ENCE 710
Design of Steel Structures

V. Lateral-Torsional Buckling of Beams

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Introduction

Following subjects are covered:

- Lateral Torsional Buckling (LTB)
- Flange Local Buckling (FLB)
- Web Local Buckling (WLB)
- Shear strength
- Lateral Bracing Design

Reading:

- Chapters 9 of Salmon & Johnson
- AISC LRFD Specification Chapters B (Design Requirements) and F (Design of Members for Flexure)

Introduction

A beam can fail by reaching the plastic moment and becoming fully plastic (see last section) or fail prematurely by:

1. LTB, either elastically or inelastically
2. FLB, either elastically or inelastically
3. WLB, either elastically or inelastically

If the maximum bending stress is less than the proportional limit when buckling occurs, the failure is elastic. Else it is inelastic.

For bending $\varphi_b M_n (\varphi_b = 0.9)$

Design of Members for Flexure (about Major Axis)

TABLE User Note F1.1
Selection Table for the Application of Chapter F Sections

Section in Chapter F	Cross Section	Flange Slenderness	Web Slenderness	Limit States
F2		C	C	Y, LTB
F3		NC, S	C	LTB, FLB
F4		C, NC, S	C, NC	Y, LTB, FLB, TFY
F5		C, NC, S	S	Y, LTB, FLB, TFY

Lateral Torsional Buckling (LTB)

- Compact Members (AISC F2)
- Failure Mode
- Plastic LTB (Yielding)
- Inelastic LTB
- Elastic LTB
- Moment Gradient Factor C_b

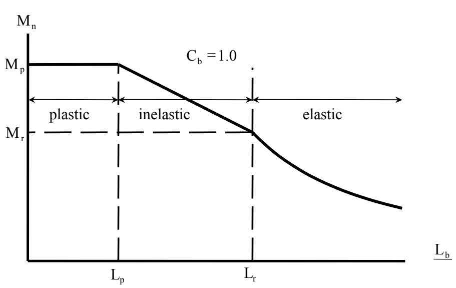
Lateral Torsional Buckling (cont.)

- Failure Mode

  A beam can buckle in a lateral-torsional mode when the bending moment exceeds the critical moment.

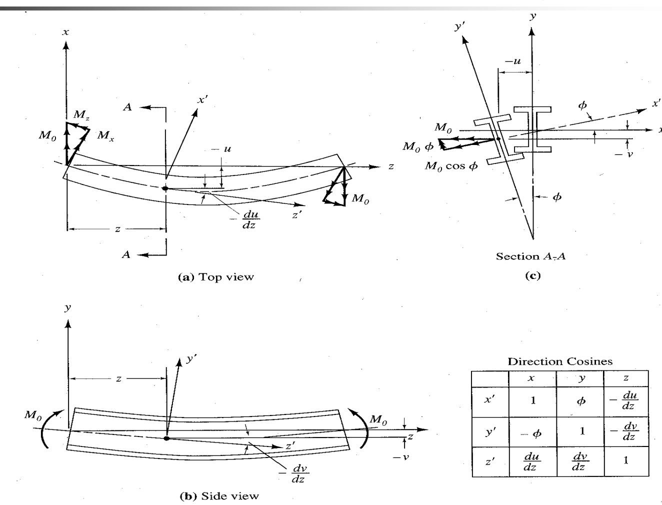
Lateral Torsional Buckling (cont.)

- Nominal Flexural Strength M_n
  - plastic when $L_b \le L_p$ and $M_n = M_p$
  - inelastic when $L_p < L_b \le L_r$ and $M_p > M_n \ge M_r$
  - elastic when $L_b > L_r$ and $M_n < M_r$



Lateral Torsional Buckling (cont.)

I-Beam in a Buckled Position



Lateral Torsional Buckling (cont.)

- Elastic LTB
  - coupled differential equations for rotation and lateral translation (8.5.10)

$$M_z = GJ\frac{d\phi}{dz} - EC_w\frac{d^3\phi}{dz^3}$$

where

M_z	= moment at location z along member axis
z	= axis along member length
φ	= angle of twist
G	= shear modulus
J	= torsional constant (AISC Table 1-1 for torsional prop.)
E	= modulus of elasticity
C_w	= warping constant (AISC Table 1-1 for warping)

Lateral Torsional Buckling (cont.)

- Plastic LTB (Yielding)
  - Flexural Strength $M_n = M_p = F_y Z$ (AISC F2-1)

where Z= plastic section modulus & F_y= section yield stress

- Limits
  - Lateral bracing limit $L_b < L_p = 1.76 r_y \sqrt{\dfrac{E}{F_y}}$ (AISC F2-5)
  - Flange and Web width/thickness limit (AISC Table B4.1)

(Note: L_{pd} in Salmon & Johnson Eq. (9.6.2) is removed from AISC 13th Ed.)

Lateral Torsional Buckling (cont.)

- Inelastic LTB $L_p < L_b \le L_r$
  - Flexure Strength (straight line interpolation)

$$M_n = C_b\left[M_p - \left(M_p - M_r\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad (9.6.4)$$

or

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(AISC F2-2)}$$

Lateral Torsional Buckling (cont.)

- Elastic LTB $L_b > L_r$
  - Flexure Strength

$$M_n = F_{cr} S_x \le M_p \qquad \text{(AISC F2-3)}$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\dfrac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(AISC F2-4)}$$

(The square root term may be conservatively taken equal to 1.0)
(c in AISC F2-8a,b for doubly symmetric I-shape, and channel, respectively)

- Limit

$$L_r = 1.95 r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_y}{E}\frac{S_x h_o}{Jc}\right)^2}} \qquad \text{(AISC F2-6)}$$

$$r_{ts}^{\ 2} = \frac{\sqrt{I_y C_w}}{S_x} \qquad \text{(AISC F2-7)}$$

Lateral Torsional Buckling (cont.)

- Moment Gradient Factor C_b
  - The moment gradient factor C_b accounts for the variation of moment along the beam length between bracing points. Its value is highest, $C_b=1$, when the moment diagram is uniform between adjacent bracing points.
  - When the moment diagram is not uniform

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \qquad (9.6.3) \ \text{(AISC F1-1)}$$

where

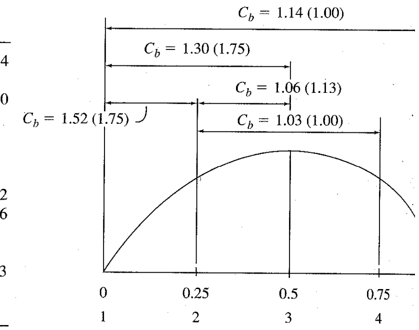
M_{max}= absolute value of maximum moment in unbraced length

M_A, M_B, M_C= absolute moment values at one-quarter, one-half, and three-quarter points of unbraced length

C_b for a Simple Span Bridge

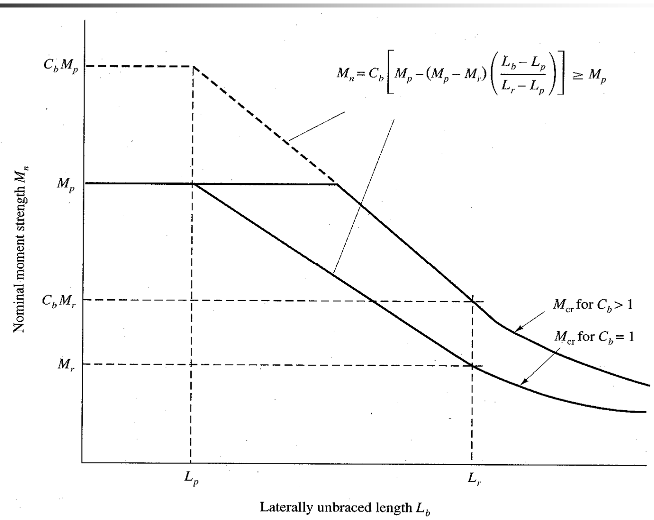
C_b FOR PARABOLIC SEGMENTS USING LRFD-F1.2a, FORMULA (C-F1-3), EQ. 9.6.11*

Case 1	Laterally braced at ends; points 1 and 5 only; $M_{\max}$ at 3	$C_b = 1.14$
Case 2	Laterally braced at ends and midspan; points 1,3, and 5 only; $M_{\max}$ at 3	$C_b = 1.30$
Case 3	Laterally braced at end and 1st quarter point; bracing at points 1 and 2; $M_{\max}$ at 2	$C_b = 1.52$
Case 4	Laterally braced at 1st and 2nd quarter points; bracing at points 2 and 3; $M_{\max}$ at 3	$C_b = 1.06$
Case 5	Laterally braced at 1st and 3rd quarter points; bracing at points 2 and 4; $M_{\max}$ at 3	$C_b = 1.03$



* Values from 1986 LRFD, Eq. 9.6.12 shown in parenthesis.

Nominal Moment Strength M_u as affected by C_b



Flange Local Buckling (FLB)

- Compact Web and Noncompact/Slender Flanges (AISC F3)
- Failure Mode
- Noncompact Flange
- Slender Flange
- Nominal Flexural strength, M_n = Min (F2, F3)

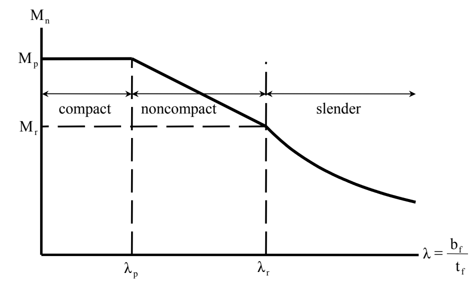
Flange Local Buckling (cont.)

- Failure Mode

  The compression flange of a beam can buckle locally when the bending stress in the flange exceeds the critical stress.

Flange Local Buckling (cont.)

- Nominal Flexural Strength M_n
  - plastic when $b_f/2t_f \le \lambda_p$ and $M_n = M_p$
  - inelastic when $\lambda_p \le b_f/2t_f \le \lambda_r$ and $M_p > M_n \ge M_r$
  - elastic when $b_f/2t_f > \lambda_r$ and $M_n < M_r$



Flange Local Buckling (cont.)

- Noncompact Flange (straight line interpolation)
  - Flexure Strength

$$M_n = M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right) \quad \text{(AISC F3-1)}$$

Flange Local Buckling (cont.)

- Slender Flange
  - Flexure Strength

$$M_n = \frac{0.9Ek_cS_x}{\lambda^2} \quad \text{(AISC F3-2)}$$

$$k_c = \frac{4}{\sqrt{h/t_w}}$$

(kc shall not be less than 0.35 and not greater than 0.76)

  - Limit (AISC Table B4.1)

Web Local Buckling (WLB)

- Compact or Noncompact Webs (AISC F4)
- Failure Mode
- Compact Web (Yielding)
- Noncompact Web
- Slender Web
- Nominal Flexural Strength, M_n=min (compression flange yielding, LTB, compression FLB, tension flange yielding)

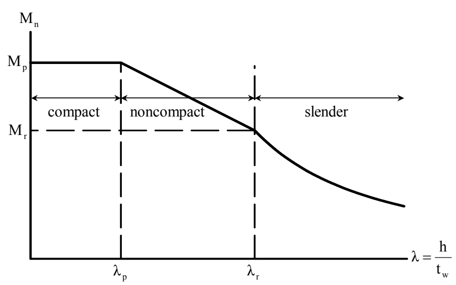
Web Local Buckling (cont.)

- Failure Mode

  The web of a beam can also buckle locally when the bending stress in the web exceeds the critical stress.

Web Local Buckling (cont.)

- Nominal Flexural Strength M_n
  - plastic when $\lambda \le \lambda_p$ and $M_n = M_p$
  - inelastic when $\lambda_p < \lambda \le \lambda_r$ and $M_p > M_n \ge M_r$
  - elastic when $\lambda > \lambda_r$ and $M_n < M_r$



Web Local Buckling (cont.)

- Compression Flange Yielding
  - Flexural Strength

$$M_n = R_{pc} M_{yc} = R_{pc} F_y S_{xc} \qquad \text{(AISC F4-1)}$$

where R_{pc}= web plasticification factor (AISC F4-9a, b) & F_y= section yield stress

  - Limits (AISC Tables B4.1)

$$L_b < L_p = 1.1 r_t \sqrt{\frac{E}{F_y}}$$

Web Local Buckling (cont.)

- LTB (Inelastic) $L_p < L_b \le L_r$
  - Flexure Strength

$$M_n = C_b\left[R_{pc}M_{yc} - \left(R_{pc}M_{yc} - F_L S_{xc}\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \le M_p \quad \text{(AISC F4-12)}$$

where F_L= a stress determined by AISC F4-6a, b

Web Local Buckling (cont.)

- LTB (Elastic) $L_b > L_r$
  - Flexure Strength

$$M_n = F_{cr}S_{xc} \le R_{pc}M_{yc} \quad \text{(AISC F4-3)}$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_t}\right)^2}\sqrt{1 + 0.078\frac{J}{S_x h_o}\left(\frac{L_b}{r_t}\right)^2} \quad \text{(AISC F4-5)}$$

  - Limit (AISC Table B4.1)

$$L_r = 1.95 r_t \frac{E}{F_L}\sqrt{\frac{J}{S_x h_o}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{F_L}{E}\frac{S_x h_o}{J}\right)^2}} \quad \text{(AISC F4-8)}$$

Web Local Buckling (cont.)

- Compression FLB (Noncompact Flange)
  - Flexure Strength

$$M_n = \left[R_{pc}M_{yc} - \left(R_{pc}M_{yc} - F_L S_{xc}\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \le M_p \quad \text{(AISC F4-12)}$$

- Compression FLB (Slender Flange)
  - Flexure Strength

$$M_n = \frac{0.9 E k_c S_x}{\lambda^2} \quad \text{(AISC F4-13)}$$

$$k_c = \frac{4}{\sqrt{h/t_w}}$$

(k_c shall not be less than 0.35 and not greater than 0.76)

Web Local Buckling (cont.)

- Tension Flange Yielding $S_{xt} < S_{xc}$
  - Flexure Strength

$$M_n = R_{pt}M_{yt} = R_{pt}F_y S_{xt} \quad \text{(AISC F4-14)}$$

R_{pt} = web plastification factor to the tension flange yielding limit

(a) $h_c/t_w \le \lambda_{pw}$ $R_{pt}=M_p/M_{yt}$ (AISC F4-15a)
(b) $h_c/t_w > \lambda_{pw}$ (AISC F4-15b)

$$R_{pt} = \left[\frac{M_p}{M_{yt}} - \left(\frac{M_p}{M_{yt}} - 1\right)\left(\frac{\lambda - \lambda_{pw}}{\lambda_{rw} - \lambda_{pw}}\right)\right] \le \frac{M_p}{M_{yt}}$$

Shear Strength

- Failure Mode
- Shear-Buckling Coefficient
- Elastic Shear Strength
- Inelastic Shear Strength
- Plastic Shear Strength

For shear $\varphi_v V_n$($\varphi_v = 0.9$ except certain rolled I-beam $h/t_w \le 2.24\sqrt{E/F_y}$, $\varphi_v = 1.0$)

$V_n = 0.6 F_y A_w C_v$ (AISC G2-1)

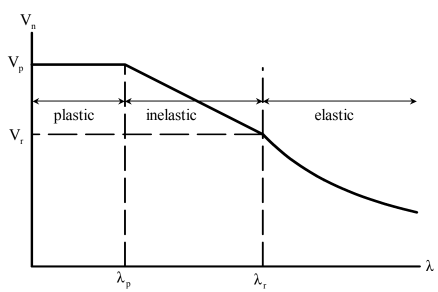
Shear Strength (cont.)

- Failure Mode

  The web of a beam or plate girder buckles when the web shear stress exceeds the critical stress.

Shear Strength (cont.)

- Nominal Shear Strength V_n ($\varphi_v = 0.9$)
  - plastic when $\lambda \le \lambda_p$ and $\tau = \tau_y$
  - inelastic when $\lambda \le \lambda_r$ and $\tau = 0.8\tau_y$
  - elastic when $\lambda > \lambda_r$ and $\tau = \tau_{cr}$



Shear Strength (cont.)

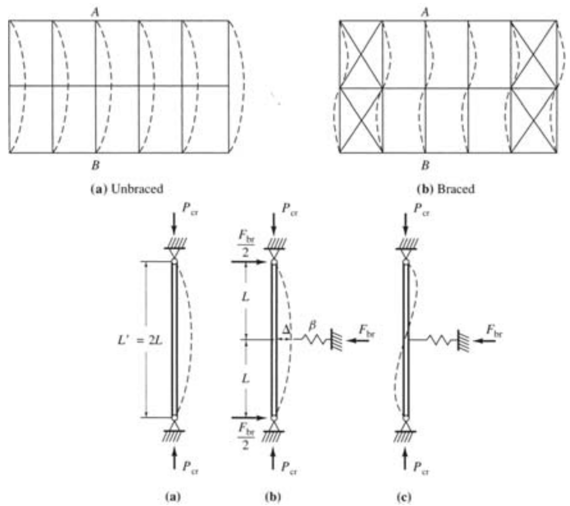
- AISC G2 Nominal Shear Strength V_n

(a) For $\frac{h}{t_w} \le 1.10\sqrt{\frac{k_v E}{F_{yw}}}$ $\quad C_v = 1.0$ (AISC G2-3)

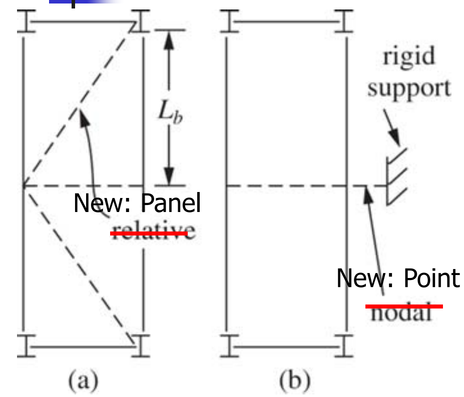
(a) For $1.10\sqrt{\frac{k_v E}{F_y}} \le \frac{h}{t_w} \le 1.37\sqrt{\frac{k_v E}{F_y}}$ $\quad C_v = \left[\frac{1.10\sqrt{k_v E / F_y}}{h / t_w}\right]$ (AISC G2-4)

(a) For $1.37\sqrt{\frac{k_v E}{F_{yw}}} \le \frac{h}{t_w}$ $\quad C_v = \left[\frac{1.51 E k_v}{\left(h / t_w\right)^2 F_y}\right]$ (AISC G2-5)

Lateral Bracing Design



Lateral Bracing Design



AISC Provisions – Stability Bracing Design for Beams

- 1. For stiffness β_{reqd},
  $\beta_{reqd} = 2\ \beta_{ideal}$
- 2. For nominal strength F_{br},
  (a) $F_{br} = \beta_{ideal}\ (2\Delta_0)$;
  (b) $F_{br} = \beta_{ideal}\ (0.004L_b)$
  Where $\beta_{ideal} = P_{cr}/L_b$

Lateral Bracing Design (cont.)

AISC Provisions – LRFD Stability Bracing Design for Columns

1. Panel bracing

$$Required\ V_{br} = 0.005P_r \quad \text{(A-6-1)}$$

Φ=0.75

$$\text{Required} \quad \beta_{rb} = \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) \quad \text{(A-6-2a)}$$

2. Point bracing

$$\text{Required} \quad P_{rb} = 0.01P_r \quad \text{(A-6-3)}$$

$$\text{Required} \quad \beta_{br} = \frac{1}{\phi}\left(\frac{8P_r}{L_b}\right) \quad \text{(A-6-4a)}$$

Lateral Bracing Design (cont.)

AISC Provisions – LRFD Stability Bracing Design for Beams

1. Lateral Bracing

1a. Panel bracing

$$Required\ V_{br} = 0.01\left(\frac{M_r C_d}{h_o}\right) \quad \text{(A-6-5)}$$

Φ=0.75

$$\text{Required} \quad \beta_{rb} = \frac{1}{\phi}\left(\frac{4M_r C_d}{L_b h_0}\right) \quad \text{(A-6-6a)}$$

1b. Point bracing

$$Required\ V_{br} = 0.02\left(\frac{M_r C_d}{h_o}\right) \quad \text{(A-6-7)}$$

$$\text{Required} \quad \beta_{br} = \frac{1}{\phi}\left(\frac{10M_r C_d}{L_b h_0}\right) \quad \text{(A-6-8a)}$$

2. Torsional Bracing