FINDING SQUARE ROOTS BY VEDIC METHODS

Square root of any number means to get a number which is multiplied by itself gives the given number. In the conventional method of finding the square root, the divisor goes on becoming larger at each step. This increases the calculation time as well as the complexity of the problem. Here, we shall try to learn some speedy Vedic Methods of finding the square roots of perfect square numbers. Before proceeding for finding square roots, let us have a look into the known facts of squares and square roots.

The basic rules for extracting square roots are :

(*i*) The given number is arranged in two-digit groups from right to left; and a single digit (if any) left over at the left and is counted as a group by itself.

(*ii*) The number of digits in the square root will be the same as the number of two-digit groups in the given number including a single digit group (if any). Thus, 36 will count as one group, 169 as two groups and 1225 as two groups.

(*iii*) If the number contains n digits then the square root will contain $\frac{n}{2}$ (when n is even) and $\frac{n+1}{2}$ (when n is odd) digits. Thus, one or two digit number will have the square root of one digit, three and four digit number will have the square root of two digits, 5 and 6 digit number will have the square root of 3 digits and so on.

(*iv*) The squares of first nine natural numbers are :

$$1^2 = 1,\ 2^2 = 4,\ 3^2 = 9,\ 4^2 = 16,\ 5^2 = 25,\ 6^2 = 36,\ 7^2 = 49,\ 8^2 = 64,\ 9^2 = 81.$$

This means :

(*a*) unit digit of the perfect square number is 1, 4, 5, 6, 9 or 0.

(*b*) a perfect square number cannot end in 2, 3, 7 or 8.

(*c*) the relation between the unit digit of a perfect square number and the unit digit of its square root is as follows :

Unit digit of the number	1	4	5	6	9	0
Unit digit of square root	1 or 9	2 or 8	5	4 or 6	3 or 7	0

(*d*) If there are odd number of zeros at the end (on right side) of a number, then it will not be a perfect square.

VILOKANAM (OBSERVATION) METHOD

Square root of one or two digit number is well known from the table given in the beginning. Now we shall learn the method of finding square root of 3 or 4 digit perfect square number by *Vilokanam* method.

Look at the unit digit of the given number and decide about the unit digit of the square root from the following data :

Unit digit of the number	0	1	4	5	6	9
Unit digit of square root	0	1 or 9	2 or 8	5	4 or 6	3 or 7

Now ignore the last two digits (unit digit and ten's digit) of the given number and find out the greatest number whose square is less than or equal to the remaining part of the given number. Then adjust the above obtained unit digits on its right side and get two numbers. Find out the unique number with its unit digit 5 which lies between these two numbers and obtain the square of this unique number. If the given number is less than this square number then the smaller number among above obtained two numbers is the square root of the given number; otherwise another one is the required square root of the given number. Let us learn it with the help of some examples.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the square root of 841.*

Solution. The given number is 841.

Its unit digit is 1, therefore, the unit digit of the square root will be 1 or 9.

Ignoring the last two digits (unit digit and ten's digit) we get 8.

The greatest number whose square is less than or equal to 8 is 2.

Adjusting above obtained two unit digits 1 or 9 to the right of 2, we get two numbers 21 and 29.

The unique number with unit digit 5 which lies between 21 and 29 is 25.

$(25)^2 = 625$

(By *Ekādhikena sūtra* : $(25)^2 = (2 \times 3)\ 25 = 625$).

Since $841 > 625$, therefore, the required square root is 29.

Hence, $\sqrt{841} = 29$.

Example 2. *Find the square root of 4356.*

Solution. The given number is 4356.

Its unit digit is 6, therefore, the unit digit of the square root will be 4 or 6.

Ignoring the last two digits (unit digit and ten's digit), we get 43.

The greatest number whose square is less than or equal to 43 is 6.

On adjusting above obtained two unit digits 4 or 6 to the right of 6, we get two numbers 64 and 66.

The unique number with unit digit 5 which lies between 64 and 66 is 65.

$(65)^2 = 4225$.

(By *Ekādhikena sūtra* : $(65)^2 = (6 \times 7)\ 25 = 4225$)

Since $4356 > 4225$, therefore, the required square root is 66.

Hence, $\sqrt{4356} = 66$.

Example 3. *Find the square root of 8649.*

Solution. The given number is 8649.

Its unit digit is 9, therefore, the unit digit of the square root is 3 or 7.

Ignoring the last two digits (unit digit and ten's digit), we get 86.

The greatest number whose square is less than or equal to 86 is 9.

On adjusting above obtained two unit digits 3 or 7 to the right of 9, we get two numbers 93 or 97.

The unique number with unit digit 5 which lies between 93 and 97 is 95.

$(95)^2 = 9025$

(By *Ekādhikena sūtra* : $(95)^2 = (9 \times 10)\ 25 = 9025$)

Since $8649 < 9025$, therefore, the required square root is 93.

Hence, $\sqrt{8649} = 93$.

Example 4. *Find the square root of 7225.*

Solution. The given number is 7225.

Its unit digit is 5, therefore, the unit digit of the square root is also 5.

Ignoring the last two digits (unit digit and ten's digit), we get 72.

The greatest number whose square is less than or equal to 72 is 8.

On adjusting above obtained two unit digits 5 to the right of 8, we get the numbers 85. Here, we are getting just one number 85, which is the required square root.

Check : $(85)^2 = (8 \times 9)\ 25 = 7225$.

Hence, $\sqrt{7225} = 85$.

Exercise 3.3 (S)

Find the square roots of the following perfect square number by *Vilokanam* method :

1. 529 **2.** 729 **3.** 1764 **4.** 5476
5. 9604 **6.** 7921 **7.** 5329 **8.** 3136
9. 5625

GENERAL VEDIC METHOD OF FINDING SQUARE ROOTS

By the general Vedic method, we can find the square root of a number consisting of any number of digits. Also, even if the number is not a perfect square number we can find its square root in decimal form up to the desired number of decimal places. However, for the present we will restrict up to six digit numbers which are perfect squares.

Vedic method of finding the square root is similar to **Straight Division**. Here, the divisor is exactly double of the first left digit of the square root. Since the biggest digit is 9, therefore, in this Vedic method of finding square root the biggest divisor can be 18. To get the actual dividend from the gross dividend, the Duplexes of the further digits of the square root (second digit onwards from left) will have to be subtracted. Thus, this method is the combination of Straight Division and Duplexes. Moreover, the quotient being a digit of a square root can never be greater than 9 and the actual dividend at any stage should not be a negative number; in that case, we shall use altered remainders. Let us learn this method with the help of some examples.

ILLUSTRATIVE EXAMPLES

Example 1. *Find the square root of 6724, use general Vedic method.*

Solution. *Procedure :*

Step 1. Form the group of two-digits from right to left. Here, we get two groups $\overline{67}\ \overline{28}$. Separate the left group *i.e.* 67 by a vertical line. Leave one line space for writing remainders and draw a horizontal line as in **straight division**. Draw a vertical line at the left side of the number to write the divisor. Thus, set the problem as shown :

Step 2. The left most group is 67. Find the maximum square of a digit that can be subtracted from this group *i.e.* 67 and write this digit below the horizontal line as the first digit of the square root.

	67	2	4
16		3	0
	8	2 ·	0

Here, we can subtract maximum square of 8 which is 64. Write 8 as the first digit of the square root and prefix remainder 67 – 64 *i.e.* 3 to the next digit 2 of the number as shown. The divisor is the double of first square root 8 *i.e.* 2 × 8 = 16, write this divisor 16 to the left of vertical line from the number.

Step 3. Next dividend is 32. This itself is actual dividend. Divide 32 by 16 to get 2 as quotient and 0 as remainder. Write this quotient digit 2 and the remainder 0 at their respective places.

Step 4. Next gross dividend = 04 *i.e.* 4 and

actual dividend = 4 – duplex (2) = $4 - 2^2 = 4 - 4 = 0$.

As the number consists of 4 digits, its square root will consist of 2 digits. Therefore, if the number is a perfect square then the work of finding square root is complete and 82 is the square root of 6724.

To confirm this, write the decimal point in the quotient *i.e.* after the digit 2 of the square root and proceed in a similar way.

Step 5. Actual dividend = 0.

Divide 0 by 16 to get 0 as quotient and 0 as remainder. This means that the work has been completed.

$\therefore \sqrt{6724} = 82$

Example 2. *Find the square root of 119716.*

Solution. *Procedure :*

Step 1. Form the groups of two digits from left to right.

Here, we have : $\overline{11}\,\overline{97}\,\overline{16}$.

Separate the left most group *i.e.* 11 from the remaining digits of the number by a vertical line and write the problem as shown :

	11	9	7	1	6
6		2	5	5	3
	3	4	6	· 0	0

Step 2. Find the maximum square of a digit that can be subtracted from 11. Here, we can subtract maximum square of 3 which is 9. Write 3 as the first digit of the square root below horizontal line and prefix remainder 11 – 9 *i.e.* 2 to the next digit 9 of the number. The divisor is 2 × 3 *i.e.* 6. Write this divisor to the left of vertical from the number.

Step 3. Next dividend is 29. This itself is actual dividend. Divide 29 by 6 to get 4 as quotient and 5 as remainder. Write the quotient 4 as the second digit of the square root and the remained 5 at their respective places.

Step 4. Next gross dividend = 57,

actual dividend = 57 – D (4) = $51 - 4^2 = 41$

Divide 41 by 6 to get 6 as quotient and 5 as remainder.

Check : Next gross dividend = 51,

actual dividend = 51 – D (46) = 51 – 2(4 × 6) = 51 – 48 = 3.

As the number consists of 6 digits, its square root will consist of 3 digits. Therefore, if the number is a perfect square then the work of finding square root is complete and 346 is the square root of 119716.

To confirm this, write the decimal point in the quotient *i.e.* after the digit 6 of the square root and proceed in a similar way.

Step 5. Divide 3 by 6 to get 0 as quotient and 3 as remainder.

Write quotient digit 0 and remainder 3 at their respective places.

Next gross dividend = 36,

actual dividend = 36 – D (460) = 36 – (2(4 × 0) + 6^2) = 0.

Divide 0 by 6 to get 0 as quotient and 0 as remainder. This means that the work is completed.

$\therefore \quad \sqrt{119716} = 346$

Example 3. *Find the square root of 405769.*

Solution. *Procedure :*

	40	5	7	6	9
12		4	9	4	4
	6	3	7 ·	0	0

Step 1. Form the groups of two digits from left to right.

Here , we have : $\overline{40}\ \overline{57}\ \overline{69}$.

Write the problem as shown : 40 – 6^2 = 4, So 6 is the first digit of the square root and 4 is the remainder; divisor is 2 × 6 *i.e.* 12.

Step 2. Actual dividend = 45

45 ÷ 12 = 3, remainder 9.

Check : Next gross dividend = 97,

actual dividend = 97 – D (3) = 97 – 3^2 = 88.

∴ 3 is the second digit of the square root and 9 is the remainder.

Step 3. 88 ÷ 12 = 7, remainder 4.

Check : Next gross dividend = 46,

next actual dividend = 46 – D (37) = 46 – 2(3 × 7) = 4.

∴ 7 is the third digit of the square root and 4 is the remainder.

As the given number consists of 6 digits, so its square root consists of 3 digits provided the given number is a perfect square. Then the work of finding the square root is complete and 637 is the square root of 405769.

To confirm this, write the decimal point in the quotient *i.e.* after the third digit 7 of the square root and proceed as above.

Step 4. 4 ÷ 12 = 0, remainder 4.

∴ Quotient is 0 and remainder 4. Write 0 and 4 at their respective places.

Next gross dividend = 49,

next actual dividend = 49 – D (370) = 49 – (2(3 × 0) + 7^2) = 0.

Divide 0 by 12 to get 0 as quotient and 0 as remainder. This means that the work is completed.

$\therefore \quad \sqrt{405769} = 637$

Example 4. *Find the square root of 361 by using general Vedic method.*

Solution. *Procedure :*

Step 1. Form the groups of two digits from right to left.

Here , we have : $\overline{3}\ \overline{61}$.

The left most group consists of only one digit. Write the problem as shown :

	3		6		1
2		2		8	
	1		9	·	0

$3 - 1^2 = 2$, so 1 is the first digit of the square root and 2 is the remainder;

divisor is 2×1 *i.e.* 2.

Step 2. Dividend = 26. It is the actual dividend.

We want to divide 26 by 2 to get quotient and remainder. But quotient being a digit of the square root must be a one digit number. The biggest one digit number is 9. Let us try 9 for the quotient. Note that here we are using the concept of altered remainder.

$26 = 2 \times 9 + 8$ *i.e.* 26 divided by 2 gives 9 as quotient and 8 as remainder.

Write 9 as quotient *i.e.* the second digit of the square root and 8 as remainder at their respective places.

Step 3. Next gross dividend = 81,

actual dividend = $81 - D(9) = 81 - 9^2 = 0$.

As the given number consists of 3 digits, its square root consists of $\frac{n+1}{2}$ (when n is odd) digits *i.e.* $\frac{3+1}{2} = 2$ digits.

If the number is a perfect square, then the work of finding the square root is complete and 19 is the square root.

To confirm this, write the decimal point in the quotient *i.e.* after the second digit 9 of the square root and proceed as above.

Step 4. Divide 0 by 2 to get 0 as quotient and 0 as remainder. It means that the work is completed.

$\therefore \quad \sqrt{361} = 19$

Example 5. *Find the square root of 56169.*

Solution. *Procedure :*

Step 1. Form the groups of two digits from left to right.

Here, we have : $\overline{5}\,\overline{61}\,\overline{69}$.

We have a single digit 5 in the left most group.

Write the problem as shown :

	5		6		1		6		9
4		1		4		4		4	
	2		3		7	·	0		0

$5 - 2^2 = 1$, so 2 is the first digit of the square root and 1 is remainder; divisor is 2×2 *i.e.* 4.

Step 2. Next dividend is 16. It is the actual dividend.

$16 \div 4 = 4$, remainder 0.

Check : Next gross dividend = 01 = 1,

actual dividend = $1 - D(4) = 1 - 4^2 = -15$, not allowed.

Therefore, we need altered remainder.

$16 = 4 \times 3 + 4$ *i.e.* 16 divided by 4 gives 3 as quotient and 4 as remainder.

$\therefore$ 3 is the second digit of the quotient and 4 is the remainder. Write 3 and 4 at their respective places.

Step 3. Next gross dividend = 41,

actual dividend = 41 – D (3) = 41 – 3^2 = 32.

32 ÷ 4 = 8, remainder 0.

Check : Next gross dividend = 06 = 6,

actual dividend = 6 – D (34) = 5 – 2(3 × 4) = –18, which is not allowed.

Therefore, we need an altered remainder.

32 = 4 × 7 + 4 *i.e.* 32 divided by 4 gives 7 as quotient and 4 as remainder.

∴ 7 is the third digit of the square root and 4 is the remainder. Write 7 and 4 at their respective places.

As the given number consists of 5 digits, so its square root consists of $\frac{n+1}{2}$ (when n is odd) digits *i.e.* $\frac{5+1}{2}$ = 3 digits. If the number is a perfect square, then the work of finding the square root is complete and 237 is the square root.

To confirm this, write the decimal point in the quotient *i.e.* after the third digit 7 of the square root and proceed as above.

Step 4. Next gross dividend = 46,

actual dividend = 46 – D (37) = 46 – 2(3 × 7) = 4.

4 ÷ 4 = 1, remainder = 0.

Check : Next gross dividend = 09 = 9,

actual dividend = 9 – D (370) = 9 – (2(3 × 0) + 7^2) = –40, not allowed.

Therefore, we need an altered remainder.

4 = 4 × 0 + 4 *i.e.* 4 divided by 4 gives 0 as quotient and 4 as remainder.

Write 0 and 4 at their respective places.

Next gross dividend = 49,

actual dividend = 49 – D (370) = 49 – (2(3 × 0) + 7^2) = 0.

Step 5. Divide 0 by 4 to get 0 as quotient and 0 as remainder.

This means that the work is completed.

∴ $\sqrt{56169}$ = 237.

Example 6. *Find the square root of 16384.*

Solution. *Procedure :*

Step 1. Form the groups of two digits from left to right.

Here, we have : $\overline{1}\ \overline{63}\ \overline{84}$.

The left most group has only 1 digit.

Write the problem as shown :

	1	6	3	8	4
2		0	2	3	6
	1	2	8 ·	0	0

1 – 1^2 = 0, so 1 is the first digit of the square root and 0 is the remainder; divisor is 2 × 1 *i.e.* 2.

Step 2. Next dividend is 6. It is the actual dividend.

6 ÷ 2 = 3, remainder 0.

Check : Next gross dividend = 03 = 3,

actual dividend = 3 – D (3) = 3 – 3^2 = –6, not allowed.

Therefore, we need an altered remainder.

6 ÷ 2 = 2, remainder is 2 *i.e.* 6 divided by 2 gives 2 as quotient and 2 as remainder.

∴ The second digit in the square root is 2 and remainder is 2. Write quotient and remainder at their respective places.

Step 3. Next gross dividend = 23,

actual dividend = 23 – D (2) = 23 – 2^2 = 19.

19 ÷ 2 = 9, remainder 1.

Check : Next gross dividend = 18,

actual dividend = 18 – D (29) = 18 – 2(2 × 9) = –18, not allowed.

Therefore, we need an altered remainder.

19 ÷ 2 = 8, remainder 3 *i.e.* 19 divided by 2 gives 8 as quotient and 3 as remainder.

∴ 8 is the third digit in the square root is and 3 is the remainder. Write 8 and 3 at their respective places.

As the number consists of 5 digits, so its square root consists of 3 digits. If the number is a perfect square then the work of finding the square root is completed and 128 is the square root.

To confirm this, write the decimal point in the quotient and proceed as above.

Step 4. Next gross dividend = 38,

actual dividend = 38 – D (28) = 38 – 2(2 × 8) = 6.

6 ÷ 2 = 3, remainder 0.

Check : Next gross dividend = 04 = 4,

actual dividend = 4 – D (283) = 4 – (2(2 × 3) + 8^2) = –62, not allowed.

Therefore, we need an altered remainder.

6 ÷ 2 = 0, remainder 6 *i.e.* 6 divide by 2 gives 0 as quotient and 6 as remainder.

Write 0 and 6 at their respective places.

Next gross dividend = 64,

actual dividend = 64 – D (280) = 64 – (2(2 × 0) + 8^2) = 0.

0 ÷ 2 = 0, remainder is 0.

This means that the work is completed.

∴ $\sqrt{16384} = 128$.

Exercise 3.4 (S)

Find the square roots of the following numbers by using the general Vedic method :

1. 144 **2.** 576 **3.** 484 **4.** 1369

5. 7744 **6.** 9216 **7.** 3249 **8.** 15129

9. 552049 **10.** 222784 **11.** 170569 **12.** 83521