Financing Costs of Additional Funds Needed: A Modified Equation Approach

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The additional funds needed (AFN) equation is a popular forecasting model for estimating additional funds requirements (Brigham [3]). Academicians regard the AFN equation as an excellent pedagogical tool; practitioners find it highly beneficial for many forecasting needs. To apply the model, an analyst needs to know the amount of assets required per dollar of sales, the amount of spontaneous liabilities available per dollar of sales, the change in sales, and additions to retained earnings. Additions to retained earnings are calculated by multiplying the net profit margin by forecasted sales and the retention ratio. The equation is shown as follows:

$$AFN = (A^*/S)\Delta S - (L^*/S)\Delta S - MS_1(1 - d) \qquad (1)$$

where:

AFN	=	additional funds needed
A^*/S	=	spontaneous assets divided by base period sales
L^*/S	=	spontaneous liabilities divided by base period sales
ΔS	=	change in sales (sales in period 1 - sales in period 0)
M	=	net profit margin
S_1	=	sales in period 1
d	=	dividend payout ratio in the base period.

The first term on the right-hand side (RHS) of equation (1) is the increase in assets required to support the change in sales. The second term subtracts the increase in spontaneous liabilities associated with the sales increase. The third term deducts additions to retained earnings.

As useful as practitioners and academicians find the AFN equation, it is subject to considerable estimation error when financing costs change substantially. The financing costs associated with additional funds needed create "feedback," requiring a firm to borrow more than the original additional funds needed in order to meet the interest and dividend obligations of the amount borrowed. Original additional funds needed is calculated from a pro forma statement as a firm's total asset projections minus initially projected liabilities and equity. The recalculation of interest

expense and dividends to accommodate the financing of original AFN causes a circular process that changes pro forma income statement and balance sheet accounts for several iterations until it converges on the actual AFN. Kester [4] recognizes the circular process as an infinite geometric progression and develops an equation to adjust the original AFN for interest, taxes, and dividends. Similarly, Boyd [2] shows that additional funds needed, including feedback financing effects, can be determined by multiplying the original additional funds needed (AFN) by a feedback multiplier. Boyd's feedback multiplier includes a weighted average of the after-tax cost of debt and the common stock dividend yield. Boyd [2] and Kester [4], however, exclude the capital gains yield in their equations. The capital gains yield, which equals the expected growth rate of dividends per share, captures the implicit increase in the cost of repurchasing common equity at the end of the period associated with the AFN. The omission of a positive growth component causes the additional funds needed to be understated, because the weighted average cost of capital (WACC) in the denominator of the feedback multiplier is not identical to the traditional WACC. In this regard, a link between AFN and WACC is not explicitly established. Nonetheless, Boyd [2] and Kester [4] contribute to an enhanced understanding of feedback financing effects.

This paper shows the derivation and application of a modified AFN equation, which offers precisely the same AFN estimates found using a multi-pass balance sheet method, while retaining the simplicity of the original AFN equation. The modified AFN equation also accommodates changes in financing costs arising from the growth component of common stock dividends. The growth component reflects an expected change in the stock repurchase price after period 0. The omission of the growth component has a notable effect on additional fund needs. In addition, the modified AFN equation shows the conceptual link between additional funds needed and the weighted average cost of capital.

I. Comparison of the AFN Equation and Projected Balance Sheet Methods

The AFN equation offers simplicity and ease of use, and an analyst is able to project funds requirements quickly, bypassing the need to construct pro forma income statements and balance sheets. The equation method also permits quick recalculation of AFN with changes in the assumptions, and in addition, other variables in the equation can be readily solved for given a target AFN.

The AFN equation has several potential disadvantages. First, it ignores economies of scale in asset utilization. The AFN equation assumes linearly increasing asset requirements. Second, costly assets are assumed to be purchased incrementally. Often, assets are "lumpy" and must be purchased in whole. The AFN equation also assumes that a firm is operating at full capacity; however, Brigham [3] shows a modification to additional funds forecasting if the firm is operating at less than full capacity. Although changes to the AFN equation are often feasible, the projected balance sheet method appears more flexible when significant forecasting changes are necessary. Nonetheless, the AFN equation is often directly applicable or easily revised to accommodate the aforementioned problems.

Even after additional fund requirements are initially estimated by the AFN equation, the funding of interest and dividend costs for AFN remains a problem. The AFN equation does directly

include the funding needs for financing costs associated with the original AFN estimation, since financing costs are indirectly embedded in the additions to retained earnings computation; this is calculated as the last term of equation (1) by multiplying forecasted sales by the net profit margin and the retention rate. Thus, when financing costs change substantially, the AFN formula can substantially under- or overstate state funds requirements, because the net profit margin and retention ratio are assumed constant from the base period.

The projected balance sheet method can accommodate additional financing costs through a multi-pass approach (Andrew [1]). In the first pass, a pro forma balance sheet is estimated, and additional funds requirements are approximated with the exclusion of AFN financing costs. In the second pass to the balance sheet, the additional funds re quirements are allocated to liability and equity accounts. The allocation to these accounts creates added interest and dividend expense, which, in turn, changes additions to retained earnings in the income statement. When the changed account for additions to retained earnings in the income statement is subtracted from the first-pass retained earnings account on the balance sheet, a "second round" of additional funds requirements is established, since the additional financing costs of the original AFN create a shortfall. Therefore, at least one or more passes are required until the "feedback" between the additions to retained earnings and the retained earnings account on the balance sheet create approximately no changes to total additional funds requirements. At the point of termination, the "feedback" of financing costs is zero.

Both the projected balance sheet and AFN equation methods typically exclude stock price changes attributable to financing with common stock. Unless dividends grow at 0 percent, the expected stock price will change after period 0, even if market conditions remain the same. Assuming a constant growth rate in dividends (g), a firm selling stock at price Po today will pay $P_1 = P_0(1 + g)$ to repurchase the stock at period 1. Typically, additional-funds forecasting methods consider only the dividend yield, and not the growth rate of dividends and its corresponding effect on the stock price.[1]

II. An Application of AFN Forecasting Methods

An example of the projected balance sheet is shown in Exhibit 1; this method is compared with the AFN equation method shown in Exhibit 2. In both examples, half of additional funds needed is raised through the sale of common stock, and the other half is raised equally through short-term and long-term debt. The balance sheet method shown assumes a sales increase of 25 percent, a short- and long-term interest rate of 12 percent, and a dividends per share increase from $1.15 in period 0 to $1.45 in period 1 .[2]

Exhibits 1 and 2 indicate that as financing costs become large, the two AFN forecast methods show considerable divergence. In particular, the equation method substantially understates

[1] Even if a firm does not repurchase the newly issued shares in period I for reasons associated with the funding of the projects in period 0, the implicit cost of repurchasing the shares should be recognized in the AFN forecast. If a firm does not repurchase the shares in period 1, the increase in repurchase costs has simply been postponed. Moreover, share repurchase of the same magnitude for other reasons, such as dividend policy or capital structure purposes, bears the same cost implications. In any case, the repurchase cost captures the present value of the constant growth in dividends. Without the repurchase component, the growth in the dividend cost of new common stock remains unfunded.

[2] The example base year income statement and balance sheet is extracted from Chapter 16 of Brigham [3].

funding needs. In the examples provided, the additional funds needed is $392.82 million with the projected balance sheet method and $379.88 million with the AFN equation, a divergence of almost $13 million.

Exhibit 1. An Application of the Multi-Pass Balance Sheet Method (Millions of Dollars)

INCOME STATEMENT

		1 + G	First Pass	Feedback	Second Pass	Feedback	Third Pass	Feedback	Final
Sales	$3000.00	1.25	$3750.00		$3750.00		$3750.00		$3750.00
Costs	2616.00	1.25	3270.00		3270.00		3270.00		3270.00
Depreciation	100.00	1.25	125.00		125.00		125.00		125.00
Total Oper. Costs	2716.00		3395.00		3395.00		3395.00		3395.00
EBIT	284.00		355.00		355.00		355.00		355.00
- Interest	88.00		88.00	$21.98	109.98	$1.48	111.46	$0.10	111.56
EBT	196.00		267.00		245.02		243.54		243.44
- Taxes	78.40		106.80	-8.79	98.01	-0.59	97.42	-0.04	97.38
NI Before Div.	117.60		160.20		147.01		146.12		146.06
Preferred Div.	4.00		4.00		4.00		4.00		4.00
NI Available to Common	$113.60		156.20		143.01		142.12		142.06
Common Div.	57.50		72.50	11.55	84.05	0.78	84.83	0.05	84.88
Additional to RE	$56.10		$83.70	$24.73	$58.97	$1.67	$57.30	$0.11	$57.18

BALANCE SHEET

		1 + G	First Pass	AFN	Second Pass	AFN	Third Pass	AFN	Final
Cash	$10.00	1.25	12.50		12.50		12.50		12.50
A/R	375.00	1.25	468.75		468.75		468.75		468.75
Inventories	615.00	1.25	768.75		768.75		768.75		768.75
Total CA	1000.00	1.25	1250.00		1250.00		1250.00		1250.00
Net P&E	1000.00	1.25	1250.00		1250.00		1250.00		1250.00
Total Assets	$2000.00		$2500.00		$2500.00		$2500.00		$2500.00
A/P	60.00	1.25	75.00		75.00		75.00		75.00
Notes Pay.	110.00		110.00	$91.57	201.57	$6.18	207.76	$0.42	208.18
Accruals	140.00	1.25	175.00		175.00		175.00		175.00
Total CL	310.00		360.00		451.58		457.76		458.18
Long-Term Bonds	754.00		754.00	91.57	845.58	6.18	851.76	0.42	852.18
Total Debt	1064.00		1114.00		1297.15		1309.52		1310.35
Preferred Stock	40.00		40.00		40.00		40.00		40.00
Common Stock	130.00		130.00	183.15	313.15	12.37	325.52	0.84	326.35
Ret. Earnings	766.00		849.70		824.97		823.30		823.30
Total Com. Eq.	896.00		979.70		1138.12		1148.81		1149.54
Tot. Liab. & Eq.	$2000.00		$2133.70		$2475.27		$2498.33		$2500.00
AFN			$366.30	$366.30	$24.73	$24.73	$1.67	$1.67	$0.11

TOTAL AFN **$392.82 = $366.30 + $24.73 + $1.67 + $0.11 + $0.01**[1]

ASSUMPTIONS:

Sales Growth (G)	25.00%	Notes Payable Interest Rate	12.00%
D_0 =	$1.15	Long-Term Bond Interest Rate	12.00%
D_1 =	$1.45	Common Stock Dividend Yield	6.30%
Initial Shares Outstanding	$50.00	w_d =	50.00%
Current Stock Price	$23.00	w_{ps} =	0.00%
Marginal Tax Rate	40.00%	w_{cs} =	25.00%

Note:

[1] The last $0.01 is the rounding of the change in the retained earnings account because of financing costs of the last $0.11 after the third pass. Using the $24.73 in added additional funds needed from pass two, and the additional funds needed of $1.67 from pass three, the change in the retained earnings account for the $0.11 after pass four is estimated as ($0.11) ($1.67/$24.73) = $0.0074, which rounds to $0.01.

Brigham [3] illustrates the projected balance sheet and equation methods, assuming a modest growth rate in sales of 10 percent, a dividend increase of 10 cents, and an average interest rate of 9 percent. Under these conditions, the projected balance sheet and AFN equation methods show virtually the same additional fund requirements, $119 million and $118 million, respectively. This example helps students visualize the similarity of estimates for the two forecasting methods.

III. A Modification of the AFN Formula

To address the divergence in funding needs sometimes encountered when comparing the projected balance sheet and equation methods, a modified AFN (MAFN) model is derived. The modified AFN equation is shown as follows:

n_{cs}	=	pre-existing shares of common stock in period 0
m_{cs}	=	new shares of common stock required to finance additional funds needed
D_{ps}	=	preferred stock dividend per share in period 0
D_{cs}	=	common stock dividend per share in period 0
ΔP_{cs}	=	change in common stock price (common stock price in period 1 minus common stock price in period 0)

The first two terms on the RHS of equation (2) are the same as in equation (1). The third term captures the after-tax interest charges of the original interest charges (I_0), plus the new interest charges (ΔI) arising from AFN funding. The last term on the RHS recognizes pre-existing and new dividend costs of preferred and common stock. Dividend costs of pre-existing preferred and common stock are $D_{ps}n_{ps}$ and $D_{cs}n_{cs}$, respectively. Dividend costs associated with selling new preferred stock are $D_{ps}m_{ps}$, and for common stock, new dividend costs are $D_{cs}m_{cs}$. These new dividend costs arise from the need to fund the additional funds needed. By expressing the stock price in period I minus the common stock price in period 0 as ΔP_{cs}, the cost of repurchasing m_{cs} shares of common stock in period 1 is $\Delta P_{cs}m_{cs}$, which is shown as the final cost component in equation (2).

To expand equation (2), let wd denote the proportion of FN financing that uses debt. If id denotes the debt interest rate, $\Delta I = i_d M\Delta FNw_d$, that is, ΔI denotes total new interest charges attributable to the portion of AFN which is debt financed. To examine the financing of AFN using equity, let P_{ps} and P_{cs} denote the price per share of preferred and of common stock, respectively. The proportions of additional funds needed that are financed by preferred and by common stock are designated w_{ps} and w_{cs}, respectively. Then the number of new shares of preferred stock required to finance w_{ps} of additional funds needed is $mps = (MAFN/P_{ps})w_{ps}$. Similarly, the number of new shares of common stock required to finance wcs of additional funds needed is $m_{cs} = (MAFN/P_{cs})w_{cs}$. Total dividend costs for preferred and for common stock, $D_{ps}m_{ps}$ and $D_{cs}m_{cs}$, can be written as $D_{ps}(MAFN/P_{ps})wps$ and Dcs(MAFN/Pcs)wcs, respectively. Since D_{cs}/P_{cs} is the dividend yield on common stock (i_{dyid}), total dividend costs of common stock

can be written as $i_{dyld}MAFNw_{cs}$. Likewise, total dividend costs on preferred stock can be written as $i_{ps}MAFNw_{ps}$.

In addition to dividend costs, however, the price of a common stock with constant growth rate is expected to change by ΔP_{cs} from period 0 to 1, which equals $(P_{1,cs} - P_{0,cs})$, where $P_{0,cs}$ and $P_{i,cs}$ denote the respective prices of common stock in periods 0 and 1. The repurchase of mcs shares at time 1 has a total dollar value of $\Delta P_{cs}m_{cs} = \Delta P_{cs}(MAFN/P_{0,cs})w_{cs}$. Note that $\Delta P_{cs} / P_{0cs}$ is the percentage change in the stock price; it is also,

Exhibit 2. An Application of the AFN Equation Method (Millions of Dollars)

$$AFN = (A^*/S)\Delta S - (L^*/S)\Delta S - MS_1(1 - d)$$

$$= (\$2000/\$3000)(\$750) - (\$200/\$3000)(\$750) - ((\$113.60/\$3000)(\$3750)(1-(\$57.50/\$113.60))$$

$$= \$500 - \$50 - \$70.125$$

$$= \$379.875$$

$$MAFN = (A^*/S)\Delta S - (L^*/S)\Delta S - ((S_1 - C_1 - D_1) - (I_0 + \Delta I))(1 - t) + (D_{ps}n_{ps} + D_{ps}m_{ps} + D_{cs}n_{cs} + D_{cs}m_{cs} + \Delta P_{cs}m_{cs}) \quad (2)$$

where:

MAFN	=	modified additional funds needed
A^*/S	=	assets required per dollar of base period sales
ΔS	=	change in sales (sales in period 1 minus sales in period 0)
L^*/S	=	spontaneous liabilities divided by base period sales
S_1	=	sales in period 1
C_1	=	operating expenses in period 1
D_1	=	depreciation in period 1
I_0	=	interest charges in period 0
ΔI	=	interest charges on the portion of debt to finance additional funds needed
t	=	marginal tax rate in period 1
n_{ps}	=	pre-existing shares of preferred stock in period 0
m_{ps}	=	new shares of preferred stock required to finance the additional funds needed

however, the constant growth rate in dividends, denoted g_{cs}. Therefore, $(P_{cs} - P_{0,cs})MAFNw_{cs} = g_{cs}MAFNw_{cs}$. Combining the dividend yield and the capital gains yield on common stock, the total dollar cost of common equity is equal to $(i_{dyld} + g_{cs})MAFNw_{cs}$. Letting $i_{cs} = i_{dyld} + g_{cs}$, the

total additional funds needed for financing common stock dividends and repurchase of stock at time 1 is $i_{cs}MAFNw_{cs}$. Making the substitutions in equation (2) for interest and dividend costs for additional funds needed in period 1 and rearranging produces the following equation (3):

$$MAFN = (A^*/S)\Delta S - (L^*/S)\Delta S - ((S_1 - C_1 - D_1) - (I_0 + i_d MAFNw_d))(1 - t) + (i_{ps}MAFNw_{ps} + i_{cs}MAFNw_{cs}) + (D_{ps}n_{ps} + D_{cs}n_{cs}). \quad (3)$$

Collecting terms for the financing costs of MAFN in equation (3) and rearranging these terms on the left-hand side (LHS) of the equal sign:

$$MAFN - MAFNi_d(1 - t)w_d - MAFNi_{ps}w_{ps} - MAFNi_{cs}w_{cs} = (A^*/S)\Delta S - (L^*/S)\Delta S - (S_1 - C_1 - D_1)(1 - t) + I_0(1 - t) + D_{ps}n_{ps} + D_{cs}n_{cs}. \quad (4)$$

The left-hand side of equation (4) can be rewritten as MAFN $[1 - (i_d(1 - t)w_d + i_{ps}w_{ps} + i_{cs}w_{cs})]$. Isolating the MAFN term on the LHS:

$$MAFN = [(A^*/S)\Delta S - (L^*/S)\Delta S - (S_1 - C_1 - D_1)(1 - t) + I_0(1 - t) + D_{ps}n_{ps} + D_{cs}n_{cs}]/[1 - [i_d(1 - t)w_d + i_{ps}w_{ps} + i_{cs}w_{cs}]] \quad (5)$$

Letting (S - CI - DI)/Si denote the operating profit margin (OPM), which is assumed constant in periods 0 and 1, and denoting the weighted average cost of capital as WACC = $i_d(1 - t)w_d + i_{ps}w_{ps} + i_{cs}w_{cs}$, equation (5) can be simplified as follows:

$$MAFN = [(A^*/S)/\Delta S - (L^*/S)\Delta S - (OPM)*(S_1)(1-t) + I_0(1-t) + D_{ps}n_{ps} + D_{cs}n_{cs}]/[1 - WACC]. \quad (6)$$

The first three terms on the RHS of equation (6) denote the additional funds needed without financing costs. The last three terms indicate the per-period cost of the existing financing. The cost of new financing is captured in the denominator. Additional funds needed are positively related to the WACC. Note that equation (6) can be rewritten as:

$$MAFN = [\ AFN\ ] \cdot 1/(1 - WACC) \quad (7)$$

where the bracketed term, the numerator, equals an original AFN without financial feedbacks, and the denominator, written as 1/(1 - WACC), is a financial feedback multiplier whose value is greater than 1. Although this model is similar to the model proposed by Boyd [2], equation (6) specifically includes the cost of preferred stock and the dividend growth component of common stock.

Exhibit 3 shows the calculation of additional funds needed using equation (6) and data from Exhibit 1. Note that the multi-pass balance sheet method and the MAFN equation show precisely the same funding requirements of $392.82 million.

IV. Additional-Funds-Needed Formulas: A Comparison

Exhibit 4 shows a comparison of forecast estimates for the AFN method and the MAFN equations, assuming sales grow at intervals between -25, -10, +10, and + 25 percent, a dividend increase of $0.10, and an average interest rate of 9 percent. The additional funds needed using the four-pass balance sheet method is identical to the MAFN equation method, assuming is = i_{dyld}. In this example, i_{dyld} = 5.435%. For conditions of modest sales growth and the financing of additional funds needed similar in cost to the previous period, AFN and MAFN provide comparable (though not identical) estimates.

However, the use of i_{dyld} misstates additional funds needed, since a firm's common equity cost should include the increase in cost of repurchasing the new common stock in period 1. If a firm decides not to repurchase the additional common stock in period 1, dividend costs will continue to escalate (or decline) at g_{cs}. Therefore, the use of i_{dyld} as the cost of common equity will normally overstate or understate is. Assuming the listed assumptions in Exhibit 4, the AFN formula overstates actual AFN by $16.04 million at a sales growth rate of -25 percent; at a sales growth rate of +25

Exhibit 3. An Application of the Modified AFN (MAFN) Equation Method (Millions of Dollars)

$$\text{MAFN} = \frac{(A^*/S)\Delta S - (L^*/S)\Delta S - (\text{OPM})(S_1)(1-t) + I_0(1-t) + D_{ps}n_{ps} + D_{cs}n_{cs}}{1 - \text{WACC}}$$

$$= \frac{(\$2000/\$3000)(\$750) - (\$200/\$3000)(\$750) - (\$284/\$3000)(\$3750)(1-0.40) + \$88(1-0.40) + \$4 + (50)(\$1.45)}{1 - [0.12(1-0.40)(0.50) + (\$1.45/\$23)(0.50)]}$$

$$= \frac{\$500 - \$50 - \$213 + \$52.80 + 4 + \$72.50}{1 - 0.06752}$$

$$= 1 - 0.06752$$

$$= \$366.30/0.93248$$

$$= \$392.82$$

Exhibit 4. A Comparison of Forecast Estimates for Equation Methods (Millions of Dollars)

Forecasting Method	-25%	-10%	0%	+ 10%	+ 25%
AFN Formula WACC = 5.417%	-$492.08	-$230.49	-$56.10	$118.29	$379.88
Modified AFN Formula [1] WACC = 5.417%	-484.76	-226.32	-54.03	118.27	376.71
Modified AFN Method WACC = 9.765 [2]	-508.12	-237.23	-56.63	123.97	394.86

Assumptions

Sales growth = 10 percent
Average before tax cost of debt = 9 percent
Marginal tax rate = 40 percent
D_0 = $1.15
D_1 = $1.25
Current Stock price = $23 per share

Notes:

[1] The projected balance sheet and MAFN equation method estimates are identical.

[2] WACC = 0.09 (1 - 0.40) (0.50) + (0.14131) (0.50)
= 0.09765
i_{cs} = ($1.25 / $23) + ($1.25 - $1.15) / $1.15
= 0.05435 + 0.08696
= 0.14131

percent, it understates actual funds requirements by $14.98 million. The projected balance sheet method provides the correct estimates for additional funds needed if the cost of repurchasing common stock is subtracted from net income, thereby reducing additions to retained earnings.

V. Conclusion

This study examines two methods of forecasting additional funds requirements and proposes modifications to the traditional AFN equation. The modifications to the AFN equation accommodate financing costs, improve its accuracy, and often eliminate the need to calculate the additional funds needed using a multi-pass AFN method. The modified AFN formula presented in this paper recognizes the cost of a stock repurchase in period 1. From a procedural perspective, the MAFN requires minimal additional calculations, and forecasts are precise and identical to a multi-pass balance sheet approach. The modified AFN equation distinguishes among the cost of funding operational requirements, the cost of pre-existing financing requirements, and the cost of new financing requirements. Conceptually, the link between AFN and the WACC is established with the MAFN equation, thereby emphasizing financial theory rather than the accounting application.

References

1. J. D. Andrew, "Implementing Kester's Solution to the Balancing Problem," Financial Management (Autumn 1988), p. 7.
2. G. M. Boyd, "A Pedagogical Note on Pro Forma Feedback Effects," Journal of Financial Education (Fall 1988), pp. 1-5.
3. E. F. Brigham, Fundamentals of Financial Management, 6th ed.,

Fort Worth, TX, Dryden Press, 1992, Chapter 16.
4. G. W. Kester, "A Note on Solving the Balancing Problem," Financial Management (Spring 1987), pp. 52-54.