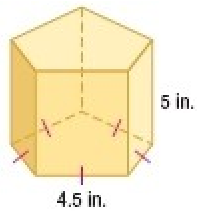
12-2 Surface Areas of Prisms and Cylinders

1. Find the lateral area of the prism.



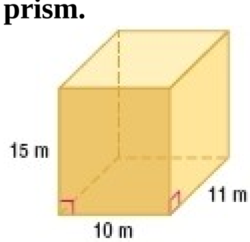
SOLUTION:

$$\begin{aligned} L &= Ph \\ &= (5 \times 4.5)(5) \\ &= 22.5(5) \\ &= 112.5 \end{aligned}$$

ANSWER:

112.5 in^2

Find the lateral area and surface area of each prism.

2. 

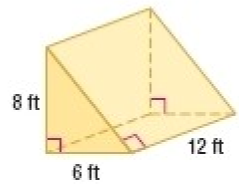
SOLUTION:

$$\begin{aligned} L &= Ph \\ &= 2(10 + 11)(15) \\ &= 42(15) \\ &= 630 \end{aligned}$$

$$\begin{aligned} S &= Ph + 2B \\ &= 630 + 2(10)(11) \\ &= 630 + 220 \\ &= 850 \end{aligned}$$

ANSWER:

Sample answer: L = 630 m^2 ; S = 850 m^2

3. 

SOLUTION:

The base of the prism is a right triangle with the legs 8 ft and 6 ft long. Use the Pythagorean Theorem to find the length of the hypotenuse of the base.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 6^2 + 8^2 \\ c^2 &= 36 + 64 \\ c^2 &= 100 \\ c &= 10 \end{aligned}$$

Find the lateral and surface area.

$$\begin{aligned} L &= Ph \\ &= (6 + 8 + 10)(12) \\ &= (24)(12) \\ &= 288 \end{aligned}$$

$$\begin{aligned} S &= Ph + 2B \\ &= 288 + 2\left[\tfrac{1}{2}(6)(8)\right] \\ &= 288 + 48 \\ &= 336 \end{aligned}$$

ANSWER:

L = 288 ft^2 ; S = 336 ft^2

4. **CARS** Evan is buying new tire rims that are 14 inches in diameter and 6 inches wide. Determine the lateral area of each rim. Round to the nearest tenth.

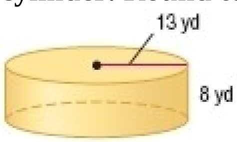
SOLUTION:

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(7)(6) \\ &= 84\pi \\ &\approx 263.9 \end{aligned}$$

ANSWER:

263.9 in^2

Find the lateral area and surface area of each cylinder. Round to the nearest tenth.

5. 

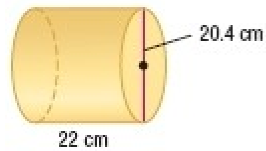
SOLUTION:

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(13)(8) \\ &= 208\pi \\ &\approx 653.5 \end{aligned}$$

$$\begin{aligned} S &= 2\pi rh + 2B \\ &= 2\pi(13)(8) + 2\pi(13)^2 \\ &= 208\pi + 338\pi \\ &= 546\pi \\ &\approx 1715.3 \end{aligned}$$

ANSWER:

$L \approx 653.5$ yd^2 ; $S \approx 1715.3$ yd^2

6. 

SOLUTION:

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(10.2)(22) \\ &= 448.8\pi \\ &\approx 1409.9 \end{aligned}$$

$$\begin{aligned} S &= 2\pi rh + 2B \\ &= 2\pi(10.2)(22) + 2\pi(10.2)^2 \\ &= 448.8\pi + 208.8\pi \\ &= 656.88\pi \\ &\approx 2063.6 \end{aligned}$$

ANSWER:

$L \approx 1409.9$ cm^2 ; $S \approx 2063.6$ cm^2

7. **FOOD** The can of soup has a surface area of 286.3 square centimeters. What is the height of the can? Round to the nearest tenth.

SOLUTION:

The surface area is the sum of the areas of the bases and the lateral surface area. Solve for the height.

$$\begin{aligned} A &= 2\pi r^2 + 2\pi rh \\ 286.3 &= 2\pi(3.4)^2 + 2\pi(3.4)h \\ 286.3 - 2\pi(3.4)^2 &= 2\pi(3.4)h \\ \frac{286.3 - 2\pi(3.4)^2}{2\pi(3.4)} &= h \\ 10.0 &\approx h \end{aligned}$$

ANSWER:

10.0 cm

8. The surface area of a cube is 294 square inches. Find the length of a lateral edge.

SOLUTION:

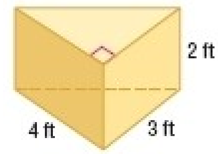
All of the lateral edges of a cube are equal. Let *x* be the length of a lateral edge.

$$\begin{aligned} S &= Ph + 2B \\ 294 &= 2(x+x)(x) + 2(x)(x) \\ 294 &= 4x^2 + 2x^2 \\ 294 &= 6x^2 \\ 49 &= x^2 \\ 7 &= x \end{aligned}$$

ANSWER:

7 in.

Find the lateral area and surface area of each prism. Round to the nearest tenth if necessary.

9. 

SOLUTION:

Find the length of the third side of the triangle.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 4^2 &= c^2 \\ 9 + 16 &= c^2 \\ \sqrt{25} &= c \\ 5 &= c \end{aligned}$$

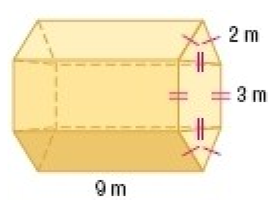
Now find the lateral and surface area.

$$\begin{aligned} L &= Ph \\ &= (3+4+5)(2) \\ &= (12)(2) \\ &= 24 \end{aligned}$$

$$\begin{aligned} S &= Ph + 2B \\ &= 24 + 2\left[\tfrac{1}{2}(4)(3)\right] \\ &= 24 + 12 \\ &= 36 \end{aligned}$$

ANSWER:

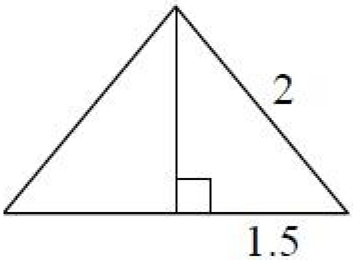
L = 24 ft^2 ; S = 36 ft^2

10. 

SOLUTION:

$$\begin{aligned} L &= Ph \\ &= [2(3) + 4(2)](9) \\ &= 14(9) \\ &= 126 \end{aligned}$$

We need to find the area of the triangle to determine the area of the bases. Use the Pythagorean Theorem to find the height of the triangles.



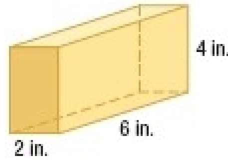
$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
1.5^2 + b^2 &= 2^2 \\
b^2 &= 2^2 - 1.5^2 \\
b^2 &= 4 - 2.25 \\
b &= \sqrt{1.75} \\
b &\approx 1.3
\end{aligned}
$$

Use $b = \sqrt{1.75}$ to calculate the surface area.

$$
\begin{aligned}
S &= Ph + 2B \\
&= L + 2\{\text{area(rectangle)} + 2[\text{area(triangle)}]\} \\
&= 126 + 2\{3(3) + 2[\tfrac{1}{2}(3)(\sqrt{1.75})]\} \\
&= 126 + 2\{9 + 3\sqrt{1.75}\} \\
&= 126 + 18 + 6\sqrt{1.75} \\
&= 144 + 6\sqrt{1.75} \\
&\approx 151.9
\end{aligned}
$$

ANSWER:

Sample answer: $L = 126$ m^2 ; $S = 151.9$ m^2

11. 

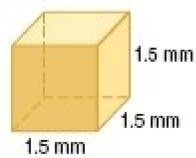
SOLUTION:

$$
\begin{aligned}
L &= Ph \\
&= 2(6 + 2)(4) \\
&= 16(4) \\
&= 64
\end{aligned}
$$

$$
\begin{aligned}
S &= Ph + 2B \\
&= 64 + 2(6)(2) \\
&= 64 + 24 \\
&= 88
\end{aligned}
$$

ANSWER:

Sample answer: $L = 64$ in^2 ; $S = 88$ in^2

12. 

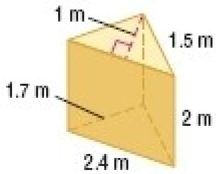
SOLUTION:

$$
\begin{aligned}
L &= Ph \\
&= 2(1.5 + 1.5)(1.5) \\
&= 6(1.5) \\
&= 9
\end{aligned}
$$

$$
\begin{aligned}
S &= Ph + 2B \\
&= 9 + 2(1.5)(1.5) \\
&= 9 + 4.5 \\
&= 13.5
\end{aligned}
$$

ANSWER:

$L = 9$ mm^2 ; $S = 13.5$ mm^2

13. 

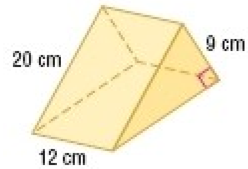
SOLUTION:

$$\begin{aligned} L &= Ph \\ &= (1.5 + 1.7 + 2.4)(2) \\ &= (5.6)(2) \\ &= 11.2 \end{aligned}$$

$$\begin{aligned} L &= Ph + 2B \\ &= (1.5 + 1.7 + 2.4)(2) + 2\left[\tfrac{1}{2}(2.4)(1)\right] \\ &= 11.2 + 2.4 \\ &= 13.6 \end{aligned}$$

ANSWER:

$L = 11.2\text{ m}^2$; $S = 13.6\text{ m}^2$

14. 

SOLUTION:

Find the other side of the base.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 9^2 + b^2 &= 20^2 \\ b^2 &= 20^2 - 9^2 \\ b^2 &= 400 - 81 \\ b &= \sqrt{319} \\ b &\approx 17.9 \end{aligned}$$

Now find the lateral and surface area.

$$\begin{aligned} L &= Ph \\ &\approx (9 + 20 + 17.9)(12) \\ &\approx 562.3 \end{aligned}$$

$$\begin{aligned} S &= Ph + 2B \\ &= (9 + 20 + \sqrt{319})(12) + 2\left[\tfrac{1}{2}(9 \times \sqrt{319})\right] \\ &\approx 562.3 + 9\sqrt{319} \\ &\approx 723.1 \end{aligned}$$

ANSWER:

$L \approx 562.3\text{ cm}^2$; $S \approx 723.1\text{ cm}^2$

15. rectangular prism: ℓ = 25 centimeters, *w* = 18 centimeters, *h* = 12 centimeters

SOLUTION:

Since the base was not specified, it can be any 2 of the 3 dimensions of the prism.

Base 1:

$$\begin{aligned} L &= Ph \\ &= 2(25+18)(12) \\ &= 86(12) \\ &= 1032 \end{aligned}$$

Base 2:

$$\begin{aligned} L &= Ph \\ &= 2(25+12)(18) \\ &= 74(18) \\ &= 1332 \end{aligned}$$

Base 3:

$$\begin{aligned} L &= Ph \\ &= 2(12+18)(25) \\ &= 60(25) \\ &= 1500 \end{aligned}$$

Note that the surface area of the solid is the same for any of the above three bases.

$$\begin{aligned} S &= Ph + 2B \\ &= 2(12+18)(25) + 2(12 \times 18) \\ &= 1500 + 432 \\ &= 1932 \end{aligned}$$

ANSWER:

L = 1032 cm^2 ; S = 1932 cm^2 (18 × 25 base); L = 1332 cm^2 ; S = 1932 cm^2 (25 × 12 base); L = 1500 cm^2 ; S = 1932 cm^2 (18 × 12 base)

16. triangular prism: *h* = 6 inches, right triangle base with legs 9 inches and 12 inches

SOLUTION:

Find the other side of the triangular base.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 9^2 + 12^2 &= c^2 \\ 81 + 144 &= c^2 \\ \sqrt{225} &= c \\ 15 &= c \end{aligned}$$

Now you can find the lateral and surface area.

$$\begin{aligned} L &= Ph \\ &= (9+12+15)(6) \\ &= (36)(6) \\ &= 216 \end{aligned}$$

$$\begin{aligned} S &= Ph + 2B \\ &= (9+12+15)(6) + 2\left[\tfrac{1}{2}(9)(12)\right] \\ &= 216 + 108 \\ &= 324 \end{aligned}$$

ANSWER:

L = 216 in^2 ; S = 324 in^2

CEREAL Find the lateral area and the surface area of each cereal container. Round to the nearest tenth if necessary.

17. 

SOLUTION:

$$\begin{aligned} L &= Ph \\ &= 2(18.6+7)(29) \\ &= 51.2(29) \\ &= 1484.8 \end{aligned}$$

$$\begin{aligned} S &= Ph + 2B \\ &= 2(18.6+7)(29) + 2(18.6)(7) \\ &= 1484.8 + 260.4 \\ &= 1745.2 \end{aligned}$$

ANSWER:

$L = 1484.8 \text{ cm}^2$; $S = 1745.2 \text{ cm}^2$

18. 

SOLUTION:

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(6.5)(24.5) \\ &= 318.5\pi \\ &\approx 1000.6 \end{aligned}$$

$$\begin{aligned} S &= 2\pi rh + 2B \\ &= 2\pi(6.5)(24.5) + 2\pi(6.5)^2 \\ &= 318.5\pi + 84.5\pi \\ &= 403\pi \\ &\approx 1266.1 \end{aligned}$$

ANSWER:

$L \approx 1000.6 \text{ cm}^2$; $S \approx 1266.1 \text{ cm}^2$

CCSS SENSE-MAKING Find the lateral area and surface area of each cylinder. Round to the nearest tenth.

19. 

SOLUTION:

The radius of the base is 3 mm and the height of the cylinder is 15 mm.

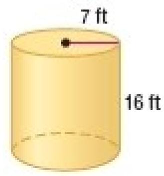
$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(3)(15) \\ &\approx 282.7 \end{aligned}$$

The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.

$$\begin{aligned} S &= 2\pi rh + 2B \\ &\approx 282.7 + 2\pi r^2 \\ &\approx 282.7 + 2\pi(3)^2 \\ &\approx 339.9 \end{aligned}$$

ANSWER:

$L \approx 282.7$ mm^2 ; $S \approx 339.3$ mm^2

20. 

SOLUTION:

The radius of the base is 7 ft and the height of the cylinder is 16 ft.

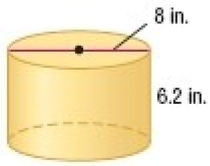
$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(7)(16) \\ &\approx 703.7 \end{aligned}$$

The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.

$$\begin{aligned} S &= 2\pi rh + 2B \\ &\approx 282.7 + 2\pi r^2 \\ &\approx 282.7 + 2\pi(7)^2 \\ &\approx 1011.6 \end{aligned}$$

ANSWER:

$L \approx 703.7$ ft^2 ; $S \approx 1011.6$ ft^2

21. 

SOLUTION:

The radius of the base is 4 in and the height of the cylinder is 6.2 in.

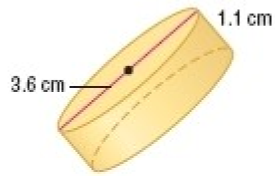
$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(4)(6.2) \\ &\approx 155.8 \end{aligned}$$

The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.

$$\begin{aligned} S &= 2\pi rh + 2B \\ &= 155.8 + 2\pi r^2 \\ &\approx 155.8 + 2\pi(4)^2 \\ &\approx 256.4 \end{aligned}$$

ANSWER:

$L \approx 155.8$ in^2 ; $S \approx 256.4$ in^2

22. 

SOLUTION:

The diameter of the base is 3.6 cm and the height of the cylinder is 1.1 cm.

$$\begin{aligned} L &= 2\pi rh \\ &\approx 2\pi(3.6)(1.1) \\ &\approx 12.4 \end{aligned}$$

The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.

$$\begin{aligned} S &= 2\pi rh + 2B \\ &\approx 12.4 + \pi r^2 \\ &\approx 12.4 + \pi(1.8)^2 \\ &\approx 32.8 \end{aligned}$$

ANSWER:

$L \approx 12.4$ cm^2 ; $S \approx 32.8$ cm^2

23. **WORLD RECORDS** The largest beverage can was a cylinder with height 4.67 meters and diameter 2.32 meters. What was the surface area of the can to the nearest tenth?

SOLUTION:

The radius is 2.32 ÷ 2 = 1.16.

$$\begin{aligned} S &= 2\pi rh + 2B \\ &= 2\pi(1.16)(4.67) + 2\pi(1.16)^2 \\ &= 10.8344\pi + 2.6912\pi \\ &= 13.5256\pi \\ &\approx 42.5 \end{aligned}$$

ANSWER:

42.5 m^2

Use the given lateral area and the diagram to find the missing measure of each solid. Round to the nearest tenth if necessary.

24. $L = 48 \text{ in}^2$

SOLUTION:

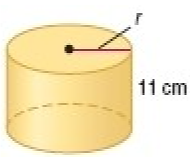
$L = 48 \text{ in}^2$, $l = 5$ in. and $w = 1$ in

$$\begin{aligned} L &= Ph \\ 48 &= [2(5) + 2(1)]h \\ 48 &= 12h \\ 4 &= h \end{aligned}$$

ANSWER:

$h = 4$ in.

25. $L \approx 635.9 \text{ cm}^2$



SOLUTION:

$$\begin{aligned} L &= 2\pi rh \\ 635.9 &= 2\pi r(11) \\ 635.9 &= 22\pi r \\ \frac{635.9}{22\pi} &= r \\ 9.2 &\approx r \end{aligned}$$

ANSWER:

$r = 9.2$ cm

26. A right rectangular prism has a surface area of 1020 square inches, a length of 6 inches, and a width of 9 inches. Find the height.

SOLUTION:

$$\begin{aligned} S &= Ph + 2B \\ S &= 2(l + w)h + 2lw \\ 1020 &= 2(6 + 9)h + 2(6)(9) \\ 1020 &= 30h + 108 \\ 912 &= 30h \\ 30.4 &= h \end{aligned}$$

ANSWER:

30.4 in.

27. A cylinder has a surface area of 256π square millimeters and a height of 8 millimeters. Find the diameter.

SOLUTION:

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ 256\pi &= 2\pi r^2 + 2\pi r(8) \\ 256\pi &= 2\pi\left(r^2 + 8r\right) \\ 128 &= r^2 + 8r \\ 0 &= r^2 + 8r - 128 \end{aligned}$$

Use the Quadratic Formula to find the radius.

$$\begin{aligned} r &= \frac{-8 \pm \sqrt{(8)^2 - 4(1)(-128)}}{2(1)} \\ &= \frac{-8 \pm 24}{2} \\ &= -16, 8 \end{aligned}$$

Since r is a length, it cannot be negative.

$r = 8$ mm and the diameter of the cylinder is 16 mm.

ANSWER:

16 mm

28. **MONUMENTS** The *monolith* shown mysteriously appeared overnight at Seattle, Washington's Manguson Park. It is a hollow rectangular prism 9 feet tall, 4 feet wide, and 1 foot thick.
a. Find the area in square feet of the structure's surfaces that lie above the ground.
b. Use dimensional analysis to find the area in square yards.

Refer to the photo on Page 835.

SOLUTION:

a. The area of surfaces that lie above the ground is the sum of the area of the upper base and the lateral surface area.

$$\begin{aligned} A &= Ph + B \\ &= 2(4+1)(9) + (4 \times 1) \\ &= 90 + 4 \\ &= 94 \end{aligned}$$

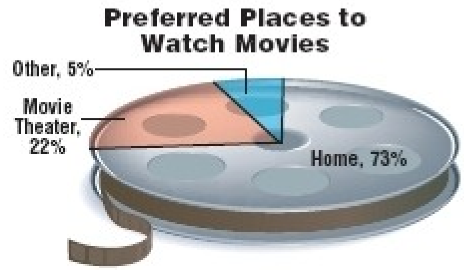
b.

$$\begin{aligned} A &= 94 \text{ ft}^2 \\ &= 94 \text{ ft} \cdot \text{ft} \\ &= 94 \cdot \tfrac{1}{3} \text{ yd} \cdot \tfrac{1}{3} \text{yd} \\ &= \tfrac{94}{9} \text{ yd}^2 \\ &\approx 10.4 \text{ yd}^2 \end{aligned}$$

ANSWER:

a. 94 ft^2
b. 10.4 yd^2

29. **ENTERTAINMENT** The graphic shows the results of a survey in which people were asked where they like to watch movies.
a. Suppose the film can is a cylinder 12 inches in diameter. Explain how to find the surface area of the portion that represents people who prefer to watch movies at home.
b. If the film can is 3 inches tall, find the surface area of the portion in part **a.**



SOLUTION:

a. First find the area of the sector and double it. Then find 73% of the lateral area of the cylinder. Next, find the areas of the two rectangles formed by the radius and height when a portion is cut. Last, find the sum of all the areas.

b.

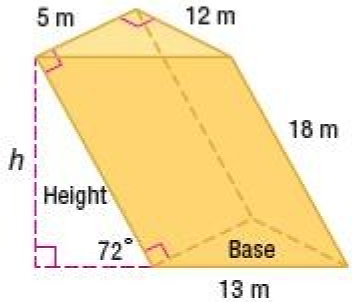
$$\begin{aligned} A &= 2\left(\tfrac{m}{360}\pi r^2\right) + 0.73(\pi dh) + 2(rh) \\ &= 2\left[0.73\pi(6)^2\right] + 0.73[\pi(12)(3)] + 2(6 \cdot 3) \\ &\approx 2(82.56) + 0.73(113.10) + 2(18) \\ &\approx 165.12 + 82.56 + 36 \\ &\approx 283.7 \end{aligned}$$

Therefore, the surface area of the portion is about 283.7 in^2.

ANSWER:

a. First find the area of the sector and double it. Then find 73% of the lateral area of the cylinder. Next, find the areas of the two rectangles formed by the radius and height when a portion is cut. Last, find the sum of all the areas.
b. 283.7 in^2

CCSS SENSE-MAKING Find the lateral area and surface area of each oblique prism. Round to the nearest tenth if necessary.

30. 

SOLUTION:

Use trigonometry to find the height.

$$\sin 72 = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 72 = \frac{h}{18}$$
$$18\sin 72 = h$$
$$17.1 \approx h$$

Use 18 sin 72 to represent *h* in order to get the most accurate surface area.

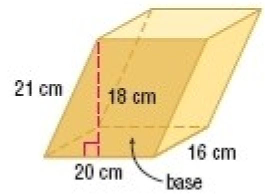
$$\begin{aligned} L &= Ph \\ &= (5+12+13)h \\ &= 30h \\ &= 30(18\sin 72) \\ &\approx 513.6 \end{aligned}$$

$$S = Ph + 2B$$
$$\begin{aligned} S &= (5+12+13)h + 2\left[\tfrac{1}{2}(5)(12)\right] \\ &= 30h + 60 \\ &= 30(18\sin 72) + 60 \\ &\approx 573.6 \end{aligned}$$

The answers in the book use $h = 17.1$ while the solution here uses 18 sin 72.

ANSWER:

513.6; 573.6

31. 

SOLUTION:

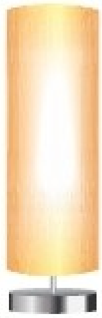
$$\begin{aligned} \text{Lateral Surface area} &= 2(21\times16) + 2(20\times18) \\ &= 2(336) + 2(360) \\ &= 1392 \text{ cm}^2 \end{aligned}$$
$$\begin{aligned} \text{Total surface area} &= 1392 + 2(20\times16) \\ &= 1392 + 2(320) \\ &= 2032 \text{ cm}^2 \end{aligned}$$

ANSWER:

Sample answer: $L = 1392 \text{ cm}^2$; $S = 2032 \text{ cm}^2$

32. **LAMPS** The lamp shade is a cylinder of height 18 inches with a diameter of $6\frac{3}{4}$ inches.

a. What is the lateral area of the shade to the nearest tenth?
b. How does the lateral area change if the height is divided by 2?



SOLUTION:

a.

$$\begin{aligned} L &= 2\pi rh \\ &= \left(6\tfrac{3}{4}\right)\pi(18) \\ &= 121.5\pi \\ &\approx 381.7 \end{aligned}$$

b. If the height is divided by 2 then the lateral area is divided by 2.

$$\begin{aligned} L &= 2\pi rh \\ &= \left(6\tfrac{3}{4}\right)\pi(9) \\ &= 60.75\pi \\ &\approx 190.85 \end{aligned}$$

ANSWER:

a. 381.7 in^2
b. The lateral area is divided by 2.

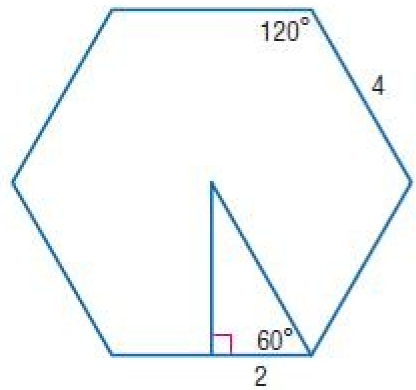
33. Find the approximate surface area of a right hexagonal prism if the height is 9 centimeters and each base edge is 4 centimeters. (*Hint:* First, find the length of the apothem of the base.)

SOLUTION:

The total surface area of the prism is the sum of the areas of the bases and the lateral surface area.
The perimeter of the base is 6(4) = 24 in.

The measure of each interior angle of a regular hexagon is $\frac{(6-2)180}{6} = 120.$

A line joining the center of the hexagon and one vertex will bisect this angle. Also the apothem to one side will bisect the side.



The triangle formed is a 30º-60º-90º triangle.
The length of the apothem is therefore $2\sqrt{3}$ cm.

Find the area of the base.

$$\begin{aligned} Area &= Pa \\ &= \frac{1}{2}(24)(2\sqrt{3}) \\ &= 24\sqrt{3} \\ &\approx 41.6 \end{aligned}$$

Now, find the surface area of the prism.

$$\begin{aligned} SA &= Ph + 2B \\ &= 24(9) + 2(24\sqrt{3}) \\ &= 216 + 48\sqrt{3} \\ &\approx 299.1 \end{aligned}$$

ANSWER:

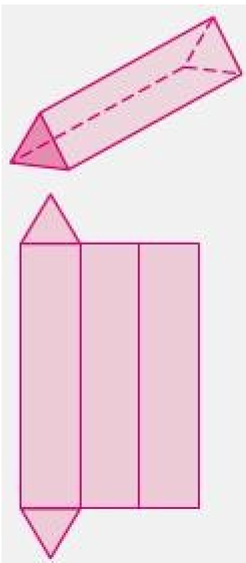
about 299.1 cm^2

34. **DESIGN** A mailer needs to hold a poster that is almost 38 inches long and has a maximum rolled diameter of 6 inches.
a. Design a mailer that is a triangular prism. Sketch the mailer and its net.
b. Suppose you want to minimize the surface area of the mailer. What would be the dimensions of the

mailer and its surface area?

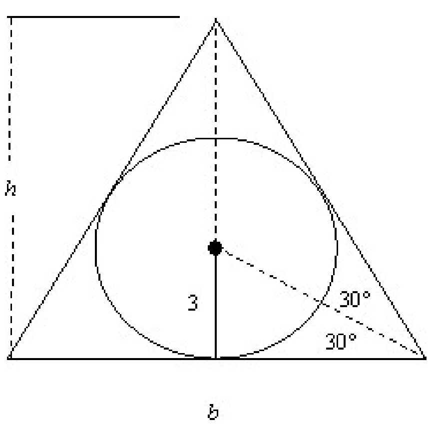
SOLUTION:

a. A triangular prism should consist of two triangles and three rectangles. They should be connected so that, when folded together they form a prism.



b. In order to minimize the surface area of the triangular prism, the triangles should be equilateral, and the side lengths of the rectangles should coincide the the length of the base of the triangle and the length of the poster. The surface area will then be $S = 3(\text{area of rectangles}) + 2(\text{area of triangles})$.

Use trigonometry to find the area of the triangles. The diameter of the poster has a maximum of 6 in. which corresponds to a radius of 3 in.



For the base we have:

$$\tan(30°) = \frac{3}{\frac{1}{2}b}$$

$$\tan(30°) = \frac{6}{b}$$

$$b = \frac{6}{\tan(30°)}$$

For the height we have:

$$\tan(60°) = \frac{h}{\frac{1}{2}b}$$

$$h = \frac{1}{2}b\tan(60°)$$

$$h = \frac{3}{\tan(30°)}\tan(60°)$$

Now calculate the area:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}\left(\frac{6}{\tan(30°)}\right)\left(\frac{3}{\tan(30°)}\tan(60°)\right)$$

$$A = 46.77$$

The area of the rectangles will be the product of the area of the base of the triangle with the length of the poster.

$$A = bl$$

$$A = \frac{6}{\tan(30°)} \cdot 38$$

$$A \approx 395$$

The total surface area can be calculated:

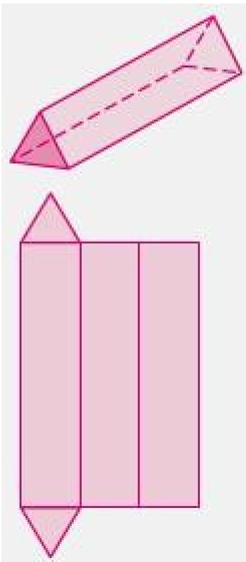
$$S = 3(\text{area of rectangles}) + 2(\text{area of triangles})$$

$$S = 3\left(\frac{6}{\tan(30°)}\right) \cdot 38 + 2\frac{1}{2}\left(\frac{6}{\tan(30°)}\right)\left(\frac{3}{\tan(30°)}\tan(60°)\right)$$

$$S = 1278 \text{ in}^2$$

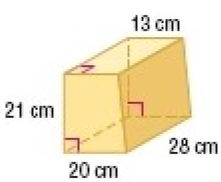
ANSWER:

a. Sample answer:



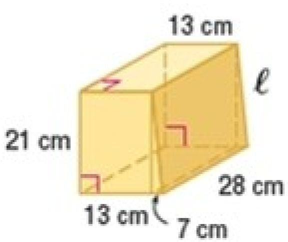
b. side lengths of triangular bases, about 10.39 in.each; height, 38 in.; 1278 in2

A composite solid is a three-dimensional figure that is composed of simpler figures. Find the surface area of each composite solid. Round to the nearest tenth if necessary.

35. 

SOLUTION:

This composite solid can be divided into a rectangular prism 13 cm by 21 cm by 28 cm and a triangular prism that has a right triangle with legs of 7 cm and 21 cm as the base and a height of 28 cm. The surface area of the solid is the sum of the surface areas of each prism without the area of the 21 cm by 28 cm rectangular face at which they are joined.



.

Rectangular prism:

$$\begin{aligned} SA &= \text{top / bottom} + \text{left} + \text{front / back} \\ &= 2(13 \cdot 28) + 21 \cdot 28 + 2(13 \cdot 21) \\ &= 728 + 588 + 546 \\ &= 1862 \end{aligned}$$

So, the surface area of five faces of the rectangular prism is 1862 cm^2.

Use the Pythagorean Theorem to find the length ℓ of the hypotenuse of the base of the triangular prism.

$$\begin{aligned} 21^2 + 7^2 &= \ell^2 \\ \sqrt{441+49} &= \ell \\ \sqrt{490} &= \ell \end{aligned}$$

Triangular prism:

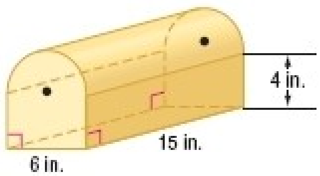
$$\begin{aligned} SA &= \text{front / back} + \text{bottom} + \text{right} \\ &= 2 \cdot \left[\frac{1}{2}(7)(21)\right] + 7 \cdot 28 + 28 \cdot \sqrt{490} \\ &= 147 + 196 + 28\sqrt{490} \\ &\approx 962.8 \end{aligned}$$

So, the surface area of four faces of the triangular prism is about 962.8 cm^2.

Therefore, the total surface area is about 1862 + 962.8 or 2824.8 cm^2.

ANSWER:

2824.8 cm^2

36. 

SOLUTION:

The solid is a combination of a rectangular prism and a cylinder. The base of the rectangular prism is 6 in by 4 in and the radius of the cylinder is 3 in. The height of the solid is 15 in.

Rectangular prism:

$$\begin{aligned} SA &= \text{bottom} + \text{left / right} + \text{front / back} \\ &= 15 \cdot 6 + 2(4 \cdot 15) + 2(6 \cdot 4) \\ &= 90 + 120 + 48 \\ &= 258 \end{aligned}$$

The surface area of five faces of the rectangular prism is 258 cm^2.

Half-cylinder:

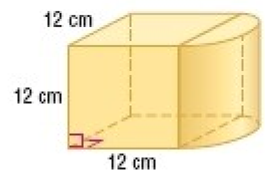
$$\begin{aligned} SA &= \frac{1}{2}\left[2\pi r^2 + 2\pi rh\right] \\ &= \frac{1}{2}\left[2\pi(3)^2 + 2\pi(3)(15)\right] \\ &= \frac{1}{2}[18\pi + 90\pi] \\ &= 54\pi \end{aligned}$$

The surface area of the half-cylinder is 54π cm^2.

The total surface area is $258 + 54\pi = 427.6$.

ANSWER:

427.6 in^2

37. 

SOLUTION:

The solid is a combination of a cube and a cylinder. The length of each side of the cube is 12 cm and the radius of the cylinder is 6 cm. The height of the solid is 12 cm.

Rectangular prism:

$$\begin{aligned} SA &= \text{bottom / top} + \text{left} + \text{front / back} \\ &= 2(12 \cdot 12) + 12 \cdot 12 + 2(12 \cdot 12) \\ &= 288 + 144 + 288 \\ &= 720 \end{aligned}$$

The surface area of five faces of the rectangular prism is 720 cm^2.

Half-cylinder:

$$\begin{aligned} SA &= \frac{1}{2}\left[2\pi r^2 + 2\pi rh\right] \\ &= \frac{1}{2}\left[2\pi(6)^2 + 2\pi(6)(12)\right] \\ &= \frac{1}{2}[72\pi + 144\pi] \\ &= 108\pi \end{aligned}$$

The surface area of the half-cylinder is 108π cm^2 and the total surface area is about 1059.3 cm^2.

ANSWER:

1059.3 cm^2

38. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the lateral area and surface area of a cylinder.

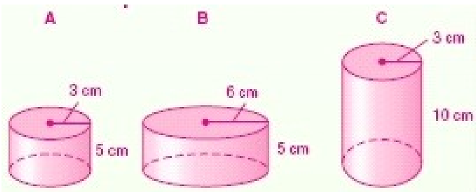
a. GEOMETRIC Sketch cylinder *A* (radius: 3 cm, height 5 cm), cylinder *B* (radius: 6 cm, height: 5 cm), and cylinder *C* (radius: 3 cm, height 10 cm).

b. TABULAR Create a table of the radius, height, lateral area, and surface area of cylinders *A*, *B*, and *C*. Write the areas in terms of π.

c. VERBAL If the radius is doubled, what effect does it have on the lateral area and the surface area of a cylinder? If the height is doubled, what effect does it have on the lateral area and the surface area of a cylinder?

SOLUTION:

a. Sketch and label the cylinders as indicated. Try to keep them to scale.



b. Calculate the lateral and surface area for each set of values.

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(3)(5) \\ &= 30\pi \\ &= 2\pi(6)(5) \\ &= 60\pi \\ &= 2\pi(3)(10) \\ &= 60\pi \end{aligned}$$

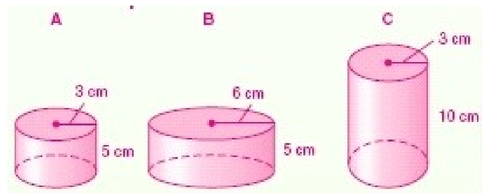
$$\begin{aligned} S &= 2\pi rh + 2B \\ &= 2\pi(3)(5) + 2\pi(3)^2 \\ &= 30\pi + 18\pi \\ &= 48\pi \\ &= 2\pi(6)(5) + 2\pi(6)^2 \\ &= 60\pi + 72\pi \\ &= 132\pi \\ &= 2\pi(3)(10) + 2\pi(3)^2 \\ &= 60\pi + 18\pi \\ &= 78\pi \end{aligned}$$

Cylinder	Radius	Height	Lateral Area	Surface Area
A	3	5	30π	48π
B	6	5	60π	132π
C	3	10	60π	78π

c. Sample answer: If the radius is doubled from 3 to 6, the lateral area is doubled from 30π to 60π and the surface area is more than doubled from 48π to 132π. If the height is doubled from 5 to 10, the lateral area is doubled from 30π to 60π, and the surface area is increased from 48π to 78π, but not doubled.

ANSWER:

a.

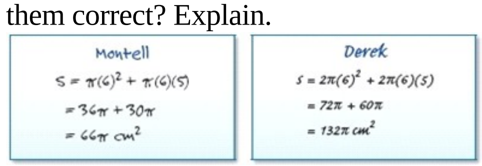


b.

Cylinder	Radius	Height	Lateral Area	Surface Area
A	3	5	30π	48π
B	6	5	60π	132π
C	3	10	60π	78π

c. Sample answer: If the radius is doubled, the lateral area is doubled and the surface area is more than doubled. If the height is doubled, the lateral area is doubled and the surface area is increased, but not doubled.

39. **ERROR ANALYSIS** Montell and Derek are finding the surface area of a cylinder with height 5 centimeters and radius 6 centimeters. Is either of them correct? Explain.



SOLUTION:

$$\begin{aligned} S &= 2\pi rh + 2B \\ &= 2\pi(6)(5) + 2\pi(6)^2 \\ &= 60\pi + 72\pi \\ &= 132\pi \end{aligned}$$

Therefore, Derek is correct.

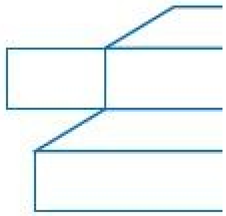
ANSWER:

Derek; sample answer: $S = 2\pi r^2 + 2\pi rh$, so the surface area of the cylinder is $2\pi 6^2 + 2\pi(6)(5)$ or $132\pi \text{ cm}^2$.

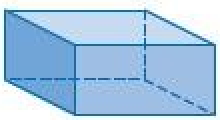
40. **WRITING IN MATH** Sketch an oblique rectangul describe the shapes that would be included in a net fo Explain how the net is different from that of a right r

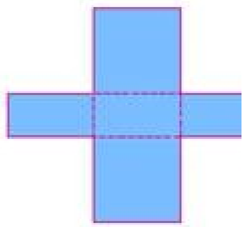
SOLUTION:

Sample answer:



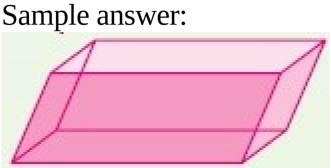
The net for the oblique rectangular prism shown abov six parallelograms and rectangles. The net of the righ prism shown below is composed of only six rectangle





ANSWER:

Sample answer:



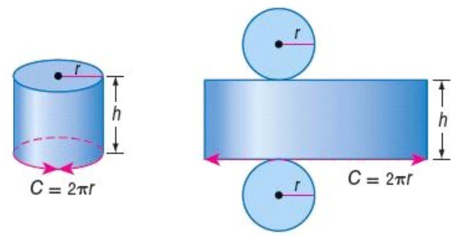
Rectangles and parallelograms; the net for a right pri rectangles only.

41. **CCSS PRECISION** Compare and contrast finding the surface area of a prism and finding the surface area of a cylinder.

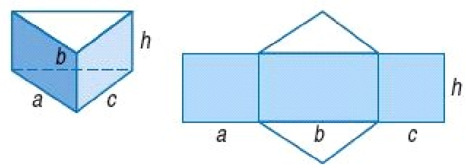
SOLUTION:

To find the surface area of any solid figure, find the area of the base (or bases) and add to the area of the sides of the figure. The faces and bases of a rectangular prism are rectangles. Since the bases of a cylinder are circles, the “side” of a cylinder is a rectangle.

Cylinder:



Prism:



ANSWER:

To find the surface area of any solid figure, find the area of the base (or bases) and add to the area of the lateral faces of the figure. The lateral faces and bases of a rectangular prism are rectangles. Since the bases of a cylinder are circles, the lateral face of a cylinder is a rectangle.

42. **OPEN ENDED** Give an example of two cylinders that have the same lateral area and different surface areas. Describe the lateral area and surface areas of each.

SOLUTION:

We need to select two cylinders where the products of the circumference and the height (the lateral area) are the same, but the areas of the bases are different. For the areas of the bases to be different, the bases must have different radii.

We have radius r_1 and height h_1 for the first cylinder, and radius r_2 and height h_2 for the second cylinder.

To solve this problem, we need $2r_1h_1 = 2r_2h_2$ and $r_1 \neq r_2$. So, we need to find two numbers whose product is equal to the product of two different numbers.

For the first cylinder, select a radius of 3 units and a height of 8 units. Any two composite numbers can be chosen for these two values. The product of 3 and 8 is 24, so we need two different numbers that make have this product. Two such numbers are 6 and 4.

$2(3)(8) = 2(6)(4)$ and $3 \neq 6$

Sample answer: A cylinder with height 8 units and radius 3 units has a lateral area of 48π square units and surface area of 66π square units; a cylinder with height 6 units and radius 4 units has a lateral area of 48π square units and surface area of 80π square units.

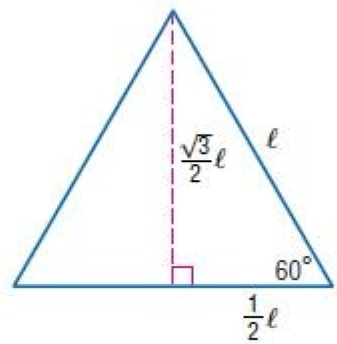
ANSWER:

Sample answer: A cylinder with height 8 units and radius 3 units has a lateral area of 48π square units and surface area of 66π square units; a cylinder with height 6 units and radius 4 units has a lateral area of 48π square units and surface area of 80π square units.

43. **CHALLENGE** A right prism has a height of h units and a base that is an equilateral triangle of side ℓ units. Find the general formula for the total surface area of the prism. Explain your reasoning.

SOLUTION:

Draw the equilateral triangle. The altitude forms two 30°-60°-90° triangles. The altitude is determined to have a length of $\frac{\sqrt{3}}{2}\ell$.



Find the area of the triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}l\left(\frac{\sqrt{3}}{2}l\right) \\ &= \frac{\sqrt{3}}{4}l^2 \end{aligned}$$

The perimeter of the triangle is 3ℓ. Find the surface area.

$$\begin{aligned} SA &= Ph + 2B \\ &= 3l(h) + 2\left(\frac{\sqrt{3}}{4}l^2\right) \\ &= 3lh + \frac{\sqrt{3}}{2}l^2 \end{aligned}$$

ANSWER:

$\frac{\sqrt{3}}{2}\ell^2 + 3\ell h$; the area of an equilateral triangle of side ℓ is $\frac{\sqrt{3}}{4}\ell^2$ and the perimeter of the triangle is 3ℓ. So, the total surface area is $\frac{\sqrt{3}}{2}\ell^2 + 3\ell h$.

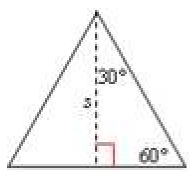
44. **WRITING IN MATH** A square-based prism and a triangular prism are the same height. The base of the

triangular prism is an equilateral triangle with an altitude equal in length to the side of the square. Compare the lateral areas of the prisms.

SOLUTION:

The lateral area of a prism is given by $L = Ph$. Let h represent the height of both prisms and s represent the length of a side of the square.
The perimeter of the square is $P = 4s$, so the lateral area of the square-based prism is given by $L = (4s)h$.

The base is of the triangular prism is an equilateral triangle with an altitude equal to the side of the square, or s. To find the perimeter of the triangle, first find the length of one of its sides.



Use the properties of the 30-60-90 right triangle to find the length of the sides.

The side opposite the 60°-angle is $\sqrt{3}$ times greater than the side opposite the 30°-angle. So, the side opposite the 30°-angle is $\frac{s}{\sqrt{3}}$. The hypotenuse is twice as long as the side opposite the 30°-angle or $\frac{2s}{\sqrt{3}}$. The perimeter of the equilateral triangle is $P = 3\left(\frac{2s}{\sqrt{3}}\right)$ or $2\sqrt{3}s$, and the lateral area of the triangular prism is $L = \left(2\sqrt{3}s\right)h$.

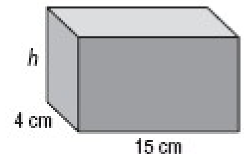
The two prisms have the same height. Compare the perimeters of their bases to compare their lateral areas.

The perimeter of the square-based prism is greater than that of the triangular prism, since $4s > 2\sqrt{3}s$. Therefore, the lateral area of the square-based prism is greater than that of the triangular prism.

ANSWER:

The lateral area of the square-based prism is greater than that of the triangular prism. The square has a perimeter of $4s$ and the triangle has a perimeter of $2\sqrt{3}s$ and $4s > 2\sqrt{3}s$.

45. If the surface area of the right rectangular prism is 310 square centimeters, what is the measure of the height h of the prism?



A 5 cm

B $5\frac{1}{6}$ cm

C 10

D $13\frac{3}{9}$ cm

SOLUTION:

$$A = Ph + 2B$$
$$310 = [2(4+15)]h + 2(4 \times 15)$$
$$310 = 38h + 120$$
$$190 = 38h$$
$$5 = h$$

Therefore, the correct choice is A.

ANSWER:

A

46. **SHORT RESPONSE** A cylinder has a circumference of 16π inches and a height of 20 inches. What is the surface area of the cylinder in terms of π?

SOLUTION:

Use the circumference to find the radius of the cylinder.

$$C = 2\pi r$$
$$16\pi = 2\pi r$$
$$\frac{16\pi}{2\pi} = r$$
$$8 = r$$

$$S = 2\pi rh + 2B$$
$$= 2\pi(8)(20) + 2\pi(8)^2$$
$$= 320\pi + 128\pi$$
$$= 448\pi$$

ANSWER:

$448\pi \text{ in}^2$

47. **ALGEBRA** The scores for a class on a 30-point math quiz are shown in the stem-and-leaf plot below. What was the mean score for this quiz?

Stem	Leaf
3	0 0
2	2 2 3 3 4 6 7 7 7 8 9
1	8 9

2|8 = 28

F 12
G 24
H 25
J 27

SOLUTION:

Read the stem and leaf plot to identify the entries. The entries are 30, 30, 22, 22, 23, 23, 24, 26, 27, 27, 27, 28, 29, 18, and 19. The mean is defined as the ratio of the sum of the entries to the number of entries. Therefore, the mean of the given set of data is $\frac{375}{15} = 25$.

The correct choice is H.

ANSWER:

H

48. **SAT/ACT** What is the value of $f(-2)$ if $f(x) = x^3 + 4x^2 - 2x - 3$?

A –31
B $-\frac{9}{2}$
C 9
D 25
E 28

SOLUTION:

Substitute $x = -2$ in $f(x)$ and evaluate.

$$f(-2) = (-2)^3 + 4(-2)^2 - 2(-2) - 3$$
$$= -8 + 16 + 4 - 3$$
$$= 9$$

Therefore, the correct choice is C.

ANSWER:

C

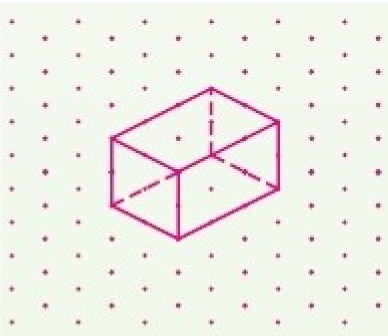
Use isometric dot paper to sketch each prism.

49. rectangular prism 2 units high, 3 units long, and 2 units wide

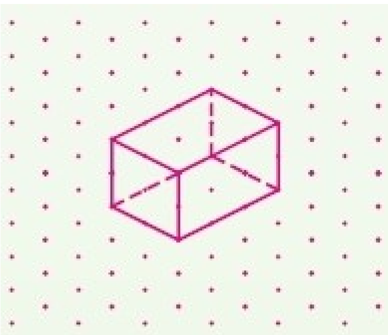
SOLUTION:

Mark the front corner of the solid. Draw 2 units down, 2 units to the left, and 3 units to the right. From the last point, draw 2 units left. Then draw a rectangle for the top of the solid.

Draw segments 2 units down from each vertex for the vertical edges. Connect the appropriate vertices using dashed lines for the hidden edges.



ANSWER:

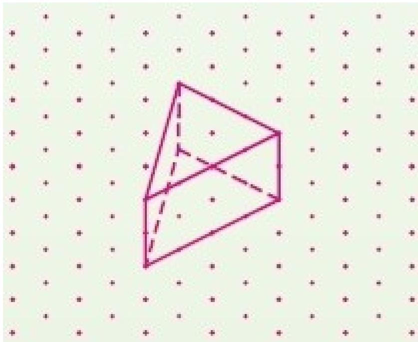


50. triangular prism 2 units high with bases that are right triangles with legs 3 units and 4 units long

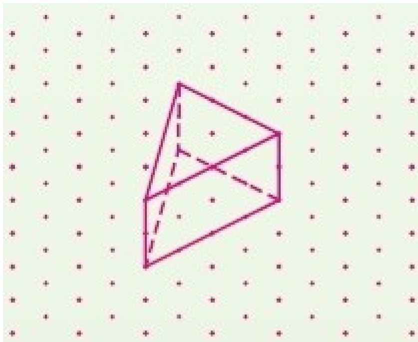
SOLUTION:

Mark the corner of the solid. Draw 2 units down, and 3 units to the right. From this point, draw 4 units left. Then draw a triangle for the top of the solid. The other side represents the hypotenuse of the base.

Draw segments 2 units down from each vertex for the vertical edges. Connect the appropriate vertices using a dashed line for the hidden edge.



ANSWER:



51. **BAKING** A bakery sells single-layer mini-cakes that are 3 inches in diameter for $4 each. They also have a cake with the same thickness and a 9-inch diameter for $15. Compare the areas of the cake tops to determine which option is a better buy, nine mini-cakes or one 9-inch cake. Explain.

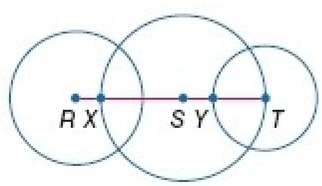
SOLUTION:

The tops of the cakes are similar circles. The ratio of the diameters of the regular cake to the mini-cake is $\frac{9}{3}$ or $\frac{3}{1}$, so the ratio of the areas of their tops is $\left(\frac{3}{1}\right)^2$ or $\frac{9}{1}$. The area of the top of the regular cake is 9 times as large as the mini-cake. Therefore, nine mini-cakes have the same area on top as one 9-inch cake, but nine mini-cakes cost 9($4) or $36 while the 9-inch cake is only $15, so the 9-inch cake is a better buy.

ANSWER:

one 9-inch cake: Nine mini-cakes have the same top area as one 9-inch cake, but nine mini-cakes cost 9($4) or $36 while the 9-inch cake is only $15, so the 9-inch cake is a better buy.

The diameters of $\odot R, \odot S,$ and $\odot T$ are 10 inches, 14 inches, and 9 inches, respectively. Find each measure.



52. *YX*

SOLUTION:

YX = *XT* – *YT*

XT = 14 since it is a diameter of a circle with a diameter of 14.
YT = 4.5 since it is a radius of a circle with a diameter of 9.

YX = 14 – 4.5 = 9.5 in.

ANSWER:

9.5 in.

53. *SY*

SOLUTION:

SY = *ST* – *YT*.

ST = 7 since it is the radius of a circle with a diameter of 14.

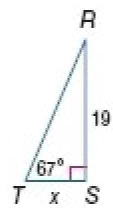
YT = 4.5 since it is a radius of a circle with a diameter of 9.

SY = 7 – 4.5 = 2.5 in.

ANSWER:

2.5 in.

Find *x*. Round to the nearest tenth.

54. 

SOLUTION:

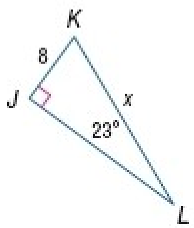
Use the tangent ratio of $\angle T$ to find the value of *x*.

$$\tan 67° = \frac{19}{x}$$

$$x = \frac{19}{\tan 67°} \approx 8.1$$

ANSWER:

8.1

55. 

SOLUTION:

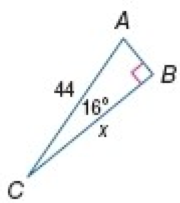
Use the sine ratio of $\angle L$ to find the value of *x*.

$$\sin 23° = \frac{8}{x}$$

$$x = \frac{8}{\sin 23°} \approx 20.5$$

ANSWER:

20.5

56. 

SOLUTION:

Use the cosine ratio of $\angle C$ to find the value of *x*.

$$\cos 16° = \frac{x}{44}$$

$$x = 44(\cos 16°) \approx 42.3$$

ANSWER:

42.3