment portfolio in terms of interest rate, rate of return and total return.** [ME, PS, R, T]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Thomas has two years before he goes off to community college. He has figured out that it will cost a total of about $10 000 for college. He invests in a GIC that pays 6% interest per year. He deposits $360.00 per month for two years. Using a TVM solver determine if Thomas will have enough money to go to college or will he have to find another way to supplement his education?

$Answer:\quad year\ 1:\ (360 \times 12) \times 1.06 = \$4579.20$

$year\ 2:\ [\$4579.20 + (360 \times 12)] \times 1.06 = \$9433.15$

$He\ will\ be\ short\ by\ \$566.85, so\ will\ need\ to\ supplement\ his\ income.$

Q Richard invested $500 at 4% interest rate. How long will it take for the $500 investment to have a future value of approximately $1000?

$Answer:\quad A = P + Prt$

$1000 = 500 + 500(0.04)t$

$t = 25 \quad \therefore\ it\ will\ take\ 25\ years\ for\ the\ investment\ to\ increase\ to\ \$1000$

Act Present the following scenario to the class and discuss as a group:

Samantha and Rick are old high school buddies who have met up again after twenty years. After some discussion they find out that they have both created investment portfolios, shown below. After the analysis, list two strengths and two weakness of each portfolios. Remember to consider interest rate, rate of return and total return when analyzing.

Samantha: Back in 1984, Samantha was an eighteen year old student. While she had time left to grow her savings, she didn't want to take a lot of risk. Samantha adopted a moderate investment profile with 50% invested in Canadian stocks (moderate risk), 40% in bonds (low risk), and 10% in cash equivalents.

Rick: In 1984, Rick was a 19-year-old professional hockey player. He thrived on taking risk. He also knew he wouldn't play hockey forever and his income might drop after he retired from the game. Rick adopted a moderately aggressive investment profile with 70% invested in Canadian Stocks (moderate risk), 20% in bonds (low risk), and 10% in cash equivalents.

The results: This table shows sample returns for Samantha and Rick, based on data from the past 20 years.

Investor	Started with (Jan 1984)	After 5 years (Jan 1989)	After 10 years (Jan 1994)	After 20 years (Jan 2004)	Average annual return
Samantha	$100 000	$154 330	$236 103	$424 785	7.5%
Rick	$100 000	$129 503	$236 736	$560 441	9.0%

Many things affected our two investors' results over this 20-year period:

- Interest rates soared in the late 1980s. The stock market on the other hand, went through a major drop in 1987 and a slow recovery. During this time, Rick's investments lagged. Samantha's investments grew faster and continued to do well throughout the ups and downs of the early 1990s.
- By 2000, the picture changed. Interest rates fell and the stock market was at a new high. Samantha's investments soon fell behind the rest.

SCO **N3**: **Analyze an investment portfolio in terms of interest rate, rate of return and total return.** [ME, PS, R, T]

- In the last five years, interest rates stayed low while the stock market went through another cycle of ups and downs. Rick's investments continued to grow the fastest, while Samantha fell further behind.

Lesson learned*: In most cases, ultra conservative portfolios will see slower, steady growth. This is the trade off for keeping money stable and secure. For more growth potential, a more aggressive asset mix, with a higher level of risk should be selected. Losses are more likely when more risk is taken when investing and it is important to ensure that there is enough time and money to recover from those losses. Registered advisers are available to determine the right asset mix for your situation.*

(adapted from http://www.getsmarteraboutmoney.ca/managing-your-money/planning/investing-basics/Pages/the-power-of-asset-mix-joan-michel-and-miriams-stories.aspx)

SCO **N4: Solve problems that involve personal budgets.** [CN, PS, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

N4: Solve problems that involve personal budgets.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
N1: Solve problems that involve unit pricing and currency exchange, using proportional reasoning. *(GMF10)* **N2:** Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay. *(GMF10)* **N3:** Demonstrate an understanding of financial institution services used to access and manage finances. *(GMF10)* **N4:** Demonstrate an understanding of compound interest. *(GMF10)* **N5**: Demonstrate an understanding of credit options, including: credit cards, and loans. *(GMF10)*	**N4:** Solve problems that involve personal budgets.	

ELABORATION

In grade 10, students gained an understanding of income and calculated gross and net pay including various tax and other deductions. They also explored in depth the services offered by financial institutions, and the impact of compound interest as related to various credit options. In this course (N2, N3) they have considered options for renting leasing and buying, and investment options.

This financial literacy outcome, N4, draws on all of this knowledge and applies it at a personal level as students explore budgets. This should be fully developed as this will be students' last chance to explore personal budgets within a mathematics course (the focus in *Financial and Workplace Mathematics 120* is on business math).

Students should explore a variety of scenarios, including short- and long-term goals, recurring and unexpected small or large expenses such as loss of a roommate, illness, fire, or loss of job.

A **balanced budget** is one in which the total income equals the total expenses. **Fixed expenses** are expenses that are unlikely to change from month to month while **variable expenses** are expenses that are likely to change from week to week or from month to month.

SCO **N4: Solve problems that involve personal budgets.** [CN, PS, R, T]

ACHIEVEMENT INDICATORS

- Identify income and expenses that should be included in a personal budget.
- Explain considerations that must be made when developing a budget; For example, prioritizing, recurring and unexpected expenses.
- Create a personal budget based on given income and expense data.
- Collect income and expense data, and create a budget.
- Modify a budget to achieve a set of personal goals.
- Investigate and analyze, with or without technology, "what if …" questions related to personal budgets.

Suggested Instructional Strategies

- Have students explore various scenarios and develop personal budgets around these scenarios.
- Use some of the free online resources available on financial literacy such as:

  www.getsmarteraboutmoney.ca

  http://www.rbcroyalbank.com/products/personalloans/budget/budget-calculator.html (a resource provided by RBC to explore the financial implications of "What if? situations such as job loss, loss of roommate, illness etc.)

  www.gailvazoxlade.com/articles.html

SCO **N4: Solve problems that involve personal budgets.** [CN, PS, R, T]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Have students use a chart similar to the following, as a guide to list all the expenses they think they might incur living on their own or with one or more roommates (this can be used as a group activity) .

Expense	*Amount ($)*
Getting started costs: One-time costs such as: hook-up fees for phone, cable or internet,; purchase of furniture, dishes, appliances.	
Rent or Mortgage	
Utilities: Electricity, telephone, heat, cable.	
Food: Staples such as flour, spices, condiments, beans; regular groceries. Home cooked meals are cheaper and usually healthier than restaurant food.	
Transportation: Public transit, bicycle, or car. If you have a car you will need to budget form insurance, gas, maintenance and parking.	
Medical/ Dental: Medical plan payments and/or costs such as glasses, contacts, prescriptions and dental care not covered by provincial Medicare or by a medical plan.	
Clothing: Consider clothes required for work, and seasonal clothes such as boots and a winter coat.	
Miscellaneous: This may include laundry, entertainment, toiletries, and cleaning supplies. Also consider purchasing gifts for birthdays and holidays.	
Other: This includes anything else that is not included in other categories such as loan payments, vacations, membership or workshop fees.	
TOTAL OF ALL ESTIMATED COSTS	

Act Present students with various scenarios – living on their own and working, living as a single parent with an infant or school aged child, going to school and working part time, working full time, living as a two income family with two children etc. Have them develop their own budget that includes all of their expenses or complete a budget such as the one found at:
http://moneyandyouth.cfee.org/en/resources/pdf/moneyyouth_chap9.pdf .

This could also be an opportunity for students to interview someone living in various circumstances listed above, to get a true picture of expenses, especially hidden ones.

SCO **A1: Solve problems that require the manipulation and application of formulas related to: slope and rate of change, Rule of 72, finance charges, the Pythagorean theorem and trigonometric ratios.** [CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Algebra

A1: Solve problems that require the manipulation and application of formulas related to: slope and rate of change, Rule of 72, finance charges, the Pythagorean theorem and trigonometric ratios.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
A1: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income, currency exchange, interest and finance charges. *(GMF10)*	**A1:** Solve problems that require the manipulation and application of formulas related to: slope and rate of change, Rule of 72, finance charges, the Pythagorean theorem and trigonometric ratios.	**G2:** Solve problems by using the sine law and cosine law, excluding the ambiguous case. *(FWM12)*

ELABORATION

Students have been modeling and solving various forms of linear equations since Grade 7 and have practiced manipulating these equations to solve for an unknown variable. In grade 10, they have solved problems that required the manipulation of formulas that related to topics of study. This skill should be further developed and the outcome addressed throughout this course as students apply formulas in a variety of contexts such as slope and rate of change, rule of 72, Pythagorean Theorem and trig ratios.

This is not an outcome that can be taught in isolation but must be integrated as a foundational concept within each of the units in the course.

ACHIEVEMENT INDICATORS

- Solve a contextual problem involving the application of a formula that does not require manipulation.
- Solve a contextual problem involving the application of a formula that requires manipulation.
- Explain and verify why different forms of the same formula are equivalent.
- Describe, using examples, how a given formula is used in a trade or an occupation.
- Create and solve a contextual problem that involves a formula.
- Identify and correct errors in a solution to a problem that involves a formula.

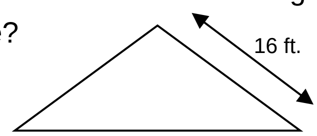
SCO **A1: Solve problems that require the manipulation and application of formulas related to: slope and rate of change, Rule of 72, finance charges, the Pythagorean theorem and trigonometric ratios.** [CN, PS, R]

Suggested Instructional Strategies

- When teaching each topic in this course, ensure that students can apply and manipulate the appropriate formulas.
- Examples of formula use can be found within the document in the "Suggested Instructional Strategies" and the "Suggested Activities for Instruction and Assessment", which addresses each topic.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Mark wants the pitch of the roof he is building to be $\frac{4}{12}$. The length of the truss is $16\ ft$. What is the rise?



$Answer: Trig: \tan\theta = \frac{4}{12} = 18.43°$ $\quad \sin 18.43° = \frac{rise}{16}$ $\quad \boldsymbol{rise = 5.06\ ft}$

$Pyth. theorem: \sqrt{hyp} = \sqrt{12^2 + 4^2}$ $\quad hyp = 12.65$ $\quad Prop. reas: \frac{12.65}{4} = \frac{16}{rise}$ $\quad \boldsymbol{rise = 5.06}$

Q Edward has saved half of what he will need to buy a car in $10\ years$. If he invests his current savings, and does not contribute additional funds, what is the rate of return that he needs to earn? $Answer: \frac{72}{x} = 10 \quad x = 7.2\ years$

Q Emma is training to be an Amazon Jungle tour guide. As part of her training, she must climb a slanted rope net into a tree that is $45\ ft$ high. How far away from the tree is the net anchored if the net is $100\ ft$ long?

$Answer: Trig.\ \sin\theta = \frac{45}{100} = 0.45 \quad sin^{-1} = 26.74° \quad \cos 26.74° = \frac{dist}{100} \quad dist. from\ tree = 89.3\ ft$

$Pyth. theorem\ 100^2 = 45^2 + x^2 \quad x = dist. from\ tree = 89.3\ ft)$

Q The most efficient operating angle for a certain conveyor belt is $35°$. If the parts must be moved a vertical distance of $19\ ft$, what length of conveyor is needed?

$Answer: \sin 35° = \frac{19}{conveyer} \quad conveyer = \frac{19}{0.5736} = 33.1\ ft$

Q A road has a rise of $6m$ in $80m$. What is the gradient angle of the road?

$Answer: \tan\theta = \frac{6}{80} = 0.075 \quad \tan^{-1} 0.075 = 4.29°$

Extension:

Q If an investment of \$25 000 earns 9% annual interest, approximate the value of the investment after 24 years.

Answer: In summary, your original investment of \$25 000 grows to \$200 000 in 24 years if it is invested at 9% and is left untouched. A way of looking at this is that you double your money every 8 years so that in 24 years you double, double again, and double one more time.
This is $2 * 2 * 2 = 8$ so that your original investment grows by 8 times (8 times \$25 000 = \$200 000)

SCO **A2: Demonstrate an understanding of slope as rise over run, as rate of change, and by solving problems.** [C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

A2: Demonstrate an understanding of slope as rise over run, as rate of change, and by solving problems.

Scope and Sequence of Outcomes:

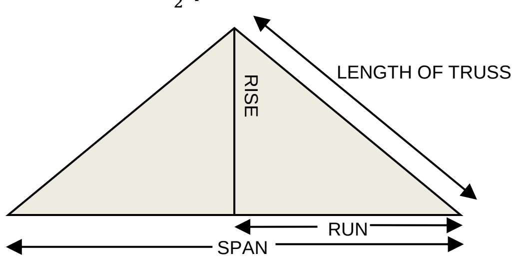
Grade Ten	Grade Eleven	Grade Twelve
RF3: Demonstrate an understanding of slope with respect to: rise and run, line segments and lines, rate of change, parallel lines, perpendicular lines. *(NRF10)*	**A2:** Demonstrate an understanding of slope as rise over run, as rate of change, and by solving problems.	

ELABORATION

Students have been exposed to the concept of slope in grade 10. Rise over run, undefined slope, zero slope and rate of change have been explored, but only with reference to the Cartesian plane.

This outcome relates slope to contextual applications such as pitch or steepness of a roof. Students will develop an understanding of slope in a variety of situations and be able to compare slopes. This outcome is directly linked to outcome G1 in which students study right triangles, angles of elevation and depression, and trigonometric ratios. This exploration of slope should be taught with reference to this previous work, and possibly directly after G1, to reinforce the important practical applications of these concepts.

The $slope = \frac{rise}{run}$ and for a roof this is called the pitch. The span of a roof is twice as long as the run, so $pitch = \frac{rise}{\frac{1}{2}span}$ or $rise = pitch \times \frac{1}{2}span$.



For additional information on pitch, or slope of a roof go to: http://roofgenius.com/roofpitch.htm

SCO **A2: Demonstrate an understanding of slope as rise over run, as rate of change, and by solving problems.** [C, CN, PS, V]

ACHIEVEMENT INDICATORS

- Describe contexts that involve slope; e.g., ramps, roofs, road grade, flow rates within a tube, skateboard parks, ski hills.
- Explain, using diagrams, the difference between two given slopes (e.g., a 3:1 and a 1:3 roof pitch), and describe the implications.
- Describe the conditions under which a slope will be either 0 or undefined.
- Explain using examples and illustrations, slope as rise over run.
- Verify that the slope of an object, such as a ramp or a roof, is constant.
- Explain, using illustrations, the relationship between slope and angle of elevation; e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4º.
- Explain the implications, such as safety and functionality, of different slopes in a given context.
- Explain, using examples and illustrations, slope as rate of change.
- Solve a contextual problem that involves slope or rate of change.

Suggested Instructional Strategies

- Have students brainstorm various applications of slope and to categorize the applications in regards to occupations and/or situations where slope would be applied.
- Take pictures of various roofs and compare pitch (slope) of the roofs. This can also be related to the angle of elevation.
- Network with various trades people in regards to how they implement the concept of slope within their occupation.
- Collaborate with carpentry teachers to find out the applications of slope that they would use in their workshop/course.
- Have students research the provincial guidelines for slopes of wheelchair ramps for home or for public buildings.
- Create a template on a transparency of various pitches ($3/12, 4/12, 5/12$ and $6/12$). Put the 12 on the horizontal and then create various similar triangles for the different pitches. Have students check the pitch of different roofs in their neighborhood using the template.

SCO **A2: Demonstrate an understanding of slope as rise over run, as rate of change, and by solving problems.** [C, CN, PS, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Recommended angles of stairs are between 30° and 35°.

a) If all stairs in a staircase are the same height and depth, the rise is $8'11"$, and the run is $12'$, is the angle of the stairway within the recommended range?

$Answer\text{: } tan^{-1}\left(8\frac{11}{12}\right)/12 = 36.6° \therefore angle\ is\ not\ within\ the\ recommended\ range$

b) Adjust either the rise or run so that the angle is within the recommended range.

$Sample\ answer\text{:}$
$tan\,32° = \frac{x}{12} = 7.5'$. $A\ rise\ of\ \ 7'6", and\ a\ run\ of\ \ 12'\ will\ give\ a\ stairway\ angle\ of\ \ 32°$

c) The recommended rise of each stair is 7" to 7 ½". If the stairway in b) has 15 stairs, is the rise of each stair within the recommended range?
$(note\text{: } will\ need\ to\ use\ the\ answer\ for\ 7.5\ ft\ from\ part\ b)\ to\ solve)$

$Answer\text{: } \frac{7.5\ feet}{15\ stairs} = \frac{90\ inches}{15\ stairs} = 6\ inches, which\ is\ less\ than\ the\ recommended\ range.$

Q The span of a roof is 30 feet *(most construction is done in imperial)* and the pitch is 1/3. What is the length of the roof?

$Answer\text{: } Step\ 1\text{: } \quad rise = pitch \times \frac{1}{2} span = 5$

$Step\ 2\text{: } \quad \sqrt{length} = 5^2 + 15^2 = 15.811'or\ 15'9\,\frac{12}{16}")$

Q The span of a roof is 22 feet and the length is 12.3 feet. What is the pitch of the roof?

$Answer\text{: } Step\ 1\text{: } \quad run = \frac{1}{2} span = 11'$

$Step\ 2\text{: } \quad rise = \sqrt{length^2 - run^2} = 5.5'$

$Step\ 3\text{: } \quad pitch = \frac{rise}{run} = \frac{5.5}{11} = \frac{1}{2}$

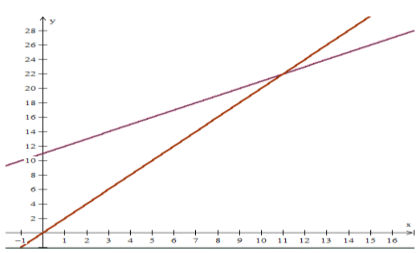
Q Magic Mountain Amusement Park offers 2 separate entry packages. The first package, which Carol chooses, charges \$11 as an entry fee and \$1 per ride. The second package, which Josh chooses, doesn't charge an entry fee but Josh will have to pay \$2 per ride.

a) Write equations representing Josh and Carol's Magic Mountain experience.
$Answer\text{: } y = 1x + 11, \quad y = 2x$

b) What is the slope of each equation and what does it represent?

c) How many rides will they have to go on to make Carol's decision better than Josh's?
$Answer\text{: } 1x + 11 = 2x \quad x = 11 \quad \therefore At\ 12\ rides\ Carol's\ decision\ is\ better.$

d) If Carol and Josh have 3½ hours at the park and each ride takes 15 min, which package would you recommend? $Answer\text{: } 14\ rides, Carol's$



SCO **A3: Solve problems by applying proportional reasoning and unit analysis.** [C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

A3: Solve problems by applying proportional reasoning and unit analysis.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
N1: Solve problems that involve unit pricing and currency exchange, using proportional reasoning. *(GMF10)* **M1:** Demonstrate an understanding of the Système International (SI) by: describing the relationships of the units for length, area, volume, capacity, mass and temperature. *(GMF10)* **M2:** Demonstrate an understanding of the imperial system by: describing the relationships of the units for length, area, volume, capacity, mass and temperature. *(GMF10)* **M3:** Solve problems, using SI and imperial units, that involve linear measurement using estimation and measurement strategies. *(GMF10)*	**A3:** Solve problems by applying proportional reasoning and unit analysis.	

ELABORATION

In grade 10, students have converted both within and between SI units and imperial units for length, area, volume, capacity, mass and temperature. A review of conversions, such as those shown in the table below, will be important as students move on to this outcome.

SI Units to Imperial Units	Imperial Units to SI Units
$1\ mm \cong \frac{4}{100} in.$	$1\ in. = 2.5\ cm$
$1\ cm = \frac{4}{10} in.$	$1\ ft. \cong 30\ cm$ $1\ ft. \cong 0.3\ m$
$1\ m \cong 39\ in.$ $1\ m \cong 3\frac{1}{4}\ ft.$	$1\ yd. \cong 90\ cm$ $1\ yd. \cong 0.9\ m$
$1\ km \cong \frac{6}{10} mi.$	$1\ mi. = 1.6\ km$

In grade 11, they will use **unit analysis** and proportional reasoning to solve problems involving compound units, such as rpm (revolutions per minute). These techniques can be applied throughout the course.

Unit analysis is one method of verifying that the units in a conversion are correct. To convert from one unit to another, expressions are multiplied by conversion factors. For example, to change $360\ inches$ into $feet$, we multiply by the conversion factor of $\frac{1\ ft.}{12\ in.}$. Because $1\ ft.$ is equal to $12\ inches$, $\frac{1\ ft.}{12\ in.}$ equals 1 so the value of the expression is not changed. All units that are the same in the numerator and denominator are removed, leaving the unit to which the term has been converted. In this case: $360\ \cancel{in.} \times \frac{1\ ft.}{12\ \cancel{in.}} = 30\ ft.$

Students should be able to clearly and methodically show how they have used unit analysis to make their conversions, showing all equivalencies and units. For example: to determine which is faster, $80\ mph$ or $40\ ft/sec$, $80\ mph$ is converted to ft/sec.

$$\frac{80\ \cancel{mi}}{1\ \cancel{hr}} \times \frac{1\ \cancel{hr}}{60\ \cancel{min}} \times \frac{1\ \cancel{min}}{60\ sec} \times \frac{5280\ ft}{1\ \cancel{mi}} = \frac{80 \times 5280}{60 \times 60}\ ft/sec = 117.33\ ft/sec.$$

Once converted to $117\ ft./sec$ it is clear to see that $80\ mph$ is faster than $40\ ft/sec$.

Examples and problems given to students should be of conversions that would typically occur in our daily lives.

SCO **A3: Solve problems by applying proportional reasoning and unit analysis.** [C, CN, PS, R]

ACHIEVEMENT INDICATORS

- Explain the process of unit analysis used to solve a problem (e.g., given km/h and time in $hours$, determine how many km; e.g., given rpm $(revolutions\ per\ minute)$, determine the number of $seconds\ per\ revolution)$.
- Solve a problem, using unit analysis.
- Explain, using an example, how unit analysis and proportional reasoning are related; e.g., to change km/h to km/min, multiply by $1h/60min$ because hours and minutes are proportional (constant relationship).
- Solve a problem within and between systems, using proportions or tables; e.g., km to m, or km/h to ft/sec.

Suggested Instructional Strategies

- Start with basic conversions before moving into compound units. An example of unit analysis used with simple units can be seen at http://www.youtube.com/watch?v=XKCZn5MLKvk.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q What are the actual dimensions of a "two by four" piece of lumber in inches? Express these dimensions in millimetres.

Q A certain alloy appears bright red at a temperature of $560°C$. What temperature is this in $°F$?

Q A Canadian driving in the United States notices her speedometer is reading $80\ km/h$. What is her speed in mph?

Q A wood lot is $25\ 000$ square feet. How many square metres is this? How many acres? How many hectares?

Q *"Peter Piper picked a peck of pickled peppers. If Peter Piper picked a peck of pickled peppers, how many pickled peppers did Peter Piper pick?"*

Convert this tongue twister to the metric system, using litres instead of pecks - one peck equal 8 quarts. "Peter Piper picked __________ litres of pickled peppers……."

Q Use metric measurements in the following sayings:

a) Give him an inch, and he'll take a mile. $(change\ inch\ to\ centimetre\ and\ miles\ to\ kilometres)$

b) I'm racing in the Indy 500. $(change\ 500\ miles\ to\ kilometres)$

c) An $ounce$ of prevention is worth a $pound$ of cure. $(change\ to\ grams)$

d) The cutting speed for a soft steel part in a lathe is $150\ ft/min$. $(change\ to\ cm/s)$

SCO **S1: Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.** [C, CN, PS, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Statistics

S1: Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.

Scope and Sequence of Outcomes:

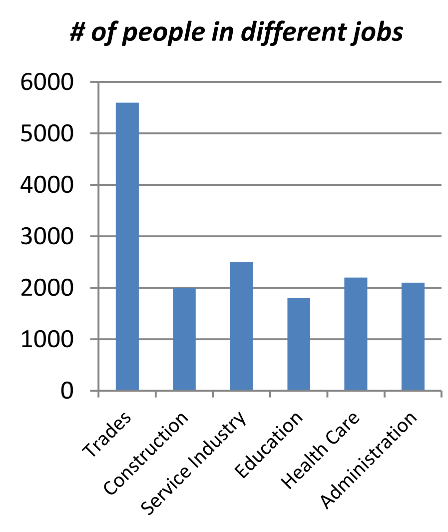
Grade Ten	Grade Eleven	Grade Twelve
RF1 Interpret and explain the relationships among data, graphs and situations. *(NRF10)*	**S1:** Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.	**A1**. Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, solving problems. *(FWM12)* **N2.** Critique the viability of small business options by considering expenses, sales, profit or loss. *(FWM12)* **S1.** Solve problems that involve measures of central tendency, including mean, median, mode, weighted mean, trimmed mean. *(FWM12)*

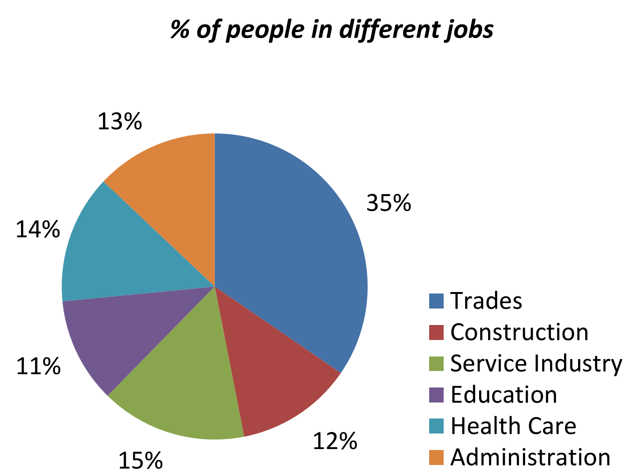
ELABORATION

Students constructed and interpreted histograms in grade 9 and were also introduced to other types of graphs.

For this outcome students will construct graphs, and be asked to compare and interpret them. The emphasis should be placed on determining the best graph for the data to be displayed and on the interpretation of data represented by the graph.

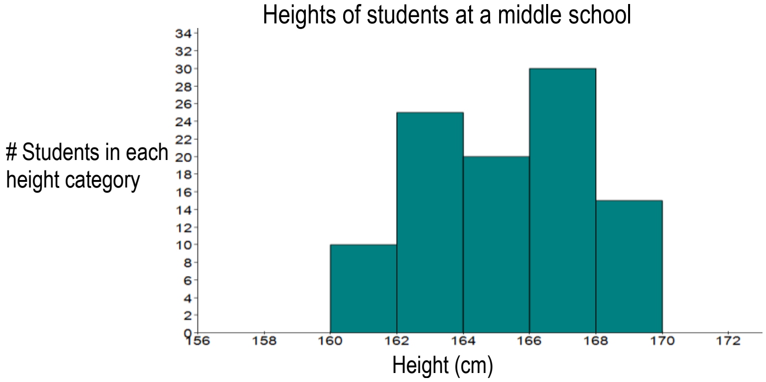
Discrete data can be represented with **circle graphs** (also known as pie charts) or **bar graphs**. Circle graphs compare numbers as a percent or part of the total. Bar graphs display categories of data as heights on a chart. These graphs can be used for discrete data such as, for example; populations in a particular year, categories of books, types of donations, or the number of people in particular categories of jobs, as shown below.





SCO **S1: Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.** [C, CN, PS, R, T, V]

Histograms present continuous data as a frequency distribution of a range of values such as, for example, time, marks or height. There are no spaces between the bars.



Line graphs are useful when following trends of continuous data. A data value can be **interpolated** which is estimation from within the range of data collected, and **extrapolated** which is estimation outside the range of the data collected.

Data and graphs are used extensively in our world. These skills will be important in day to day life, in the workplace and in business math to effectively represent and interpret data collected.

ACHIEVEMENT INDICATORS

- Determine the possible graphs that can be used to represent a given data set, and explain the advantages and disadvantages of each.
- Create, with and without technology, a graph to represent a given data set.
- Describe the trends in the graph of a given data set.
- Interpolate and extrapolate values from a given graph.
- Explain, using examples, how the same graph can be used to justify more than one conclusion.
- Explain, using examples, how different graphic representations of the same data set can be used to emphasize a point of view.
- Solve a contextual problem that involves the interpretation of a graph.

SCO **S1: Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.** [C, CN, PS, R, T, V]

Suggested Instructional Strategies

- Collect various examples of graphs from newspapers, magazines, school data reports, etc.
- Statistics Canada website has lots of data and suggested activities for teachers and students to use. Some of this data with lesson plans is posted on the Portal under Grade 9 Math Resources, and more recent data can be found at www.statcan.gc.ca/edu
- Graphs can be created using free online websites such as www.chartgo.com, software such as Excel, or graphing calculators.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Worksheet/activity for circle graphs can be found at:
http://www.superteacherworksheets.com/graphing/pie-graph-hard-3.pdf

Q The annual budgets for two families are given in the chart. Compare the budgets of these two families.

a) What type of graph(s) would best display this data and compare the budgets? Why?

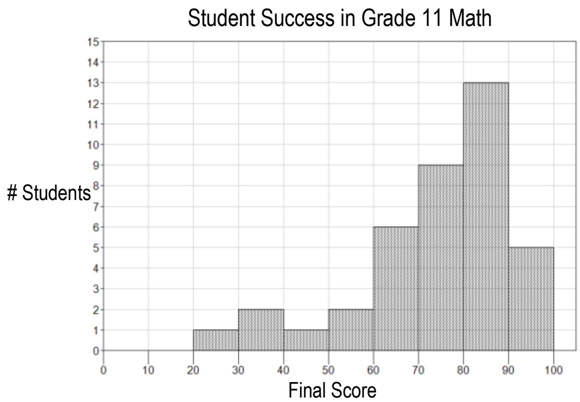
b) Construct a graph(s).

c) What conclusions do you draw from the graphs about each family's budget? How do the graphs support your conclusions?

	Brown Family	*Smith Family*
Food	$3000	$2400
Housing	$4000	$3600
Operating Expenses	$2800	$2400
Clothing	$1200	$1400
Charities	$1000	$600
Medical Expenses	$1800	$800
Miscellaneous	$600	$1200
Savings	$600	$2600

SCO **S1: Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.** [C, CN, PS, R, T, V]

Q Final marks for *Financial and Workplace Mathematics 110* are displayed in the following histogram.



a) How many students are in the class?

b) How many students achieved a passing mark (60% or more)?

c) What is the highest mark?

d) Could this data be displayed effectively on a different type of graph? Explain.

e) What conclusions can be made from this graph?

f) If there are 86 students in total enrolled in this course, how many would you predict will pass the course?

SUMMARY OF CURRICULUM OUTCOMES

Financial and Workplace Mathematics 110

Mathematical Processes
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Geometry

General Outcome: Develop spatial sense

Specific Outcomes
G1. Solve problems that involve two and three right triangles. [CN, PS, T, V]
G2. Solve problems that involve scale. [PS, R, T, V]
G3. Model and draw 3-D objects and their views. [CN, R, V]
G4. Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects. [CN,V]

Number

General Outcome: Develop number sense and critical thinking skills.

Specific Outcomes
N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]
N2. Analyze costs and benefits of renting, leasing and buying. [CN, PS, R, T]
N3. Analyze an investment portfolio in terms of interest rate, rate of return, total return. [ME, PS, R, T]
N4. Solve problems that involve personal budgets. [CN, PS, R, T]

Algebra

General Outcome: Develop algebraic reasoning.

Specific Outcomes
A1. Solve problems that require the manipulation and application of formulas related to:
- slope and rate of change
- Rule of 72
- Finance charges
- The Pythagorean theorem and Trigonometric ratios [CN, PS, R]

A2. Demonstrate an understanding of slope:
- as rise over run
- as rate of change
- by solving problems [C, CN, PS, V]

A3. Solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]

Statistics

General Outcome: Develop statistical reasoning.

Specific Outcome
S1. Solve problems that involve creating and interpreting graphs, including:
- bar graphs
- histograms
- line graphs
- circle graphs [C, CN, PS, R, T, V]

REFERENCES

Alberta Education, System Improvement Group. *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings*. Edmonton, AB:

Alberta Education, 2006. Available at http://www.wncp.ca/math/report_2006.pdf (Accessed September 20, 2007).

Armstrong, Thomas. *7 Kinds of Smart: Identifying and Developing Your Many Intelligences*. New York, NY: Plume, 1993.

Banks, J. A. and C. A. M. Banks. *Multicultural Education: Issues and Perspectives*. 2nd ed. Boston, MA: Allyn and Bacon, 1993.

British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*. Victoria, BC: British Columbia Ministry of Education, 2000.

Caine, Renate Nummela and Geoffrey Caine. *Making Connections: Teaching and the Human Brain*. Alexandria, VA: Association for Supervision and Curriculum Development, 1991.

Hope, Jack A. et al. *Mental Math in the Primary Grades*. Palo Alto, CA: Dale Seymour Publications, 1988.

McAskill, B. et al. *WNCP Mathematics Research Project: Final Report*. Victoria, BC: Holdfast Consultants Inc., 2004. Available at http://www.wncp.ca/math/Final_Report.pdf (Accessed September 20, 2007).

National Council of Teachers of Mathematics. *Computation, Calculators, and Common Sense: A Position of the National Council of Teachers of Mathematics*. May 2005. http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/computation.pdf (Accessed September 20, 2007).

Rubenstein, Rheta N. “Mental Mathematics beyond the Middle School: Why? What? How?” *Mathematics Teacher* 94, 6 (September 2001), pp. 442–446.

Shaw, J. M. and M. J. P. Cliatt. “Developing Measurement Sense.” In P. R. Trafton (ed.), *New Directions for Elementary School Mathematics: 1989 Yearbook* (Reston, VA: National Council of Teachers of Mathematics, 1989), pp. 149–155.

Steen, L. A. *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, DC: Mathematical Sciences Education Board, National Research Council, 1990.

Western and Northern Canadian Protocol for Collaboration in Education. *The Common Curriculum Framework for K–9 Mathematics* May 2006. http://www.wncp.ca/english/subjectarea/mathematics/ccf.aspx

Western and Northern Canadian Protocol for Collaboration in Education. *The Common Curriculum Framework for Grades 10-12 Mathematics* January 2008. http://www.wncp.ca/english/subjectarea/mathematics/ccf.aspx