Firm and Worker Dynamics in an Aging Labor Market*

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Abstract

I develop an idea flows theory of firm and worker dynamics in order to assess the consequences of aging. Older people are less likely to attempt entrepreneurship and switch employers because they have found better jobs. Consequently, aging reduces firm and worker dynamics through three channels. First, entry and mobility fall due to a composition effect. Second, the fact that potential hires are in better jobs dissuades entry. Third, fewer start-ups imply fewer new job opportunities for workers. Aging accounts for 37 and 81 percent of the declines in entry and mobility, respectively, since 1986. Cross-state evidence supports these predictions.

Keywords: Demographics; Employment; Economic Growth; Labor Turnover; Entrepreneurial choice

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1 Introduction

The aging of the labor force is fundamentally changing the US labor market. Constituting less than 30 percent of the labor force throughout the 1980s, the share of workers aged 45 years and older has increased to almost 45 percent today. The primary factor behind these shifts in the age composition is substantial declines in fertility decades earlier.[1] What are the consequences of aging on such a large scale for the US labor market?

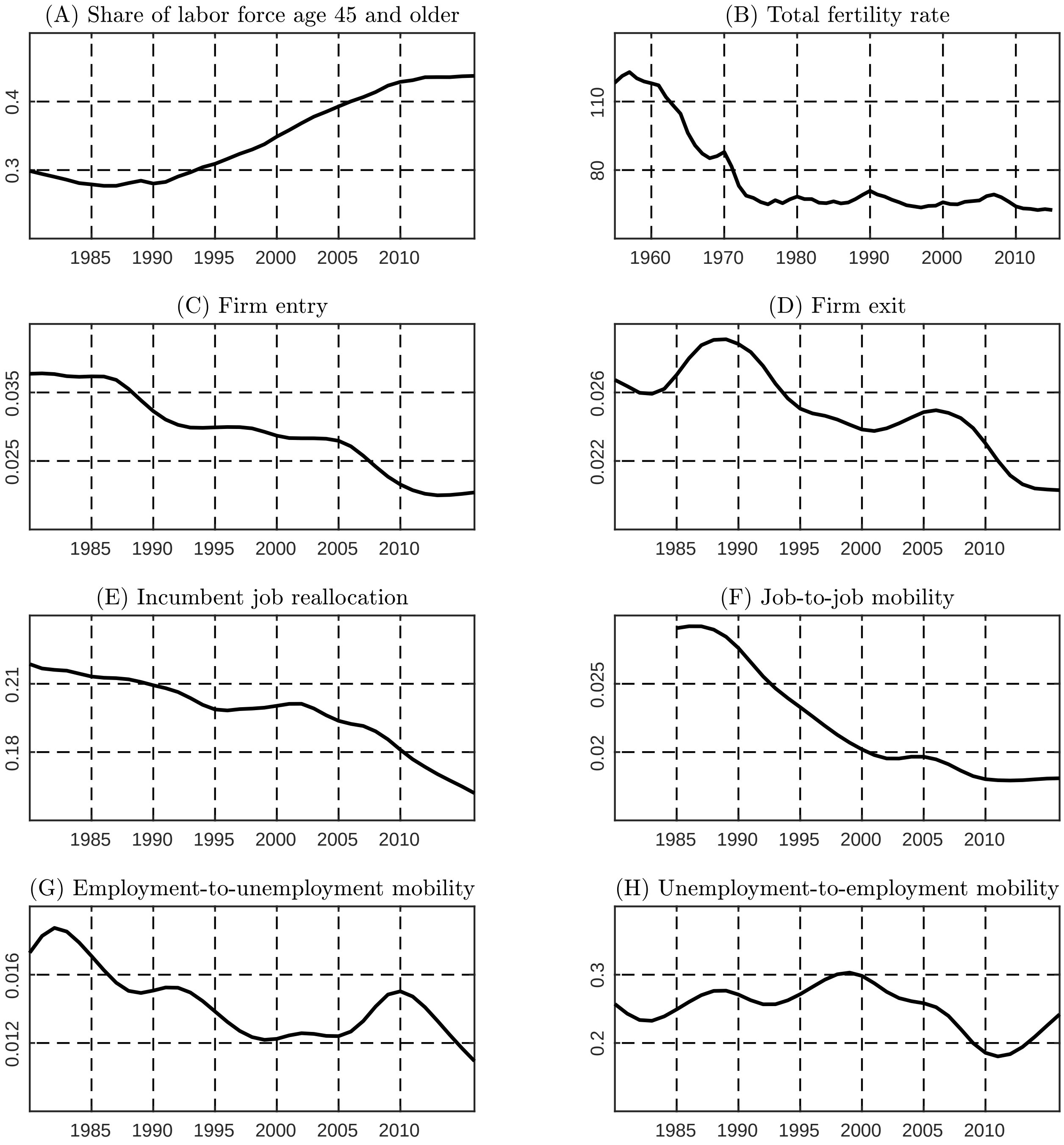
Over the same period, the rates at which jobs and workers reallocate across firms have trended down. The declines are pervasive and began long before the Great Recession, as highlighted by Figure 1. On the firm side, the entry rate, exit rate, and incumbent dynamics have all fallen. On the worker side, job-to-job mobility and job loss have decreased. Moreover, economic growth has been low at least since the turn of the century (Fernald, 2014). Understanding the causes of these declines is critical to determining whether and how policy should respond to them. This paper argues that an aging labor force accounts for a significant share of these facts.

Why might aging affect the labor market? This paper starts with the observation that older individuals are less likely to move between employers and enter entrepreneurship. The explanation put forth for these patterns is that through on-the-job search, individuals gradually find their way to better jobs. As a result, older individuals on average have to sacrifice more valuable jobs to switch employers or become entrepreneurs, making them more reluctant to do so. By shifting the composition of the labor force toward older, less mobile individuals, aging reduces job-to-job mobility and entry through a composition effect. These compositional effects in turn have equilibrium effects on the labor market. On the one hand, young workers constitute a disproportionate share of hires, so the presence of fewer young people who have not yet settled into good jobs further dissuades job creation and entry. On the other hand, young firms are a major source of job creation (Haltiwanger et al., 2013), so the lower entry rate implies fewer new, better job opportunities for workers. Consequently, worker mobility declines further. In this sense, firm and worker dynamics interact in equilibrium to amplify the affects of aging.

In order to quantify the impact of aging, I embed this intuition in an equilibrium theory of joint firm and worker dynamics that combines elements from three literatures. First, I adopt the *quality ladder* view of firm dynamics and growth through creative destruction (Aghion and Howitt, 1992; Grossman and Helpman, 1991). Second, I incorporate a *job ladder* model of worker mobility (Bur-

[1]These shifts in fertility are so large, in fact, that they swamp other factors, such as changes in labor force participation by age, the retirement age, or age of labor market entry.

FIGURE 1. AGING, FIRM DYNAMICS AND WORKER DYNAMICS OVER TIME



Note: Data from Business Dynamic Statistics (BDS), Current Population Survey (CPS), Survey of Income and Program Participation (SIPP) and Centers for Disease Control and Prevention (CDC). Fraction of the labor force aged 16 and older that is 45 years and older. Annual employment-weighted entry and exit rates of *firms*. Annual incumbent job reallocation is the sum of job creation and destruction of firms that remain in business between two years. Job-to-job mobility is the fraction of employed workers in a month who are with a different employer in the subsequent month. Employment-to-unemployment (unemployment-to-employment) mobility is the fraction of employed (unemployed) workers in a month who are unemployed (employed) in the subsequent month. Data are annual and HP-filtered with the usual smoothing parameter.

dett and Mortensen, 1998), by which I refer to a ranking of firms that workers gradually climb through on-the-job search. Third, I add an *entrepreneurial choice* (Lucas, 1978). A group of individuals enter the labor market, build their careers and eventually retire, only to be replaced by their offspring. At a point in time, one of these individuals may be unemployed, working for someone else, or running her own business. While unemployed or employed, she sometimes encounters potential jobs and business opportunities. If so, she weighs the benefits of pursuing these opportunities against the costs of quitting what she is currently doing. If she enters entrepreneurship, she may build on the innovations of incumbent firms. As an entrepreneur, she decides how many workers to try to hire, trading off the benefit of producing more against the cost of hiring. Over time, the entry of new, more productive firms gradually prices her out of the market, such that at some point she finds it optimal to shut down her firm and return to unemployment.

The view that the opportunity cost of wage employment is an important determinant of the decision to become an entrepreneur departs from the traditional focus in the vast literature on entrepreneurial choice. That literature has tended to stress the importance of financial constraints (Evans and Jovanovic, 1989) or differences in risk aversion (Kihlstrom and Laffont, 1979) in determining entrepreneurship, among others. Empirical work, however, has struggled to find support for the former (Hurst and Lusardi, 2004). Furthermore, recent evidence in Hacamo and Kleiner (2016) highlights that job displacement is associated with a significant increase in the probability of entering entrepreneurship without notably affecting the quality of new firms.[2] Whereas it is difficult to explain this pattern by sorting on risk aversion, it is easily interpretable within the context of a job ladder theory of the labor market, where a fall off the ladder is associated with a lower opportunity cost of doing something else.[3] At a conceptual level, this view is reminiscent of Atkeson and Kehoe (2007), who stress that because incumbent plants have accumulated knowledge of how to use old technologies, they may be more reluctant to adopt new technologies. In a similar spirit, I argue that since older workers have invested significant time in finding good jobs, they may be less eager to give them up to innovate through entrepreneurship.[4]

[2]This does not imply that the average firm started by an unemployed worker is as good as that started by an employed worker, because of the selection of who becomes unemployed. These authors carefully address such selection issues. The authors also argue that this is not simply a shift in the timing of entry, in the sense that these firms would not have been started had the founder not become displaced. Similar evidence has recently been presented for Sweden (Nykvist, 2008; von Greiff, 2009), Norway (Roed and Skogstrom, 2014), and Canada (Galindo da Fonseca, 2018).

[3]In this sense, I develop a suggestion in Coles and Mortensen (2016) that modeling the entrepreneurship decision as endogenous in the context of a frictional labor market would be "both realistic and worth pursuing" (see footnote 2 of NBER Working Paper 18022; sadly this footnote did not make it to the published article).

[4]In many aspects, the predictions from search theory are close to those from theories of firm-specific human capital. I prefer the search-theoretic approach because it is tractable and provides a natural way to connect to data on worker

In any case, the goal of this paper is not to develop a comprehensive model of all the potential factors behind a person's decision to enter entrepreneurship. Rather, it is to emphasize the notion that prior labor market events may play an integral role in such a decision, and to assess its quantitative importance in driving life cycle firm and worker dynamics. In order to do so, I estimate the model by generalized method of moments targeting productivity dispersion across firms, average productivity of entrants, the firm size distribution, the firm exit rate, average worker mobility, and estimates of the effect of displacement on the probability of entering entrepreneurship. I *do not* target life cycle dynamics of firms and workers. Instead, I use these outcomes as a test of the quantitative relevance of the forces highlighted. The proposed mechanism accounts for a large share of life cycle patterns of firms and workers. New firms enter small, exit at a high rate and only slowly grow large conditional on survival, as a result of random growth and labor market frictions. Older individuals have had more time to find good jobs, and are as a consequence less likely to switch employers, lose their jobs and enter entrepreneurship. Conditional on entry, however, post-entry survival and growth rates are unrelated to the age of the founder.

I subsequently use the theory to quantify the effects of aging. Holding all other parameters fixed, I ask what is the effect on the balanced growth path (BGP) of reducing the rate at which new individuals enter the labor market in order to match the increase in the share of older workers in the US between the 1980s and now? I design this experiment to hold fixed labor supply, thus isolating the role of shifts in the age composition of the labor force as distinct from the quantity of labor supplied. While the growth rate of labor supply is a key ingredient in many growth models and the effect of its decline on firm entry has been analyzed recently by Karahan et al. (2016) and Hopenhayn et al. (2018), much less is known about the effect of the age composition of the workforce (or more generally a measure of the quality of labor). This motivates my focus.

Aging gives rise to large declines in firm dynamics. The aggregate entry rate falls as older potential entrepreneurs have more to lose from attempting entrepreneurship since they on average are in better jobs, and it is harder to recruit workers in an older labor market. The decline in entry in turn lowers the rate of technological obsolescence, reducing the need to reallocate labor across production units. As a consequence, job reallocation falls, accounting for 64 percent of the large empirical decline over this period. As in the data, this is due to declines in entry, exit, and incumbent dynamics. In fact, in relative terms, aging accounts for the smallest share of the fall in entry, highlighting that other factors are also behind the large decline in entry over this period.

reallocation rates, but one could reinterpret this aspect of the theory in terms of firm-specific human capital.

The fall in labor supply growth is likely one such force (Karahan et al., 2016; Hopenhayn et al., 2018). Because technological obsolescence serves to reduce productivity dispersion across firms, its decline leads to an increase in cross-sectional productivity dispersion, in line with the data.

The biggest effect of aging is on the rate at which workers move between employers, accounting for 81 percent of the large empirical decline in job-to-job mobility over this period. In a pure accounting sense, 38 percent of this decline is accounted for by the shift in the aggregate labor force toward older, less mobile individuals. The remainder takes place conditional on an individual's age. This finding highlights the importance of modeling the endogenous process that reallocates jobs and workers in order to correctly assess the consequences of aging. Such mobility is driven by the desire to take advantage of new, better job opportunities as more productive entrants gradually replace incumbent firms. By reducing incentives to start firms and innovate, aging slows this process. The rate of job loss also declines, as incumbent firms do not become obsolete as fast as before, while the unemployment-to-employment mobility rate falls by less. The implication is a decline in the unemployment rate, in line with the US experience over this period. Finally, increased productivity dispersion across firms, together with reduced labor market dynamism, results in higher residual wage dispersion, matching 82 percent of the empirical increase over this period. On the other hand, income volatility is lower in the less dynamic economy, accounting for a third of the empirical decline in the standard deviation of annual income innovations.

Aging leads to a fall in annual economic growth of 0.30 percentage points across BGPs. At the same time, it has a positive level effect on output. Older workers are further up the job ladder. Moreover, the decline in the rate of obsolescence in response to aging results in workers being higher up the job ladder also conditional on age. In order to assess the relative importance of the growth and the level effect, I proceed to conduct a transition experiment. Solving for the full transition path is computationally infeasible,[5] so I follow the lead of Jones and Kim (2018) in approximating it by assuming that some of the decision rules jump, while other decision rules and the distributions evolve dynamically. This suggests that aging has a non-monotone effect on growth over this period, rising through the 1990s and subsequently falling.[6]

[5]Solving for the transition path requires tracking the evolution of the distributions of employment and firms, because these enter as states in the firms' and workers' problems. That is, whereas in many models agents care about these distributions only because it helps them to forecast the path for an interest rate or a competitive wage, here these distributions enter directly into the agents' problem. For this reason, random search models such as the current framework are notoriously difficult to solve outside steady state. Further complicating the problem, the growth rate is endogenous.

[6]This experiment also suggests that most of the changes in firm and worker dynamics take place within the 35-year sample period, confirming an argument in the search literature that transition dynamics are fast (at least relative to

In the final part of the paper, I provide reduced-form support for the hypothesis that aging has lowered firm and worker dynamics by exploiting variation in the age composition of the labor force across US states between 1978 and 2016. Aging is negatively correlated with a range of measures of firm and worker dynamics across states, although only some of the estimates for worker reallocation are statistically significant. If these correlations reflect a causal link, they imply an economically large effect of aging, of a magnitude similar to that implied by the model. Furthermore, while labor supply growth is positively correlated with firm entry, it does not alter the conclusion that the share of older people is strongly negatively correlated with entry.

Related literature. This paper builds on an earlier literature that studies the effect of embodied technical change on the labor market when the latter is characterized by frictions (Aghion and Howitt, 1994; Mortensen and Pissarides, 1998; Hornstein et al., 2007).[7] I introduce three key novelties to these earlier works. First, as in Michau (2013), I allow for on-the-job search, motivated by the dual observations that job-to-job mobility is a critical component of life cycle career dynamics (Topel and Ward, 1992), and that it has declined substantially over the past decades. Second, I model technological innovation as endogenously determined by the selection process associated with entry and exit, building on a vast literature stretching back to Schumpeter (1942). In particular, I build on the quality ladder models in Aghion and Howitt (1992) and Grossman and Helpman (1991). The closest paper in this regard is Aghion and Howitt (1994), who introduce endogenous growth in a frictional labor market. They abstract, however, from a job ladder and an entrepreneurial choice, and hence do not highlight the interaction between firm and worker dynamics stressed in the current paper. In terms of modeling, I also relate to a recent literature on technology diffusion (Luttmer, 2007, 2012; Lucas and Moll, 2014; Perla and Tonetti, 2014; Sampson, 2016). Third, I introduce an entrepreneurial choice as in Lucas (1978). Perhaps surprisingly given the vastness of the two literatures on labor market search and entrepreneurship, a dynamic entrepreneurial choice has not previously been introduced in a job ladder environment.[8]

My project also contributes to a literature that assesses the effects of demographics on the labor market.[9] Shimer (2001) studies the effect of the age composition on the unemployment rate,

slow-moving demographics). Evidently, these flows are related more to other parts of the distribution than the right tail, which converge much faster. On the other hand, the convergence of the right tail of the productivity distribution is slow, confirming findings in the random growth literature (Gabaix et al., 2016).

[7] See also Postel-Vinay (2002), Michelacci and Lopez-Salido (2007), and Pissarides and Vallanti (2007), among others.

[8] Fonseca et al. (2001) is perhaps closest in this regard, but they abstract from on-the-job search and hence a job ladder (and also do not model growth).

[9] Also related are several papers that correlate aging with economic growth using either cross-country or cross-state

highlighting as in the current paper that having a larger share of old, well-matched labor market participants may disincentivize job creation. Karahan et al. (2016) and Hopenhayn et al. (2018) argue that lower labor supply growth has contributed to the decline in the firm start-up rate. Three reasons lead me to instead explore the role of the age composition of the labor force, holding the quantity of labor fixed. First, these theories predict no change in firm dynamics conditional on firm age. As such, they cannot speak to large declines in incumbent job reallocation also conditional on firm age over this period, which account for over 80 percent of the overall decline in incumbent job reallocation. Second, the growth rate of labor supply primarily declined in the 1980s and the first part of the 1990s, while dynamism and growth have continued to decline as the population has aged. Third, I find that the share of older people correlates strongly with dynamism across US states, also controlling for labor supply. Together with the time series correlation, this suggests an important role for the age composition of the labor force over and above the quantity of labor supplied. Liang et al. (2016) document, in line with the hypothesis in this paper, that entrepreneurship entry is lower at all points in the life cycle in older countries.

Finally, several recent works propose explanations behind some aspects of the decline in dynamism. Salgado (2017) and Jiang and Sohail (2019) argue that changes in skill prices have reduced entry into entrepreneurship, Mercan (2017) proposes that better information has lowered job-to-job mobility, and Bornstein (2018) finds that an older, inertial pool of consumers has dissuaded entry. Given that the mechanism emphasized here accounts for only some of the declines in dynamism (particularly for entry), I view these explanations as complementary to the forces highlighted in this paper.

Outline. Section 2 develops a joint theory of firm and worker dynamics, which Section 3 brings to the data. Section 4 applies the theory to quantify the effect of aging on firm and worker dynamics. Section 5 provides additional empirical support, and Section 6 concludes.

2 The Model

This section develops an idea flows theory of firm and worker dynamics. In order to quantify the potential interactions between firm and worker dynamics and explore their link to growth, I combine elements from three benchmark theories. The first is a *quality ladder* model of firm dynamics

data, including Feyrer (2007), who finds a negative correlation between aging and growth using a large sample of developed and developing countries; Acemoglu and Restrepo (2017), who find no relationship in a shorter panel of countries; and Maestas et al. (2016), who find a negative correlation across US states.

and growth through creative destruction based on Aghion and Howitt (1992) and Grossman and Helpman (1991). The second is a *job ladder* model of worker mobility in a frictional labor market, as in Burdett and Mortensen (1998), and the third is a dynamic *entrepreneurial choice* (Lucas, 1978).

2.1 Environment

Time is continuous, and I abstract from aggregate shocks. The economy consists of a unit mass of ex ante identical individuals, a positive mass of firms, and an amount of land λ.[10]

Demographics and preferences. Individuals enter the labor market, build their careers and eventually retire at rate κ, at which point they are replaced by their offspring. They have dynastic preferences over a unique consumption good discounted at rate ρ, $\mathbb{E}_0 \int_0^\infty e^{-\rho t} c\Big(t \Big) dt$.[11]

Technology. The economy produces a multipurpose consumption good through production units that I refer to as firms. A firm is an idea for how to combine labor to produce output. Entrepreneur founders own and implement these ideas. In order to stay active, a firm needs a unit of land. If the entrepreneur sells her land, the idea is permanently lost and she returns to unemployment.

Denote by Z the idiosyncratic productivity of the idea of the entrepreneur founder—henceforth, the productivity of the firm. If a firm with productivity Z hires n workers, it produces a flow

$$Y = e^Z n$$

of the consumption good. The idiosyncratic productivity Z evolves according to a diffusion

$$dZ(t) = \mu \, dt + \sigma \, dW$$

where W is a standard Brownian motion, σ an exogenous intensity of shocks, and μ an exogenous drift. In order to hire workers, a firm advertises vacancies v at cost $C(v, t)$.

[10]While I think of this fixed factor as land, alternative interpretations are managers, as in Atkeson and Kehoe (2007), or human capital, as in Aghion and Howitt (1994). The assumption of a fixed amount of land restricts growth to exclusively take the form of quality improvements, in the spirit of the quality ladder literature (Aghion and Howitt, 1992; Grossman and Helpman, 1991). Although it may be interesting to expand the model to also feature growth through expanding varieties, as in Romer (1990), Garcia-Macia et al. (2016) estimate that varieties grow at the same rate as both employment and the number of firms over this period, and that such growth accounts for less than 10 percent of overall growth. Hence I focus on quality improvements.

[11]I assume that parameter values are such that this rate is greater than the growth rate of the economy.

Labor market search. At a point in time, an individual may be unemployed, employed, or an entrepreneur. She enters the labor market as unemployed, except if her parent was an entrepreneur in which case she inherits the firm. As both unemployed and employed, she searches for jobs, in the latter case with exogenous relative search intensity ϕ.[12] Search is random, in the sense that an individual cannot direct her search toward a particular segment of the market (Burdett and Mortensen, 1998). As employed, she becomes unemployed at exogenous rate δ, because she chooses to quit, or because her firm exits. As unemployed, she enjoys value of leisure $B(t)$.

Entrepreneurial choice. At exogenous rate π, unemployed and employed individuals learn of potential business opportunities. In order to pursue it, she has to do three things. First, she has to devote some time $\varepsilon \overset{i.i.d}{\sim} \tilde{\Gamma}(t)$ to start the business. It represents time needed to develop the idea, obtain required licenses, and so on, and is idiosyncratic to the idea. Second, she has to set up shop, i.e. she needs to acquire a unit of land. Land is traded in a competitive market. Third, entrepreneurship is a full-time commitment, so she has to quit whatever she is currently doing.[13]

Having borne the costs associated with entry, she realizes her initial productivity. That is, ideas are *experience goods* in the sense that only by trying the idea will she figure out how good it is. Three observations motivate this assumption. First, it is consistent with the high exit rates of new firms observed in the data, which at face value suggest substantial initial uncertainty about the viability of an idea. Second, it is consistent with the evidence cited above that displacement significantly increases the probability of entry to entrepreneurship without notably affecting the success of new start-ups. Relatedly, recent work by Hombert et al. (2017) finds that providing incentives to unemployed workers who start businesses substantially increases entry without worsening the quality of new entrants. In both cases, if instead ideas had been "inspection goods," one would have expected such marginal firms to be worse. Third, within the context of the current model, if individuals had prior information about the quality of their idea, it would lead to selection patterns into entrepreneurship over the life cycle and, as a result, systematic differences in post-

[12]While it could be interesting to endogenize search intensity, it would arguably, if anything, amplify the effects of aging as people respond to the lack of better jobs by searching less. To focus on the issue at the core of this paper—the equilibrium interaction between firm and worker dynamics—I leave this for future research.

[13]One could imagine that where the entrepreneur was previously employed could affect her prospects if she were to return to the labor market after a spell of entrepreneurship (including perhaps returning to her previous employer). While this would lead me to estimate a different parameter value to match the same cross-sectional moment, I do not believe that it would lead to very different effects of aging (conditional on matching the same cross-sectional moment). In any case, it seems highly uncertain that one could quit one's job only to have the same job back months later. Note also that the model is about *residual* dispersion (and is estimated to match this in the next section). One should not think of this as a previous investment banker returning to flip burgers at McDonalds.

entry performance by age of the founder. As I show in the next section, there is no evidence for this in the data. Notice also the assumption that there is a market failure for ideas in the sense that only the individual who receives the idea can pursue it. That is, potential ideas cannot be traded.

An entrant at time t starts with productivity $Z(t)$ that is related to the distribution of incumbents at time t, summarized by some moment $\overline{Z}(t)$, plus an idiosyncratic component $Z \overset{i.i.d.}{\sim} \zeta$

$$Z(t) \;=\; \overline{Z}(t) \,+\, Z, \qquad Z \sim \zeta$$

Because entrants build upon the ideas of incumbents, the economy may grow forever. In contrast to an earlier generation growth models, however, innovation here does not take place at the frontier (Grossman and Helpman, 1991; Aghion and Howitt, 1992). Instead, as in the recent idea flows literature, spillovers are related to the entire distribution of incumbents (Luttmer, 2007; Lucas and Moll, 2014). This approach is motivated by the empirical observation that entrants are almost exclusively small, typically low productive, and exit at high rates, which appears at odds with entrants entering at the frontier. The microfoundation is based on epidemic models of diffusion, whereby technology spreads through random meetings between producing agents.

This view of the entrepreneurial entry process abstracts from liquidity constraints and wealth accumulation (Evans and Jovanovic, 1989; Cagetti and De Nardi, 2006). My choice is motivated by recent empirical work that has struggled to find support for such factors being key drivers of entrepreneurship entry (Hurst and Lusardi, 2004). Furthermore, empirical evidence suggests that being displaced raises the probability of entry into entrepreneurship, without notably affecting the quality of new start-ups (Hacamo and Kleiner, 2016). This leads me to instead study the role of prior labor market events in shaping the entrepreneurial entry decision. In this sense, I relate more closely to a literature that focuses on the risk of entering entrepreneurship (Kihlstrom and Laffont, 1979), where here the risk associated with entry is in terms of forsaken wage employment.

Matching. The number of meetings in the labor market is a function of the efficiency mass of searching workers, S, and the number of open jobs, V, represented by a constant returns to scale matching function $\chi(S, V)$. The job finding rate p and the worker finding rate q are, respectively,

$$p \;=\; \frac{\chi\big(S\,,\,V\big)}{S} \qquad q \;=\; \frac{\chi\big(S\,,\,V\big)}{V} \tag{1}$$

Bargaining. Values are split between the firm and worker following the alternating offers bargaining framework developed by Dey and Flinn (2005) and Cahuc et al. (2006). This results in an unemployed worker getting a slice β of the surplus when meeting a firm. When an employed worker meets another potential employer, the incumbent and poaching employer first Bertrand compete for the worker. This is won by the bidder with the higher valuation of the worker's services. The worker subsequently bargains with the winning employer, using the runner-up firm as her threat point. She gets a slice β of the differential surplus between the two matches. The contract may be renegotiated whenever one party has a credible threat to leave a bilaterally efficient match, which ensures that matches maximize their joint surplus. In particular, a worker who receives an entrepreneurship opportunity may use it to renegotiate her wage with her current employer. In such cases, the firm is assumed to make the worker a take-it-or-leave-it offer. Similarly, because productivity evolves, there may be instances when either the firm or the worker has a credible threat to terminate a bilaterally efficient match under the current contract. In such instances, I assume that the party that did not initiate the renegotiation makes a take-it-or-leave-it offer. Values are delivered to the worker through a fixed wage.

2.2 A transformed stationary economy

Instead of studying the growing, non-stationary economy, it is convenient to analyze a transformed, stationary version of the model. To this end, a particularly convenient transformation is to denominate all variables in the lowest productivity at time t, $\underline{Z}(t)$. That is, if $Z(t)$ is the productivity of the firm at time t, then the normalized productivity of this firm $z(t)$ is

$$z(t) \quad = \quad Z(t) \; - \; \underline{Z}(t)$$

In the transformed economy, the productivity of an incumbent firm $z(t)$ follows the diffusion

$$z(t) \quad = \quad -m \, dt \; + \; \sigma \, dW$$

where $m = M - \mu$ is the difference between the overall endogenous growth rate of the economy, M, and the exogenous growth rate of incumbent firms, μ. This *rate of obsolescence* characterizes how fast incumbent firms fall behind on the quality ladder on the BGP.

I use similarly transformed variables throughout the exposition of the model. I assume that the vacancy cost, cost of entry, and flow value of leisure can be expressed as a function of the lowest productivity at time t, $C(v,t) = \underline{Z}(t)c(v)$, $\tilde{\Gamma}(t) = \underline{Z}(t)\Gamma$, and $B(t) = \underline{Z}(t)b$, where $c(v)$ is strictly convex. This captures the notion of some outside option with an associated time value. In order to obtain a BGP, these objects must grow at the rate of the economy.[14] While in my empirical implementation I follow Sampson (2016) to assume that knowledge spillovers are represented by the mean of the incumbent distribution, I assume for now that entrants innovate on the least productive firm at time t, as in Luttmer (2012). This simplifies the exposition of the model.

Denote by U the value of being unemployed, $J(z)$ the value of a match between a firm with productivity z and a worker, E the expected value of entry into entrepreneurship, and L the equilibrium price of land. They solve the following set of Hamilton-Jacobi-Bellman (HJB) equations.

Unemployed individual. The value of unemployment equals,

$$rU = b + p\beta \int_{\underline{z}}^{\infty} \Big(J(z) - U \Big)^{+} dF(z) + \pi \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big(E - L - \varepsilon - U \Big)^{+} d\Gamma(\varepsilon) \tag{2}$$

where $r = \rho - M$ is the discount rate in the transformed economy and $x^{+} = \max\{x, 0\}$. An unemployed individual enjoys flow value of leisure b. While she dies at rate κ, she has dynastic preferences, so this does not change her value. At job finding rate p, she meets a potential employer, drawn from offer distribution F. If the job is sufficiently valuable, she enters employment and gets a slice β of the surplus. Finally, at rate π the individual encounters an entrepreneurship opportunity. If she draws a low enough idiosyncratic entry cost, she attempts entrepreneurship.

The recursion (2) defines a reservation threshold $\bar{z}^{w}$ such that if she meets a firm with productivity $z \geq \bar{z}^{w}$, she enters employment. This threshold is given by

$$J\Big(\underline{z}^{w}\Big) = U \tag{3}$$

The recursion (2) also defines a cost threshold $\bar{\varepsilon}(0)$ such that if she draws an idiosyncratic cost $\varepsilon \leq \bar{\varepsilon}(0)$, she attempts entrepreneurship. This is given by

$$E = L + \bar{\varepsilon}\Big(0\Big) + U \tag{4}$$

[14] An alternative would be to link these costs to, for instance, the average wage in the economy. As will become clear, aging pushes up average wages relative to the exit threshold. Consequently, such an assumption would lead to an increase in the cost of hiring, entry, and the value of leisure, exacerbating the declines in dynamism. To avoid overstating the effects, I make the conservative assumption that these objects are linked to the exit threshold.

That is, she enters entrepreneurship if the expected return from entry, E, exceeds the cost, consisting of the cost of a unit of land, L, the idiosyncratic entry cost ε, and the opportunity cost U.

Employed individual. Because of linearity in production, matches can be analyzed in isolation, as long as the value of leisure b is high enough that workers quit to unemployment before the entrepreneur wants to shut down the firm. I assume that this always holds, and think of it as workers abandoning a "sinking ship."[15] For $z \geq \underline{z}^w$, the value of a match solves the HJB equation,

$$\begin{aligned} r\,J\Big(z\Big) \;&=\; e^z + (-m)\,J'\Big(z\Big) + \frac{\sigma^2}{2}\,J''\Big(z\Big) \\ &+\; \Big(\delta+\kappa\Big)\Big(U - J(z)\Big) \\ &+\; \phi\,p\,\beta \int_{\underline{z}}^{\infty} \Big(J(z') - J(z)\Big)^{+} dF(z') \\ &+\; \pi \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big(E - L - \varepsilon - J(z)\Big)^{+} d\Gamma(\varepsilon) \end{aligned} \tag{5}$$

subject to the boundary conditions

$$J\Big(\underline{z}^w\Big) = U, \qquad J'\Big(\underline{z}^w\Big) = 0 \tag{6}$$

I discuss each term of (5) in sequence. The match produces output e^z. Productivity falls behind the market at the rate of obsolescence m and is hit with shocks at intensity σ. The worker exogenously quits at rate δ, and dies at rate κ. In the latter case, her offspring enters as unemployed. There is no change if the entrepreneur dies since her offspring takes over the firm. The worker finds a new job at rate ϕp, which is drawn from distribution F. She accepts the job if it provides a higher value than her current job, and gets a share β of the differential value between the two matches.[16] At rate π, the worker learns of an entrepreneurship opportunity. She pursues

[15]Failure of this restriction to hold requires stipulating a multilateral bargaining protocol, which introduces a range of complications without conceptually changing the forces highlighted in this paper. This is because the shadow value of land effectively works like a fixed cost for the firm. Determining how to split the burden of this fixed cost between the firm's owner and its many workers requires a multilateral bargaining protocol. Under this parameter restriction, on the other hand, a firm has no employees at the point of exit, and hence exit is a unilateral decision by the firm's owner. In Bilal et al. (2019), we undertake the complex task of developing such a multilateral bargaining protocol. Based on insights from that project, I believe that loosening this parameter restriction would not change the mechanism and conclusions in this paper, but would come at the cost of substantial added complexity.

[16]While the incumbent firm loses some value when the worker leaves the firm, the worker is perfectly compensated for this loss by the poaching firm under the stipulated bargaining protocol. From the perspective of the match, these

it if its expected value exceeds the cost of entry, consisting of the price of a unit of land, the time cost of starting the business and the opportunity cost of forsaking her current job. Because the employment contract can be renegotiated whenever one party of the match has a credible threat to abandon it, all decisions are bilaterally optimal, including the entrepreneurship entry decision.

The mobility decision on the job is to accept all offers $z' > z$. Hence the two interesting decisions defined by (5) are the maximum cost $\bar{\varepsilon}(z)$ the worker is willing to pay to enter,

$$E \;=\; L + \bar{\varepsilon}(z) + J\Big(z\Big) \tag{7}$$

and a threshold $\underline{z}^w$ defined by (3) such that the worker quits to unemployment if productivity falls below this level.

Entrepreneur. Because of linearity in production and the parameter restriction on b, the problem of the firm of how many workers to hire and when to exit is independent of how many workers the entrepreneur currently employs. Hence, denote by $Q(z)$ the expected value of hiring additional workers to the firm. It solves for $z \geq \underline{z}$ the HJB equation,

$$\begin{aligned} r\,Q\Big(z\Big) \;&=\; \max_{v \geq 0} \Bigg\{ -\,c(\,v\,) + v\,q \\ &\times\;\Big(1-\beta\Big)\left(\frac{u}{S}\Big(J(\,z\,)-U\Big)^{+} + \frac{\phi\,e}{S}\int_{\underline{z}}^{\infty}\Big(J(\,z\,)-J(\,z'\,)\Big)^{+} dG\Big(z'\Big)\right)\Bigg\} \\ &+\;(-m)\,Q'\Big(z\Big) + \frac{\sigma^2}{2}\,Q''\Big(z\Big) \end{aligned} \tag{8}$$

subject to value matching and smooth pasting,

$$Q\Big(\underline{z}\Big) \;=\; U + L\,, \qquad \text{and} \qquad Q'\Big(\underline{z}\Big) \;=\; 0 \tag{9}$$

where u is the mass of unemployed, e the mass of employed, and $G(z)$ the distribution of employed workers over firms. That is, the entrepreneur attempts to hire new workers up to the point where the marginal cost of trying to hire more workers equals the expected value of meeting a worker. The firm falls behind the market at the rate of obsolescence m, receives shocks with intensity σ, and optimally exits at a point where the value of staying in the market equals the return

two terms offset each other, which explains why neither shows up in the above recursion for the value of the match.

to selling the land and returning to unemployment. Because the transformed economy is denominated in the exit threshold, consistency requires that the equilibrium price of land is such that $\underline{z} = 0$, that is

$$L \;=\; Q\Big(\,0\,\Big) \;-\; U \tag{10}$$

In addition to the reservation threshold $\underline{z}$, the recursion (8) defines an optimal vacancy policy, $v(z)$, from the first-order condition

$$v(z) \;=\; c'^{-1}\left\{ q\Big(1-\beta\Big)\left(\frac{u}{S}\Big(\,J(z) \;-\; U\,\Big)^{+} \;+\; \frac{\phi\, e}{S}\int\limits_{\underline{z}}^{\infty}\Big(\,J(z) \;-\; J(z')\,\Big)^{+} dG(z')\right)\right\} \tag{11}$$

The expected value of entry into entrepreneurship equals $E \;=\; \int_{\underline{z}}^{\infty} Q(\,z\,)\, d\zeta(\,z\,)$.

2.3 Laws of motion

In order to solve for the equilibrium, I also need to specify how the distributions of workers and firms evolve. To that end, denote by $h(z)$ the distribution of firms over productivity and by x the entry rate of firms. Then h solves for $z \geq \underline{z}$ the Kolmogorov forward equation (KFE)

$$0 \;=\; -\,(-m)\,h'(z) \;+\; \frac{\sigma^2}{2}\,h''(z) \;+\; x\,\zeta(z) \tag{12}$$

subject to the boundary conditions

$$h(\underline{z}) \;=\; 0, \qquad\qquad 1 \;=\; \int\limits_{\underline{z}}^{\infty} h(z)\,dz, \qquad\qquad x \;=\; \frac{\sigma^2}{2} h'(\underline{z}) \tag{13}$$

Productivity falls behind the market at the rate of obsolescence m, is subject to shocks with intensity σ, and new entrants enter at rate x according to the distribution ζ. The first boundary condition in (13) requires that the density is zero at the exit threshold, since firms exit when they hit it. The second condition imposes that h is a density. The final condition can be seen by integrating the KFE from $\underline{z}$ to infinity and imposing the first and second conditions.[17]

[17]Specifically, $0 = -m \int_{\underline{z}}^{\infty} h'(z)dz + \frac{\sigma^2}{2}\int_{\underline{z}}^{\infty} h''(z)dz + x\int_{\underline{z}}^{\infty}\zeta(z)dz = -\frac{\sigma^2}{2}h'(\underline{z}) + x$ since the second condition requires that $\lim_{z\to\infty} h(z) \;=\; 0$ and $\lim_{z\to\infty} h'(z) \;=\; 0$, the first requires that $h(\underline{z}) \;=\; 0$, and $\zeta(z)$ integrates to one by nature of being a density.

For $z \geq \underline{z}^w$, the evolution of employment, g, is given by the KFE

$$
\begin{aligned}
0 &= -(-m)\, g'(\, z\,) + \frac{\sigma^2}{2} g''(\, z\,) \\
&- \Big(\, \kappa + \delta + \phi p(\, 1 - F(z)\,) + \pi\, \Gamma(\, \bar{\varepsilon}(z)\,)\, \Big)\, g(\, z\,) \\
&+ p\, f(\, z\,) \left(\frac{u}{S} + \frac{\phi e}{S} G(\, z\,) \right)
\end{aligned} \tag{14}
$$

The first and second terms are due to the drift of and shocks to productivity z. The third term is outflows due to death, exogenous separations, endogenous mobility up the job ladder, and entry to entrepreneurship. The last term is inflows from unemployed and employed workers who accept a job at a firm with productivity z. This is subject to the boundary conditions

$$
\begin{aligned}
0 &= g\Big(\, \underline{z}^w \,\Big), \qquad 1 = \int_{\underline{z}}^{\infty} g\Big(\, z\,\Big)\, dz \\
0 &= -\Big(\, p\,(\, 1 - F(\, \underline{z}^w\,)\,) + \pi\, \Gamma\Big(\, \bar{\varepsilon}(\, 0\,)\,\Big) + \kappa\, \Big)\, u \\
&+ \left(\, \delta + \frac{\sigma^2}{2} g'(\, \underline{z}^w\,)\, \right) \Big(\, 1 - u - \lambda\, \Big) + x\,\lambda + \kappa\,(\, 1 - \lambda\,)
\end{aligned} \tag{15}
$$

The last condition reflects flows out of unemployment to employment, to entrepreneurship, or out of the labor force, and into unemployment from employment, entrepreneurship, or not in the labor market. Recall that the offspring inherit the firms of their parents.

2.4 Equilibrium

The offer distribution F is the distribution of firms h weighted by their vacancy decisions $v(z)$,

$$
F\Big(\, z\,\Big) = \frac{\lambda}{V} \int_{\underline{z}}^{z} v\Big(\, \tilde{z}\,\Big)\, dH\Big(\, \tilde{z}\,\Big), \qquad V = \lambda \int_{\underline{z}}^{\infty} v(z) dH(z) \tag{16}
$$

where H is the cumulative distribution function (cdf) of h. The entry rate x equals

$$
x = \frac{\pi}{\lambda} \left(u\, \Gamma\Big(\, \bar{\varepsilon}(\, 0\,)\,\Big) + \Big(\, 1 - u - \lambda\,\Big) \int_{\underline{z}}^{\infty} \Gamma\Big(\, \bar{\varepsilon}(\, z\,)\,\Big)\, dG(\, z\,) \right) \tag{17}
$$

Individuals receive entrepreneurship opportunities at rate π. A mass u of unemployed workers enter if the idiosyncratic cost is sufficiently low. A mass $1 - u - \lambda$ of employed workers are distributed over the job ladder according to G and enter if the cost is sufficiently low. To obtain the entry rate of firms, the flow of entrant individuals has to be divided by the mass of firms, λ.

Definition 1. *A stationary equilibrium consists of value functions U, J, and Q; decision rules $\underline{z}^w$, $\bar{\varepsilon}\left(z\right)$, $v(z)$, and $\underline{z}$; finding rates p and q, an aggregate mass of vacancies V, a price of land L, and an aggregate entry rate x; a distribution of workers $\left\{g, u\right\}$ and a vacancy-weighted distribution of firms f; and a distribution of firms h and a rate of obsolescence m such that*

1. *The value function U is given by (2), the reservation threshold $\underline{z}^w$ by (3), and the entry policy $\bar{\varepsilon}\left(0\right)$ by (4);*

2. *The value function J and reservation policy $\underline{z}^w$ solve the stopping time problem (5)–(6), and the entry policy $\bar{\varepsilon}\left(z\right)$ is given by (7);*

3. *The value function Q and the exit threshold $\underline{z}$ solve the stopping time problem (8)–(9), and the vacancy policy $v(z)$ is given by (11);*

4. *The finding rates p and q are given by (1), the aggregate stock of vacancies V by (16), the price of land L satisfies (10) and the aggregate entry rate x is given by (17);*

5. *The distribution of workers $\left\{g, u\right\}$ is given by (14)–(15), and the vacancy-weighted distribution of firms f by (16); and*

6. *The distribution h and rate of obsolescence m are given by (12)–(13).*

I provide a brief discussion of how the equilibrium is determined in Appendix A.

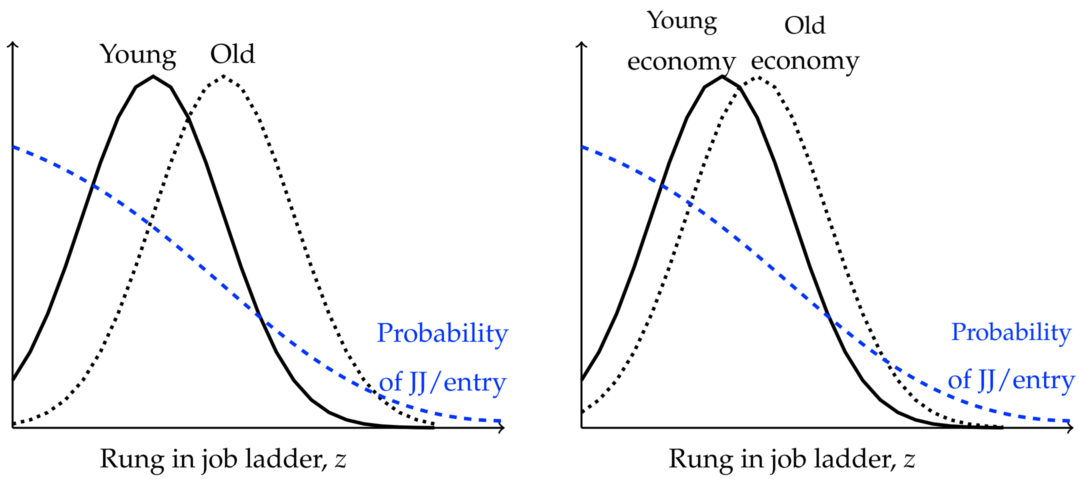
2.5 The effects of aging on the equilibrium

Before bringing the theory to the data, it is useful to illustrate the channels through which aging affects the labor market. My view of aging is a decline in the rate at which dynasties are reincarnated, κ.[18] The effects of aging can be classified as either composition or equilibrium effects. The

[18] Because a change in κ changes both the entry and exit rates from the labor market, it affects the age composition but also the amount of time an individual expects to stay in the market. In my baseline experiment, I hold fixed individuals' expectations for how long they expect to remain in the market to avoid the second effect. Also letting expectations adjust, however, makes little quantitative difference to the results. This is presumably due to the dynastic preferences and the fact that matches terminate at rates that are orders of magnitude higher than the reincarnation rate.

former arise because older individuals are higher up the job ladder since they have had more time to climb it. Consequently, they have a higher opportunity cost of doing something else, including switching to another employer or entering entrepreneurship. Labor force aging shifts the composition of the labor force toward individuals higher up the job ladder, reducing the aggregate job-to-job (JJ) rate and entry rate. Figure 2 illustrates.

FIGURE 2. DISTRIBUTION OF YOUNG AND OLD INDIVIDUALS, AGGREGATE DISTRIBUTION IN YOUNG AND OLD ECONOMY, AND PROBABILITY OF A JJ MOVE/ENTRY



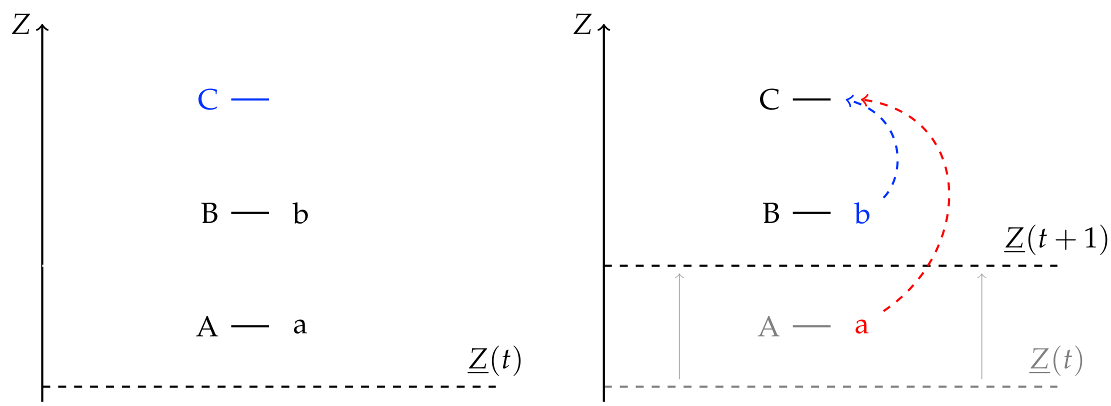
These composition effects in turn affect the labor market through two key equilibrium channels. On the one hand, the fact that the aggregate labor force is higher up the job ladder makes it more expensive for firms to hire. Effectively, a firm needs to spend more resources to locate individuals who are "movable." Moreover, conditional on finding such an individual, the firm has to offer her more to move since she has on average a better outside option. This discourages job creation and entry. Consequently, entry falls conditional on a potential entrepreneur's age. That is, in equilibrium the decline in worker dynamics further amplifies the fall in firm dynamics.

On the other hand, the lower entry rate implies fewer new, better job opportunities for workers. Effectively, the rate at which new, better rungs are added to the job ladder is lower. Figure 3 illustrates the intuition. Suppose two firms are currently in the market—A and B—and two workers—a and b—are working for them. At some point, a new firm C enters. Suppose for simplicity that it enters at the top of the quality ladder.[19] It offers new, better job opportunities to workers, who try to reallocate to it. In this sense, JJ mobility plays a critical role in taking advantage of new, better ways to produce. The prospect of finding a job at C induces worker a to quit to unemployment to devote her full effort toward finding a job at C. Moreover, firm A finds it

[19]In the full model, it would enter somewhere in the middle of the distribution, but the intuition would be the same.

optimal to exit the market due to the higher land price. The exit threshold shifts up. The economy keeps going like this forever. If the entry rate is lower, worker mobility is lower. That is, in equilibrium the decline in firm dynamics amplifies the fall in worker dynamics.

FIGURE 3. THE QUALITY LADDER AND THE JOB LADDER



3 Estimation

This section brings the theory to the data. I estimate the parameters of the model by generalized method of moments (GMM) targeting a set of key moments in 2016. I target the latest available years because of data availability. Importantly, I *do not* target life cycle moments of firms and workers. Instead, I use these as a test of the quantitative importance of the forces highlighted in this paper in driving the life cycle dynamics of firms and workers. The next section subsequently considers the effect of a younger labor force on firm and worker dynamics in the estimated model.

3.1 Empirical extensions

Before bringing the theory to the data, I introduce three extensions. First, to avoid overstating the role of climbing the job ladder in driving life cycle dynamics, I incorporate general human capital. A worker with human capital h working for a firm with productivity z produces output $y = e^z h$. An unemployed worker enjoys flow value of leisure bh. Individuals enter the labor market with low human capital, h_l, and jump to high human capital as employed at Poisson rate ξ. Unemployed individuals with high human capital fall back to low human capital at the same Poisson rate, and previous entrepreneurs reenter the labor market with low human capital.[20] Second, I

[20]The last assumption avoids the need to keep track of human capital for the entrepreneur. I do not, however, believe that it is particularly crucial, for the following reason. If instead human capital reverted to, say, h_h, this would

allow the separation rate to vary with a firm's productivity, $\delta(z) = \delta_0 - \delta_1 z$. This captures in reduced form heterogeneity in separation rates across establishments, which the recent literature argues is a key feature of the data (Jarosch, 2015).[21] Third, I assume that a firm's productivity falls to zero at some Poisson rate d, which allows the model to match the weighted firm exit rate.

3.2 The data

The moments on firm dynamics are based on the Business Dynamics Statistics (BDS), which are aggregate statistics based on the Longitudinal Business Database micro data. The data are available annually from 1978 to 2016 and cover the universe of private sector, incorporated firms with at least one employee. They are also provided broken down by firm age and firm size groups. All moments are HP filtered with a smoothing parameter of 100 in order to extract the secular trend.

For worker mobility, I use micro data from the Survey of Income and Program Participation (SIPP) and merged Current Population Survey (CPS) basic monthly files. I focus on private sector workers aged 16 and older. Both data sources are monthly. The SIPP is available 1984–2013, although with several years of missing data, while the CPS is available 1978–2018. The JJ series, however, can only be constructed in the CPS since the 1994 redesign of the survey. To stick as closely as possible to the existing literature, I use data from the CPS with the exception of the JJ mobility series, which I take from the SIPP. This allows me in the next section to have a consistent series back to 1985 (reassuringly, however, the JJ series from the SIPP and the CPS are very similar).[22] I compute monthly averages of worker mobility measures during the year and HP filter the resulting annual data to obtain the secular trend.

Finally, to study entrepreneurship, I rely on two data sources. For entry by age, I use the Global Entrepreneurship Monitor (GEM) 2001–2012. The GEM was designed explicitly to capture various forms of entrepreneurship, which allows me to focus on entrepreneurship for the purpose of making money, as distinct from, for instance, people entering self-employment because they want to be their own boss. Second, to link subsequent firm performance to the founder, I use data

incentivize entry everything else equal. To match the same data targets, this would in turn imply a higher estimate of the cost of entry, but would leave the same all economic forces highlighted in this paper.

[21] An earlier version of this paper endogenized this outcome by assuming that matches were hit by match-idiosyncratic productivity shocks. As a result, workers at firms close to the separation threshold were more likely to separate to unemployment. This added one state to the problem, but neither contributed to the insights in this paper nor changed its conclusions. Hence, I proceed under this reduced-form approach.

[22] An important redesign of the SIPP in 1996 introduced a break in the series for job-to-job mobility. I splice the SIPP in the break by projecting it on a pre-1996 dummy and the same series in the CPS (which is available without break from 1994) and using the estimated coefficient on the dummy to adjust the pre-1996 SIPP series for the 1996 break. I use the same linear projection on the corresponding CPS series to extend the JJ series in the SIPP through 2016.

from the Kauffman Firm Survey (KFS) 2004–2011. The KFS follows a cohort of firms started in 2004 for up to seven years after entry, recording annually measures of firm performance.

3.3 Externally set parameters

In order to reduce the dimensionality of the parameter search, I start by externally calibrating eight parameters. The frequency of the model is monthly.[23] I set the discount rate to the equivalent of a five percent annual real interest rate and the growth rate to the equivalent of two percent annually. The latter only matters for the effective discount rate on the BGP. The more relevant moment for the purposes of this paper is the composition of the growth rate, which I do not pre-set.

I assume that the matching function is Cobb-Douglas, $m(S, V) = \chi S^{\alpha} V^{1-\alpha}$, and that the cost of vacancies is iso-elastic, $c(v) = c_0 v^{1+\eta}/(1+\eta)$. Absent vacancy data, matching efficiency χ and the scalar c_0 are not separately identified, so I normalize $c_0 = 1$. I set both the elasticity of the matching function with respect to vacancies and workers' bargaining power to 0.5. This would be consistent with a Hosios (1990) condition, although there is no expectation that such a condition in general holds here. The birth rate κ is set to match the share of the labor force that is 45 years and older in 2016 and the amount of land λ to match the average firm size in 2016. Finally, I normalize low human capital $h_l = 1$. Table 1 summarizes the exogenously calibrated parameter values.

TABLE 1. EXTERNALLY CALIBRATED PARAMETERS

	Parameter	Value	Target
ρ	Discount rate	0.0041	Annual real interest rate of 5%
M	Overall growth rate	0.0017	Post-WWII growth in GDP per capita of 2%
c_0	Scalar in vacancy cost	1	Normalization
α	Elasticity of matching function	0.5	Petrongolo and Pissarides (2001)
β	Bargaining power	0.5	Hosios (1990) condition
κ	Birth rate	0.0042	Share of labor force aged 45+ in 2016
λ	Mass of land	1/23	Average firm size in 2016
h_l	Low human capital	1	Normalization

Note: The frequency is monthly. Age composition of the labor force is from the CPS; average firm size is from the BDS.

3.4 Internally estimated parameters

I estimate the remaining 14 parameters by minimizing the equally weighted sum of squared percentage deviations between 18 moments in the model and the data. While the estimation is joint, some moments are particularly informative about some parameters. Heuristically, the cross-sectional standard deviation of TFP across firms and the cross-sectional standard deviation of

[23]Since the model is set in continuous time, however, I can correctly time-aggregate all moments to any desired frequency. Hence, the assumed frequency is inconsequential.

residual log wages provide information about the standard deviation of shocks, σ, since a higher σ is associated with greater dispersion in productivity across firms.[24] I follow Sampson (2016) to assume that entrants draw an initial productivity that is linked to the average of incumbent firms. It turns out to be difficult to pin down the amount of productivity dispersion among entrants given the available data, and in any case it has very little effect on any of the results in this paper. Hence, I set the innovation distribution ζ to be degenerate such that all entrants start with a productivity that is a fraction of the average productivity of incumbent firms. The average productivity of entrants is informed by the average productivity of firms age less than one year to those 10 years and older. The productivity measures are taken from Decker et al. (2017) and Foster et al. (2016) and refer to within-sector TFPR, estimated in a standard fashion. Residual wage dispersion is from the March CPS after controlling flexibly for worker observables.

The entry rate informs the arrival rate of entrepreneurship opportunities π, which also pins down the exit rate, since exit has to equal entry in a stationary equilibrium. The employment-weighted exit rate informs the death probability of firms, d. If this is larger, more large firms exit, increasing the weighted relative to the unweighted exit rate. I target for the shape of the hiring cost η the size distribution of firms, summarized by the share of firms with less than 100 employees, the share of firms with more than 500 employees, the share of employment at firms with less than 100 employees, and the share of employment at firms with more than 500 employees. All moments characterizing firm dynamics are from the BDS in 2016 (after HP-filtering).

The UE rate informs matching efficiency, χ, and the ratio of the JJ to UE rate the relative search efficiency of the employed, ϕ. The intercept in the separation rate, δ_0, is informed by the aggregate EU rate and the slope δ_1 by the relative EU rate of high-tenured workers. Since tenure is positively correlated with productivity, the difference in separation rate by tenure is informative about the extent to which δ falls in z. These moments are from the CPS (EU and UE) and SIPP (JJ and EU by tenure). Within each estimation loop, I set the flow value of leisure b such that individuals want to quit to unemployment prior to the firm shutting down. This is effectively a normalization and results are not sensitive to the exact value for b conditional on satisfying the restriction.

I target for the probability of accumulating human capital, ξ, the difference in average wages between ages 16–44 and 45+ in the 2016 cross section of the March CPS. To avoid one additional parameter, I set the level of high human capital, h_h, such that it takes an employed worker with

[24]The empirical measure of productivity is TFPR. The model-based measure is the sum of output at the firm divided by total employment. In both the data and the model, these measures are employment-unweighted.

low human capital 30 years on average to move to high human capital. This essentially amounts to choosing a grid for human capital, and any reasonably choice is inconsequential for the results of this paper. This is the only life cycle moment that I target.

This leaves one parameter to estimate, the time cost of entry, ε. I parameterize this as $\varepsilon = \psi - \epsilon$, where ψ is a common component and ϵ is an i.i.d. component drawn from a Pareto distribution with shape γ, $\epsilon \sim Pareto(\gamma)$. I normalize the common component ψ such that an unemployed individual with low human capital enters with probability one conditional on receiving an entrepreneurship opportunity. The important parameter is the shape γ, because it governs how less likely individuals are to enter entrepreneurship as they climb the job ladder. I target for this the estimate in Hacamo and Kleiner (2016) of the effect of displacement on the probability of entering entrepreneurship. In particular, I use their estimate that it raises the probability of starting an employer firm by a factor of four. This is a conservative target given that they report that it increases the probability of overall self-employment by a factor of six to seven. As in their data, the comparison group consists of workers with at least four years of tenure with their prior employer.

Given that three parameters can be normalized, 11 parameters remain to estimate. Table 2 shows the estimated parameters as well as the targeted moments in the model and the data. The fit is in general good, including the model's ability to match the firm size distribution. The main exception is the cross-sectional dispersion in productivity, which the model somewhat struggles to reconcile with observed dispersion in residual wages. One interpretation is greater measurement error in productivity in the data. As I discuss in Appendix B, these parameters appear to be well informed by the targeted moments.

I briefly comment on the estimated parameter values. Entrants are significantly less productive than incumbent firms. Nevertheless, they are more productive than the exiting incumbents they replace, as in the data (Baily et al., 1992). The selection process associated with entry and exit is a powerful source of growth, accounting for about half of US economic growth. Empirical attempts to decompose growth, however, suggest that growth is almost exclusively due to incumbent improvements, in line with the data (see Appendix B). The reason is the combination of random growth and endogenous employment adjustment and exit, which looks like incumbent innovation. Employed workers are estimated to search with a relatively high efficiency. This is because of the "slippery" nature of the job ladder, which implies that workers high up the ladder are relatively shielded from job loss. They consequently stay there for a long time, and they reject many job offers (see Jarosch (2015) for a similar estimate of the relative search efficiency from employment). As a result of the high search efficiency from employment, as well as the fact that employment builds human capital, the estimated flow value of leisure is reasonably high at 31 percent of average flow output. About 40 percent of workers have high human capital, and this is associated with 13 percent higher productivity.

TABLE 2. INTERNALLY ESTIMATED PARAMETERS

	Parameter	Value	Target	Data	Model
Panel A: Firm dynamics					
σ	St.d. of shocks	0.024	St.d. of productivity	0.420	0.267
			St.d. of residual wages	0.559	0.498
ζ	Entrants relative to incumbents	0.296	Prod. of young/mature firms	-0.353	-0.320
d	Exogenous death rate of firms	0.002	Employment-weighted exit rate	0.020	0.018
η	Curvature of hiring cost	5.389	Share of employment $n < 100$	0.339	0.273
			Share of employment $n \geq 500$	0.520	0.515
			Share of firms $n < 100$	0.979	0.973
			Share of firms $n \geq 500$	0.004	0.006
Panel B: Worker dynamics					
χ	Matching efficiency	1.151	UE rate	0.247	0.227
ϕ	Relative search efficiency	0.800	JJ/UE	0.076	0.094
δ_0	Separation rate, max $\{\delta_0 - \delta_1 z, 0\}$	0.030	EU rate	0.011	0.012
δ_1	Separation rate, max $\{\delta_0 - \delta_1 z, 0\}$	0.018	EU, high / low tenure	1.902	1.827
ξ	Rate of human capital accumulation	0.000	Wage older / young workers	0.307	0.362
h_h	High human capital	1.125	30 year average transition	Normalization	
b	Flow value of leisure	0.875	Indifference at exit threshold	Normalization	
Panel C: Entrepreneurship entry					
π	Arrival rate of opportunities	0.001	Entry rate	0.082	0.097
ψ	Cost of entering	42.690	Conditional entry rate at $h = h_0$	Normalization	
ϵ	Dispersion in i.i.d. entry cost	0.295	Effect of displacement on entry	4.00	3.81

Panel A: Productivity refers to within-sector TFPR estimated in a standard fashion from Decker et al. (2017) (overall st.d.) and Foster et al. (2016) (young/old firms). Young firms are ≤ 1 year old, old firms 10+ years old. Annual firm reallocation measures are from the BDS. *Panel B:* Monthly worker reallocation measures are from merged basic monthly CPS and the SIPP, hourly log real wages are from the March CPS. *Panel C*: Annual entry rate is from the BDS, effect of displacement on entry is from Hacamo and Kleiner (2016). All moments refer to 2016, except for productivity measures, which are for 2010 (the latest years available). All empirical moments are HP-filtered with the standard annual smoothing parameter. All moments are constructed identically in the data and model.

3.5 Validation

I now turn to a series of validation exercises structured around three key predictions of the theory: a life cycle of firms, high "movability" of young workers, and declining entrepreneurship entry as individuals age. These components form critical ingredients of the equilibrium effects of aging in the next section. I stress that none of these objects is targeted in the estimation.

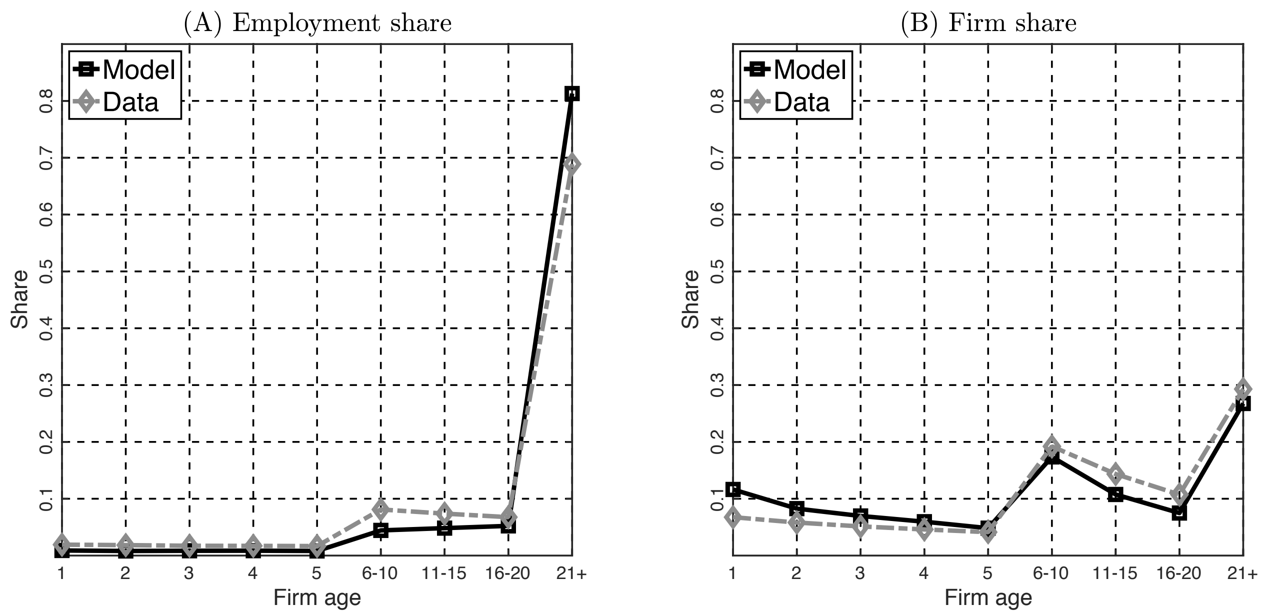
As the goal of this paper is to assess the effect of aging on reallocation rates, I start by validating that the theory matches aggregate firm dynamics in the data. Table 3 illustrates that while targeting the unweighted entry rate (i.e. also the exit rate in steady-state)[25] and the weighted exit rate,[26] the model also matches reasonably well broader measures of job reallocation, understating somewhat these moments. Moreover, the model also predicts a significant amount of *churn*—hires and separations of workers over and above that due to changes in net employment. Key is that firms simultaneously separate workers up the job ladder and hire from firms below them.[27]

TABLE 3. AGGREGATE DYNAMICS, DATA AND MODEL

	Data	Model
Annual job reallocation	0.245	0.175
Job creation incumbents	0.114	0.078
Job destruction incumbents	0.090	0.059
Job creation entry	0.020	0.010
Job destruction exit	0.020	0.028
Quarterly job reallocation	0.097	0.072
Quarterly churn	0.162	0.134

Note: Annual firm dynamics are from the BDS in 2016; quarterly job and worker flows are from the Quarterly Workforce Indicators in 2014. Churn is the difference between worker and job reallocation. All moments are constructed identically in the data and model.

FIGURE 4. EMPLOYMENT AND FIRM SHARES BY FIRM AGE



Note: Data are from the BDS in 2016. Share of total employment/firms in that firm age bin. Firms refer to firms and not establishments.

[25]While the unweighted exit and entry rates need not be equal in the data, they are both 0.08 in 2016.

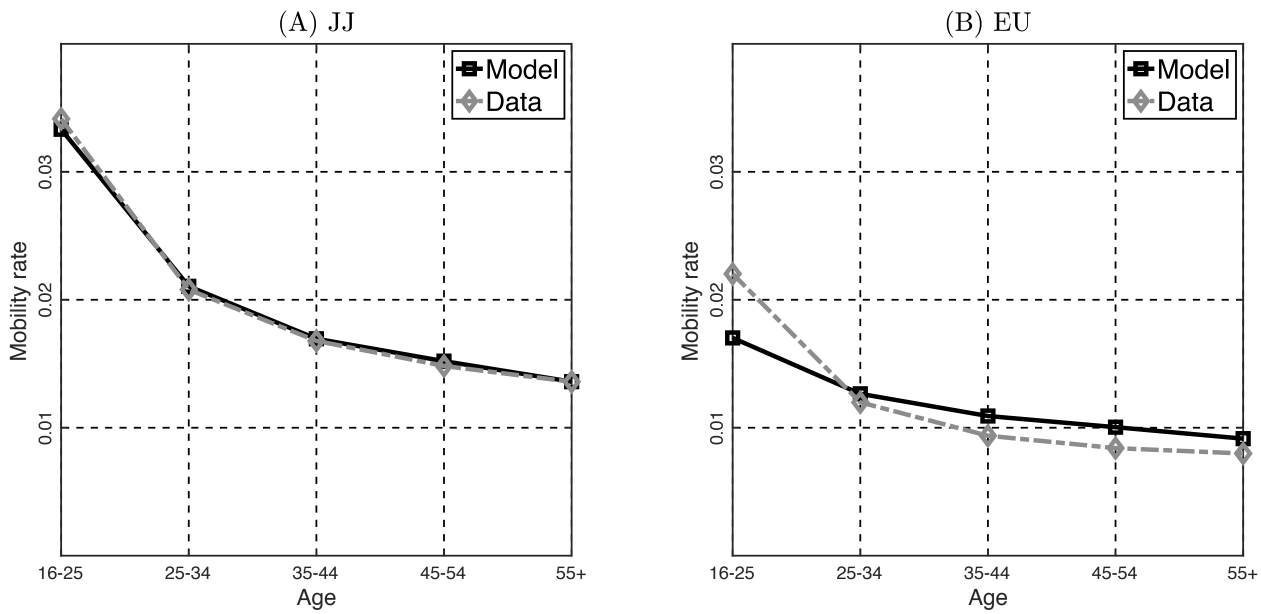
[26]The weighted exit rate in Table 3 differs from Table 2 because the former is simulated while the latter is exact (but correctly time aggregated). The simulated number is higher because of the discreteness of workers, which the exact measure ignores. Specifically, a law of large numbers does not hold at the firm level, particularly so for small firms. While ideally, I would have simulated the model in the estimation step, this is prohibitively expensive computationally. Hence, I use the exact measure in estimation and later simulate the model under the estimated parameters.

[27]Some of this is accounted for by time aggregation—some firms that contract in one month expand in the subsequent month such that in a quarter there are some additional hires and separations over and above job flows—but primarily it is due to replacement hiring within a month.

A life cycle of firms plays a key role in the theory: new firms enter, a few remain and gradually grow large, but eventually all firms exit as new more productive firms sooner or later drive them out of business. To assess the model's ability to replicate this pattern, Figure 4 plots the distribution of employment and firms by firm age. In both the model and the data, firms older than 21 years employ over 70 percent of workers. Individuals exclusively start small firms, only innovating somewhat on the least productive incumbents. Very few of these start-ups eventually become big, through a long sequence of good shocks. Nevertheless, as highlighted by Perla and Tonetti (2014), such gradual improvement on firms in the middle of the distribution offers potentially significant scope for growth because there is a large mass of such firms.

The second key element of the theory is the notion that over time, workers find their way to better firms, making them less willing to make subsequent moves. Figure 5 shows that the mechanism accounts for a significant share of life cycle worker dynamics in the data. Older individuals are less likely to make a JJ transition, as they gradually find their way to more productive employers. They are also less likely to make an EU transition, as they move away from firms that are close to the exit threshold. In contrast, the UE rate is, to a first order, flat with age in both the model and the data (see Appendix B). Appendix B also illustrates that the model matches well the tenure profiles of mobility as well as the life cycle profile of average wages (targeted) and inequality.

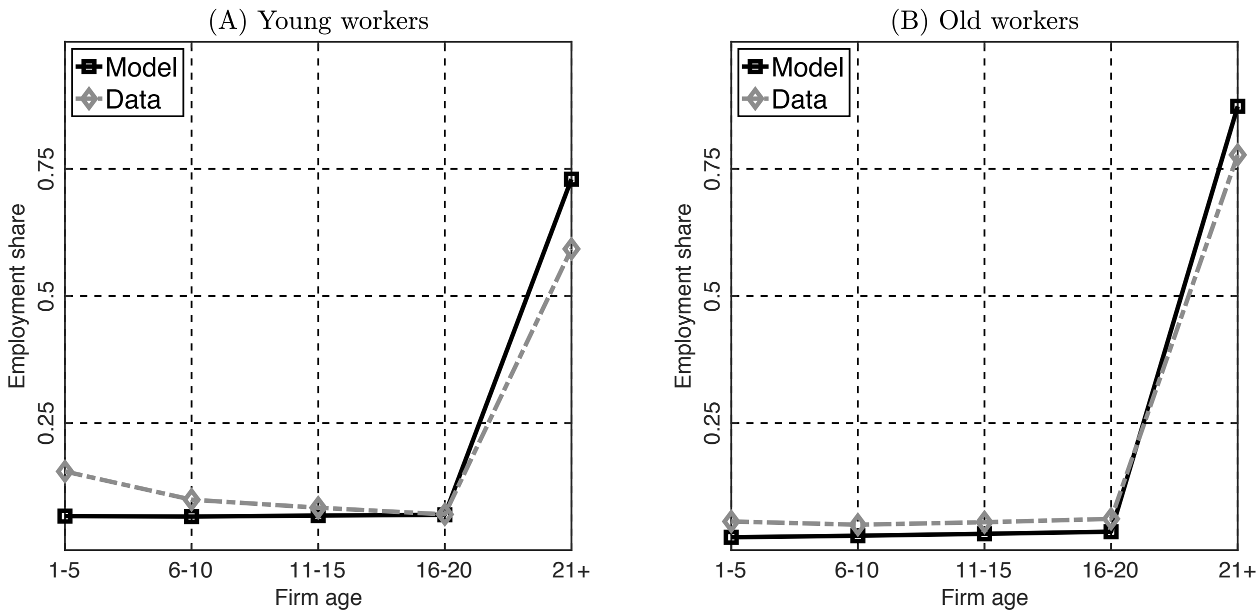
FIGURE 5. LIFE CYCLE WORKER MOBILITY



Note: Data from merged basic CPS in 2016. Monthly transition rate, constructed identically in the model and data. Model-based moments are rescaled to match the empirical age unconditional mean to enable a comparison of the life cycle profiles (see Table 2 for the levels).

The model highlights that an older pool of potential hires may reduce incentives to create jobs and enter by driving up the cost of hiring workers. This mechanism is motivated by the empirical observation that young workers constitute a disproportionate share of hires, particularly at young firms (Ouimet and Zarutskie, 2014). To assess the extent to which the theory matches this observation, Figure 6 plots the distribution of young individuals (age 16–34) and old individuals (age 55+) over firms by the age of the firm.[28] Young firms are estimated to enter with low productivity and are hence at the bottom of the job ladder. Young individuals are at the bottom of the ladder since they have not yet had time to find a good job. The model matches these patterns pretty well.

FIGURE 6. DISTRIBUTION OF YOUNG AND OLD WORKERS OVER FIRMS BY FIRM AGE



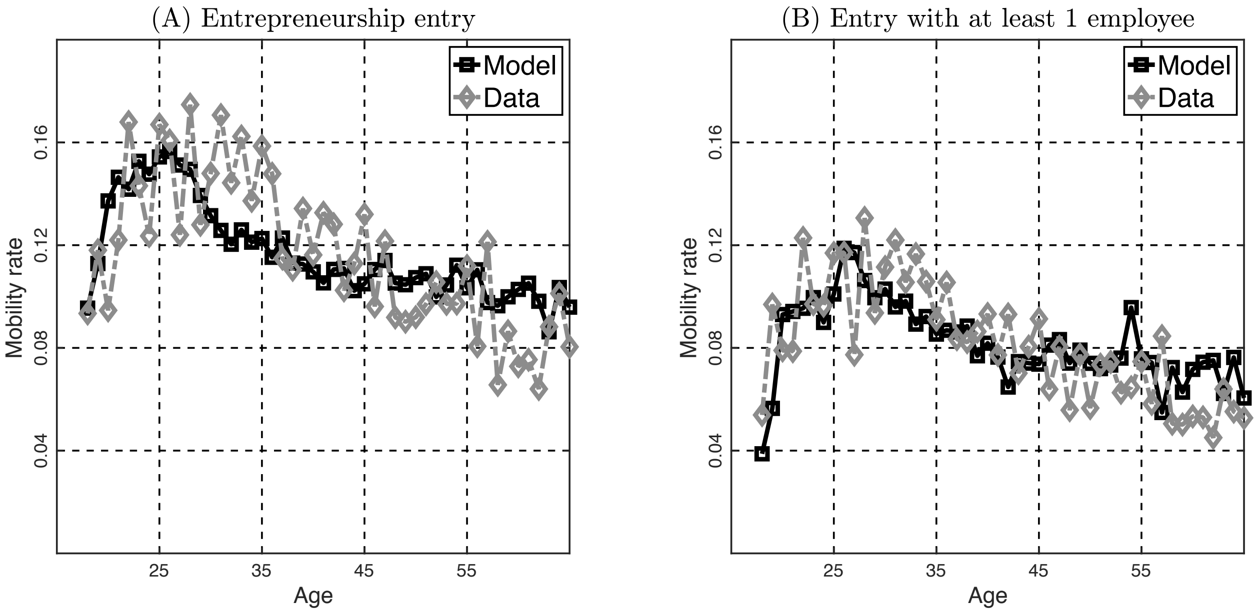
Note: Employment shares by firm age by worker age groups from Ouimet and Zarutskie (2014). Their reported worker age distributions conditional on firm age are converted to firm age distributions conditional on worker age using employment shares by firm age from the BDS in 2016. "Young" refers to the unweighted average of workers aged 16–24 and 25–34, "old" to workers aged 55+. All moments are constructed identically in the model and data.

The third key prediction of the theory is that entrepreneurship entry also declines as individuals age since they find better wage employment. Figure 7 plots the fraction of individuals who are currently running a start-up business that has been active for at most 3.5 years. The right panel additionally conditions on employing at least one worker. None of these is targeted in the estimation. The mechanism accounts for a significant share of the life cycle pattern of entrepreneurship entry in the data. Entry increases initially, for a mechanical reason. Having not been in the labor force for long, young people have not had much time to accumulate entrepreneurship opportuni-

[28]The other age groups are convex combinations of these profiles and not shown to preserve space.

ties. Hence, the measured entry rate initially increases. It subsequently declines somewhat too fast at first and somewhat too slow later in careers in the model relative to the data. Nevertheless, the mechanism accounts for a significant share of the life cycle dynamics of entrepreneurship entry.

FIGURE 7. ENTREPRENEURSHIP ENTRY BY AGE



Note: Data are from the GEM 2001–2010. Entrepreneurship entry is the fraction of the population of that age group that is involved in a new firm that has been active at most 3.5 years. Panel B additionally conditions on employing at least one worker.

Anticipating the analysis in the next section of the effects of labor force aging on the economy, a relevant question is to what extent the age of the founder influences firm performance post entry. Appendix B finds no systematic variation in the data in post-entry performance by age of the founder in terms of survival rates, the probability of hiring a worker, average size conditional on hiring a worker, and job creation and destruction measures.[29] The model matches these findings.

4 Quantifying the Effects of Aging

This section quantifies the effects of labor force aging on the labor market. Specifically, I consider the following hypothetical experiment. Holding all parameters fixed at their estimated values, what is the effect of changing the rate at which individuals enter and exit the labor market, κ, so as to achieve a change in the share of labor force participants who are 45 years and older that mimics the change in the US between the 1980s and now? I think of these shifts in κ as representing

[29] In unreported results, I also find no systematic difference in revenues post entry.

the effect of past changes in fertility, motivated by the empirical observation that changes in the retirement age, labor force participation by age, and average age of labor market entry were of second order in terms of driving the shifts in the age composition of the labor force over this period. By holding the size of the labor force fixed, this experiment isolates the importance of shifts in the age composition—a measure of the *quality* of the labor force—as distinct from the *quantity* of labor supplied. I purposefully pursue this approach since the effect of labor supply on the growth rate has been studied before, whereas the effect of the age composition is novel.[30]

4.1 The effects of aging

Aging leads to substantial declines in firm dynamics, as highlighted by Table 4. Aging accounts for 37 percent of the fall in firm creation over this period. A consequence of less entry is a decline in the rate of technological obsolescence. By reducing the need to reallocate labor, this in turn also lowers incumbent dynamics and exit. Overall, aging accounts for 64 percent of the decline in job reallocation between 1986 and 2016. While aging gives rise to a shift in employment towards older firms that is well in line with the US time trend, the aging of firms only accounts for about a third of the decline in incumbent job reallocation.[31] Instead, in both the model and the data, most of the decline in incumbent job reallocation takes place conditional on firm age.

The largest effect of aging is on the rate at which workers move across employers. The JJ rate declines by 32 percent, accounting for 81 percent of the empirical decline over this period. The EU rate falls by 33 percent in both the model and the data. These declines are significantly larger than what can be accounted for, in a pure accounting sense, by the declines in job reallocation. About half of them are due to a reduction in worker churning—worker reallocation over and above job reallocation. A similar pattern is evident in the data (Davis and Haltiwanger, 2014).

In contrast to the large decline in the JJ rate, the fall in the UE rate is less pronounced. In fact, the predicted fall in the UE rate due to aging is larger than the raw decline in the data over this period. The more interesting aspect, however, is the differential behavior of the JJ rate versus the UE rate. If the decline in the JJ rate was driven only by firms posting fewer vacancies, one may

[30] As noted earlier, a change in κ affects the age composition but also the amount of time an individual expects to stay in the market. In my baseline experiment, I hold fixed individuals' expectations for how long they expect to remain in the market, so that they end up exiting at a slightly higher rate than they expected. Also letting expectations adjust makes little difference to the results. This is presumably because the rate at which dynasties are reincarnated, κ, is orders of magnitude lower than worker flows, and firm ownership is inherited.

[31] Because of data limitations, this is relative to 1988 and firms age 11 and older are grouped into one bin. Model moments are constructed equivalently.

have expected a proportional decline in the UE rate. Part of the larger decline in the JJ rate is due to the fact that older individuals have a lower JJ rate relative to UE rate, since they are higher up the job ladder and hence less likely to accept an outside offer. But the pattern is also evident conditional on age, for the following reason. The lower firm creation rate in the older economy implies that there are fewer new, better job opportunities for employed individuals at any point in time. Unemployed individuals, on the other hand, are less affected by this decline since they continue to accept jobs at old, incumbent firms.

TABLE 4. EFFECT OF AGING

	1986		2016		Change		Model/
	Data	Model	Data	Model	Data	Model	data
Panel A: Firm dynamics							
Annual job reallocation	0.345	0.238	0.245	0.175	-0.100	-0.063	64%
Job creation incumbents	0.151	0.103	0.114	0.078	-0.038	-0.026	68%
Job destruction incumbents	0.128	0.085	0.090	0.059	-0.038	-0.026	68%
Job creation entry	0.037	0.016	0.020	0.010	-0.017	-0.006	37%
Job destruction exit	0.028	0.034	0.020	0.028	-0.008	-0.006	72%
Panel B: Worker dynamics							
JJ mobility	0.031	0.031	0.018	0.021	-0.012	-0.010	81%
EU mobility	0.016	0.018	0.011	0.012	-0.005	-0.006	110%
UE mobility	0.260	0.262	0.242	0.227	-0.018	-0.035	194%
Panel C: Other outcomes							
Annual growth rate in labor productivity		0.020		0.017		-0.003	
St.d. of within-sector TFP	0.350	0.232	0.420	0.267	0.070	0.035	50%
St.d. of residual log hourly wage	0.439	0.399	0.559	0.498	0.121	0.099	82%
St.d. of annual income innovations	0.55	0.478	0.50	0.461	-0.05	-0.016	32%
Labor share	0.650	0.849	0.600	0.824	-0.050	-0.026	52%

Note: All moments are constructed identically in the data and model. *Panel A*: Annual employment weighted firm reallocation measures are from the BDS. *Panel B*: Monthly worker reallocation measures are from merged basic monthly CPS (EU and UE) and SIPP (JJ). JJ series from the SIPP is spliced in the 1996 break and extended to 2017 based on a linear projection on the corresponding series in the CPS. All moments are constructed identically in the data and model. *Panel C*: Productivity is from Decker et al. (2017), wages are from the March CPS, dispersion in annual income innovations is from Guvenen et al. (2014), the labor share is from the OECD (in 2011 because of data availability).

The growth rate declines by 0.30 percentage points annually, as the fall in entry slows the process of creative destruction. Technological obsolescence prevents some firms from becoming very productive, in a relative sense. Consequently, its decline leads to an increase in productivity dispersion across firms. Aging accounts for half of the empirical increase in the dispersion in productivity across firms over this period. The increased productivity dispersion, together with the decline in labor market dynamism, results in increased worker inequality, matching 82 percent of the increase in residual wage dispersion over this period. While some of this increase is because of the shift in composition toward older workers, who endogenously display greater dispersion in wages (see Appendix 3), a majority of it takes place conditional on age. On the other hand, income volatility is lower in the less dynamic, older economy, accounting for a third of the empirical decline since 1986 (note also that the volatility of income is not targeted, yet the model matches it very well). Finally, the labor share declines as employment gravitates up the ladder. Firms higher up the ladder are able to pay workers a lower fraction of output because they face less competition from other firms for workers.

4.2 Understanding the effects of aging

The effects of aging can be decomposed into composition and equilibrium effects. To do so, I consider the following counterfactual exercises, focusing on the entry and JJ rates.[32]

Composition and equilibrium effects. I define the composition effect of aging as that resulting from changing the rate at which individuals enter and exit the labor market, κ, holding decision rules and the aggregate entry rate, x, fixed at their initial levels. Denote by G_c the composition-adjusted employment distribution, by u_c and e_c the composition-adjusted unemployment and employment rate, respectively, and by p_c and q_c the composition-adjusted finding rates. Finally, denote by $\bar{\varepsilon}_1$, $v_1(z)$, etc., the optimal policies under the young age distribution. The composition-adjusted entry and JJ mobility rates are[33]

$$x_c = \frac{\pi}{\lambda}\left(\frac{u_c}{S_c}\bar{\varepsilon}_1(0) + \frac{e_c}{S_c}\int_{\underline{z}}^{\infty}\Gamma\Big(\bar{\varepsilon}_1(z)\Big)dG_c\Big(z\Big)\right), \qquad JJ_c = \phi p_c\int_{\underline{z}}^{\infty}\Big(1-F_1(z)\Big)dG_c\Big(z\Big)$$

where $F_1(z) = \frac{\lambda}{V_1}\int_{\underline{z}}^{z} v_1(\tilde{z})dH_1(\tilde{z})$ and $V_1 = \lambda\int_{\underline{z}}^{\infty} v_1(z)dH_1(z)$. This is a partial equilibrium exercise in the sense that individuals' entry decisions do not aggregate to the aggregate entry rate in the economy. This disjoint holds the growth rate fixed.

In equilibrium, the aging of the pool of potential hires reduces incentives for potential entrepreneurs to enter conditional on their rung in the job ladder. To quantify this, I first resolve the

[32]For this measure, I use the monthly employment-unweighted entry rate. Table 4 reports its annual employment-weighted counterpart, time-aggregated following the US Census Bureau's procedure for constructing the comparable empirical moments. This accounts for the difference between Table 4 and 5. The employment-weighted entry rate has also declined by more in the data.

[33]In order to highlight the intuition behind these counterfactuals, I abuse notation and do not include human capital as a state. I also account for human capital in the implementation of these counterfactuals.

problem of an incumbent firm under the composition-adjusted distribution of potential hires,

$$
\begin{aligned}
r\, Q_w\Big(z\Big) \;&=\; \max_{v\geq 0}\; \Big\{ \,-\, c(\,v\,) \,+\, v\, q_c \\
&\times\;\; \Big(1-\beta\Big)\left(\frac{u_c}{S_c}\,\Big(\,J_1(\,z\,)-U_1\,\Big)^{+} \,+\, \frac{\phi\, e_c}{S_c}\int_{\underline{z}}^{\infty}\Big(\,J_1(\,z\,)-J_1(\,z'\,)\,\Big)^{+}\, dG_c\Big(\,z'\,\Big)\right)\Big\} \\
&+\;\; (-m_1)\, Q_w'\Big(\,z\,\Big) \,+\, \frac{\sigma^2}{2}\, Q_w''\Big(\,z\,\Big)
\end{aligned}
$$

subject to $Q_w\Big(\underline{z}\Big) = U_1 + L_w$ and $Q_w'\Big(\underline{z}\Big) = 0$ where $L_w = Q_w(0) - U_1$. This again holds the growth rate fixed at its initial level. I subsequently define the effect of an aging pool of potential hires on firm dynamics as the resulting change in the aggregate entry rate, computed by integrating the new entry policy against the composition-adjusted distribution of potential entrepreneurs, G_c,

$$
x_w \;=\; \frac{\pi}{\lambda}\left(\frac{u_c}{S_c}\,\bar{\varepsilon}_w(0) \,+\, \frac{e_c}{S_c}\int_{\underline{z}}^{\infty}\Gamma\Big(\,\bar{\varepsilon}_w(z)\,\Big)dG_c\Big(\,z\,\Big)\right)
$$

The aging of the pool of potential entrepreneurs lowers firm creation, resulting in a lower arrival rate of new job opportunities for workers. To assess its importance, I first compute the growth rate m_c and distribution of firms h_c associated with the composition-adjusted aggregate entry rate x_c induced by the aging of potential entrepreneurs. Based on these objects, I resolve the worker's problem (2) and (5)–(6). The effect of an aging pool of potential entrepreneurs on worker dynamics is the JJ rate associated with this updated optimal behavior on the part of workers.

Table 5 summarizes the results from these counterfactuals. The composition effect accounts for 45 percent of the total decline in the entry rate and 38 percent of the decline in the JJ rate. Hence, the significant aging over this period, combined with large life cycle differences in behavior, implies non-trivial composition effects.[34] An aging pool of potential hires reduces the value to potential entrepreneurs of entering by driving up the cost of hiring labor. This exacerbates the decline in entry, causing a further 15 percent decline. Moreover, the aging of potential entrepreneurs results in a lower entry rate, reducing the arrival rate of new, better job opportunities for workers. This contributes an additional 10 percent decline in JJ mobility.

The equilibrium interaction between the above effects partly counters the decline in entry,

[34] As the model matches a large share of the life cycle declines in entry and JJ mobility, the composition effect in the model lines up well with that in the data.

through two channels. First, the lower entry rate effectively reduces the cost of entry relative to the productivity of entrants. Specifically, the distribution of firm productivity fans out in response to the lower rate of obsolescence. Since entrants innovate relative to the mean of this distribution, while costs are linked to an outside value represented by the least productive firm, potential entrepreneurs are incentivized to enter. Second, firms last longer, which also incentivizes entry. Interaction effects lead to a modest further decline in JJ mobility. The reason is that in equilibrium, the entry rate falls by more than the composition effect. This in turn implies even fewer new job opportunities for workers relative to the effect of an aging pool of potential entrepreneurs.

TABLE 5. DECOMPOSING THE EFFECT OF AGING

	Entry		JJ	
	Marginal	% of total	Marginal	% of total
Composition effects	-9.2%	45.0%	-11.9%	37.5%
Effect of older pool of potential hires	-15.3%	74.9%		
Effect of older pool of potential entrepreneurs			-9.7%	30.6%
Further equilibrium interaction	4.1%	-19.9%	-10.1%	31.9%
Total decline	-20.5%	100%	-31.7%	100%

Note: Composition effect: Effect of changing κ holding all decisions and the aggregate entry rate fixed. Effect of older pool of potential hires: Effect of changing age composition of potential hires holding aggregate entry rate fixed. Effect of older pool of potential entrepreneurs: Effect of changing aggregate entry rate based on the composition effect on worker mobility.

What moments inform the effects. To highlight what in the data led me to infer large effects of aging, I consider the following counterfactual experiment. Suppose that one data moment differed from its actual value in 2016, while all the other moments remained the same. I reestimate the model to target this new set of moments and compute the effect of aging. This exercise suggests that the EU rate is the most important worker dynamics moment governing the effects of aging. In particular, the larger is the EU rate, the smaller is the predicted effect of aging. The reason is that a lower idiosyncratic job destruction rate leaves more scope for the endogenous rate of obsolescence in determining where individuals are on the job ladder, amplifying the equilibrium mechanism highlighted in this paper. On the firm side, the most important moment is the size distribution of firms. In particular, the larger is the share of employment at big firms, the greater is the effect of aging. The reason is that this implies a more elastic vacancy cost function. I discuss these and related robustness results in greater detail in Appendix C.

Appendix C also highlights two additional important takeaways. On the one hand, the estimated effect of aging is not hardwired into the structure of the model, in the sense that there are empirical values for the targeted moments that would have led me to conclude that the effect of aging is significantly larger or smaller. On the other hand, given the targeted moments, my point estimates of the effect of aging appears to be precise, in the sense that the gradient of the objective function around the point estimates is steep. In other words, while the model could generate very different effects of aging, it would come at the cost of substantially missing the targeted moments.

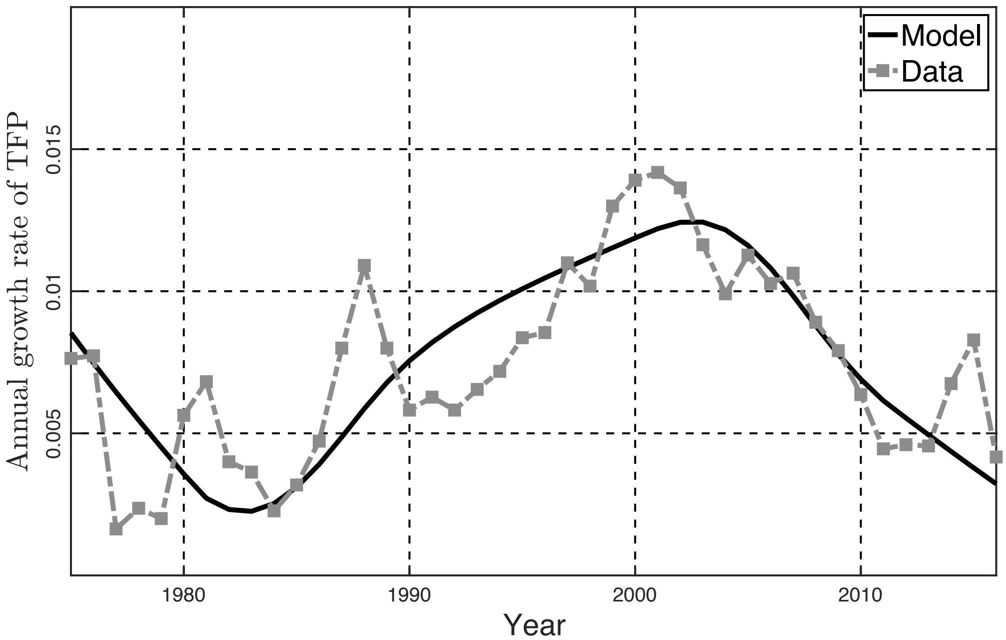
4.3 Transition effects

Aging has both a level and a growth effect on output, and they generally go in opposite directions. To assess their relative importance, I consider a transition experiment. As in the comparative static exercise above, I hold all parameters fixed at their estimated values except the rate at which individuals exit and enter the labor market, κ, which I now assume is time varying. Starting from the young steady state in 1975, I assume that individuals suddenly realize that the share of the labor force that is 45 years and older will evolve as it has done since then and as it is projected to evolve until 2060. After that, it converges to its older steady-state. Solving for the dynamic economy involves keeping track of the evolution of the distribution of employment and firms, which are both high-dimensional objects. As this is a computationally infeasible problem, I adopt the approximation proposed by Jones and Kim (2018). Specifically, I assume that the entry policy and workers' separation threshold jump to their new steady-state values, while the other policies and distributions evolve dynamically.

The predicted evolution of the growth rate is non-monotone, in contrast to the gradual declines in dynamism throughout the past 30 years. The influx of a large number of young, poorly matched workers in the late 1970s and early 1980s has a negative level effect on output, which is amplified by an endogenous shift in workers down the job ladder as the rate of obsolescence is higher. The level effect is in fact sufficiently strong that measured growth falls. As the labor force ages, it shifts up the job ladder and builds human capital such that the measured growth rate booms in the 1990s, even though dynamism continues to decline and the growth effect continues to fall. Again, the compositional effect of aging on the level of output is amplified by the endogenous movement of workers up the job ladder conditional on age in response to the decline in technological obsolescence. Since the turn of the century, the level effect teeters off and the measured growth rate enters a prolonged, gradual decline. Figure 8 illustrates. This, of course, is only an approximation to the true transition path, so these findings should be interpreted cautiously.[35]

[35] This transition experiment also reveals that most of the changes in firm and worker dynamics take place within the 35-year period I consider. This is in line with the typical argument in the search literature that convergence is fast, at least relative to slow-moving secular trends in demographics, and is not surprising given the life cycle profiles in

FIGURE 8. GROWTH RATE, 1975–2016



Note: Labor-augmenting TFP growth is from the BLS. Model is the annual growth rate of total output divided by total employment. Both data and model are smoothed using an 11-year rolling average.

5 Additional Evidence on the Effect of Aging

This section offers additional reduced-form evidence that aging is associated with lower firm and worker dynamics. I construct an annual data set at the level of a US state from 1978 to 2016 on firm dynamics, worker mobility, the age composition of the labor force, and lagged birth rates. The respective data sources are the BDS, the merged basic monthly CPS, the US Census Bureau's Vital Statistics of the United States (VSUS), and Intercensal censi from the CDC. I regress measures of firm or worker dynamics in state s in year t, $y_{s,t}$, on the share of the labor force that is 45+ years old, $older_{s,t}$, state fixed effects, ξ_s, and year effects, ξ_t,[36]

$$\log y_{s,t} \quad = \quad \log\left(older_{s,t} \right) \; + \; \xi_s \; + \; \xi_t \; + \; \varepsilon_{s,t} \tag{18}$$

Figure 5, which suggest that over a period of 35 years, worker dynamics have mostly settled down. The exception is dispersion in productivity, and to a lesser extent wages, which are slower to adjust.

[36] I have considered a range of robustness specifications, including having both the independent and dependent variables in levels, using the age composition of the workforce age 16–65, and adding controls for GDP per capita, the share female, the share black, the share with a college degree, the share in nine aggregate industries, the bindingness of the minimum wage, and the effective tax rate at the state level. The latter two attempt to control for policy endogeneity in response to aging that may affect dynamism. None of these specifications leads to substantially changed conclusions.

I adjust standard errors to account for autocorrelated errors within a state up to order n depending on the autocorrelation structure of the errors.

Identification of (18) exploits the fact that although all states saw increases in the share of older people since the 1980s, the magnitude and timing of these changes differ importantly across states. An important assumption, however, is that such shifts in the age composition are exogenous to dynamism. This would be violated if workers move across states in response to variation in dynamism.[37] To address such concerns, I instrument for the share of older workers in a state with the sum of births in the state 16–44 years ago and the sum of births 45–65 years ago. The exclusion restriction is that fertility 16+ years ago is not related to current dynamism through channels other than the current age distribution. In line with Shimer (2001)'s original findings, these instruments predict well the current age composition.

Table 6 presents the OLS and IV estimates based on equation (18). Across the board, aging is negatively correlated with firm and worker dynamics, although the point estimates in the worker dynamics regressions are only borderline statistically significant. This may partly reflect measurement error given that the measures are computed at the state-year level based on relatively small survey data. Typically, the IV regressions show a larger point estimate, which, as argued in Shimer (2001), could happen if the age composition is measured with error. One should be very careful with applying cross-sectional estimates to understand national time trends. Nevertheless, it is useful to give a sense of the economic magnitude of the cross-sectional correlation. If the cross-state relationship is informative of the national time trend over this period, the OLS estimate suggests that aging over this period has reduced job reallocation by 18 percent. The US saw a 27 percent decline in this measure between 1986 and 2016. Hence, if these estimates reflect causality, they indicate large effects of aging, of a magnitude similar to that predicted by the theory.

Table 7 additionally includes the growth rate of the labor force in that state and year, constructed based on the CPS. Since it is difficult to simultaneously instrument for the age composition and labor supply growth, I only present OLS specifications. As noted by Karahan et al. (2016), labor supply growth is negatively correlated with the entry rate. Hence, to the extent that these correlations reflect a causal relationship, the decline in labor supply growth may have contributed

[37]As noted by Shimer (2001), the worry is not as simple as, for instance, older people always moving to Florida, since that would be accounted for by the state effects. The concern is if one particular age group disproportionately moves in response to *temporary* variation in dynamism, such as if, for instance, a boom in firm entry in Florida induces disproportionately many young people to move into Florida in the years of the boom. A similar concern arises if, say, older people disproportionately drop out of the labor force in response to a decline in firm entry. I find similar results using the age composition of the working-age population, however, suggesting that this concern is second-order.

to reduced firm creation over the past 30 years. Nevertheless, it only marginally affects the point estimate on the share of older labor force participants, which remains negative, statistically significant, and economically large. Notice also that by the same logic, the decline in labor supply growth would predict *increases* in the exit rate and the EU rate over this period, whereas aging predicts declines across the board in firm and worker dynamics.

TABLE 6. AGING AND DYNAMISM ACROSS US STATES, 1978–2016

Panel A: Firm reallocation

	Job reallocation		Entry		Exit		Incumbent reallocation	
	OLS	IV	OLS	IV	OLS	IV	OLS	IV
Older	-0.384	-1.174	-0.635	-2.668	-0.608	-2.597	-0.315	-0.610
	(0.045)	(0.172)	(0.097)	(0.403)	(0.102)	(0.468)	(0.044)	(0.146)
N	1,950	1,950	1,850	1,850	1,911	1,911	1,811	1,811
R2	0.897	0.851	0.883	0.799	0.757	0.610	0.862	0.853
R2 (within)	0.095		0.066		0.053		0.064	
F statistic		34.007		24.405		23.535		31.034

Panel B: Worker reallocation

	JJ		EU		UE	
	OLS	IV	OLS	IV	OLS	IV
Older	-0.242		-0.174	-0.271	-0.115	-0.537
	(0.127)		(0.092)	(0.336)	(0.079)	(0.319)
N	1,150		1,950	1,950	1,950	1,950
R2	0.808		0.711	0.711	0.719	0.713
R2 (within)	0.004		0.003		0.002	
F statistic			34.007		34.007	

Note: Annual firm reallocation measures are from the BDS. Age composition of the labor force and average monthly worker reallocation rates are from the CPS. Lagged fertility is from the VSUS and the CDC. An observation is a state-year. All regressions control for state and year effects. 50 US states from 1978 to 2016, apart from entry and incumbent reallocation (to 2015), and JJ mobility (from 1994). The independent variable is the log of the share of the labor force aged 16 years and older that is 45 years and older. The instruments are the sum of births in the state 16–44 years earlier and 45–65 years earlier. Standard errors are robust to heteroscedasticity and for autocorrelation up to order 2 or 3 depending on tests of the autocorrelation of the residual. Exit and incumbent reallocation are dropped in that state-year if it is less than 20 percent or greater than 180 percent in the prior year.

TABLE 7. AGING, LABOR SUPPLY GROWTH, AND DYNAMISM ACROSS US STATES, 1978–2016

	JR	Entry	Exit	Inc.	JJ	EU	UE
Older	-0.355	-0.591	-0.604	-0.285	-0.236	-0.189	-0.085
	(0.044)	(0.100)	(0.101)	(0.042)	(0.126)	(0.094)	(0.077)
ΔLF	0.114	0.559	-0.569	0.151	0.453	-0.981	1.376
	(0.083)	(0.151)	(0.168)	(0.088)	(0.214)	(0.193)	(0.193)
N	1,900	1,800	1,862	1,762	1,150	1,900	1,900
R2	0.905	0.884	0.757	0.875	0.809	0.720	0.731
R2 (within)	0.095	0.070	0.058	0.064	0.009	0.019	0.044

Note: Annual firm reallocation measures are from the BDS. Age composition of the labor force and average monthly worker reallocation rates are from the CPS. An observation is a state-year. All regressions control for state and year effects. 50 US states from 1978 to 2016, apart from entry and incumbent reallocation (to 2015), and JJ mobility (from 1994). The independent variable is the log of the share of the labor force aged 16 years and older that is 45 years and older. Standard errors are robust to heteroscedasticity and for autocorrelation up to order 2 or 3 depending on tests of the autocorrelation of the residual. Exit and incumbent reallocation are dropped in that state-year if it is less than 20 percent or greater than 180 percent in the prior year.

6 Conclusion

What are the consequences of substantial aging of the US labor force over the past 30 years? To address this question, I develop an equilibrium theory of joint firm and worker dynamics that captures the notion that older individuals are less likely to switch employers and enter entrepreneurship because they have had more time to find a good job. I show that this mechanism accounts for a significant share of empirical life cycle dynamics. Furthermore, I highlight that firm and worker dynamics interact importantly in equilibrium to amplify the effect of aging. When potential hires age, it discourages potential entrepreneurs from entering conditional on their age by driving up the cost of hiring. When potential entrepreneurs age, the fall in firm creation implies fewer new, better jobs for workers, such that worker mobility falls conditional on a worker's age. Quantitatively, aging accounts for 64 percent of the large decline in job reallocation, and 81 percent of the decline in JJ mobility over this period.

Three avenues seem particularly fruitful for future research. First, anecdotal evidence suggests that aging has contributed to poor labor market performance in other countries, including Japan. Given rapid aging across the world, more research is needed to understand its effects on labor market performance. Second, while empirical research has made progress on linking firm and worker dynamics, theoretical models that speak to both are still in their infancy. In subsequent work, we further pursue this (Bilal et al., 2019). Third, more work is needed—both theoretical and empirical—on the link between entrepreneurship and labor market events.

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A Additional Details on Model

An equilibrium with endogenous growth must satisfy that the growth rate is given by the mass of entrants, while the mass of entrants is optimally determined given the growth rate. I refer to the first condition as the *demand for entrants*, since it characterizes how much entry is required to sustain a particular growth rate. I refer to the latter as the *supply of entrants* because it gives the amount of entry that individuals optimally want to undertake for a given growth rate of obsolescence m. An equilibrium is when the supply of entrants equals the demand for entrants.[38]

Demand for entrants. Suppose that the innovation distribution ζ is exponential with rate parameter ζ (i.e. it is Pareto in levels with shape ζ). Then the stationary distribution of firms is

$$h(z) \quad = \quad \frac{x}{(-m) \,+\, \frac{\sigma^2}{2}\zeta}\left(e^{\frac{2(-m)}{\sigma^2}z} \,-\, e^{-\zeta z} \right) \tag{19}$$

and the rate of obsolescence is[39]

$$m \quad = \quad \frac{x}{\zeta} \tag{20}$$

More entry increases the rate of economic growth by speeding up the selection rate of firms, which implies that incumbent firms fall behind faster. A thinner tail of the entry distribution implies that entrants on average are less productive, which lowers the rate of economic growth and obsolescence for a given entry rate.

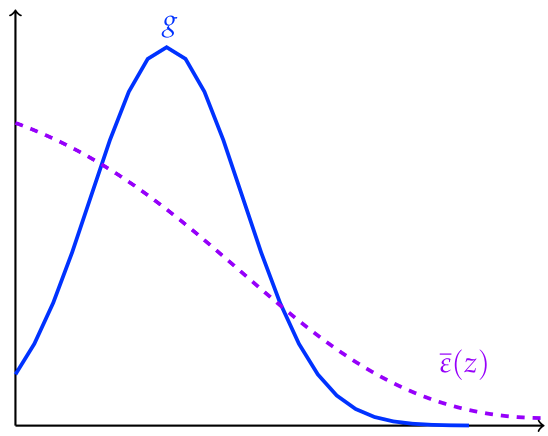
Supply of entrants. The supply of entrants (17) can be broken into two components. First, the distribution of individuals on the job ladder—u and $G(z)$—and second, individuals desire to enter conditional on a particular rung in the job ladder, $\bar{\varepsilon}(z)$. Figure 9 illustrates these two components of the entry rate. The maximum entry cost an individual is willing to pay, and hence the probability of entering entrepreneurship, declines in the productivity of the current firm since better matched

[38]There are several technical requirements for a candidate BGP equilibrium. First, as noted above the overall growth rate, $M = \mu + m$, cannot be larger than the discount rate ρ because utility would then be infinite. Second, the rate of obsolescence cannot be too low, because then a stationary firm productivity distribution would not exist. See, for instance, Luttmer (2012) for a rigorous treatment of such issues in a related environment. While a general existence proof is unavailable in the current environment, Luttmer (2012) shows in a model that shares some of the same features as the current one that a unique equilibrium exists if the elasticity of entry is sufficiently low (i.e., if in the current environment the dispersion in the idiosyncratic cost of entry, ε, is sufficiently high). There is good reason to expect that a similar argument is also applicable here (my numerical analysis in the next section appears to support this).

[39]In order for the stationary distribution to be well defined I require that $m < \frac{\sigma^2}{2}\zeta$.

individuals have to forgo a more valuable match in order to enter.

FIGURE 9. DISTRIBUTION OF INDIVIDUALS ON THE JOB LADDER AND ENTRY POLICY



To understand the slope of the supply of entrants, consider first the effect of a higher rate of obsolescence on the distribution of individuals on the job ladder, G. I refer to this as the *mismatch effect*. To focus on the interaction between the quality ladder and the job ladder, let me abstract from unemployment by setting the idiosyncratic separation rate to zero, $\delta = 0$, and assume that entrepreneurship opportunities arrive very infrequently, $\pi \to 0$. Then because there is essentially no entry, there is also no exit. Let me also assume that $\phi = 1$ and $b = 0$ so that no option value is forsaken by entering employment. Hence individuals never separate to unemployment (recall that productivity is always positive). For simplicity, also assume away idiosyncratic productivity shocks, $\sigma = 0$. Then the distribution of employment can be solved for in closed form:

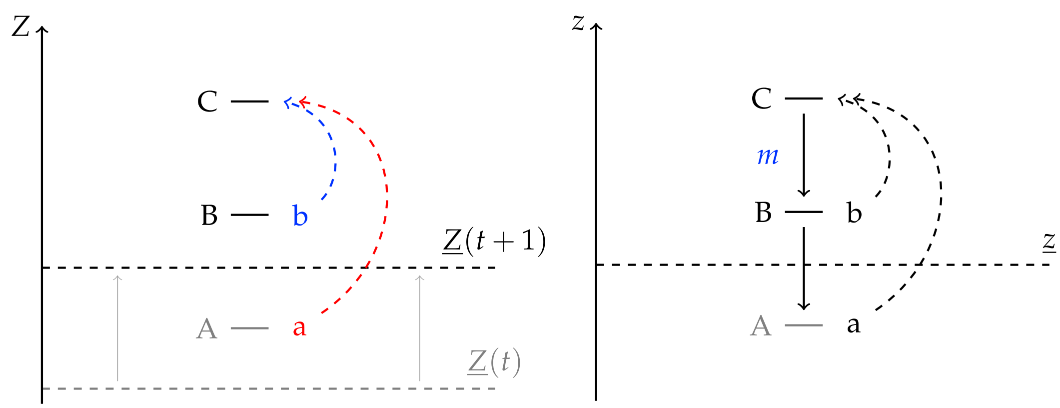
$$G\Big(z \, ; \, m \Big) \quad = \quad \exp\left(- \frac{p}{m} \int_{\underline{z}}^{\bar{z}} 1 \; - \; F(\, z \,) \, dz \right)$$

Hence, in partial equilibrium with p and F treated as parameters, a higher rate of obsolescence shifts the stationary distribution of employment down the job ladder, $\partial G(\, z \, ; \, m \,)/\partial m > 0$. To illustrate the intuition behind this, Figure 10 considers an economy consisting of workers a and b employed by firms A and B. Suppose a new more productive firm C enters, adding a new rung to the job ladder. The increased competition pushes firm A out of business, reflected in an increase in the exit threshold $\underline{z}(t)$. Worker a becomes unemployed and tries to find a new job at firms B or C. Firm B remains in business and worker b remains employed, but she is no longer at the top of the job ladder. Consequently, she tries to reallocate through on-the-job mobility to firm C. Over time, workers gradually move toward the new firm, but because of frictions, this process takes time. Eventually, the entry of another new firm pushes firm B out of business, and so on.

The right panel of Figure 10 illustrates this in the transformed BGP environment, in which firms drift toward the exit threshold at the rate of obsolescence. The job ladder is similar to a standard model (Burdett and Mortensen, 1998). But while in standard job ladder models workers try to move up a fixed rung of firms, here the underlying rung of firms gradually turns over. It is as though workers are trying to make their way up an escalator the wrong way in small stochastic steps, being gradually brought down as technology evolves. If the escalator moves slower, workers on average find themselves further toward the top of it. Everything else fixed, this reduces entry, since better-matched individuals are less likely to attempt entrepreneurship.

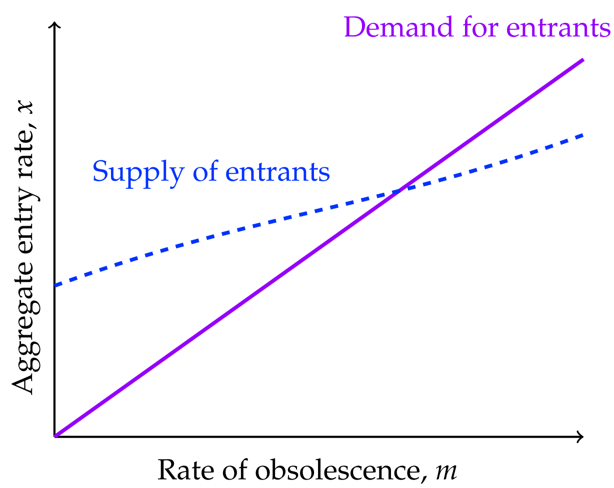
FIGURE 10. TECHNOLOGICAL INNOVATION AND THE JOB LADDER

(A) NON-TRANSFORMED ECONOMY (B) TRANSFORMED ECONOMY



A change in the rate of obsolescence also affects incentives to enter conditional on the rung in the job ladder, $\bar{\varepsilon}(z)$. I refer to this as the *incentive effect*. On the one hand, the lower rate of obsolescence leads to a fanning out of the stationary distribution of productivity, which encourages entry by increasing the average productivity of entrants (Luttmer, 2007). Furthermore, the lower rate of obsolescence implies that new firms are expected to last longer, also encouraging entry. On the other hand, the decline in the rate of obsolescence means that incumbent matches are also expected to last longer. This raises the value of an incumbent match, and since a potential entrepreneur has to sacrifice her current match in order to enter, it discourages entry. Furthermore, by making the labor market better matched, it raises the cost of hiring workers and hence discourages entry. Because of these offsetting forces, my estimates in the next section imply that the *supply of entrants* increases in the rate of obsolescence, as illustrated by Figure 11.

FIGURE 11. DEMAND FOR AND SUPPLY OF ENTRANTS

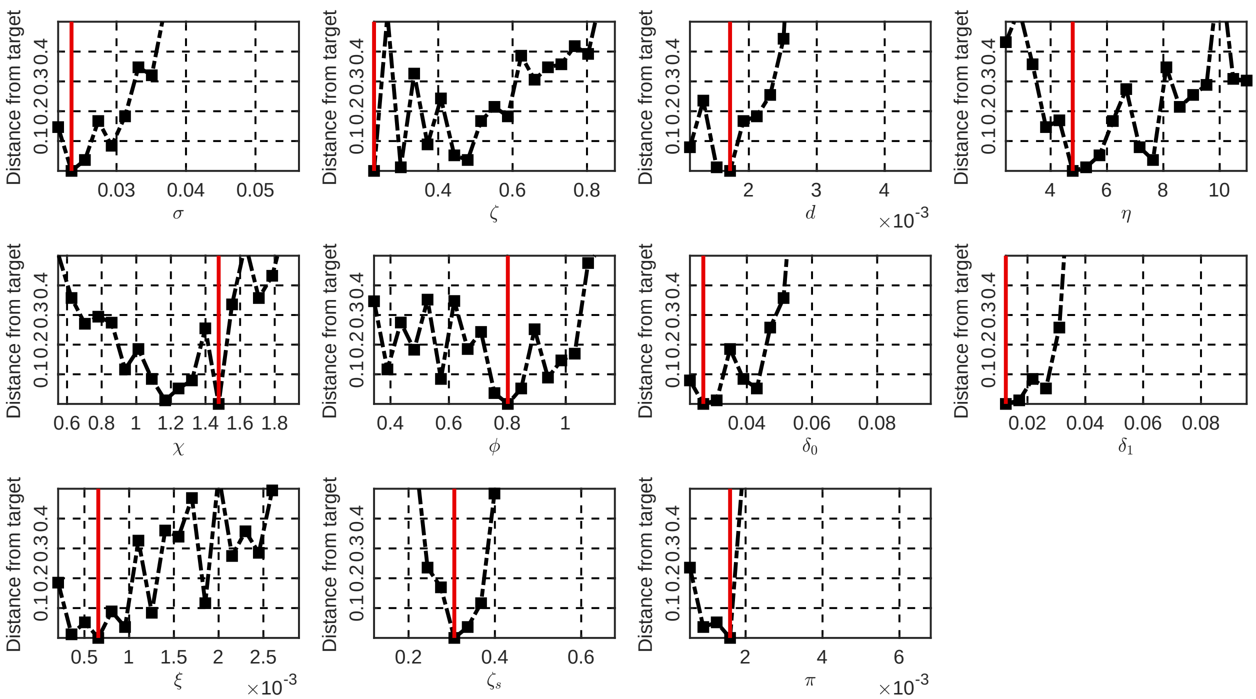


B Additional Details on Estimation

B.1 Identification

Figure 12 plots the minimum distance of the objective function as a function of each of the estimated parameters.

FIGURE 12. MINIMUM DISTANCE AS FUNCTION OF PARAMETERS



Note: Minimum distance between targeted moments in model and data as a function of each of the estimated parameters.

B.2 Additional moments

Worker dynamics. Figure 13 shows that the UE rate is to a first-order flat in both the model and the data.

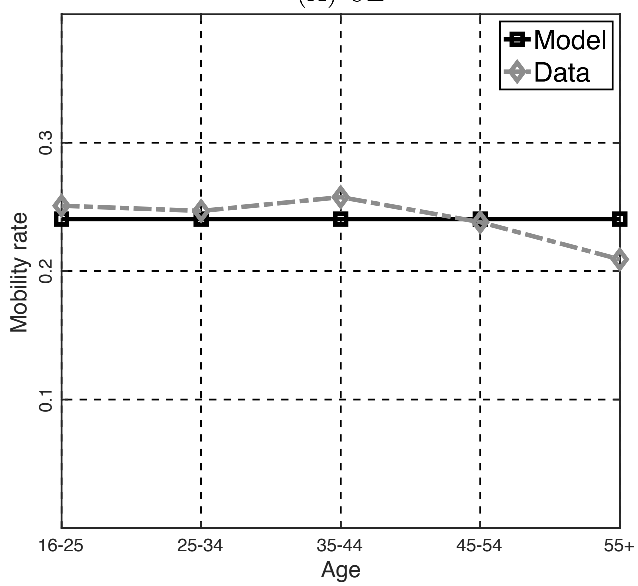
Because individuals are more likely to remain longer at better firms, the theory also makes predictions about worker mobility with tenure. Figure 14 highlights that the model matches closely the declines in JJ and EU mobility by tenure. Recall, however, that I target in the estimation the relative EU hazard at tenure 1-2 years to that at tenure 5+ years, and hence the fit of the EU hazard is partly by construction.

Figure 15 illustrates that the model matches well the first two moments of wages over the life cycle. Recall, however, that I target the difference in the average wage between age 16–44 and 45+

in the data, and hence the left panel is perhaps not surprising. The model matches patterns of residual wage inequality over the life cycle almost perfectly.

FIGURE 13. UE RATE BY AGE

(A) UE

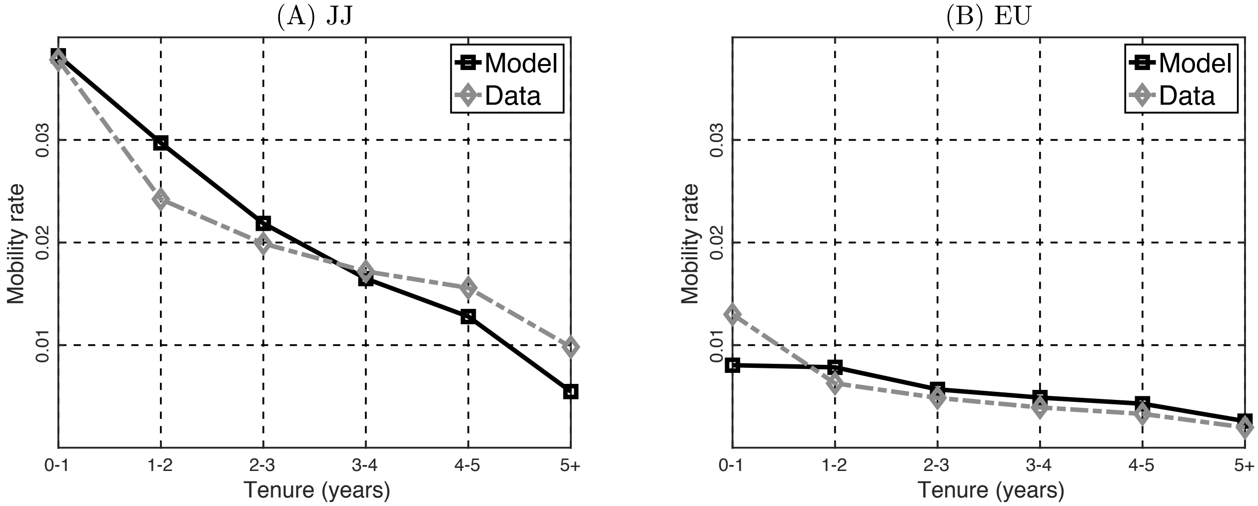


Note: Merged basic CPS in 2016 (after HP-filtering). Mobility transition rates. Model-based moments are rescaled to match the empirical unconditional mean to enable visual comparison (see Table 2 for the levels of these rates).

FIGURE 14. WORKER DYNAMICS BY TENURE

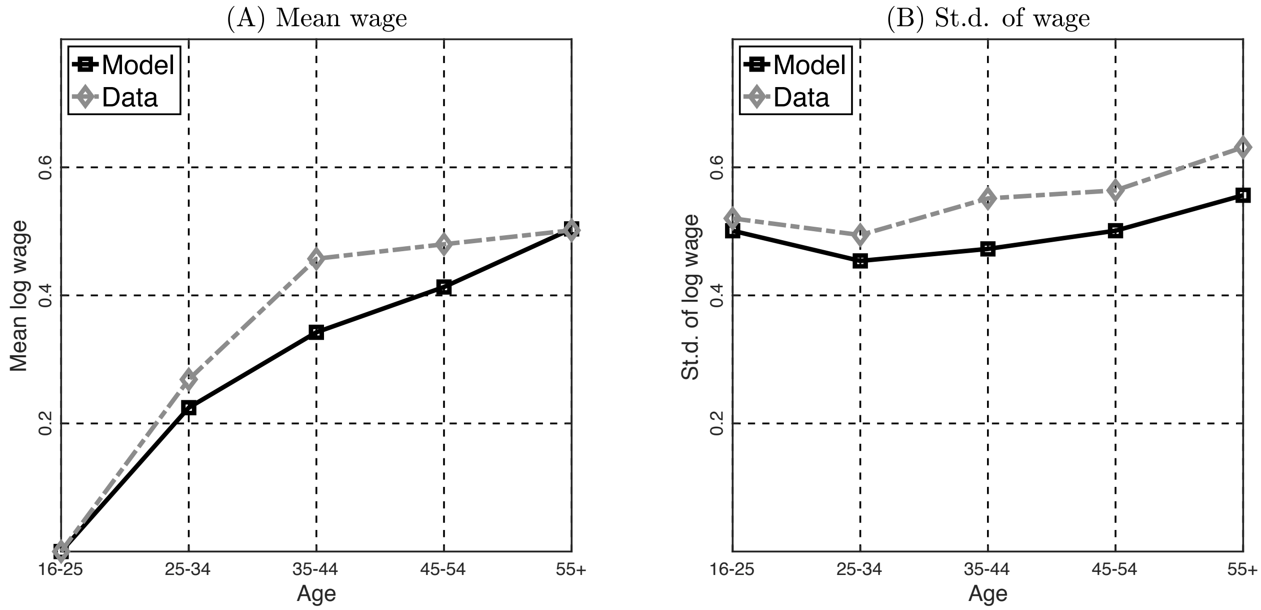
(A) JJ

(B) EU



Note: Merged basic CPS and SIPP in 2016 (after HP-filtering). Mobility transition rates. Model-based moments are rescaled to match the empirical unconditional mean to enable visual comparison (see Table 2 for the levels of these rates).

FIGURE 15. LIFE-CYCLE WAGE DYNAMICS



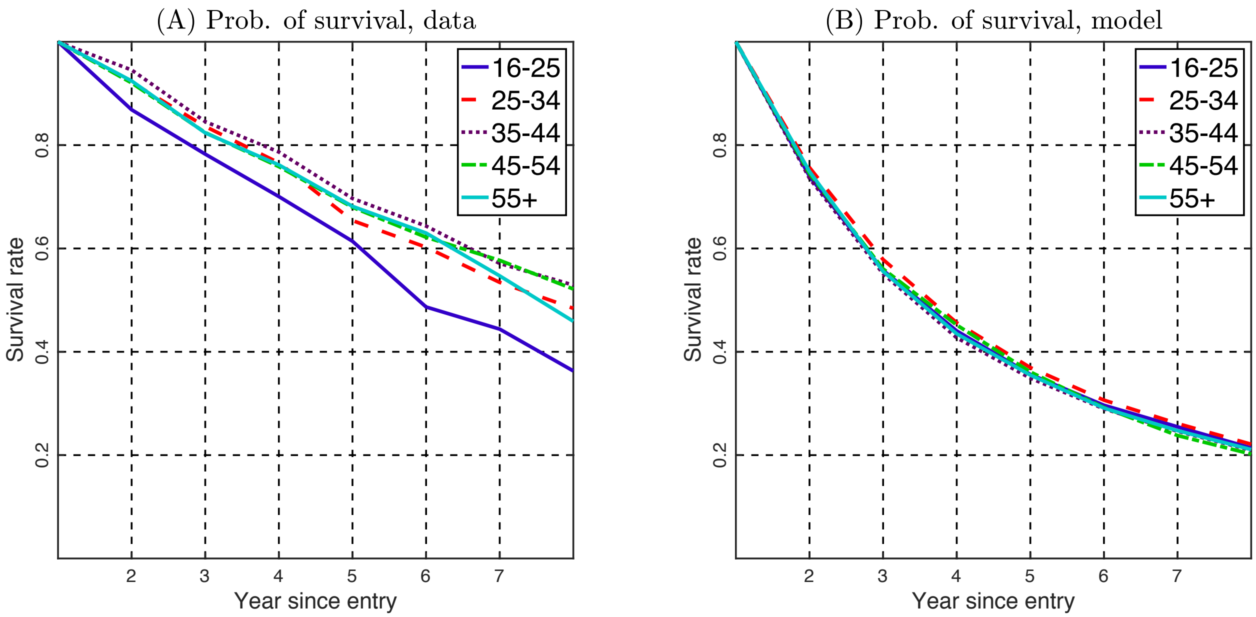
Note: March CPS in 2016 (after HP-filtering). Residual wages after controlling flexibly for gender interacted with age, race and education.

Post-entry performance. Figure 16 plots the survival probability of new firms by age of the founder based on the KFS for up to eight years after entry. Firms exit at a high rate in both the model and the data. The model somewhat overstates this pattern. Figure 17 plots the probability that a firm will hire at least one worker conditional on survival by age of the founder. In both the model and the data, only a fraction of remaining firms hire any worker. The model again overstates this pattern. Figure 18 plots the average log number of employees conditional on hiring at least one worker. Post-entry growth is slow even conditional on hiring, in both the model and the data, although the model overstates post-entry growth. Figure 19 plots job creation, which falls substantially as firms age. New firms create somewhat too many jobs relative to the data. Finally, Figure 20 plots job destruction, which to a first order is flat with firm age.

One possible factor for why firms grow a little too fast in the model is if the data are "contaminated" by some firms that are started for reasons outside the model, such as the desire to be one's own boss. To the extent that such firms are likely to stay in business no matter what and they do not grow their employment much, this may account for the less pronounced up-or-out dynamics in the data. Nevertheless, given that none of these moments are targeted in the estimation, the model does a reasonably good job at matching these patterns.

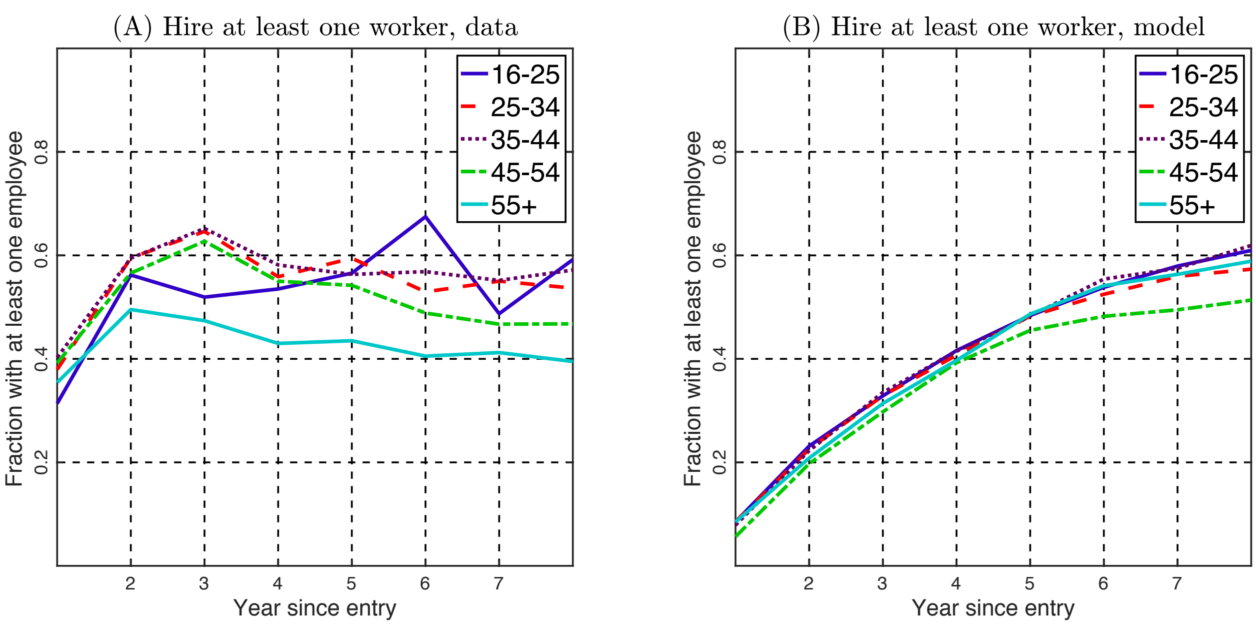
While the survey is limited in scope and sample size, it provides little evidence of systematic differences in post-entry performance of firms by age of the founder.[40] The model matches this by construction.

FIGURE 16. SURVIVAL PROBABILITY OF NEW FIRMS BY AGE OF FOUNDER



Note: Post-entry performance of new start-ups in year 2004 over the subsequent seven years by age of the founder from the KFS 2004–2011. Firm size is the log of the number of employees. All moments are constructed identically in the model and data.

FIGURE 17. PROBABILITY OF HIRING AT LEAST ONE WORKER BY AGE OF FOUNDER



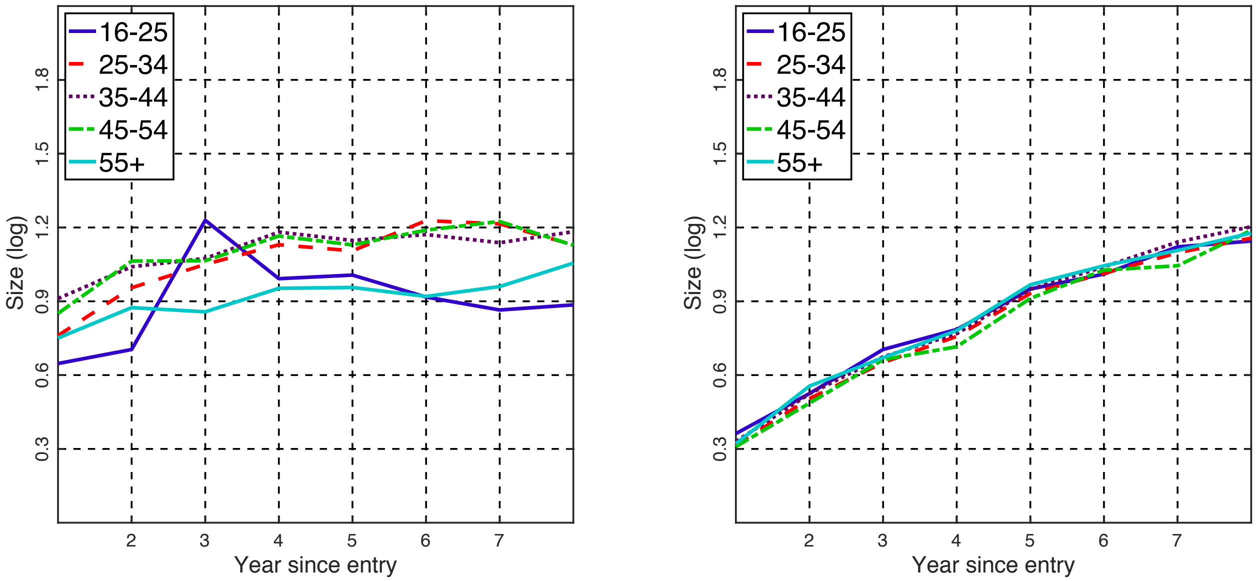
Note: Post-entry performance of new start-ups in year 2004 over the subsequent seven years by age of the founder from the KFS 2004–2011. Firm size is the log of the number of employees. All moments are constructed identically in the model and data.

[40]The same is true for revenues; these data are available on request.

FIGURE 18. AVERAGE FIRM SIZE BY AGE OF FOUNDER

(A) Size, data

(B) Size, model

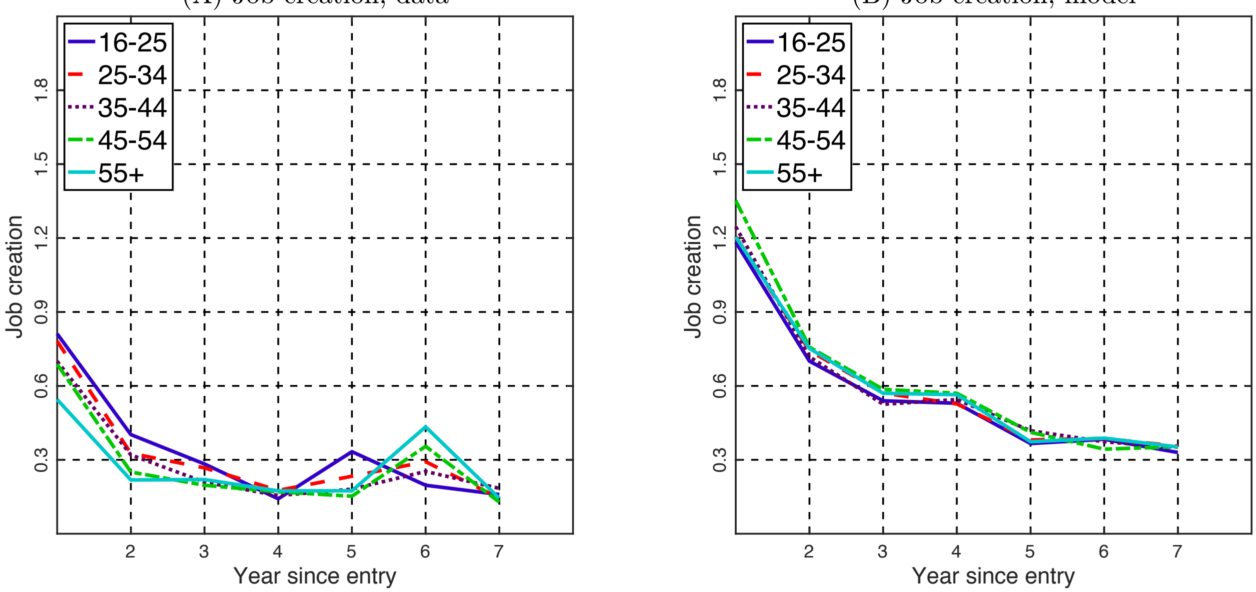


Note: Post-entry performance of new start-ups in year 2004 over the subsequent seven years by age of the founder from the KFS 2004–2011. Firm size is the log of the number of employees. All moments are constructed identically in the model and data.

FIGURE 19. JOB CREATION BY AGE OF FOUNDER

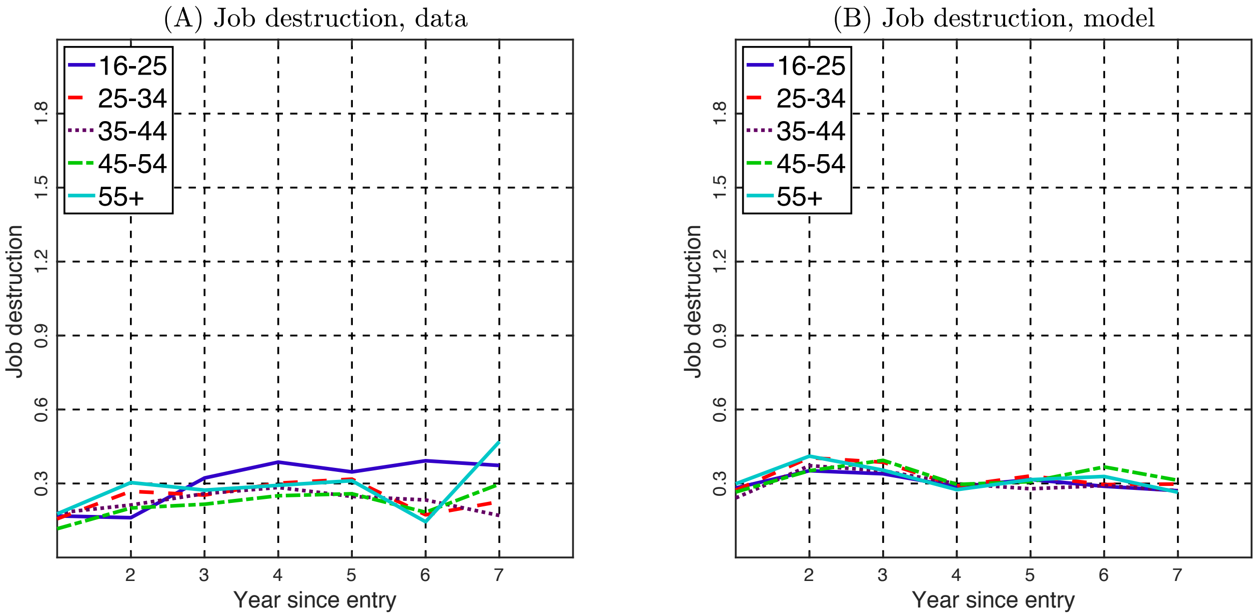
(A) Job creation, data

(B) Job creation, model



Note: Post-entry performance of new start-ups in year 2004 over the subsequent seven years by age of the founder from the KFS 2004–2011. Firm size is the log of the number of employees. All moments are constructed identically in the model and data.

FIGURE 20. JOB DESTRUCTION BY AGE OF FOUNDER



Note: Post-entry performance of new start-ups in year 2004 over the subsequent seven years by age of the founder from the KFS 2004–2011. Firm size is the log of the number of employees. All moments are constructed identically in the model and data.

B.3 The sources of economic growth

The estimates imply that a large share of US economic growth fundamentally is due to the Schumpeterian selection process involved in entry and exit. To assess whether this is consistent with the data, I employ a standard empirical methodology to decompose the sources of economic growth developed originally by Baily et al. (1992).[41] Specifically, denote by $s_{it} = N_{it}/N_t$ a firm's share of employment in year t, by $\theta_{it} = Y_{it}/N_{it}$ a firm's value added per worker in year t, and by $\Theta_t = \sum_i s_{it}\theta_{it}$ the total value added per worker in year t. I follow Lentz and Mortensen (2008) to decompose annual growth as

$$\Delta\Theta_t = \underbrace{\sum_{i\in C_t} s_{it-1}\Delta\theta_{it}}_{\text{Within}} + \underbrace{\sum_{i\in C_t} \theta_{it-1}\Delta s_{it}}_{\text{Between}} + \underbrace{\sum_{i\in C_t} \Delta\theta_{it}\Delta s_{it}}_{\text{Cross}} + \underbrace{\sum_{i\in E_t} \theta_{it}s_{it}}_{\text{Entry}} - \underbrace{\sum_{i\in X_t} \theta_{it-1}s_{it-1}}_{\text{Exit}}$$

where C_t are continuing firms in period t, E_t entering firms and X_t exiting firms.[42]

[41] While the literature has recognized that it is difficult to structurally interpret this decomposition, I employ it for lack of a better alternative.

[42] As in Lentz and Mortensen (2008), in the implementation I normalize the between and entry/exit terms by Θ_{t-1},

$$\Delta\Theta_t = \sum_{i\in C_t} s_{it-1}\Delta\theta_{it} + \sum_{i\in C_t} \left(\theta_{it-1} - \Theta_{t-1}\right)\Delta s_{it} + \sum_{i\in C_t} \Delta\theta_{it}\Delta s_{it} + \sum_{i\in E_t} \left(\theta_{it} - \Theta_{t-1}\right)s_{it} - \sum_{i\in X_t} \left(\theta_{it-1} - \Theta_{t-1}\right)s_{it-1}$$

Table 8 presents the decomposition based on model-generated data. It suggests that a majority of economic growth is driven by within incumbent innovation, in line with typical empirical findings. The reason is that random growth plus endogenous adjustment of employment and exit look like incumbent innovation. To see why, suppose half of firms get a good shock while the other half get a bad shock, so that on average there is no incumbent productivity growth. Firms that get a good shock, however, grow, while those that get a bad shock shrink and exit. As a result, incumbent productivity grows once the endogenous choices of firms are taken into account.

In contrast, reallocation of labor contributes a smaller fraction of growth, and the "cross" term is positive. The former is again in line with typical findings in the literature, while the latter is typically found to be negative. Note, however, that this effectively captures the correlation between the change in value added per worker and the change in firm size. Any measurement noise in size or random fluctuations in size for non-modeled reasons would bias this coefficient down. Exit adds to growth because exiting firms are less productive than the average firm in the economy, while entry subtracts from growth for the same reason. This is again consistent with typical empirical findings.

TABLE 8. DECOMPOSITION OF ECONOMIC GROWTH

Within	0.0365
Between	0.0068
Cross	0.0190
Entry	-0.0243
Exit	0.0258

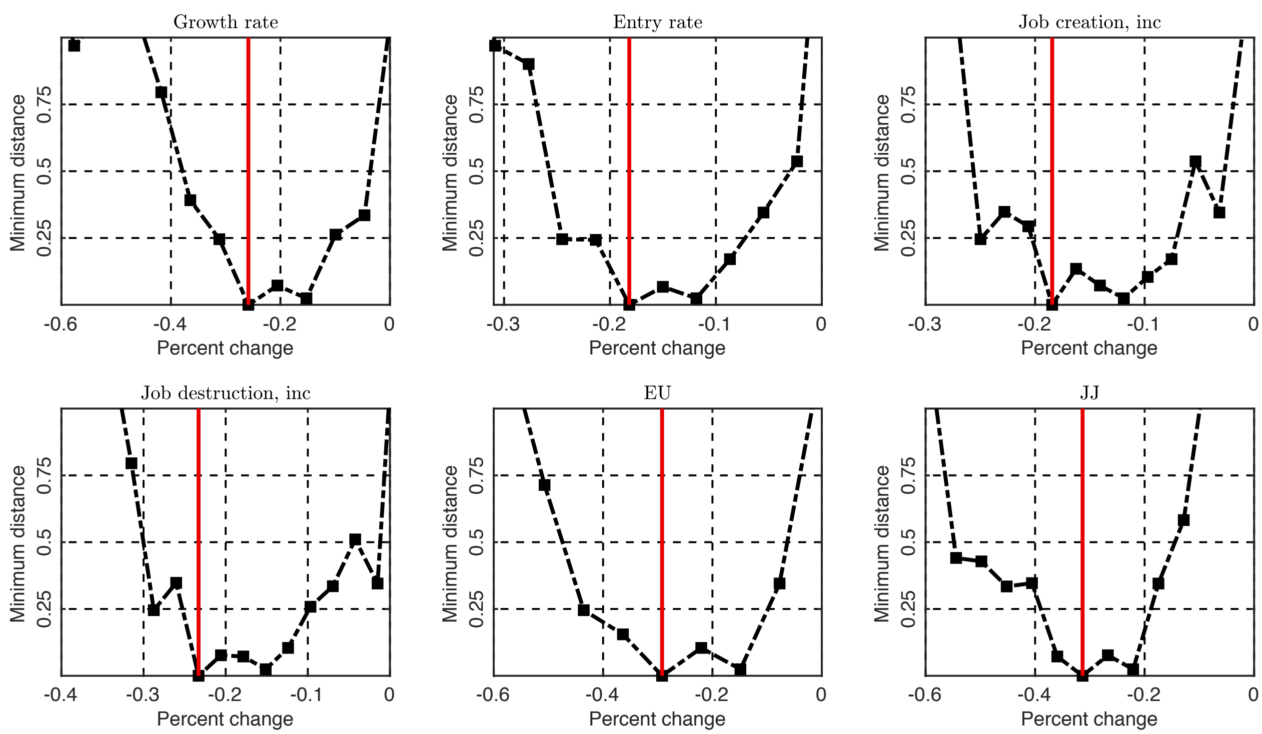
Note: Baily et al. (1992) decomposition based on simulated annual data.

C Additional Details on Results

C.1 Robustness of estimated effects of aging

To illustrate the robustness of the estimated effects of aging, Figure 21 plots how the minimum distance between the targeted moments in the model and the data varies as a function of the estimated effects of aging. Effectively, one can think of this as plotting the gradient of the objective function around the predicted effect of aging. If this is steep, the predicted effect is well identified and vice versa. The exercise highlights two things. First, the model may generate a quite different magnitude effect of the same magnitude of aging if the estimated parameter values had been different. Hence, the large effects of aging that I find are not hardwired into the model. Second, the minimum distance increases rapidly around the estimated effect of aging, suggesting that the point estimates are relatively precise.

FIGURE 21. MINIMUM DISTANCE TO TARGETED MOMENTS RELATIVE TO THE PREDICTED EFFECT OF AGING

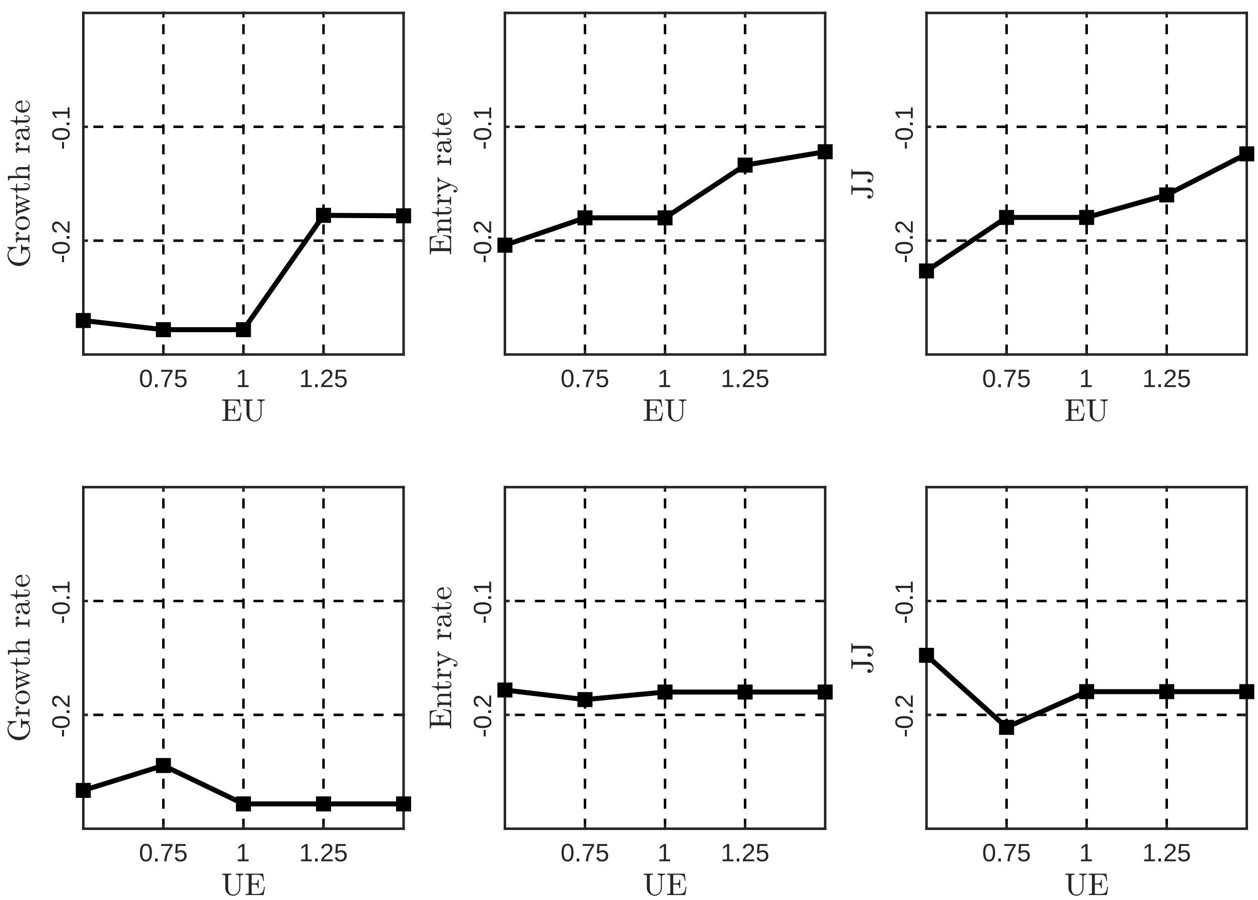


Note: Minimum distance to targeted moments as a function of the predicted effect of aging on the moment on the x-axis.

C.2 Effect of moments on outcomes

Figure 22 plots the results from the robustness exercise described in Section 4.2. In particular, it varies the two key moments on the worker dynamics side, focusing on the implied change in the growth rate, unweighted entry rate, and JJ rate in response to different targeted values for the EU rate and the UE rate. The larger is the EU rate, the smaller is the predicted effect of aging on the growth rate, firm dynamics, and worker dynamics. The reason is that the smaller is the idiosyncratic risk that a match terminates, the more important is the endogenous rate of obsolescence in determining where individuals are on the job ladder. This leaves greater scope for the equilibrium mechanisms highlighted here. For the same reason, the higher is the UE rate (hence also the JJ rate), the larger are the effects of aging.

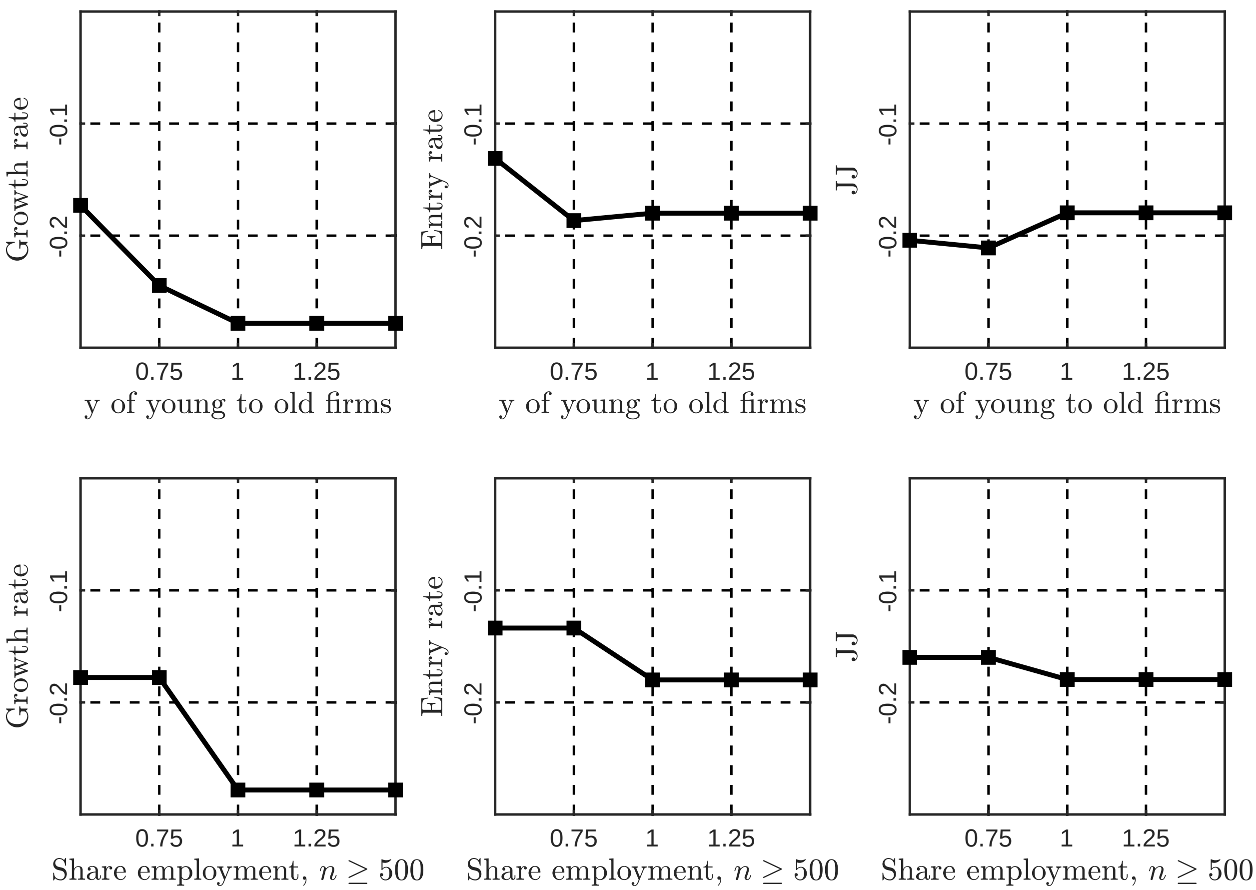
FIGURE 22. EFFECT OF AGING WHEN TARGETED MOMENT ON THE X-AXIS VARIES BETWEEN 0.5 AND 2 TIMES ITS EMPIRICAL VALUE



Note: Predicted percent change in key outcomes in response to aging of the same magnitude as in the US between the 1980s and the present when all moments are the same as in the baseline apart from the moment on the x-axis, which varies between 0.5 and 2 times its actual empirical value.

Figure 23 considers the same experiment, but this time varying the two key moments on the firm side. In particular, I vary the productivity of entrants to mature firms and share of employment at large firms. The more productive are entrants relative to mature firms, the larger is the effect of aging on growth, firm dynamics, and worker dynamics, although the effect is fairly mild. The reason is that the same magnitude decline in entry has a larger effect on growth when entrants are more productive. The larger is the share of employment at large firms, the larger is the effect of aging. The reason is that a greater share of large firms leads me to infer a more elastic job creation margin, which amplifies the effects of aging.

FIGURE 23. EFFECT OF AGING WHEN TARGETED MOMENT ON THE X-AXIS VARIES BETWEEN 0.5 AND 2 TIMES ITS EMPIRICAL VALUE



Note: Predicted percent change in key outcomes in response to aging of the same magnitude as in the US between the 1980s and the present when all moments are the same as in the baseline apart from the moment on the x-axis, which varies between 0.5 and 2 times its actual empirical value.