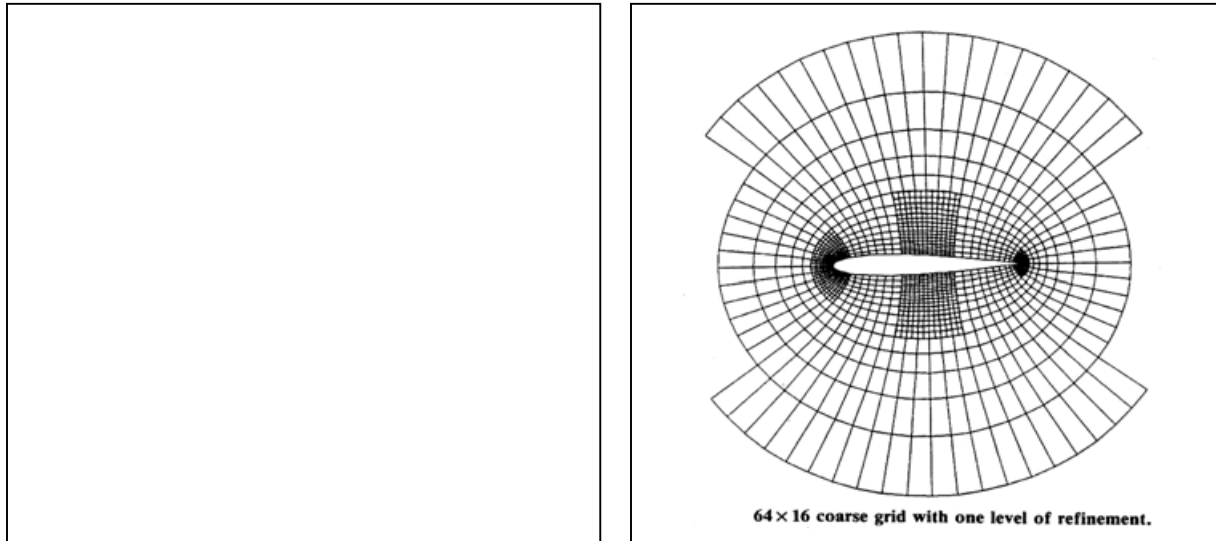


FINITE ELEMENT METHOD IN FLUID MECHANICS & HEAT TRANSFER

AERSP-560



Department : Aerospace Engineering

Instructor : Dr. Cengiz Camci

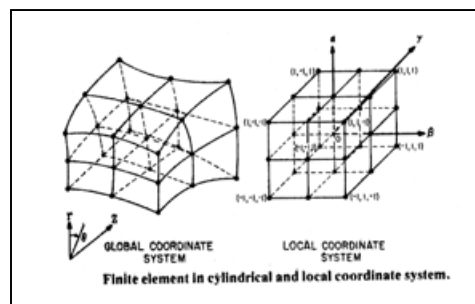
Time and Place : T & R , 4:15 - 5:30 pm 173 Willard Building

Prerequisites : Fluid Mechanics, Thermodynamics, advanced calculus, matrix algebra, computer programming (Fortran or C++)

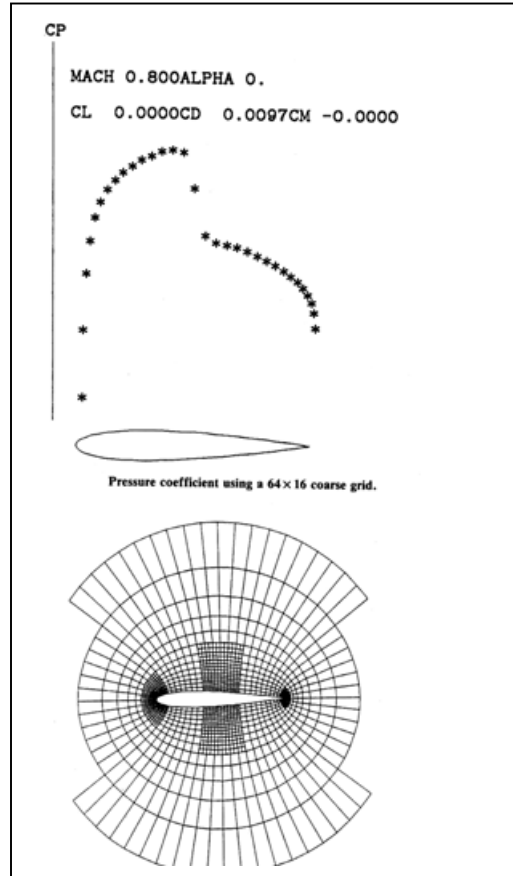
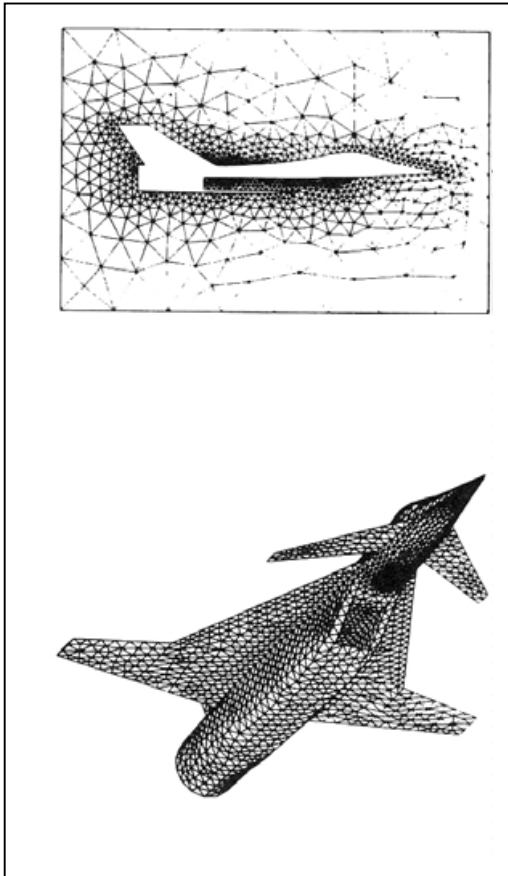
Grade : Computer assignments and a final exam

Credits : three credits

This course is intended to provide a thorough introduction to the basic ideas employed in the application of finite element techniques to especially fluid flow and heat transfer problems. A student who successfully completed this course should be able to perform quick analysis of small problems using the finite element method and write full sized application codes for analyzing fluid flow and heat transfer problems. Additionally, mastery of the material provided in this course will enable the student to more intelligently use commercially available codes based on the finite element method.



PLEASE CONTACT CENGIZ CAMCI IF YOU HAVE INTEREST IN THIS COURSE



TYPICAL SEQUENCE OF LECTURES

Introduction to finite element method, interpolations
 Numerical integration, iso-parametric elements
 Variational principles, method of variations
 Method of weighted residuals
 Potential flow solutions
 Transient heat conduction, parabolic diffusion problems
 Full potential equation solutions
 Incompressible viscous flow through the solution of Navier-Stokes equations
 Coupled heat transfer/flow solutions
 Natural/forced convection
 Density dependent convective diffusion
 Penalty method
 Flow/Heat transfer solutions for turbomachinery internal flow configurations

AEROSPACE 560

Finite Element Method in Fluid Mechanics and Heat Transfer

A. Bulletin Listing

1. Designation: AERSP
2. Number: 560
3. Title: Finite Element Method in Fluid Mechanics and Heat Transfer
4. Abbreviated title : Finite Elements in Thermo-fluids Engineering
5. Credits,class periods, practicum periods: 3,3,0
6. Description: A thorough introduction to the basic ideas employed in the application of finite element techniques to especially fluid flow and heat transfer problems encountered in aerospace engineering and mechanical engineering .

B. Course Outline

Introduction

Mathematical Tools

Foundations of Fluid Mechanics and Heat Transfer

An Introduction to Finite Element Analysis Using "Galerkin Weak Statement"

A Model One Dimensional Problem

The Weak Statement

Derivation of a Symmetric Weak Formulation

The Galerkin Procedure

Removal of the Arbitrariness

The Galerkin Procedure and Finite Element Discretization

Construction of the Trial Space Set

Finite Element Matrix Calculations

Development of the Local Coordinate System

Element Conductivity Matrix

Element Load Matrix

Assembly Procedure

A Solution for the Model Problem

A Higher Order Formulation

Flow Between Two Parallel Plates (Poiseville Flow)

A Galerkin Weak Statement Solution

Couette Flow

A Galerkin Weak Statement Solution

Cubic Basis Functions

A Comparison of Linear, Quadratic and Cubic Basis Functions

Another Finite Element Strategy Based on Euler's Equation of Variational Calculus

A Direct Minimization Technique

Euler's Theorem of Variational Calculus

Application of Euler's Equation in Finite Element Analysis

Generalized Form of Euler's Equation in Three Dimensional Space

Example problem: Steady State 3D Heat Conduction

Generalization of the Direct Minimization Technique to a Multi-Dimensional Problem

Example Problem : $u''+x.u=1$, $u(x=0)=0$, $u(x=1)=1$

Example Problem: Couette Flow Problem Using Direct Minimization Technique

Example Problem: Heat Conducting Bar with Convection on the Lateral Surface Using Direct Minimization

Determination of the Energy Functional for the Equations Belonging to a Sturm-Liouville System

Accuracy and Error Considerations in One-Dimensional Problems

Finite Element Analysis in Multi-Dimensions

Linear Triangular Basis

Quadratic Triangular Basis

Two Dimensional Finite Element Algorithm Development

Galerkin Weak Statement (GWS) for 2D Heat Conduction Problem

The Discrete Weak Statement

Implementation of the Linear Basis

Basis Derivatives in Local Coordinates

Coordinate Transformation

Element Matrix System

Boundary Conditions

(GWS) for Steady Heat Conduction with Boundary Convection

Aerodynamic Incompressible Potential Flow, A GWS solution

Steady Potential Flow over an Elliptic Cylinder

Compressible Subsonic Potential Flow

Linearized Theory For Thin Airfoils

A (GWS) Solution to Prandtl-Glauert Equation

Unsteady Transport with Fluid Motion

Unsteady Energy Equation + An Imposed 1D Velocity Field

Galerkin Weak Statement and Discretized (GWS)

Family of Single Step Time Iterative Algorithms

Explicit Euler, Trapezoidal, Backward Euler

Viscous Incompressible Unsteady Flow (Laminar) in 2D

A Stream-function/Vorticity Formulation of 2D Navier Stokes Equations

(GWS) for the Equation $L^2\psi = -\omega$

Removal of Arbitrariness

(GWS) for the Vorticity Transport Equation

Removal of Arbitrariness

Element Matrix Formulation for the Stream Function/Vorticity Formulation

Development of the Time Marching Procedure

Calculation of Stream Function Values at Nodal Points

Calculation of Velocity Components

Remarks on the Recovery of the Pressure Field

Examples of Finite Element Solutions of 2D Unsteady Viscous Flow

Unsteady Viscous Flow Around a Cylinder

Unsteady Viscous Flow Facing a Back Step

Unsteady Viscous Flow Around a Cylindrical Obstacle on its bed

Wind Driven Flow in a Rectangular Cavity

Flow in a Channel of Finite Width with a Rectangular Obstruction

Thermal Fluid Flow (2D, Viscous, Incompressible, Unsteady)

Natural Convection in an Enclosure

A Stream Function Vorticity Formulation and Thermal Energy Equation with
Buoyancy Force Terms

(GWS) and Interpolations on Stream Function, Vorticity and Temperature

An Incompressible Turbulent Boundary Layer Solution using the Finite Element Method

(GWS) for x Momentum Equation,

Removal of Arbitrariness, Boundary Conditions

Calculation of Cross Stream Velocities

A (GWS) Solution for the Continuity Equation

Computer Programming Aspects of Finite Element Analysis in 2D

Finite Element Grid Generation
Typical Data Structure of A Finite Element Program
Assembly Procedure
Implementation of Boundary Conditions
Solution of a System of Linear Algebraic Equations

2D Quadrilateral Elements (Bi-linear and Quadratic Elements)

Pure Rectangular Element (Bi-Linear)
Generic Quadrilateral Element (Bi-Linear)
Implementation of Bi-Linear Basis in Steady State Diffusion Equation
Transformation of Differential Line Element into Local Coordinates
Transformation of Differential Area Element into Local Coordinates
Numerical Integration in 2D, Gausssian Quadrature

Finite Element Analysis in Three Dimensional Space

Example Problem : 3D Steady Heat Conduction with Boundary Convection
Problem Statement
(GWS) and Global Matrix System
Isoparametric/Quadratic 3D Elements
Conduction Matrix in 3D
Coordinate Transformation
Numerical Integration over a Three Dimensional Cube
Convection Contribution in the Conduction Matrix
Numerical Integration over a Curved Convective Surface
Right Hand Side Column Matrix Resulting From Convective Boundaries
Assembly Procedure for the 3D Global System
Prescribed Boundary Temperatures
Solution of The Global System

Penalty Formulation, Fractional Step Method and A Solution Method in Primitive
Variables

Finite Element Strategies for the Flow Field Analyses in Rotating Machinery

C. Justification Statement

1. Instructional Objectives:

The objectives of this course are to cover the basic ideas employed in the application of finite element techniques to especially fluid flow and heat transfer problems. Although there are many existing courses dealing with finite element analysis, non of the current offerings specifically deal with fluid flow and heat transfer problems in depth. Almost all of the existing courses are focussed on structural mechanics and dynamics applications with minimal coverage on viscous flow and heat transfer. The techniques discussed in this course have wide application potential in Aerospace Engineering, Mechanical Engineering, Nuclear Engineering (thermo-hydraulics) and Chemical Engineering. A student who successfully completed this course will be able to perform quick analysis of small scale problems using the finite element method and to write full sized application codes for analyzing fluid flow and heat transfer problems. Additionally, mastery of the material provided in this course will enable the student to more intelligently use commercially available codes based on the finite element method.

2. Evaluation Methods of the Course:

Homework assignments, computer projects, midterm and final exams

3. Relationship of Course to Other Courses :

This is the only finite element course specifically dealing with fluid mechanics and heat transfer including viscous unsteady flow and convective heat transfer . Other course offerings in the area of Finite Element Method at the College of Engineering are as follows :

| | |
|-------------|--|
| ME 461 | Applied Finite Element Analysis |
| Emch 560 | An Introduction to the Finite Element Method of Analysis |
| Emch/ME 563 | Nonlinear Finite Element Analysis |

The course outlines of the courses listed above are enclosed to this document.

4. Relationship of Course to Major/Minor/Option or General Education:

This course would be required for Aerospace Engineering Students specializing in Thermal Turbomachinery research and Heat Transfer. It

would be optional for Aerospace and Mechanical Engineering Students who desire some background in the basics of Finite Element Method of Solution in Fluid Mechanics, Aerodynamics and Heat Transfer.

7. Consultation with other Departments and Academic Support Units:
Faculty in Mechanical Engineering and Engineering Science and Mechanics who offer courses in related areas were consulted to ensure there would be no significant overlap of material. A memo of support and suggestions from Dr. H.R.Jacobs is attached. This memo was obtained in 1993 during the first offering of the course.
6. Frequency of Offering and Enrollment:
The course will be offered every two years during the spring semester. This course was offered four times (Fall 1993, Spring 1995 , Spring 1997 and Fall 1998). The enrollment was 12, 10, 12 and 12 respectively.

D. Effective Date

Spring Semester 1999