first steps®

First Steps in Mathematics

Number Course Book

PEARSON

Government of Western Australia
Department of Education and Training

FSIM012 | First Steps in Mathematics: Number Course Book

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OPERATIONS

Addition and Subtraction

Multiplication and Division

COMPUTATIONS

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From Fractions to Decimals
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Units 1–4 Whole and Decimal Numbers
These units use the Key Understandings from Chapter 2 in the *Number Sense* Resource Book to familiarize you with the mathematical patterns of our numeration system; what places some children at risk from both a teaching and a learning point of view; and how to move students along.

Units 6–7 Operations
These units use the Key Understandings from Chapter 2 in the *Operation Sense* Resource Book to familiarize you with choosing and using operations to mathematically model situations, that is, to use operations and numbers alone to represent situations. This information is then used to look at what places some children at risk from both a teaching and a learning point of view, and how to move students along.

Unit 8 Computations
This unit uses the Key Understandings from Chapter 3 in the *Operation Sense* Resource Book to familiarize you with: building and selecting strategies from a repertoire of computation strategies; what places some children at risk from both a teaching and a learning point of view; and how to move students along.

Unit 5 Classroom Research: Task Review and Unit 10 Conclusion: Developing a Plan
These units use knowledge, experience, and evidence from both the *Number Sense* and *Operation Sense* Resource Books to analyze work samples and plan teaching and learning programs.

All units need to be included in the Teacher Course. However, the timetable will be developed in negotiation with the school or group of participants

Professional Development Messages

Probably nothing has more impact on students than the personal, professional growth of their teacher.

Barth (1990), p. 49

These 4 key messages underpin the *First Steps in Mathematics* Professional Development Resources:

1. **Students may get the right answers for the wrong reasons.**
2. **Good questions and tasks provoke students to show us what they know and understand.**
3. **Understanding the mathematics helps us make better professional decisions.**
4. **Seeing the pattern in students' responses helps us plan effective learning experiences.**

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UNIT

1 Whole and Decimal Numbers

INTRODUCTION

Desired Outcomes

Participants will

- be introduced to the beliefs that underpin *First Steps in Mathematics*
- become familiar with the philosophy and rationale that underpins the *First Steps in Mathematics* Resource Books and professional learning process
- begin to learn about the structure of the Diagnostic Map: Number in the Emergent, Matching, and Quantifying phases

Unit Contents

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Related Reading

First Steps in Mathematics:
***Number Sense* Resource Book**

Chapter 1: An Overview of First Steps in Mathematics
Beliefs about Teaching and Learning
Learning Mathematics: Implications for the Classroom
Understanding the Elements of First Steps in Mathematics
How to Read the Diagnostic Map

Chapter 2: Whole and Decimal Numbers
Key Understanding 4, pp. 52–63

Diagnostic Map: Number

Emergent phase
Matching phase
Quantifying phase

Suggestions for Further Reading

Munns, P. "Children's beliefs about counting." In *Teaching and Learning Early Number*, edited by Ian Thompson. Open University Press, Philadelphia, 1998.

Willis, S. "Numeracy and Outcomes in the Early Years." *Every Child*, 5(1), Autumn 1999.

Classroom Planning Aid

Two low-achieving students:

Student 1_____ **Student 2** _____

Key Understanding(s)	Mathematical Focus	Sample Learning Activities	Focus Questions	Observations Anecdotes
	concept *e.g., a dot point in KU* **diagnostic tasks**			**post task**

Two average-achieving students:

Student 1_____ **Student 2** _____

Key Understanding(s)	Mathematical Focus	Sample Learning Activities	Focus Questions	Observations Anecdotes
	concept **diagnostic tasks**			**post task**

Two high-achieving students:

Student 1_____ **Student 2** _____

Key Understanding(s)	Mathematical Focus	Sample Learning Activities	Focus Questions	Observations Anecdotes
	concept **diagnostic tasks**			**post task**

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Beliefs about Teaching and Learning

First Steps in Mathematics (*FSiM*) is underpinned by the following beliefs. Indicate the extent to which you currently share these beliefs by checking the appropriate box.

1. Learning is improved if the whole school has a shared understanding of the mathematics outcomes, and a commitment to achieving them.

   agree ☐ ☐ ☐ ☐ ☐ disagree

2. All students can be successful learners of mathematics.

   agree ☐ ☐ ☐ ☐ ☐ disagree

3. Learning is not simply about the transfer of knowledge from one person to another.

   agree ☐ ☐ ☐ ☐ ☐ disagree

4. Students need to construct their own knowledge in their own way.

   agree ☐ ☐ ☐ ☐ ☐ disagree

5. If all students are to succeed to the best of their ability in achieving mathematics outcomes, differentiated instruction will be not only possible but necessary.

   agree ☐ ☐ ☐ ☐ ☐ disagree

6. Teaching cannot be reduced to a set of instructions about what to do in any given situation.

   agree ☐ ☐ ☐ ☐ ☐ disagree

7. Robust learning that focuses on students achieving mathematics outcomes fully and in depth is essential for long-term sustained learning.

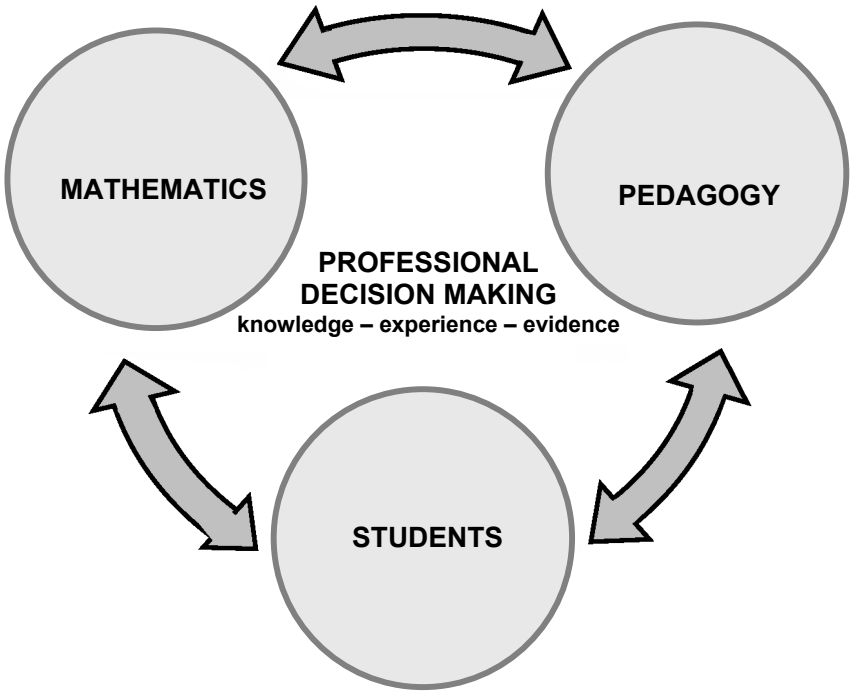
   agree ☐ ☐ ☐ ☐ ☐ disagree

8. A focus on short-term performance or procedural knowledge places students at risk of not progressing in later years.

   agree ☐ ☐ ☐ ☐ ☐ disagree

Professional Decision Making

Components of Professional Decision Making



> Professionalism has one essential feature: practice requires the exercise of complex high level judgments [which] involve mixes of specialized knowledge; high level cognitive skills; sensitive and sophisticated personal skills; broad and relevant background and tacit knowledge.
>
> Preston, B., *Independent Education*, 23(4), pp. 4–12, 1993.

Professional Development Cycle
Number Sense Resource Book, p. 16

Discussion

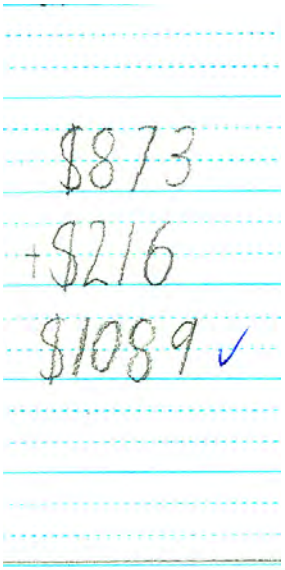
How do you currently make professional decisions about mathematics, pedagogy, and students?

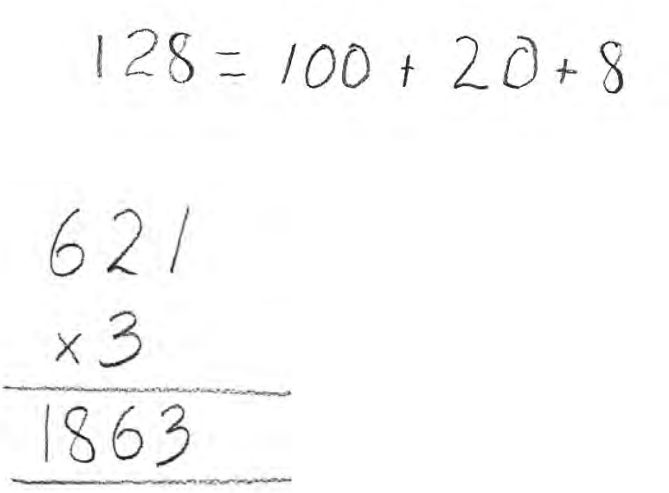
Making Professional Decisions about Student Learning

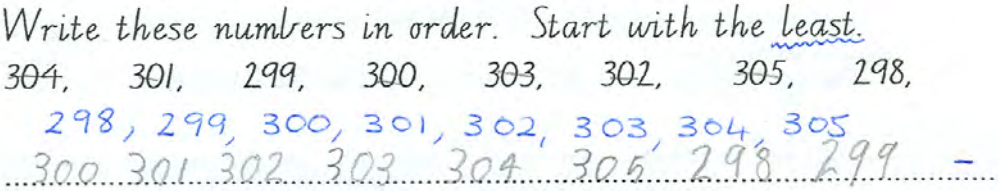
Sarah's Story

FSiM researchers met Sarah when she was an eight-year-old student in Grade 3. While not falling behind in mathematics, she wasn't thriving and had begun to dislike and fear mathematics lessons. She was usually able to calculate correct answers for "sums" set out for her to copy and solve.

Sarah's Work Samples







Consider what Sarah might not understand.
Note: "least" was underlined by the teacher;
the middle line of numbers is the teacher's correction.

The Teacher's Professional Decisions about Sarah's Understanding

1. The teacher's observation and interpretation of what Sarah knew:
    - Sarah can add and multiply into the thousands. She uses standard place value. However, she is not a robust learner when it comes to mathematics.
2. The teacher's decision about the mathematics that would help Sarah move on:
    - Sarah is now ready to learn to order numbers through the hundreds and the meaning of "least."
3. The teacher's decision about learning activities and focus questions:
    - Show Sarah that 200s come before 300s. Give her a lot of practice with addition and multiplication.

Teacher/Parent Interview

Teacher's comments:

- lovely child, very eager to please
- does not have a lot of stamina when it comes to math
- tires easily

Teacher's advice to parents:

- Focus on what she can do, not on what she can't do.
- She'll get there in the end, they all do.
- Practise tables and sums.

What Does Sarah Actually Know?

The following are extracts from the researcher's interview with Sarah.

Sample A Part 1

I: Sarah, I am going to start counting from 95. When I stop I want you to keep on going. Are you ready?
S: (Nodding)
I: 95, 96, 97…..
S: 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 200

Sample A Part 2

The interviewer engaged Sarah in counting on the calculator using the constant function (see *Number Sense* Resource Book, p. 55, Number Scrolls). The interviewer keyed in *98, +, 1,* = and asked Sarah to predict the result. Sarah was taught to use the constant function (see table below), predicting each time what the next number would be.

S: 108? (Pressing.) Yes! 109? (Pressing.) Yes! 200? (Pressing.) What? This thing's wrong. Look what it did!
I: What did it say?
S: It put a ten there.
I: It shows one hundred ten.
S: (Picking up the calculator and motioning to bang it on the table as though it is broken.)
I: Well, I'll start you again at say…102. OK?

Sarah repeated predicting the counting and arrived at 110 again. The interviewer spoke.

I: Sarah, what do you think the next number the calculator will say if it thinks this is the next number?
S: I don't know.
I: Well, try it and see.
S: One hundred eleven. Boy, is it wrong!
I: If it keeps on going with these numbers, what do you think the next number will be?
S: Well, it has to be one hundred twelve. Strange isn't it? (Continues predicting to 119 and then says 200.) Nope, it's going through the twenties too.

The number on the screen reached 128.

S: What is that?
I: (Puzzled.) What…the number?
S: It's not a number. What is it?

Teacher Reference: How to Use the Constant Function

Directions	**Examples**
♦ Key in a number.	**107**
♦ Key in an operation.	+, ÷, ×
♦ Key in a number by which you want to increase/decrease your original number.	107 + **1**
♦ Press the equal key.	107 + 1 =
♦ To increase/decrease the number by a constant amount, repeatedly press the equal key.	107 + 1 = **108 = 109 = 110** 107 **–** 1 = **106 = 105 = 104**

Discussion

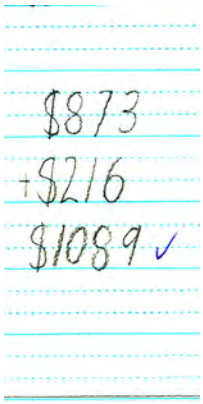
What preconceptions, partial conceptions, and misconceptions have you noticed as children learn the sequence of number names?

Sarah's Misconceptions

Further questioning revealed 128 was not in the counting sequence Sarah had been using. She did not realize that all of the numbers between 109 and 200 were there and that those numbers described quantities. Those numbers did not exist for Sarah.

She had a partial concept and so had to invent ideas to make sense of the mathematics in her class when working with numbers in the hundreds. These ideas were not the correct ideas but they enabled her to get the right answers for the wrong reasons. These types of ideas are called misconceptions and are typical of many students.

Sample B

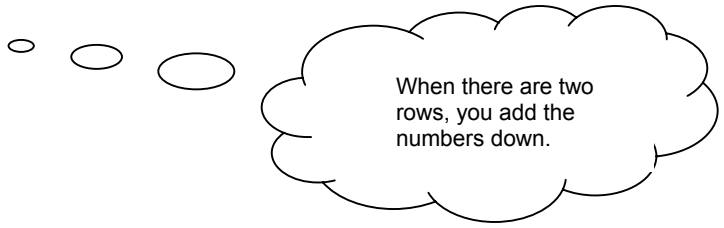


I: Sarah, tell me how you found this answer.
S: Well, this is the short way…
I: What is the long way?
S: Well, you pull the numbers apart like this. (writing three single digit additions)

$$\begin{array}{rrr} 8 & 7 & 3 \\ +\,2 & +1 & +6 \\ \hline 10 & 8 & 9 \end{array}$$

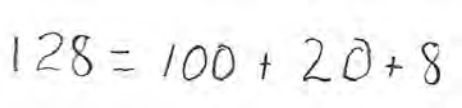
S: See, that's what you do, but we do it the short way.

Pattern Sarah Sees



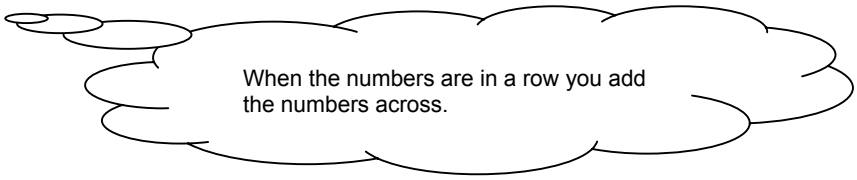
Sample C

I: Tell me how you found this answer.
S: I love these … you pull the numbers apart like this.



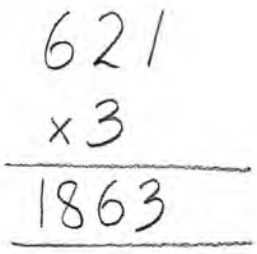
Then you don't put any zeros on the first one, you put one zero on the next one and you just keep adding zeros. If there are more numbers, you put more zeros.
I: Oh, I see.
S: When the numbers are in a line, you add them across like that.

Patterns Sarah Sees



Sample D

I: (Pointing to the *621*.)
Sarah, see this number on the top.
How much is that?



S: Oh, that's 9.

Rules Sarah Uses

1. When there are two rows of numbers you add the numbers down.
2. When the numbers are in a row you add the numbers across.

Discussion

Look at Sarah's sequencing of numbers from least to greatest. How has she applied her rules?

300 301 302 303 304 305 298 299

Review the Teacher's Decisions

A look back at Sarah's ordering of numbers between 298 and 305 shows that Sarah was adding the digits across. For example, 3 + 0 + 0 = 3; 3 + 0 + 1 = 4…

> Students do misunderstand, but it is seldom because they cannot understand; most often it is because they understand something else.
>
> Tripp, D.,*Critical Incidents in Teaching*

PROFESSIONAL DECISION MAKING	Teacher's decision, made without *FSiM*	Your decision, made using *FSiM*
1. **STUDENTS** **Observe and interpret what Sarah does and says.**	"Sarah can add and multiply into the thousands. However, she is not a robust learner when it comes to mathematics."	What set of numbers is Sarah working with? What does Sarah think? (See the first bullet in By the End of the Matching Phase.)
2. **MATHEMATICS** **Decide on the mathematics needed to move Sarah on.**	"Sarah is now ready to learn to order numbers through the hundreds."	What KU describes the precise mathematics Sarah needs to progress?
3. **PEDAGOGY** **Decide on learning activities and focus questions.**	"Show Sarah that 200s come before 300s. Give her a lot of practice with addition and multiplication."	What Sample Learning Activities will help Sarah progress?

Discussion

Record your decision about what Sarah knows and how you might advance her thinking.

Searching For Patterns

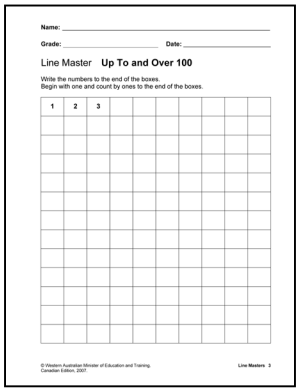
Key Understanding 4: The whole numbers are in a particular order, and there are patterns in the way we say them that helps us remember the order. (*Number Sense* Resource Book, p. 52)

Key Understanding 5: There are patterns in the way we write whole numbers that help us remember their order. (*Number Sense* Resource Book, p. 64)

DIAGNOSTIC TASK: Up To and Over 100 & Up To and Through the Hundreds

KU 4 for Grades 2–7 (ages 8–12)

(See below and Course Book, p. 171.)



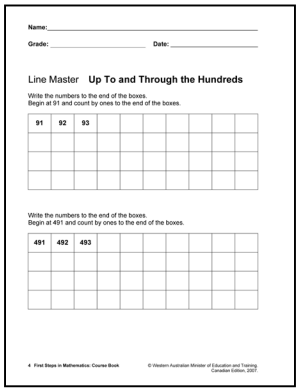
Name: _____

Grade: _____ Date: _____

Line Master **Up To and Over 100**

Write the numbers to the end of the boxes.
Begin with one and count by ones to the end of the boxes.

1	2	3								



Name: _____

Grade: _____ Date: _____

Line Master **Up To and Through the Hundreds**

Write the numbers to the end of the boxes.
Begin at 91 and count by ones to the end of the boxes.

91	92	93								

Write the numbers to the end of the boxes.
Begin at 491 and count by ones to the end of the boxes.

491	492	493								

Purpose

To see if students know the pattern in the way we say numbers up to and over 100

Materials

- Line Master: Up To and Over 100
- Line Master: Up To and Through the Hundreds

Instructions

1. To see if students know the decade sequence, give them the Up To and Over 100 line master. Ask them to fill it in, counting by ones, beginning at one.

2. To see if students know that the decade sequence extends through the hundreds, give students the Up To and Through the Hundreds line master and ask them to fill it in.

3. Interview individual students when

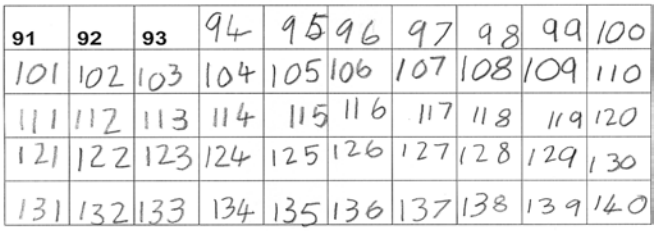
    - the student writes an incorrect number or writes a number incorrectly. Ask him or her to "say" that part of the sequence so that you are able to hear what the student actually thinks the pattern is.
    - the student generally experiences difficulty when working with numbers.

Student Work Samples

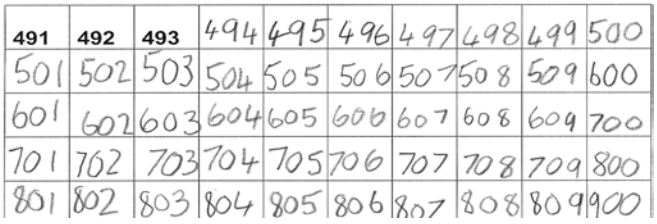
Analyzing the Evidence (Students A–E)

Do the students	A	B	C	D	E
know the words for the numbers 1–13 in sequence?					
know the 14–19 part of the sequence?					
repeat the 1–9 sequence within each decade?					
follow the 1–9 sequence to name the decades?					
repeat the decade sequence and 1–9 sequence within each hundred?					
repeat the 1–9 sequence to name the hundreds (*Number Sense* Resource Book, p. 52)?					
understand and use the ones, tens, hundreds cyclical pattern to read any whole number (*Number Sense* Resource Book, p. 53)?					

Student A: Maddison Grade 4 (aged 9)



91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140

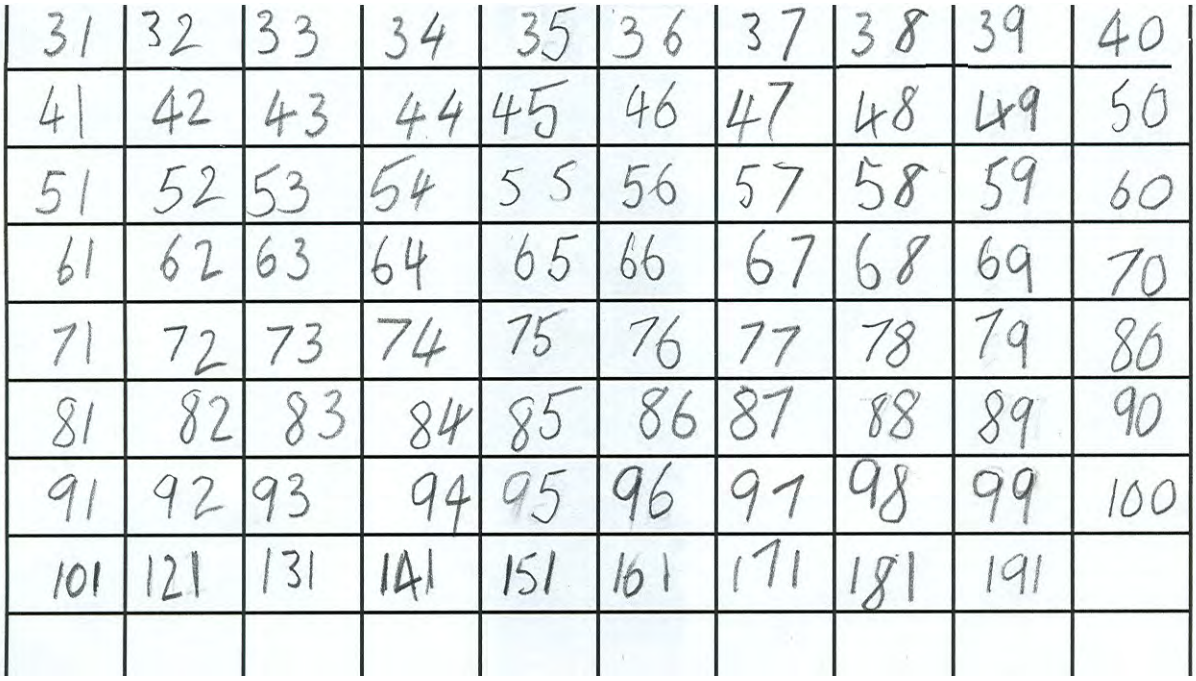


491	492	493	494	495	496	497	498	499	500
501	502	503	504	505	506	507	508	509	600
601	602	603	604	605	606	607	608	609	700
701	702	703	704	705	706	707	708	709	800
801	802	803	804	805	806	807	808	809	900

Analyzing the Evidence (Student A)

What does this child believe about the counting sequence and why?	
What KU and bullet does she need to understand?	
What phase of development is she in?	

Student B: Con Grade 5 (aged 10)



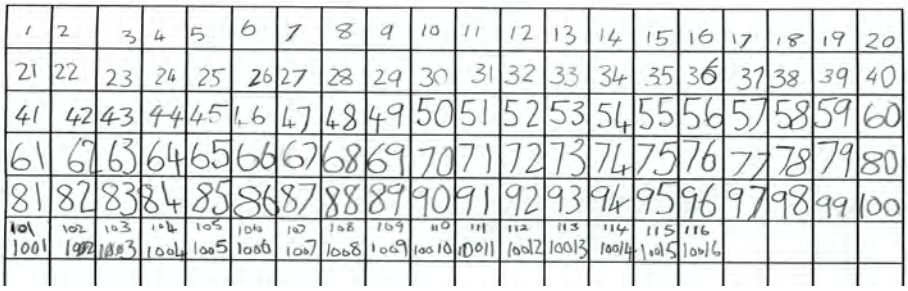
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	121	131	141	151	161	171	181	191	

Analyzing the Evidence (Student B)

What does this child believe about the counting sequence and why?	
What KU and bullet does he need to understand?	
What phase of development is he in?	

Student C: Des Grade 4 (aged 9)

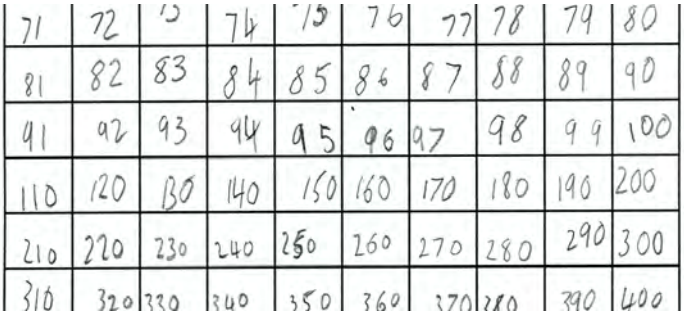
The teacher noted what Des said as he wrote the numbers beyond 100; Des said 101, 102, 103.



Analyzing the Evidence (Student C)

What does this child believe about the counting sequence and why?	
What KU and bullet does he need to understand?	
What phase of development is he in?	

Student D: Ben Grade 6 (aged 11)

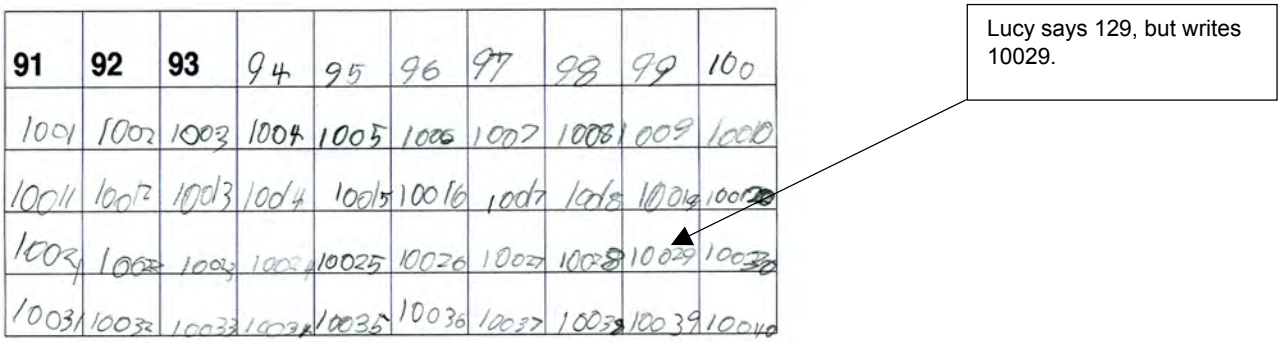


Remember:
Children need to be exposed to the patterns in the number sequence from the earliest years if they are to progress through the Quantifying phase.

Analyzing the Evidence (Student D)

What does this child believe about the counting sequence?	
What KU and bullet does he need to understand?	
What phase of development is he in?	

Student E: Lucy Grade 2 (aged 7)



Analyzing the Evidence (Student E)

What does this child believe about the counting sequence and why?	
What KU and bullet does she need to understand?	
What phase of development is she in?	

Bridging to Practice

Which of the six students you identified on page 3 of this Course Book might not yet understand all the mathematics in Key Understanding 4?

Identify students with whom you work who might need to learn the same mathematics.

UNIT

2

Whole and Decimal Numbers

FROM COUNTING TO PARTITIONING
PART 1: COUNTING TO QUANTIFYING
PART 2: SUBITIZING TO PARTITIONING

Desired Outcomes

Participants will

- look at the development of counting
- become familiar with the patterns in our numeration system
- learn the importance and usefulness of subitizing and partitioning
- become familiar with the mathematics of Whole and Decimal Numbers Key Understandings 1, 2, 4, and 5
- learn about the Emergent, Matching, Quantifying, and Partitioning phases of the Diagnostic Map: Number

Unit Contents

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Related Reading | ***First Steps in Mathematics: Number Sense* Resource Book**

Chapter 2: Whole and Decimal Numbers
Key Understanding 1, pp. 24–35
Key Understanding 2, pp. 36–41
Key Understanding 4, pp. 52–63
Key Understanding 5, pp. 64–71

Diagnostic Map: Number

Emergent phase
Quantifying phase
Matching phase
Partitioning phase

Suggestions for Further Reading

Ross, S. "Parts, wholes, and place value: A developmental view." *Arithmetic Teacher*, 36(6), pp. 47–51, 1989.

PART 1: COUNTING TO QUANTIFYING

Principles Of Counting

Key Understanding 1: We can count a collection to find out how many are in it. (*Number Sense* Resource Book, p. 24)

DIAGNOSTIC TASK: Counting Principles
KU 1 for Grades K–2 (ages 4–7)
(See Course Book, p. 167.)

Knowing What Students Know

1. Observe a student counting.
2. Indicate the counting principles the student understands.
3. With which principle(s) is the student struggling?

Principles of Counting	Michael	Kiah	Joseph	Daniel	My Students
1. Each object to be counted must be touched or "included" exactly once as the numbers are said.					
2. The numbers must be said once and always in the conventional order.					
3. a) The objects can be touched in any order.					
b) The starting point and order in which the objects are counted does not affect how many there are.					
4. The arrangement of the objects does not affect how many there are.					
5. The last number said tells "how many" are in the whole collection; it does not describe the last object touched.					

- Principles 1 through 4 — Diagnostic Task: Counting Principles (Course Book, p. 167)
- Principle 5 — Diagnostic Task: Get Me Task (Course Book, p. 167)

Discussion

What does a child who is through the Matching phase still need to learn about counting in order to progress through the Quantifying phase?

Bridging to Practice: Counting by Ones

Moving Students Along

Select Sample Learning Activities from pages 26 to 31 of the *Number Sense* Resource Book for:

- Constructing the principles of counting

- Using and practising the principles

Children's experience of counting games with adults is often simply playful. The quantitative goal is absent or is not made explicit.

Munns, P. (1998) *Children's beliefs about counting*. Teaching and learning early number. Philadelphia: Open University Press, p. 15

Ginsburg found that children often treated numbers as names or labels, rather than as signifying quantities.

He cites an example of a child, Charles, who counted a set of four red marbles and one blue marble. The blue marble was last in the line and so was number five. From that time on he called all blue marbles "five."

Ginsburg, H. (1982) *Children's Arithmetic*

Discussion

What does a student who is able to apply the principles of counting still need to learn about counting in order to trust that counting by groups gives the same result as counting by ones?

DIAGNOSTIC TASK: Skip Counting

KU 2 for Grades 1–4 (ages 6–9)

(See Course Book, p. 170.)

DIAGNOSTIC TASK: Get Me Task

KU 1 for K–Grade 2 (ages 4–7)

(See Course Book, p. 167.)

Get Me Task Research

They have to remember the number word, then count out the number of items, and monitor the requested number while they count. If children do not know the number string well, their working memory is taken up with remembering the sequence and they tend to forget the number they were asked to get.

Baroody, A.J., and Wilkins, J.L.M. (1999) "The development of informal counting, number and arithmetic skills and concepts." In *Mathematics in the Early Years, Birth to Five,* edited by J. Copely. Reston, VA.: National Council of Teachers of Mathematics, pp. 48–65

DIAGNOSTIC TASK: Ice Cream (Equal Sets)

KU 1 for Grades 1–3 (ages 6–8)

(See Course Book, p. 169.)

Bridging to Practice: Skip Counting

Knowing What Students Know

- Use Diagnostic Task: Skip Counting (Course Book, p. 170).
- Analyze your Skip Counting work samples.

What counting principles do these students need to understand to be able to count by twos or to count in other small groups?

Discussion

What activities or tasks could give a true indication of children's understanding of counting? Use the table below to record your ideas.

Analyzing the Evidence for Counting

Phase	As they enter ...	During ...	By the end ...
Emergent Phase			
Matching Phase			
Quantifying Phase			

Moving Students Along

Remember:
Students need to demonstrate their understanding of the principles of counting to progress through both the Matching and Quantifying phases for whole and decimal numbers.

Select Sample Learning Activities on pages 27 to 29 of the *Number Sense* Resource Book that help students do the following:

- Trust that counting by twos gives the same result as counting by ones

_____

_____

- Use and practise group counting for real purposes

_____

_____

Bridging to Practice: Patterns in the Way We Say Numbers

Key Understanding 4: The whole numbers are in a particular order, and there are patterns in the ways we say them that help us to remember the order. (*Number Sense* Resource Book, p. 52)

Students	**Use the Course Book Diagnostic Tasks to know what students know.**	**Use Sample Learning Activities from the *Number Sense* Resource Book to move students along.**
Grades 2–4 *(ages 7–9)*	• Up To and Through the Hundreds (p. 171)	• Number Cards (p. 57) • Number Scrolls (p. 55)
Grades 4–7 *(ages 9–12)*	• Saying the Number Sequence by Ones and Tens (p. 171) • What's Next? (p. 178)	• Bicycle Odometer (p. 56)

Select Sample Learning Activities from the *Number Sense* Resource Book (pp. 54–59) for:

Remember:
Seeing the patterns in students' responses helps us to plan effective learning experiences.

- Mathematical ideas about the sequence of number names into the millions

- Predicting and checking ideas about the patterns in the sequence of number names

Discussion

What is the connection between Key Understanding 4 and place value?

Remember:
Place value is the key to understanding how we say, read, write, and do computations with whole numbers. It is the pattern in the way we put the digits together that enables us to write an infinite sequence of whole numbers and to easily put any two whole (or decimal) numbers in order.

Background Notes: Understand Whole and Decimal Numbers

Linking Counting to "How Many"

Generally children learn to use the number names one, two, and three through a range of family/cultural practices where number words are used almost as an adjective in much the way blue is used in "blue shoes." The idea of number begins to emerge as children recognize pairs of things, and learn to name them as "two," perhaps pointing and saying "two eyes," "two ears," and comparing this with only "one nose." In a similar way, through familiar rhymes and stories they learn to recognize and name three things at a glance: "three pigs," "three blind mice."

The emergent linking of quantity with number names

As the idea of "oneness," "twoness," and "threeness" emerges, children develop the concept of number. They come to see the sense in which "one," "two," and "three" are alike (they all represent "set size" or quantity) and yet different (they represent different set sizes or quantities).

Most children can relate small numbers to each other without actually counting. They "know" that two is more than one even if they do not have the language to describe that knowledge. Being able to express the basic idea that a collection of two (always) has more than a collection of one, that a collection of three always has more than a collection of two, and so on, is the basis for ordering those numbers and hence for connecting them with the counting sequence, 1, 2, 3, ...

Trying to teach children to use the number names to "count" a collection is likely to be unsuccessful if they are unable to see the difference in size between small collections or have not learned to use the number words "one," "two," and "three" to name the difference in size. This would be like trying to teach children to read before they know what books are for or that text conveys the message and has permanent meaning.

Since, for many children, the capacity to distinguish small quantities and to use the first few counting numbers to name those quantities develops before they come to school, it is easy to overlook its significance. However, some children, particularly among those with intellectual delay or disability, may not develop this specific "capacity" as early or readily. Such children will need experiences in the early grades which focus explicitly on learning to distinguish small quantities and use the first few counting numbers to name the quantities. Without this, they may then learn to count in a technical sense but may not be able to make much sense of the process and hence will not learn to use counting for themselves to answer questions.

Children's early experience of numbers

In many families, learning to recite the number names in order is the focus of many informal and playful activities. Other activities are focused on counting actions—steps, spoonfuls, jumps, and finger points at objects. Children at first imitate and coordinate the actions and words of counting and only over an extended period learn to see that this tells them "how many" things there are.

In order to systematically count a collection, children need both to remember the counting sequence and know how to use the sequence in one-to-one correspondence with the items in the collection. However, the order in which children learn these two things will vary (just as some children learn many separate words before they attempt to say a whole sentence and others hardly say an individual word until suddenly they come out with a whole sentence).

One child may recite the number names correctly up to 40 or 50 or even more and yet not be able to reliably count 8 or 9 things unassisted. This child needs to learn the counting process, that is, how to *use* the number names one-to-one to count a collection (as described in KU 1). Another child may only remember the number names to ten or twelve but be able to use these numbers one-to-one to decide how many there are in a collection of 8 or 9 items. This child probably does not need to learn *how* to count a collection, but needs help to *remember* more of the number sequence (as described in KU 4) in order to extend the repertoire to which they can apply their understanding of the counting process.

In each of the above scenarios, children use the sequence of the numbers to count how many are in a collection. It seems like "common sense" that if you cannot "count" to 8 (in the sense of saying the number names in order), you will not be able to "count" a collection of 8 objects and the way in which teaching and testing programs are sequenced often assume this.

Learning to recite the numbers in order is not equally valued in all families. For example, some Aboriginal communities may find reciting sequences of number out of context an odd thing to do and not teach their children counting songs and games. This does not necessarily mean that children are not encouraged to develop a sense of number, however, since other social activities may help children recognize "how many" are in a scattered collection just by looking. That is, within some Aboriginal communities in Australia, subitizing is the focus of informal and playful activities. These are different from, but parallel to, the counting-oriented activities that many majority-culture children experience. Such children may not "count," in the sense that they are unable to say the numbers in order up to eight or nine, but may well have learned to tell that there are six birds flying overhead, recognizing "sixness" in the same way that other children recognize "threeness."

For such children, learning to count may require that they investigate collections which they recognize "by looking" as having "five" or "six" or "seven" things in them, comparing and equalizing quantities and talking about what they have done. The aim is that they should be able to place these collections in order so that each is bigger than the one before (each is *one* bigger than the one before). Generalizing from this is the basis for understanding why we say the numbers 5, 6, 7 in the counting sequence in that order: each number in the sequence is one more than the one before.

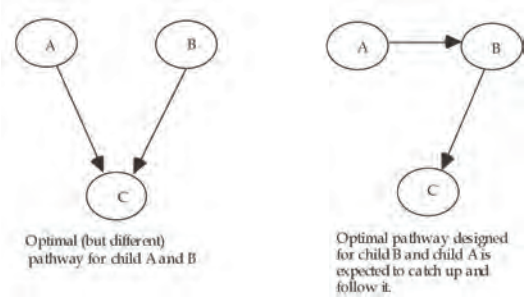
Sequencing learning activities to link quantity with the order of the number names

The fact that we want children as far as possible to achieve a common learning outcome does not mean that they should all experience the same activities or curriculum, sequenced in the same way.

Just the opposite. As indicated above, children are different from one another and will come to school with varied experiences.

Some children may begin by knowing the counting sequence and need to learn to use it to work out "how many." Other children may begin by seeing how many and need to learn the counting sequence from it. Neither order is better or preferable. However, if the questions we ask of children and the way we sequence learning activities assumes that learning "naturally" proceeds in the way it does for the majority of children, then we are likely to place at educational risk the minority of children whose learning sequence may be different.

We might imagine some children living in place A and others living in place B, all having to get to a third place C. We can provide each with the best pathway to C, or we can require those who live at A to travel to B and then take the path from B to C. If we do the latter, then the children who start at A clearly have farther to go. Is it any wonder they fall behind? In an analogous way, a "common" input curriculum *may* cause educational disadvantage.



In such cases, the risk does not lie in some characteristic of the children or their backgrounds but rather in the inappropriate match of the curriculum to their knowledge. Thus we may not recognize that the children *can* tell how many, think of them as "behind" and move them through the learning pathways with which WE are most familiar and comfortable rather than those likely to be most helpful for their learning. By failing to respect and build upon their existing strategies we actually undermine these children. The challenge this provides us with is to ensure that typical developmental sequences of the majority of children do not dominate and thus become the mechanism by which certain children are *put* at educational risk rather than the means by which educational risk is reduced or removed. We *do* want children to learn to link the counting sequence to quantity but they do not all have to learn this in the same way or order. Children do not all have to make the same journey, rather we want to them all to arrive at the same destination.

In short, most children are likely to learn "to count" in the sense of chanting the number names in order and *then* learn to count a collection by 1-1 matching of the number names in order with the items. Others may learn to recognize "how many" are in small collections by looking, that is, they may recognize "sixness" before they can chant number names in order to six. Either way, the two ideas must come together so that children see the link between the order in which we say the number names and the size of collections.

Notes

Use this page to summarize the key points of the Background Notes.

PART 2: SUBITIZING TO PARTITIONING

Subitizing

Key Understanding 2: We can often see how many are in a collection just by looking and also by thinking of it in parts. (*Number Sense* Resource Book, p. 36)

Subitizing precedes counting and underpins it. Seeing 3 as more than 2 and 2 as more than 1 helps children learn that counting numbers go up by one each time.

It is through the use of the counting sequence with the subitized amounts 1, 2, and 3 that children begin to realize when they are counting that we are referring to the same idea, the idea of quantity.

What arrangements are easy to subitize? More challenging? Draw some examples below.

Easy	More challenging

DIAGNOSTIC TASK: Subitizing

KU 2 for K–Grade 1 (ages 4–6)

(See Course Book, p. 175.)

Analyzing the Evidence

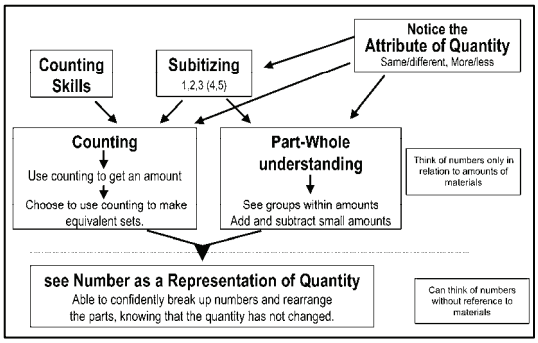
Phase	As they enter ...	During ...	By the end ...
Emergent Phase			
Matching Phase			
Quantifying Phase			

Subitizing And Partitioning

Discussion: Noah (Video)

What does Noah do with numbers to solve the problem?

Links Between Counting, Subitizing, and Partitioning



Treacy K., & Willis, S. "A Model of Early Number Development." In *Mathematics Education Research: Innovation, Networking, Opportunity: Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australasia.* Ed. Bragg, L. Campbell, C. Herbert, G. & Mousley, J., Geelong, July 2003, pp. 674–681.

Discussion

What mathematical understandings and skills underpin partitioning? Refer to the *Number Sense* Resource Book, pages 36 to 37.

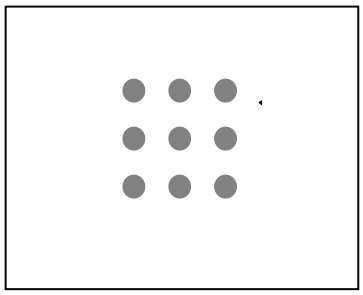
Subitizing

Use subitizing to tell how many dots are in each set. Describe what you subitized. How did you calculate the total?

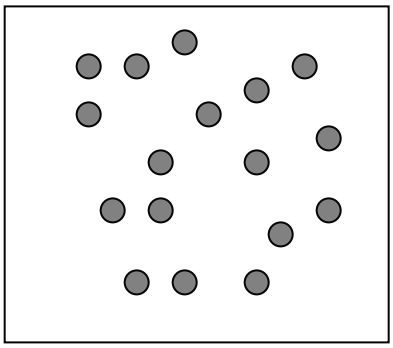
Remember:
The skill of seeing "how many" at a glance could form the basis for further number work much as counting does for other children.

Number Sense
Resource Book, p. 36

SET 1



SET 2



DIAGNOSTIC TASK: How Did You Do It?

KU 5 and KU 6 for Grades 3–7 (ages 8–12)
(See Course Book, p. 177.)

How would you partition the numbers below to make mental calculation easier? Refer to:

- *Number Sense* Resource Book, Chapter 2, Key Understanding 6, p. 72
- *Operation Sense* Resource Book, Chapter 3, Key Understanding 2, p. 126

26 + 37

My partition:

My partner's partition:

Other useful partitions:

Did you find other ways easier?
Why or why not?

Remember:
Part-whole understanding helps students to think of numbers as compositions of other numbers.

Three pictures hang in front of a six-month-old child. The first shows two dots, the others show one dot and three dots. The infant hears three drumbeats. Her eyes move to the picture with three dots.

Clements, 1999, p. 400

FSIM012 | First Steps in Mathematics: Number Course Book

Bridging to Practice: Subitizing

Knowing What Students Know

- Use Diagnostic Task: Subitizing (see Course Book, p. 175)

Moving Students Along

Select Sample Learning Activities from pages 38–43 of the *Number Sense* Resource Book for

- Recognizing random and regular groups of small numbers without counting

- Using subitizing when counting small groups of objects

Bridging to Practice: Partitioning

Knowing What Students Know

- Use Diagnostic Task: Animals (ages 7–9) (see Course Book, p. 176)
- Use Diagnostic Task: How Did You Do It? (ages 8–12+) (see Course Book, p. 177)
- Adapt the contexts and numbers to suit your students.

Moving Students Along

Select Sample Learning Activities from pages 38 to 43 of the *Number Sense* Resource Book and from KU 6 (*Number Sense* Resource Book, pp. 74–79) for the following:

- Learning partitions of numbers to 10

- Using what they know to work out unfamiliar partitions

- Partitioning double-digit numbers into standard place-value partitions to make calculation easier

UNIT

3

Whole and Decimal Numbers

FROM PARTITIONING TO PLACE VALUE

Desired Outcomes

Participants will

- become familiar with the patterns in our numeration system
- become familiar with the mathematics of Whole and Decimal Numbers Key Understandings 4, 5, and 6
- learn about the progression through the Quantifying, Partitioning, and Factoring phases of the Diagnostic Map: Number

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps in Mathematics: Number Sense* Resource Book**

Chapter 2: Whole and Decimal Numbers
Key Understanding 5, pp. 64–71
Key Understanding 6, pp. 72–79

Diagnostic Map: Number

Quantifying phase
Partitioning phase
Factoring phase

Suggestion for Further Reading

Ross, S. "Parts, wholes, and place value: A developmental view." *Arithmetic Teacher*, 36(6), pp. 47–51, 1989.

Reading, Writing, and Saying Numbers

DIAGNOSTIC TASK: Read, Write, and Say Numbers
KU 5 for Grades 3–7 (ages 8–12+)
(See Course Book, p. 188.)

Student Work Samples

Students were asked to write the following large numbers as the teacher said them.

A. sixty-three
B. one thousand twenty
C. twenty-six thousand fifteen
D. five hundred six thousand fifteen
E. one million five
F. five billion thirty-six million four hundred seven thousand four

Claire (Grade 3)

A. 63
D. 500,6015
B. 1020
E. 1,0000,05
C. 26,015
F. 5,36,0000,400,7000,04

Hussain (Grade 3)

A. 63
D. 500 6015
B. 1020
E. 10005
C. 26 015
F. 5D0000360000400704

Marty (Grade 5)

A. 63
D. 500,6000,15
B. 1000,20
E. 10000,5
C. 26000,15
F. 5000000,360000,400 7000,14

Anne-Marie (Grade 5)

A. 63
D. 5600015
B. 1020
E. 10005
C. 26000015
F. 5000003600004004

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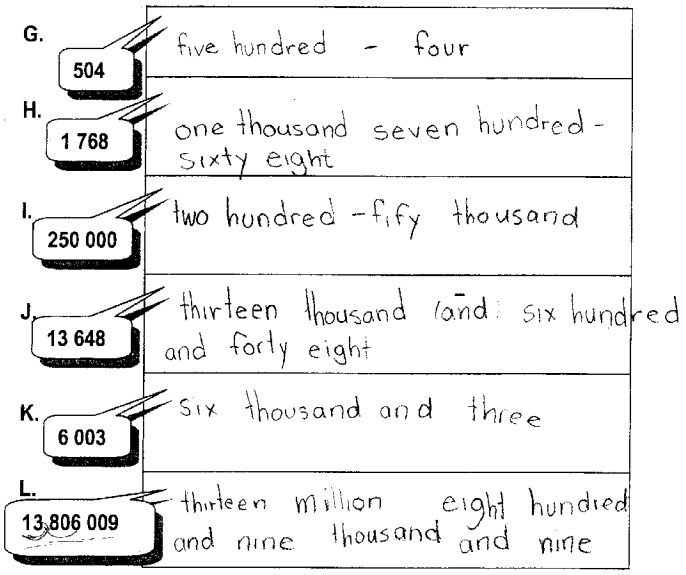
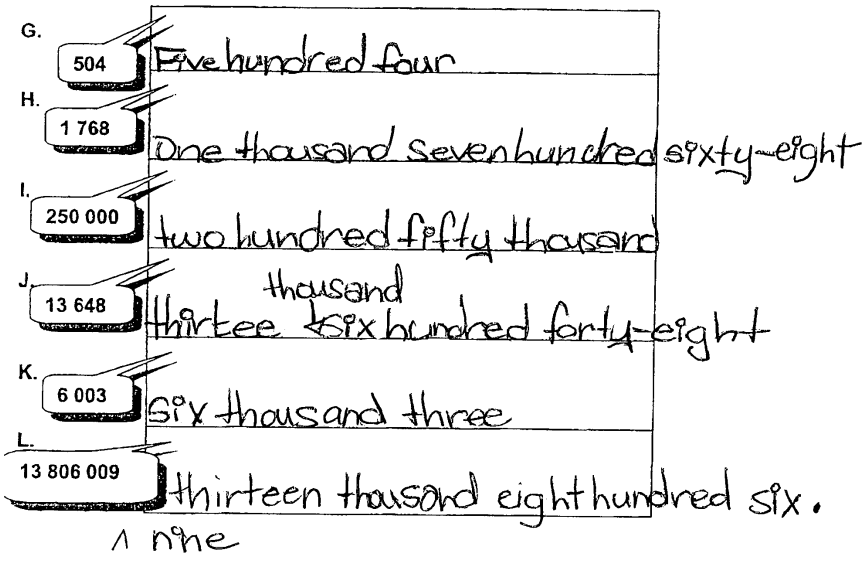
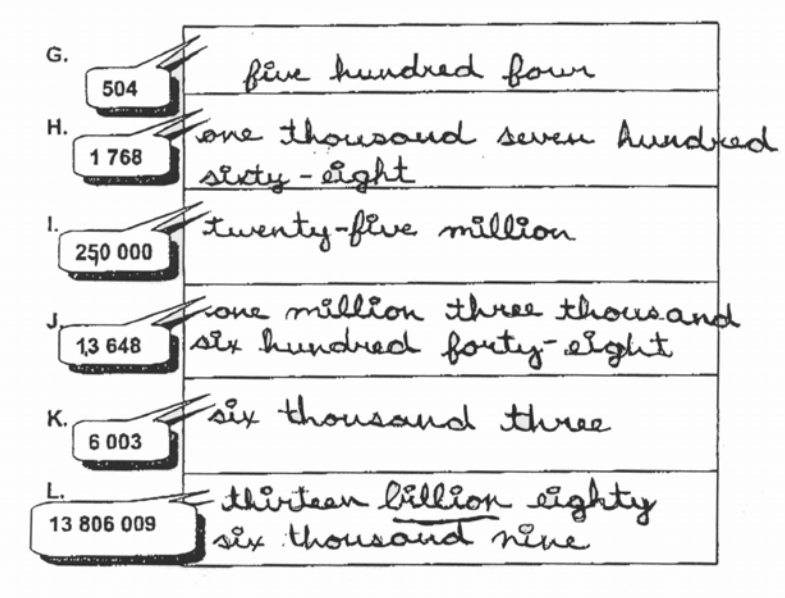
Claire (Grade 3)



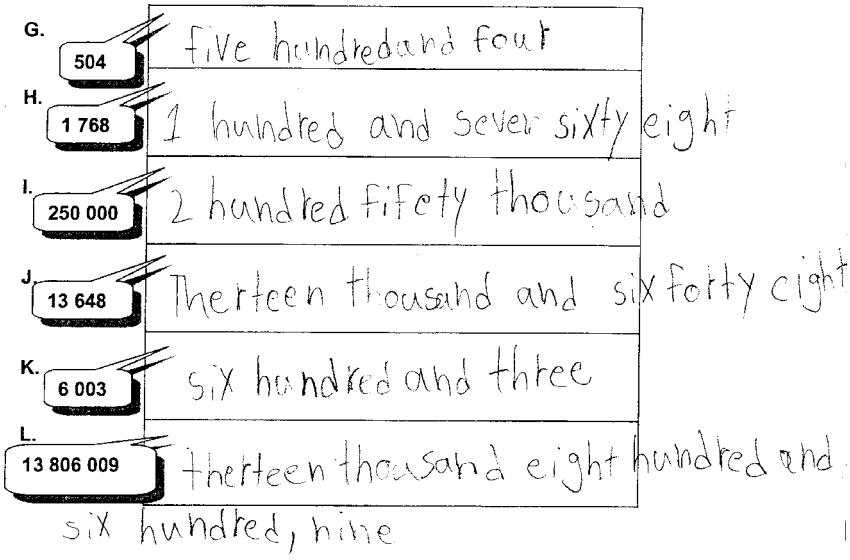
Anne-Marie (Grade 5)



Marty (Grade 5)



Hussain (Grade 3)



Discussion

What patterns are students using to read, write, and say numbers?

Bridging to Practice: Patterns in the Way We Write Numbers

Knowing What Students Know

Use Diagnostic Task: Read, Write, and Say Numbers on page 188 of this Course Book.

Moving Students Along

Select Sample Learning Activities from pages 66 to 71 of the *Number Sense* Resource Book for:

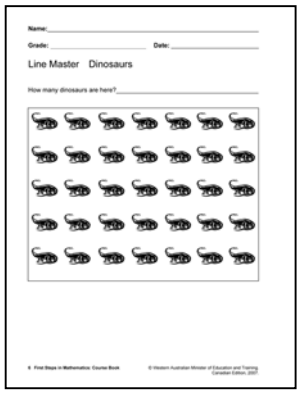
- Understanding the "ones, tens, hundreds" cycle in writing large numbers

Understanding the Value of Place

Key Understanding 5: There are patterns in the way we write whole numbers that help us to remember their order. (*Number Sense* Resource Book, p. 64)

DIAGNOSTIC TASK: Dinosaurs

KU 5 for Grades 2–7 (ages 7–12+)
(See below and Course Book, p. 180)


Name:_____
Grade:_____ Date:_____
Line Master Dinosaurs
How many dinosaurs are here?_____

Purpose

To examine student understanding of the meaning of the individual digits in a two-digit number (KU 5).

Materials

- Line Master: Dinosaurs
- Pencils or pens in two different colours, e.g., green and red

Note: If your students' work is being examined in a group, it is essential that the entire group uses the same colours for the task.

Instructions

1. Ask students to work out how many dinosaurs there are on the page. Students write how many in the space provided.

2. Talk about the number of dinosaurs until everyone agrees there are 35. Write the number 35 on the board.

3. Point to the 5 and say "Use a green pencil and put a circle around what this part of the number means in the set of dinosaurs." Do not say the word "five."

4. Point to the 3 and say "Use a red pencil and put a circle around what this part of the number means in the set of dinosaurs." Do not say the word "three."

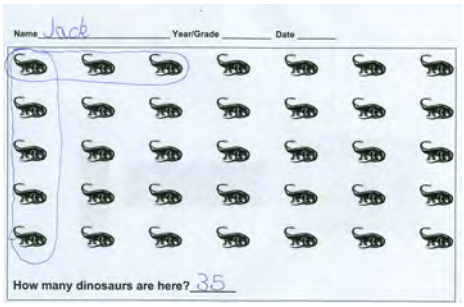
Student Work Samples

Refer to Key Understanding 5 as you analyze the work samples on the following pages.

- Which of the dot points does each child know?
- Which of the dot points does each child need to know next?

FSIM012 | First Steps in Mathematics: Number Course Book

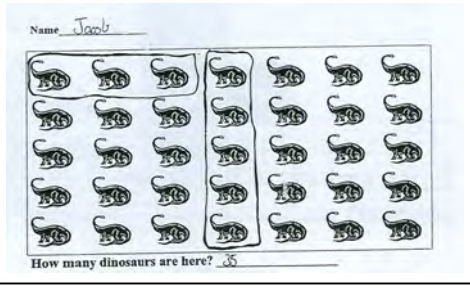
Jack



Analyzing the Evidence

Knows	
Doesn't know yet	
Phase	
Evidence from Map	

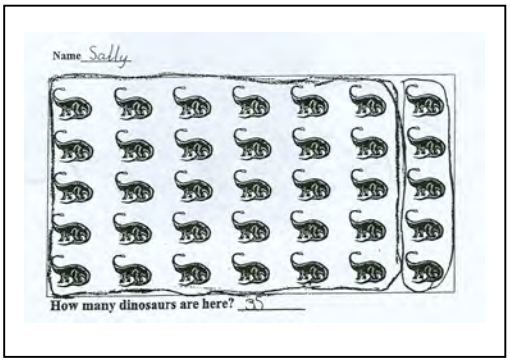
Jacob



Analyzing the Evidence

Knows	
Doesn't know yet	
Phase	
Evidence from Map	

Sally



Analyzing the Evidence

Knows	
Doesn't know yet	
Phase	
Evidence from Map	

Game: Can I Have All Your…?

1. Play this game with a partner.
2. Each person should enter a 3-digit number into a calculator, keeping it hidden from his or her partner.
3. Partner A asks: "Can I have all your 8s [or 3s, or 2s, etc.]?"
4. a) If Partner B does not have that digit, he or she says, "You can have nothing."

   b) If Partner B has that digit, he or she says, "You can have 80 [or 808, or 88, depending on place value]."
5. Partner B subtracts the value. Partner A adds the value.
6. Partner B now asks: "Can I have all your..." and play continues.
7. The winner is the first to reach 1000 or make his or her opponent reach 0.

Extension: Play the game using decimals.

DIAGNOSTIC TASK: 52 Candies

KU 5 for Grades 3–7 (ages 8–12+)
(See below and Course Book, p. 183.)


Name: _____
Grade: _____ Date: _____
Line Master Candies (Set A)
Candies can be bought as single candies or in rolls of ten as shown here.
How many candies are shown here? _____
Task based on Ross, Sharon. Arithmetic Teacher, 1989
© Western Australian Minister of Education and Training, Canadian Edition, 2007.
Line Masters 7

Purpose

To explore student understanding of the meaning of the individual digits in a two-digit number when confronted by both standard and non-standard groupings of objects (KU 5).

Materials

- Line Master: Candies (Set A)
- Pencils or pens in red and blue

Instructions

Provide each student with a copy of the 52 Candies worksheet. Read the introductory sentence with them to make sure that students understand that these candies can be bought as single candies or in rolls of ten.

1. Ask: "How many candies are represented altogether?"

2. Talk with the students about their answers until all students agree that there are 52. Observe students as they record 52 on their pages.

3. Write 52 on the board in view of all students. When giving the following instructions it is important that you do not say the words "five" or "fifty" or "two."

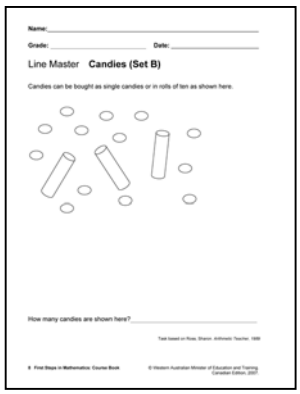
   a) Point to the 2 in 52 and say, "Use a blue pen to colour in what this part of the 52 means in the drawing."

   b) Point to the 5 in the 52 on the board and say, "Use a red pen to colour in what this part of 52 means in the drawing."

Based on ideas by Sharon Ross (1989)

DIAGNOSTIC TASK: 43 Candies

KU 5 for Grades 3–7 (ages 8–12+)
(See below and Course Book, p. 184.)


Name:_____

Grade:_____ Date:_____

Line Master **Candies (Set B)**

Candies can be bought as single candies or in rolls of ten as shown here.

How many candies are shown here?_____

Task based on Ross, Sharon. *Arithmetic Teacher*, 1989

Purpose

To explore student understanding of the meaning of the individual digits in a two-digit number when confronted by both standard and non-standard groupings of objects (KU 5).

Materials

- Line Master: Candies (Set B)
- Pencils or pens in red and blue

Instructions

Provide each student with copies of the 43 Candies worksheet. Introduce as for 52 Candies.

1. Ask: "How many candies are represented altogether?"

2. Talk with the students about their answers until all students agree that there are 43. Observe students as they record 43 on their pages.

3. Write 43 on the board in view of all students. When giving the following instructions it is important that you do not say the words "four" or "forty" or "three."

   a) Point to the 3 in 43 and say, "Use a blue pen to colour in what this part of the 43 means in the drawing."

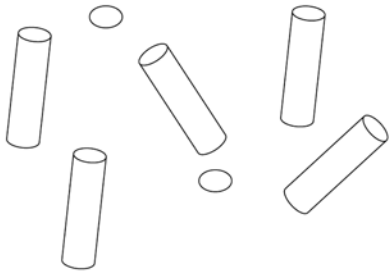
   b) Point to the 4 in the 43 on the board and say, "Use a red pen to colour in what this part of 43 means in the drawing."

Name: _____

Grade: _____ Date: _____

Line Master **Candies (Set A)**

Candies can be bought as single candies or in rolls of ten as shown here.



How many candies are shown here?_____

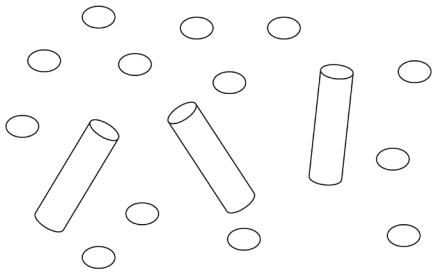
Task based on ideas by Sharon Ross. *Arithmetic Teacher, 1989*

Name: _____

Grade: _____ Date: _____

Line Master **Candies (Set B)**

Candies can be bought as single candies or in rolls of ten as shown here.



How many candies are shown here?_____

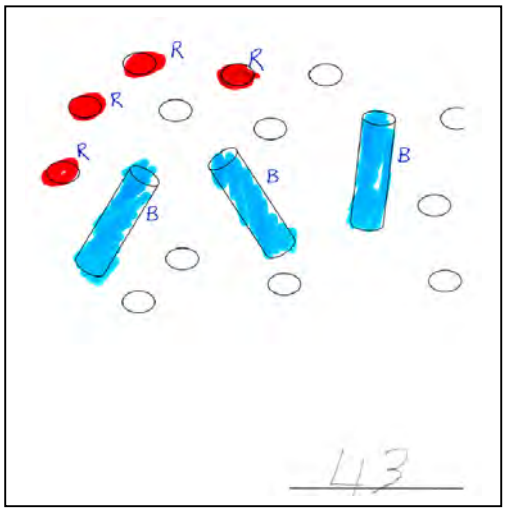
Task based on ideas by Sharon Ross. *Arithmetic Teacher, 1989*

Student Work Samples

Remember:
Students may get the right answers for the wrong reasons.

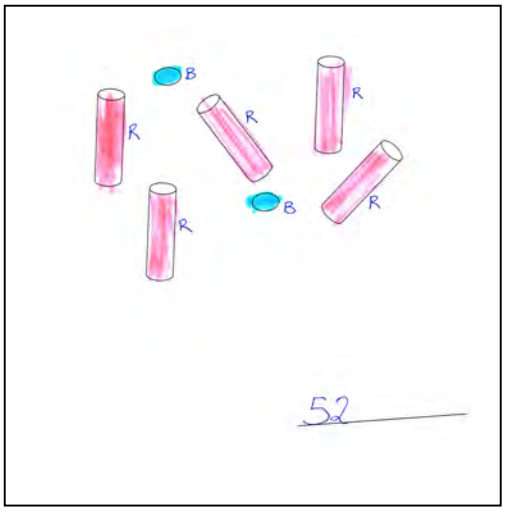
Child A

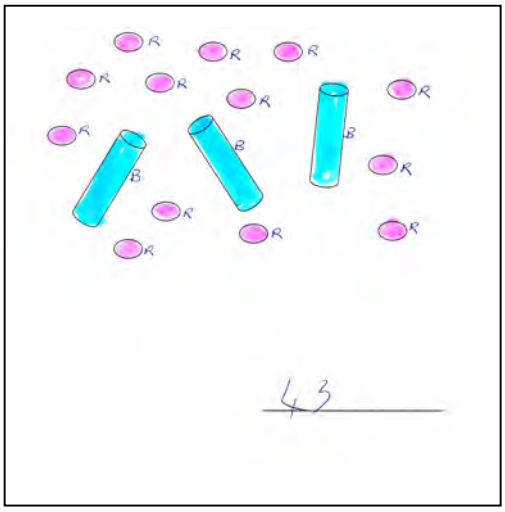
52 Candies—coloured rolls red (R) and singles blue (B)
43 Candies—coloured rolls blue and 4 singles red



Child B

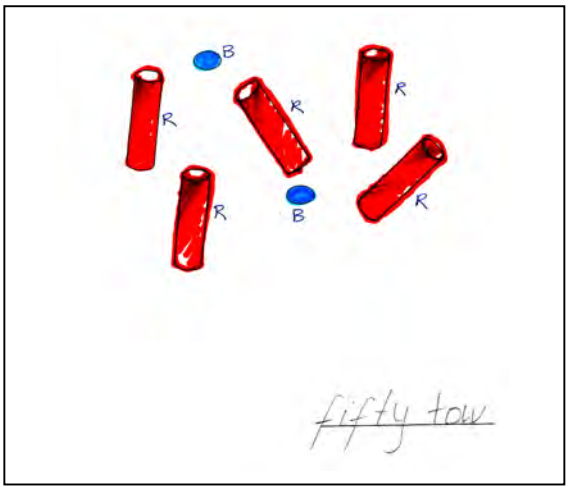
52 Candies—coloured rolls red (R) and singles blue (B)
43 Candies—coloured rolls blue and all singles red

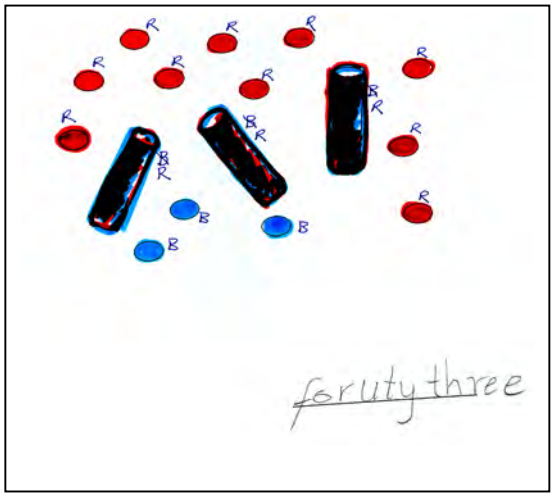




Child C

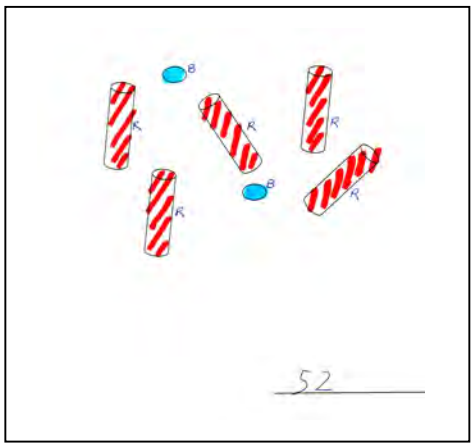
52 Candies—coloured rolls red (R) and singles blue (B)
43 Candies—at first coloured rolls blue, began the singles in red, changed mind and coloured over the rolls in red, coloured 3 singles blue, and the rest in red

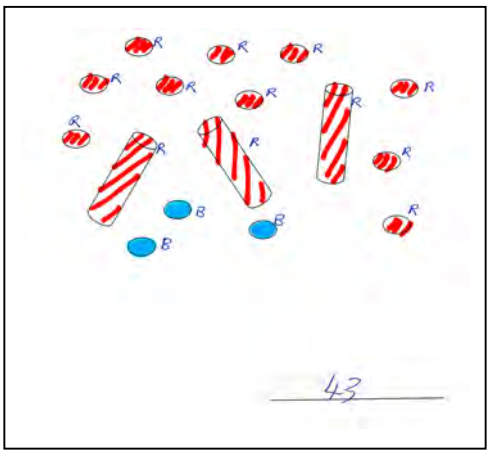




Child D

52 Candies—coloured two singles blue (B) and rolls red (R)
43 Candies—coloured three singles blue and the rest red





A Traditional Worksheet

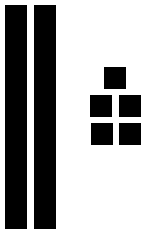
1. Fill in the missing numbers.

   3486 = ____ thousands + ____ hundreds + _____ tens + _____ ones

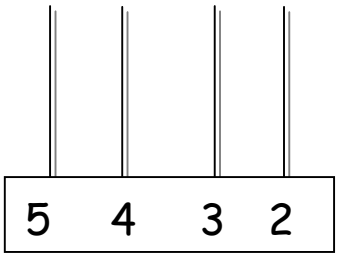
   546 = _____ hundreds + _____ tens + _____ ones

Remember:
Students do not need to understand place value to answer the questions on this page correctly.

2. Complete the chart.

Number	**Model with Base Ten Blocks. Draw what you used.**	**Write the number in words.**
		twenty-five
136		
6 973		six thousand nine hundred seventy-two

3. Show the number 5432 on the abacus.



If we introduce materials that have been designed to embody base ten groupings before students have constructed appropriate quantitative meanings for the individual digits, we may unwittingly provoke or reinforce a face value interpretation of digits.

Ross, Sharon. *Arithmetic Teacher*, 1989.

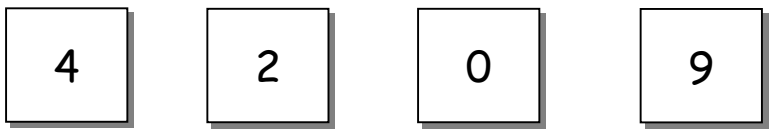
4. What number is

   6 thousands + 4 hundreds + 0 tens + 8 ones? _____

   9 thousands + 8 hundreds + 7 tens + 0 ones? _____

   8 hundreds + five tens + 2 ones?_____

5. What is the biggest number you can make with these number cards?



Block Towers Comparisons

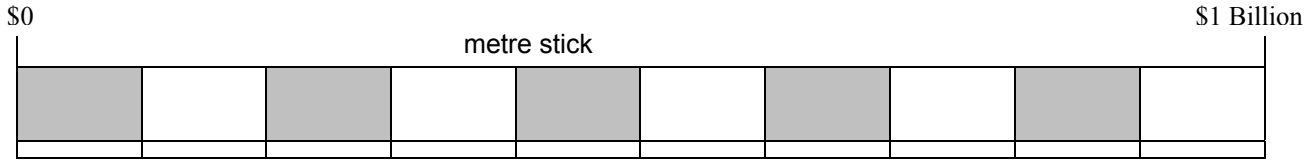
What is different about the explanations?

Sophia	Jenna

Magnitude of a Million and a Billion

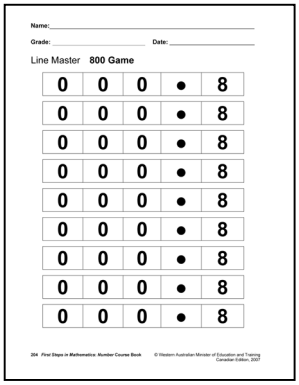
Suppose a metre stick is a number line that represents a range from someone who have zero dollars to a billionaire.

Where would a millionaire be on this number line? Use the metre stick below to mark your answer.



DIAGNOSTIC TASK: 800 Game

KU 5 for Grades 5–7 (ages 10–12+)
(See below and Course Book, p. 195.)


Name:_____
Grade:_____ Date:_____
Line Master 800 Game

0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8

204 *First Steps in Mathematics: Number Course Book* © Western Australian Minister of Education and Training Canadian Edition, 2007

Purpose

To see the extent of students' understanding of the relationship between the places. For example, do the students know that 8 is ten times greater than 0.8 and ten times smaller than 80 (KU 5)?

Materials

- Line Master: 800 Game
- Scissors
- Calculators

Note: To make the game easier, use only two zeros with the eight and omit the decimal point.

Instructions

Copy and cut out cards; distribute one set (3 zeros, decimal point, and 8) to each student. Students are to work with partners.

Instructions for players:

1. Make a number with your cards.
2. Decide how you could change the value of your number so that it is equal to the value of your partner number.
3. Use a calculator to test your suggestion.
4. Try some more examples.
5. Talk to other players.
   a) Are they doing it the same way as you?
   b) How do you account for any differences?

Observe students as they play and talk. Record what they understand about the multiplicative relationship between the places.

Note: See *Number Sense* Resource Book, p. 70, for additional ideas to bring out the mathematics and to extend this diagnostic task into a learning activity.

Discussion

In each of the following phases, what numbers and strategies would a student be able to use in playing this game?

	Partitioning	**Factoring**	**Operating**
Numbers			
Strategies			

Using Place Value

Key Understanding 6: Place value helps us to think of the same whole number in different ways and this can be useful. (*Number Sense* Resource Book, p. 72)

What are the important ideas in KU 6?

Teaching place value concepts separately as a prerequisite to double digit addition and subtraction is ineffective and unnecessary….

In fact, manipulative materials may actually detract from thinking because tasks are too easy to do with the materials.

Ross, Sharon, *Arithmetic Teacher*, 1989.

What can students who have reached the end of the Partitioning phase do in relation to place value?

What can students who have reached the end of the Factoring phase do in relation to place value?

Bridging to Practice: Patterns in the Way We Write Numbers

Knowing What Students Know

- Use Diagnostic Task: Dinosaurs (Course Book, p. 180)
- Use Diagnostic Task: 52 Candies (Course Book, p. 183)
- Use Diagnostic Task: 43 Candies (Course Book, p. 184)

Moving Students Along

Select Sample Learning Activities from the *Number Sense* Resource Book, pp. 66–71 for:

- Understanding the value of each digit in 3-digit numbers

UNIT

4 Whole and Decimal Numbers

FROM FRACTIONS TO DECIMALS

Desired Outcomes

Participants will

- become familiar with the mathematical ideas of Whole and Decimal Numbers Key Understanding 7
- examine how the phases in the Diagnostic Map: Number relate to decimal understanding
- investigate the development of decimal understanding
- explore learning experiences that develop decimal understanding

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps In Mathematics: Number Sense* Resource Book**

Chapter 2: Whole and Decimal Numbers
Key Understanding 7, pp. 80–85

Chapter 3: Fractions
Key Understanding 2, pp. 112–123
Key Understanding 6, pp. 152–159

Diagnostic Map: Number

Partitioning phase
Factoring phase
Operating phase

Understanding Decimals

Key Understanding 7: We can extend the patterns in the way we write whole numbers to write decimals. (*Number Sense* Resource Book, p. 80)

Activity: Decimal Place Value

Wholes		Parts of One Whole			
Ones	•	Tenths	Hundredths	Thousandths	Ten thousandths
	•				

In the space above, record the decimal number your model represents.

- What links are there among the ways we record whole and decimal numbers?

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Decimal Development

This table demonstrates that students' understanding of decimals depends on what they understand about whole number place value and fractions.

Whole Numbers	Decimals	Fractions
By the end of the Quantifying phase, students • trust that whole numbers tell them how many are in a collection. • name the places as Hundreds, Tens, and Ones, but don't realize what this means.	By the end of the Quantifying phase, students typically think • the decimal point is a decoration or a punctuation. • a half is 0.5 and so think a quarter is 0.55.	By the end of the Quantifying phase, students • talk about halving but think of halves as one of two bits. • may say, "*You have the big half. I don't want it.*" • think halves have to look the same.
By the end of the Partitioning phase, students • understand the magnitude of hundreds, tens, and ones, and know for example that 4 in the tens place is 40. • use standard-place value to partition numbers.	By the end of the Partitioning phase, students typically hold one or many of these ideas (i.e., they think digits to the right of the decimal point have): 1. Whole Number Place Values a) They think of the decimal point as separating two sets of whole numbers. The ones on the left are the big whole numbers and the ones on the right are the little whole numbers. For example, they think of dollars and cents as collections where whole number PV applies. b) They think the decimal point separates 2 units of money or measures (e.g., 6.125 as \$6.125, which is really 7.25). 2. Some students come to see numbers on the right of the decimal point are different in some way. For example, c) they think the numbers on the right are the reverse of the whole number place-value system, so think the 1 in 0.125 are *oneths*, the 2 are tenths, and the 5 are hundredths and so think decimals are the *th*s. d) they think that the more digits, the smaller the number. e) they think the fewer digits, the smaller the number. 3. Parts or Fractions of the Previous Digit to Left f) Many students begin to link decimals to fractions but may hear teachers say decimals are fractions and so think, e.g., "point six" is really "one sixth". g) Many students know 0.5 is a half and think 0.05 is a half of a half and so call 0.05 a quarter. h) At first, students think that hundredths are hundredths of the tenths, not of hundredths of the whole.	By the end of the Partitioning phase, students • understand and partition a whole into equal portions and name each of the portions as a unit fraction. (e.g., If I cut up this apple into 4 equal parts, each part will be one quarter. If I have two parts, that will be two quarters.)

Whole Numbers	Decimals	Fractions
By the end of the Factoring phase, students • know the value of each digit in a number by the position it is from the left of the decimal point. • think of numbers as non-standard place values (e.g., 582 can be thought of as 382 + 200).	By the end of the Factoring phase, students typically • correctly link the unit fraction symbol to the decimal fraction symbol as represented on the calculator (e.g., 1/10 = 0.1). • may think decimal fractions are negative numbers.	By the end of the Factoring phase, students • know that if one is divided by ten, the result will be tenths. • know that if a tenth is divided by ten, the result will be a hundredth of the whole. • know that if a hundredth is divided by ten, the result will be a thousandth of the whole they began with, and so on.
By the end of the Operating phase, students • understand that each place to the left is ten times greater than the previous place. • write any whole and decimal number using their knowledge of the thousands cycle.	By the end of the Operating phase, students • sensibly round 3 or more decimal places to the nearest two-place decimal number. • think flexibly between decimal fractions, common fractions, and percentages.	By the end of the Operating phase, students • understand equivalence. • add unlike fractions by converting to common fraction sizes.

Notes

Student Misconceptions

DIAGNOSTIC TASK: Decimals

KU 7 for Grades 5–9 (ages 10–14)
(See Course Book, p. 197.)

Use this task to become familiar with the decimal misconceptions students may develop.

By the end of the Quantifying phase ...

Students think the decimal point is a decoration or punctuation.

Compare and Order

Circle the biggest number in each group of three:

i)	**5 436**	or	547	or	56
ii)	**6.78**	or	45.6	or	345
iii)	**3.521**	or	3.6	or	3.75
iv)	15.4	or	15.56	or	**15.327**
v)	4.09	or	4.7	or	**4.008**

Swan, M. Decimals

By the end of the Partitioning phase ...

Students use whole number place value reasoning.

a) The decimal point separates two sets of whole numbers. The ones on the left are the big whole numbers and the ones on the right are the little whole numbers.

Compare and Order

Circle the biggest number in each group of three:

i)	5 436	or	547	or	56
ii)	6.78	or	45.6	or	345
iii)	3.521	or	3.6	or	3.75
iv)	15.4	or	15.56	or	15.327
v)	4.09	or	4.7	or	4.008

Swan, M. Decimals

By the end of the Partitioning phase ...

Students use whole number place value reasoning.

b) The decimal point separates 2 units of money or measures, e.g., 6.125 means $6.125, which is really 7.25.

Compare and Order

Circle the biggest number in each group of three:

i)	5 436	or	547	or	56
ii)	6.78	or	45.6	or	345
iii)	3.521	or	3.6	or	3.75
iv)	15.4	or	15.56	or	15.327
v)	4.09	or	4.7	or	4.008

Swan, M. Decimals

By the end of the Partitioning phase ...

Students realize decimals have some sort of different value.

c) Students come to see numbers on the right of the decimal point are different in some way (e.g., reverse of the whole number place-value system).

Compare and Order

Circle the biggest number in each group of three:

i)	5 436	or	547	or	56
ii)	6.78	or	45.6	or	345
iii)	3.521	or	3.6	or	3.75
iv)	15.4	or	15.56	or	15.327
v)	4.09	or	4.7	or	4.008

Swan, M. Decimals

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By the end of the Partitioning phase ...

Students realize decimals have some sort of different value.

d) The more digits a decimal has, the smaller the number, and so the less digits, the greater the number.

Compare and Order

Circle the biggest number in each group of three:

i)	5 436	or	547	or	56
ii)	6.78	or	45.6	or	345
iii)	3.521	or	3.6	or	3.75
iv)	15.4	or	15.56	or	15.327
v)	4.09	or	4.7	or	4.008

Swan, M. Decimals

By the end of the Partitioning phase ...

Students realize decimals have some sort of different value.

e) The fewer digits a decimal has, the smaller the number and so the more digits it has, the greater the number.

Compare and Order

Circle the biggest number in each group of three:

i)	5 436	or	547	or	56
ii)	6.78	or	45.6	or	345
iii)	3.521	or	3.6	or	3.75
iv)	15.4	or	15.56	or	15.327
v)	4.09	or	4.7	or	4.008

Swan, M. Decimals

By the end of the Partitioning phase ...

Students realize decimals are fractions.

f) The decimal number describes the denominator, not the numerator, and students may think "point six" is really "one sixth."

Compare and Order

Circle the biggest number in each group of three:

i)	5 436	or	547	or	56
ii)	6.78	or	45.6	or	345
iii)	3.521	or	3.6	or	3.75
iv)	15.4	or	15.56	or	15.327
v)	4.09	or	4.7	or	4.008

Swan, M. Decimals

By the end of the Factoring phase ...

g) Students correctly link the unit fraction symbol to the decimal symbol as represented on the calculator (e.g., 1/10 = 0.1).

Compare and Order

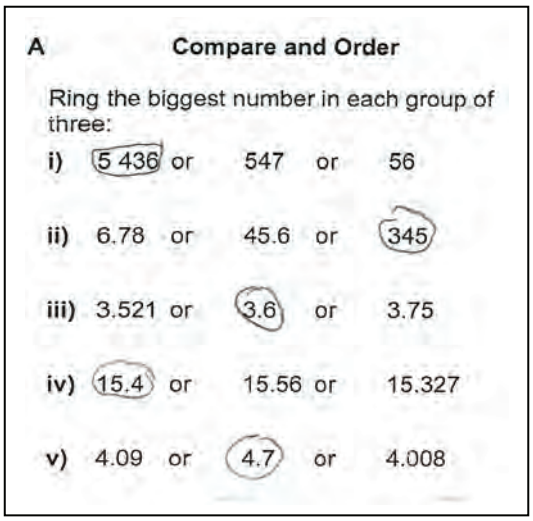
Circle the biggest number in each group of three:

i)	5 436	or	547	or	56
ii)	6.78	or	45.6	or	345
iii)	3.521	or	3.6	or	3.75
iv)	15.4	or	15.56	or	15.327
v)	4.09	or	4.7	or	4.008

Swan, M. Decimals

Student Work Samples

Zac's work samples are shown here. Zac is 14 years old.



A **Compare and Order**

Ring the biggest number in each group of three:

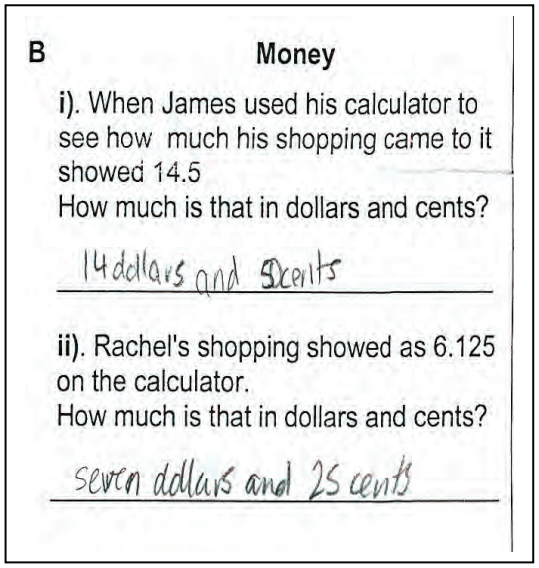
i) 5 436 or 547 or 56

ii) 6.78 or 45.6 or 345

iii) 3.521 or 3.6 or 3.75

iv) 15.4 or 15.56 or 15.327

v) 4.09 or 4.7 or 4.008



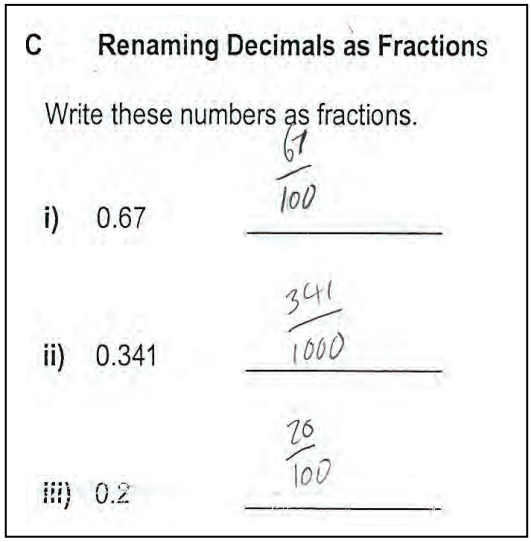
B **Money**

i). When James used his calculator to see how much his shopping came to it showed 14.5
How much is that in dollars and cents?

14 dollars and 5cents

ii). Rachel's shopping showed as 6.125 on the calculator.
How much is that in dollars and cents?

seven dollars and 25 cents



C **Renaming Decimals as Fractions**

Write these numbers as fractions.

i) 0.67 — 67/100

ii) 0.341 — 341/1000

iii) 0.2 — 20/100

Through the Quantifying phase
Students think the decimal point is a decoration or punctuation.

During and through the Partitioning phase
Students develop misconceptions based on partial concepts about place value and fractions.

Whole number place value:

a) decimal point *separates two sets of whole numbers*, the ones on the left are the big whole numbers and the ones on the right are the little whole numbers.
b) decimal point separates 2 units of money or measures, e.g., 6.125 as $6.125 which is really 7.25.

Some sort of different value:

c) reverse of whole number system and so students think the 1 in 0.125 are *oneths*, etc…
d) the more digits, the smaller the number
e) the less digits, the smaller the number

Parts or fractions of the previous digit to left:

f) link decimals to fractions but think, e.g., "point six" is really "one sixth"
g) know 0.5 is a half but think 0.05 is a half of a half
h) hundredths of tenths not hundredths of one whole

During and through the Factoring phase
Students know unit fraction symbol 1/10 is shown as decimal fraction symbol 0.1 but may think decimal fractions are negative numbers.

During and through the Operating phase
Students

- round 3 or more decimal places to the nearest two-place decimal number because it makes sense to do so.
- think flexibly between decimal fractions, common fractions, and percentages.

D Naming Digits after the Decimal Point.

i) What does the 3 mean in 0.236?

3hundredths

ii) What does the 2 mean in 0.236?

2 tenths

iii) What does the 6 mean in 0.236?

6 thousandths

E Ordering

Sonya said, "When we put books on the library shelf we put 65.6 before 65.125 because 6 is less than 125." but Tao didn't agree.

Who is right? Sonya

Why do you think that?

Write your explanation in this box.

The numbers go up each time up to 9 then the number before the dot goes up one meaning 6.999 + 1 = 7.000 + 1 = 7.001

F Write down the next 2 numbers in each sequence

a) 0.2, 0.4, 0.6, 0.8, 1 (add on 0.2 each time)

b) 0.3, 0.6, 0.9, 1.2, 1.5 (add on 0.3 each time)

c) 0.92, 0. 94, 0.96, 0.98, 1, 1.2 (add on 0.02 each time)

d) 1.13, 1.12, 1.11, 1.10, 1.09 (take away 0.01 each time)

G Quantity

Paper clips come on boxes of 1 000. Abi counted the loose paper clips in a tray and said there were 1 260. Jeremy said that's 1.26 boxes of paper clips. Could they both be right?

Yes ☑ No ☐

Why do you think that?

because it is because 1 means 1000 and 26 means 260 because you add a zero. example. 1.20m = 120cm

H Number Sequence

How would you use a calculator to generate this number sequence?

2.0, 0.2, 0.02, 0.002

you would put 2.0 - 1.8 = 0.2 then clear then put 0.2 - 10 which gives us the pattern

Activity

Show what Zac understands about the magnitude of decimals by linking each set of responses to the appropriate phase descriptors.

Discussion: Reflection on Creating a Model

- What did you think about as you planned your model?

- What did you have to know to create your own model?

- What was easy about creating your own model? What was challenging?

- What discoveries did you make when predicting the value of another person's model?

Bridging to Practice: Decimals

Knowing What Students Know

- Use Diagnostic Task: Decimals, found on page 197 of this Course Book.

Moving Students Along

Select Sample Learning Activities from the *Number Sense* Resource Book, pages 88 to 93, for:

- Understanding the value of each digit in numbers with decimals to two places

- Understanding the ten times greater, ten times smaller relationship between the places for whole and decimal numbers

- Understanding that there are numbers between whole numbers

Background Notes: Progress Of Decimal Understanding

These notes relate to Key Understanding 7 (*Number Sense* Resource Book, p. 80).

1. A decimal point is like a punctuation or a decoration.

Students think the decimal point can be ignored so that 2.25 really means 225. In the early years when students are first learning about the counting system, it is normal to expect students to simply ignore what they don't make sense of. Later they may think you can choose to put in a decimal point or leave it out, and it just changes the way you say the number, not the quantity.

Note that students are often taught that removing the decimal point in $2.25 gives you the number of cents—225 cents—which has the same value as $2.25. Therefore, there is logic in the idea that students can believe it is just about changing how you say the number because 225 cents is the same amount of money as $2.25.

2. A decimal point separates two whole numbers.

Students think the numbers on each side of the decimal point are independent whole numbers referring to two different sets of objects. They may therefore believe you can drop the zeros between the decimal point and digits to the right, and may assign value to the zeros after the digits, i.e., they follow the same whole number place value rules separately for each of the two "whole numbers"; e.g., they think that 0.5, 0.05 are both 5, but 0.50 is 50. Or they may think 0.5 is 50 "because someone left the zero off." These ideas about decimals are often strongly tied to their knowledge of money and measurement.

During the Partitioning phase students may progress through different levels of meaning:

a) Students may initially believe that the decimal point can separate any two sets of objects, with the objects on the left being larger than the objects on the right. (Partitioning phase a)

- Big apples and little apples
- Adults and children
- Cars and bicycles

The students may think that you can add the two numbers together because, in the examples above, they are all apples, all people, or all vehicles; e.g., 85.6 apples means 85 big apples and 6 smaller apples, which would be 91 apples altogether.

b) They may then come to think that the decimal point can only separate units of money or measurements, not just any two sets of objects.
(Partitioning phase b)

- Dollars and cents
- Metres and centimetres
- Kilograms and grams—but may over-generalize and think this can also include:
- Hours and minutes
- Years and months
- Even goals and points

They generally still read the units and sub-units as two independent whole numbers and therefore think that \$4.125 is \$4 and 125 cents, or \$5.25. Note there is some logic in this—consider adding \$4.95 and 30 cents: in the process of adding the amounts they can logically and correctly be thought of as \$4 and 125 (95 + 30) cents; for the child who thinks of the decimal point as a separator between dollars and cents, it is sensible to read \$4.125 as 4 dollars and 125 cents.

c) Eventually students realize decimal notation is restricted to measurements where the two types of units have an exchange value of either 10s, 100s or 1000s (e.g., 100 ¢ = \$1, 10 mm = 1 cm, 1000 g = 1 kg).

They may still believe the digits to the right of the decimal point make up a separate whole number of subunits and so make errors like 3.25 cm = 3 cm, and 25 mm = 5 cm 5 mm. However, they often learn that 1 decimal place is always about 10 units (e.g., mm in cm: 0.9 means 9 mm and 1 more must be 1.0 read 1 cm and no mm, rather than 0.10); that 2 decimal places are always about 100 units (e.g., cm in m, cents in \$s); and 3 decimal places are about 1000 units (e.g., grams in kg, metres in km). The important thing to note here is that although students continue to believe the digits on the right are a whole number of subunits, they can learn to correctly exchange subunits for units and so calculate with most measurements accurately.

Students in this stage can think that decimals *are* metric measures rather than understand that we have used decimal place value relationships to construct the metric measurement system to make calculations and conversions easier. Knowing how metric measurements work is not the same as understanding how fractional numbers are represented in decimal notation.

Note that students can develop these misconceptions while they are still using Quantifying ideas—the way decimals and metric measurements are introduced and taught may have an impact on which ideas precede the others, and indeed, with careful teaching, it may be possible for students to skip the often unhelpful "rule" based on misconceptions that typify Partitioning thinking. (That is, they may be able to move straight from thinking of decimals as metric units to understanding the more helpful idea of a decimal point separating whole numbers from a part of a whole.)

3. **In decimals the numbers on the right are different in some way**
(Partitioning phases c, d, e)

Students come to think that the digits on the right do not represent a "normal" whole number—they still may think they are like a whole number but believe they behave differently than expected and look for rules to explain how they work. They may even think that decimals out of a context behave differently than in money and measurement (i.e., ideas developed while moving through the Quantifying and Partitioning phases may develop and continue in parallel). Giving students "rules" to operate on decimals may exacerbate their misconceptions.

- "The more digits, the smaller the number, or, the fewer digits, the larger the number." They may have been told the place values get smaller the further from the decimal point, but they have confused the values of the places with the values of the digits in the places.
- "The place value columns reverse after the decimal point."
They may add a "oneth" place to keep symmetry around the decimal point, then tenths place and hundredths place—often reading the digits backwards believing that hundredths place is about "hundreds" back to front.
- Students may believe the decimal point separates whole numbers from negative numbers.

4. **In decimals the numbers on the right are about parts or fractions**
(Partitioning phases f, g, h)

Students recognize that the decimal point separates whole numbers from parts or fractions of some kind, but may not see how the parts are derived from the numbers, e.g.:

- They may think the digits after the decimal point are denominators of unit fractions.
- They may think the digits refer to a number of halves or quarters.
- They learn that 0.5 is a half, but think that 0.05 is half of a half or a quarter.

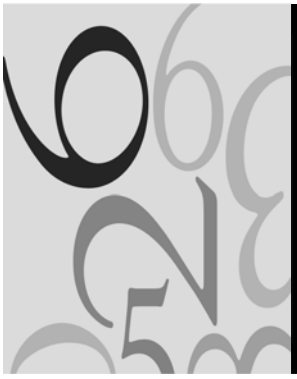
If students understand fractions sufficiently well, they may learn that the digits in the "tenths" column describe a number of tenths (just as the digit in the 10s column describes a number of 10s) so they may be able to make conventional sense of decimal numbers in an additive way and be able to say that 0.26 means 2 tenths and 6 hundredths; they may also recognize it as 26 out of 100 (as in cents or cm) but not be able to link the two, or understand why 26 divided by 10 must be 2.6 and 26 divided by 100 must be 0.26. They may know that 0.25 is equal to ¼ and say this is because 25 is a quarter of 100, but not understand how this relates to 2 tenths and 5 hundredths.

5. **Decimal fractions extend whole number place relationships to represent numbers between the whole numbers.**
(During and through the Factoring and Operating phases)

Students realize that the places after the decimal point have the same "times 10" relationships as whole numbers and link this to the idea that a unit can be divided and re-divided into increasingly smaller quantities. They recognize that the decimal point indicates which digit is in the units place, and that the value of all other places are derived multiplicatively from the position of the units.

They understand the links between decimal fractions, common fractions, percentages, multiplication and division by powers of 10, the cohesive whole of the number system, and metric measurements. They can flexibly move between different representations of the same number and understand the connections.

Notes



UNIT

5

Classroom Research

TASK REVIEW

Desired Outcomes

Participants will

- become more familiar with the phases of the Diagnostic Map
- identify patterns in student responses
- identify the Key Understandings that students need to learn
- use the *Number Sense* Resource Book to plan learning activities and focus questions to move students on

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps In Mathematics: Number Sense* Resource Book**

Chapter 1: An Overview of *First Steps in Mathematics*

How to Read the Diagnostic Map, p. 12

Planning with *First Steps in Mathematics*, p. 15

Diagnostic Map: Number

Emergent phase
Quantifying phase
Matching phase
Partitioning phase

Suggestion for Further Reading

Fennema, E., Franke, M.L. et al. "Using Children's Mathematical Knowledge in Instruction." *American Educational Research Journal*, 30(3): 555–583, 1993.

Understanding the Research Task

Bridging to Practice: Focusing on Students

Think about the Diagnostic Tasks you have examined so far. Some tasks can be used as a whole class activity or with a smaller group of students. Other tasks require an individual interview. Do the same set of tasks with at least six students in your class. Do not prompt or teach students as they do the task. Record what they say and do.

Selecting Diagnostic Tasks

What tasks will you use with your students to find out what they know about the Whole and Decimal Numbers Key Understandings? Predict what responses students will give. Be as specific as possible.

Preparing for the Task Review

1. Sort your student work samples into two groups: correct and incorrect responses.

2. Sort the incorrect responses group into common responses. How you sort them is up to you, but you will need to explain your groupings in the next session. Think about the following:
   - what the student knows
   - what the student needs to know (which Key Understanding)
   - in what phase of the Diagnostic Map: Number the student may be
   - what math concepts the student needs to learn next (which dot points in the Key Understanding)

Use sticky notes to annotate students' work and to prepare for the sharing session with your colleagues.

Note: Remember to bring your sorted work samples to the next workshop.

Analyzing Students' Work

1. In your small group, sort through your work samples and put them into groups of common responses.

- What math do the students in each group know?

_____

_____

- What math do they need to learn next?

_____

_____

- Which Key Understandings will help?

_____

_____

- In which phase of the Diagnostic Map are these students?

_____

_____

2. Create a chart or sheet to share with the rest of the participants. The chart should show the phase each group of students is in and the indicators that helped you decide on the phase. See the sample charts provided below for ideas.

Task Review Chart for: [Diagnostic Task, e.g., Dinosaurs]			
Student Name	**Observations**	**Phase**	**What's next?**
Nula			
Diem			

Task Review Chart for: [Student name, e.g., Nula]			
Diagnostic Task	**Observations**	**Phase**	**What's next?**
Dinosaurs			
800 Game			

Selecting Sample Learning Activities

What Sample Learning Activities will you use with your students?

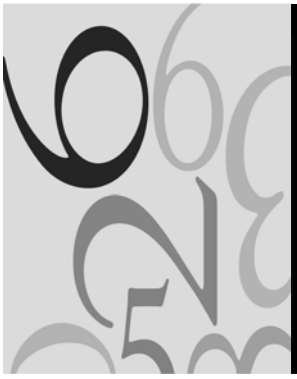
What is the mathematical focus or focuses of the lesson(s)?

Reflecting On Professional Decision Making

What did you learn about the math?

What did you learn about your students?

What did you learn about your teaching?



UNIT

6 Operations

FROM SITUATION TO OPERATION

Desired Outcomes

Participants will

- begin to distinguish between understanding operations and computation
- gain an understanding of the early development of concepts of addition and subtraction
- further develop their understanding of the phases of the Diagnostic Map: Number

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps In Mathematics: Operation Sense* Resource Book**

Chapter 1: An Overview of *First Steps in Mathematics*

Beliefs about Teaching and Learning. Learning Mathematics: Implications for the Classroom, p. 2

Understanding the Elements of *First Steps in Mathematics*, p. 8

How to Read the Diagnostic Map, p. 12

Chapter 2: Operations

Background Notes, pp. 22–29

Key Understanding 1, pp. 32–33

Key Understanding 2, pp. 40–41

Key Understanding 8, pp. 94–95

Diagnostic Map: Number

Emergent phase through to Operating phase

Suggestion for Further Reading

Fennema, E., Franke, M.L., et al. "Using Children's Mathematical Knowledge in Instruction." *American Educational Research Journal*, 30(3), pp. 555–583, 1993.

Operations and Computations

Operations and Computations Outcomes

Operations

- Understand the meaning, use, and connections between addition, multiplication, subtraction, and division. (*Operation Sense* Resource Book, p. 21)

Computations

- Choose and use a repertoire of mental, paper, and calculator computational strategies for each operation, meeting needed degrees of accuracy and judging the reasonableness of results. (*Operation Sense* Resource Book, p. 107)

Ruth's Story

A bright seven-year-old, Ruth, asked me to help her with some problems. They were taken from a rack of cards that were grouped according to difficulty and Ruth proudly told me that she was doing the hardest ones. On checking the last five problems she had done I found they were all correct, so I assumed Ruth wanted attention and praise for the work rather than assistance.

"I thought you wanted me to help you. You've got them all right," I said.

"Yes," she replied, "I know how to do them but why did I have to add in this one and times in that one?"

She pointed to two of the cards, which read: *Jon had 22 marbles and his brother had 35 marbles. How many marbles did they have altogether*? and *Seven children had six sweets each. How many sweets were there altogether?*

Teacher: How did you manage to get them right? Did someone tell you how to do them?

Ruth: No. This one's a times because I've done things like six times seven and these are hard cards so it wouldn't be six add seven, or a take-away sum.

Teacher: What about the question about the marbles?

Ruth: We haven't done "timesing" big numbers so it can't be that. It must be an "add-up."

Teacher: Is there anything else it could be?

Ruth: Well, it's not a take-away.

Teacher: Why not?

Ruth: It says altogether and altogether sums are add-ups.

Hollands, R., *Mathematics Teachers Forum*, Bulletin 41, Jan/Feb 1997

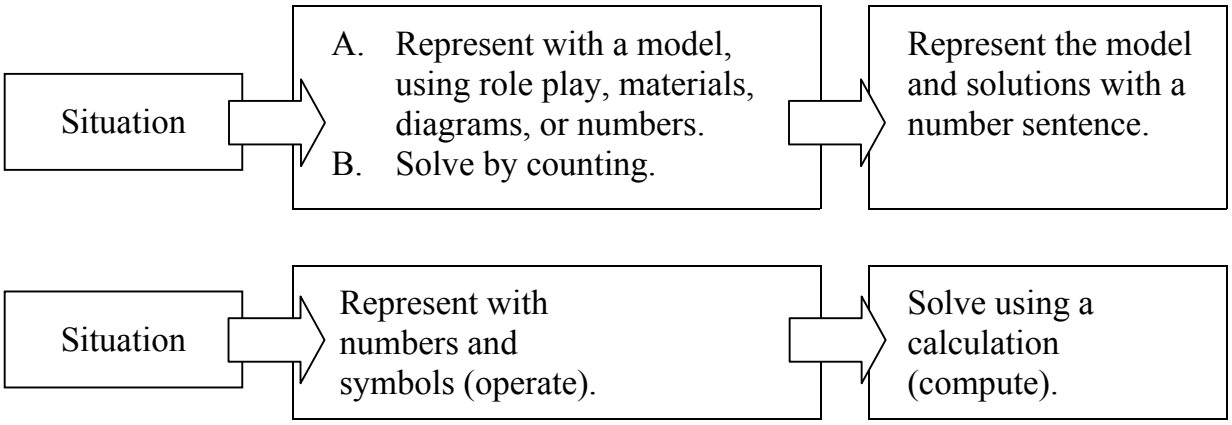
Analysis of Ruth's Story

	Ruth knows	Ruth doesn't know
Compute		
Operate		

What are some factors that impede students in developing a robust understanding of the meaning of, use of, and connections between addition, multiplication, subtraction, and division?

Moving from Situation to Operation

Key Understanding 1: Adding and subtracting numbers are useful when we: change a quantity by adding more or taking some away; think of a quantity as combined of parts; and equalize or compare two quantities. (*Operation Sense* Resource Book, p. 32)



Notes

FSIM012 | First Steps in Mathematics: Number Course Book

Student Work Samples: Comparing Bananas

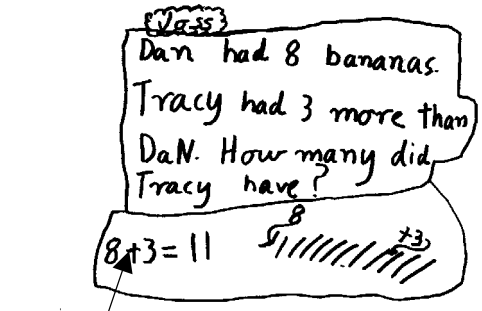
Ellen

Solve?	
Represent quantity?	
Represent action?	
Numbers and operation?	
Phase?	

Evidence from Diagnostic Map:

Joss

Solve?	
Represent quantity?	
Represent action?	
Numbers and operation?	
Phase?	



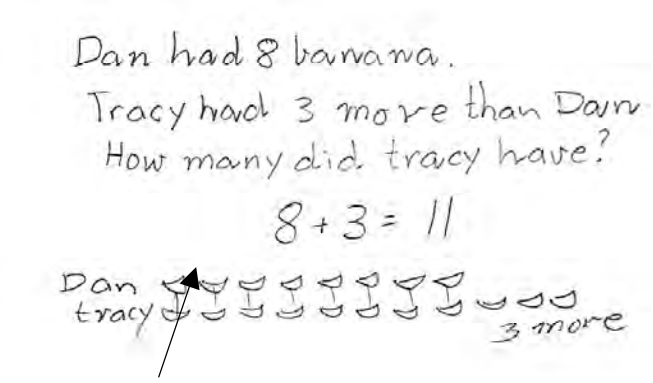
Joss wrote the number sentence after prompting by the teacher.

Evidence from Diagnostic Map:

Chew En

Solve?	
Represent quantity?	
Represent action?	
Numbers and operation?	
Phase?	

Evidence from Diagnostic Map:



Chew En represented the Bananas with numbers and used the number sentence to show that she solved the problems in her head using basic facts.

Danny

Solve?	
Represent quantity?	
Represent action?	
Numbers and operation?	
Phase?	

Evidence from Diagnostic Map:

Bridging to Practice: Situation to Operation

Knowing What Students Know

Diagnostic Tasks

K–Grade 3 *(ages 4–8)*

- "Messages" from Sample Learning Activities in the *Operation Sense* Resource Book, p. 34
- Think Board in the *Operation Sense* Resource Book, p. 23
- Sample Learning Activities in the *Operation Sense* Resource Book, pp. 56–58

Grades 3–7 *(ages 8–12+)*

- Think Boards, pp. 229–230 in this Course Book

Case Study

K–Grade 3 *(ages 4–8)*

- "Messages" task in the *Operation Sense* Resource Book, p. 56

Notes

One Situation, Different Operations

Key Understanding 8: Thinking of a problem as a number sentence often helps us to solve it. Sometimes we need to re-write the number sentence in a different but equivalent way. (*Operation Sense* Resource Book, p. 94)

What are the important ideas in Key Understanding 8?

Remember:
Students need a lot of experience in representing problems in ways that enable them to deal with the problems mathematically.

(*Operation Sense* Resource Book, p. 94, paragraph 1)

DIAGNOSTIC TASK: How Much Taller?

KU 1 and 2 for Grades 4–7 (ages 10–12+)
(See below and Course Book, p. 208.)

Name: _____

Grade: _____ Date: _____

Line Master How Much Taller?

Jesse and Sylvia were chatting on the Net. Jesse said that she was 154 cm tall and Sylvia said she was 132 cm. Jesse said, "I am taller than you." Sylvia said, "Yes, but not by much."

1. How much taller is Jesse than Sylvia? _____

2. Explain how you worked out the answer.

3. Write a number sentence that you could put into a calculator to work it out.

Diagnostic Tasks 209

Purpose

To see if students are able choose an appropriate operation to solve a comparison problem

Materials

- Line Master: How Much Taller?

Instructions

Read the question aloud to the students while they follow on the sheet. Ask them to paraphrase the question so that you are sure they understand what you are asking.

Ensure that the students understand the phrase "number sentence" by writing one or two examples on the board: e.g., 7 + 3 = 10 or 10 – 7 = 3. Make sure that you do not use the numbers from the problem.

- Do **not** allow students to use calculators for this task.

If using this as a whole class task, follow-up interviews to clarify what some students are thinking may be necessary.

Student Work Samples

Tanya

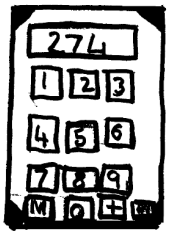
Name Tanya Grade 5

Jesse and Sylvia were chatting on the Net. Jesse said that she was 154 cm tall and Sylvia said she was 132 cm. Jesse said, "I am taller than you." Sylvia said, "Yes, but not by much."
How much taller is Jesse than Sylvia? 22cm taller

Explain how you worked out the answer.
If Jesse is taller than Sylvia she is 22 cms taller. I worked it out by counting it on my fingers.

Write a number sentence that you could use in a calculator to work it out.
If you could have a guess of which number goes between it, you might be right that's if it's 22.

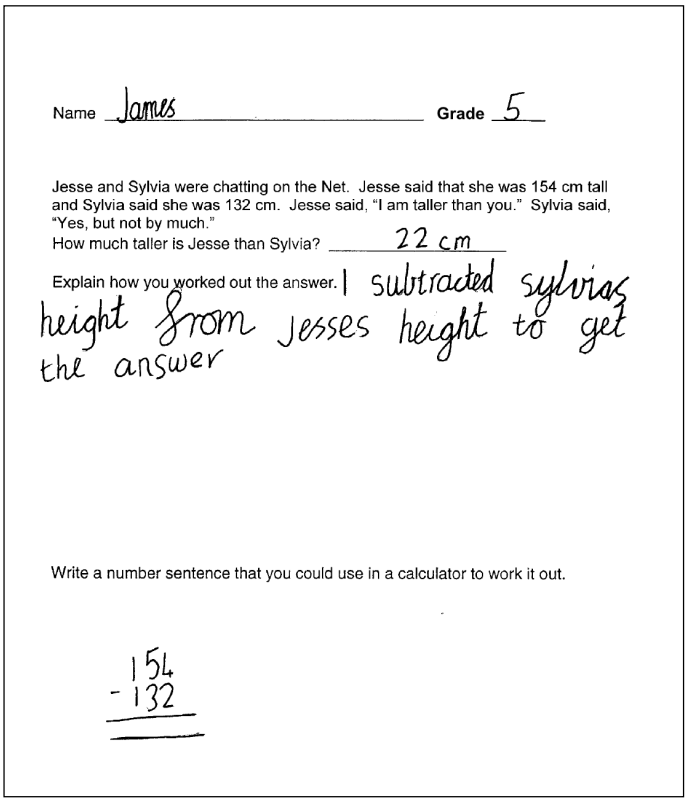
132
+22
154



Analyzing the Evidence

Is Tanya able to solve the problem? How?	
How does she represent each quantity?	
How does she represent the action?	
Is Tanya able to use numbers and an operation to solve the problem? How?	
Record behaviours from the Diagnostic Map that you think relate to Tanya.	

James



Name James Grade 5

Jesse and Sylvia were chatting on the Net. Jesse said that she was 154 cm tall and Sylvia said she was 132 cm. Jesse said, "I am taller than you." Sylvia said, "Yes, but not by much."
How much taller is Jesse than Sylvia? 22 cm

Explain how you worked out the answer. I subtracted sylvias height from jesses height to get the answer

Write a number sentence that you could use in a calculator to work it out.

154
-132

Analyzing the Evidence

Is James able to solve the problem? How?	
How does he represent each quantity?	
How does he represent the action?	
Is James able to use numbers and an operation to solve the problem? How?	
Record behaviours from the Diagnostic Map that you think relate to James.	

Moving Students Along

Select Sample Learning Activities from pages 34 to 39 in the *Operation Sense* Resource Book for:

- The introduction to addition and subtraction

- Recognizing a wide range of problem types to which addition and subtraction apply

Notes

Equivalent Number Sentences

Knowing that the same situation can be represented by different operations is critical for success with addition and subtraction.

Students find it more difficult to work with representations that do not match the actions in the problem. They need many opportunities to become confident that transformed number sentences are always "asking the same question."

If they don't trust that the inverse relationship always works, they are forced to rely on trial and error or rote procedures to solve problems such as ? + 16 = 18.

Look at the following example:

- We ate 16 biscuits between us and there are 18 left in the packet. How many were in the packet to start with?

  ? – 16 = 18

  18 + 16 = ?

Why are these two number sentences equivalent?

FSIM012 | First Steps in Mathematics: Number Course Book

Bridging to Practice: One Situation, Different Operations

Knowing What Students Know

Diagnostic Tasks

K–Grade 3 *(ages 4–8)*

- How Many Want Jelly Beans? (Problem B) (Course Book, p. 220)

Grades 3–7 *(ages 8–12+)*

- Calculator Number Sentences (p. 217)
- How Much Taller? (Course Book, p. 208)

Moving Students Along

Select KU 8 Sample Learning Activities from pages 96–101 in the *Operation Sens*e Resource Book for:

- Learning to represent situations as number sentences
- Knowing how to write equivalent number statements

How would you introduce each activity to your students? How would you focus your students on the mathematical idea?

Talking to Parents

Make some notes you could use in talking to parents about the Operations outcome. What is it about? Why is it important? How is it different from doing computations?

UNIT

7 Operations

TYPES OF ADDITION AND SUBTRACTION PROBLEMS

Desired Outcomes

Participants will

- develop their understanding of the range of addition and subtraction problem types
- develop an understanding of the connections between addition and subtraction
- develop an understanding of the Matching, Quantifying, and Partitioning phases of the Diagnostic Map: Number

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps In Mathematics: Operation Sense* Resource Book**

Chapter 1: An Overview of *First Steps in Mathematics*

Beliefs about Teaching and Learning, p. 2
Understanding the Elements of *First Steps In Mathematics*, p. 8
How to Read the Diagnostic Map, p. 12

Chapter 3: Operations

Background Notes, p. 22
Key Understanding 3, p. 48
Key Understanding 4, p. 60
Key Understanding 6, p. 82

Diagnostic Map: Number

Emergent phase through to Operating phase

Suggestion for Further Reading

Fennema, E., Franke, M.L., et al. "Using Children's Mathematical Knowledge in Instruction." *American Educational Research Journal*, 30(3), pp. 555–583, 1993.

Types of Addition and Subtraction Problems

Key Understanding 1: Adding and subtracting numbers are useful when we

- change a quantity by adding more or taking some away
- think of a quantity as combined of parts
- equalize or compare two quantities

(*Operation Sense* Resource Book, p. 32)

Remember:
Students should be helped to see how these types of problems can all be thought of in terms of part-part-whole, and can be solved using the same operations.

What mathematics do students need to learn to understand the meaning and use of addition and subtraction?

The Nature of the Unknown Part

Write a word problem for each of these representations.

Type of Problem	Numeric Representation	Word Problem
unknown result	9 + 7 = ?	
unknown change	9 + ? = 16	
unknown start	? + 7 = 16	

Which of these problem types do you think students see most often?

Which of these problem types do you think students find easiest to deal with and why?

Remember:
Students need exposure to the full range of common problem types. Students cannot develop their understanding of the connections between the operations if they are not exposed to problems that require them to use these connections.

Change Tasks

Write numbers and symbols that match the *semantics* of each problem below.

1. Anna had 76 trading cards and then her brother gave her 39. How many does she have now?

2. Anna has 117 jelly beans but would like to have 310. How many more does she need to get?

3. Anna had some marbles and then her brother gave her 93. Now she has 108. How many did she have to start with?

4. Anna had 87 trading cards and then she gave her brother 38. How many does she have left?

5. Anna had 157 jelly beans and then she gave her brother some. Now she has 78 left. How many did she give her brother?

6. Anna had some candies and gave her brother 38 of them. Now she has 77 left. How many did she have to start with?

Discussion

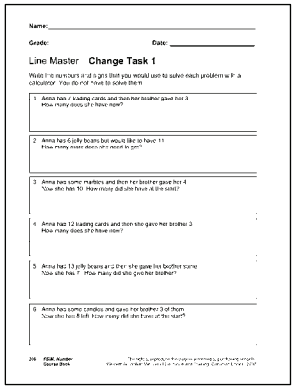
What impact has the availability of calculators had on young students' early ideas about numbers?

What are the implications of this for teaching?

Remember:
Using a calculator helps students to understand the inverse relationship between addition and subtraction, and multiplication and division.

DIAGNOSTIC TASK: Change Task

KU 2 for Grades 1–7 (ages 6–12+)
(See below and Course Book, p. 205.)



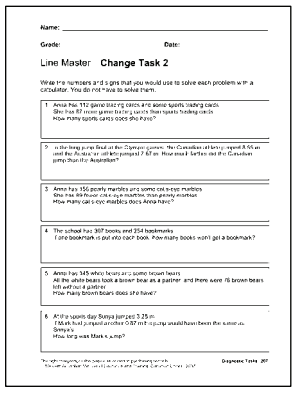
Name: _____

Grade: Date: _____

Line Master Change Task 1

Write the numbers and signs that you would use to solve each problem with a calculator. You do not have to solve them.

1. Anna has 7 trading cards and then her brother gave her 3. How many does she have now?
2. Anna has 6 jelly beans but would like to have 11. How many more does she need to get?
3. Anna has some marbles and then her brother gave her 4. Now she has 10. How many did she have at the start?
4. Anna had 12 trading cards and then she gave her brother 3. How many does she have now?
5. Anna has 13 jelly beans and then she gave her brother some. Now she has 7. How many did she give her brother?
6. Anna has some candies and gave her brother 9 of them. Now she has 8 left. How many did she have at the start?



Name: _____

Grade: Date:

Line Master Change Task 2

Write the numbers and signs that you would use to solve each problem with a calculator. You do not have to solve them.

1. Anna has 112 game playing cards and some sports trading cards. She has 87 more game playing cards than sports trading cards. How many sports cards does she have?
2. In the long jump final at the Olympic games, the Canadian athlete jumped 8.66 m and the Australian athlete jumped 7.87 m. How much farther did the Canadian jump than the Australian?
3. Anna has 156 pearly marbles and some cat-eye marbles. She has 89 more cat-eye marbles than pearly marbles. How many cat-eye marbles does Anna have?
4. The school has 307 books and 254 bookmarks. If one bookmark is put into each book, how many books won't get a bookmark?
5. Anna has 345 white bears and some brown bears. All the white bears took a brown bear as a partner and there were 78 brown bears left without a partner. How many brown bears does she have?
6. At the sports day Sonya jumped 3.25 m. Mark had jumped another 0.87 m and would have been the winner. Sonya's How long was Mark's jump?

- Use Change Task 1 with Grades 1–5 (*ages 6–10*).
- Use Change Task 2 with Grades 5–7 (*ages 10–12+*).

Purpose

To see if students are able to use the inverse relationship between addition and subtraction when solving word problems with a calculator

Materials

- Line Master: Change Task 1
- Line Master: Change Task 2

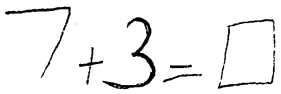
Instructions

Begin with Change Task 1. If the students are successful on this task, then, at a later time, ask them to complete Change Task 2. Do *not* allow students to use calculators for these tasks.

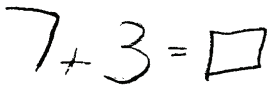
Student Work Samples

Teegan (8 years old)

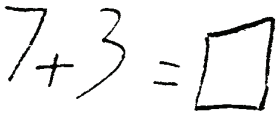
1. Anna had 7 trading cards and then her brother gave her 3. How many does she now have?



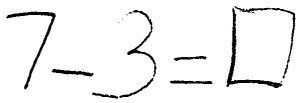
2. Anna had 7 jelly beans but would like to have 10. How many more does she need to get?



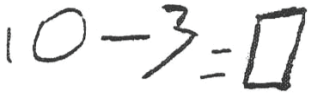
3. Anna had some marbles and then her brother gave her 3. Now she has 10. How many did she have to start with?



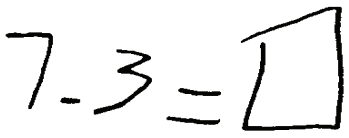
4. Anna had 7 trading cards and then she gave her brother 3. How many does she now have?



5. Anna had 10 jelly beans and then she gave her brother some. She now has 7. How many did she give her brother?



6. Anna had some candies and gave her brother 3 of them. Now she has 7 left. How many did she have to start with?



Discussion: Looking at Teegan's Work

In a small group, discuss Teegan's work.

1. Which operation has Teegan chosen for each problem? Why?

2. Which part of the problem is the unknown quantity?

3. Why has Teegan chosen to use addition for the second and third problems?

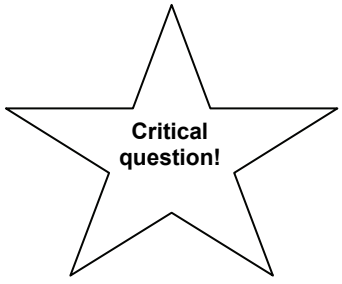
4. Why would students find it easier to solve problems that are result unknown?

5. What difference does the use of a calculator make in the choice of operation?

6. Use the Diagnostic Map: Number to decide in which phase of thinking Teegan may be.

7. What evidence from the map supports your judgement?

8. In which phase do students *routinely* use the inverse relationship between addition and subtraction?



Part-Part-Whole Relationships

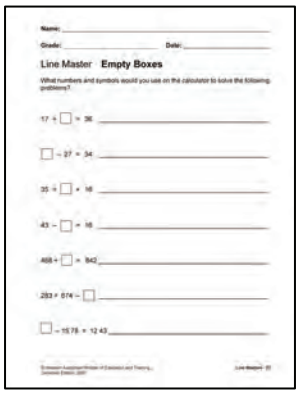
Key Understanding 2: Partitioning numbers into part-part-whole helps us relate addition and subtraction and understand their properties.
Operation Sense Resource Book, p. 40

Remember:
The part-part-whole relationship is also the key to your students seeing *why* addition is commutative and *why* subtraction is not.
Operation Sense Resource Book, p. 40, par. 2

Draw a diagram to illustrate how part–part-whole relationships show how addition and subtraction are related, with subtraction being the inverse of addition.

DIAGNOSTIC TASK: Empty Boxes

KU 2 for Grades 5–7 (ages 10–12+)
(See Course Book, p. 210.)


Name: _____
Grade: _____ Date: _____
Line Master Empty Boxes
What numbers and symbols would you use on the calculator to solve the following problems?
17 + ☐ = 36 _____
☐ − 27 = 34 _____
35 + ☐ = 16 _____
43 − ☐ = 16 _____
468 + ☐ = 842 _____
283 + 674 − ☐ _____
☐ − 1578 + 12 43 _____

Purpose

To see if students are able to use the inverse relationship between addition and subtraction to solve open number problems

Materials

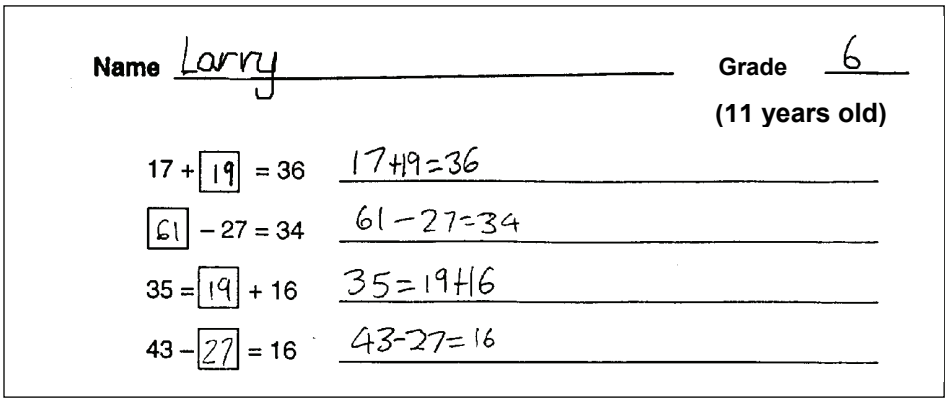
- Line Master: Empty Boxes

Instructions

1. Explain to students that they are to write what they would put into a calculator to solve the problem, not just write the answer.
   - Do *not* allow students to use calculators for this task.
2. If using this as a whole-class task, conduct follow-up interviews to clarify what some students are thinking, if necessary.

Student Work Samples

Students were asked: What numbers and symbols would you use on the calculator to solve the following problems?



Name Larry Grade 6 (11 years old)

17 + [19] = 36 17+19=36

[61] − 27 = 34 61−27=34

35 = [19] + 16 35=19+16

43 − [27] = 16 43-27=16

Analyzing the Evidence

Knows	
Doesn't know yet	
Phase	
Evidence from Map	

Notes

Name Shami Grade 6

17 + ☐ = 36	17+19=36
☐ − 27 = 34	61-27=34
35 = ☐ + 16	35=19+16
43 − ☐ = 16	43-59=16

Analyzing the Evidence

Knows	
Doesn't know yet	
Phase	
Evidence from Map	

Name Laurel Grade 6

17 + ☐ = 36	17 + 19 =
☐ − 27 = 34	7 − 27 =
35 = ☐ + 16	16 + 19 =
43 − ☐ = 16	43 − 27 =

Analyzing the Evidence

Knows	
Doesn't know yet	
Phase	
Evidence from Map	

Name Tassie Grade 6

17 + [19] = 36 17−36=19

[61] − 27 = 34 34+27=61

35 = [ ] + 16 16−35=18

43 − [23] = 16 16+23=43

Analyzing the Evidence

Knows	
Doesn't know yet	
Phase	
Evidence from Map	

Notes

Bridging to Practice: Key Understanding 2

Knowing What Students Know

Diagnostic Tasks
K–Grade 3 (*ages 4–8*)

- "Messages" from Sample Learning Activities (*Operation Sense* Resource Book, p. 34)
- Think Board (Course Book, p. 229)
- Change Task 1 (Course Book, p. 206)

Grades 3–7 (*ages 8–12+*)

- Think Board (*Operation Sense* Resource Book, p. 88)
- Change Task 2 (Course Book, p. 207)
- How Much Taller? (Course Book, p. 208)
- Empty Boxes (Course Book, p. 210)

Case Study

- K–Grade 3 (*ages 4–8*): "Messages" (*Operation Sense* Resource Book, p. 56)

Select Sample Learning Activities from pages 42–46 in the *Operation Sense* Resource Book for:

- Understanding the part-part-whole relationship of numbers

- Knowing the inverse relationship between addition and subtraction

Addition and Subtraction Problem Types

Use the Background Notes on page 24 of the *Operation Sense* Resource Book to clarify your understanding of each of these common addition and subtraction problem types.

Write some examples of each type for your students. Use contexts that are familiar and relevant to your students.

Change Students have to transform one quantity by adding to or taking away from it (includes an *action*).

Remember:
Students do not need to learn the names of the different problem types.

Combine Students have to consider two *static* quantities either separately or combined.

Compare Students compare two quantities (that are *static*).

Remember:
As students get older, we need to include problems with greater numbers and decimals in the activities we plan for them.

Equalize Students equalize two quantities (includes an *action*).

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Writing Story Problems

Grades	Change	Combine	Equalize	Compare
K–Grade 3 (*ages 4–8*)				
Grades 3–5 (*ages 8–10*)				
Grades 5–8 (*ages 10–12+*)				

Different Situations, Same Operation

Subtraction

Students need to recognize the range of situations that can be represented with subtraction. Look at the following questions.

- What's the difference between 7 and 2?
- How much more is 7 than 2?
- What's left if I take 2 from 7?
- If I count back 2 from 7 on a number line, what number will I come to?
- I spent $2 and I started with $7. How much do I have left?
- I have $2 and I started with $7. How much did I spend?

Looking at a wide range of problems and asking "What is the same?" helps students to understand what subtraction is and why it can be used in different situations.

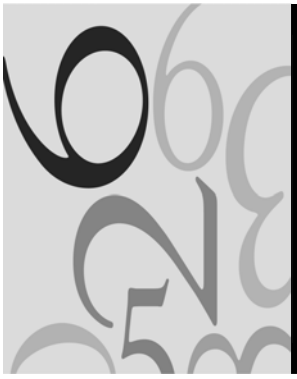
Diagnostic Map and Developmental Phases

Which problem types are mentioned in the Diagnostic Map?

What sort of problems could students be dealing with by the end of the following phases? (See the description of phases in KU 2 notes.)

Matching	Quantifying	Partitioning

Which of the problems you have written would be suitable for students who are working towards the end of the Matching, Quantifying, and Partitioning phases?



UNIT

8 Operations

THE MEANING OF MULTIPLICATION AND DIVISION

Desired Outcomes

Participants will

- explore Key Understandings 3 and 4 and the Background Notes for multiplication and division
- analyze students' learning to identify what they know and what experiences they need to develop their thinking and to move them on, Key Understanding 8
- identify the phase of a student's progress using the Diagnostic Map: Number
- analyze and develop the different types of multiplication and division problems

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps in Mathematics: Operation Sense* Resource Book**

Chapter 1: An Overview of *First Steps in Mathematics*

Beliefs about Teaching and Learning, p. 2
Understanding the Elements of *First Steps in Mathematics*, p. 8
How to Read the Diagnostic Map, p. 12

Chapter 3: Operations

Background Notes, p. 22
Key Understanding 3, p. 48
Key Understanding 4, p. 60
Key Understanding 6, p. 82

Diagnostic Map: Number

Emergent phase through to Operating phase

Suggestion for Further Reading

Fennema, E., Franke, M.L. et al. "Using Children's Mathematical Knowledge in Instruction." *American Educational Research Journal*, 30(3), pp. 555–583, 1993.

Multiplication

Key Understanding 3: Multiplying numbers is useful when we
- repeat equal quantities
- use rates
- make ratio comparisons or changes (scales)
- make arrays and combinations
- need products of measures.

(*Operation Sense* Resource Book, p. 48)

Remember:
To solve a problem using multiplication means that
- the problem or situation has equal groups
- we need to identify the number of equal groups
- we need to identify the size of the group in order to use multiplication

Discussion

Refer to the Background Notes on page 25 of the *Operation Sense* Resource Book. Draw a simple diagram to represent each type of problem.

The two divisions for each multiplication are of different types.

Multiplication	**Division** (partitioning/sharing)— know how many portions	**Division** (quotition/grouping)— know the size of the portions
Repeat equal quantities.		
Use rates.		
Make ratio comparisons or changes (e.g., scale).		

The two divisions for each multiplication are not of different types.

Make arrays and combinations.	
Need products of measures.	
	Repeat equal quantities

Tracking Multiplication and Division Through the Phases

Read the "During" and "By the end of" statements to see what students understand about multiplication and division.

Phase	Multiplication	Division
Emergent		
Matching		
Quantifying		
Partitioning		
Factoring		
Operating		

DIAGNOSTIC TASK: Story Problems—Sausages

KU 2 and 3 for K–Grade 2 (ages 4–7)

(See Course Book, p. 212.)

Dad said, "We're going to have visitors for a barbecue. That means there will be 9 people. We'll have 3 sausages each." Dad sent the children to the shop to buy the sausages. How many sausages will they need to buy?

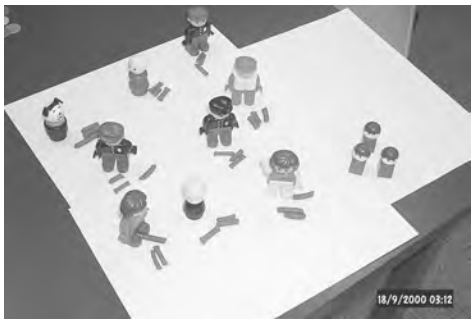
Student Work Samples

Jenny (Grade 1)

• Solve? • Represent quantities? • Represent action? • Numbers and operation?	Jenny tried but said paper was too hard. She used "pretend people" and set up this scene. She picked up, moved, and chatted to the pieces to coincide with her recount of the story (problem). She put 3 dolls at the butcher's and took out pretend sausages from a container. When asked, she said she had brought back 3 sausages. She pretended to cut up the 3 sausages and handed a piece to each character including the butcher and the dog.

Phase	

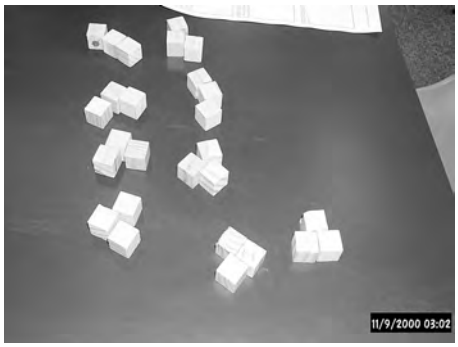
Jed (Grade 1)



• Solve? • Represent quantities? • Represent action? • Numbers and operation?	Jed said it was too hard to work out with blocks, counters, pencil, and paper. He used these toys as "nine visitors" and pretended to go to the butcher where he handed 3 sausages to each visitor. When asked how many sausages he needed he counted all the sausages and said 27.

Phase	

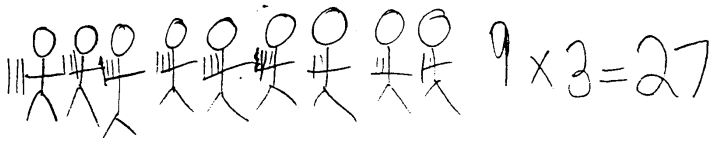
Joel (Grade 3)



• Solve? • Represent quantities? • Represent action? • Numbers and operation?	Joel set out 7 groups of 3. He said, "There are 7 of them, so you need 2 more of them." Joel counted: "3, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27."

Phase	

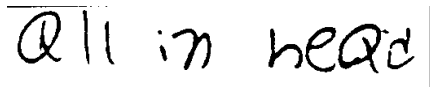
Cassie (Grade 3)



- Solve?
- Represent quantities?
- Represent action?
- Numbers and operation?

Phase	

Elliot (Grade 3)

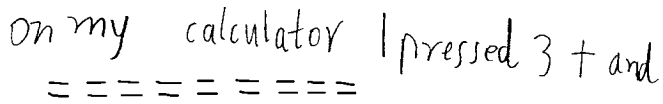


• Solve? • Represent quantities? • Represent action? • Numbers and operation?	Elliot wrote "27" and the words "All in head." Teacher: What did you say in your head? Elliot: 3, 6, 9, 12, 15, 18, 21, 24, 27. Teacher: How did you know when to stop counting? Elliot: I was counting two things at once. I stopped when I counted 9 in the other count.

Phase	

Georgina (Grade 3)

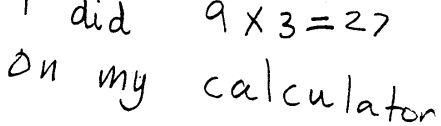
Georgina said, "27."



- Solve?
- Represent quantities?
- Represent action?
- Numbers and operation?

Phase	

George (Grade 3)



- Solve?
- Represent quantities?
- Represent action?
- Numbers and operation?

Phase	

Bridging to Practice: Key Understanding 3

Knowing What Students Know

Diagnostic Tasks

K–Grade 3 (ages 4–8)

- "Sausages" question from Story Problems (1) (Course Book, p. 213)
- "Messages" from Sample Learning Activity (*Operation Sense* Resource Book, p. 34)
- Think Board (Course Book, p. 229)

Grades 3–7 (ages 8–12+)

- Story Problems (2) (Course Book, p. 214)
- Calculator Number Sentences (Course Book, p. 217)
- Finding Factors (Course Book, p. 215)
- Think Board (Course Book, p. 229)

Case Study

- "Messages" (ages 4 to 8), p. 36 in the *Operation Sense* Resource Book

Moving Students Along

Select Sample Learning Activities from pages 50 to 54 in the *Operation Sense* Resource Book for:

- The introduction to multiplication for K–Grade 3 students

- Recognizing a wide range of problem types to which multiplication applies

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Discussion

How is the role numbers play in a multiplication problem different from the role they play in an addition problem?

Division

Key Understanding 4: Dividing numbers is useful when we:
- share or group a quantity into a given number of portions
- share or group a quantity into portions of a given size or
- need the inverse of multiplication.

(*Operation Sense* Resource Book, p. 60)

In a similar way to addition, subtraction, and multiplication, students should learn to recognize a wide range of problem types to which division applies.

There are two types of division across the problem types:
- sharing
- grouping

Types of Division Problems

	Partitive	Quotative
Common Name		
Known Part		
Unknown Part		

Examples of Division Problems

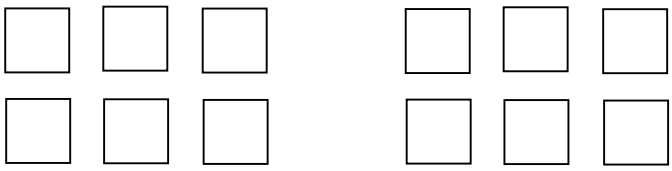
Key Understanding 6: The same operation can be said and written in different ways. (*Operation Sense* Resource Book, p. 82)
Refer to the Background Notes on page 25 of the *Operation Sense* Resource Book.

What are the important ideas of KU 6?

What is the difference between what students at the Quantifying and Partioning phases can do in connection with KU 6?

Student Work Samples: Block Piles

A pile of 12 blocks was placed in front of each student to see but not to handle. The students were asked to say how many piles of three could be made from the pile of blocks.



Oscar (8 years old): OK—so I see 6 which is 2 piles of threes. There's another 6—so that's another 2 piles. So that's 4 piles.

Nathan (9 years old): Four.
Teacher: That was quick. How did you work that out so quickly?
Nathan: Well I know three 3s are 9 so four 3s are 12, so it must be 4.

Analyzing the Evidence

What operation did Oscar use?

What operation did Nathan use?

Paraphrase each student's response.

Go back to the Sausages work samples on page 113 of this Course Book. Work with a partner to paraphrase each of the student's responses.

Remember:
One situation can be represented by different operations.

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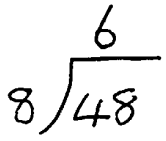
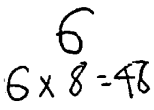
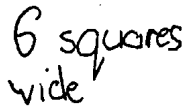
Student Work Samples: The Chocolate Problem

A bar of chocolate has 48 squares in it. If it is 8 squares long, how wide is it?

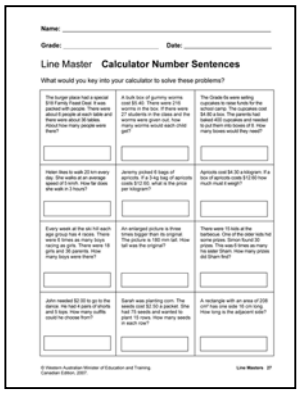
Student work samples	Strategies used by the student to represent the problem	Suggest paraphrasing to move this student along
Pia (Grade 7) 		
Abi (Grade 6)  Teacher: How did you work it out? Abi: I said to myself, "What times table is 48 and 8?" and I said, "6."		
Ted (Grade 6) "8 times 6 is 48." 		
Chrystal (Grade 7) "8 times what is 48?" "8 times 6 is 48." "So it's 6."		
Lee (Grade 6) "48 goes into 8 six times."		
Jeremy (Grade 5) 		

DIAGNOSTIC TASK: Calculator Number Sentences

KU 8 for Grades 4–7 (ages 10–12+)
(See below and Course Book, p. 217.)

Materials

- Line Master: Calculator Number Sentences
- A calculator for each child


Name: _____

Grade: _____ Date: _____

Line Master **Calculator Number Sentences**

What would you key into your calculator to solve these problems?

The burger place had a special $18.6 Family Feast Deal. It was packed with people. There were about 6 people at each table and there were about 36 tables. About how many people were there?	A bulk box of gummy worms cost $3.40. There were 216 worms in the box. If there were 27 students in the class and the worms were given out, how many worms would each child get?	The Grade 6s were selling cupcakes to raise funds for the school camp. The cupcakes cost $4.80 a box. The parents had baked 400 cupcakes and needed to put them in boxes of 6. How many boxes would they need?
Helen likes to walk 20 km every day. She walks at an average speed of 5 km/h. How far does she walk in 3 hours?	Jeremy picked 6 bags of apricots. If a 3-kg bag of apricots costs $12.00, what is the price per kilogram?	Apricots cost $4.30 a kilogram. If a box of apricots costs $12.90 how much must it weigh?
Every week at the ski hill each age group has 4 races. There were 8 times as many boys racing as girls. There were 16 girls racing. How many boys were there?	An enlarged picture is three times bigger than its original. The picture is 180 mm tall. How tall was the original?	There were 10 kids at the barbecue. One of the older kids had some prizes. Simon found 30 prizes. This was 6 times as many as Sharon found. How many prizes did Sharon find?
John needed $2.50 to go to the dance. He had 4 pairs of shorts and 5 tops. How many outfits could he choose from?	Sarah was planting corn. The seeds cost $2.50 a packet. She had 75 seeds and wanted to plant 15 rows. How many seeds in each row?	A rectangle with an area of 208 cm^2 has one side 16 cm long. How long is the adjacent side?

Line Masters 37

Instructions

Individual Interview

Ask the student if she/he would like to have the problems read aloud or to read the problems herself or himself and write in the numbers sentence as she/he goes. Remind her/him that it is not the answer to the problem that is required but the number sentence she/he would need to key into the calculator.

Some students may use a trial-and-error approach. Note those students who use trial and error to choose the operation; which operations they try; and how they arrive at their final decisions.

Whole-Class Activity

Read the problems to the students if they need this level of support. Remind them that it is not the answer to the problem that is required but the number sentence they would need to key into the calculator. Note any trial-and-error approaches as above.

Notes

Calculator Number Sentences

Name Mia **Grade** 7F **Date** Nov 16

What would you key into your calculator to solve these problems?

The burger place had a special $18 Family Feast Deal. It was packed with people. There were about 6 people at each table and there were about 36 tables. About how many people were there? 6 x 36	A bulk box of gummy snakes cost $5.40. There were 216 snakes in the box. If there were 27 students in the class and the snakes were given out, how many snakes would each child get? 216 ÷ 27	The Grade 6s were selling cup cakes to raise funds for the school camp. The cakes cost $4.80 a box. The canteen had cooked 400 cakes and needed to put them into boxes of 8. How many boxes would they need? 400 x 8
Helen likes to walk 20 km every day. She walks at an average speed of 5 km/h. How far does she walk in 3 hours? 20 x 5	Jeremy picked 6 bags of apricots. If a 3 kg bag costs $12.60, what is the price per kilo? 6 ÷ 12,60	Apricots cost $4.30 a pound. If a sack of apricots cost $12.60, how much must it weigh? 4.30 x 1260
Every week at ski hill each age group has 4 races. There were 6 times as many boys racing as girls. There were 18 girls and 36 parents. How many boys were there? 6 x 18	A picture, which has been enlarged three times its original size, is now 180 mm high. What was its original height? 3 ÷ 180	There were 15 kids at the barbecue. One of the older kids hid some prizes. Simon found 30 prizes. This was 6 times as many as his sister Sharn. How many prizes did Sharn find? 30 ÷ 6
John needed $2.00 to go to the T Ball disco. He had 4 pairs of shorts and 5 tops. How many outfits could he choose from? 5 x 4	Sarah was planting corn. The seeds cost $2.50 a packet. She had 75 seeds and wanted to plant 15 rows. How many seeds in each row? 75 x 15	A rectangle of area of 208 cm² has one-side 16 cm long. How long is the adjacent side? 16 x 208

Bridging to Practice: Key Understanding 4

Remember:
Students associate division mainly with sharing. It is important that problem types vary and that different language patterns are used from the beginning.

Knowing What Students Know

Diagnostic Tasks

K–Grade 4 (ages 4–9)

- Use the problem types for Repeat Equal Quantities problems (Background Notes, *Operation Sense* Resource Book, p. 25) as tasks.

Grades 3–7 (ages 8 to 12+)

- Use each of the problem types (Background Notes, *Operation Sense* Resource Book, p. 25) as tasks.
- Use the Diagnostic Activity in Did You Know? (*Operation Sense* Resource Book, p. 71)

Moving Students Along

Read Case Study 2: "Relay" (ages 8 to 10), *Operation Sense* Resource Book, p. 68.
Select Sample Learning Activities from pages 62–67 in *Operation Sense* Resource Book for:

- Recognizing that both sharing and grouping are division situations
- Recognizing a wide range of problem types to which division applies
- Recognizing the inverse relationship between multiplication and division

Notes

Properties of Operations

Key Understanding 5: Repeating equal quantities and partitioning a quantity into equal parts help us relate multiplication and division and understand their properties. (*Operation Sense* Resource Book, p. 72)

Key Understanding 7: Properties of operations and relationships between them can help us to decide whether number sentences are true. (*Operation Sense* Resource Book, p. 86)

Key Understanding 9: We make assumptions when using operations. We should check that the assumptions make sense for the problem. (*Operation Sense* Resource Book, p. 102)

What are the important ideas in KU 5?

What are the important ideas in KU 7?

What are the important ideas in KU 9?

Bridging to Practice: Key Understanding 5

Knowing What Students Know

Diagnostic Tasks

Record the Diagnostic Tasks, activities, and sample lessons you would select for your students in order to find out what they know about KU 5.

Moving Students Along

Select Sample Learning Activities from pages 74–80 in the *Operation Sense* Resource Book for:

- The introduction to partitioning numbers
- Recognizing a wide range of problem types to which multiplication applies

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Bridging to Practice: Key Understanding 6

Knowing What Students Know

Diagnostic Tasks

Record the Diagnostic Tasks and activities you would select for your students in order to find out what they know about KU 6.

Case Study 1

- "Messages" (ages 4–8), *Operation Sense* Resource Book, p. 56.

Moving Students Along

Select Sample Learning Activities from pages 83–85 in the *Operation Sense* Resource Book for

- The introduction to recording number sentences
- Knowing the concise way to write mathematics

Bridging to Practice: Key Understanding 7

Knowing What Students Know

Diagnostic Tasks

Record the Diagnostic Tasks and activities you would select for your students in order to find out what they know about KU 7.

Moving Students Along

Select Sample Learning Activities from pages 88–93 in the *Operation Sense* Resource Book for

- The introduction to inverse relationships
- Knowing and using the commutative and associative properties for multiplication

---

Bridging to Practice: Key Understanding 9

Knowing What Students Know

Diagnostic Tasks

Record the tasks and activities you would choose for your students in order to find out what they know about KU 9.

Moving Students Along

Select Sample Learning Activities from pp. 104–106 in the *Operation Sense* Resource Book for:

- Regular reflection on the appropriateness of choosing particular ways to represent situations

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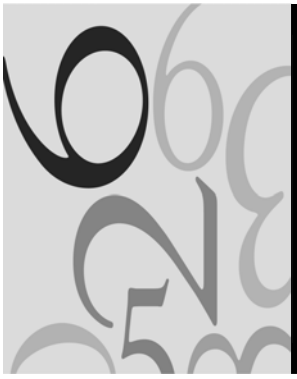


UNIT

9 Computations

BUILDING A REPERTOIRE

Desired Outcomes

Participants will

- become familiar with the mathematics of the Computations Key Understandings, particularly 1, 2, 3, 4, 5, and 6
- become familiar with how students learn to develop a repertoire of mental strategies for doing computations
- revisit the Diagnostic Map to see the links between Number, Operations, and Computations
- analyze and develop the different mental computation techniques

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps in Mathematics: Operation Sense* Resource Book**

Chapter 1: An Overview of *First Steps in Mathematics*

Beliefs about Teaching and Learning, p. 2
Understanding the Elements of *First Steps in Mathematics*, p. 8
How to Read the Diagnostic Map, p. 12

Chapter 3: Computations

Background Notes, p. 108
Key Understanding 4, p. 142
Key Understanding 1, p. 118
Key Understanding 5, p. 152
Key Understanding 2, p. 126
Key Understanding 6, p. 164
Key Understanding 3, p. 134

Diagnostic Map: Number

Emergent phase
Matching phase
Quantifying phase
Partitioning phase

Suggestions for Further Reading

Kamii, C., (1990), "Constructivism and beginning Arithmetic (K-2)" In *Teaching and Learning Mathematics in the 1990s*, edited by Cooney, T. & Hirsch, C. NCTM, Virginia, U.S.A.

Kamii, C, Joseph, L. (1988). Teaching Place Value and Double-Column Addition, *Arithmetic Teacher* 35 (6), pp. 48–52, February

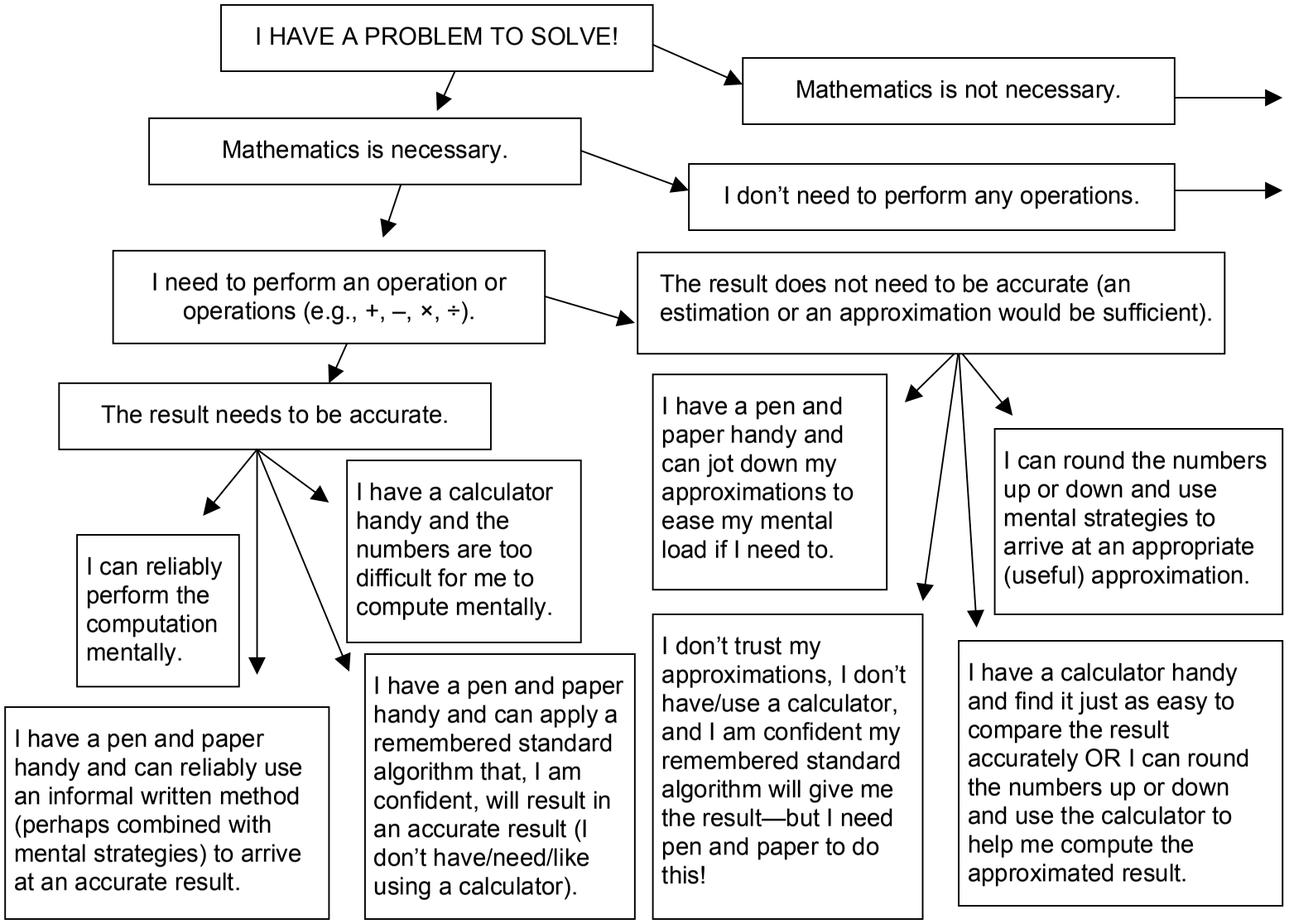
Understanding Computations

The Computations Outcome

Choose and use a repertoire of mental, paper, and calculator computational strategies for each operation, meeting needed degrees of accuracy and judging the reasonableness of results (*Operation Sense* Resource Book, p. 107).

Notes

Computational Choices



With permission: Dianne Tomazos (1998)

Use the Computational Choices diagram on the facing page to track your decisions in the following situations.

1. The total at the checkout is $159.64. If you ask for $200 extra cash, how much should be debited from your account?

2. Seventeen people on a school staff share a $359,037.45 lottery prize.
   How much does each lucky teacher win?

3. Your parking fee is $6.40. You give the attendant $10. How much change should you receive?

4. Paint costs $85 for a 4-L can and $60 for a 2-L can. Should I paint the living room?

5. Thirty-two children paid $4.50 for an excursion. How much should their teacher have collected?

6. A store is offering 15% off all bed linen. How much would you pay for a set of sheets that would normally cost $89.99?

7. How much do you need to pay for four tickets costing $45 each?

8. You are knitting a sweater and need to know how many balls of wool to buy.
   How can you work it out?

9. Approximately 60 people will be attending your daughter's 21st birthday party.
   How much beer, wine, and pop should you purchase?

10. I deposit two cheques at the ATM. One is for $37.48 and the other is for $144.79. How much should I key in as the deposit?

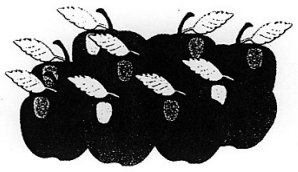
DIAGNOSTIC TASK: Buying Apples

Computations for Grades 4–7 (ages 9–12+)

(See Course Book, p. 227.)

Student Work Samples

Apples
$3.20 per kilogram



Andrew (9 years old)

Evan bought 0.4 kilograms.
How much did he pay? $1.28
Explain your answer.

10 32c goes into $3.20. half of $3.20 is $1.60 take 32c is $1.28

Apples
$3.20 per kilogram

Lilly (12 years old)

Evan bought 0.4 kilograms.
How much did he pay? $12.80
Explain your answer.

$3.20
× 0.4
12.80
+ 00.00
$ 12.80

I timed $3.20 by 0.4 and got $12.80

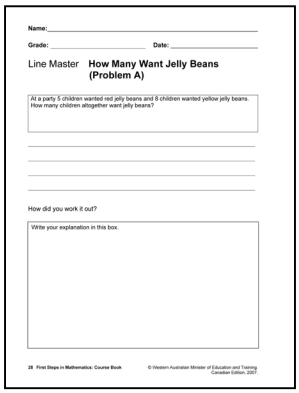
What mathematics has Andrew used to solve the problem?

What mathematics has Lilly used to solve the problem?

FSIM012 | First Steps in Mathematics: Number Course Book

DIAGNOSTIC TASK: How Many Want Jelly Beans?

Computations for K–3 for Grades 1–3 (ages 4–8)
(See below and Course Book, p. 220.)


Name:_____

Grade:_____ Date:_____

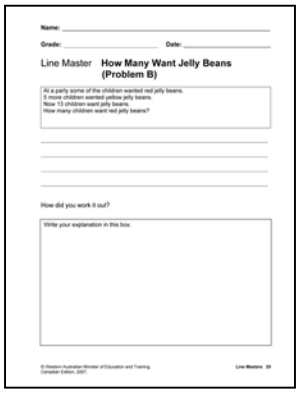
Line Master **How Many Want Jelly Beans (Problem A)**

At a party 5 children wanted red jelly beans and 8 children wanted yellow jelly beans. How many children altogether want jelly beans?

How did you work it out?

Write your explanation in this box.

28 First Steps in Mathematics: Course Book © Western Australian Minister of Education and Training Canadian Edition, 2007.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Grade:_____ Date:_____

Line Master **How Many Want Jelly Beans (Problem B)**

At a party some of the children wanted red jelly beans.
5 more children wanted yellow jelly beans.
Now 13 children want jelly beans.
How many children want red jelly beans?

How did you work it out?

Write your explanation in this box.

© Western Australian Minister of Education and Training Canadian Edition, 2007. Line Masters 29

- How Many Want Jelly Beans? (Problem A)
  Grades 1–2 (ages 6–7)
- How Many Want Jelly Beans? (Problem B)
  Grades 2–3 (ages 7–8)

Materials

- Line Master: How Many Want Jelly Beans? (Problem A)
- Line Master: How Many Want Jelly Beans? (Problem B)

Instructions

Provide each student with the appropriate Line Master. Read it aloud to them to make sure they understand what they need to find out.

Explain that they can solve it mentally or work it out by drawing or using some materials.

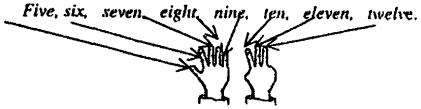
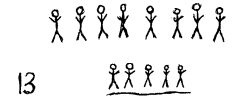
After each student has found an answer, ask them to say how they worked it out and record their descriptions on their page.

Notes

FSIM012 | First Steps in Mathematics: Number Course Book

Student Work Samples: How Many Want Jelly Beans? (Problem A)

At the beginning of the school year, students in a Grades 1 and 2 class answered this question:
At a party, 5 children wanted red jelly beans and 8 wanted yellow jelly beans. How many children want jelly beans?

Student	What the student said or did	Analysis
A **Claire**	"Well, I looked around and counted the daffodils." (She is referring to daffodil cutouts displayed on the window.) "I counted out 5 daffodils, then I counted out 8, then I counted all of them and I got 13."	Claire is using a counting strategy: count the first set, then the second set, then join them and count all.
B **Elisa**	Elisa counted using her fingers. First she decided to recall the 5 in her head and hold up 8 fingers to add onto the 5. However, she began her count at 5 instead of 6 and so her result was 12.  	Elisa is trying to use a "count on" strategy, but doesn't yet understand that the last number in the count tells the quantity rather than the place in the sequence of counting numbers. She is not yet a quantifier because she does not yet trust the count. (See *Computations* KU 1 and 2 and *Whole and Decimal Numbers* KU 1 and 2.)
C **Luke**	"I counted out 5 fingers, then I put 5 in my head and counted 8 more and that is 13."	Luke knows that if he counts out 5 fingers, then the next time he counts the same set of fingers the result will be 5. He knows he doesn't have to count them again and, as he says, he thinks of the five and counts on. He has begun to use "counting on" forwards as a strategy.
D **Jessica**	 	Jessica drew 8 people, then 5 more in separate sets. She underlined the 5 and said, "That's 5." Then she counted on from 5 to add the other 8 and wrote the total below.
E **Brenden**	"I just know that 4 and 8 is 12 and one more is 13."	Brenden has begun to compensate; that is, partition, and rearrange the numbers into manageable parts and sizes so that he can use basic facts. He has partitioned 5 into 4 + 1 because he knows 4 + 8 = 12. Then he has added the 1 to bring the 4 up to 5.

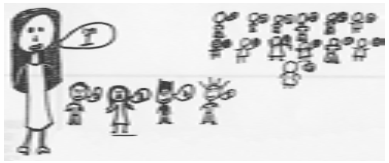
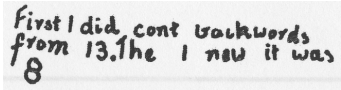
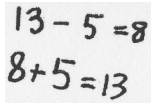
Order the students' responses from least to most sophisticated response.

Discussion: How Many Want Jelly Beans? (Problem A)

Student	Look at the Diagnostic Map. List the concepts each student understands.	Look at the Key Understandings. Decide what mathematical ideas each student used.
Claire		Number Operations
Elisa		Number Operations
Luke		Number Operations
Jessica		Number Operations
Brenden		Number Operations

Student Work Samples: How Many Want Jelly Beans? (Problem B)

At the beginning of the school year, students in a Grades 1 and 2 class answered this question:
At a party, some children want red jelly beans. Then 5 more want yellow jelly beans. Now 13 children want jelly beans.
How many children want red jelly beans?

Student	What the student said or did	Analysis
A **Laura**		Laura has not been able to work out what action to take to calculate this situation. She has drawn a collection of people to represent each of the numbers she heard, but has not realized the 13 is the whole and 5 is a part of the 13. (See *Operations* KU 2.)
B **Megan**	 Megan drew 13 objects to represent 13 children. She began to cross out the last 5 but realized that she would get the same result by crossing out the first 5. Then she counted the remainder, saying her result was 8. She then wrote the number sentence 13 – 5 = 8 to describe her actions.	Megan has begun to use her understanding that 13 will always be 13 (whether it describes people or objects) and that taking off from the start will give the same result as taking off from the end. The situation is helping her see the relationships between addition and subtraction. She realized the total was 13 and subtracted 5 to find the missing starting number of unknown jelly beans. She is developing the idea that ? + 5 = 13 can be thought of as 13 – 5 = ?
C **Andrew**		Andrew understands the part-part-whole nature of numbers to about 20 and thinks of ? + 5 = 13 as 13 – 5 = ?. He has used a "counting on" backwards strategy.
D **Victor**	"I just know that 4 and 8 make 12 and so 5 and 8 make 13."	Like Brenden in the Problem A samples, Victor has begun to compensate; that is, partition, and rearrange the numbers into manageable parts and sizes so that he can use basic facts. He has partitioned 5 into 4 + 1 because he knows 4 + 8 = 12, then he has added the one to bring the 4 up to 5.
E **Gloria**	 "The answer is 8 because I know 8 and 5 equals 13."	Gloria has extended her basic facts to numbers beyond 10 + 10 and trusts the inverse relationship between addition and subtraction.

Order the students' responses from least to most sophisticated response.

Discussion: How Many Want Jelly Beans? (Problem B)

Student	Look at the Diagnostic Map. List the concepts each student understands.	Look at the Key Understandings. Decide what mathematical ideas each student used.
Laura		Number Operations
Megan		Number Operations
Gloria		Number Operations
Andrew		Number Operations
Victor		Number Operations

From Basic Facts to Flexible Computations

Key Understanding 4: Place value and basic number facts together allow us to calculate with any whole or decimal numbers. (*Operation Sense* Resource Book, p. 142)

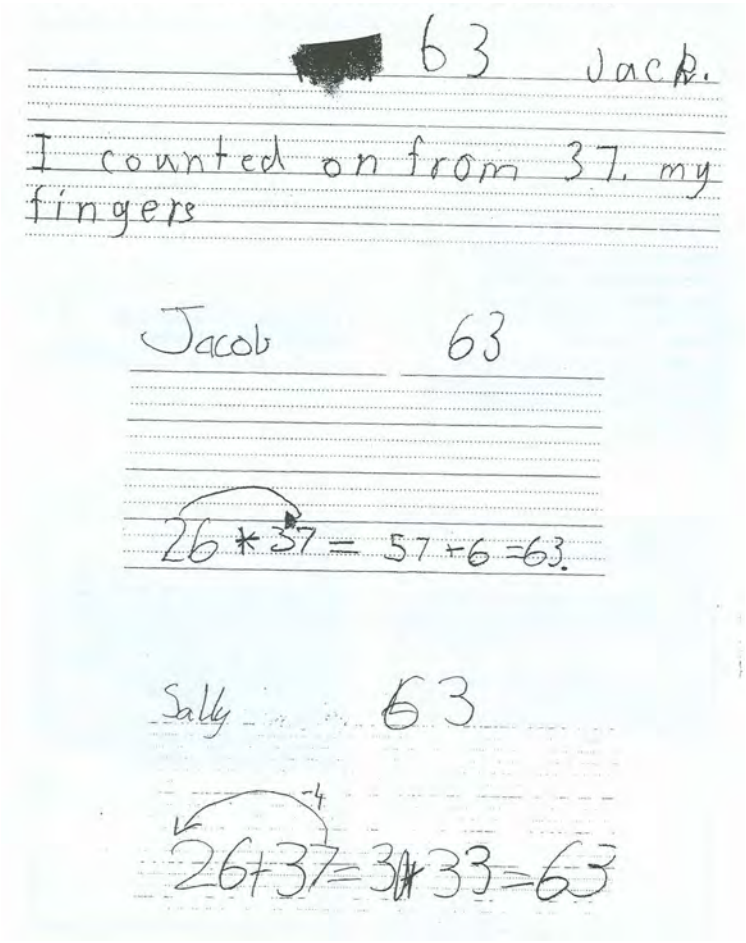
Key Understanding 5: There are strategies we can practise to help us do calculations in our head. (*Operation Sense* Resource Book, p. 152)

Discussion: Counting On Strategies

1. Compare the computation strategies used by Jack, Jacob, and Sally to solve this problem:

   $$26 + 37$$

   What do students need to know to solve problems like this mentally?



2. How would you do it?

Remember:
If students are not familiar with the backwards counting string, they will not be able to count backwards for subtraction.

3. What is it about counting on as a computational strategy that students find difficult?

4. Use counting on to solve this problem: 16 + 27
   Describe to your partner the process that you used.

   a) What did you need to keep track of?

   b) How did you know when to stop counting?

5. Is counting an efficient and reliable strategy?

6. What phase do students have to be in before they can use counting on?

7. What phase do students need to be through before they can make sense of basic facts? Why do you think so?

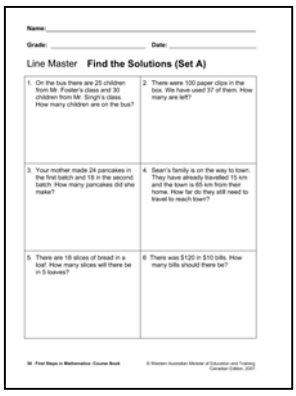
Remember:
Counting on involves double counting. It requires students to keep track of the next number in the sequence and the numbers that have been counted.

DIAGNOSTIC TASK: Find the Solutions

Grades 3–7 (ages 8–12+)

(See below and Course Book, p. 223.)

- Find the Solutions (Sets A and B), *Grades 3–4 (ages 8–9)*
- Find the Solutions (Sets B and C), *Grades 5–7 (ages 10–12+)*

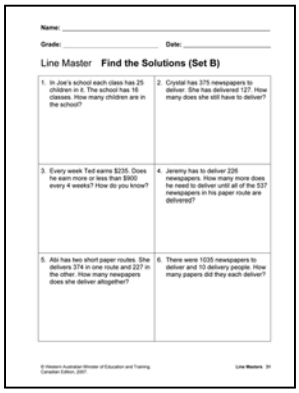


Name:_____

Grade: _____ Date: _____

Line Master **Find the Solutions (Set A)**

1. On the bus there are 25 children from Mr. Foster's class and 30 children from Mr. Singh's class. How many children are on the bus?	2. There were 100 paper clips in the box. We have used 37 of them. How many are left?
3. Your mother made 24 pancakes in the first batch and 18 in the second batch. How many pancakes did she make?	4. Sean's family is on the way to town. They have already travelled 15 km and the town is 65 km from their home. How far do they still need to travel to reach town?
5. There are 16 slices of bread in a loaf. How many slices will there be in 5 loaves?	6. There was $120 in $10 bills. How many bills should there be?

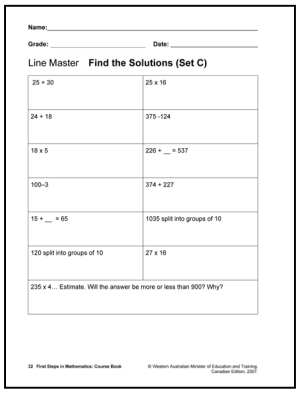


Name: _____

Grade: _____ Date: _____

Line Master **Find the Solutions (Set B)**

1. In Joe's school each class has 25 children in it. The school has 16 classes. How many children are in the school?	2. Crystal has 375 newspapers to deliver. She has delivered 127. How many does she still have to deliver?
3. Every week Ted earns $235. Does he earn more or less than $900 every 4 weeks? How do you know?	4. Jeremy has to deliver 226 newspapers. How many more does he need to deliver until all of the 537 newspapers in his paper route are delivered?
5. Abi has two short paper routes. She delivers 374 in one route and 227 in the other. How many newspapers does she deliver altogether?	6. There were 1035 newspapers to deliver and 10 delivery people. How many papers did they each deliver?



Name:_____

Grade: _____ Date: _____

Line Master **Find the Solutions (Set C)**

25 + 30	25 x 16
24 + 18	375 -124
18 x 5	226 + __ = 537
100–3	374 + 227
15 + __ = 65	1035 split into groups of 10
120 split into groups of 10	27 x 16
235 x 4… Estimate. Will the answer be more or less than 900? Why?	

Materials

- Line Master: Find the Solutions (Set A)
- Line Master: Find the Solutions (Set B)
- Line Master: Find the Solutions (Set C)

Instructions

1. Explain to the students that they need only work on the problems that they think they can do. It is important to end the interview when the child indicates the questions are getting too hard.You may wish to copy only the left-hand set of questions for the students you believe might feel inadequate for not progressing to the end.

2. Provide each student with the appropriate Line Master(s). Read the questions with them to make sure they understand what they need to find out.

Individual Interview

Sit with a student as he/he works out each question. When she/he has an answer, ask how she/he worked it out. The focus is on revealing some of the repertoire the student has developed; commenting on whether the answer is right or wrong is inappropriate in this situation. It is important to end the interview when the student indicates that the questions are getting too hard. Record the numbers the students uses as she/he partitions, re-arranges, orders, and operates on a separate copy.

Whole Class Task

Provide each student in the class with a copy of the questions. Ask them to read the question, and work out the answer in their head. Interview a few students of different abilities to find out how they thought of the numbers and how they worked each question out. Record the numbers they use as they partition, re-arrange, order and operate on a separate copy.

Student Work Samples: Find the Solutions (Set A)

2. There were 100 paper clips in the box. We have used 37 of them. How many are left?

Child A: It's 63. 10 take 3 is 7 so in hundreds it's 100 – 30 is 70 and minus 7 is 63.

Child B: It's 63.
I know 7 + 3 is 10 ...
So 70 + 30 is 100.
So 30 from 100 is 70 take away 7 is 63.

Child C: It's 63. 100 minus 30 is 70 and minus 7 is 63.

3. Mom made 24 pancakes in the first batch and 18 in the second batch. How many pancakes did she make?

Child A: It's 38. I started with 24 and counted on 18 by ones.

Child B: It's 42. 6 from 8 is 30.
2 from 8
2 more and 10 more.

Child C: It's 42. I changed the 24 to 22, then I added the 2 to the 18 so 22 and 20 is 42.

4. Sean's family are on the way to town. They have already travelled 15 km and the town is 65 km from their home. How far do they still need to travel to reach town?

Child A: 50. (The child kept a tally for counting on from 15 by ones.)

Child B: 50. Because 10 add 50 equals 60 add 5 = 65.

Child C: It's 50. You just take 15 from 65.

5. There are 18 slices of bread in a loaf. How many slices will there be in 5 loaves?

Child A: (Counted out 5 on her fingers 18 times.
The child then got lost halfway through and began again using a tally.)

Child B: 90. I counted by fives, 18 times using a tally to keep track.

Child C: 90 because 12 × 5 are 60,
6 × 5 are 30.
It's 60 + 30.

Child D: 90 because ten eighteens are 180.

Discussion

Record how the students on the facing page thought of the numbers.

Student Work Samples: Find the Solutions (Set B)

1. In Joe's school each class has 25 children in it. The school has 16 classes. How many children in the school?

Child A: That's 25 times 16 and there's practically no sums like that. I'm going to change it to, take off 11 from the 16 and added it to the 25. So 36 x 5 is 180.

Child B: 300. So I put 25 + 25 that's one 50 and then I did that 8 times.

Child C: 400: 4 × 25 is 100, 8 × 25 is 200, 12 × 25 is 300, 16 × 25 is 400.

Child D: It's 400, because 4 × 25 make 100 and there are 4 × 4 in 16 for it's four hundreds.

2. Crystal had 375 papers to deliver. She has delivered 127. How many does she still have to deliver?

Child A: It's 392.

Child B: Take 100 away from 300 = 200
Take 20 away from 70 = 50
I can't do the rest because you can't take 7 from 5.

Child C: If I make the 127 into 130 then that makes the 375 back to 372 and so that's 200 and 50, and 2. It's 252.

Child D: It's 275, then it's 255, then it's 248.

3. Every week Ted earns $235. Does he earn more than or less than $900 every 4 weeks? How do you know?

Child A: 4 × 2s are 8 So that's 800 and 4 × 35 is more than 100 because 4 × 3 =12 so it's more than 900.

Child B: More because 4 times 200 = 800, then 4 times 35....
4 × 3 =12, add 0, that's 120.
4 × 5 = 20: more than 900.

Child C: Well, there are 4 × 225 is 900 so it's more.

4. Jeremy has delivered 226 papers. How many more does he need to deliver until all of the 537 papers in his route are delivered?

Child A: 311

Child B: That's really 537 – 226. 200 from 500, That's 300, 20 from 30 that's 10 and 6 from 7 that's 1. So it's 311.

Child C: The difference is about 300. Let's see. 300 and 226 is 526 and you need 11 more. So the answer is 311.

Discussion

Record how the students on the facing page thought of the numbers.

Algorithms

Key Understanding 6: There are some special calculating methods that we can use for calculations we find hard to do in our head. (*Operation Sense* Resource Book, p. 164)

What do you see as an appropriate approach to teaching standard algorithms?

Compare your ideas to those in Key Understanding 6.

Danielle and Cathy (Video)

What is each student able to do? What is each student not able to do yet? Record your answers below.

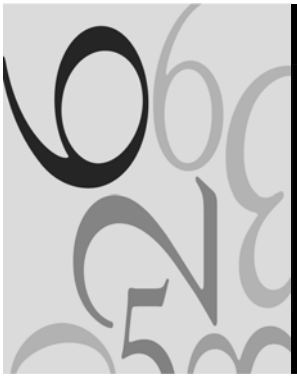
Danielle	**Cathy**
is able to	is able to
is not yet able to	is not yet able to

Bridging to Practice: Building a Repertoire

What opportunities do you currently provide for your students to develop a flexible repertoire of mental, paper, and calculator strategies?

Where is most of your emphasis? What might you incorporate more or less of in your planning?

Notes



UNIT 10

Conclusion

DEVELOPING A PLAN

Desired Outcomes

Participants will

- use the *First Steps in Mathematics* Planning Cycle to develop a practical classroom plan for a starting point
  - identify a Key Understanding on which to focus once back in the classroom
  - choose a Diagnostic Task that will uncover learning patterns and student needs, and a Learning Activity that will address those needs
  - predict and anticipate student responses to both the Diagnostic Task and the Learning Activity based on an understanding of the phases of the Diagnostic Map
  - formulate follow-up focus questions and paraphrases that will move all students on
- become familiar with *First Steps in Mathematics* support resources, such as tracking sheets, organizers, and correlations
- recognize the value of on-going teacher discussions about how to address student needs, and brainstorm possible structures in the school day where similar discussions could take place

Unit Contents

FSIM012 | First Steps in Mathematics: Number Course Book

Related Reading | ***First Steps in Mathematics: Operation Sense*** **Resource Book**

Chapter 1: An Overview of *First Steps in Mathematics*

Understanding the Elements of *First Steps in Mathematics*, p. 8

"Planning with *First Steps in Mathematics*," p. 15

Suggestions for Further Reading

West, L., Staub, F.C. *Content-Focused Coaching: Transforming Mathematics Lessons.* Heinemann, 2003

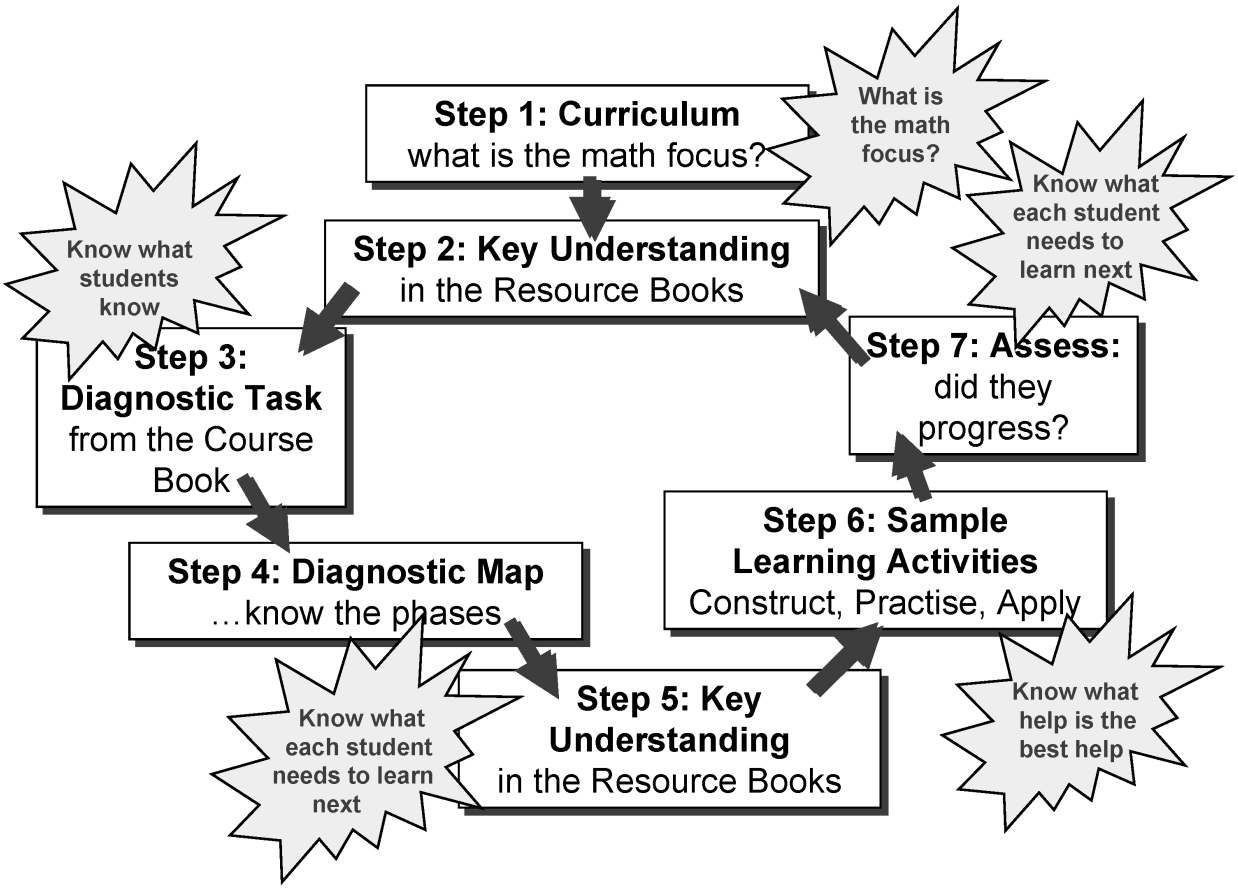
Goldenberg, Claude. *Successful School Change: Creating Settings to Improve Teaching and Learning.* Teachers College Press, 2004

Professional Decision Making: Towards Precise Instruction

"... a breakthrough will occur in which the education community as a whole focuses on improving classroom instruction and adopts processes for turning it into a more precise, validated, data-driven expert activity that can respond to the learning needs of individual students."

M. Fullan, P. Hill, C. Crevola, *Breakthrough,* Sage Publications, 2006, p. xv

Using *First Steps in Mathematics* to Advance Every Student



Discussion

How could *First Steps in Mathematics* enable you to respond more precisely to the learning needs of your students?

Planning to Advance a Whole Class in One Mathematical Idea (Key Understanding)

To plan how you will use *First Steps in Mathematics* in your classroom, you can work with the Planning Masters on the following pages or the Planning Master on page 192 of the *Number Sense* Resource Book (also found on page 304 of the *Operation Sense* Resource Book).

The Planning Masters in this Course Book are reproduced on pages 153–155. You may wish to enlarge the pages and photocopy them on 11 × 17 paper to provide more space for writing.

Notes

Planning Master: Grades K–1

1. What is my teaching goal?

Curriculum expectation/outcome:

Key Understanding:

2. How will I find out what my students know and believe?

Diagnostic Task:

3. What do my students know and believe?

Diagnostic Map: Phases of thinking summaries

Emergent: The child recognizes that numbers may be used to signify quantity.
- At the end of the Emergent phase for this KU, students...

Matching: The child uses one-to-one relations to share and count out.
- At the end of the Matching phase for this KU, students...

Quantifying: The child uses part-part-whole relations for numerical quantities.
- At the end of the Quantifying phase for this KU, students...

Partitioning: The child uses additive thinking to deal with many-to-one relations.
- At the end of the Partitioning phase for this KU, students...

5. What evidence of learning do I see and hear?

Observations and next steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Resource Book and page:
Focus Questions and paraphrasing:

…to move students from the Emergent phase to the Matching phase:

…to move students from the Matching phase to the Quantifying phase:

…to move students from the Quantifying phase to the Partitioning phase:

Planning Master: Grades 2–5

Curriculum expectation/outcome:

1. What is my teaching goal?

Key Understanding:

2. How will I find out what my students know and believe?

Diagnostic Task:

What do my students know and believe?

Diagnostic Map: Phases of thinking summaries

Matching: The child uses part-part-whole relations for numerical quantities.
- At the end of the Matching phase for this KU, students...

Quantifying: The child uses additive thinking to deal with many-to-one relations.
- At the end of the Quantifying phase for this KU, students...

Partitioning: The child thinks both additively and multiplicatively about numerical quantities.
- At the end of the Partitioning phase for this KU, students...

Factoring: The child thinks both additively and multiplicatively about numerical quantities.
- At the end of the Partitioning phase for this KU, students...

5. What evidence of learning do I see and hear?

Observations and next steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Resource Book and page:
Focus Questions and paraphrasing:

…to move students from the Matching phase to the Quantifying phase:

…to move students from the Quantifying phase to the Partitioning phase:

…to move students from the Partitioning phase to the Factoring phase:

Planning Master: Grades 4–8

Curriculum expectation/outcome:

1. What is my teaching goal?

Key Understanding:

2. How will I find out what my students know and believe?

Diagnostic Task:

3. What do my students know and believe?

Diagnostic Map: Phases of thinking summaries

Quantifying: The child uses part-part-whole relations for numerical quantities.
- At the end of the Quantifying phase for this KU, students...

Partitioning: The child uses additive thinking to deal with many-to-one relations.
- At the end of the Partitioning phase for this KU, students...

Factoring: The child thinks both additively and multiplicatively about numerical quantities.
- At the end of the Partitioning phase for this KU, students...

Operating: The child can think of multiplications and divisions in terms of operators.
- At the end of the Partitioning phase for this KU, students...

5. What evidence of learning do I see and hear?

Observations and next steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Resource Book and page:
Focus Questions and paraphrasing:

…to move students from the Quantifying phase to the Partitioning phase:

…to move students from the Partitioning phase to the Factoring phase:

…to move students from the Factoring phase to the Operating phase:

Linking the Key Understandings, the Diagnostic Tasks, and the Phases of the Diagnostic Map

Number Sense Resource Book

Key Understandings	Diagnostic Tasks	Students' typical responses by the end of a phase:					
***Number Sense* Resource Book**	***Number* Course Book**	***Emergent phase*** These students recognize that numbers may be used to signify quantity. **Does the student:**	***Matching phase*** These students use one-to-one relations to share and count out. **Does the student:**	***Quantifying phase*** These students use part-part-whole relations for numerical quantities. **Does the student:**	***Partitioning phase*** These students use additive thinking to deal with many-to-one relations. **Does the student:**	***Factoring phase*** These students think both additively and multiplicatively about numerical quantities. **Does the student:**	***Operating phase*** These students can think of multiplications and division in terms of operators. **Does the student:**
Whole and Decimal Numbers ***KU 2*** K–Grade 3 SLA K–Grade 3 SLA Grades 3–5 SLA	**More** p. 166	…use "bigger", "smaller", "the same" to describe differences between small collections of like objects and between easily compared quantities?	…use most of the principles of counting if they count the collections to check after selecting the card with more dots?	…use counting without prompting as a strategy to solve problems such as, "Who has more?" where the collections are large or different sized objects?			
Whole and Decimal Numbers ***KU 1*** K–Grade 3 SLA K–Grade 3 SLA: "Matching" & "Enough for all" Case Study 2 K to Grade 3 SLA: "Matching" & "Enough for all" Case Study 2	**Subitizing** p. 175	…recognize collections of 1, 2, & 3?	…recognize collections up to 5?				
	Animals p. 176		…use numbers alone, moving "one" from this number to that number?	…use materials or visualize to decompose small numbers into parts empirically; e.g., 8 is the same as 5 with 3? …use known facts to work out other partitions?	…describe half of the possible partitions and use the inverse relationship to say why, e.g., 8 is "0 + 8, 1 + 7, 2 + 6 , 3 + 5, 4 + 4 and you don't need to say the others because it's just those turned around"?		
	Get Me Task p. 167	…take a handful, not focusing on the amount requested?	…choose to use counting to help find the right amount? …understand that it is the last number said which gives the count?				
	Counting Principles p. 167	…say a string of number names in order but not connect them to how many are in the collection?	…include each item only once? …say the number string on order? …attempt to keep track of the starting point?	…count from the middle when asked and include all of the items in the collection?			
	Ice Cream p. 169	…take a handful, without counting either the bears or the ice creams?	…give out the ice creams one at a time, without counting?	…count the starting group and use this count to count out the correct collection of things?			
	Skip Counting p. 170	…say a string of number names in order but not connect them to how many are in the collection?	…say the number string by twos without connecting it to the quantity? …include the last single object as two rather than one?	…count the collection by twos and correctly arrive at 15?			

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Key Understandings	Diagnostic Tasks	Emergent phase	Matching phase	Quantifying phase	Partitioning phase	Factoring phase	Operating phase
Whole and Decimal Numbers ***KU 4*** • K–Grade 3 SLA • Grades 3–5 SLA	**1–9 Repeating Sequence** p. 171 **Up To and Through the Hundreds** p. 171	…memorize the 1 to 10 (and may be to13) words in sequence?	…hear the 4 to 9 part of the sequence in 14 to 19? …predict and name the decades by following the 1 to 9 sequence? …repeat the 1 to 9 sequence within each decade?	…repeat the decade sequence and 1 to 9 sequence within each of the hundreds?	…repeat the hundreds, decade sequence and 1 to 9 sequence within each of the thousands? …readily use the names of the first several places from right (ones, tens, hundreds, ones of thousands)?		
Whole and Decimal Numbers ***KU 4*** • Grades 3–5 SLA • Grades 5–8 SLA • Case Study 3 • Did You Know, p. 63	**Read, Write, and Say Numbers** p. 188			…write most of these numbers correctly but use invented rules which help them get right answers? …write 500 6000 015 for five hundred and six thousand and fifteen?	…give explanations based on correct place value understanding?		
Whole and Decimal Numbers ***KU 4*** • Grades 3–5 SLA • Grades 5–8 SLA • Sample Lesson 3	**What's Next?** p. 178			…repeat the decade sequence and 1 to 9 sequence within each of the hundreds and correctly say 489, 490?	…repeat the hundreds, decade sequence, and 1 to 9 sequence within each of the thousands and correctly say 489–490; 999–1000; 1099–1100, 2999–3000; 4909–4910?	…repeat the hundreds, decade sequence, and 1 to 9 sequence within each of the thousands and correctly say 489–490; 999–1000; 1099–1100, 2999–3000; 4909–4910; 9999–10 000; 199 999–200 000; 1 099 999–1 100 000?	
Whole and Decimal Numbers ***KU 4*** • Grades 3–5 SLA • Sample lesson 3 • Grades 5–8 SLA ***Whole and Decimal Numbers*** ***KU 5*** • Grades 3–5 SLA • Grades 5–8 SLA	**Saying the Number Sequence by Ones and Tens** p. 174				…count up and down in tens from a starting number like 23 or 79 (into the 1000s)? …readily use the names of the first several places from right (ones, tens, hundreds, ones of thousands)?	…count forwards and backwards from any whole number? …use the cyclical pattern in whole numbers and so can read numbers into the billions?	
Whole and Decimal Numbers ***KU 5*** • K–Grade 3 SLA • Grades 3–5 SLA ***Whole and Decimal Numbers*** ***KU 1*** • K–Grade 3 SLA • Grades 3–5 SLA	**Dinosaurs** p. 180		…attempt to count every dinosaur by ones using principles of counting? …think the 3 in 35 mean 3 dinosaurs?	…count dinosaurs by fives or sevens? …think the 3 in 35 mean 10 dinosaurs because it is in the tens place?	…count dinosaurs by fives or sevens? …circle 30 dinosaurs to mean the 3 in 35?	…use multiplication to count the dinosaurs, e.g., says seven fives or five sevens?	
Whole and Decimal Numbers ***KU 5 & 6*** • Grades 3–5 SLA • Grades 5–8 SLA • Did You Know, p. 67	**52 Candies, 43 Candies** pp. 182–185			…count by tens and correctly arrive at 52 and 43? …circle 5 rolls of ten for 5 in 53 and 3 rolls of ten for the 3 in 43?	…count by tens to arrive at 52 and 43? …circle 5 rolls of ten for 5 in the 52 question and 3 rolls of ten for the 3 in the 43 question? …may have a change of mind and then correct the 43 question by circling 3 singles for the 3 and 3 rolls plus 10 singles for 40 in 43?	…correctly circle 5 rolls to mean 50 in the 52 question and 3 singles and 3 rolls of ten plus 10 singles to mean 40 in 43?	

Key Understandings	Diagnostic Tasks	Emergent phase	Matching phase	Quantifying phase	Partitioning phase	Factoring phase	Operating phase
Whole and Decimal Numbers ***KU 6*** • Grades 3–5 SLA • Grades 5–8 SLA	**800 Game** pp. 195				…use additive thinking to change, e.g., 8 to 800 by first mentally calculating the difference then adding 792 to 8 on the calculator; or 8000 to 800 by mentally calculating the difference and then subtracting 7200 from 8000 on the calculator?	…fully understand and flexibly use the multiplicative relationship between the places in whole-number place value, e.g., divides by 10 to change 8000 to 800 and multiplies by 100 to change 80 to 8000?	…fully understand and flexibly use the multiplicative relationship between the places in decimal number place value, e.g., divides by 10 to change 8 to 0.8 and multiplies by 100 to change 0.08 to 8?
Whole and Decimal Numbers ***KU 5 & 6*** Grades 3–5 SLA Grades 5–8 SLA ***Whole and Decimal Numbers*** ***KU 7*** Grades 3–5 SLA Grades 5–8 SLA	**Circle the Biggest** p. 190			…say that the bigger number has more numbers or refer to putting a zero on makes it bigger?	…say the 3 in 370 is in the hundreds and the 3 in 37 is in the tens or explanations that relate in a similar way to place value for each of the whole number questions?	…hold a misconception about the magnitude of decimals (see Course book p. 35)?	…describe the value of each number correctly using place value, e.g., 0.37 as 37/100 and 0.0037 as 37/10 000?
Whole and Decimal Numbers ***KU 6*** Grades 3–5 SLA Grades 5–8 SLA	**Flexible Numbers** p. 192				…use standard place value to generate one correct combination for each number, e.g., 312 uses 3 hundreds, one ten, and 2 ones? …use non-standard partitioning to generate 2 digit numbers, e.g., make 61 with 5 tens and 11 ones?	…are flexible in mental partitions confident that the quantity has not changed? …use their knowledge to generate alternate partitions, e.g., use non-standard place value for 454—use 3 hundreds, 6 tens, 4 tens, 3 tens, 10 ones, and 14 ones?	
Whole and Decimal Numbers ***KU 6*** Grades 3–5 SLA Grades 5–8 SLA ***Computations*** ***KU 6*** Grades 3–5 SLA Grades 5–8 SLA Background Notes	**How Did You Do It?** p. 177				…use compensation strategies and decomposing, e.g., 62 as 60 + 2 and 23 as 20 + 3 then add 60 + 20 and 2 + 3? …for 26 + 37 at first try mental strategies then use jottings or written algorithm to find the result?	…draw on a considerable range of strategies to mentally calculate 2 digit addition, e.g., think of 26 + 37 as 26 + (4 + 33) 26 + 4 =30 30 + 33 = 63?	…use non-standard place value, e.g., think of 84 as 70 +14 and 67 as 60 + 7, first deal with the tens then the 7 from 14? …change the number by subtracting the same amount, e.g., think of 84 – 67 as 87 – 70?
Whole & Decimal Numbers ***KU 7*** Grades 3–5 SLA Grades 5–8 SLA ***Course Book*** Whole, decimal fractional link Notes: Progress of Decimal Understanding	**Decimals** p. 197			…ignore the decimal point or think of it as a decoration or punctuation?	…use the ideas based on daily use of money or measures, so may think the decimal point separates two whole numbers, where the whole numbers refer to different-sized units: e.g., when referring to money, think 6.125 is $7.25? …think decimals have some sort of different value? …may rightly think of decimals as another way to represent fractional numbers but, for example, think 0.6 is one sixth?	…link the unit fraction symbol to the decimal fraction notation as represented on the calculator, e.g., 1/10 can be renamed as 0.1? …use the idea of splitting a whole into parts to understand, for example, that 2.4 is 2 + 4/10 and that 2.45 is 2 + 45/100?	…use their understanding of the relationship between successive places to order decimal numbers regardless of the number of places? …flexibly partition decimal numbers in multiple ways; e.g., knowing that 0.36 is 0.3 + 0.06 and also 0.2 + 0.16, and so on?

Operation Sense Resource Book

Key Understandings	Diagnostic Tasks	Students' typical responses by the end of a phase:					
***Operation Sense* Resource Book**	***Number* Course Book**	***Emergent phase*** These students recognize that numbers may be used to signify quantity. **Does the student:**	***Matching phase*** These students use one-to-one relations to share and count out. **Does the student:**	***Quantifying phase*** These students use part-part-whole relations for numerical quantities. **Does the student:**	***Partitioning phase*** These students use additive thinking to deal with many-to-one relations. **Does the student:**	***Factoring phase*** These students think both additively and multiplicatively about numerical quantities. **Does the student:**	***Operating phase*** These students can think of multiplications and division in terms of operators. **Does the student:**
Operations ***KU 1*** • K–Grade 3 SLA	**Hide the Jelly Beans** p. 219	…count when asked to count the jelly beans there are, and may arrive at the right number? …count the number not hidden but not focus on those that are hidden?	…count when asked to find how many jelly beans there are? …solve small number story problems which require them to add some, take some away, or combine two amounts by imagining or role-playing the situation and counting the resulting quantity? … solve subtraction problems when there is a specific action suggested?	…use known facts to solve the problem?			
	How Many Want Jelly Beans? (Problem A) p. 221	…say a string of number names in order but not connect them to how many are in the collection?	…count out each set then count all to find the solution but may need prompting?	…select counting as a strategy without prompting? …find it obvious that when combining or joining collections counting on will give the same result as counting all?	…use part-whole reasoning without needing to see or visualize physical collections and so use known facts to solve the problem?		
Operations ***KU 1*** • K–Grade 3 SLA: "Matching" & "Enough for all" • Case Study 2	**How Many Want Jelly Beans? (Problem B)** p. 222	…say a string of number names in order but not connect them to how many are in the collection?	…draw or represent each set of people or jelly beans but unable to solve "start unknown" change problems?	…think of "+" and "-" problems in terms of as the whole and two parts and which is missing; e. g., use materials, draw, or visualize 13 jelly beans in two parts 5 yellow and count backwards; use known facts or use inverse relationship for small or familiar numbers?	…use part-whole reasoning without needing to see or visualize physical collections and so use known facts to solve the problem?		
Operations ***KU 1*** • Grades 3–5 SLA • Grades 5–8 SLA	**How Much Taller?** p. 208			…use modelling or counting to solve compare problems? …can think of "+" and "-" situations in terms of the whole and the parts and which is missing and so count on from 132 to 154?			

Key Understandings	Diagnostic Tasks	Emergent phase	Matching phase	Quantifying phase	Partitioning phase	Factoring phase	Operating phase
Operations ***KU 5 & 6*** • Grade 3–5 SLA • Grade 5–8 SLA	**Empty Boxes** p. 210			…use diagrams or counting to solve inverse addition/ subtraction problems? …write number sentences to match how they think about the story line (sematic structure) for small number "+"and "-", problems, counting back for some problems? …may be able to use the inverse relationship between "+" and "-" to choose the more efficient of counting on or counting back?	…use the inverse relationship between addition and subtraction to make a direct calculation possible: e.g., re-interpret 43 – ☐ = 16 as "what do you have to add to 16 to get 43?" and so count on by tens and ones?	…use the inverse relationship between addition and subtraction to make a direct calculation possible: e.g., re-interpret 17 + ☐ = 36 as 36 – 17 = ☐?	
Operations ***KU 5 & 6*** Grades 3–5 SLA Grades 5–8 SLA	**Change Task** p. 205			…can think of addition and subtraction situations in terms of the whole and the two parts and which is missing with 2 digit easily calculated numbers?	…routinely use inverse relationship between addition and subtraction to solve change problems which are not straight-orward? …write suitable number sentences for a range of addition and subtraction situations to solve on the calculator?		
Operations ***KU 3*** K–Grade 3 SLA Grades 3–5 SLA Background Notes	**Story Problems** **Sausages** p. 212	…use realistic materials to show a situation involving people, sausages, sharing and using one or some of the numbers in the problem?	…solve small number story problems which require adding up equal groups of 3s using counting by ones to solve the problem?	…can count equal groups by physically or mentally laying out each group, but think of and treat each group as direct from the others?	…choose multiplication to solve repeated addition situations?		
	Story Problems **Tennis Balls** p. 212			…need help with the meaning of "five times as many"?	…can write multiplication number sentences for problems which they can think of as "lots of", may only solve other types of multiplication problems with materials or by counting?	…interpret multiplication situations as "times as much" and so can see that 5 times as many tennis balls is 5 x 3?	
Operations ***KU 3*** K–Grade 3 SLA Grades 3–5 SLA Background Notes	**Story Problems** **Flavours** p. 212			…use diagrams and addition to represent and solve combination problems?	…can write multiplication number sentences for problems which they can think of as "lots of", may only solve other types of multiplication problems with materials or by counting?	…select an appropriate multiplication or division operation on whole numbers including for problems that are not easily interpreted as "lots of"; e.g., combinations?	
Operations ***KU 4*** K–Grade 3 SLA	**Story Problems** **Animals** p. 212	…distinguish spoken numerals from other spoken words? …use realistic materials to show a situation involving animals and pens or cages, sharing and using one or some of the numbers in the problem?	…share realistic objects into 2 drawn paddocks or cages by dealing out an equal number of items or portions to each paddock/cage, cycling around the paddocks/cages one at a time, or handing out two or three at a time?	…use materials, e.g., blocks/counters, or diagrams to represent sharing situation?			

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Key Understandings	Diagnostic Tasks	Emergent phase	Matching phase	Quantifying phase	Partitioning phase	Factoring phase	Operating phase
Operations ***KU 5*** • Grades 3–5 SLA • Grades 5–8 SLA ***Computations*** • Background Notes • Multiplication facts • Background Notes • Use relationships (commutativity and inverse)	**Finding Factors** p. 215			…need to use materials and counting to find equal groups?	…use some known multiplication facts?	…understand that a number can be decomposed and recomposed into its factors in a number of ways without changing the quantity? …use known facts, inverse relationships, and commutativity; to work out all of the factors: e.g., divide by 3 to check if it is a factor; knows 27 x 3 gives the same result as 3 x 27 and so both are factors?	…use multiplicative reasoning; e.g., knows 9 is a factor of 81, and so 3 must be because it is a factor of 9 and that 18 can't be because multiples of 8 never end in 1 whereas 27 could be for the same reason, check to find out?
Operations ***KU 3*** • Grades 3–5 SLA • Grades 5–8 SLA • Background Notes • Multiplication and division problems ***Operations*** ***KU 4*** • Grades 3–5 SLA • Grades 5–8 SLA • Sample Lesson 2 • Did You Know? ***Operations*** ***KU 5*** • Grades 3–5 SLA • Grades 5–8 SLA	**Calculator Number Sentences** p. 217			…attempt to solve using sharing and grouping of materials?	…select multiplication or division for straightforward "repeat equal quantities" problems?	…understand why sharing and grouping problems can be solved by the same division process? …select an appropriate multiplication or division operation on whole numbers including for problems that: - they can visualize as arrays - are not easily interpreted as "lots of": e.g., combination and comparison problems ?	…recognize the need to multiply or divide where the multiplier is a fractional number? …deal with proportional situations; e.g., ratio comparison and changes (scale)? …use division in situations where divisors are decimal and fractional numbers and may be bigger than the number being divided into?

Key Understandings	Diagnostic Tasks	Emergent phase	Matching phase	Quantifying phase	Partitioning phase	Factoring phase	Operating phase
Computations • Background Notes: Techniques for mental calculation ***Operations*** ***KU 4*** K to Grade 3 SLA Grades 3–5 SLA Case Study 2	**Find the Solutions** **(Set A)** p. 188			…use tallies to count on? …select an appropriate operation to solve on the calculator? …may be unable to use the inverse relationship between addition and subtraction to choose the more efficient of count on or count back for solving particular problems?	...partition at least two- and three-digit numbers into standard component parts; e.g., 25 in to 20 + 5 and add 20 + 30 + 5? …count up and down in tens from any starting point then front load? …double count in multiplicative situations by representing one group and counting repeats of the same group, simultaneously keeping track of the number of groups and the number in each group?		
Computations Background Notes: Techniques for mental calculation ***Operations*** ***KU 5*** Grades 3–5 SLA Grades 5–8 SLA Case Study 2	**Find the Solution** **(Sets B & C)** p. 188			…use tallies to count on? …select an appropriate operation to solve on the calculator? …may be unable to use the inverse relationship between addition and subtraction to choose the more efficient of count on or count back for solving particular problems?	...partition at least two- and three-digit numbers into standard component parts; e.g., 25 in to 20 + 5 and add 20 + 30 + 5? …count up and down in tens from any starting point then front load?	...use an extended calculation repertoire for reducing memory load? …mentally add and subtract two-digit numbers and mentally multiply and divide by single-digit numbers and multiples of	…use non-standard partitioning, number facts, and inverse relationships to mentally calculate each problem? …use properties of operations such as commutativity, and distribution of multiplication
Operations ***KU 8*** Grades 3–5 SLA Grades 5–8 SLA					…double count in multiplicative situations by representing one group and counting repeats of the same group, simultaneously keeping track of the number of groups and the number in each group?	ten for “easy” numbers such as 4 x 25	over division to rearrange the numbers to make calculation easier?
Computations ***KU 4*** Grades 3–5 SLA Sample Lesson 3 Grades 5–8 SLA ***Computations*** ***KU 5*** Grades 3–5 SLA Grades 5–8 SLA	**Buying Apples** p. 189					…know they can choose between multiplication and division to make calculation easier? …fail to select an appropriate operation in situations where they cannot think of the multiplier or divisor as a whole number?	…recognize the need to multiply where the multiplier or divisor is a fractional amount? …record stages in calculating with fractions that they cannot complete mentally?

Diagnostic Tasks

Instructions and Line Masters

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Diagnostic Tasks

WHOLE AND DECIMAL NUMBERS

Counting

Partitioning

Place Value

OPERATIONS

Addition and Subtraction

Multiplication and Division

COMPUTATIONS

Using the Whole and Decimal Numbers Tasks

If you teach Kindergarten through to Grade 2 students, start with Get Me Task and follow up with Counting Principles and Subtizing. If students have trouble with these tasks, then try the More task. If they succeed on the Get Me task, then try the Ice Cream task, the Animals task, and the Skip Counting task.

If you teach Grade 2 through to Grade 4 students, start with the Ice Cream task, the Skip Counting task, and the Animals task. If students struggle with these tasks, try the tasks suggested for K–2 students above.

COUNTING

DIAGNOSTIC TASK: More

Whole and Decimal Numbers
KU 2 for K–Grade 1 (ages 4–6)

Purpose

To assess students' ability to compare small collections and say which is bigger without using numbers

Materials

- Cards with 1–6 dots on each

Instructions

This task is a card game for two players. Distribute all the cards to the players. Players hold their cards face down in a pile. Both players turn over a card at the same time. They look at the cards. The player with the most dots is the winner and gets both cards.

Ask: "Who has the most dots?" or "Which card has more dots?"

If the same number of dots is turned up, then a match is declared and both players turn up another card simultaneously. If one is larger, the winner takes both pairs of cards.

DIAGNOSTIC TASK: Get Me Task

Whole and Decimal Numbers
KU 1 for K–Grade 2 (ages 4–7)

Purpose

To see if the student chooses to use counting when asked to get a number of items

Materials

- General classroom equipment

Instructions

1. Tell the student that you need some blocks to make a house.
   Ask: "Can you get me 7 blocks?"
   Observe: Does the student
   - choose to use counting to help find the right amount?
   - take a handful, ignoring the amount requested?

2. Use all 7 blocks to build a house.

3. Ask: "Can you get me 12 counters so I can make a path to my house, please?"
   Observe the student again using the above criteria.

4. Set out the path around the house and engage the student in a short playtime.

DIAGNOSTIC TASK: Counting Principles

Whole and Decimal Numbers
KU 1 for K– Grade 2 (ages 4–7)

Purpose

To assess student understanding of the principles of counting

Materials

- General classroom equipment

Instructions

1. *Counting Principles 1, 2, and 3(a)*: Show a student a scattered collection of 8 items.
   Ask: "Can you tell me how many [e.g., animals] are here?"
   Observe: Does the student
   - include each item only once?
   - say the number names in the right order?
   - move the items or keeps track of her/his starting point?

2. *Counting Principle 5*: When the student has completed the count, notice whether she/he emphasizes the last number. Even if it has been emphasized, ask the next question.

   Ask: "How many [e.g., animals] are there?"
   Observe: Does the student
   - recount from the start?
   - repeat the last number word?
   - look at you as though the question doesn't make sense?

3. *Counting Principle 4*: Place the items in a line.

   Ask: "How many [animals] are there?"
   Observe: Does the student
   - recount from the start?
   - restate the number without counting?
   - get the same number as she/he did with the first count?

   If the student finishes the count with a different number from the original count, say: "When you counted them before you said there were [say number] and now you say there are [say number]."

   Ask: "So, are there [the first number student said] or are there [the second number student said]?"
   Observe: Does the student know that he/she should have the same number for both counts?

4. *Counting Principle 3(b)*: Take a new collection of items. Place them in a line and ask the student to count the items starting in the middle. Point to an item in the middle.
   Ask: "How many [blocks] do we have? When you count the [blocks], begin with this one [pointing to the middle item]. It is number one."

   Observe: Does the student
   - count from the middle item. and includes all of the items in the count?
   - count from the middle moving from left to right to the end of the row, omitting the first few items in the row?

DIAGNOSTIC TASK: Ice Cream (Equal Sets)

Whole and Decimal Numbers
KU 1 for Grades 1–3 (ages 6–8)

Purpose

To see if the student chooses to use counting in order to make an equivalent set

Materials

- A box filled with cut-out pictures of ice cream cones
- A picture of 6 children scattered around the page
- A picture of 10 children standing close together in a line
- A picture of 14 children scattered around the page

Instructions

1. Show the box of ice cream cones, place it on a desk some distance away from the student, and say it is an ice cream parlour.

2. Say to the student, "You are the [Mom/Dad] of this group of children [shown on a page in front of them] and they all want ice cream. Will you go to the ice cream parlour and get an ice cream cone for each of the children?"

3. Repeat the instruction for each of the different-sized groups.

   Observe: Does the student
   - count the starting group?
   - use the count of the starting group to count the number of ice cream cones?
   - just grab a handful of ice cream cones?
   - choose to give out one ice cream cone at a time, with no counting?

DIAGNOSTIC TASK: Skip Counting

Whole and Decimal Numbers
KU 1 for Grades 1–4 (ages 6–9)

Purpose

To find out if the student knows that counting in groups gives the same result as counting by ones

Materials

- Any collection of 15 small items (e.g., beads)
- Any collection of 50 small items (e.g., popsicle sticks)

Instructions

This task may be carried out during lessons while a student is involved in making and counting collections.

1. Give a student 15 small things, such as beads.

   Ask: "How many beads have I given you? How did you decide that?"
   Observe: If the student counts to 14 accurately by twos, then adds the one, stop the interview here.
   If the student counts by ones, go on to step 2.

2. Ask: "Will you get the same answer if you count by twos? Count by twos to find out."
   Observe: Notice how the student keeps track of the "twos" and what he/she does when he/she reaches the remaining single bead.

   At the end of the count, notice if the student calls the single bead the next number in the "twos" sequence, even though there is only one there. For example, does the student point and say "12, 14, 16?" or "12, 14, 15" (arriving at 15 for the answer)?

   If by now you are sure the student knows that counting by twos gives the same result as counting by ones, stop the task. If you are still unsure, continue with step 3.

3. Empty out a container that holds more than 50 objects, e.g., craft sticks.

   Ask: "How many craft sticks do you think are there? How could you know exactly how many are there?"
   Observe: Watch to see if the student begins and then continues to count the whole collection by ones. If he/she does, ask: "How many craft sticks will there be if you count by fives?"

DIAGNOSTIC TASK: 1–9 Repeating Sequence

Whole and Decimal Numbers
KU 4 for K–Grade 4 (ages 4–9)

Purpose

To find out how far the student knows the sequence of numbers used when counting

Instructions

Interview (ages 4–7): Listen to the student say the counting numbers as far as she/he can go. Help the student with the decade names if necessary.

Interview (ages 7–9): Listen to the student say the counting numbers from 80 to over 100. Say you want her/him to take over the counting where you leave off. Begin the count with "80, 81, 82, 83, 84, 85, …" having student continue to, say, 140.

DIAGNOSTIC TASK: Up To and Over 100; Up To and Through the Hundreds

Whole and Decimal Numbers
KU 4 for Grades 2–7 (ages 7–12+)

Purpose

To see if students know the pattern in the way we say numbers up to and over 100

Materials

- Line Master: Up To and Over 100
- Up To and Through the Hundreds

Instructions

1. Provide each student with a blank 10 × 20 grid and ask them to fill it in, counting by ones, beginning at one.

2. Interview individual students when:
   - the student writes an incorrect number or writes a number incorrectly. Ask him/her to "say" that part of the sequence so that you are able to hear what the student actually thinks the pattern is.
   - the student generally experiences difficulty when working with numbers.

Name: _____

Grade: _____ **Date:** _____

Line Master **Up To and Over 100**

Write the numbers to the end of the boxes.
Begin with one and count by ones to the end of the boxes.

1	**2**	**3**							

Name:_____

Grade: _____ **Date:** _____

Line Master **Up To and Through the Hundreds**

1. Write the numbers to the end of the boxes.
   Begin at 91 and count by ones to the end of the boxes.

91	**92**	**93**							

2. Write the numbers to the end of the boxes.
   Begin at 491 and count by ones to the end of the boxes.

491	**492**	**493**							

DIAGNOSTIC TASK: Saying the Number Sequence by Ones and Tens

Whole and Decimal Numbers
KU 4 and KU 6 for Grades 4–7 (ages 9–12+)

Purpose

To see if students can use the patterns in the way we say numbers to count on and back by tens from any starting point. The counting by ones gives information about KU 4. The counting by tens aspect of this task gives us information about KU 6.

Instructions

Interview: Ask the student to take over saying the number sequence where you leave off. For example, say "74, 75, 76, 77, 78, …" Repeat this process for each of the examples below.

1. Say the number sequence going forward by ones
   a) starting from 79
   b) starting from 985

2. Say the number sequence going backwards by ones
   a) starting from 75
   b) starting from 1010

3. Say the number sequence going forward by tens
   a) starting from 180
   b) starting from 34

4. Say the number sequence going backwards by tens
   a) starting from 520
   b) starting from 146

DIAGNOSTIC TASK: Subitizing

Whole and Decimal Numbers
KU 2 for K–Grade 1 (ages 4–6)

Purpose

To assess student ability to subitize quantities up to six

Materials

- Six blocks (two-centimetre cubes)
- An ice-cream tub or similar container

Instructions

1. Place two blocks in a container.
2. Show it briefly to the student. Ask: "How many blocks are there?"
3. Add blocks or remove blocks to show the following numbers: 2, 3, 1, 5, 4, and 6, asking the student to tell you how many each time.

Note: Remember to give the student only a brief look so that she/he is forced to use subitizing rather than counting.

DIAGNOSTIC TASK: Animals

Whole and Decimal Numbers
KU 2 for Grades 2–4 (ages 6–9)

Purpose

To find out if the student can partition using materials or with numbers

Materials

- Sheet of paper
- Collections of countable objects if necessary
- Children's picture book about groups of animals (e.g., rabbits in or out of a garden patch)

Instructions

Have the student create partitions in response to the story about animals, using the same number of animals as appeared in the story. Ask the student to show all the different ways she/he could put (e.g., 12) animals in two places (e.g., garden or cage). At first limit the student to paper and pen. If the student struggles, suggest using some materials to help.

Observe: Does the student

- use numbers alone, moving "one" from this number to that number?
- draw lines, dots, or other symbols and count, and then record her/his partitioning?
- use materials to count and then record his/her partitioning?

DIAGNOSTIC TASK: How Did You Do It?

Whole and Decimal Numbers
KU 5 and KU 6 for Grades 3–7 (ages 8–12+)

Purpose

To see if student needs to count to solve a computation mentally, or can use partitioning based on place value or his/her own written methods

Instructions

This lesson enables students to share the strategies they use to carry out a mental computation. The lesson needs to be modelled a number of times with simple computation examples to give students practice in thinking about and recording their mental strategies.

Listen to and record student responses on the board including counting strategies if students mention them.

If the students cannot do the computation mentally, they should find an answer by using pencil and paper. Students should indicate on their page whether they worked it out mentally or used pencil and paper or materials.

Present students with either the Grades 3–5 or 5–8 Sample Learning Activity from Whole and Decimal Numbers KU 6.

Grades 3–5: Math Method (*Number Sense* Resource Book, p. 79)	**Grades 5–8: Different Strategies** (*Number Sense* Resource Book, p. 79)
62 – 23 26 + 37	Your grandpa is 84 but you only have 67 candles for his birthday cake. How many more candles do you need?

Students write the problem on their page, solve it mentally, and then record their strategy. If students cannot solve the problem mentally, tell them to work it out on the paper in a way that makes sense to them, or to use materials if they need to.

Note: Record what the students actually say and do. This will help you to reflect on what they are thinking rather than what you *assume* they are thinking.

DIAGNOSTIC TASK: What's Next?

Whole and Decimal Numbers
KU 4 and KU 5 for Grades 4–7 (ages 9–12+)

Purpose

To see to what extent the student understands the number sequence (to be used with those students who show that they know the sequence of numbers to 200)

Materials

- Line Master: What's Next?

Instructions

Direct students to complete the set of questions by writing in the next number.

Name: _____

Grade: _____ **Date:** _____

Line Master **What's Next?**

The turnstiles at the stadium gates count the people as they go through one by one. What will be the number on each of the gates when one more person goes through?

489 _____	999_____
1 099_____	2 999 _____
4 909_____	9 999 _____
10 999_____	13 999 _____
199 999_____	1 099 999_____

DIAGNOSTIC TASK: Dinosaurs

Whole and Decimal Numbers
KU 5 for Grades 2–7 (ages 7–12+)

Purpose

To examine student understanding of the meaning of the individual digits in a two-digit number

Materials

- Line Master: Dinosaurs
- Pencils or pens in two different colours, e.g., green and red

Note: If your students' work is being examined in a group, it is essential that the entire group uses the same colours for this task.

Instructions

1. Ask students to work out how many dinosaurs there are on the page. Students write how many in the space provided.

2. Talk about the number of dinosaurs until everyone agrees there are 35. Write the number 35 on the board.

3. Point to the 5 and say "Use a green pencil and put a circle around what this part of the number means in the set of dinosaurs." Do not say the word "five."

4. Point to the 3 and say "Use a red pencil and put a circle around what this part of the number means in the set of dinosaurs." Do not say the word "three."

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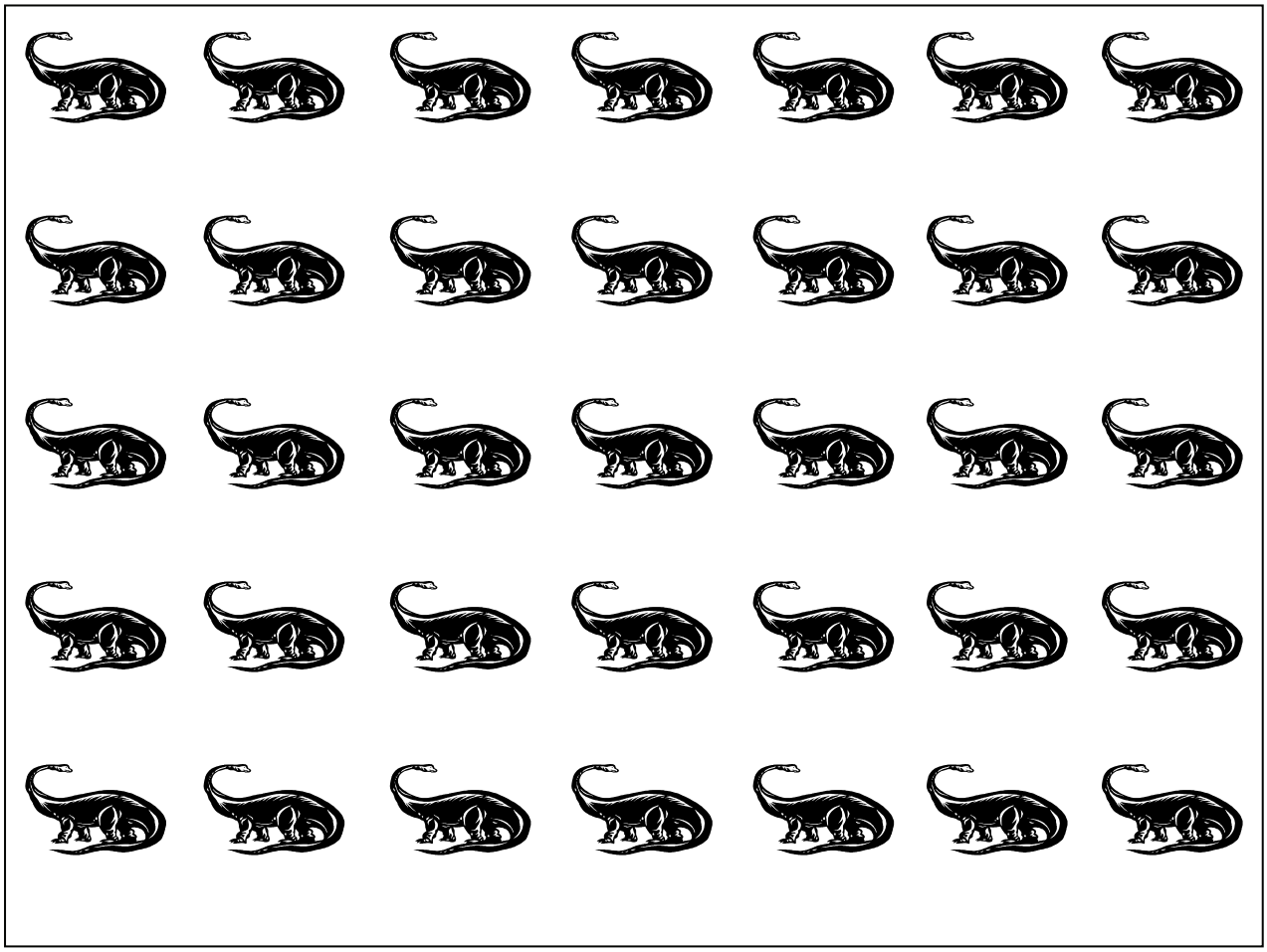
Name:_____

Grade: _____ **Date:** _____

Line Master **Dinosaurs**

How many dinosaurs are here?_____



DIAGNOSTIC TASK: 52 Candies

Whole and Decimal Numbers
KU 5 for Grades 3–7 (ages 8–12+)

Purpose

To explore student understanding of the meaning of the individual digits in a two-digit number when confronted by both standard and non-standard groupings of objects

Materials

- Line Master: Candies (Set A)
- Pencils or pens in red and blue

Instructions

1. Provide each student with a copy of the line master for this task.

2. Read the introductory sentence with them to make sure that students understand that these candies can be bought as single candies or in rolls of ten.

3. Ask: "How many candies are represented altogether?"

4. Talk with the students about their answers until all students agree that there are 52 candies. Observe students as they record 52 on their pages.

5. Write 52 on the board in view of all students. When giving the following instructions it is important that you do not say the words "five" or "fifty" or "two."
   a) Point to the 2 in 52 and say, "Use a blue pen to colour in what this part of the 52 means in the drawing."
   b) Point to the 5 in the 52 on the board and say, "Use a red pen to colour in what this part of the 52 means in the drawing."

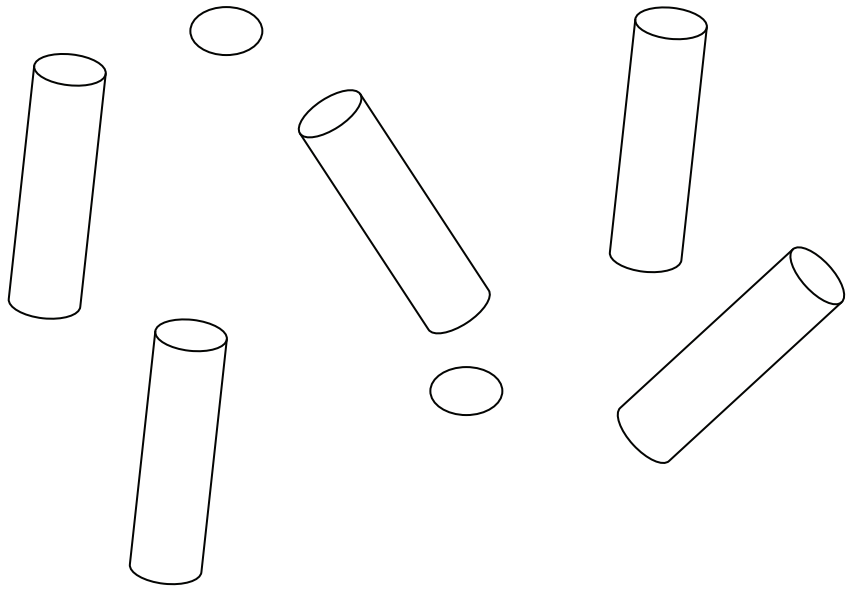
Based on ideas by Sharon Ross (1989)

Name: _____

Grade: _____ **Date:** _____

Line Master **Candies (Set A)**

Candies can be bought as single candies or in rolls of ten as shown here.



How many candies are shown here?_____

Task based on ideas by Sharon Ross. *Arithmetic Teacher, 1989*

DIAGNOSTIC TASK: 43 Candies

Whole and Decimal Numbers
KU 5 for Grades 3–7 (ages 8–12+)

Purpose

To explore student understanding of the meaning of the individual digits in a two-digit number when confronted by both standard and non-standard groupings of objects

Materials

- Line Master: Candies (Set B)
- Pencils or pens in red and blue

Instructions

1. Provide each student with copies of the line master for this task.

2. Introduce as for 52 Candies.

3. Ask: "How many candies are represented altogether?"

4. Talk with the students about their answers until all students agree that there are 43 candies. Observe students as they record 43 on their pages.

5. Write 43 on the board in view of all students. When giving the following instructions it is important that you do not say the words "four" or "forty" or "three."
   a) Point to the 3 in 43 and say, "Use a blue pen to colour in what this part of the 43 means in the drawing."
   b) Point to the 4 in the 43 on the board and say, "Use a red pen to colour in what this part of 43 means in the drawing."

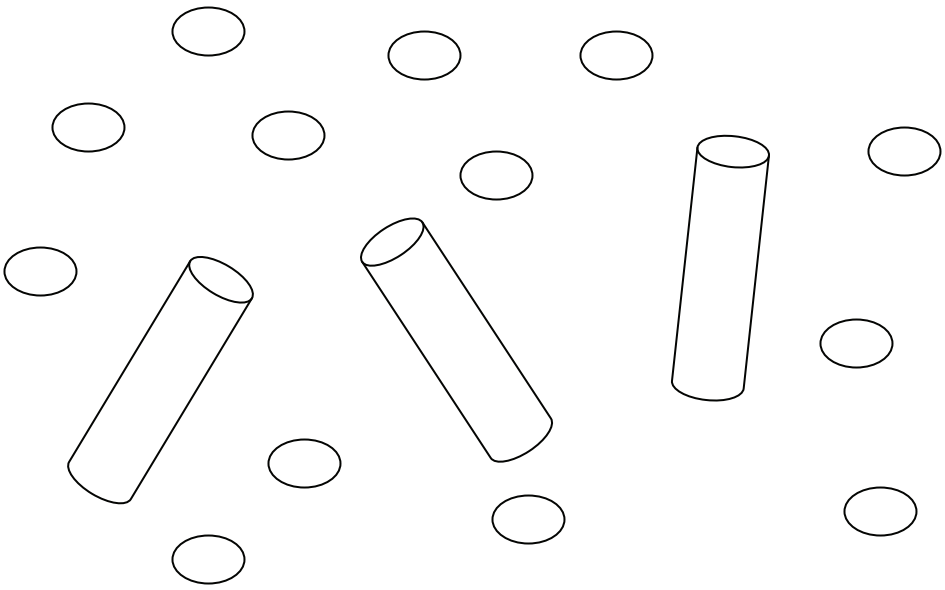
Based on ideas by Sharon Ross (1989)

Name:_____

Grade: _____ **Date:** _____

Line Master **Candies (Set B)**

Candies can be bought as single candies or in rolls of ten as shown here.



How many candies are shown here?_____

Task based on ideas by Sharon Ross. *Arithmetic Teacher, 1989*

DIAGNOSTIC TASK: 116 Candies

Whole and Decimal Numbers
KU 2 and KU 6 for Grades 3–7 (ages 8–12+)

Purpose

To explore if and how students can produce standard and non-standard partitions of a quantity

Materials

- Line Master: 116 Candies

Instructions

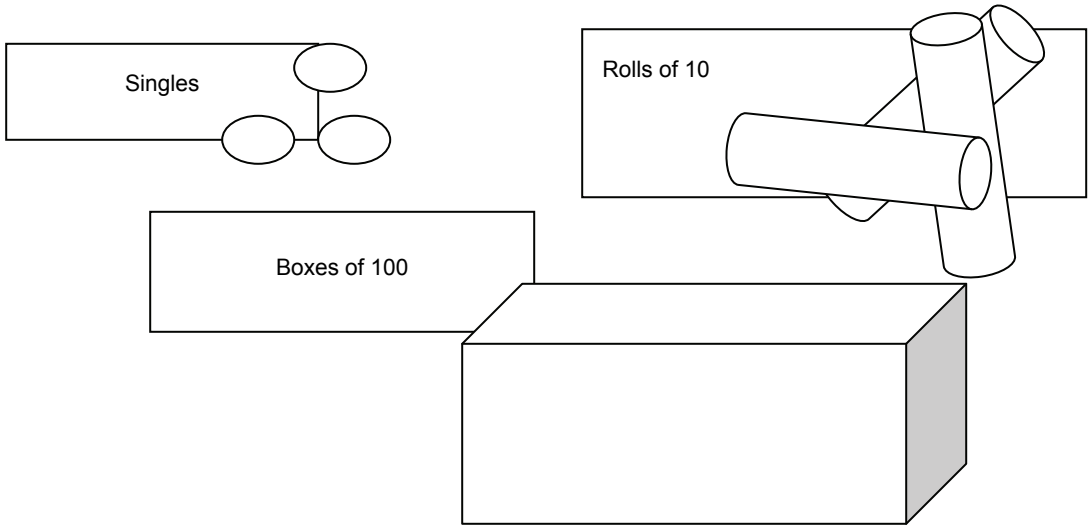
Provide each student with a copy of the 116 Candies task. Read the introductory sentence to make sure all students understand that candies can be bought as single candies, in rolls of ten, or boxes of 100 (10 rolls of ten). Students complete the task independently.

Name: _____

Grade: _____ **Date:** _____

Line Master **116 Candies**

Candies can be bought as single candies, in rolls of ten, or in boxes of 100 as shown here.



How many different ways could you make up an order for 116 candies?

Draw or write your answer in this box.

DIAGNOSTIC TASK: Read, Write, and Say Numbers

Whole and Decimal Numbers
KU 5 for Grades 3–7 (ages 8–12+)

Purpose

To explore the limits of students' writing of large numbers and to expose their personal rules or misconceptions when writing such numbers

Materials

- Line Master: Read, Write, and Say Numbers

Instructions

1. Provide each student with a copy of the line master for this task.

2. Call out the following numbers for students to write for questions 1–6:
   - sixty-three
   - one thousand twenty
   - twenty-six thousand fifteen
   - five hundred six thousand fifteen
   - one million five
   - five billion, thirty-six million, four hundred seven thousand four

3. Students complete the rest of the sheet independently.

4. Interview some individual students and ask them to explain how they knew to write the number in the way that they did. The purpose of the interview is to uncover any invented rules that students may be using.

Name:_____

Grade: _____ **Date:** _____

Line Master **Read, Write, and Say Numbers**

Write the numbers the teacher says. Here is an example:

If the teacher says ***nineteen***, you write ***19***.

A._____ **D.** _____

B._____ **E.** _____

C._____ **F.** _____

Write these numbers in words.

G. 504

H. 1 768

I. 250 000

J. 13 648

K. 6 003

L. 13 806 009

DIAGNOSTIC TASK: Circle the Biggest

Whole and Decimal Numbers
KU 5 for Grades 6 and up (ages 11+)

Purpose

To see if students are able to compare numbers using multiplicative relationships

Materials

- Line Master: Circle the Biggest

Instructions

1. Distribute the line master for this task.
2. Ask students to write a full explanation of the reasoning behind each choice. You may need to conduct some individual interviews if students' reasoning is not clear from the written explanation.

Name: _____

Grade: _____ **Date:** _____

Line Master **Circle the Biggest**

1. Circle the bigger number.

37 **370**

How do you know it is bigger?

How many times bigger is it?

2. Circle the smaller number.

647 **6470**

How do you know it is smaller?

How many times smaller is it?

3. Circle the bigger number.

0.37 **0.0037**

How do you know it is bigger?

How many times bigger is it?

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DIAGNOSTIC TASK: Flexible Numbers

Whole and Decimal Numbers
KU 2 and KU 6 for Grades 6–9 (ages 11–14)

Purpose

To explore students' understanding that numbers can be partitioned in many ways (KU 2), and to find out if students can produce non-standard partitions of a number (KU 6)

Materials

- Line Master: Flexible Numbers (1)
- Line Master: Flexible Numbers (2)
- Scissors

Instructions

Have students complete the task individually. Interview some individual students, asking them to explain how they knew to make the number in the way that they did.

1. Have students cut the Flexible Numbers (2) page into separate cards and use the cards to make each of these numbers in as many ways as they can: 312, 400, 454, 401, 204, 61.

2. Direct students to record the ways they made each number on the worksheet, then put the cards back into the centre to make the other numbers.

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Name:_____

Grade: _____ **Date:** _____

Line Master **Flexible Numbers (1)**

Use the cards on the Flexible Numbers (2) Line Master to make each number in as many ways as you can. Record the different ways as you go.
Put the cards back into a pile to use for the next number.

For example, you can make up the number **532** using these cards:

5 hundreds	**3 tens**	**2 ones**

Now put the number cards in these boxes to make up the numbers shown.

61

312

454

Name: _____

Grade: _____ **Date:** _____

Line Master **Flexible Numbers (2)**

NUMBER CARDS Cut-Out Sheet

14 ones	**1 one**	**4 ones**
2 ones	**4 hundred**	**10 ones**
11 ones	**12 ones**	**3 tens**
4 tens	**5 tens**	**40 tens**
41 tens	**42 tens**	**1 ten**
45 tens	**6 tens**	**3 hundreds**
31 tens	**2 hundreds**	**11 tens**

DIAGNOSTIC TASK: 800 Game

Whole and Decimal Numbers
KU 5 for Grades 5–7 (age 10–12+)

Purpose

To see the extent of students' understanding of the relationship between the places. For example, do the students know that 8 is ten times greater than 0.8 and ten times smaller than 80?

Materials

- Line Master: 800 Game
- Scissors
- Calculators

Instructions

Copy and cut out cards; distribute one set (3 zeros, one decimal point, and one 8) to each student. Students should work with partners.

Observe students as they play and talk.
Record what they understand about the multiplicative relationship between the places.

Instructions for players:

1. Make a number with your cards.

2. Decide how you could change the value of your number so that it is equal to the value of your partner's number.

3. Use a calculator to test your suggestion.

4. Try some more examples.

5. Talk to other players.
   a) Are they doing it the same way as you?
   b) How do you account for any differences?

Notes:
- To make the game easier, use only two zeros and omit the decimal point.
- See the description in the *Number Sense* Resource Book, page 70, for additional ideas about how to bring out the mathematics and extend this Diagnostic Task into a learning activity.

Name:_____

Grade: _____ **Date:** _____

Line Master **800 Game**

0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8
0	0	0	•	8

DIAGNOSTIC TASK: Decimals

Whole and Decimal Numbers
KU 7 for Grades 5–9 (ages 10–14)

Purpose

To see the extent of students' understanding of decimals and to uncover preconceptions and misconceptions

Materials

- Line Masters: Decimals (Sets A–H)

Instructions

Have students complete the questions individually. You may need to conduct some individual interviews where students' reasoning is not clear from the written explanation.

Note: Students usually find some questions easier than others. Therefore, you may choose to break up the worksheets and present sections one at a time.

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Name: _____

Grade: _____ **Date:** _____

Line Master **Decimals (Sets A–D)**

A Compare and Order

Circle the biggest number in each group of three.

i) 5 436 or 547 or 56

ii) 6.78 or 45.6 or 345

iii) 3.521 or 3.6 or 3.75

iv) 15.4 or 15.56 or 15.327

v) 4.09 or 4.7 or 4.008

Swan, M. (1983)

B Money

i) When James used his calculator to see how much his shopping came to it showed 14.5.
How much is that in dollars and cents?

_____

ii) Rachel purchased 4 balls.
She worked out the price for one ball on the calculator.
The result was 6.125. How much is that in dollars and cents?

_____

C Renaming Decimals as Fractions

Write these numbers as fractions.

i) 0.67 _____

ii) 0.341 _____

iii) 0.2 _____

D Naming Digits after the Decimal Point

i) What does the 3 mean in 0.236?

_____

ii) What does the 2 mean in 0.236?

_____

iii) What does the 6 mean in 0.236?

_____

Name:_____

Grade: _____ **Date:** _____

Line Master **Decimals (Sets E–F)**

E Ordering

Sonya said, "When we put books on the library shelf we put 65.6 before 65.125 because 6 is less than 125," but Tao didn't agree.

Who is right? _____

Why do you think that?

Write your explanation in this box.

F Counting On and Back by Decimal Numbers

Write down the next 2 numbers in each sequence.

a) 0.2, 0.4, 0.6, _____, _____
(add 0.2 each time)

b) 0.3, 0.6, 0.9, _____, _____
(add 0.3 each time)

c) 0.92, 0.94, 0.96, 0.98,
_____, _____
(add 0.02 each time)

d) 1.13, 1.12, 1.11,
_____, _____
(take away 0.01 each time)

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Name: _____

Grade: _____ **Date:** _____

Line Master **Decimals (Sets G–H)**

G Quantity

Paper clips come in boxes of 1000. Abi counted the loose paper clips in a tray and said there were 1260. Jeremy said, “That's 1.26 boxes of paper clips.”
Could they both be right?

Yes ☐ No ☐

Why do you think that?

H Number Sequence

How would you use a calculator to generate this number sequence?

2.0, 0.2, 0.02, 0.002

_____

_____

_____

_____

_____

_____

_____

_____

_____

FSIM012 | First Steps in Mathematics: Number Course Book

DIAGNOSTIC TASK: Numbers

Operations
KU 7 for Grades 5–9 (ages 10–14)

Purpose

To see the extent of students' understanding of decimals and to uncover any possible preconceptions and misconceptions they may have

Materials

- Line Master: Numbers

Instructions

Have students complete the questions individually. You may need to conduct some individual interviews where students' reasoning is not clear from the written explanation.

Name: _____

Grade: _____ **Date:** _____

Line Master **Numbers**

What do you think this number means? **85.6**

Explain how many apples you think you would have to give me if I asked for **85.6** apples.

DIAGNOSTIC TASK: Money

Operations
KU 7 for Grades 5–9 (ages 10–14)

Purpose

To see the extent of students' understanding of decimals and to uncover any possible preconceptions and misconceptions they may have

Materials

- Line Master: Money

Instructions

Have students complete the questions individually. You may need to conduct some individual interviews where students' reasoning is not clear from the written explanation.

Name: _____

Grade: _____ **Date:** _____

Line Master **Money**

Jacob had to share $33 among 8 people.

He used his calculator and pressed **33 ÷ 8 =** and this is what he saw on his calculator:

4.125

How much money should he give each person? _____

Explain how you decided.

Will there be any money left over? _____

If so, how much? _____

Operations

ADDITION AND SUBTRACTION

DIAGNOSTIC TASK: Change Task

Operations
KU 2 for Grades 1–7 (ages 6–12+)

- Use Change Task 1 with Grades 1–5 (ages 6–10).
- Use Change Task 2 with Grades 5–7 (ages 10–12+).

Purpose

To see if students are able to use the inverse relationship between addition and subtraction when solving word problems with a calculator

Materials

- Line Master: Change Task 1
- Line Master: Change Task 2

Instructions

Begin with Change Task 1. If the students are successful on this task, then, at a later time, ask them to complete Change Task 2. Do **not** allow students to use calculators for this task.

1. Explain that students are to write what they would put into a calculator to solve the problem, not just to write the answer.

2. Read out all of the problems while the students follow on the sheet.

3. If you are using this as a whole-class task, follow up interviews may be necessary to clarify what some students are thinking.

Note: You might like to modify the contexts of these problems to make the stories more relevant to your students.

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Name:_____

Grade: _____ **Date:** _____

Line Master **Change Task 1**

Write the numbers and signs that you would use to solve each problem with a calculator. You do not have to solve them.

1. Anna had 7 trading cards and then her brother gave her 3.
   How many does she have now?

2. Anna has 6 jelly beans but would like to have 11.
   How many more does she need to get?

3. Anna had some marbles and then her brother gave her 4.
   Now she has 10. How many did she have at the start?

4. Anna had 12 trading cards and then she gave her brother 3.
   How many does she have now?

5. Anna had 13 jelly beans and then she gave her brother some.
   Now she has 7. How many did she give her brother?

6. Anna had some candies and gave her brother 3 of them.
   Now she has 8 left. How many did she have at the start?

Name: _____

Grade: _____ **Date:** _____

Line Master **Change Task 2**

Write the numbers and signs that you would use to solve each problem with a calculator. You do not have to solve them.

1. Anna has 112 game trading cards and some sports trading cards.
   She has 87 more game trading cards than sports trading cards.
   How many sports cards does she have?

2. In the long jump final at the Olympic games, the Canadian athlete jumped 8.55 m, and the Australian athlete jumped 7.67 m. How much farther did the Canadian jump than the Australian?

3. Anna has 156 pearly marbles and some cat's-eye marbles.
   She has 89 fewer cat's-eye marbles than pearly marbles.
   How many cat's-eye marbles does Anna have?

4. The school had 307 books and 254 bookmarks.
   If one bookmark is put into each book, how many books won't get a bookmark?

5. Anna has 145 white bears and some brown bears.
   All the white bears took a brown bear as a partner, and there were 78 brown bears left without a partner.
   How many brown bears does she have?

6. At the sports day Sonya jumped 3.25 m.
   If Mark had jumped another 0.87 m his jump would have been the same as Sonya's.
   How long was Mark's jump?

DIAGNOSTIC TASK: How Much Taller?

Operations
KU 1 and KU 2 for Grades 4–7 (ages 10–12+)

Purpose

To see if students are able choose an appropriate operation to solve a comparison problem

Materials

- Line Master: How Much Taller?

Instructions

Do **not** allow students to use calculators for this task.

1. Read the question aloud to the students while they follow on the sheet. Ask them to paraphrase the question so that you are sure they understand what you are asking.

2. Ensure that the students understand the phrase "number sentence" by writing one or two examples on the board: e.g., 7 + 3 = 10 or 10 – 7 = 3. Make sure that you do not use the numbers from the problem.

3. If using this as a whole-class task, follow-up interviews to clarify what some students are thinking may be necessary.

Name:_____

Grade: _____ **Date:** _____

Line Master **How Much Taller?**

Jesse and Sylvia were chatting on the Net. Jesse said that she was 154 cm tall and Sylvia said she was 132 cm. Jesse said, “I am taller than you.” Sylvia said, “Yes, but not by much.”

1. How much taller is Jesse than Sylvia?_____

2. Explain how you worked out the answer.

_____

_____

_____

_____

_____

_____

3. Write a number sentence that you could put into a calculator to work it out.

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DIAGNOSTIC TASK: Empty Boxes

Operations
KU 2 for Grades 5–7 (ages 10–12+)

Purpose

To see if students are able to use the inverse relationship between addition and subtraction to solve open number problems

Materials

- Line Master: Empty Boxes

Instructions

Do **not** allow students to use calculators for this task.

1. Explain to students that they are to write what they would put into a calculator to solve the problem, not just write the answer.

2. If using this as a whole-class task, follow-up interviews to clarify what some students are thinking may be necessary.

Name: _____

Grade: _____ **Date:** _____

Line Master **Empty Boxes**

What numbers and symbols would you use on the calculator to solve the following problems?

17 + ☐ = 36 _____

☐ – 27 = 34 _____

35 = ☐ + 16 _____

43 – ☐ = 16 _____

468 + ☐ = 842 _____

283 = 674 – ☐ _____

☐ – 15.78 = 12.43 _____

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DIAGNOSTIC TASK: Story Problems

Operations
KU 3and KU 4 for K–Grade 4 (ages 4–9)

- For K–Grade 2 (ages 4–7), use Story Problems 1.
- For Grades 3 and 4 (ages 8 and 9), use Story Problems 2. You may choose to alter the numbers.

Purpose

To see what strategies students use to solve problems to which multiplication and division apply

Materials

- Line Master: Story Problems 1
- Line Master: Story Problems 2
- A calculator for each student
- Blocks or counters
- Objects that realistically represent the items in the problem; e.g., toy people and tiny clay sausages for the Sausage Problem

Instructions

Individual Interviews: Interviews would be appropriate for K to Grade 2 (5- to 7-year-old) students or for Grade 3 to 4 (8- and 9-year-old) students at risk.

1. Read the first problem from the worksheet.

2. Ask the student to restate the problem in his/her own words to make sure he/she understands the problem, and to get a sense of what that understanding is.

3. Offer the following items in order. (Withhold materials until it becomes obvious that a child cannot proceed without them.)

    - A calculator
    - The problem sheet or a larger blank sheet of paper and a pencil (if they cannot use a calculator)
    - Blocks or counters (if they cannot proceed)
    - Realistic objects appropriate to the problem

Whole Class Activity: Administering this task with the whole class would be appropriate for Grades 3 and 4 (8- and 9-year-old) students.

1. Have the students follow as you read the problems.

2. Offer the following items in order. (Withhold materials until it becomes obvious that a student cannot proceed without them. Then provide that student with the appropriate materials.)
    - Calculator, worksheet and pencil
    - Counters or blocks
    - Realistic objects selected for that problem, e.g., cardboard shapes representing the types of cones and circles of card representing the flavours of ice cream

Note: Students in Grades 3 or 4 who cannot answer Story Problems 1 questions using diagrams, counters, or blocks may require individual interviews with the more realistic materials.

Name: _____

Grade: _____ **Date:** _____

Line Master **Story Problems 1**

Work out the answer to each story problem. Show how you worked it out.

Animals

The farmer wants to separate his 24 animals into small pens. He wants to put 6 animals in each pen. How many pens does he need?

Sausages

Dad said, “We’re going to have visitors for a barbecue. That means there will be 9 people. We’ll have 3 sausages each.” Dad sent the children to the store to buy the sausages. How many sausages will they need to buy?

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Name:_____

Grade: _____ **Date:** _____

Line Master **Story Problems 2**

Flavours

Katie went to the store. There were 4 ice cream flavours—strawberry, vanilla, chocolate, and bubblegum. There were three types of cones—chocolate, vanilla, and waffle. How many choices of single ice cream cones did Katie have?

Tennis Balls

Sam has 3 tennis balls. Hanna has 5 times as many tennis balls as Sam. How many balls does a Hanna have?

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DIAGNOSTIC TASK: Finding Factors

Operations
KU 5 for Grades 5–7 (ages 10–12+)

Purpose

To see if students know what factors are and how to find them

Materials

- Line Master: Finding Factors
- A calculator for each student

Instructions

The question in the box at the bottom of the page is the crucial part of this task and will give you the most significant information about what students know. It may be necessary to remind students what a factor is and let them practise with easier numbers like 12 or 15.

The students could be given some factor activities on the board, which are similar to the first two top boxes, to enable them to become familiar with the idea of factors and multiples.

Note: This task can be used as an individual interview or as a whole class activity. It may be beneficial to give the top part of the task to the whole class but withhold the last box from the sheet and use it in an individual interview.

Name: _____

Grade: _____ **Date:** _____

Line Master **Finding Factors**

Find factors for these numbers.

81 _____

Which numbers did you try? _____

Which ones were hardest to find? _____

How did you work it out? _____

_____

105 _____

Which numbers did you try? _____

Which ones were hardest to find? _____

How did you work it out? _____

_____

Sam wondered if 13 was a factor of 105 but did not know what to put into the calculator to find out.

Explain to Sam what he could do to find out.

DIAGNOSTIC TASK: Calculator Number Sentences

Operations
KU 8 for Grades 4–7 (ages 10–12+)

Purpose

To see if students can think of problems as number sentences. To see if students know how to rewrite number sentences in different but equivalent ways so they can solve them using a calculator

Materials

- Line Master: Calculator Number Sentences
- A calculator for each child

Instructions

Individual Interview: Ask the student if she/he would like to have the problems read aloud or to read the problems independently and write in the numbers sentence as she/he goes. Remind the student that it is not the answer to the problem that is required but the number sentence that would need to be keyed into the calculator.

Some students may use a trial-and-error approach. Note those students who use trial and error to choose the operation; which operations they try; and how they arrive at their final decisions.

Whole Class Activity: Read the problems to the students if they need this level of support. Remind them that it is not the answer to the problem that is required but the number sentence they would need to key into the calculator. Note any trial-and-error approaches as above.

Name: _____

Grade: _____ **Date:** _____

Line Master **Calculator Number Sentences**

What would you key into your calculator to solve these problems?

The burger place had a special $18 Family Feast Deal. It was packed with people. There were about 6 people at each table and there were about 36 tables. About how many people were there?	A bulk box of gummy worms cost $5.40. There were 216 worms in the box. If there were 27 students in the class and the worms were given out, how many worms would each child get?	The Grade 6 students were selling cupcakes to raise funds for the school camp. The cupcakes cost $4.80 a box. The parents had baked 400 cupcakes and needed to put them into boxes of 8. How many boxes would they need?
Helen likes to walk 20 km every day. She walks at an average speed of 5 km/h. How far does she walk in 3 hours?	Jeremy picked 6 bags of apricots. If a 3 kg bag of apricots costs $12.60, what is the price per kilogram?	Apricots cost $4.30 a kilogram. If a box of apricots costs $12.60, how much must it weigh?
Every week at the ski hill each age group has 4 races. There were 6 times as many boys racing as girls. There were 18 girls and 36 parents. How many boys were there?	An enlarged picture is three times taller than its original. The picture is 180 mm tall. How tall was the original?	There were 15 kids at the barbecue. One of the older kids hid some prizes. Simon found 30 prizes. This was 6 times as many his sister Sharn. How many prizes did Sharn find?
John needed $2.00 to go to the dance. He had 4 pairs of shorts and 5 tops. How many outfits could he choose from?	Sarah was planting corn. The seeds cost $2.50 a packet. She had 75 seeds and wanted to plant 15 rows. How many seeds in each row?	A rectangle with an area of 208 cm² has one side 16 cm long. How long is the adjacent side?

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DIAGNOSTIC TASK: Hide the Jelly Beans

Computations
KU 1 and KU 2 for K–Grade 1 (ages 4–6)

Purpose

To see what students know about the numbers involved and what strategies they use to compute

Materials

- An opaque container
- Jelly beans or other collection of small objects all in one colour

Instructions

This is a game for two students to play together; both students can be observed at the same time.

Start with a small number of objects, e.g., five or six. Both students count the objects to agree on how many there are. One student closes his/her eyes while the other student hides some of the objects under the upturned container. The first student opens his/her eyes and says how many are hidden.

Observe the child doing the computation to see what he/she knows about the numbers and the strategy he/she uses to calculate how many are hidden. Increase or decrease the number of objects in the collection depending on the numbers the students are familiar with.

DIAGNOSTIC TASK: How Many Want Jelly Beans?

Computations
KU 2 for Grades 1–3 (ages 6–8)

- How Many Want Jelly Beans? (Problem A) is suitable for Grades 1–2 (ages 6–7).
- How Many Want Jelly Beans? (Problem B) is suitable for Grades 2–3 (ages 7–8).

Purpose

To see if students know how to think of a number as a sum or difference in different ways

Materials

- Line Master: How Many Want Jelly Beans? (Problem A)
- Line Master: How Many Want Jelly Beans? (Problem B)

Instructions

Provide each student with the appropriate line master. Read it aloud to them to make sure they understand what they need to find out. Explain that they can just do it in their head or work it out by drawing or using some materials. After each student has found an answer, ask them to say how they worked it out and record their descriptions on their page.

Name:_____

Grade: _____ **Date:** _____

Line Master **How Many Want Jelly Beans? (Problem A)**

At a party 5 children wanted red jelly beans and 8 children wanted yellow jelly beans. How many children altogether want jelly beans?

_____

_____

_____

_____

How did you work it out?

Write your explanation in this box.

Name: _____

Grade: _____ **Date:** _____

Line Master **How Many Want Jelly Beans? (Problem B)**

At a party some of the children wanted red jelly beans.
5 more children wanted yellow jelly beans.
Now 13 children want jelly beans.
How many children want red jelly beans?

_____

_____

_____

_____

How did you work it out?

Write your explanation in this box.

DIAGNOSTIC TASK: Find the Solutions

Computations
KU 1 through KU 6 for Grades 3–7 (ages 8–12+)

- Find the Solutions (Sets A and B) are suitable for Grades 3–4 (ages 8–9)
- Find the Solutions (Sets B and C) are suitable for Grades 5–7 (ages 10–12+)

Purpose

To see what strategies students use to solve problems requiring addition, subtraction, multiplication, or division

Materials

- Line Master: Find the Solutions (Set A)
- Line Master: Find the Solutions (Set B)
- Line Master: Find the Solutions (Set C)

Instructions

1. Provide each student with the appropriate line master(s). Read the questions with them to make sure they understand what they need to find out.

2. Explain to the students that they need only work on the problems that they think they can do. It is important to end the interview when the child indicates the questions are getting too hard. You may wish to copy only the left-hand set of questions for the students you believe might feel inadequate for not progressing to the end.

Individual Interview: Sit with a student as she/he works out each question. When she/he has an answer, ask about the strategy the student used. The focus is on revealing some of the repertoire the student has developed; commenting on whether the answer is right or wrong is inappropriate in this situation. It is important to end the interview when the student indicates that the questions are getting too hard. Record the numbers the students uses as she/he partitions, re-arranges, orders, and operates on a separate copy.

Whole-Class Task: Provide each student in the class with a copy of the questions. Ask them to read the question, and work out the answer in their head. Interview a few students of different abilities to find out how they thought of the numbers and how they worked each question out. Record the numbers they use as they partition, re-arrange, order, and operate on a separate copy.

Name:_____

Grade: _____ **Date:** _____

Line Master **Find the Solutions (Set A)**

1. On the bus there are 25 children from Mr. Foster's class and 30 children from Mr. Singh's class. How many children are on the bus?	2. There were 100 paper clips in the box. We have used 37 of them. How many are left?
3. Your mother made 24 pancakes in the first batch and 18 in the second batch. How many pancakes did she make?	4. Sean's family is on the way to town. They have already travelled 15 km and the town is 65 km from their home. How far do they still need to travel to reach town?
5. There are 18 slices of bread in a loaf. How many slices will there be in 5 loaves?	6 There was $120 in $10 bills. How many bills should there be?

Name: _____

Grade: _____ **Date:** _____

Line Master **Find the Solutions (Set B)**

1. In Joe's school each class has 25 children in it. The school has 16 classes. How many children are in the school?	2. Crystal has 375 newspapers to deliver. She has delivered 127. How many does she still have to deliver?
3. Every week Ted earns $235. Does he earn more or less than $900 every 4 weeks? How do you know?	4. Jeremy has to deliver 226 newspapers. How many more does he need to deliver until all of the 537 newspapers in his paper route are delivered?
5. Abi has two short paper routes. She delivers 374 in one route and 227 in the other. How many newpapers does she deliver altogether?	6. There were 1035 newspapers to deliver and 10 delivery people. How many papers did they each deliver?

Name:_____

Grade: _____ **Date:** _____

Line Master **Find the Solutions (Set C)**

25 + 30	25 x 16
24 + 18	375 – 124
18 x 5	226 + __ = 537
100 – 3	374 + 227
15 + __ = 65	1035 split into groups of 10
120 split into groups of 10	27 x 16
235 x 4 Estimate. Will the answer be more or less than 900? Why?	

DIAGNOSTIC TASK: Buying Apples

Computations for Grades 4–7 (ages 9–12+)

Purpose

To see if students know how to use place value and basic number facts to calculate with whole and decimal numbers

Materials

Line Master: Buying Apples

Instructions

Provide each student with a copy of the problem and ask them to record how they solved it. Record the computational strategies each student used and what each student knows about numbers and operations.

Name: _____

Grade: _____ **Date:** _____

Line Master **Buying Apples**

Apples
$3.20 per kilogram

Evan bought 0.4 kilograms of apples.

How much did he pay? _____

Explain your answer.

_____

_____

_____

_____

_____

Think Board

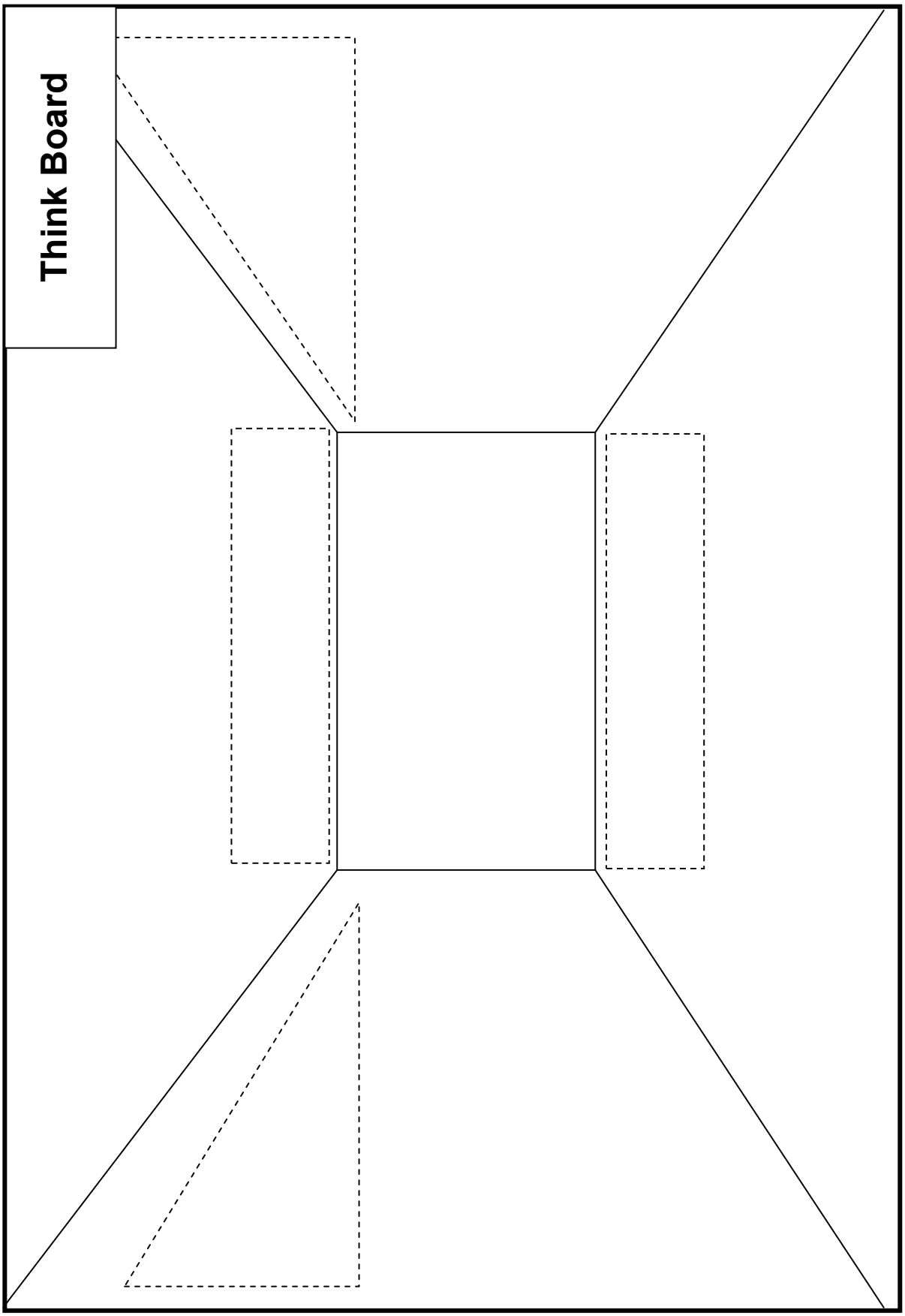
Diagram
Use marks or dots to show the numbers of things, then use a sign to show what to do with the marks or dots to solve the problem.

Show the problem
Draw a picture or use things to show the problem part of the story.

Other number sentences to solve the same problem

THE STORY PROBLEM

Write a number sentence
Write numbers and signs to show how to solve the problem part of the story.



Planning Master: Grades K–1

Curriculum expectation/outcome:

1. What is my teaching goal?

Key Understanding:

2. How will I find out what my students know and believe?

Diagnostic Task:

3. What do my students know and believe?

Diagnostic Map: Phases of thinking summaries

Emergent: The child recognizes that numbers may be used to signify quantity.
- At the end of the Emergent phase for this KU, students...

Matching: The child uses one-to-one relations to share and count out.
- At the end of the Matching phase for this KU, students...

Quantifying: The child uses part-part-whole relations for numerical quantities.
- At the end of the Quantifying phase for this KU, students...

Partitioning: The child uses additive thinking to deal with many-to-one relations.
- At the end of the Partitioning phase for this KU, students...

5. What evidence of learning do I see and hear?

Observations and next steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Resource Book and page:
Focus Questions and paraphrasing:

…to move students from the Emergent phase to the Matching phase:

…to move students from the Matching phase to the Quantifying phase:

…to move students from the Quantifying phase to the Partitioning phase:

Planning Master: Grades 2–5

Curriculum expectation/outcome:

1. What is my teaching goal?

Key Understanding:

2. How will I find out what my students know and believe?

Diagnostic Task:

5. What evidence of learning do I see and hear?

Observations and next steps:

What do my students know and believe?

Diagnostic Map: Phases of thinking summaries

Matching: The child uses one-to-one relations to share and count out.
- At the end of the Matching phase for this KU, students...

Quantifying: The child uses part-part-whole relations for numerical quantities.
- At the end of the Quantifying phase for this KU, students...

Partitioning: The child uses additive thinking to deal with many-to-one relations.
- At the end of the Partitioning phase for this KU, students...

Factoring: The child thinks both additively and multiplicatively about numerical quantities.
- At the end of the Partitioning phase for this KU, students...

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Resource Book and page:
Focus Questions and paraphrasing:

…to move students from the Matching phase to the Quantifying phase:

…to move students from the Quantifying phase to the Partitioning phase:

…to move students from the Partitioning phase to the Factoring phase:

Planning Master: Grades 4–8

Curriculum expectation/outcome:

1. What is my teaching goal?

Key Understanding:

2. How will I find out what my students know and believe?

Diagnostic Task:

3. What do my students know and believe?

Diagnostic Map: Phases of thinking summaries

Quantifying: The child uses part-part-whole relations for numerical quantities.
- At the end of the Quantifying phase for this KU, students...

Partitioning: The child uses additive thinking to deal with many-to-one relations.
- At the end of the Partitioning phase for this KU, students...

Factoring: The child thinks both additively and multiplicatively about numerical quantities.
- At the end of the Partitioning phase for this KU, students...

Operating: The child can think of multiplications and divisions in terms of operators.
- At the end of the Partitioning phase for this KU, students...

5. What evidence of learning do I see and hear?

Observations and next steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Resource Book and page:
Focus Questions and paraphrasing:

…to move students from the Quantifying phase to the Partitioning phase:

…to move students from the Partitioning phase to the Factoring phase:

…to move students from the Factoring phase to the Operating phase: